EXPERIMENTAL EVALUATION OF THE DRAG TORQUE, DRAG FORCE AND MAGNUS FORCE ACTING ON A ROTATING PROLATE SPHEROID

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ABSTRACT. The drag torque, drag force and Magnus force acting on a spheroid rotating around its axis of symmetry and moving perpendicularly to this axis in calm water, were studied using experimental data and numerical simulation. The prolate plexiglas spheroid with ratio of the axes 3/4 was sped up in special device, which ensured the required rotational and translational velocity in the given plane. A video system was used to record the spheroid motion in water. Using the video records the spheroid translational and angular velocities and trajectory of its centre were determined and compared with the results of the numerical simulation. The dependences of the coefficients of the drag torque, drag force and Magnus force on the Reynolds number and dimensionless angular velocity were obtained.
Key words: prolate spheroid, drag torque, drag force, Magnus force

1. INTRODUCTION

For calculations of the conveyance of solid particles, models of spherical particles moving in fluid are usually used (e.g., Kholpanov and Ibyatov, 2005; Lukerchenko et al., 2006, 2008). However, the influence of the particle shape on its motion in fluid flow sometimes is significant. For example, in the case of particle saltation in water flowing over a rough bed, the elongated shape of the particles leads to an increase of the angular velocity (Nino and Garcia, 1998) which must be taken into account in numerical models. The endless variety of the particle shapes does not allow this problem to be solved using common case; therefore the special cases play an important role. After all a spheroid is the simplest shape of all the particles but can give more information in comparison with a sphere; in particular, the influence of the particle elongation can be evaluated.

Due to collisions with flow boundaries and other particles, the particle in a fluid flow not only moves translationally but also rotates. The following forces and torque act on the particle that moves and simultaneously rotates in a fluid flow:

- submerged gravitational force – difference between the gravitational force and Archimedean force;
- drag force $\boldsymbol{F}_{\mathrm{d}}$;
- force due to added mass;
- Magnus force $\boldsymbol{F}_M$;
- history force;
- torque due to added mass;
- drag torque $\boldsymbol{M}$.

The calculation for the submerged gravitational force is simple. The formulas for the tensor of added mass for ellipsoid are known (e.g., Kochin et al., 1955).

The expressions for the history force were dirived for spherical particles only (e.g., Basset, 1888; Mei et al., 1991; Mei and Adrian, 1992; Kim et al., 1998), a formula of the history force for ellipsoid is yet to be researched.

The formulas for the drag torque $\boldsymbol{M}$, drag force $\boldsymbol{F}_d$ and Magnus force $\boldsymbol{F}_M$ include the particle translational and angular velocity relatively fluid, therefore they can be studied for the particle moving in the calm fluid. In this case these velocities are absolute because the fluid velocity is zero. The present study deals with the drag torque, drag force and Magnus force acting on the ellipsoid of revolution - prolate spheroid with semi-axes a_0 = 0.020 m (the axis of symmetry axis) and b_0 = 0.015 m) that rotates about its axis of symmetry and moves perpendicularly to this axis in calm water. The spheroid motion is characterized by two dimensionless parameters: translational Reynolds number (or Reynolds number) $Re = 2a_0\,|\boldsymbol{V}|\,/\,\nu$ and rotational Reynolds number $Re_\omega = |\boldsymbol{\omega}|\ b_0^2\,/\,\nu$, where ν is the fluid kinematical viscosity, $\boldsymbol{V}$ is the vector of the translational velocity of the particle centre of mass, which coincides with its geometrical centre in the case of homogeneous particles, and $\boldsymbol{\omega}$ is the vector of angular velocity of the particle rotation about its axis of symmetry. Sometimes the dimensionless angular velocity $\Gamma = b_0\,|\boldsymbol{\omega}|\,/\,|\boldsymbol{V}|$ is used instead of the rotational Reynolds number Re_ω (Tanaka et al., 1990; Oesterle and Bui Dinh, 1998). Any pair of these three dimensionless parameters, Re, Re_ω, and Γ is equivalent to the others and $\Gamma=2q^{-1}Re_\omega\,/Re$, where $q = b_0\,/\,a_0$ is the ratio of the spheroid semi-axes.

Information about the drag torque is known only for the spherical particle rotating in fluid without translational motion (Sawatzki, 1970) and with translational motion (Lukerchenko et al., 2008).

The drag force acting on a spheroid moving in fluid perpendicularly of its axis of symmetry without rotation is known (Clift et al., 1978).

The theoretical evaluation of the Magnus force F_M for a spheroid that rotates about its axis of symmetry and moves perpendicularly to this axis is given by Lukerchenko (2005):

$$\boldsymbol{F}_M=2\Omega_0\,\rho\,|\boldsymbol{\omega}|\,|\boldsymbol{V}|, \tag{1}$$

where ρ is the fluid density, $\Omega_0 = (4/3)\pi a_0\,b_0^2$ is the volume of the spheroid.

Tanaka et al. (1990) investigated experimentally the drag and Magnus forces acting on sphere, prolate and oblate spheroids that rotate about an axis perpendicular to the free stream. They used the wind tunnel of an open-circuit type. The spheroids were made to rotate about the axis perpendicular to the axis of symmetry. The Reynolds number Re of the test bodies ranges from 60000 to 150000. They revealed for the sphere and oblate spheroid, the shape of which was near sphere, the negative Magnus effect, which was observed at the Reynolds number Re higher than 100000 and a certain range of the dimensionless angular velocity Γ. Thus, experimental and theoretical data on the drag torque, drag force and Magnus force in the case when the spheroid moves translationally and rotates simultaneously is not described sufficiently in the literature for use in numerical models. In this case the mutual influences of the translational and rotational particle movements should be taken into account.

The basic relationships for these forces and torque can be expressed from dimensional analysis and the dimensionless coefficients should be determined experimentally in dependence on pair of dimensionless parameters Re and Re_ω or Re_ω and Γ.

For a sphere, the dependence of the drag torque and drag force coefficients on Re and Re_ω were found experimentally by Lukerchenko et al. (2008) and the dependence of the Magnus force coefficient on Re_ω and Γ was obtained by Oesterle and Bui Dinh (1998).

The aim of the present study is to obtain the dependence of the drag torque, drag force and Magnus force coefficients on the both Reynolds numbers Re and Re_ω (or Reynolds number Re and dimensionless angular velocity Γ) for the prolate spheroid ($q = 3/4$) moving perpendicularly to its axis of symmetry and simultaneously rotating about this axis in initially quiescent fluid.

2. DRAG TORQUE, DRAG FORCE, AND MAGNUS FORCE

The *drag torque* acting on a spheroid rotating about its axis of symmetry in fluid is given by the formula:

$$\boldsymbol{M} = - C_\omega\, 0.5\rho b_0^5 |\boldsymbol{\omega}| \boldsymbol{\omega}, \tag{2}$$

Where $C_\omega = C_\omega\,(q,\ Re,\ Re_\omega)$ is the dimensionless drag torque coefficient that depends on the ratio of the semi-axes q, Reynolds number Re and rotational Reynolds number Re_ω or dimensionless angular velocity Γ. The reliable experimental and theoretical data for sphere in the case $\boldsymbol{V} = 0$: $C_\omega = C_\omega\,(q = 1,\ Re,\ Re_\omega) = C_{\omega 10}\,(Re_\omega)$ are described by Sawatzki (1970)), where the data are obtained for a sphere rotating with constant angular velocity in viscous fluid, which is at rest at a large distance from the rotating sphere. For a rotating sphere in the case $\boldsymbol{V} \neq 0$: $C_\omega = C_\omega\,(q = 1,\ Re,\ Re_\omega) = C_{\omega 1}\,(Re,\ Re_\omega)$ the expression for the drag torque coefficient was obtained by Lukerchenko et al. (2008):

$$C_\omega = C_{\omega 1}\,(Re,\ Re_\omega) = C_{\omega 10}\,(Re)\,(1+0.0044\, Re^{\,0.5}). \tag{3}$$

The value of the *drag force* acting on a spheroid moving in fluid perpendicularly of its axis of symmetry can be described by the following formula:

$$\boldsymbol{F}_d = - C_d\, 0.5\rho\, S_0 |\boldsymbol{V}| \boldsymbol{V}, \tag{4}$$

where $S_0 = \pi\, a_0\, b_0$ is the square of the spheroid midlength section and $C_d = C_d\,(q,\ Re,\ Re_\omega)$ is the dimensionless drag force coefficient that depends on the ratio of the semi-axes q, Reynolds number Re and rotational Reynolds number Re_ω or dimensionless angular velocity Γ. In the case $q = 1$ (sphere) and $Re_\omega = 0$ (the motion without rotation) the drag force coefficient $C_d = C_d\,(q = 1,\ Re,\ Re_\omega = 0) = C_{d10}\,(Re)$ is a well known function of the Reynolds number (e.g., Nino and Garcia, 1994):

$$C_{d10} = \frac{24}{Re}\left(1 + 0.15 Re^{\frac{1}{2}} + 0.017 Re\right) - \frac{0.208}{1 + 10^4 Re^{-0.5}}. \tag{5}$$

If $q \neq 1$ (spheroid) and $Re_\omega = 0$ (the motion without rotation) the drag force coefficient $C_d = C_d\,(q,\ Re,\ Re_\omega = 0) = C_{d0}\,(q,\ Re)$ can be expressed as (Clift et al., 1978):

$$C_{d0}=0.445[1+1.63(1-q)^2] \qquad \text{if } 980 < Re < 10\,000, \tag{6}$$

$$C_{d0}=0.445[1+1.63(1-q)] \qquad \text{if } Re > 40\,000. \tag{7}$$

For values of the Reynolds number $10^4 < Re < 4{\cdot}10^4$ the value of the drag force coefficient can be taken using the linear interpolation.

The lateral *Magnus force* is defined by the next expression:

$$\boldsymbol{F}_M = C_M\, \Omega_0\, \rho [\boldsymbol{\omega}, \boldsymbol{V}], \tag{8}$$

where C_M is the dimensionless Magnus force coefficient.

3. MATHEMATICAL MODEL OF THE SPHEROID MOTION IN FLUID

Let us consider the spheroid that moves and simultaneously rotates in fluid in a vertical plane. Its axis of symmetry is the axis of rotation and is normal to the plane of motion during the period of the observation. The coordinate axis Ox is horizontal and the coordinate axis Oy is vertical. The system of equations describing the spheroid motion in initially quiescent fluid is:

$$\Omega_0 \rho_p \frac{d\boldsymbol{V}}{dt} = \boldsymbol{F}_g + \boldsymbol{F}_d + \boldsymbol{F}_m + \boldsymbol{F}_M, \tag{9}$$

$$J \frac{d\boldsymbol{\omega}}{dt} = M, \tag{10}$$

where ρ_P is the particle density and J is the particle momentum of inertia:

$$J = \frac{8\pi}{15} \rho_p a_0 b_0^4, \tag{11}$$

$\boldsymbol{F}_g$ is the submerged gravitational force:

$$\boldsymbol{F}_g = \Omega_0 \left(\rho_P - \rho \right) \boldsymbol{g}, \tag{12}$$

$\boldsymbol{g}$ is the vector of the gravitational acceleration, $\boldsymbol{F}_m$ is the force due to added mass:

$$\boldsymbol{F}_m = -\Omega_0\, \rho C_m \frac{d\boldsymbol{V}}{dt}, \tag{13}$$

C_m is the added mass coefficient.

If the spheroid velocity is perpendicular to the axis of symmetry, the coefficient C_m can be written (Loitsianskii, 1973):

$$C_m = \frac{1/e^2 - (1-e^2)/2e^3 \cdot \ln((1+e)/(1-e))}{2 - 1/e^2 + (1-e^2)/2e^3 \cdot \ln((1+e)/(1-e))}, \tag{14}$$

where $e = \sqrt{1-q^2}$ is the eccentricity. For $q = 3/4$: $C_m = 0{,}587$.

The Basset history force is neglected in the present study because the expression of this force for the spheroid is not known and because a relatively large body is used (a_0 =20 mm and b_0 =15 mm), for which the Basset history force is small in comparison with other forces (Bombardelli et al, 2008).

The change in the modulus of the angular velocity ω allows us to evaluate the drag torque, that is, the drag torque coefficient C_ω. The change in the modulus of the translational velocity $\boldsymbol{V}$ allows us to evaluate the drag force, that is, the drag force coefficient C_d. The trajectory curvature allows us to evaluate the Magnus force, that is, the Magnus force coefficient C_M.

4. EXPERIMENTAL PROCEDURE

The experimental set up is depicted in Fig. 1. The experiments were carried out in a rectangular glass vessel 0.780 m long, 0.580 m wide, and 0.980 m high. The water depth was about 0.800 m. The measured spheroid (plexiglas, a_0 = 0.020 m and b_0 =0.015 m) was sped up in a special device (developed in the Institute of Hydrodynamics AS CR, v. v. i.) situated above the water surface. The device ensured that the required spheroid rotation in the given plane.

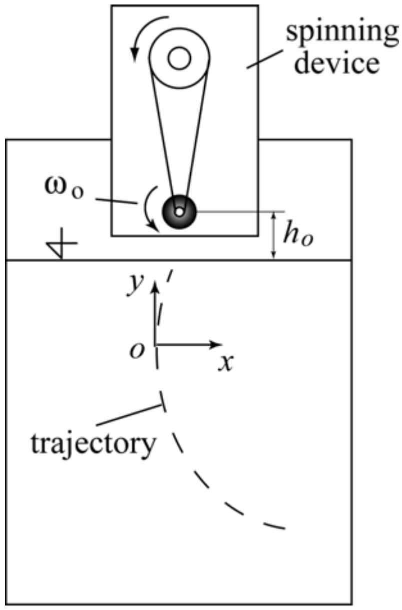

Fig. 1 Experimental setup

and translational velocity of the spheroid were reached by free fall. The device allowed the spheroid to spin up to 6500 revolutions per minute (rpm). The translational velocity of the spheroid was given by the height h_0 of the axis of rotation of the device above the water surface. The initial height h_0 and angular spheroid velocity ω_0 can be chosen independently from one other.

In the experiments the device was situated as that the axis of the rotation was at height h_0 of 17 mm and 30 mm above water surface what determined the initial translational velocity with which the spheroid enters to water. An unsteady entrance region can be observed when the particle enters water. Therefore, only the experimental data outside this region were used for the data analysis. The ball movement in water was recorded using the digital video camera NanoSenze MKIII+ with a frequency up to 1000 frames per second.

Hairlines were drawn along two perimeters of the spheroid with the angle of 90° to make it possible to visualize the particle rotation. Only experiments in which the plane of the particle trajectory was parallel to the plane of the video camera objective were chosen.

The initial angular spheroid velocity ω_0 was initially maximum 6500 rpm and then in the next experiments was decreased with step 500 rpm. The spheroid motion was stable, i.e. the axis of the rotation (axis the symmetry) kept its direction during the motion, for large values of the ω_0 whereas for $\omega_0 < 1000$ the axis of rotation changed usually its direction. Therefore, in the present study the value of the initial angular spheroid velocity ω_0 was ranged from 1000 to 6500 rpm. The measured values of the dimensionless angular velocity were from 3 to 12, the measured values of the

Reynolds number were from 10000 to 20000. For each pair of h_0 and ω_0 at the least three experiments were carried out.

The geometric and kinematical properties of the particle motion were found from the image sequences.

5. RESULTS

The system of equations (9)–(10) describing the spheroid motion in fluid was solved numerically. The experimental and calculated trajectories and the particle kinematical parameters were plotted as functions of time. The values of the drag torque coefficient C_ω , drag force coefficient C_d and Magnus force coefficient C_M were found using the method of best fit of the experimental data.

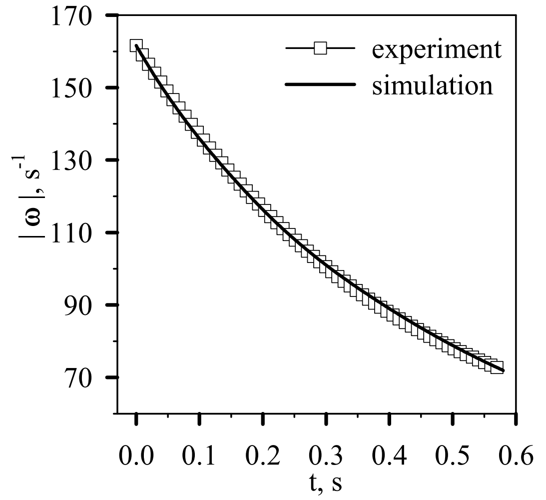

Fig. 2. Absolute value of the angular velocity (h_0 = 0.017 m; ω_0 = 2000 rpm)

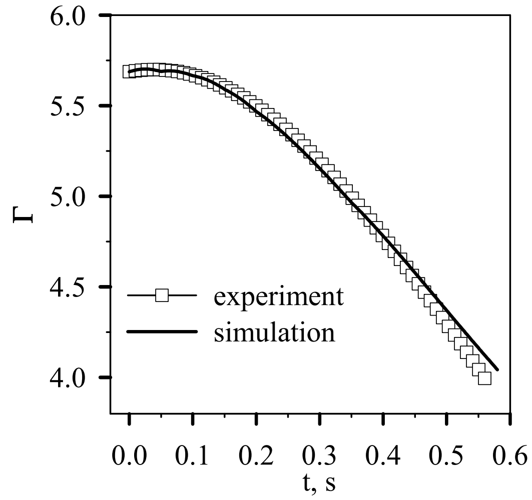

Fig. 3. Dimensionless angular velocity (h_0 = 0.017 m; ω_0 = 2000 rpm)

The initial part of the spheroid trajectory immediately after its entry into the water was neglected because its motion was unsteady in this part. The added forces act on the spheroid in the initial part due to the different disturbances (particle transition from air to water, effect of air bubbles on particle surface, etc.) in the water.

Fig. 2 and Fig. 3 show the absolute value of the angular velocity and dimensionless angular velocity of the spheroid versus time, respectively. The good agreement the calculated and experimental data is reached if the following formula for the drag torque coefficient is used in the numerical model:

$$C_\omega = C_{\omega 10}\,(1 + k_\omega \Gamma^{0.5}), \tag{15}$$

where $C_{\omega 10}$ is calculated using the data of Sawatski (1970), k_ω is constant, its average value and standard deviation obtained using 15 experiments is 0.226 and 0.010, respectively.

The calculated and experimental values of the modulus of the translational velocity versus time are depicted in Fig. 4.

The drag force coefficient C_d was sought as:

$$C_d = \lambda_d\, C_{d0}\,, \tag{16}$$

where C_{d0} was calculated using the expression (6) and (7). The value of the factor λ_d was chosen in each time interval 0.05 s as that the calculated values of the translational velocity are near to the experimental ones. The acceptable formula for the factor λ_d is not yet found. The values of the factor λ_d for some values of Re and Γ are represented in Table 1.

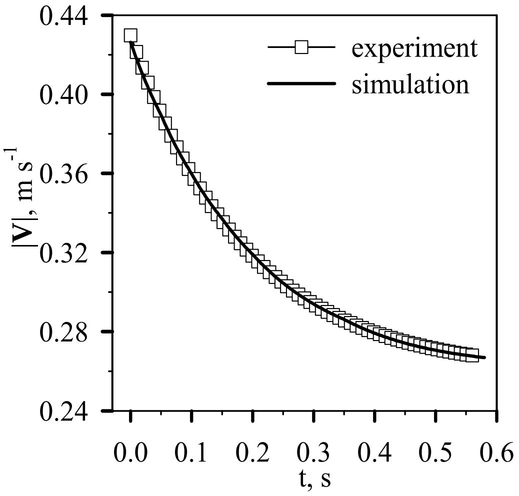

Fig. 4. Modulus of the translational velocity (h_0 = 0.017 m; ω_0 = 2000 rpm)

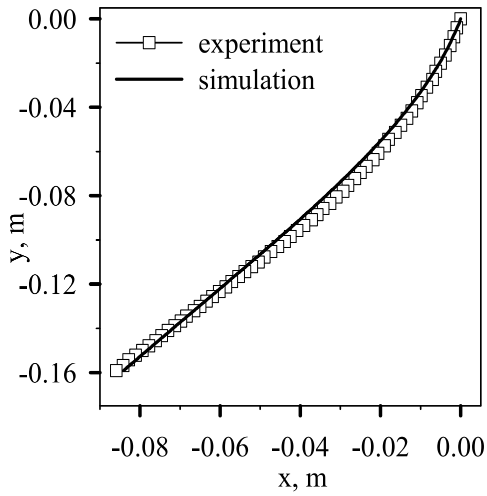

Fig. 5. Trajectory of the spheroid centre (h_0 = 0.017 m; ω_0 = 2000 rpm)

In a qualitative sense the dependence of the drag force coefficient on the parameter Γ for a spheroid is like of that for a sphere. The accurate dependence for C_d in the form of a formula need added experimental data but the following tendencies can be noted. For the values of $\Gamma < \sim 5$ the drag force coefficient increases with increasing of Γ, for $\Gamma > \sim 5$ the drag force coefficient decreases with increasing of Γ. For $\Gamma \sim 10$ the drag force coefficient is approximately equal to that for $\Gamma = 0$ (motion without rotation) and for $\Gamma > 10$ it is even less than that for the case of the motion without rotation.

Table 1.

λ_d	$Re*10^{-3}$	Γ	λ_d	$Re*10^{-3}$	Γ	λ_d	$Re*10^{-3}$	Γ	λ_d	$Re*10^{-3}$	Γ	λ_d	$Re*10^{-3}$	Γ
1.00	19.8	9.96	1.05	18.7	9.52	1.25	17.5	8.89	1.40	17.6	6.92	1.50	17.5	5.23
1.05	17.7	9.51	1.10	16.7	9.12	1.30	15.6	8.70	1.50	15.4	7.09	1.60	15.2	5.51
1.15	16.1	9.12	1.15	15.4	8.72	1.35	14.2	8.41	1.60	13.7	7.14	1.70	13.5	5.66
1.15	14.8	8.71	1.20	14.3	8.31	1.40	13.2	8.11	1.60	12.6	7.07	1.80	12.4	5.72
1.20	13.8	8.28	1.27	13.3	7.95	1.45	12.3	7.80	1.65	11.8	6.91	1.80	11.5	5.67
1.20	13.0	7.89	1.33	12.6	7.62	1.50	11.7	7.48	1.65	11.2	6.71	1.70	11.0	5.51
1.25	12.4	7.49	1.37	11.9	7.28	1.55	11.1	7.17	1.65	10.7	6.44	1.70	10.7	5.29
1.25	11.9	7.11	1.45	11.4	6.97	1.55	10.7	6.85	1.65	10.4	6.13	1.60	10.5	5.04
1.30	11.5	6.75	1.50	11.0	6.68	1.60	10.4	6.51	1.65	10.2	5.83	1.60	10.4	4.76
1.30	11.2	6.41	1.55	10.6	6.37	1.65	10.2	6.19	1.65	10.1	5.53	1.60	10.4	4.49
1.35	11.0	6.05	1.65	10.2	5.97	1.70	9.9	5.80	1.65	9.9	5.12	1.60	10.3	4.25

λ_d	Re*10^{-3}	Γ	λ_d	Re*10^{-3}	Γ	λ_d	Re*10^{-3}	Γ	λ_d	Re*10^{-3}	Γ	λ_d	Re*10^{-3}	Γ
1.70	17.7	4.03	1.70	19.7	2.60	0.85	18.0	11.7	1.00	18.3	10.3	1.00	17.0	9.46
1.75	14.9	4.42	1.80	15.9	3.03	0.90	16.9	10.6	0.95	16.9	9.63	1.10	15.8	8.85
1.80	13.2	4.65	1.90	13.6	3.33	1.00	15.8	9.79	1.05	15.7	9.02	1.15	14.7	8.34
1.85	12.0	4.76	2.00	12.1	3.54	1.00	14.9	9.14	1.08	14.7	8.49	1.15	13.9	7.87
1.80	11.3	4.76	1.95	11.1	3.63	1.10	14.0	8.57	1.20	13.8	8.07	1.20	13.3	7.45
1.75	10.8	4.67	1.85	10.6	3.63	1.15	13.3	8.11	1.22	13.0	7.71	1.20	12.7	7.04
1.70	10.5	4.51	1.75	10.3	3.55	1.25	12.6	7.73	1.35	12.3	7.38	1.25	12.3	6.67
1.65	10.3	4.31	1.65	10.1	3.42	1.25	12.1	7.35	1.37	11.7	7.09	1.30	11.9	6.34
1.60	10.2	4.10	1.65	10.1	3.28	1.35	11.6	7.01	1.45	11.3	6.80	1.30	11.6	6.02
1.55	10.2	3.88	1.65	10.0	3.14	1.35	11.4	6.81	1.50	10.9	6.51	1.40	11.3	5.72
1.50	10.2	3.64	1.65	10.0	3.00				1.50	10.6	6.22	1.37	11.0	5.24

λ_d	Re*10^{-3}	Γ	λ_d	Re*10^{-3}	Γ	λ_d	Re*10^{-3}	Γ	λ_d	Re*10^{-3}	Γ	λ_d	Re*10^{-3}	Γ
1.10	16.2	8.44	1.22	15.8	6.73	1.35	15.7	5.70	1.70	15.9	4.46	1.80	16.2	3.16
1.15	15.2	7.90	1.34	14.7	6.55	1.45	14.4	5.68	1.70	14.0	6.68	1.85	14.0	3.44
1.25	14.3	7.48	1.45	13.7	6.38	1.50	13.4	5.63	1.80	12.8	4.78	1.90	12.6	3.60
1.30	13.6	7.13	1.50	12.9	6.22	1.55	12.6	5.53	1.80	11.9	4.80	1.90	11.7	3.67
1.40	12.9	6.82	1.65	12.2	6.08	1.60	12.0	5.40	1.70	11.4	4.72	1.80	11.1	3.66
1.45	12.3	6.55	1.65	11.6	5.90	1.60	11.5	5.24	1.70	11.0	4.58	1.80	10.8	3.59
1.50	11.8	6.29	1.70	11.2	5.69	1.60	11.2	5.06	1.70	10.7	4.41	1.75	10.5	3.49
1.50	11.4	6.01	1.70	10.8	5.47	1.65	10.9	4.87	1.70	10.6	4.23	1.75	10.4	3.37
1.50	11.1	5.71	1.55	10.7	5.21	1.65	10.7	4.68	1.60	10.5	4.03	1.70	10.3	3.24
1.45	10.9	5.42	1.50	10.6	4.91	1.65	10.5	4.47	1.60	10.4	3.83	1.70	10.2	3.11
1.37	10.8	5.01	1.45	10.7	4.55	1.65	10.4	4.20	1.60	10.4	3.57	1.70	10.2	3.94

Good agreement of the calculated and experimental data is obtained for the trajectory of the spheroid centre (Figure 5) if the coefficient of Magnus force was calculated according to the formula:

$$C_M = k_M / \Gamma, \qquad (17)$$

where the factor k_M is constant, its average value and standard deviation obtained using 15 experiments is 0.177 and 0.010, respectively.

6. CONCLUSIONS

The mutual influence of the translational and rotational movements in fluid of the prolate spheroid was studied using the experimental data and numerical simulation. The spheroid moves perpendicularly of its axis of symmetry and rotates about this axis. The drag torque, drag force and Magnus force coefficients of a spheroid were evaluated for the Reynolds numbers range $10^4 < Re < 2{\cdot}10^4$ and dimensionless angular velocity range $3 < \Gamma < 12$.

For the *drag torque coefficient* the following formula was obtained:

$$C_\omega = C_{\omega 10}\,(1 + k_\omega \Gamma^{0.5})$$

where $C_{\omega 10}$ is calculated using the data of Sawatski (1970), $k_\omega = 0.226 \pm 0.010$.
For the *Magnus force coefficient* the following formula is valid:

$$C_M = k_M / \Gamma,$$

where $k_M = 0.177 \pm 0.010$.

The dependence of the *drag force coefficient* C_d on the Reynolds number Re and dimensionless angular velocity Γ is obtained in the tabular form. The accurate dependence for C_d in the form of a formula need added experimental data but the following tendencies can be noted. For the values of $\Gamma < \sim 5$ the drag force coefficient increases with increasing of Γ, for $\Gamma > \sim 5$ the drag force coefficient decreases with increasing of Γ, for $\Gamma \sim 10$ the drag force is less than for motion without rotation.

The results are useful for numerical simulation of the particle motion in fluid and modelling of the particle-laden flow and sediment transport of sand in rivers.

7. ACKNOWLEDGEMENTS

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8. NOMENCLATURE

a_0 semi-axis of the spheroid, corresponding to its axis of symmetry, m

$b_0 = c_0$ semi-axes of the spheroid normal to its axis of symmetry, m

C_d drag force coefficient, -

C_{d0} drag force coefficient of the spheroid moving in fluid without rotation (ω=0), -

C_{d10} drag force coefficient of the sphere ($q = 1$) moving in fluid without rotation ($\omega = 0$), -

C_m added mass coefficient, -

C_M Magnus force coefficient, -

C_ω drag torque coefficient, -

$C_{\omega 1}$ drag torque coefficient for the sphere ($q = 1$) rotating in fluid with the translational motion ($V \neq 0$),-

$C_{\omega 10}$ drag torque coefficient for the sphere ($q=1$) rotating in fluid without the translational motion($\boldsymbol{V}$=0),-
e eccentricity of the spheroid, m
$\boldsymbol{F}_d$ drag force, N
$\boldsymbol{F}_g$ submerged gravitational force, N
$\boldsymbol{F}_m$ force due to added mass, N
$\boldsymbol{F}_M$ Magnus force, N
$\boldsymbol{g}$ vector of the gravitational acceleration, m/s^2
h_0 height of the axis of rotation of the device above the water surface, m
J particle momentum of inertia, kg m^2
$\boldsymbol{M}$ drag torque, N m
q ratio of the semi-axes of the spheroid, -
Re translational Reynolds number, -
Re_ω rotational Reynolds number, -
S_0 square of the spheroid midlength section, m^2
$\boldsymbol{V}$ vector of the spheroid translational velocity, m s^{-1}
Γ dimensionless particle angular velocity, -
ν kinematical fluid viscosity, m^2 s^{-1}
ρ fluid density, kg m^{-3}
ρ_P particle density, kg m^{-3}
$\boldsymbol{\omega}$ vector of the spheroid angular velocity, s^{-1}
ω_0 initial spheroid angular velocity in the spinning device, revolutions per minute (rpm)
Ω_0 volume of the spheroid, m^3

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