Paluš and Novotná Reply

Timmer in his Comment [1] on our Letter [2] emphasizes the well-known fact that the surrogate data methods for testing nonlinearity (see [3] and refs. within) suffer from some limitations. Technical problems in surrogate data can be avoided by taking necessary precautions [4,5] or by using more sophisticated (and computationally costly) surrogate construction methods [3]. On the other hand, we observe [6], in accordance with Timmer [7,8], that a formal rejection of the null hypothesis of a (transformed) linear, Gaussian, stationary stochastic process (LGSSP thereafter) does not automatically imply a nonlinear deterministic system underlying studied data. Therefore the test of significance of amplitude-frequency correlation (AFC thereafter) was our attempt to identify in the sunspot data a feature typical for nonlinear oscillators, in addition to standard nonlinearity tests which brought positive results [4,6], however, which suffer from limited physical interpretability [2,6]. Our null hypothesis was not a (transformed) LGSSP, but independence of instantaneous amplitude and frequency. Realizing the surrogates using transformations of LGSSP we attempted to constrain some properties of the surrogates according to the sunspot cycle [2]. The statistical evidence by rejecting the null hypothesis was obtained for the AFC, a driven nonlinear oscillator has been proposed as its physical explanation. Such a process can naturally be considered as a projection from a spatio-temporal field described by nonlinear partial differential equations, i.e., our result does not contradict to magnetohydrodynamic dynamo models.

Considering possible concurrent explanations by stochastic processes exhibiting AFC we are aware of exponential autoregressive models [9]. Such nonlinear time series models are, however, closely related to nonlinear stochastic dynamical systems [10], i.e., physically we have again the randomly driven nonlinear oscillators. The Timmer's "possible alternatives" [7,8] (refs. [3,4] in [1]) present processes able to violate the (transformed) LGSSP null hypothesis in general and do not deal specifically with the AFC. The second order linear stochastic (AR2) process with time-variable frequency (Eqs. (6, 8, 11, 12) in [7], without the variance adjustment (Eqs. 13, 14) in [7]) (Fig. 1a) possesses a complex, analytically given (Eqs. 7, 9 in [7]) relation between its variance and theoretical frequency (Fig. 1b), which however, is not necessarily reflected in a systematic AFC (Fig. 1c, note that the actual instantaneous frequency differs from the theoretical one due to natural frequency fluctuations). Thus the most probable AFC values in such processes are close to zero (Fig. 1d) and the AFC values obtained from the sunspot numbers lie close to the 90th percentile of the related absolute AFC distribution estimated for this AR2 process (Fig. 1e). Moreover, such a process can be easily rejected by using, e.g., a statistic for temporal asymmetry [11].

Although it might be possible to construct a special stochastic process possessing a systematic AFC, its statistical and physical plausibility should be carefully assessed before it can be considered as a concurrent alternative to the driven nonlinear oscillator proposed in our Letter [2] as a mechanism underlying the sunspot cycle.

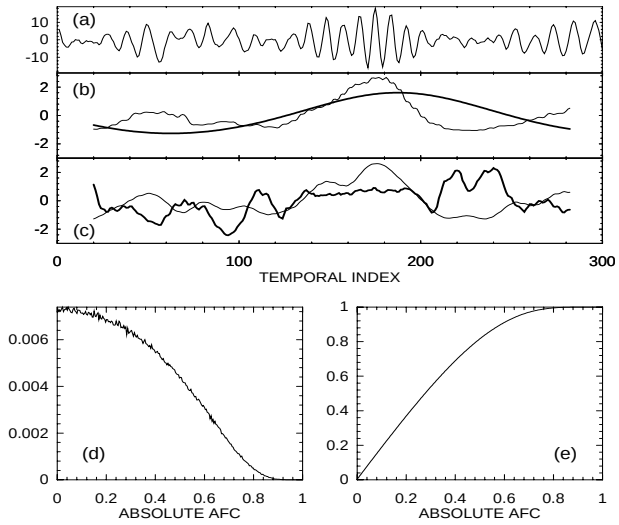

FIG. 1. (a) A realization of the AR2 process with the periodically modulated frequency (b, thick line), 30-sample moving variance (b, thin line) of this process, and (c): its actual (estimated) instantaneous frequency (thick line) and amplitude (thin line). The series in (b) and (c) are rescaled into a zero mean and a unit variance. A histogram (d) and a cumulative histogram (e) of the absolute amplitude-frequency correlation of this AR2 process obtained from its 1.5 million realizations.

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