127 (2002) MATHEMATICA BOHEMICA No. 2, 301–310
Proceedings of EQUADIFF 10, Prague, August 27–31, 2001

SOME COMMON ASYMPTOTIC PROPERTIES OF SEMILINEAR PARABOLIC, HYPERBOLIC AND ELLIPTIC EQUATIONS

P. Poláčik, Bratislava

Abstract. We consider three types of semilinear second order PDEs on a cylindrical domain $\Omega \times (0,\infty)$, where Ω is a bounded domain in $\mathbb{R}^N$, $N \geqslant 2$. Among these, two are evolution problems of parabolic and hyperbolic types, in which the unbounded direction of $\Omega \times (0,\infty)$ is reserved for time t, the third type is an elliptic equation with a singled out unbounded variable t. We discuss the asymptotic behavior, as $t \to \infty$, of solutions which are defined and bounded on $\Omega \times (0,\infty)$.

Keywords: parabolic equations, elliptic equations, hyperbolic equations, asymptotic behavior, center manifold

MSC 2000: 35B40, 35K55, 35L70, 35J25

1. The equations and their gradient-like structure

Let Ω be a bounded domain in $\mathbb{R}^N$ with a C^2-boundary, and let $g\colon \overline{\Omega} \times \mathbb{R} \to \mathbb{R}$ be a sufficiently regular function (assume g is of class C^1 at least). We consider the following three types of semilinear problems in the cylindrical domain $\Omega \times (0,\infty)$:

$$\text{(PP)} \qquad \begin{cases} u_t = \Delta u + g(x,u), & x \in \Omega,\ t > 0, \\ u(x,t) = 0, & x \in \partial\Omega,\ t > 0, \\ u(x,0) = u_0(x), & x \in \Omega, \end{cases}$$

with $u_0 \in H_0^1(\Omega) \cap L^\infty(\Omega)$;

$$\text{(HP)} \qquad \begin{cases} u_{tt} + \alpha u_t = \Delta u + g(x,u), & x \in \Omega,\ t > 0, \\ u(x,t) = 0, & x \in \partial\Omega,\ t > 0, \\ u(x,0) = u_0(x), & x \in \Omega, \\ u_t(x,0) = v_0(x), & x \in \Omega, \end{cases}$$

with $(u_0, v_0) \in (H_0^1(\Omega) \cap L^\infty(\Omega)) \times L^2(\Omega)$ and $\alpha > 0$; and

$$
\text{(EP)} \qquad \begin{cases} -u_{tt} + \alpha u_t = \Delta u + g(x,u), & x \in \Omega,\ t > 0, \\ u(x,t) = 0, & x \in \partial\Omega,\ t > 0, \\ u(x,0) = u_0(x), & x \in \Omega, \\ u_t(x,0) = v_0(x), & x \in \Omega, \end{cases}
$$

with $(u_0, v_0) \in (H_0^1(\Omega) \cap L^\infty(\Omega)) \times L^2(\Omega)$ and $\alpha \neq 0$.

In all cases we are interested in solutions defined on $\Omega \times (0, \infty)$ for which

$$
\sup_{t>0} \{ \|u(\cdot,t)\|_{H^1(\Omega)}, \|u(\cdot,t)\|_{L^\infty(\Omega)} \} < \infty. \tag{1.1}
$$

We want to understand the possible behavior of such solutions as $t \to \infty$.

Problems (PP), (HP) are evolution problems of parabolic and hyperbolic type, respectively. The given initial-value problems are well-posed (in the case of (HP), under additional growth conditions on g), and the solutions define a local dynamical system on a suitable state space (see [1], [10], [29]). For these evolution problems the objective we have set ourselves is a rather standard one—to understand the asymptotic behavior of bounded solutions.

The elliptic problem (EP) is usually viewed as a static, rather than an evolution one, in particular the initial-value problem (EP) is in general ill posed (note, however, that dynamical system ideas have proved very useful in the study of (EP), see [4], [17], [19], [20], for example). In any case, the behavior of bounded solutions as $t \to \infty$ is of interest. In particular, one would like to know whether each such solution has to asymptotically settle down to some fixed profile $\varphi(x)$, $x \in \Omega$, or whether there can be oscillations for large values of t.

All the three problems share a common property that significantly restricts the way bounded solutions may behave. Namely, each of the problems has a gradient-like structure, which is to say that it admits a Lyapunov functional. For the parabolic problem, the functional is given by the usual "energy":

$$
V_P(u) := \frac{1}{2} \int_\Omega |\nabla u(x)|^2 \,\mathrm{d}x - \int_\Omega G(x, u(x)) \,\mathrm{d}x,
$$

where

$$
G(x,u) = \int_0^u g(x,\xi) \,\mathrm{d}\xi.
$$

For problems (HP) and (EP) the functionals are given, respectively, by

$$
V_H(u) := V_P(u) + \frac{1}{2} \int_\Omega u_t^2(x) \,\mathrm{d}x
$$

and

$$V_E(u) := \left(V_P(u) - \frac{1}{2} \int_\Omega u_t^2(x)\,\mathrm{d}x \right) \operatorname{sign} \alpha.$$

A standard computation shows that if u is a solution of any of the above problems that satisfies (1.1) and is not constant in t, then for the corresponding functional V, the function $t \mapsto V(u(\cdot,t))$ is strictly decreasing on $(0,\infty)$. Using this property, one can show (see for example [4], [10], [20]) that as $t \to \infty$, any solution $u(\cdot,t)$ satisfying (1.1) approaches a set of solutions of the elliptic problem on the cross-section Ω:

$$\text{(EC)} \qquad \begin{cases} \Delta v + g(x,v) = 0, & x \in \Omega, \\ v(x) = 0, & x \in \partial\Omega. \end{cases}$$

To say this more precisely, assume $u(x,t)$ is a solution of one of the problems (PP), (HP), (EP) satisfying (1.1). Then $\{u(\cdot,t)\colon t > 0\}$ is relatively compact in $H_0^1(\Omega)$ and its ω-limit set,

$$\omega(u) = \{z \in H_0^1(\Omega)\colon u(\cdot,t_n) \to z \text{ in } H_0^1(\Omega) \text{ for some } t_n \to \infty\},$$

is a connected subset of $H_0^1(\Omega)$ consisting of solutions of (EC). One can also show that $u_t(\cdot,t) \to 0$ (in $L^2(\Omega)$ at least). Hence if oscillations occur at all, they must slow down.

2. Stabilization or not?

Having said the above, the basic question concerning the asymptotic behavior of a bounded solutions is now whether $\omega(u)$ must be a single function, or whether it can be a nontrivial continuum of solutions of (EC). In the former case, we say the solution u is *stabilizing* or *convergent*; otherwise u is said to be *nonstabilizing* or *nonconvergent*. In literature, one can find various sufficient conditions for stabilization of all bounded solutions. Let us list a few such conditions that are common to the three problems.

- One-dimensional domains. If $N = 1$, bounded solutions have been proved to stabilize (converge to a single solution of (EC)). The proofs are given in [31], [18], [11] for (PP), [11] for (HP) and [2], [20] for (EP) (see [2], [3], [5], [6], [8] for related results in 1D).
- Analytic nonlinearities. If g is real analytic in u, then bounded solutions for each of the three problems stabilize. The proof is given in [26] for (PP) and (EP), and in [14] for (HP) (see [15], [12], [9], [28] for other results based on similar ideas).
- Positive solutions on a ball. If Ω is a ball and $u(\cdot,t)$ is a positive bounded solution of any of the three problems, then it stabilizes to a single radially

symmetric solution of (CE). The proofs for all three problems are given in [13]; see also [7] for a related convergence result in periodic-parabolic equations.

We thus have a rather complete understanding of the behavior of bounded solutions if $N = 1$ or if $N \geqslant 1$ and g is analytic. In case $N > 1$ and g is merely smooth, the situation is more complicated. The solutions may no longer stabilize. The following theorem gives a specific statement to that effect.

Theorem 1. *Let Ω be any bounded domain in $\mathbb{R}^N$ $(N = 2, 3)$ with a C^2 boundary. There exists a C^∞ function $g\colon \bar{\Omega} \times \mathbb{R} \longrightarrow \mathbb{R}$ such that each of the problems* (PP), (HP) *and* (EP) *has (for suitable initial functions u_0 or (u_0, v_0)) a solution $u(\cdot, t)$ which is bounded in $H^2(\Omega)$ and whose ω-limit set $\omega(u)$ is a continuum in $H^1_0(\Omega)$ homeomorphic to S^1.*

We restrict our attention to dimensions $N = 2, 3$ for simplicity, so that we can treat all the three problems simultaneously and work in the L^2-setting.

The theorem has been proved in [25] for (PP) (see [24] for an earlier weaker result) and in [23] for (HP) and (EP). Notice the curious fact that the nonstabilizing solutions occur for each of the problems with the *same function g*. This is an interesting example of similarities in the behavior of solutions of the three problems. It may be instructive to discuss certain common features of these problems, and, in doing so, explain why one function g yields nonconvergent examples in all the three problems.

3. Common features of (PP), (HP) and (EP)

As already mentioned above, a consequential feature common to (PP), (HP) and (EP) is the *presence of a Lyapunov functional.* This forces bounded solutions to approach a set of "steady-states", that is t-independent solutions. Such solutions are given by (EC), the *equation for steady-states shared by* (PP), (HP) *and* (EP).

The next common property is associated with the linearization at steady-states. Namely, the three problems have *a common central part of the spectrum of the linearization.* To be more specific, let us write the nonlinearity g in the form

$$g(x, u) = a(x)u + f(x, u).$$

One can think of this as the linearization around a solution φ of (EC), in which case $a(x) = g_u(x, \varphi(x))$ and $f(x, u) = g(x, \varphi(x) + u) - a(x)u - \Delta\varphi(x)$, so that $f(x, 0) \equiv f_u(x, 0) \equiv 0$. We rewrite each of the problems in the abstract form

$$U' = AU + \hat{f}(U). \tag{3.1}$$

This is done as follows. Let $E = L^2(\Omega)$ for (PP) and $E = H_0^1(\Omega) \times L^2(\Omega)$ for (HP) and (EP). We define a closed operator A on E with the domain $\mathrm{Dom}(A) = X$ ($X := H^2(\Omega) \cap H_0^1(\Omega)$ for (PP) and $X := (H^2(\Omega) \cap H_0^1(\Omega)) \times H_0^1(\Omega)$ for (HP) and (EP)) as follows:

$$
\begin{aligned}
&\text{for (PP):} && Au = (\Delta + a(x))u && (u \in X),\\
&\text{for (HP):} && AU = \begin{pmatrix} 0 & I \\ \Delta + a(x) & -\alpha I \end{pmatrix} U && \left(U = \begin{pmatrix} u \\ v \end{pmatrix} \in X\right),\\
&\text{for (EP):} && AU = \begin{pmatrix} 0 & I \\ -\Delta - a(x) & \alpha I \end{pmatrix} U && \left(U = \begin{pmatrix} u \\ v \end{pmatrix} \in X\right),
\end{aligned}
$$

where I is the identity operator. The nonlinearity $\hat{f}$ is defined in the following way:

$$
\begin{aligned}
&\text{for (PP):} && \hat{f}(u)(x) = \tilde{f}(u)(x) := f(x, u(x)) && (u \in X)\\
&\text{for (HP):} && \hat{f}(U) = \begin{pmatrix} 0 \\ \tilde{f}(u) \end{pmatrix} && \left(U = \begin{pmatrix} u \\ v \end{pmatrix} \in X\right),\\
&\text{for (EP):} && \hat{f}(U) = \begin{pmatrix} 0 \\ -\tilde{f}(u) \end{pmatrix} && \left(U = \begin{pmatrix} u \\ v \end{pmatrix} \in X\right).
\end{aligned}
$$

Observe that $\hat{f}\colon X \to X$ is a smooth function if f is smooth (this is due to our assumption $N \leqslant 3$, see [27]).

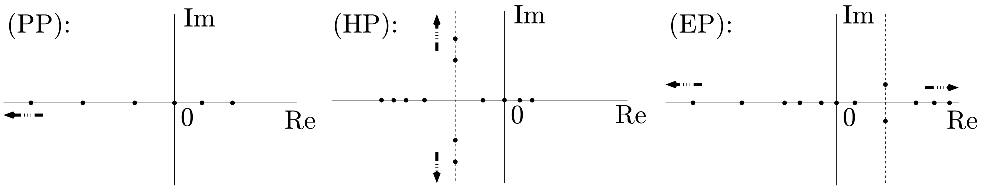

Figure 1. $\sigma(A)$: the arrows indicate directions in which $\sigma(A)$ is unbounded.

Let us now examine the spectrum of the operator A (see Fig. 1). In the parabolic case, $\sigma(A)$ consists of real eigenvalues $\mu_1 > \mu_2 > \mu_3 > \ldots$ accumulating at $-\infty$; each of them has the same algebraic and geometric multiplicity and the multiplicity is finite. In the hyperbolic case, $\sigma(A)$ consists of the eigenvalues

$$\frac{1}{2}\big(-\alpha \pm \sqrt{\alpha^2 + 4\mu_k}\big) \quad (\text{the roots of } \lambda^2 + \alpha\lambda = \mu_k), \quad k = 1, 2, \ldots.$$

In particular, the only possible eigenvalue on the imaginary axis is $\lambda = 0$; it occurs if and only if $\mu_k = 0$ for some k and then the algebraic multiplicity of this eigenvalue coincides with the geometric multiplicity and is equal to the multiplicity of the eigenvalue μ_k of $\Delta + a(x)$. This can be seen by examining the spectral projection associated with the eigenvalue $\lambda = 0$ (see [23]).

In the elliptic case, $\sigma(A)$ consists of the eigenvalues

$$\frac{1}{2}\big(\alpha \pm \sqrt{\alpha^2 - 4\mu_k}\big) \quad \text{(the roots of } \lambda^2 - \alpha\lambda = -\mu_k\text{)}, \quad k = 1, 2, \ldots .$$

Again, the only eigenvalue on the imaginary axis occurs if $\mu_k = 0$, and it is the eigenvalue $\lambda = 0$ with the same multiplicity as μ_k.

Although the structure of the spectra is quite different in the three cases, they have the same intersection with the imaginary axis: if nonempty, it is just the eigenvalue $\lambda = 0$ which has the same multiplicity for all the three problems. This is what we have meant by the common central part of the spectrum.

An important consequence of the above property is that *center manifolds of steady states have the dimension independent of the type of the problem.* The center manifold plays a crucial role in the proofs of Theorem 1, so let us discuss it in some detail. We do so in terms of the abstract equation (3.1).

Assume 0 is an eigenvalue of A and let $P \in \mathcal{L}(E)$ denote the spectral projection onto the corresponding eigenspace E_1 (note that the eigenspace coincides with $\ker A$). Let further E_2 denote the range of $I - P$ (I is the identity on E) and let $X_2 = X \cap E_2$. One can then prove (see [30], for example) that for any given positive integer k, if $\hat{f} \in C_b^k$ (that is, all its Frechet derivatives up to order k are continuous and bounded) and if the Lipschitz constant of $\hat{f}$ is sufficiently small, then there is a C_b^k map $\sigma_f\colon\, E_1 \to X$ with its image contained in X_2 such that the manifold

$$\mathcal{W}_f = \{U_1 + \sigma_f(U_1)\colon\, U_1 \in E_1\}$$

has the following invariance property (see Fig. 2). If $U_0 = U_1^0 + \sigma_f(U_1^0) \in \mathcal{W}_f$ and $U_1(t)$ is the solution of

$$\begin{aligned} \dot{U}_1 &= P\hat{f}\big(U_1(t) + \sigma_f(U_1(t))\big), \\ U_1(0) &= U_1^0, \end{aligned} \tag{3.2}$$

then $U(t) := U_1(t) + \sigma_f(U_1(t)) \in \mathcal{W}_f$ is a solution of (3.1) (with $U(0) = U_0$). Thus

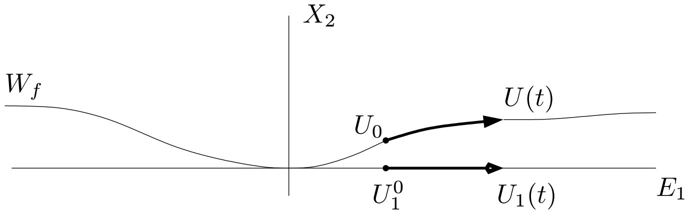

Figure 2. The center manifold of (3.1).

the *center manifold* $\mathcal{W}_f$ consists of solutions of (3.1) which are defined for all $t \geqslant 0$ (since the nonlinearity in (3.2) is globally Lipschitz). Note on passing that the elliptic initial value problem (EP) is well posed if the initial data are chosen in the center manifold. Equation (3.2), usually referred to as the *center manifold reduction*, is an ODE on the finite dimensional space E_1. Using real coordinates on E_1, we can rewrite it as an ODE on $\mathbb{R}^n$ with $n = \dim E_1$:

$$\dot{\xi} = h_f(\xi). \tag{3.3}$$

This ODE depends on the nonlinearity and on the type of problem we consider. However, for all the three problems (PP), (HP) and (EP), the ODE is posed on the same Euclidean space $\mathbb{R}^n$.

We now come to the last (in our list) common feature of problems (PP), (HP) and (EP). It concerns the gradient-like structure of the center manifold reduction. As mentioned above, each of the problems admits a Lyapunov functional V, which can be viewed as a functional on X (identifying $U = (u, v)$ with (u, u_t) for (HP), (EP)). From the invariance property of the center manifold, it follows that the composition $U_1 \mapsto V(U_1 + \sigma_f(U_1))$ is a Lyapunov functional for the reduction (3.3). Thus in real coordinates on E_1, equation (3.3) has a Lyapunov functional H_f. For the parabolic problem (PP), one can even prove (see [24]) that h_f in (3.3) is the gradient of H_f with respect to a Riemannian metric on $\mathbb{R}^n$. This is not necessarily true for (HP) and (EP), but still all equilibria of (3.3) are critical points of H_f (see [23]). Again, H_f depends on the type of the problem. However, inspecting formulas for H_f, one discovers a term which is independent of the type and which is, in some sense, the most important one. Specifically, if $(\varphi_1, \dots, \varphi_n)$ denotes an $L^2(\Omega)$-orthonormal basis of $\ker(\Delta + a(x))$ (under the Dirichlet boundary condition), then one has (see [24], [25], [23])

$$H_f(\xi) = \int_\Omega F(x, \xi \cdot \varphi(x))\, \mathrm{d}x + \dots \tag{3.4}$$

where $F(x, u) = \int_0^u f(x, s)\, \mathrm{d}s$, $\varphi = (\varphi_1, \dots, \varphi_n)$ and "$\cdot$" stands for the usual scalar product in $\mathbb{R}^n$. The missing terms in this formula are of higher order than the first term, which roughly speaking means that if we take $f = \varepsilon f_0$ with $\varepsilon \to 0$, then those terms are of order $O(\varepsilon^2)$ (note that the integral term is linear in f). The missing terms depend on the type of the problem and they involve the function σ_f from the center manifold. On the other hand, the first term is common to all the three problems. In addition, it does not involve σ_f, hence it is much easier to control when one wants to construct examples.

Let us now indicate how the proofs of Theorem 1 in [25], [23] utilize (3.4). First the function $a(x)$ is taken such that $n = \dim(\ker(\Delta + a(x)) = 2$ and such that the eigenfunctions φ_1, φ_2 satisfy additional conditions that we do not specify here.

One then wants to show that f can be chosen such that

$$\int_\Omega F(x, \xi \cdot \varphi(x))\,\mathrm{d}x = H_0(\xi), \tag{3.5}$$

where $H_0(\xi)$ is the smooth function on $\mathbb{R}^2$ given by

$$H_0(\varrho\cos\gamma, \varrho\sin\gamma) = \begin{cases} b\mathrm{e}^{1/(1-\varrho M(\gamma))}\sin(1/(\varrho M(\gamma)-1)-\gamma) & \text{if } \varrho M(\gamma) > 1, \\ 0 & \text{if } \varrho M(\gamma) \leqslant 1, \end{cases}$$

where $b \in \mathbb{R} \setminus \{0\}$ and $\gamma \mapsto M(\gamma)$ is a smooth positive 2π-periodic function. This is a modification of a function used in [21] in an example of a planar gradient vector field with a nonconvergent bounded trajectory. By careful estimates of the missing terms in (3.4), one can show that if (3.5) holds, then H_f has certain geometric properties which are not affected by the missing terms and which guarantee that (3.3) has a nonconvergent bounded trajectory. This information is then "lifted" to the center manifold $\mathcal{W}_f$ and one obtains a bounded nonconvergent solution of the original PDE. Thus to prove Theorem 1, it is sufficient to show that (3.5) can be solved for F (and then set $f = F'$). Note that (3.5) is a linear integral equation with respect to the unknown F, but it is not of any standard form. It actually requires quite a bit of technical work to solve it (the method of solution relies on properties of the eigenfunctions φ_1, φ_2 for a suitably chosen $a(x)$). The existence of a smooth solution F is established in [25], where parabolic equations are considered. Taking advantage of the fact that the leading term of the Lyapunov functional on the center manifold is common to all the three problems, the construction of [25] is used in [23] in the proof of the theorem for (HP) and (EP).

We remark that for the evolution equations (PP), (HP) one can use invariant foliations to prove that there actually exist infinite-dimensional manifolds of initial conditions that give nonconvergent bounded trajectories.

As the above discussion briefly outlines, our method for constructing examples of nonconvergent bounded solutions relies on "controlling" the center manifold reduction by adjusting the nonlinearity in the PDE. Similar ideas are of course not limited to gradient-like equations. In more general problems a similar method can be used to reveal even more interesting dynamics. See [22] for a survey of applications of this method.

References

[1] *A. V. Babin, M. I. Vishik*: Attractors of Evolution Equations. North-Holland, Amsterdam, 1992.

[2] *P. Brunovský, X. Mora, P. Poláčik, J. Solà-Morales*: Asymptotic behavior of solutions of semilinear elliptic equations on an unbounded strip. Acta Math. Univ. Comenian. (N.S.) *60* (1991), 163–183.

[3] *P. Brunovský, P. Poláčik, B. Sandstede*: Convergence in general periodic parabolic equations in one space dimension. Nonlinear Anal. *18* (1992), 209–215.

[4] *A. Calsina, X. Mora, J. Solà-Morales*: The dynamical approach to elliptic problems in cylindrical domains, and a study of their parabolic singular limit. J. Differ. Equations *102* (1993), 244–304.

[5] *X.-Y. Chen*: Uniqueness of the ω-limit point of solutions of a semilinear heat equation on the circle. Proc. Japan Acad. Ser. A Math. Sci. *62* (1986), 335–337.

[6] *X.-Y. Chen, H. Matano*: Convergence, asymptotic periodicity, and finite-point blow-up in one-dimensional semilinear heat equations. J. Differ. Equations *78* (1989), 160–190.

[7] *X.-Y. Chen, P. Poláčik*: Asymptotic periodicity of positive solutions of reaction diffusion equations on a ball. J. Reine Angew. Math. *472* (1996), 17–51.

[8] *E. Feireisl, P. Poláčik*: Structure of periodic solutions and asymptotic behavior for time-periodic reaction-diffusion equations on R. Adv. Differ. Equations *5* (2000), 583–622.

[9] *E. Feireisl, F. Simondon*: Convergence for degenerate parabolic equations. J. Differ. Equations *152* (1999), 439–466.

[10] *J. K. Hale*: Asymptotic Behavior of Dissipative Systems. American Mathematical Society, Providence, RI, 1988.

[11] *J. K. Hale, G. Raugel*: Convergence in gradient-like systems with applications to PDE. J. Applied Math. Physics (ZAMP) *43* (1992), 63–124.

[12] *A. Haraux, M. A. Jendoubi*: Convergence of solutions of second-order gradient-like systems with analytic nonlinearities. J. Differ. Equations *144* (1998), 313–320.

[13] *A. Haraux, P. Poláčik*: Convergence to a positive equilibrium for some nonlinear evolution equations in a ball. Acta Math. Univ. Comenian. (N.S.) *61* (1992), 129–141.

[14] *M. A. Jendoubi*: Convergence of global and bounded solutions of the wave equation with linear dissipation and analytic nonlinearity. J. Differ. Equations *144* (1998), 302–312.

[15] *M. A. Jendoubi*: A simple unified approach to some convergence theorems of L. Simon. J. Funct. Anal. *153* (1998), 187–202.

[16] *M. A. Jendoubi, P. Poláčik*: Nonstabilizing solutions of semilinear hyperbolic and elliptic equations with damping. Preprint.

[17] *K. Kirchgässner*: Wave-solutions of reversible systems and applications. J. Differ. Equations *45* (1982), 113–127.

[18] *H. Matano*: Convergence of solutions of one-dimensional semilinear parabolic equations. J. Math. Kyoto Univ. *18* (1978), 221–227.

[19] *A. Mielke*: Hamiltonian and Lagrangian Flows on Center Manifolds with Applications to Elliptic Variational Problems. Springer, Berlin, 1991.

[20] *A. Mielke*: Essential manifolds for an elliptic problem in infinite strip. J. Differ. Equations *110* (1994), 322–355.

[21] *J. Palis, W. de Melo*: Geometric Theory of Dynamical Systems. Springer, New York, 1982.

[22] *P. Poláčik*: Parabolic equations: asymptotic behavior and dynamics on invariant manifolds. Handbook on Dynamical Systems III: Towards Applications. Elsevier (B. Fiedler, ed.). To appear.

[23] *M. A. Jendoubi, P. Poláčik*: Nonstabilizing solutions of semilinear hyperbolic and elliptic equations with damping. Preprint.
[24] *P. Poláčik, K. P. Rybakowski*: Nonconvergent bounded trajectories in semilinear heat equations. J. Differ. Equations *124* (1996), 472–494.
[25] *P. Poláčik, F. Simondon*: Nonconvergent bounded solutions of semilinear heat equations on arbitrary domains. Preprint.
[26] *L. Simon*: Asymptotics for a class of nonlinear evolution equations, with applications to geometric problems. Annals Math. *118* (1983), 525–571.
[27] *T. Valent*: Boundary value problems of finite elasticity. Springer, New York, 1988.
[28] *P. Takáč*: Stabilization of positive solutions of analytic gradient-like systems. Preprint.
[29] *R. Temam*: Infinite-dimensional dynamical systems in mechanics and physics. Springer, New York, 1988.
[30] *A. Vanderbauwhede, G. Iooss*: Center manifold theory in infinite dimensions. Dynamics Reported: Expositions in Dynamical Systems. Springer, Berlin, 1992, pp. 125–163.
[31] *T. I. Zelenyak*: Stabilization of solutions of boundary value problems for a second order parabolic equation with one space variable. Differ. Equations *4* (1968), 17–22.

Author's address: *P. Poláčik*, Institute of Applied Mathematics, Comenius University, Mlynská dolina, 842 48 Bratislava, Slovakia; e-mail: `polacik@fmph.uniba.sk`.