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A symmetry preserving contact treatment in isogeometric analysis

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Micro Abstract
In this contribution an isogeometric contact treatment by the penalty method is presented. The symmetry preserving formulation, also known as the two-half-pass formulation, is utilized together with the Gauss-point-to-segment discretization. A particular attention is payed to the contact detection, i. e. to the closest point projection of a point onto a NURBS patch. The problem of contact pressure post-processing is also presented in more detail.

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Introduction

Contact analysis based on traditional finite elements utilizes element facets to describe a contact surface. The facets are C^0-continuous so that surface normals can experience jump across facets boundaries leading to artificial oscillations in normal and tangent contact forces. The geometric non-smoothness is usually attacked by contact smoothing techniques, where contact interface is approximated by various species of splines [8, 9, 11].

Another remedy to the geometric discontinuity provides isogeometric analysis (IGA) [3]. The fundamental idea is to accurately describe a physical domain of interest by a proper mathematical representation (e.g. B-Spline, Non-Uniform Rational B-Spline (NURBS), etc.) and then utilize the same basis for the analysis. This is in contrast with the classical finite element method where the basis is given in advance by the element type and the geometry of physical domain is then only approximated with the aid of isoparametric mapping. Isogeometric NURBS-based contact analysis [1] has some additional advantages: preserving geometric continuity, facilitating patch-wise contact search, supporting a variationally consistent formulation, and having a uniform data structure for the contact surface and the underlying volumes.

In this contribution a simple yet powerful symmetry preserving contact formulation [2] is employed in the IGA framework. It is based on the Gauss-point-to-segment discretization where contact constraints are enforced at the Gaussian quadrature point level by the penalty method. The topics of contact detection and contact pressure post-processing are also briefly mentioned. The performance of the overall algorithm is presented by means of Hertzian contact problem.

Two-half-pass Gauss-point-to-segment contact algorithm

An essential contact kinematic quantity is the normal gap function

$$g_{\mathrm{N}}^{(i)}(\mathbf{x}^{(i)}) = -\left(\mathbf{x}^{(i)} - \bar{\mathbf{x}}^{(k)}\right) \cdot \bar{\mathbf{n}}^{(k)}, \qquad \forall \mathbf{x}^{(i)} \in \gamma_{\mathrm{c}}^{(i)}, \tag{1}$$

which can be understood as the signed distance function. Here $\mathbf{x}^{(i)} \in \gamma_{\mathrm{c}}^{(i)} \in \mathbb{R}^{n_{\mathrm{sd}}}$, where n_{sd} indicates the number of spatial dimensions, is the position vector of a point on the potential contact boundary $\gamma_{\mathrm{c}}^{(i)}$. The superscript $(i), i \in 1, 2$ denotes the index of the body. To generalize the notation, let us introduce another body index as $(k) := \{1, 2\} \setminus (i)$, i.e. (k) is the complement body to the body (i). The quantities denoted by $(\bar{\bullet})$ are related to the closest point projection of the point $\mathbf{x}^{(i)}$. Finally $\bar{\mathbf{n}}^{(k)} \in \mathbb{R}^{n_{\text{sd}}}$ is the normal vector. Note that according to our definition, the gap is open, i.e. bodies are not in contact, when the normal gap function is negative. Conversely, a positive value of the normal gap function indicates penetration of the bodies. The physical behaviour at the contact boundary in the normal direction can be described using the Hertz-Signoriniho-Moreau conditions

$$p_{\text{c}}^{(i)} \geq 0, \qquad g_{\text{N}}^{(i)} \leq 0, \qquad p_{\text{c}}^{(i)} g_{\text{N}}^{(i)} = 0, \qquad \forall \mathbf{x}^{(i)} \in \gamma_{\text{c}}^{(i)}. \tag{2}$$

where $p_{\text{c}}^{(i)}(\mathbf{x}^{(i)}) \in \mathbb{R}$ is the contact pressure. This conditions are known in mathematical optimization as the Karush–Kuhn–Tucker (KKT) conditions. A particularly simple formulation can be obtained by the regularization of the contact pressure by the penalty method. It consists in replacing the unknown contact pressure field with a new one which is dependent on the displacement field. The contact pressure is prescribed by the function

$$p_{\text{c}}^{(i)} = \epsilon_{\text{N}} \langle g_{\text{N}}^{(i)} \rangle, \tag{3}$$

where $\epsilon_{\text{N}} \in \mathbb{R}$ is the normal penalty parameter and $\langle \bullet \rangle := \frac{|\bullet| + \bullet}{2}$ are so-called Macaulay's brackets. Employing the well known principle of virtual work, the balance of linear momentum with consideration of the contact constraints can be symbolically formulated in the weak sense as

$$\delta \Pi = \sum_{i=1}^{2} \left(\delta \Pi_{\text{int}}^{(i)} + \delta \Pi_{\text{ext}}^{(i)} + \delta \Pi_{\text{c}}^{(i)} \right) = 0, \tag{4}$$

where $\delta \Pi_{\text{int}}^{(i)}$ consists of stress divergence term, $\delta \Pi_{\text{ext}}^{(i)}$ comprises of terms expressing virtual work due to volume and surface forces, and $\delta \Pi_{\text{c}}^{(i)}$ denotes virtual work done by contact forces, which is defined as

$$\delta \Pi_{\text{c}}^{(i)} = - \int_{\gamma_{\text{c}}^{(i)}} \delta \mathbf{u}^{(i)} \cdot \epsilon_{\text{N}} \langle g_{\text{N}}^{(i)} \rangle \bar{\mathbf{n}}^{(k)} \, \mathrm{d}\gamma^{(i)}. \tag{5}$$

After spatial discretization by the finite element method, one ends up with non-linear system of equations

$$\mathbf{F}_{\text{int}}(\mathbf{d}) + \mathbf{F}_{\text{ext}}(\mathbf{d}) + \sum_{i=1}^{2} \mathbf{F}_{\text{c}}^{(i)}(\mathbf{d}) = \mathbf{0}, \tag{6}$$

where $\mathbf{d}$ is the vector of nodal displacements, $\mathbf{F}_{\text{int}}$ is the vector of equivalent internal nodal forces, $\mathbf{F}_{\text{ext}}$ is the vector of equivalent external nodal forces, and $\mathbf{F}_{\text{c}}^{(i)}$ are the vectors of equivalent contact nodal forces. The last mentioned are assembled by the standard finite element procedures from the local counterparts

$$\mathbf{F}_{\text{c}}^{s(i)} = - \int_{\gamma_{\text{c}}^{s(i)}} \epsilon_{\text{N}} \left\langle g_{\text{N}}^{(i)} \right\rangle \mathbf{N}^{s(i)} \, \mathrm{d}\gamma^{s(i)}, \tag{7}$$

where superscript s indicates the number of contact segment of the element, and $\mathbf{N}^{s(i)}$ is the matrix defined as

$$\mathbf{N}^{s(i)} = \begin{bmatrix} R_1 \bar{\mathbf{n}}^{(k)} \\ \vdots \\ R_{n_{\text{scp}}} \bar{\mathbf{n}}^{(k)} \end{bmatrix}, \tag{8}$$

where $R_j, j = 1, \dots, n_{\text{scp}}$ are the contact segment NURBS basis functions, and n_{scp} indicates the number of contact segment control points. Note that the global vectors of the control point contact forces are then assembled in a conventional manner.

Contact detection

By the contact detection it is understood the evaluation of the normal gap function (1) in all Gaussian quadrature points of the potential contact interface $\gamma_{\mathrm{c}}^{(i)}$. In particular, for each Gauss-point $\mathbf{x}^{(i)}$, it is necessary to calculate the closest point projection $\bar{\mathbf{x}}^{(k)} = \mathbf{x}^{(k)}(\bar{\boldsymbol{\xi}})$, which has to satisfie the orthogonality condition

$$\left(\mathbf{x}^{(i)} - \mathbf{x}^{(k)}(\bar{\boldsymbol{\xi}})\right) \cdot \frac{\partial \mathbf{x}^{(k)}(\bar{\boldsymbol{\xi}})}{\partial \boldsymbol{\xi}} = \mathbf{0}, \tag{9}$$

where $\boldsymbol{\xi} \in \mathbb{R}^{n_{\mathrm{npd}}}$ are the convective coordinates which parametrizes the potential contact boundary $\gamma_{\mathrm{c}}^{(k)}$, and n_{npd} indicates the number of parametric dimensions. The importance of this procedure results from the fact that this evaluation must be performed in each iteration of the non-linear solver. Computational time of this procedure determines the resulting time demands of the whole contact algorithm. Indeed, the simplest contact detection procedure consists in solving (9) by a non-linear solver, such as the Newton-Raphson method, in the loop over slave quadrature points and master contact patches. The ability to perform loop over contact patches, instead of contact segments, is one of the most important benefits of the isogeometric contact analysis.

It is obvious that searching for the closest point in a cycle over all quadrature points is not optimal, because if the bodies are not in contact, the exact value of the gap function is not necessary. Instead, only logical value is sufficient to indicate that no contact occurs. Therefore, the contact detection is usually divided into two phases: a global search, which consists of effectively detecting and sorting all potential candidate slave points and their corresponding candidate master segments, and a local search for closest point projections of slave points onto master segments.

The closest point projection of a point onto a free-form-surface (FFS), as spline surfaces are sometimes called, also poses a common problem to computer graphics. Therefore, these methods can be directly employed in the isogeometric contact analysis. Computational methods for the orthogonal projection of points in CAD/CAM applications were presented in a review paper [5]. In order to guarantee robustness of the calculation, it is recommended to initially employ a subdivision based global scheme, followed by a Newton-type iteration method. An interesting alternatives to the Newton-type iteration method are the geometric iteration methods [6]. In particular the torus approximation method [7] has proven to be a very powerful technique, especially for NURBS surfaces of higher order where the costly evaluation of base functions and their derivations by recurrent formulas must be carried out.

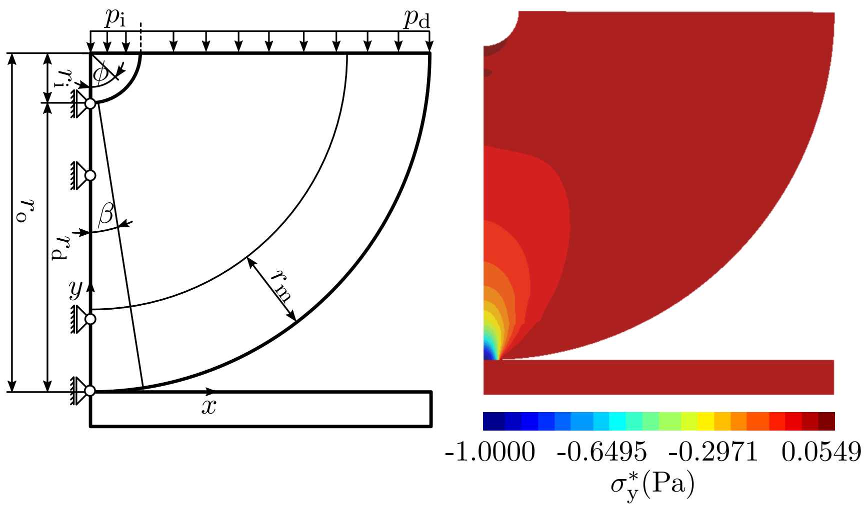
Contact pressure post-processing

Here employed post-processing scheme for the contact pressure was originally proposed by Sauer [10]. The idea is to calculate control point contact pressures p_{c_A} to get post-processed contact pressure in the form

$$p_{\mathrm{c}}^{\mathrm{p}(i)} = \sum_{A=1}^{n_{\mathrm{cp}}} R_A p_{\mathrm{c}_A}^{(i)}. \tag{10}$$

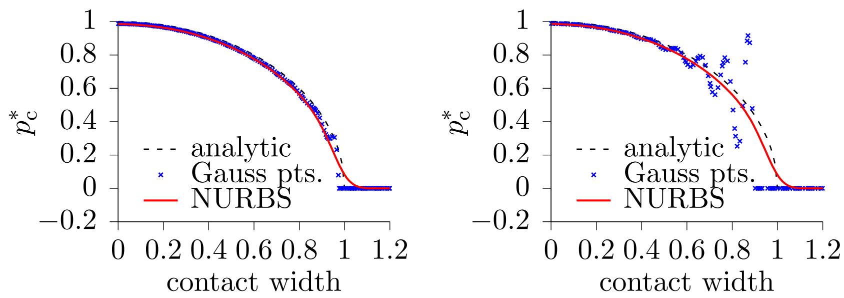
The calculation of the control point values of the contact pressure is inspirited by the mortar method [4]. In particular, control point pressures are evaluated as a weighted average

$$p_{\mathrm{c}_A}^{(i)} = \frac{\int_{\gamma_{\mathrm{c}}^{(i)}} R_A \epsilon_{\mathrm{N}} \left\langle g_{\mathrm{N}}^{(i)} \right\rangle \mathrm{d}\Gamma^{(i)}}{\int_{\Gamma_{\mathrm{c}}^{(i)}} R_A \, \mathrm{d}\gamma^{(i)}}. \tag{11}$$



(a) Parameters of the IGA mesh. (b) Contours of σ_y^* stress component.

Figure 1. Hertzian contact problem.



(a) $p = 4, \epsilon_N = 10^4$ (b) $p = 4, \epsilon_N = 10^6$

Figure 2. Contact pressure distribution.

Numerical test: Hertzian contact problem

As the numerical example, the classic Hertzian problem is investigated. In particular, the plane strain contact of cylinder with rigid plane is considered (see Fig. 1(a)). The material parameters, geometry, and boundary conditions as well as IGA mesh parametrization are taken from the work of Temizer et al. [4].

For illustration, the contours of the normalized vertical stress component $\sigma_y^* = \sigma_y / p_o$ are showed in Fig. 1(b). The detail study of contact pressure distribution in dependence on the contact area radius a is presented in Fig. 2. In this figure, the plot on the left was obtained for the penalty parameter $\epsilon_N = 10^4\,\mathrm{N/m^3}$ whereas the plot on the right was obtained for $\epsilon_N = 10^6\,\mathrm{N/m^3}$. In each the sub-plots of Fig. 2 there is analytical solution, the contact pressure at Gauss points $p_{c_g} = \epsilon_N g_{N_g}$, and a post-processed contact pressure denoted in legend as NURBS.

One can see in Fig. 2 that post-processed contact pressure shows very good agreement with the analytical solution. On the other hand, the contact pressure at Gauss points exhibits oscillations that increase with the polynomial order of the basis functions. This is the expected result because the GPTS formulation leads to the over-constrained system of equations. Note that calculation of the weighted average quantities is the bottom line of the mortar-KTS algorithm [4]. While Eq. (11) was used only once in a post-processing solution, the mortar FE algorithm uses this formula in each iteration of the non-linear solution to obtain the correct number of contact constraints.

Conclusions

In this paper, a simple large deformation frictionless contact formulation [2] was presented in the IGA framework. An essential feature of this formulation is an unbiased treatment of the contact residual, which means that there is no need to distinguish between master and slave bodies. The proposed algorithm fully exploits the features of the isogeometric analysis, namely that the local contact searching can be done in the loop over patches rather than over elements. The performance of the resulting IGA contact algorithm was tested by means of the Hertz contact problem. It was demonstrated that oscillations of the Gauss point solution can be efficiently eliminated in post-processing.

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References

[1] L. De Lorenzis, P. Wriggers, and T. J. R. Hughes. Isogeometric contact: a review. *GAMM-Mitteilungen*, 37(1):85–123, July 2014.

[2] D. Gabriel, J. Plešek, and M. Ulbin. Symmetry preserving algorithm for large displacement frictionless contact by the pre-discetization penalty method. *International Journal for Numerical Methods in Engineering*, 61(15):2615–2638, December 2004.

[3] T. J. R. Hughes, J. A. Cottrell, and Y. Bazilevs. Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement. *Computer Methods in Applied Mechanics and Engineering*, 194(39–41):4135–4195, 2005.

[4] İlker Temizer, P. Wriggers, and T. J. R. Hughes. Contact treatment in isogeometric analysis with nurbs. *Computational Methods in Applied Mechanics and Engineering*, 200:1100–1112, 2011.

[5] K. Ko and T. Sakkalis. Orthogonal projection of points in cad/cam applications: an overview. *Journal of Computational Design and Engineering*, 1(2):116–127, February 2014.

[6] J. Kopačka, D. Gabriel, J. Plešek, and M. Ulbin. Assessment of methods for computing the closest point projection, penetration, and gap functions in contact searching problems. *International Journal for Numerical Methods in Engineering*, 105(11):803–833, March 2016.

[7] X.-M. Liu, L. Yang, J.-H. Yong, H.-J. Gu, and J.-G. Sun. A torus patch approximation approach for point projection on surfaces. *Computer Aided Geometric Design*, 26(5):593–598, June 2009.

[8] D. M. Neto, M. C. Oliveira, L. F. Menezes, and J. L. Alves. A contact smoothing method for arbitrary surface meshes using nagata patches. *Computer Methods in Applied Mechanics and Engineering*, 299:283–315, February 2016.

[9] M. A. Puso and T. A. Laursen. A 3d contact smoothing method using gregory patches. *International Journal for Numerical Methods in Engineering*, 54(8):1161–1194, July 2002.

[10] R. A. Sauer. Local finite element enrichment strategies for 2d contact computations and a corresponding post-processing scheme. *Computational Mechanics*, 52:301–319, 2013.

[11] M. Tur, E. Giner, F. J. Fuenmayor, and P. Wriggers. 2d contact smooth formulation based on the mortar method. *Computational Methods in Applied Mechanics and Engineering*, 247–248:1–14, November 2012.