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Detecting phase synchronization in noisy systems

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Abstract

A quantitative method for automatic detection of phase synchronization in noisy experimental bivariate time series is proposed, based on the fact that instantaneous phases of phase-synchronized (sub)systems are mutually dependent in a specific way irrespectively of a relation between the original time series. The level of dependence between the instantaneous phases is quantified by a statistical dependence parameter, which also reflects the strength of the systems' phase synchronization. Ranges of the parameter values, for which the detection of the phase synchronization can be considered reliable, are estimated by using the technique of surrogate data. Possible applications of the proposed method are demonstrated by using both numerically generated and real experimental data, namely solutions of two coupled Rössler systems, mammalian cardio-respiratory data, and long-term recordings of surface atmospheric temperature and sunspot numbers.

1 Introduction

Synchronization of oscillatory systems is a phenomenon intensively studied in various areas of science and technology. The strongest definition of synchronization requires that the difference between states of synchronized systems asymptotically vanishes. This definition is called *identical synchronization* [1], while the notion of *generalized synchronization* requires that states of coupled systems are (asymptotically) related by some (possibly complex) function [2, 3]. In the classical case of *periodic* self-sustained oscillators, *phase* synchronization is usually defined as locking of phases $\phi_{1,2}$:

$$n\phi_1 - m\phi_2 = \text{const.}, \tag{1}$$

for integer n and m, while the amplitudes can be different. Recently, Rosenblum et al. [4] have discovered the phase synchronization in a case of coupled *chaotic* systems, where the phase entrainment (locking) is described as

$$|n\phi_1 - m\phi_2| < \text{const.}, \tag{2}$$

while the amplitudes of the two systems may be completely uncorrelated, i.e., linearly independent.

The phase of a signal $s(t)$ can be determined[1] by using the analytic signal concept of Gabor [5], introduced into the context of chaotic synchronization by Rosenblum et al. [4]. The analytic signal $\psi(t)$ is a complex function of time defined as

$$\psi(t) = s(t) + j\hat{s}(t) = A(t)e^{j\phi(t)}, \tag{3}$$

where the function $\hat{s}(t)$ is the Hilbert transform of $s(t)$

$$\hat{s}(t) = \frac{1}{\pi}\,\text{P.V.}\int_{-\infty}^{\infty}\frac{s(\tau)}{t-\tau}d\tau. \tag{4}$$

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[1] For a detailed discussion of this problem and alternative methods see Refs. [4, 6].

(P.V. means that the integral is taken in the sense of the Cauchy principal value.) The instantaneous phase $\phi(t)$ of the signal $s(t)$ is then

$$\phi(t) = \arctan \frac{\hat{s}(t)}{s(t)}. \tag{5}$$

Let us consider the same example of the phase synchronization as described in Rosenblum et al. [4] – two coupled Rössler systems [7]

$$\begin{aligned} \dot{x}_{1,2} &= -\omega_{1,2} y_{1,2} - z_{1,2} + \epsilon(x_{2,1} - x_{1,2}), \\ \dot{y}_{1,2} &= \omega_{1,2} x_{1,2} + 0.15 y_{1,2}, \\ \dot{z}_{1,2} &= 0.2 + z_{1,2}(x_{1,2} - 10). \end{aligned} \tag{6}$$

The frequencies $\omega_{1,2}$ are defined as $\omega_{1,2} = 1 \pm \Delta\omega$, where the frequency mismatch $\Delta\omega = 0.015$ is used in this article. For this system $m = n = 1$ and the instantaneous phase difference for the components $x_{1,2}$ is

$$\Delta\phi(t) = \phi_1(t) - \phi_2(t) = \arctan \frac{\hat{x}_1(t) x_2(t) - x_1(t) \hat{x}_2(t)}{x_1(t) x_2(t) + \hat{x}_1(t) \hat{x}_2(t)}. \tag{7}$$

With the coupling strength $\epsilon = 0.035$, a phase-synchronized regime occurs, in which the phase difference $\Delta\phi$ is confined inside a limited interval around 1 (approximately (0.6, 1.4), Fig. 1a). For $\epsilon = 0.001$ the subsystems are not synchronized and the phase difference $\Delta\phi$ varies in the whole range[2] $(-\pi, \pi)$ allowed by Eq. (7). (Fig. 1b.)

By computing and plotting the phase difference $\Delta\phi = m\phi_1 - n\phi_2$, the phase synchronization was observed in chaotic systems modelled on digital [4] or analog [1] computers,[3] as well as in bivariate experimental data such as those registered from a mammalian cardio-respiratory system [8]. In general, however, the evaluation of the instantaneous phase difference $\Delta\phi(t)$ alone can be insufficient for answering the question about existence of the phase synchronization between (sub)systems under study.

Consider, for instance, that the time series generated in the phase-synchronous regime of the above coupled Rössler systems ($\epsilon = 0.035$) are mixed with additive "measurement" Gaussian white noise.[4] The condition (2) is broken since the phase differences attain any value from the full range $(-\pi, \pi)$ (Fig. 1c). In Fig. 1d we illustrate the phase differences between two independent linear stochastic processes, asynchronously oscillating with the same frequencies as the phase-synchronized Rössler series.

In an experimental situation it could be hard if not impossible, even for an expert, to decide from plots of $\Delta\phi$, whether analyzed systems are phase-synchronized but noisy, or no synchronization is present. And in many practical analyses an expert's eye cannot participate, e.g., when a large amount of data is being processed automatically and segments of phase-synchronous regimes should be detected.

In this Letter we propose a simple quantitative method for detecting the phase synchronization in experimental bivariate time series.

2 The method

Let us rewrite the condition (2) for the phase entrainment as

$$n\phi_1 - m\phi_2 \approx C. \tag{8}$$

[2] Here we do not use the definition of monotonically increasing phase/phase difference like Rosenblum et al. [4] (cf. Fig. 1 in [4]), since the latter is not suitable for the method presented below. The range of the standard function arctan(X) (e.g., in FORTRAN implemented as ATAN(X)) is $(-\pi/2, \pi/2)$. If the argument X can be written as $X = X_1/X_2$, such as in the case presented here, the FORTRAN function ATAN2(X_1, X_2) can be used, the range of which covers the whole circle $(-\pi, \pi)$. The latter representation of the phases as functions in $(-\pi, \pi)$ is used below. The results of the statistical procedure presented below, however, do not depend on the particular implementation used, i.e., one can work in the range $(-\pi/2, \pi/2)$ using ATAN, or in the range $(-\pi, \pi)$ using ATAN2.

[3] The authors of [1] investigated oscillators with unidirectional coupling, while in [4] and here (Eq. 6) bidirectionally coupled systems were studied.

[4] I.e., the noise has been added to the already generated data. A dynamical noise affecting evolution of systems is not considered in this case. The term "30% of noise" means that the standard deviation (SD) of the noise is equal to 30% of SD of the original noise-free data.

Then, one of the instantaneous phases, say ϕ_2, can be written as a linear function of the other instantaneous phase ϕ_1

$$\phi_2 \approx A + B\phi_1, \tag{9}$$

for $A = -C/m$ and $B = n/m$, i.e., if the two (sub)systems are phase-synchronized, their instantaneous phases are approximately related by the simple linear function (9), although their amplitudes can be linearly independent [4].

Formally, a relation such as (9) holds also in cases when no phase synchronization is present. Then, however, the intercept A is not constant, but contains a time-dependent term of the type αt. (E.g., $\alpha = 2\Delta\omega$ in the case of the above coupled Rössler systems (6).) Using the representation of the phases as monotonically increasing functions (as used in [4]) a graph of ϕ_2 plotted against ϕ_1 will always be a straight line with the slope $B + \alpha$. Distinction of phase synchronized regimes ($\alpha = 0$) from asynchronous states ($\alpha \neq 0$) would require knowledge about particular value of $B = n/m$ and can be statistically unreliable, especially for small α. Moreover, the representation of the phases as monotonically increasing functions requires an addition of 2π to the phase value obtained from Eq. (5) after each cycle, due to the fact that Eq. (5) always gives the phase values in the range $-\pi, \pi$.[5] This operation can be very problematic when processing experimental data with noise and/or fluctuating periods.

On the other hand, using directly the phase values obtained from Eq. (5), the proposed method does not require any additional information and/or data preprocessing and, as we will see below, provides a very clear and statistically robust distinction between phase synchronous and asynchronous states. The temporal evolution of instantaneous phases in this representation remains a sawtooth shape – the phase linearly increases from $-\pi$ to π, then jumps down to $-\pi$ and continues to increase to π, jumps down to $-\pi$, etc.

Let us study the phase relation of the coupled Rössler systems (6) in the phase synchronized state ($\epsilon = 0.035$) by plotting ϕ_2 against ϕ_1 (Fig. 2a). Inside a particular cycle, the relation $\phi_2 \approx A + B\phi_1$ holds, with $A \approx 1$ and $B \approx 1$. When the phase of the phase-advanced system starts a new cycle and jumps to $-\pi$, the intercept A changes to $A \approx 1 - 2\pi$. Then the phase of the phase-delayed system jumps to $-\pi$ and the intercept A returns to $A \approx 1$. As a result of this behaviour, the phase pairs (ϕ_1, ϕ_2) are confined inside two strips $\phi_2 \approx A + B\phi_1$ with the same slope B but with two intercepts, which differ by 2π.

In asynchronous states such as that for $\epsilon = 0.01$ (Fig. 2b), the phase relation $\phi_2 \approx A + B\phi_1 + \alpha t$ holds. Between the phase jumps, the pairs (ϕ_1, ϕ_2) draw lines increasing with the slope $B + \alpha$. After each phase jump, however, a new line starts with an "effective" instantaneous intercept $A + \alpha t$ (or $A + \alpha t - 2\pi$, with its absolute value taken to mod π). After a sufficient number of cycles, the phase pairs (ϕ_1, ϕ_2) fill almost homogeneously the whole planar interval $(-\pi, \pi) \times (-\pi, \pi)$ (Fig. 2b) and thus effectively make the phases ϕ_1 and ϕ_2 *statistically independent.*

In Figure 2c the phase pairs (ϕ_1, ϕ_2) are plotted, obtained from the noisy phase-synchronous Rössler series ($\epsilon = 0.035$, 30% of noise), the phase differences of which were plotted in Fig. 1c. Figure 2d presents the phase pairs (ϕ_1, ϕ_2) for the asynchronous linear stochastic oscillators, which phase differences were drawn in Fig. 1d. The distinction of synchronous (a,c) and asynchronous (b,d) states in Fig. 2 is much clearer than in Fig. 1. Moreover, the level of dependence between the phases ϕ_1, ϕ_2 can be quantitatively measured by a statistical or information-theoretic functional of probability distributions, which can be used for objective distinction between synchronous and asynchronous states even in more complicated[6] and/or more noisy situations than those presented above, and in automatic detection of phase synchronization.

Let $p_1(\phi_1)$ and $p_2(\phi_2)$ be probability distributions of the phases ϕ_1 and ϕ_2, respectively, and $p_{1,2}(\phi_1, \phi_2)$

[5] Providing we use the FORTRAN function ATAN2. When the standard function ATAN is used, an addition of π is necessary after each half-cycle.

[6] In the cases when $n \neq m > 1$, the phase pairs can be confined into more then two (and less or equal to $m + n$) strips in the planar interval $(-\pi, \pi) \times (-\pi, \pi)$.

be their joint distribution. The mutual information[7]

$$I(\phi_1, \phi_2) = \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} p_{1,2}(\phi_1, \phi_2) \log \frac{p_{1,2}(\phi_1, \phi_2)}{p_1(\phi_1)p_2(\phi_2)} d\phi_1 d\phi_2 \tag{10}$$

is one of suitable statistics for testing the dependence between the phases $\phi_1(t)$, $\phi_2(t)$. Theoretically, independence of the phases, i.e., the absence of the phase synchronization means $I(\phi_1, \phi_2) = 0$; while for the phase synchronization, i.e., a mutual dependence of the phases, $I(\phi_1, \phi_2) > 0$ holds.

Processing experimental data, when the existence of the phase synchronization is investigated, it is questionable whether any positive value of $I(\phi_1, \phi_2)$ can be considered as evidence for the phase synchronization. In order to prevent "spurious" detections of the phase synchronization, we propose to use the technique of surrogate data [10, 11] for establishing the range of $I(\phi_1, \phi_2)$ values, which can be obtained from bivariate processes having some properties similar to the studied data, however, not possessing the searched phase synchronization. A number of realizations of the surrogate data are numerically generated according to a given prescription (a null hypothesis) and their $I(\phi_1, \phi_2)$ are evaluated. Then the mutual information $I(\phi_1, \phi_2)$ obtained from the studied data is compared with a set of $I(\phi_1, \phi_2)$ values obtained from the surrogates, and, if there is a significant difference, the null hypothesis is rejected. One can use several types of surrogate data according to properties and quality of the data under study.

1. IID1 surrogates are realizations of mutually independent IID (independent identically distributed) stochastic processes (white noises) with the same mean, variance and histogram as the series under study. The IID1 surrogates are constructed by "scrambling" the original series, i.e., the elements of the original series are randomly permutated in temporal order, in each realization different random permutations are used for the two components of the bivariate series. This randomization destroys any temporal structure, if present in the original series. The IID1 surrogates present the null hypothesis of independent white noises, i.e., nor synchronization neither oscillations are considered.

2. IID2 surrogates are realizations of IID stochastic processes (white noises), which count for possible cross-dependence between the two components of the bivariate series. In each realization, the same random permutation is used for both components of the bivariate series.[8] The IID2 surrogates present the null hypothesis of mutually dependent white noises, i.e., the two series are "synchronized" in a sense of mutual dependence given, e.g., by crosscorrelations (for instance, spatial correlations of series sampled in different locations from the same spatially extended process), however, the specific phenomenon of the phase synchronization of oscillatory processes, as well as other temporal structures are absent.

The IID surrogates are suitable in the case of very noisy data, when a narrow band-pass filtering is applied before detecting the phase synchronization. It is important to generate the IID surrogates before the filtering[9] and then to evaluate $I(\phi_1, \phi_2)$ from both filtered data and filtered IID surrogates.

3. FT1 surrogates are independently generated for each of the two components of the studied bivariate series as realizations of linear stochastic processes with the same sample power spectra as the series under study. The FT1 surrogates are obtained by computing the Fourier transform (FT) of the series, keeping unchanged the magnitudes of the Fourier coefficients (the spectrum), but the phases of the Fourier coefficients are randomized and the inverse FT into the time domain is performed [10, 11]. The FT1 surrogates realize the null hypothesis of two linear stochastic processes which asynchronously oscillate with the same frequencies (power spectra) as the original series under study.

4. FT2 surrogates are realizations of a bivariate linear stochastic process which mimics individual spectra of the two components of the original bivariate series as well as their cross-spectrum. Constructing the FT2

[7] A simple box-counting method for estimating the mutual information is described in [11, 14], correlation integrals are used in [15].

[8] Consider a "toy" example – two three-sample series $\{a, b, c\}$ and $\{A, B, C\}$. A realization of the IID1 surrogates can be, e.g., $\{b, a, c\}$ and $\{C, B, A\}$, while a realization of the IID2 surrogates should look like, e.g., $\{b, a, c\}$ and $\{B, A, C\}$.

[9] Especially, before the filtering which enhances or "uncovers" the studied oscillations. Some kinds of preprocessing, such as removing slow phenomena or trends (nonstationarities) could be, and in some cases, should be done before generating the surrogates.

surrogates not only the spectra but also differences between phases of the Fourier coefficients of the two series for particular frequency bins must be kept unchanged. Thus the phase randomization is performed by adding the same random number to the phases of both coefficients of the same frequency bin. (For more details see [12].) The FT2 surrogates preserve (a part of) synchronization, if present in the original data, which can be explained by a bivariate linear stochastic process. This "linear stochastic phase synchronization" is usually weaker than the phase synchronization in the original data from nonlinear (chaotic) systems.

3 A numerical example

Let us return to the two coupled Rössler systems (6) with the frequency mismatch $\Delta\omega = 0.015$ and evaluate the mutual information $I(\phi_1, \phi_2)$ for the coupling strength ϵ increasing from $\epsilon = 0$ to $\epsilon = 0.1$ (Fig. 3). In the asynchronous regime $(0 \leq \epsilon < 0.03)$ $I(\phi_1, \phi_2)$ is close to zero. For $\epsilon > 0.03$ the phase synchronization emerges and $I(\phi_1, \phi_2)$ rises to the values[10] between 1.4 and 1.9 in the case of the noise-free data (Figs. 3a,b, thick solid line), and about 0.8 in the case of the data with 30% of additive "measurement" noise (Fig. 3c,d, thick solid line). The FT1 surrogates (Fig. 3a,c, thin solid line and thin dashed lines illustrate mean and mean $\pm$ SD, respectively, of a set of fifteen realizations of the surrogates for each value of ϵ) give $I(\phi_1, \phi_2)$ mostly coinciding with $I(\phi_1, \phi_2)$ from the Rössler data in the asynchronous regime (i.e. $I(\phi_1, \phi_2) \approx 0$), while in the phase-synchronous regime the mutual information $I(\phi_1, \phi_2)$ from the FT1 surrogates does not reach over 0.7, i.e., $I(\phi_1, \phi_2)$ of the phase-synchronized Rössler series is clearly different from $I(\phi_1, \phi_2)$ of the FT1 surrogates. Thus the phase-synchronized Rössler series are discernible, with a very high level of statistical significance, from the asynchronous isospectral oscillations, even in the noisy case (Fig. 3c).

In the phase-synchronous regime, the mutual information $I(\phi_1, \phi_2)$ obtained from the FT2 surrogates reaches higher values than that of the FT1 surrogates, however, it is still clearly smaller than $I(\phi_1, \phi_2)$ of the phase-synchronized Rössler series (Fig. 3b,d). The bivariate linear stochastic process can only partially reproduce the phase synchronization of the chaotic oscillators, and the two phenomena are still discernible with a high statistical significance.

Let us assign the values of the mutual information $I(\phi_1, \phi_2)$ for the examples of the phase differences in Fig. 1 and the phase plots in Fig. 2: The mutual information for the phase-synchronous regime in Fig. 1a/2a can be found in Fig. 3a for $\epsilon = 0.035$ as $I(\phi_1, \phi_2) \approx 1.5$, for the asynchronous regime (Fig. 1b/2b) $I(\phi_1, \phi_2) \approx 0$ (Fig. 3a, $\epsilon = 0.01$), for the synchronous ($\epsilon = 0.035$) series with noise (Fig. 1c/2c) $I(\phi_1, \phi_2) \approx 0.8$ (Fig. 3c), and the mutual information of asynchronous linear stochastic oscillations (Fig. 1d/2d) can be found in the range 0 – 0.7 (Fig. 3a). (The latter were generated as a realization of the FT1 surrogates of the noise-free phase-synchronized Rössler series. Note that $I(\phi_1, \phi_2)$ obtained from the FT1 surrogates of the noisy phase-synchronized Rössler series does not reach over 0.4.)

4 Two examples of experimental data

Synchronization between respiratory movements (breathing) and heart rate (instantaneous heartbeat frequency) is a phenomenon intensively studied by physiologists (see [8] and references therein). Here we demonstrate a possible application of the proposed methodology in detecting segments of phase-synchronous regimes in long-term recordings from the cardio-respiratory system of a sleeping piglet. The time series of respiratory movements (RM) and heart rate (HR) are described in detail and analyzed by different methods in [8]. In the example of the analysis presented here (Fig. 4), we do not estimate the mutual information between the phases of the whole record of RM and HR, but inside a 400-sample window, which is moved along the series, in order to detect short segments of a possible phase synchronization. The surrogates were constructed from the whole record and processed by using the same "moving mutual information" approach. Therefore we do not average $I(\phi_1, \phi_2)$ obtained from the surrogates, but we plot the results from all realizations of the surrogates[11] in order to compare local maxima of $I(\phi_1, \phi_2)$ obtained from the data and from the surrogates.

[10] The mutual information is in *nats*, since the natural logarithm $\log_e$ was used in (10). These values can be converted into *bits* by multiplying them by $\log_e 2$.

[11] For better readability only $I(\phi_1, \phi_2)$ from five surrogate realizations are plotted in Fig. 4 and 5 using various dashed and dotted lines.

In the presented example (Fig. 4), the mutual information $I(\phi_1, \phi_2)$ provide a strong statistical evidence for the presence of the phase synchronization in most parts of the studied series. These results are physiologically meaningful – the phase synchronous regime corresponds to the intervals of the NON-REM sleep (see the sleep stages classification in the top panel of Fig. 4), while the asynchronous (or weakly synchronous) segments correspond to the intervals of the REM sleep or of the undetermined sleep stages. (See [8] for detailed discussion.) A visual inspection of the phase difference plot, obtained from the data (Fig. 4, the second panel from the top), cannot provide such a clear distinction between the states, although some differences from the phase difference plot, obtained from a realization of the surrogates (Fig. 4, the third panel from the top) are visible.

In the bottom panel of Fig. 4, we can see that in the synchronous segments even the FT2 surrogates are rejected with a high statistical significance. This result agrees with the positive identification of nonlinearity in the cardio-respiratory synchronization [8], obtained using the method for detection of nonlinearity in multivariate time series [13].

There was no need to apply the IID surrogates in the above cases of the Rössler data and the cardio-respiratory data with oscillations clearly visible without any preprocessing, unlike in the following example from a study of low-frequency components of atmospheric dynamics.[12] After an appropriate band-pass filtering of surface atmospheric temperature series of length of almost 200 years, recorded at several European locations, oscillatory phenomena with a period close to a decade can be observed. Are these cycles related to the sunspot cycles [16], which have a similar (11-year) periodicity ?

Monthly sampled differences from the long-term monthly average temperatures were used. The subtraction of the long-term monthly averages removed the yearly periodicity (seasonality). Then these temperature data, as well as monthly sunspot numbers and related surrogate data were band-pass filtered using a phase-neutral moving-average (MA) filter. First, moving averages from a 137-point window were subtracted from the data in order to remove slower phenomena. Finally, a 35-point window MA was used to obtain the filtered series, examples of which are illustrated in Fig. 5: Reading from the top to the bottom, there are band-pass filtered sunspot numbers, surface atmospheric temperatures recorded at the Prague–Klementinum station, and examples of band-pass filtered realizations of the FT1 and IID1 surrogates, obtained from the temperature series.

The same approach of the moving window for the detection of the segments of a possible phase synchronization as above has been applied, i.e., the surrogates were constructed from the whole records (actually from a subset of 2048 monthly samples), then both the data and the surrogates were band-pass filtered and the mutual information $I(\phi_1, \phi_2)$ was estimated inside a moving 240-sample window. The IID1 null[13] has been reliably rejected in one segment only (1940 – 1960, Fig. 5, the second panel from the bottom), a similar result has been obtained using the IID2 surrogates (not presented). The FT1 surrogates, however, produce local $I(\phi_1, \phi_2)$ maxima comparable with the maximum $I(\phi_1, \phi_2)$ obtained from the data, i.e., the asynchronous FT1 null hypothesis has not been rejected (Fig. 5, the bottom panel). Therefore we conclude that using this approach *no* evidence for phase synchronization between the sunspot cycles and the studied surface atmospheric temperature series has been found. (Similar results have been obtained using temperature records from several other European locations.)

5 Conclusion

A quantitative method for detection of phase synchronization in experimental bivariate time series has been proposed. The method is based on the fact that the instantaneous phases of phase-synchronized systems are mutually dependent, while the instantaneous phases of phase-asynchronous systems are effectively statistically independent. The level of a dependence between the instantaneous phases is quantified by mutual information, statistically measuring a general mutual dependence between the phases. This measure of the phase dependence also reflects the strength of the systems' phase synchronization. In order to avoid spurious

[12] A detailed report on that study will be published elsewhere. Here we present only a small part of preliminary results in order to demonstrate an application of the proposed method.

[13] Not-rejecting the IID1 null hypothesis in this test, however, is not equivalent to a proof of the IID property and to the rejection of the existence of the cycles. The existence of the near-decadal cycles in the processed surface atmospheric temperature records can be proven by comparing periodograms of the temperature series and related IID surrogates.

detections of the phase synchronization, the value of the mutual information $I(\phi_1, \phi_2)$ obtained from the data under study is compared with a set of $I(\phi_1, \phi_2)$ values obtained from realizations of surrogate data, representing appropriate null hypotheses. The IID1 surrogates present the null hypothesis of independent white noises, i.e., neither synchronization, nor oscillations are considered; the IID2 surrogates allow for a mutual dependence (cross-correlated white noises) but do not contain oscillations. The FT1 surrogates present the null hypothesis of asynchronous oscillatory processes with the same spectra as the studied data. A rejection of all IID1, IID2 and FT1 null hypotheses can be considered as the evidence for a phase synchronization in the studied data/systems. The FT2 surrogates can possess some level of the phase synchronization which is consistent with a bivariate linear stochastic process. A rejection of the FT2 null hypothesis means the detection of phase-synchronized deterministic nonlinear[14] systems.

The proposed method, possible applications of which were demonstrated using both numerically generated and real experimental data, is suitable for automatic detection of (segments of) phase-synchronized regimes in noisy experimental bivariate time series.

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[14] Note that this test is *not* generally equivalent to tests for nonlinearity in multivariate time series [12, 13].

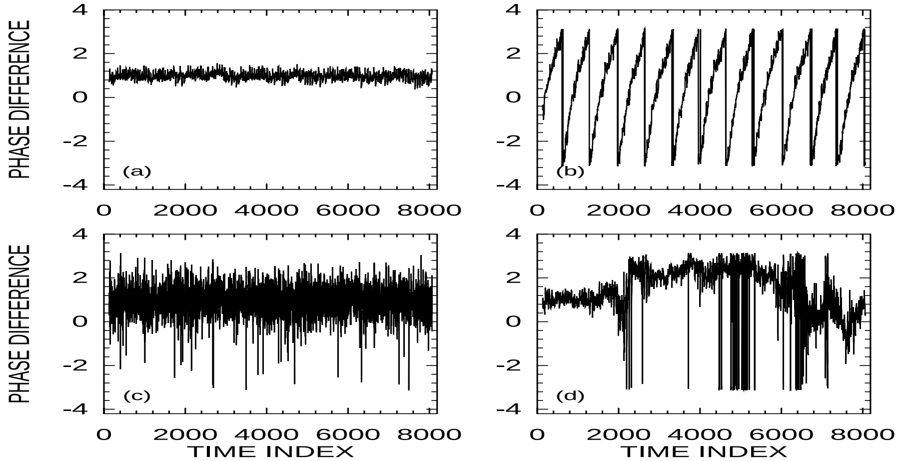

Figure 1: *Instantaneous phase differences for (a) the phase-synchronized Rössler systems ($\epsilon = 0.035$), (b) asynchronous Rössler systems ($\epsilon = 0.01$), (c) the series from the synchronized Rössler systems ($\epsilon = 0.035$) contaminated with 30% of noise, (d) asynchronous linear stochastic processes isospectral to the Rössler series (FT1 surrogates).*

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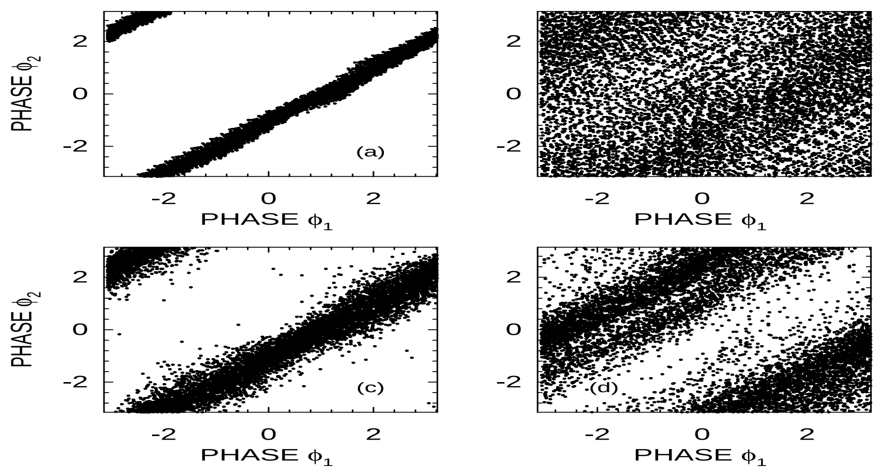

Figure 2: *Plots of the instantaneous phase ϕ_2 against the instantaneous phase ϕ_1 for (a) the phase-synchronized Rössler systems ($\epsilon = 0.035$), (b – upper right plot) asynchronous Rössler systems ($\epsilon = 0.01$), (c – lower left plot) the series from the synchronized Rössler systems ($\epsilon = 0.035$) contaminated with 30% of noise, (d – lower right plot) asynchronous linear stochastic processes isospectral to the Rössler series (FT1 surrogates).*

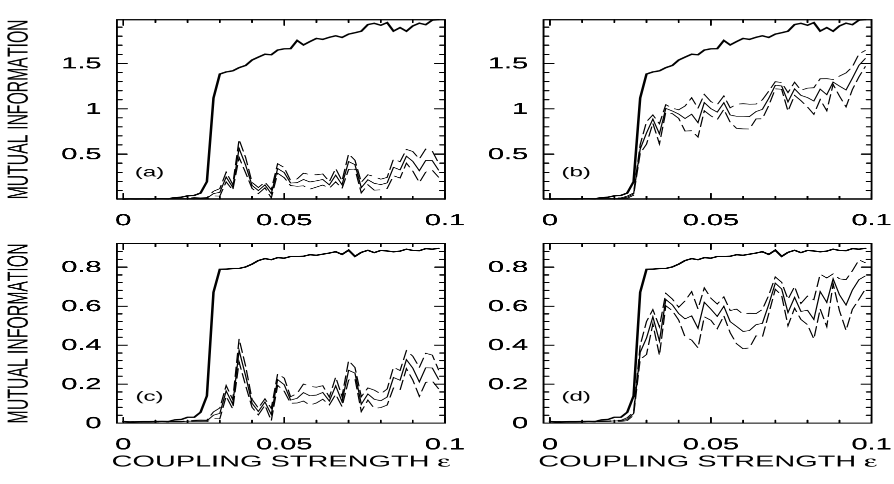

Figure 3: *Mutual information $I(\phi_1, \phi_2)$ as the measure of the mutual dependence between the phases ϕ_1, ϕ_2, plotted as the function of the coupling strength ϵ, for the data from the coupled Rössler systems (thick solid lines) and surrogates (thin solid lines and thin dashed lines illustrate mean and mean ± SD, respectively, of a set of fifteen realization of the surrogates), (a,c) the asynchronous FT1 surrogates, (b,d) the bivariate FT2 surrogates; for (a,b) the numerically generated noise-free data, and (c,d) the data contaminated with 30% of Gaussian noise.*

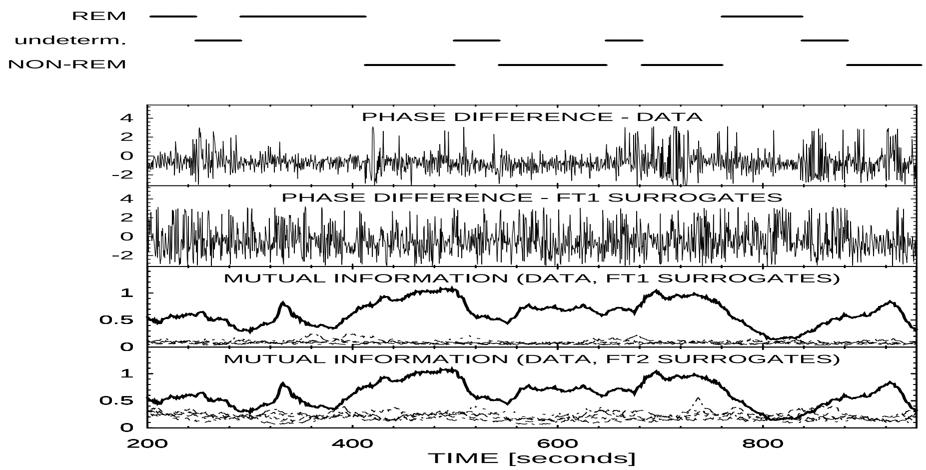

Figure 4: *From top to bottom: Classification of intervals of sleep stages (REM, undetermined, NON-REM sleep); plot of phase differences between respiratory movements (RM, 8 Hz sampling) and heart rate (HR, resampled to 8 Hz) of a sleeping piglet (both series high-pass filtered by subtracting a 45-sample moving average); plot of phase differences obtained from a realization of FT1 surrogates (phase differences obtained from FT2 surrogates look very similar); mutual information* $I(\phi_1, \phi_2)$ *as the measure of the mutual dependence between the phases* ϕ_1, ϕ_2 *obtained from a moving 400-sample window from the phases of the RM and HR series (thick solid line) and from five realizations of the asynchronous FT1 surrogates (thin dashed and dotted lines); mutual information* $I(\phi_1, \phi_2)$ *as the measure of the mutual dependence between the phases* ϕ_1, ϕ_2*, obtained from a moving 400-sample window from the phases of the RM and HR series (thick solid line) and from five realizations of the bivariate FT2 surrogates (thin dashed and dotted lines).*

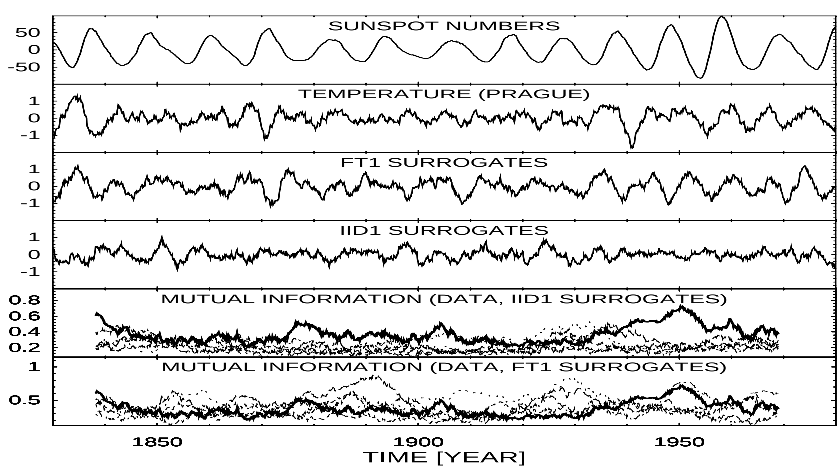

Figure 5: *From top to bottom: Monthly sampled time series of sunspot numbers; differences from long-term monthly means of surface atmospheric temperature, Prague–Klementinum station; FT1; and IID1 surrogate realizations constructed from the temperature data. All the series were band-pass filtered. The second panel from the bottom: The mutual information* $I(\phi_1, \phi_2)$ *as the measure of the mutual dependence between the phases* ϕ_1, ϕ_2 *obtained from a moving 240-sample window from the phases of the band-pass filtered sunspot and temperature series (thick solid line) and from five realizations of the band-pass filtered IID1 surrogates (thin dashed and dotted lines). The bottom panel: The same as in the previous panel, but the asynchronous isospectral FT1 surrogates were used.*