Title: Upper and lower sharp bounds for Neumann eigenvalues of the Hermite operator.

Abstract: Denote by $\mu_1(\Omega)$ the first nontrivial eigenvalue of the problem

$$(1) \qquad \begin{cases} -\Delta u + x \cdot \nabla u = \mu u & \text{in} \quad \Omega \\ \\ \frac{\partial u}{\partial \nu} = 0 & \text{on} \quad \partial\Omega, \end{cases}$$

where Ω is a smooth and possibly unbounded domain of $\mathbb{R}^n$ and ν is the outward normal to $\partial\Omega$. We firstly prove that among all smooth domains Ω of $\mathbb{R}^n$ symmetric about the origin, having prescribed Gaussian measure, $\mu_1(\Omega)$ is maximum if and only if Ω is the euclidean ball centered at the origin (see [1]).
We will then consider (1) when Ω is a convex and possibly unbounded domain in $\mathbb{R}^2$ having an axis of symmetry $\widetilde{r}$ passing through the origin. Denoted with $\mu_1^{\text{odd}}(\Omega)$ the lowest eigenvalue of (1) having a corresponding eigenfunction odd with respect to $\widetilde{r}$, we will provide (see [2]) a lower bound for $\mu_1^{\text{odd}}(\Omega)$ in terms of the first eigenvalue of a suitable Sturm-Liuville problem.
Finally, time permitting, we will discuss the inequality

$$\mu_1(\Omega) \geq 1,$$

where Ω is a convex domain of $\mathbb{R}^n$ (see [3] and [4]).

References

[1] F. Chiacchio - G. di Blasio, *Isoperimetric estimates for the first Neumann eigenvalue of Hermite differential equations,* Ann. Inst. H. Poincaré Anal. Non Linéaire. Volume 29, Issue 2, 199–216, (2012).

[2] B. Brandolini -F. Chiacchio - C. Trombetti, *A sharp lower bound for some Neumann eigenvalues of the Hermite operator*, Differential and Integral Equations 26 (2013), 639-654.

[3] B. Brandolini - F. Chiacchio - A. Henrot - C. Trombetti, *An optimal Poincaré-Wirtinger inequality in Gauss space*, Math. Research Letters 20 (2013), 449-457.

[4] Brandolini B. - Chiacchio F. - Krejcirik D. - Trombetti C., The equality case in a Poincaré-Wirtinger type inequality, http://arxiv.org/abs/1410.0676.