Labor-Managed vs Profit-Maximizing Monopsony in the Labor Market

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The first email address is much more reliable than the others; still, please use **all three** addresses, and send **three separate identical** messages, to confirm the arrival of my packet.

I should also be most grateful to you if you send this email confirmation as soon as possible, say by Monday, June 23rd - for the right reasons, the grantors have approved the 6 months, no cost, extension of my grant, which expires by the end of June.

I will of course notify you as soon as I re-establish a full control of my email addresses, but I cannot commit myself to finish this job before that critical June, 23rd.

ACKNOWLEDGMENTS. This research was supported by a grant from CERGE-EI Foundation under a program of the Global Development Network. Additional funds for grantees in the Balkan countries have been provided by the Austrian Government through WIIW, Vienna. All opinions expressed are those of the author and have not been endorsed by CERGE-EI, WIIW, or the GDN. I thank Milan Drazic and Pavle Petrovic for helpful comments and suggestions. The usual disclaimer applies.

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ABSTRACT

This paper compares the efficiency of a labor-managed and a profit-maximizing firm (LMF and PMF) at the monopsonistic labor market.

We demonstrate that, both locally and globally, a LMF can (efficiency) dominate a PMF, where the local and global dominance are respectively linked with a single inverse labor supply or wage function and a single family of such functions.

While the global dominance concept is novel, and has no counterpart in the literature, the result on the local dominance is, to a certain extent, complementary to the well-known possibility result on a LMF's welfare superiority to a PMF in monopolistic competition (Neary 1985, 1992). The issue of how to privatize non-wage-taking firms is addressed.

JEL Classification Numbers: J42, J54, L20

Keywords. Labor Market; Labor-Managed vs Profit-Maximizing Monopsony; Efficiency Dominance.

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I. Introduction

The labor-controlled or, more familiar (though less precise), labor-managed (LM) firms are normally linked with various organizational and property rights structures, provided that control rights are vested in a firm's labor.[1]

Initially, labor-managed firms (LMFs) used to be identified with the Western type producer cooperatives and partnerships in the service sector (Meade, 1972; Bonin (1984); Dreze, 1989), collective farms of the Soviet Union (Domar, 1966), and almost all industrial firms of the 'self-managed' era in the SFR Yugoslavia (Ward, 1958; Vanek, 1970; Estrin, 1983).

Nowadays, LMFs may also be linked with many employee-controlled firms that have emerged during the transition process in Russia, Ukraine, Latvia, Georgia, Belarus and Slovenia and, to a smaller extent, in Poland, Estonia, Hungary, Romania and Bulgaria (Lissovolik, 1997; Uvalic, 1997; Uvalic and Vaughan-Whitehead, 1997; Jones *et al.*, 1998; Earle and Estrin, 1996).

Finally, some forms of partial labor control and/or risk bearing, like codetermination, internal bargaining or wage cuts in the ailing firms[2], are becoming a fact in the not negligible number of (previously) conventional proprietorships of industrialized economies.

Starting with Ward (1958), it is most often assumed that one of the basic features of LMFs is their maximand of income per labor unit or of a 'full' wage (see, for example, Bonin and Putterman, 1987; Estrin, 1983; Ireland and Law, 1982; Jossa and Cuomo, 1997), which clearly distinguishes such enterprises from conventional, profit-maximizing firms (PMFs) [3].

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[1] The excellent, comprehensive survey of the vast literature on LMFs is given in Bonin and Putterman (1987). See also the review monograph by Bartlett and Uvalic (1986) and a more recent book by Jossa and Cuomo (1997). For a concise review, focusing on a certain gap between the theory and evidence on LMFs, see Bonin, Jones and Putterman (1993). A fully-fledged textbook on the LM firms is Ireland and Law (1982).

[2] The first allocation suboptimal model of a codetermined firm has been constructed by Svejnar (1982). For some extensions of the model, performed within the internal-bargaining framework, see Miyazaki (1986).

[3] In this connection it is worth mentioning that the so far most systematic empirical study of the LMF behavior (Pencavel and Craig, 1994) has at least not rejected the wage-maximizing hypothesis; but see also Bonin, Jones and Putterman (1993) and Craig and Pencavel (1993). Some more recent examples of adopting the wage-maximizing assumption are Baniak (2000), Futagami and Okamura (1996), and Neary and Ulph (1997).

As regards the comparison of LMFs' and PMFs' performance under various conditions, it seems to be the dominant view that the wage-maximizing behavior by labor-managed firms is, at least on average, inferior to traditional profit-maximization.

Still, a significant number of the results have been obtained - mostly, if not exclusively, within the price taking environments - where the LMF behavior and its effects are, fully or partially, equalized with those of conventional profit-maximizing firms.[4]

However, few cases have also been detected (Neary, 1985; Neary, 1992; Neary and Ulph, 1997)[5] which show that - under some forms of the output market imperfections - LMFs can, in one way or another, be superior to PMFs.

The aim of this paper is to point to an additional example of a LMF's possible superiority to a PMF, which refers to an important case of the input market imperfection, that of the monopsony in the labor market or, more broadly, to the non-wage-taking firms, which face an upward sloping wage curve[6].

Thus, in a sense, the present paper is due to Domar's (1966) model of, effectively, a labor-managed (LM) monopsony in the labor market. However, while Domar was interested in the comparative statics of such an enterprise, our sole concern is its efficiency, as compared with that of a corresponding profit-maximizing (PM) monopsonistic firm.

The structure of the paper is as follows.

In part II we first define, in section 1, a typical family of increasing and convex inverse labor supply or wage functions - obtained by systematically varying the degree of labor scarcity - which enable both a LMF and a PMF to earn nonnegative profit.

To motivate the reader not interested in the labor-management per se, we then introduce one numerically generated graphic of this family to focus, in section 2 of part II, on the (always existing) family-member wage function that yields exactly the Pareto optimal equilibrium of a no loss making LMF. Of course, this surely means that in the considered case a LM monopsony Pareto dominates a PM monopsony, since the latter, as is well-understood, can never reach the Paretian norm.

In part III we represent the well-known PM monopsony equilibrium in the form appropriate for straightforward efficiency comparisons with the corresponding LM

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[4] See, for instance, Bonin (1981;1984), Domar (1966), Dreze (1989), Estrin (1982), Greenberg (1979), McCain (1977), Miyazaki and Neary (1983).

[5] See also Futagami and Okamura (1996).

[6] A valuable initial source for assessing the relevance of the non-wage-taking phenomenon is Boal and Ransom (1997).

equilibrium. Then, we formally characterize the two types of the latter equilibrium, first considered by Domar (1966).

In part IV we show that the family of wage functions, defined in part II, is always divided, by some neutral member-function, in its upper and lower subfamily, where the former implies the efficiency dominance of a LMF over a PMF, while within the latter the converse is true.

In part V we discuss some of the results of numerical simulations, performed to obtain an idea about the relative size of the LMF and the PMF dominance regions, and fully presented in Appendix 2. In Appendix 1, we graphically represent the three numerical simulations analyzed in part V, which test the sensitivity of the LMF/PMF dominance relation on the curvature of the considered wage functions.

Summary and conclusion, where the latter also addresses the issue of privatising a non-wage-taking enterprise, are left for part VI.

II. The Typical Family of Wage Functions and the Case When a LM Monopsony Pareto Dominates a PM Monopsony

1. *The S family of inverse labor supply or wage functions*

In order to define the one-parameter family of all inverse labor supply or wage functions, which yields nonnegative profit to a non-wage-taking firm, we first introduce the function of firm's (non-capital) income per unit of labor or a 'full' wage, y:

$$y = \frac{X(L) - C}{L} \tag{1}$$

where $X(L)$ and L are the short-run production function and the labor input, and where, by suitably choosing the measure of X, its (constant) price, p, is normalized to unity. Finally, C stands for fixed (capital) costs.

The reader familiar with the theory of a labor-managed firm (LMF) - see, for example, Dreze (1989), Bonin and Putterman (1987), Ireland and Law (1982) - will recognize in (1) the most frequently assumed objective function of such an enterprise. Here, the y function - depicted in figures 1 and 2 below – will, inter alia, serve to define the steepest wage function that yields zero profits both to a labor-managed and to a conventional, profit-maximizing firm (PMF).

In the monopsonistic labor market the typical (inverse) labor supply or wage function faced by a firm may be represented as:

$$W_k = f(L, a_k) \equiv W_k(L) \;\; , \quad L > 0 \quad , \tag{2}$$

where W_k is a wage rate or a supply price of labor, L is a firm's demand for labor and a_k is (nonnegative) parameter, which represents a measure of labor scarcity experienced by a firm.

In what follows a_k will be varied so as to cover all relevant degrees of labor scarcity, displayed in relation (3) and the related part of the text.

The f function is further characterized as follows:

$$\frac{\partial f}{\partial L} \equiv f' > 0 \tag{2a}$$

$$\frac{\partial^2 f}{\partial L^2} \equiv f'' \geq 0 \tag{2b}$$

$$\frac{\partial f}{\partial a_k} \equiv f_{a_k} > 0 \tag{2c}$$

$$\frac{\partial f'}{\partial a_k} \equiv f'_{a_k} > 0 \tag{2d}$$

In (2a) the positivity of f' says that the wage rate is increasing in the demand for labor.

In (2b) the nonegativity of f'' says that – perhaps, due to a rising marginal disutility of labor - the wage function is convex or, at least, linear.

In (2c) the positivity of f_{a_k} means that the wage rate is increasing with labor scarcity, for any given demand for labor.

Finally, in (2d) the positivity of f'_{a_k} means that an increase in the labor scarcity makes a greater increase in the wage rate, given any (infinitesimal) increase in the demand for labor.

By varying the a_k parameter within the interval defined in (3) below, we obtain the one-parameter family of W_k functions, denoted by S,

$$S = \{W_k = f(L, a_k) \equiv W_k(L) \ , \ a_k \in (a_e, a_z)\} \ , \tag{3}$$

where the S family is bounded from below by the horizontal entry-wage schedule W_e, depicted in figure 1[7],

$$W_e = f(L, a_e) = const > 0 \quad , \tag{4}$$

and where a_e generates the equilibrium labor use by a hypothetical, wage taking PMF, L_e, such that:

$$L_e = \arg\left[X'(L) - f(L, a_e) = 0\right], \ \forall a_e \mid f(L, a_e) = const > 0 \tag{4a}$$

At the same time, the upper boundary of S reduces to the function W_z,

$$W_z = f(L, a_z) \equiv W_z(L) \mid [W_z(L) = y(L)] \wedge [W_z'(L) = y'(L)] \ , \tag{5}$$

where the equilibrium employment, $L = L_z$, implied by a_z, may be written as:

$$L_z = \arg\left[W_z'(L) - y'(L) = 0\right] \wedge \arg\left[W_z(L) - y(L) = 0\right] \tag{5a}$$

Thus, as already mentioned, relation (4) defines the hypothetical case of a wage-taking enterprise - i.e., of zero labor scarcity faced by a single firm - while (5) defines the steepest relevant wage function which, by definition, yields zero profit both to a PMF and a LMF – see the $W_z(L)$ function of figure 1.

[7] Note that in figure 1 – as in most of simulations performed in part V - we assume, for simplicity, that the entry-wage is insensitive to the value of the a_k parameter. This however does not affect our main result, on the alternating dominance of a LMF and a PMF, summarized by proposition 1 of part IV, nor does it influence the possibility result on a LMF's (PMF's) global dominance over a PMF (LMF), obtained via numerical simulations fully displayed in Appendix 2, and partially reported and discussed in part V below.

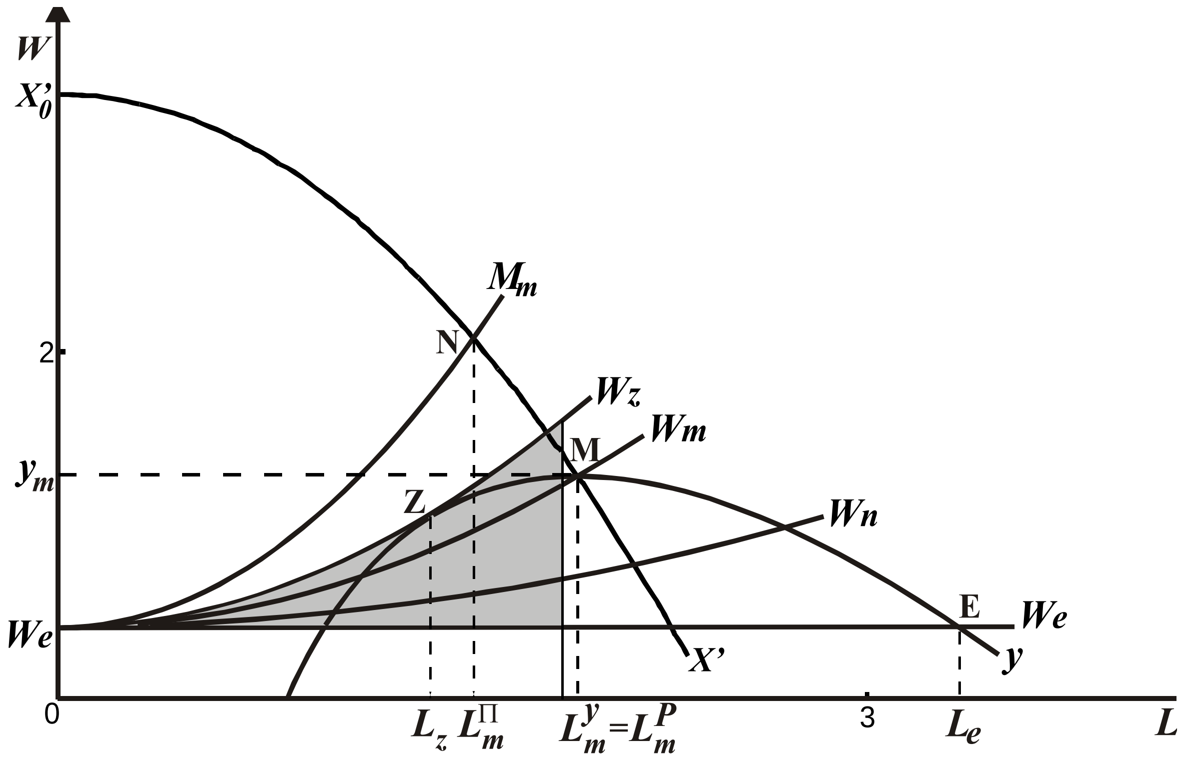

Figure 1. The S family of wage functions of (3), represented by the shaded area bordered by the horizontal line W_e of (4) and the W_z function of (5). The functions W_n and W_m appear in (17) and (12), while labor's marginal cost, M_m, is: $M_m = W_m + LW'_m$. The functions y and X' are those of (1) and (7). The M point is defined by the unconstrained maximum of y, L_m^y, given in (14a), and the corresponding (maximal) value of y, $y = y_m$: $\text{M} = (L_m^y, y_m)$; the PMF labor use, L_e, is of (4a). The Pareto optimal labor use, L^P, identical in the considered case with the LM monopsony labor use, L_m^y - see also section 2 of this part - is defined by eq. (5b), and for the typical family-member function, $W_k(L)$, in relation (21) below. $X' = 3.5\text{-}0.6L^2$, C=2.85, $y = 3.5 - 0.2L^2 - C/L$, W_e=0.4, $W_n = 0.4 + 0.0803L^2$, $W_m = 0.4 + 0.237L^2$, $W_z = 0.4 + 0.344L^2$, $M_m = 0.4 + 0.711L^2$

Note finally that, when coupled with the inequality $L > 0$ of (2), eqs. (2a)-(2d), which describe one well-behaved inverse labor supply or wage function, also ensure that any two member-functions of the S family do not have any common point. Among other things, this implies that any member-function divides S in the two disjoint subfamilies.

2. *The Pareto dominance of a LM monopsony over a PM monopsony: The example*

As mentioned in the Introduction, in this section we provide the numerically generated example in which the labor-managed monopsony achieves exactly the Pareto optimum and thus, by definition, Pareto dominates the profit-maximizing monopsony, which never reaches the Pareto optimum.

When labor is the only variable input, the LMF maximand of income-per-worker, y, is defined as in eq. (1) above.

At the same time, the unconstrained maximum of y in L is defined by the well-known LMF's equilibrium condition, $X'(L) = y(L)$, displayed in relation (14) below, where $X'(L)$ and $y(L)$ are respectively labor's marginal product and income-per-labor-unit functions, and where the product price, p, is taken to be the numeraire, $p = 1$.

Suppose now that one family-member wage function - denoted by $W_m(L)$, defined in eq. (12), and depicted in figure 1 – intersects the $y(L)$ function of eq. (1) just at its maximum, L_m^y, of eq. (12a).

Therefore, we have:

$$W_m(L) = X' \tag{5b}$$

Thus, for the $W_m(L)$ wage function, the LM monopsony equilibrium, L_m^y, coincides with the Pareto optimal equilibrium, L_m^P - see also figure 1.

On the other hand, and by definition, this means that a labor-managed monopsony Pareto dominates its profit-maximizing twin[8].

Thus, in the considered case, the labor-managed monopsony's extra output - exclusively due to the LMF objective of income maximization - emerges as significantly greater (about 12%) than the profit-maximizing monopsony's (net) output.

III. The Equilibrium of a PM and a LM Monopsony

1. *The two forms of the PM monopsony equilibrium*

Starting from (2), the economic profit of the PM monopsony, Π, may be represented as:

$$\Pi = X(L) - LW_k(L) - C\,, \tag{6}$$

[8] To the best of our knowledge, this is the unique case in the world of imperfect competition that *some* type of a firm's *un*constrained maximizing behavior (which also frequently appears in the literature) can generate the Pareto optimum.

where $X(L)$ and C appear in (1).

The standard first order condition for the maximum of Π in L reads:

$$X'(L) - [W_k(L) + LW'_k(L)] = 0 \,, \tag{7}$$

where X' and $W_k' \left(\equiv f'\right)$ respectively denote the first derivatives of X and W_k with respect to L, and where, due to (6), the maximum of Π in L may be written as:

$$L^{\Pi} = \arg\max\,[\Pi(L) \equiv X(L) - LW_k(L) - C] \tag{7a}$$

The standard PM monopsony equilibrium condition, obtained from (7), reads:

$$X'(L) = W_k(L) + LW'_k(L) \equiv M_k(L) \;, \tag{8}$$

where the R.H.S. of eq. (8) is the marginal labor cost of a PM monopsony, denoted by $M_k(L)$.

To simplify the efficiency comparison of a PMF and a LMF – performed in part IV below – we will write the monopsony equilibrium of (8) in the form:

$$W_k(L) = X'(L) - LW'_k(L) \equiv g_k(L) \tag{8a}$$

Note that the defined $g_k(L)$ function is smaller than $X'(L)$ for any $L>0$, and is obviously decreasing with L:

$$g_k(L) < X'(L) \;,\; L>0 \,, \tag{8b}$$

$$\frac{\partial g_k}{\partial L} \equiv g_k'(L) < 0 \,, \quad L \geq 0$$

Finally, the corresponding second order condition, derived from (7), is

$$X'' - 2W_k' - LW_k'' < 0 \tag{9}$$

or, using (2), (2a) and (2b):

$$X'' - 2f' - Lf'' < 0, \tag{9a}$$

where X'' and $W_k''(\equiv f'')$ respectively denote the second derivatives of X and W_k.

2. *The LM monopsony constrained equilibrium*

Depending on the degree of labor scarcity, the monopsonistic LMF is characterized by the two types of equilibrium, initially considered, though for different reasons, by Domar (1966).

The first type of the LMF equilibrium is the constrained one. Here the wage function $W_k(L)$ is binding on the maximum of $y(L)$ – see point C in figure 2 below - where this maximum reduces to

$$L_c^y = \sup \arg\left[y(L) - W_k(L) = 0\right], \tag{10}$$

where:

$$W_k(L) \equiv f(L, a_k) \quad , \quad a_k \in (a_m, a_z) \tag{10a}$$

Thus a LMF attains the constrained maximum L_c^y of (10) within the open interval

$$L_z < L_c^y < L_m^y \ , \tag{11}$$

where L_z and L_m^y are given in (5a) and (12a), while the a_m value of the labor scarcity parameter of (10a) – which generates the maximum of y in L - is defined as follows:

$$a_m \mid f(L, a_m) \equiv W_m(L) = y(L_m^y), \tag{12}$$

where the maximum of y in L – depicted in figure 2 - will be denoted by L_m^y:

$$L_m^y = \arg\max y(L) \tag{12a}$$

Finally, the entire subfamily of $W_k(L)$ functions of (10a) – which is a subfamily of S - may be written as:

$$S_y^c = \{W_k(L) \mid W_m(L) < W_k(L) < W_z(L)\} \quad , \tag{13}$$

where $W_m(L)$ is of (12), and where the $W_z(L)$ function is defined in (5).

As will be demonstrated in part IV below, within the S_y^c subfamily, a LM monopsony exhibit a higher efficiency than a PM monopsony – as illustrated in figure 2 below.

3. *The LM monopsony unconstrained equilibrium*

The second type of the LM monopsony equilibrium is obtained when the wage function $W_k(L)$ is not binding on the maximum of $y(L)$ of (1):

$$X' = \frac{X(L) - C}{L} \equiv y \tag{14}$$

The subfamily of wage functions, which yield the LMF unconstrained equilibrium of (14), is generated by varying the a_k parameter between its values a_m of (10a) and (12), and a_e of (4a), and will be denoted by S_y^u , where

$$S_y^u = S \setminus S_y^c$$

(14a)

As will bee shown in part IV below, within the S_y^u subfamily, a LM monopsony efficiency dominates a PM monopsony, just like within the LMF constrained equilibrium region, S_y^c, defined in the previous section.

Just like the S_y^c subfamily, S_y^u can be defined via the typical wage function W_k,

$$W_k = W_k(L) \equiv f(L, a_k) \ , \ a_k \in (a_n, a_m] \ , \tag{15}$$

$$S_y^u = \{W_k(L) \mid W_n(L) < W_k(L) < W_m(L)\} \ , \tag{16}$$

where the $W_m(L)$ function is of (12), and where the $W_n(L)$ function - which will be labeled the neutral wage function – will be written as:

$$W_n = f(L, a_n) \equiv W_n(L) \tag{17}$$

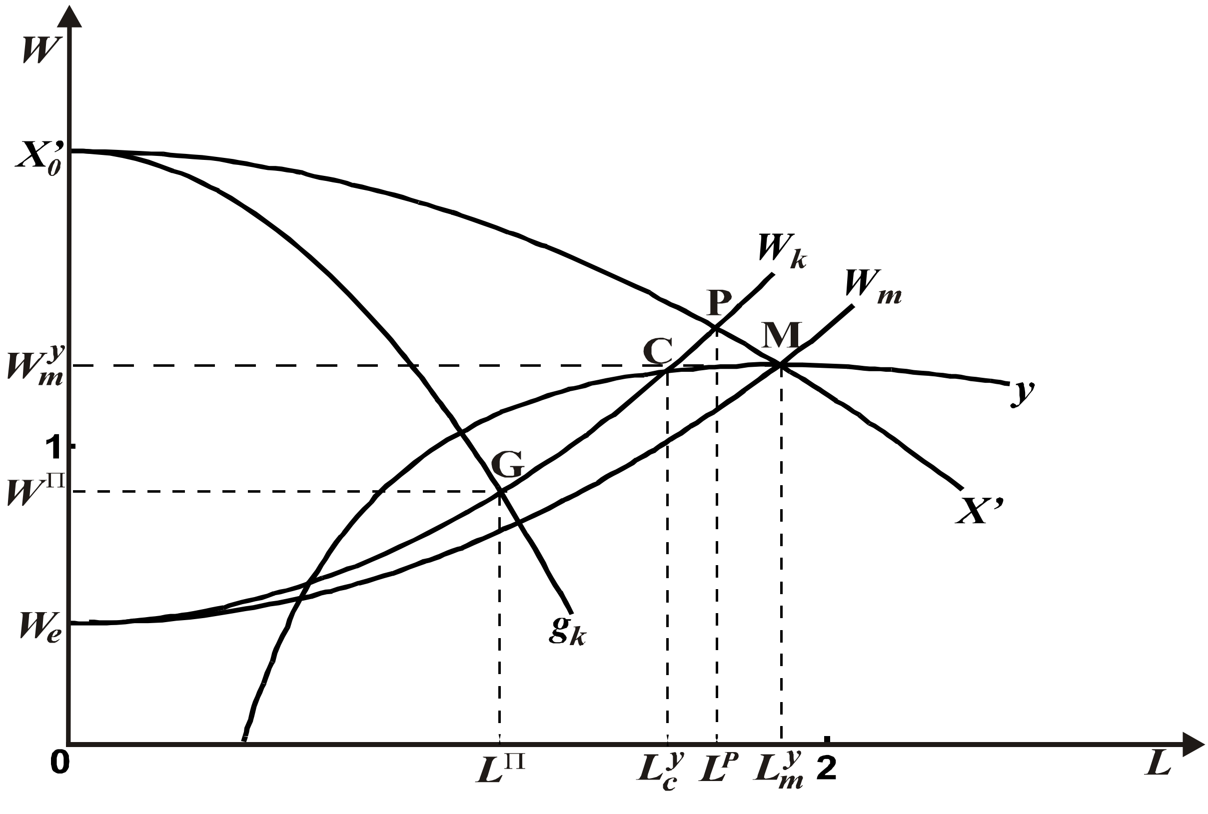

Figure 2. The PMF equilibrium, L^{Π}, is of (7a); the LMF constrained, L_c^y, and unconstrained, L_m^y, equilibria are of (10) and (14a), while the Paretian norm, L^P, is of (21) below. The functions W_k and g_k are of relations (2) and (8a); other functions as in figure 1.

The desired feature of the neutral $W_n(L)$ function, the existence of which is demonstrated in subsection 1.2 of part IV below[9], is that it generates the maximum of $\Pi(L)$, L_n^{Π}, equal to the *un*constrained maximum of y, given in (12a):

$$\begin{aligned} L_n^{\Pi} &= \arg\max\left[X(L) - LW_n(L) - C\right] \\ &= L_m^y \end{aligned} \tag{18}$$

Thus a_n of (15) and (17) represents the degree of labor scarcity which yields the identical equilibrium of a PMF and a LMF.

In what follows, we will denote by S_y the family that consists of disjoint subfamilies S_y^c and S_y^u:

$$S_y = S_y^c \cup S_y^u, \quad S_y^c \cap S_y^u = \emptyset \tag{19}$$

Taking account of the definitions of S_y^c and S_y^u of (13) and (16), the S_y family may also be written as:

$$S_y = \{W_k = f(L, a_k) \equiv W_k(L) \;, \;\; a_k \in (a_n, a_z)\} \;, \tag{19a}$$

where a_z is that of (5).

Finally, we introduce the remaining subfamily of S, denoted by S_Π, where

$$S_\Pi = S \setminus S_y \tag{20}$$

or, via the typical wage function W_k,

$$S_\Pi = \{W_k = f(L, a_k) \equiv W_k(L) \;, \;\; a_k \in (a_e, a_n)\} \;, \tag{20a}$$

where a_e and a_n respectively appear in (4), (15) and (17).

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[9] See equation (33) and the related part of the text.

IV. The Alternating Efficiency Dominance of a LM and a PM Monopsony

1. *The dominance of a LMF over a PMF within the S_y subfamily of S*

1.1 *The dominance of a LMF over a PMF within the S_y^c subfamily of S*

In the model, for the typical wage function of *S*, the Pareto optimal equilibrium L^P, depicted in figures 1 and 2 above, is defined by the standard condition:

$$L^P = \arg[X'(L) - W_k(L) = 0] \ , \qquad W_e \leq W_k(L) \leq W_z(L) \tag{21}$$

At the same time, the local dominance of a LMF over a PMF, or *vice versa*, is defined as follows:

Definition 1 - The Local Efficiency Dominance. Given the *S* family of wage functions, which all yield nonnegative profit both to a LMF and a PMF, a LMF (PMF) is defined to locally efficiency dominate a PMF (LMF) iff, for some function of *S*, a LMF (PMF) employs more labor, and thus produces more output, than a PMF (LMF).

Now, starting from (1), we may write the first derivative of $y(L)$ as:

$$y'(L) \equiv \frac{X'(L) - y(L)}{L} \tag{22}$$

Also, solving (22) for $X'(L)$ and substituting the latter into (8a), we can write the PMF equilibrium of (8a) in the form appropriate for efficiency comparisons:

$$W_k(L) = y(L) - L[W'_k(L) - y'(L)] \equiv g_k(L) \ , \tag{23}$$

where in (23) the $g_k(L)$ function appears in a slightly different form than in (8a).

On the other hand, within the S_y^c subfamily of (13), the LMF constrained equilibrium of (10) always satisfies the condition – see also point C in figure 2:

$$W_k(L) = y(L) \tag{24}$$

Now, within the relevant interval, already given in (11), we have:

$$W_k'(L) > y'(L) > 0 \quad , \quad L \in \left(L_z, L_m^y\right) \tag{25}$$

Hence, due to (25), it follows that in (23) the $g_k(L)$ function satisfies the following inequality:

$$g_k(L) < y(L) \quad , \quad L \in \left(L_z, L_m^y\right) \tag{26}$$

The monopsonistic PMF equilibrium, L^{Π}, obtained via the $g_k(L)$ function, is depicted in figure 2 above, where the G point of this figure may be written as G = (L^{Π}, W^{Π}):

$$L^{\Pi} = \arg\,[g_k(L) - W_k(L) = 0] \quad , \quad W^{\Pi} = W_k(L^{\Pi}) \tag{27}$$

Now, since W_k is increasing in L we have - due to (24), (25), (26) and (21) - that for any wage function of S_y^c, a LMF uses more labor, and thus produces more output, than a PMF, though less than required to reach the Pareto optimum:

$$L^P > L_c^y > L^{\Pi} \quad , \tag{28}$$

where L_c^y is given in (10).

We therefore conclude that the results of this subsection may be summarized by the following lemma:

Lemma 1. Within the S_y^c subfamily of wage functions a LMF efficiency dominates a PMF.

The efficiency dominance of the LMF equilibrium, L_c^y, over the PMF equilibrium, L^{Π}, within the S_y^c subfamily of wage functions is depicted in figure 2 above, where L^P is the Pareto optimal equilibrium of (21).

1.2 *The dominance of a LMF over a PMF within the* S_y^u *subfamily of* S

Now we focus on the S_y^u subfamily of wage functions, which allow a LMF to reach its unconstrained equilibrium but, as will be easily seen, still ensure the dominance of such a firm over a conventional PMF.

The striking feature of the upper boundary function of S_y^u, which is $W_m(L)$, is that - due to (12a) (14) and (12) – this function generates exactly the Pareto optimal equilibrium of a LMF monopsony[10] :

$$\begin{aligned} X_y'\left(L_m^y\right) &= y\left(L_m^y\right) \\ &= W_m\left(L_m^y\right) , \end{aligned} \tag{29}$$

where X_y' denotes the LMF labor's equilibrium marginal product.

At the same time, for the $W_m(L)$ wage function, a PMF is still behind a LMF since, due to (8) and $k = m$, we have:

$$X_\Pi'(L) = W_m(L) + LW_m'(L) > W_m(L) = X_y'(L), \tag{30}$$

that is,

$$X_\Pi'(L) > X_y'(L) \;\Rightarrow\; L_m^\Pi < L_m^y = L^P , \tag{30a}$$

where X_Π' denotes the PMF labor's equilibrium marginal productivity and L^P is the Pareto optimal labor use of (21).

Thus, it appears that there always exist some wage function - the $W_m(L)$ function in our case - for which the non-wage-taking LMF, reaches the Pareto optimum, and thus Pareto dominates the non-wage-taking PMF.

[10] Note that the non-wage-taking PMF, *un*like the corresponding LMF, never reaches the Pareto optimum.

Furthermore, a decrease in the a_k parameter from its a_m level, will not affect the LMF equilibrium, $L^y = L^y_m$.

On the other hand, a decrease in the a_k (expectedly) increases the PMF equilibrium labor use, L^Π. To verify this, we write the PMF equilibrium of (8) as:

$$X'(L) = f(L, a_k) + Lf'(L, a_k) \tag{31}$$

Then, we differentiate (31) with respect to a_k and use the envelope theorem to obtain, due to (2c), (2a) and (9a):

$$\frac{dL}{da_k} = \frac{f_{a_k} + Lf'_{a_k}}{X'' - 2f' - Lf''} < 0 \tag{31a}$$

Thus, with a_k decreasing from a_m of (12) to a_e of (4a), the PMF equilibrium labor use, L^Π, is strictly monotonically increasing, until it (hypothetically) reaches its wage-taking level L^Π_e of (4a), where, due to $X'' < 0$, we have:

$$L^\Pi_e > L^y_m \tag{32}$$

But, this further implies that there always exists some value a_n of the a_k parameter, where $a_e < a_n < a_m$, and the corresponding neutral wage function $W_n(L) \equiv f(L,a_n)$, already introduced in (17), which yield:

$$\begin{aligned} L^\Pi_n &= \arg\max\left(X(L) - LW_n(L) - C\right) \\ &= L^y_m \end{aligned} \tag{33}$$

In other words, within the S^u_y family of wage functions there always exists a single, neutral wage function, $W_n(L)$, which equalizes the PMF and the LMF equilibrium and, thus implies identical efficiency of the two types of a firm.

We can now collect the results of this subsection to obtain the following lemma:

Lemma 2. Within the S_y^u subfamily of wage functions a LMF efficiency dominates a PMF.

Finally, we integrate Lemma 1 and Lemma 2, to obtain:

Lemma 3. Within the S_y family of wage functions a LMF efficiency dominates a PMF, where $S_y = S_y^c \cup S_y^u$, $S_y^c \cap S_y^u = \emptyset$.

2. The dominance of a PMF over a LMF within the S_Π subfamily of S and The Alternating Dominance Theorem

Due to the results of the previous section, for the remaining S_Π subfamily of wage functions, defined in (20) and (20a), we instantly obtain the following lemma:

Lemma 4. Within the S_Π subfamily of wage functions a PMF efficiency dominates a LMF, where $S_\Pi = S \setminus S_y$

To conclude this part, we will write the relation (20) in the following form:

$$S = S_y \cup S_\Pi \quad , \quad S_y \cap S_\Pi = \emptyset \quad , \tag{33a}$$

which simply states that S_y and S_Π are disjoint subfamilies.

Finally, we integrate Lemma 3 and Lemma 4 to get the general proposition on the alternating (efficiency) dominance of a LMF and a PMF:

Proposition 1 - The Alternating Dominance Theorem. Given the income-per-worker function $y = y(L)$, the S family of wage functions is divided by one, neutral member-function, $W_n(L)$, in the two disjoint subfamilies, S_y and S_Π , where for any function of S_y a LMF dominates a PMF, while for any function of S_Π the converse is true.

V. The LMF/PMF Efficiency Dominance Ratio δ and the Size of the LMF and the PMF Dominance Regions

As mentioned in the title, we now aim to compare the size of the relevant subfamilies of S. To accomplish this, here and in Appendices 1 and 2, we assume that S is no more continuous but rather a *discrete* family, characterized with the (small) uniform step in the a_k scarcity parameter, $\Delta a_k = \Delta a$, where $k = 1,\ldots,n$, and where n is big.

This will generate the (big number of) wage functions, evenly spread across the S family. Thus, measuring the (approximate) relative size of relevant subfamilies will simply reduce to counting wage curves that belong to S and to its relevant subfamilies.

In order to measure the LMF/PMF dominance relation, we first introduce what seems to be its natural definition:

Definition 2. The LMF/PMF dominance relation is identified with the δ ratio, where the numerator and denominator of δ respectively reduce to the shares of S_y and S_Π in S, and where these shares, denoted by $N(S_y)$ and $N(S_\Pi)$, represent the size of the LMF and the PMF dominance regions,

$$\delta = \frac{N(S_y)}{N(S_\Pi)} \ , \tag{33b}$$

where:

$$N(S_y) = \frac{\delta}{1+\delta} \quad , \qquad N(S_\Pi) = \frac{1}{1+\delta} \tag{33c}$$

The construction of the δ dominance ratio seems also to naturally call for introducing the concept of global (efficiency) dominance, as distinct from the already defined local efficiency dominance:

Definition 3. Given the S family of wage functions, a LMF (PMF) is defined to globally efficiency dominate a PMF (LMF) iff the δ dominance ratio is greater (smaller) than unity.

To get the idea about possible magnitudes of the δ ratio, we below partially present the three types of numerical simulations, which are fully displayed in Appendix 2 and quite selectively graphically presented in Appendix 1.

The Type 1 simulations examine the sensitivity of δ on the type of the employed (convex) technology, while the Type 2 simulations analyze the dependence of this relation on the type of the employed (inverse) labor supply or wage functions. Finally, the Type 3 simulations examine the sensitivity of δ on the magnitude of the entry-wage, W_e.

1.*The Type* 1 *simulations: The LMF/PMF dominance relation under different types of (convex) technology*

Here, we have combined the 'unbiased' family of quadratic wage functions $W_k(L) = W_e + a_k L^2$ with the three types of convex technology, T1, T2, and T3, respectively characterized with concave, linear, and convex function of labor's marginal productivity.

The employed technologies are commensurable in the sense that, given the price of output $p(=1)$, fixed capital costs, C, and the entry-wage, W_e, they yield the income per worker functions, characterized with (almost) the same maximum, $L_m^y (\approx 1.98)$, and the same (maximal) value of income per worker computed at this maximum, $y_m(=1.26)$.

As for the entry-wage W_e, we have first chosen it to be 0.4, i.e., somewhat below 50% of the average market wage, which itself is assumed to comprise 66% of maximal income per worker $y_m(=1.26)$.

The results of the Type 1 simulations – obtained for $W_e = 0.4$ - are presented in the second row of table 1 of Appendix 2[11]:

$$\begin{array}{ll} \text{Concave } X'(L) \text{ - Technology T1:} & \delta_{21} = 3.28 \\ \text{Linear } X'(L) \text{ - Technology T2:} & \delta_{22} = 10.9 \\ \text{Convex } X'(L) \text{ - Technology T3:} & \delta_{23} = 22.5 \end{array} \tag{34}$$

(quadratic wage functions $W(L)$, entry-wage $W_e = 0.4$)

The complete results - which, for $W_e = 0.4$, also include linear and cubic wage functions - are presented in table 1 of Appendix 2.

[11] The values of δ_{ij}s of relations (34) – (39) and of Appendix 2 are approximate, where the computational error can be made arbitrarily small. The complete calculation procedure is available from the author on request.

Finally, we represent the LMF/PMF dominance relation results - analogous to that of (34) - obtained for the entry-wage $W_e = 0.63$, and the entry-wage higher than the average market wage, $W_e = 1$:

$$\begin{array}{ll} \text{Concave } X'(L) \text{ - Technology T1:} & \delta_{21} = 2.88 \\ \text{Linear } X'(L) \text{ - Technology T2:} & \delta_{22} = 8.10 \\ \text{Convex } X'(L) \text{ - Technology T3:} & \delta_{23} = 16.2 \end{array} \tag{35}$$

(quadratic wage functions $W(L)$, entry-wage $W_e = 0.63$)

$$\begin{array}{ll} \text{Concave } X'(L) \text{ - Technology T1:} & \delta_{21} = 2.27 \\ \text{Linear } X'(L) \text{ - Technology T2:} & \delta_{22} = 4.23 \\ \text{Convex } X'(L) \text{ - Technology T3:} & \delta_{23} = 7.50 \end{array} \tag{36}$$

(quadratic wage functions $W(L)$, entry-wage $W_e = 1.0$)

The complete results - which also include linear and cubic wage functions - are presented in table 2 ($W_e = 0.63$) and table 3 ($W_e = 1$) of Appendix 2.

Few notes seem to be apposite here.

First, it appears that a LMF strongly dominates PMF for all types of the employed technology, in the sense that in all cases the LMF dominance region is significantly greater than the PMF one.

Second, this domination, measured by the δ ratio, increases by switching from technologies with concave functions of labor's marginal productivity to those characterized with linear labor's marginal product functions and, finally, to technologies with convex functions of labor's marginal productivity.

Third, the δ ratio also increases by switching from families of (convex) functions with smaller curvature to families of more convex wage functions.

Fourth, greater δ ratios are associated with smaller values of the entry-wage.

2. *The Type 2 simulations: The LMF/PMF dominance relation under different types of inverse labor supply or wage functions*

In these simulations, assuming first $W_e = 0.4$, we have combined the 'unbiased' linear labor's marginal product function with the three families of inverse labor supply or wage functions - S1, S2 and S3 - which are respectively composed of linear, quadratic and cubic (wage) functions, $W_k = W_e + a_k L^n$, n=1,2,3.

The results of the Type 2 simulations - obtained for W_e = 0.4 - are graphically presented in figures A1.1-A1.3 of Appendix 1 and in the second column of table 1 of Appendix 2.

In these figures, the darker and the lighter shaded areas approximate, respectively, the S_y and S_Π subfamilies of wage functions, where the relative size of these subfamilies $\delta_{i2}\ (i=1,2,3)$ reduces to:

$$
\begin{array}{llll}
\text{Figure A1.1} & \rightarrow & \text{Linear wage functions S1:} & \delta_{12} = 3.64 \\
\text{Figure A1.2} & \rightarrow & \text{Quadratic wage functions S2:} & \delta_{22} = 10.9 \\
\text{Figure A1.3} & \rightarrow & \text{Cubic wage functions S3:} & \delta_{32} = 45.0
\end{array}
\tag{37}
$$

(linear $X'(L)$, entry-wage $W_e = 0.4$)

As in the case of the Type 1 simulations, we now display the LMF/PMF dominance ratios - analogous to that of (37) - obtained for the entry-wages $W_e = 0.63$, relation (38), and $W_e = 1$, relation (39):

$$
\begin{array}{ll}
\text{Linear wage functions S1:} & \delta_{12} = 1.85 \\
\text{Quadratic wage functions S2:} & \delta_{22} = 8.10 \\
\text{Cubic wage functions S3:} & \delta_{32} = 29.2
\end{array}
\tag{38}
$$

$$\bar{\delta}_{i2} \approx 7.96 \quad (i=1,2,3)$$

$$\bar{N}_{i2}(S_y) = \frac{\bar{\delta}_{i2}}{1+\bar{\delta}_{i2}} \approx 0.89$$

(linear $X'(L)$, entry-wage $W_e = 0.63$)

Note that we have also computed in this case, on the basis of eq. (33b), the corresponding average dominance ratio, $\bar{\delta}_{i2}$, and, on the basis of eq. (33c), the corresponding average size of the LMF dominance region, $\bar{N}_{i2}(S_y)$.

$$
\begin{array}{ll}
\text{Linear wage functions S1:} & \delta_{12} = 1.33 \\
\text{Quadratic wage functions S2:} & \delta_{22} = 4.23 \\
\text{Cubic wage functions S3:} & \delta_{32} = 10.3
\end{array}
\tag{39}
$$

(linear $X'(L)$, entry-wage $W_e = 1.0$)

The second simulation of (38), which yields $\delta = \delta_{22} = 8.10$, is graphically presented in figure 3.

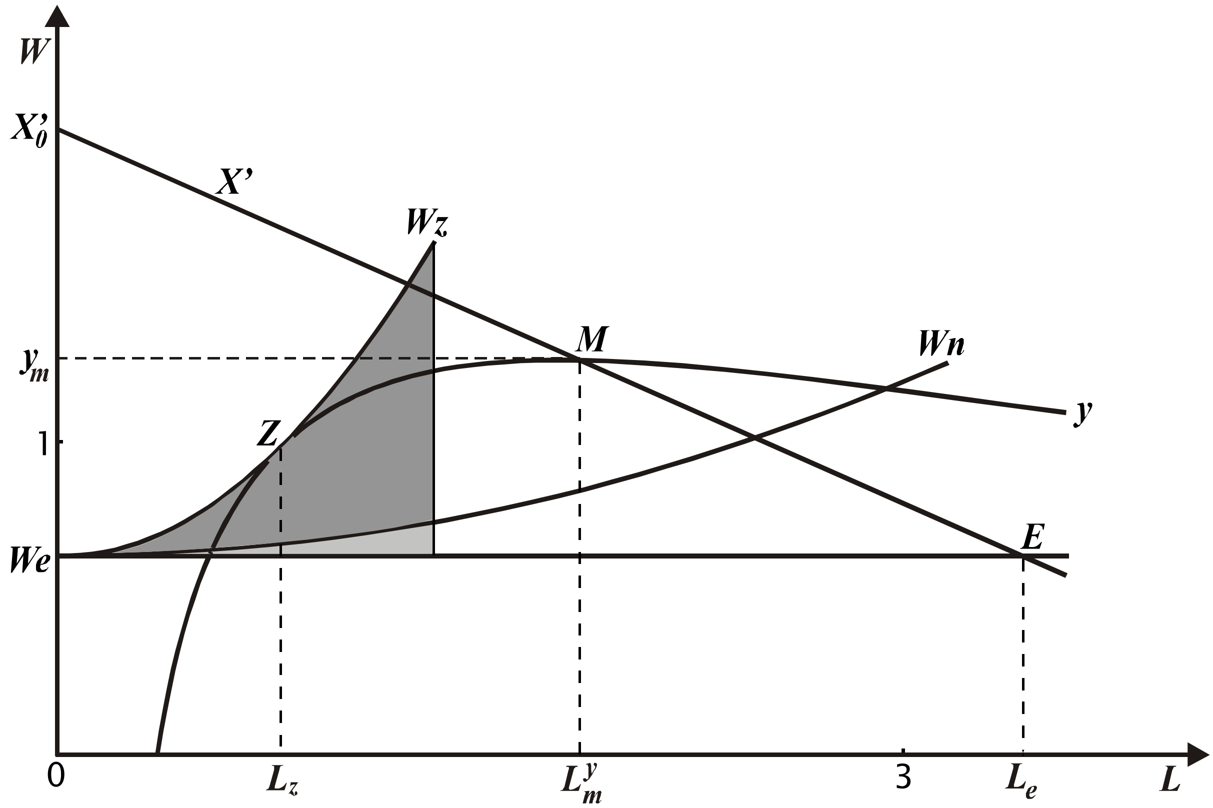

Figure 3. The LMF and the PMF dominance regions, identified with the S_y and S_Π subfamilies of S, are approximated by the darker and lighter shaded areas, bordered by the W_z, W_n, and W_e functions: The case of 2nd simulation of (38), $\delta = \delta_{22} = 8.10$.

$X = 2L - 0.2L^2$	- production function
$X' = 2 - 0.4L$	- labor's marginal product
$y = 2 - 0.2L - C/L$	- income per worker
$C = 0.68$	- fixed costs
$W_e = 0.63$	- entry wage horizontal line, $a_k = 0$
a_k	- the (varying) labor scarcity parameter, $a_k \in (0, a_z)$
$W_k = W_e + a_k L^2$	- typical wage function
$W_n = W_e + a_n L^2$	- neutral wage function that implies equal equilibrium of LMF and PMF, $a_n = 0.0624$
$W_z = W_e + a_z L^2$	- zero-profit wage function that implies zero-profit and equal equilibrium of LMF and PMF, $a_z = 0.564$

Similar to the case of Type 1 simulations, few notes are in order.

First, as in the case of Type 1 simulations, it appears that a LMF strongly dominates PMF.

Second, we observe that the change in the LMF/PMF dominance ratio is again a systematic one – in the sense that this ratio increases by switching from families typical of convex wage functions with smaller curvature, to families with more convex wage functions.

Third, greater LMF/PMF dominance ratios are associated with smaller values of the entry-wage[12].

VI. Summary and Conclusion

In this paper we have used a standard model of the monopsony in the labor market to compare the efficiency of a labor-managed and conventional, profit-maximizing firm (LMF and PMF) in non-wage-taking environments.

To accomplish this, we have first defined the local efficiency dominance, according to which one firm dominates the other when, for a single inverse labor supply or wage function, the former produces more output than the latter, provided that both firms are able to make no losses.

For a well-behaved, increasing and convex typical wage function, we have then systematically varied a suitably defined labor scarcity parameter from zero to its zero-profit level. Given a turned U-shaped income-per-worker schedule, the latter level defines the steepest wage curve that yields zero profit both to a LMF and a PMF, and thus have the tangency point with the above schedule.

This procedure has generated the continuous family of wage functions, which all ensure nonnegative profit to a LMF and a PMF and where, by definition, the number of such functions is infinite.

Finally, we have demonstrated that this family is always divided by, some neutral member-function, in its upper and lower subfamily, where for any function of the former a LMF (locally) dominates a PMF, while for any function of the latter the converse is true. Thus, we have also shown that, on the level of a single wage function, a LMF can efficiency dominate a PMF, and vice versa.

After detecting this alternating LMF/PMF dominance relation, we have focused on getting the idea about the relative size of the LMF and the PMF dominance regions,

[12] To test the relevance of the performed simulations, we have also done the three modified exercises, where the (previously parametric) entry-wage has been modeled as an increasing function of the labor scarcity parameter. These new simulations have been designed so as to be fully comparable with the three arguably most relevant parametric entry-wage simulations, summarized by relation (38). However, it has emerged that these additional exercises have not altered the tenor of the previous results - the LMF dominance region has, on average, decreased pretty modestly, from 89% to 87%.

identified with the ratio of shares of the corresponding subfamilies in the above defined family of wage functions.

To achieve this, we have had to temporarily assume that this family is discrete, and that its member-functions (the number of which is big) are evenly spread across the family. Also, this has required to establishing the concept of global dominance, where one firm has been defined to globally dominate the other when the former locally dominates the latter for more than a half of all wage functions which constitute the (entire) family.

After that, we have performed 27 (carefully selected) numerical simulations, which combine three types of technology, three types of wage functions, and three levels of the entry-wage.

First, the simulations indicate that the LMF/PMF dominance relation - identified with the ratio of the LMF and the PMF dominance region - systematically increases by switching from technologies with concave labor's marginal product to those characterized by convex labor's marginal productivity. Second, the LMF dominance region also clearly (relatively) increases by switching from families of linear wage functions, to families of (strictly) convex functions with smaller curvatures and, finally, to families that consist of more convex (wage) functions. Third, the above ratio is greater for lower levels of the entry wage.

The basic result of the performed simulations is that, on average, a LMF (strongly) globally dominates a PMF, where the average size of the LMF dominance region amounts to 94% of all considered wage functions, and where just one of 27 simulations yields a (relatively weak) PMF's dominance - see relation (A2.2) and tables 1-3 of Appendix 2.

Finally, two notes are in order.

The first one refers to the (novel) concept of the global efficiency dominance, which should obviously not be restricted to the present LMF/PMF case of monopsonistic labor markets and could, in principle, be applied in various situations and under different market structures.

However, in the present case, and when considered on the empirical level, the concept would require each family-member function to be weighted by the probability of its occurrence at the specific labor market. Still, on the theoretical level the (implicitly) assumed equal probability of all relevant wage functions is acceptable, if not for the fact that all these functions enable both a LMF and a PMF to make no losses and thus, almost by definition, should be non-discriminatory taken into account when comparing the (global) efficiency of a labor-managed and a profit maximizing monopsony.

The second note may be of relevance for the theory and policy of privatizing non-wage-taking firms. If, say, in the context of post-socialist transition, the econometric evidence reveals the local dominance of some insider-controlled firm (assumed to behave like a canonical LMF) over the corresponding outsider-privatized PMF, a higher local efficiency of the former - due to its objective of wage maximization - ought to be weighed against the possibly superior technical productivity of the latter, observed, for example, in the case of the outsider-privatized firms across Central-Europe.[13] This, among other things, should be taken into account when defining the strategy of how to privatize a non-wage-taking firm.

In any case, and irrespective of these remarks, the key result of the paper clearly points to the fact that in non-wage-taking environments, and with equal technical and market opportunities, the labor-managed firm can be more efficient, both locally and globally, than the conventional, profit-maximizing enterprise.

[13] See Frydman, Gray, Hessel and Rapaczynski (1999), where the revenue performance of such firms, not of interest on the present occasion, has also been analyzed.

Appendix 1. The graphical presentation of the LMF/PMF δ dominance ratio for the three types of wage functions[14]

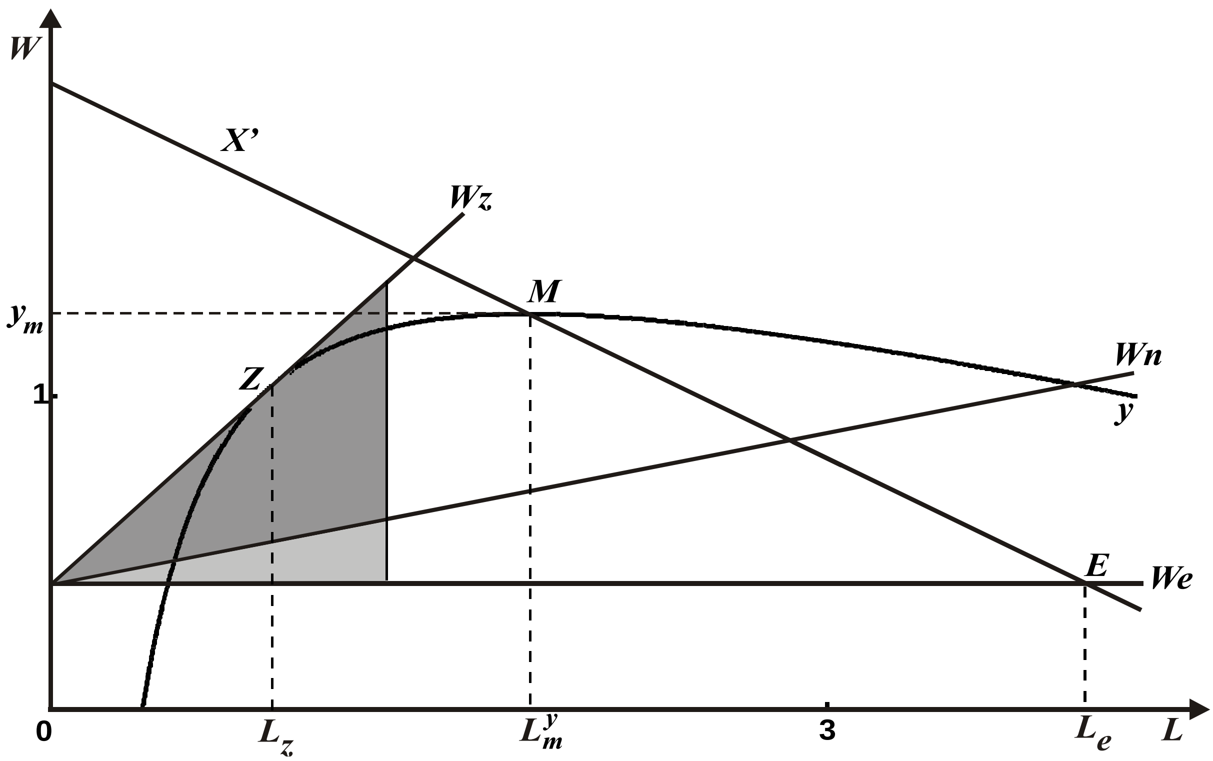

Figure A1.1 The LMF and PMF dominance regions, identified with the S_y and S_Π subfamilies of the discrete S family, are approximated by the darker and lighter shaded areas, bordered by the W_z, W_n, and W_e functions: The case of linear labor's marginal product and linear wage functions, $\delta_{12} = 3.64$, see table 1 of Appendix 2.

$X = 2L - 0.2L^2$	- production function
$X' = 2 - 0.4L$	- labor's marginal product
$y = 2 - 0.2L - C/L$	- income per worker
$C = 0.68$	- fixed costs
$W_e = 0.4$	- entry wage horizontal line, $a_k = 0$
$a_k = n\,\Delta a_k$	- the (varying) labor scarcity parameter a_k, where $\Delta a_k = \Delta a$, $k = 1,\ldots,n$
$W_k = W_e + a_k L^2$	- typical wage function
$W_n = W_e + a_n L$	- neutral wage function that implies equal equilibrium of LMF and PMF, $a_n = 0.160$
$W_z = W_e + a_z L$	- zero-profit wage function that implies zero-profit and equal equilibrium of LMF and PMF, $a_z = 0.743$

[14] Note that in Appendix 1 the entry-wage is $W_e = 0.4$, while in both Appendix 1 and Appendix 2 the product price is p=1. Also, as already mentioned in part V, the maximum point of income per worker is the same in all figures: M = (L_m^y , y_m) $\approx$ (1.98, 1.26).

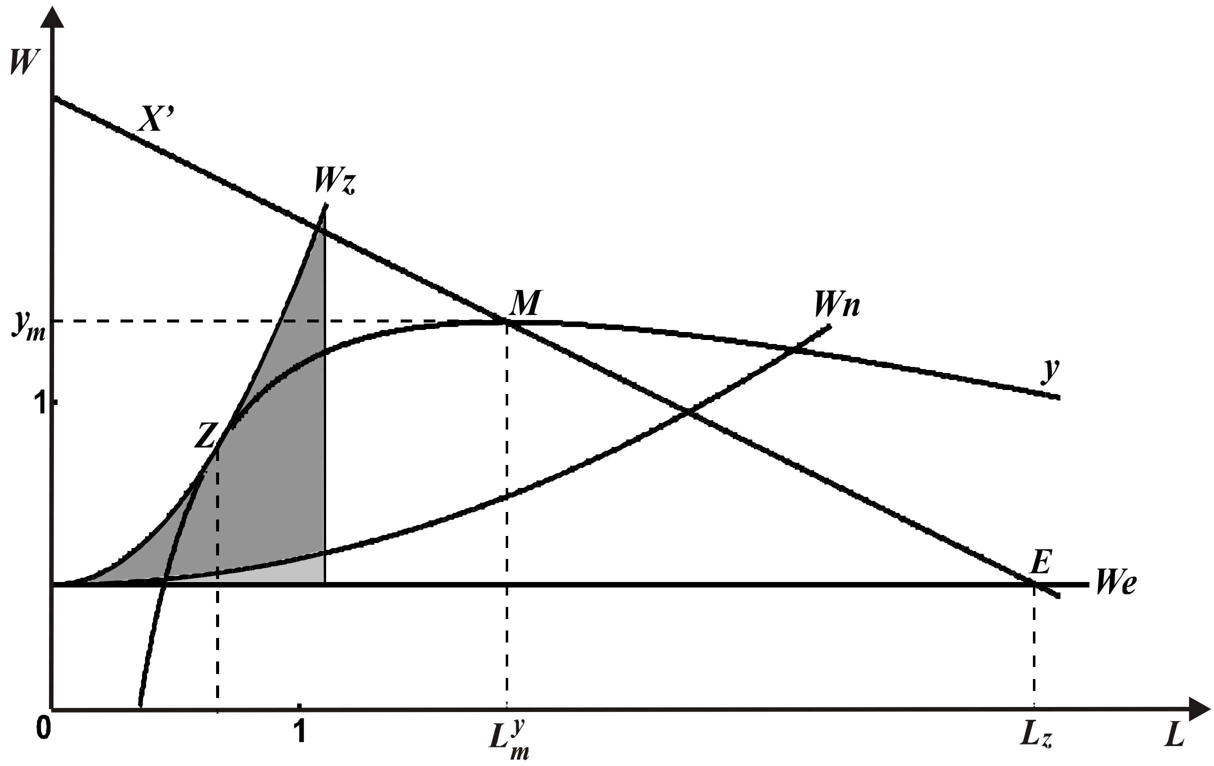

Figure A1.2 The LMF and PMF dominance regions, identified with the S_y and S_Π subfamilies of the discrete S family, are approximated by the darker and lighter shaded areas, bordered by the W_z, W_n, and W_e functions: The case of linear labor's marginal product and quadratic wage functions, $\delta_{22} = 10.9$, see table 1 of Appendix 2.

X, X', y, C, W_e, a_k, and W_k	- as in figure A1.1
$W_n = W_e + a_n L^2$	- neutral wage function, defined as in figure A1.1, $a_n = 0.085$
$W_z = W_e + a_z L^2$	- zero-profit wage function, defined as in figure A1.1, $a_z = 1.01$

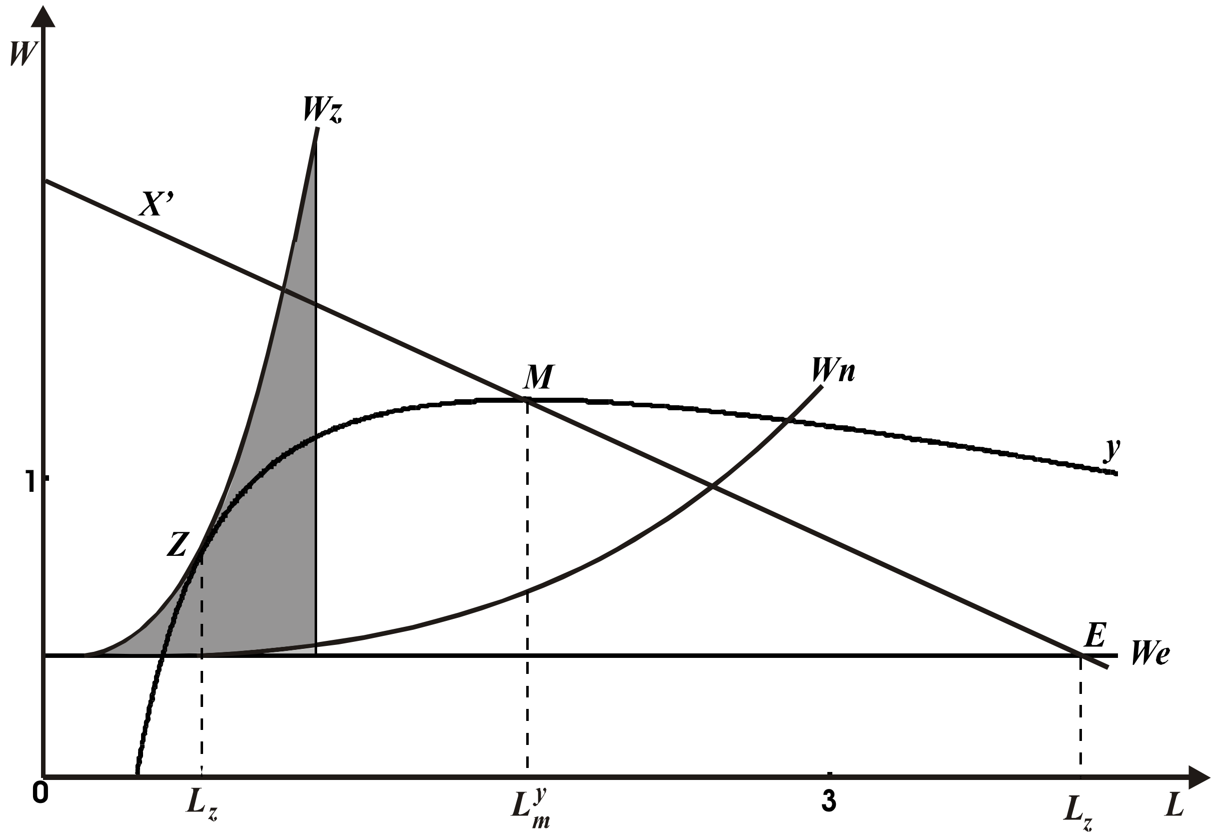

Figure A1.3 The LMF and PMF dominance regions, identified with the S_y and S_Π subfamilies of the discrete S family, are approximated by the darker and lighter shaded areas, bordered by the W_z, W_n, and W_e functions: The case of linear labor's marginal product and cubic wage functions, $\delta_{32} = 45.0$, see table 1 of Appendix 2.

X, X', y, C, W_e, a_k, and W_k	- as in figure A1.1
$W_n = W_e + a_n L^3$	- neutral wage function, defined as in figure A1.1, $a_n = 0.0347$
$W_z = W_e + a_z L^3$	- zero-profit wage function, defined as in figure A1.1, $a_z = 1.61$

Appendix 2. The values of the LMF/PMF δ dominance ratio and of the average size of the LMF dominance region $\overline{N}(S_y)$ [15]

A. The values of the δ dominance ratio, resulted from 27 numerical simulations

[15] The values of $\delta = \delta_{ij}$ (i,j=1,2,3), and thus of $\overline{N}(S_y)$, are approximate where, as already mentioned in footnote 11 above, the computational error can be made arbitrarily small.

$We = 0.4$	T1 $X' = 3.5 - 0.6L^2$ $y = 3.5 - 0.2L^2 - C/L$ $C = 2.85$	T2 $X' = 2 - 0.4L$ $y = 2 - 0.2L - C/L$ $C = 0.680$	T3 $X' = 2.08/(1.45L^{0.2})$ $y = (2.6/1.45L^{0.2}) - C/L$ $C = 0.600$
S1 $W_n = W_e + a_n L$ $W_z = W_e + a_z L$	$a_n = 0.231$ $a_z = 0.505$ $\delta_{11} = 1.19$	$a_n = 0.160$ $a_z = 0.743$ $\delta_{12} = 3.64$	$a_n = 0.227$ $a_z = 0.980$ $\delta_{13} = 3.32$
S2 $W_n = W_e + a_n L^2$ $W_z = W_e + a_z L^2$	$a_n = 0.0803$ $a_z = 0.344$ $\delta_{21} = 3.28$	$a_n = 0.0850$ $a_z = 1.01$ $\delta_{22} = 10.9$	$a_n = 0.0790$ $a_z = 1.86$ $\delta_{23} = 22.5$
S3 $W_n = W_e + a_n L^3$ $W_z = W_e + a_z L^3$	$a_n = 0.0314$ $a_z = 0.261$ $\delta_{31} = 7.31$	$a_n = 0.0350$ $a_z = 1.61$ $\delta_{32} = 45.0$	$a_n = 0.0314$ $a_z = 4.15$ $\delta_{33} = 131$

Table 1. The values of the δ dominance ratio for the entry-wage $W_e = 0.4$

Most of the above dominance ratios $\delta = \delta_{ij}$ (i,j=1,2,3), where δ is of Definition 2 of part V, and where the entry-wage is $W_e = 0.4$, already appear in part V. Technologies T1, T2 and T3, and the wage functions S1, S2 and S3, are also defined in part V, respectively in sections 1 and 2.

$We = 0.63$	T1 $X' = 3.5 - 0.6L^2$ $y = 3.5 - 0.2L^2 - C/L$ $C = 2.85$	T2 $X' = 2 - 0.4L$ $y = 2 - 0.2L - C/L$ $C = 0.6800$	T3 $X' = 2.08/(1.45L^{0.2})$ $y = (2.6/1.45L^{0.2}) - C/L$ $C = 0.600$
S1 $W_n = W_e + a_n L$ $W_z = W_e + a_z L$	$a_n = 0.171$ $a_z = 0.363$ $\delta_{11} = 1.12$	$a_n = 0.172$ $a_z = 0.490$ $\delta_{12} = 1.85$	$a_n = 0.166$ $a_z = 0.628$ $\delta_{13} = 2.78$
S2 $W_n = W_e + a_n L^2$ $W_z = W_e + a_z L^2$	$a_n = 0.0595$ $a_z = 0.231$ $\delta_{21} = 2.88$	$a_n = 0.0620$ $a_z = 0.564$ $\delta_{22} = 8.10$	$a_n = 0.0580$ $a_z = 1.00$ $\delta_{23} = 16.2$
S3 $W_n = W_e + a_n L^3$ $W_z = W_e + a_z L^3$	$a_n = 0.0232$ $a_z = 0.161$ $\delta_{31} = 5.94$	$a_n = 0.0250$ $a_z = 0.756$ $\delta_{32} = 29.2$	$a_n = 0.0230$ $a_z = 1.86$ $\delta_{33} = 79.9$

Table 2. The values of the δ dominance ratio for the entry-wage $W_e = 0.63$

Most of the above dominance ratios $\delta = \delta_{ij}$ (i,j=1,2,3), where δ is of Definition 2 of part V and where the entry-wage is $W_e = 0.63$, already appear in part V. Technologies T1, T2 and T3, and the wage functions S1, S2 and S3, are also defined in part V, respectively in sections 1 and 2.

$We = 1$	T1 $X' = 3.5 - 0.6L^2$ $y = 3.5 - 0.2L^2 - C/L$ $C = 2.85$	T2 $X' = 2 - 0.4L$ $y = 2 - 0.2L - C/L$ $C = 0.680$	T3 $X' = 2.08/(1.45L^{0.2})$ $y = (2.6/1.45L^{0.2}) - C/L$ $C = 0.600$
S1 $W_n = W_e + a_n L$ $W_z = W_e + a_z L$	$a_n = 0.0750$ $a_z = 0.149$ $\delta_{11} = 0.987$	$a_n = 0.0720$ $a_z = 0.168$ $\delta_{12} = 1.33$	$a_n = 0.0690$ $a_z = 0.195$ $\delta_{13} = 1.83$
S2 $W_n = W_e + a_n L^2$ $W_z = W_e + a_z L^2$	$a_n = 0.0260$ $a_z = 0.085$ $\delta_{21} = 2.27$	$a_n = 0.0260$ $a_z = 0.136$ $\delta_{22} = 4.23$	$a_n = 0.0240$ $a_z = 0.204$ $\delta_{23} = 7.50$
S3 $W_n = W_e + a_n L^3$ $W_z = W_e + a_z L^3$	$a_n = 0.0102$ $a_z = 0.0510$ $\delta_{31} = 4.00$	$a_n = 0.0110$ $a_z = 0.124$ $\delta_{32} = 10.3$	$a_n = 0.00900$ $a_z = 0.245$ $\delta_{33} = 26.2$

Table 3. The values of the δ dominance ratio for the entry-wage $W_e = 1$

Most of the above dominance ratios $\delta = \delta_{ij}$ (i,j=1,2,3), where δ is of Definition 2 of part V, and where the entry-wage is $W_e = 1$, already appear in part V. The technologies T1, T2 and T3, and the wage functions S1, S2 and S3, are also defined in part V, respectively in sections 1 and 2.

B. The $\bar{\delta}$ average dominance ratio, resulted from 27 numerical simulations

(A2.1) $$\bar{\delta} = \frac{1}{27}\sum_{i=1}^{3}\sum_{j=1}^{3}\delta_{ij} = 16.1 ,$$

where δ_{ij}s are of tables 1-3.

C. The average size of the LMF dominance subfamily, $\bar{N}(S_y)$, obtained via $\bar{\delta}$ of (A2.1)

(A2.2) $$\bar{N}(S_y) = \frac{\bar{\delta}}{1+\bar{\delta}} = 0.942 ,$$

where (A2.2) is analogous to (33c) of the text.

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