

SIXTY YEARS OF PROFESSOR FRANTIŠEK NEUMAN

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An outstanding Czech mathematician, Prof. RNDr. František Neuman, DrSc., one of the leading personalities of Brno's mathematics and a prominent specialist in the theory of linear differential equations, celebrated his sixtieth birthday on May 28, 1997.



František Neuman was born in Brno, where he also attended primary and secondary school. Already as a secondary school student he showed his mathematical talent, being twice among the winners of the Czechoslovak Mathematical Olympiad. In 1960 he graduated at the Faculty of Science of J. E. Purkyně University (now Masaryk University) in Brno and started to work at the Department of Mathematics at this faculty. In 1965 he received his Candidate of Science degree (CSc.) and one year later he was appointed Associate Professor of Mathematical Analysis. In 1974 he left university for the Brno branch of Mathematical Institute of the Academy of Sciences just established, where since 1991 he has been working as its head. After the change F. Neuman has continued in his pedagogical activities at the Faculty of

Science by reading special lectures for undergraduate students and has devoted a lot of his time to postgraduate students. In 1980 he received the title Doctor of Science (DrSc.) and in 1991 was appointed Professor of Mathematical Analysis.

The scientific activities of F. Neuman are closely connected with the qualitative theory of differential equations. This orientation was strongly influenced by Professor Borůvka who was his supervisor during postgraduate studies. At the beginning of the scientific career F. Neuman concentrated his attention on the second order linear differential equations. He proved a series of new results concerning the distribution of zero points, periodicity, asymptotic behaviour and extremal properties of solutions of these equations. Among these results let us mention the paper [26], where conditions are established which guarantee that all solutions of a given second order linear differential equation are periodic and an explicit description of periodic solutions is offered. These papers attracted considerable attention of mathematical community since they reveal a close relationship between qualitative theory of linear differential equations, affine geometry and theory of functional equations.

In late sixties F. Neuman turned his attention to the third and higher order differential equations. Using an ingenious combination of algebraical and geometrical methods with methods typical for investigation of differential and functional equations he created in next 20 years a unified theory of global properties of linear differential equations. This global theory of linear differential equations made it possible to resolve several until that time open problems and its results find applications in many related mathematical disciplines, as theory of functional and functional-differential equations. It is rather difficult to describe the basic results of Neuman's theory in a few sentences. Let us mention here at least the paper [40] where, using a category theory approach, the algebraic structure of global transformations of linear differential equations is established, and papers [53, 59] where F. Neuman investigated the problem of global canonical forms of mutually transformable differential equations and offered (up to a certain particular exception) an effective criterion of global equivalence of two linear differential equations. These papers are valuable also from the historical point of view since they reveal the "localness" of investigation of linear differential equations in the 19-th century.

The main results of Neuman's global theory of linear differential equations are summarized in the monograph [M2]. This monograph has attracted a considerable international attention and became one of the basic references in the field—see the recent monograph [E].

Recently F. Neuman has been dealing with qualitative properties of functional-differential equations. From the transformation point of view he investigates algebraic and geometrical aspects of this problem. It is also worth mentioning that in addition to his fundamental work in the theory of differential equations, F. Neuman

has achieved remarkable results in other mathematical disciplines as well. For example, the paper [6] gives a complete characterization of the trees whose square is a Hamiltonian graph. A graph with this property is now usually called Neuman's tree. In the theory of functional equations his papers [49, 50, 51, 74] represent basic results for decomposition of functions of two variables into finite sums of products of functions of single variables.

František Neuman has been for many years one of the leading mathematical personalities in Brno. He organizes the seminar on differential equations at Masaryk University and acts as a member of Editorial Boards of several international journals. He also significantly contributes to the organization of Equadiff Conferences which are periodically held in Prague, Brno and Bratislava (he was the chairman of the Scientific and Organizing committee of Equadiff 9 held in Brno in August 97). F. Neuman was supervisor of several postgraduate students who now are well known mathematicians.

Based on his scientific and teaching activities, F. Neuman has obtained many invitations to lecture at universities abroad and to plenary lectures at international conferences. He has also obtained several scientific distinctions, among them let us mention the Bolzano medal awarded to distinguished scientists by the Presidium of the Czech Academy of Sciences.

On behalf of the whole Czech mathematical community we take the opportunity to wish Professor František Neuman good health and every success in his personal life and his scientific work.

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