A Dynamic Model of the Investment Behavior of a Joint-Stock Company and Impacts of Investment Incentives

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Abstrakt

Hlavním cílem této práce je konstrukce a analýza adaptivního dynamického matematického modelu investičního chování typické akciové společnosti, který může usnadnit pochopení vlivu investičních stimulů na úroveň její investiční aktivity. Chování modelu se analyzuje pomocí Portrjaginova principu maxima ve čtyřech krocích, které jsou postupnými aproximacemi komplikovaného investičního chování reálných firem. Analytická řešení modelu jsou zčásti interpretována v analýze vlivu investičních stimulů, které byly zavedeny v české ekonomice v rámci nového daňového systému, na úroveň investiční aktivity akciových společností zformovaných v první vlně kupónové privatizace.

Abstract

The main goal of this paper is to formulate and analyze an adaptive dynamic mathematical model of the investment behavior of a typical joint-stock firm, which may facilitate the understanding of the influence of investment incentives on the level of its investment activity. The behavior of the model is analyzed by means of Portryagin's maximum principle in four steps, which are successive approximations of the complicated investment behavior of real firms. The analytical solutions of the model are partially interpreted in analyzing influences of the investment incentives, which were introduced into the Czech economy in the framework of the new tax system, on the level of investment activity of joint-stock companies formed in the first wave of voucher privatization.

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Introduction

The joint-stock firm is the prevailing type of the Czech privatized industrial firms which were formed in the process of the economic transformation after 1991. This group of firms includes the large companies that form the core of the Czech industry and also the core of the investment problems of the Czech economy.

The critical contemporary problem, which started after a fall in the volume of investment in the year 1991, is the low level of the investment activity of firms. This low level of investment, which continued in 1992 and also in 1993, is a difficult problem that is solved by the Czech economic policy in the second period of the economic transformation. Some investment incentives were introduced into the framework of the tax reform in 1993.

In this paper, we try to construct the mathematical model of a typical joint-stock firm formed in the first wave of voucher privatization, which may facilitate the understanding of its investment dynamics and analyzing the possible impacts of the new investment incentives and the other measures that are oriented on the investment performance of firms.

Our modelling of the representative Czech joint-stock firm is based on the general system methodology of modelling investment dynamics of a firm that we developed in the research paper [ŠTACH-NEDOMA (1988)], on the microeconomic and macroeconomic theories of investment and on the observation of the performance of Czech joint-stock companies, investment funds and stock markets.

The paper is organized as follows. Section 1 deals with problem bounding. In Section 2 we formulate the basic assumptions and relations of the model. In Section 3 we complete the general model and analyse its behavior. In Section 4 we deal with the economic interpretation of the model solutions.

1. Formulation of the Problem

The formulation of the problem that is investigated in this paper is based on the analysis of the historical development and contemporary state of the investment performance of Czech firms in the framework of the mechanism of functioning the Czech economy (see ŠTACH(1993)).

There are many problems in the area of the microeconomic analysis of the investment behavior of firms. This research study deals with one general theoretical problem, the solution of which can facilitate solving existing problems of the overall investment process in the Czech economy. These problems arose after the fall in the volume of investment in the year 1991.

According to the macroeconomic analysis (see ŠUJAN (1993)), the fall of real gross investment (31.8 %) was caused by the restrictive macroeconomic policy at the beginning of economic reform (13.9 %), liberalization of prices (4.1 %), devaluation of currency (9.8 %) and by the collapse of the CMEA (3.9 %).

The fall in real private consumption (23.9 %) after the massive liberalization of prices, the decrease in exports to Eastern European countries and the increase in the average interest rate on banking credits (from 6.6 % in 1990 to l4.5 % in 1991) caused the decrease in the real cash flows of state owned enterprises. Sales fell short, real revenues declined and real expenses, especially on imported raw materials and fuel, increased. The cost of funds increased and availability of funds was worse than in previous years, because banks oriented the portfolios of their credits on loans for the privatization projects. The volume of banking investment credits decreased. Firm's perception of risk increased and their willingness to bear risk decreased. Firms anticipated low future demand for their products and the low profitability of the investment projects. They reduced risk and uncertainty by delaying investment projects.

In 1992, under uncertainty before the split of the CSFR, the volume of investment increased only by 16,5 % and in 1993, after the split, the increase in investment in the Czech economy was only small and it was caused especially by foreign investors.

It is necessary to solve the problems of stimulating investment activities. These solutions have to be based on deep understanding of the investment behavior of the existing types of firms in the Czech economy. Since the prevailing form of the Czech privatized industrial firms is a joint-stock company, it is appropriate to start the investigation of the investment behavior by the modelling the investment performance of this type of a firm. This research can also facilitate the construction of the macroeconomic investment function for the econometric model of the Czech economy.

Deeper investigation into the impact of investment incentives can be facilitated by means of the dynamic mathematical models, which can describe the characteristic features of the investment behavior of the given type of firm and relation betwen the goal of a firm, investment incentives and the level of its investment activity (investment demand).

Our research is directed especially at the influence of the investment incentives that were introduced in the framework of the new tax system in 1993. The main question we try to answer is: What is the influence of these investment incentives on the level of investment activity of the Czech joint-stock firms?

For this investigation we construct an adaptive dynamic model, the behavior of which is similar to the behavior of the real firms. The behavior of the model is analysed in four steps, which are succesive approximations of the complicated behavior of the real system that is modelled.

2. The Basic Assumptions and Relations of the Model

2.1. The Goal of a Joint-Stock Company

The joint-stock companies that were formed in the first wave of voucher privatization are predominantly controlled by privatization investment funds, which acquired the majority of shares in individual firms. Remaining equity shares are owned by small individual shareholders, municipalities, banks and the Fund of National Property. Some selected joint-stock firms (e.g. energy

companies) are still controlled by the Fund of National Property and in several companies the majority of equities is owned by foreign investors.

We can distinguish different types of privatization investment funds. They differ especially in the percentage of reinvestment of the yield of capital. According to this criterion we can distinguish: growth funds, dividend funds and mixed funds. All the privatization investment funds that were founded for the first wave of voucher privatization have the institutional form of a joint-stock company.

The growth funds are oriented on the growth of stockholders' property by reinvesting of the yields of their equities. The dividend funds are oriented on the maximization of paying dividends to their shareholders. The mixed funds combine both these orientations. They pay some dividends but they also reinvest some percentage of their yields. Nevertheless all the investment funds have the same basic goal that consists in maximizing the market value of existing shareholders' equity. This goal is consistent with both the maximization of the growth of property and with the maximization of dividends. It is also consistent with the goals of investment companies that are the financial administrators of privatization investment funds. The reward of administrators for their services depends on the market value of their equities (the maximum is 2 per cent of this value).

The goals of the other shareholders in privatized joint stock firms resembles the fund's goals. Consequentely we may claim that the investment performance of a typical Czech joint-stock firms is aimed at the maximization of the market value of existing shareholders' equity. This goal resembles the goal of the other joint-stock firms.

2.2. Production Function

For the description of the technology of a firm, we use the aggregate production function as a relation that describes the long-term stochastic causal dependence of the net aggregate output of the firm on the main factors of production

(agregate inputs). Generally we use the instantaneous production function of the type

$$Y=f(L,K,t) \tag{1}$$

where Y is the net (of material consumption) output of the firm per unite time, K is the stock of capital employed by the firm, L is the amount of labor per unit time (in hours worked per unit time) and t is the time variable. The variables Y, L and K should each be thought of as functions of time, i.e. $Y(t), L(t), K(t)$.

Particularly we use the Cobb-Douglas function with constant returns to scale. According to Jorgenson's conclusion (see [JORGENSON (1973)]) from econometric studies, this function is the appropriate description of technology for a model of investment behavior.

Generally, we assume that the production function is characterized by positive though diminishing marginal products of capital and labor and a direct dependence of the marginal product of capital (employment of labor) on employment of labor (capital):

$$f_K, f_L > 0; f_{KK}, f_{LL} < 0, f_{KL} > 0.$$

Particularly we assume that the production function f is linearly homogenous in L and K so that

$$\mu f(L,K,t) = f(\mu L, \mu K, t), \mu > 0.$$

Let us consider the production function of the Cobb-Douglas type

$$Y = A_0 e^{\lambda t} L^{\alpha} K^{\beta}, \tag{2}$$

where A_0 is a positive constant, λ is the parameter of technological change and the other factors and α, β are elasticies of output with respect to labor and

capital. If we assume constant returns to scale $(\alpha + \beta = 1)$ then this function is linearly homogeneous and we can derive the dependence of labor productivity $y = Y/L$ on the ratio of capital to labor $k = K/L$ of the form

$$y = A_0 e^{\lambda t} k^{\beta}. \tag{3}$$

In the first version of the dynamic optimization model we use (for simplicity of mathematical analysis) the general production function of the type

$$Y(t) = f(L(t), K(t)),$$

without the influence of technological change. If this function is linearly homogeneous, it can be transformed in the form

$$y(t) = F(k(t)) = f(1, K(t)/L(t)). \tag{4}$$

We assume that $F'(k) > 0,\ F''(k) < 0,$ i.e. it is increasing and concave.

2.3. The Equation of the Change in the Capital Stock

We describe the change in the stock of capital by means of the differential equation

$$\frac{dK(t)}{dt} = I_r(t) - \delta K(t) \tag{5}$$

where $I_r(t) = I(t)/p(t)$ is the level of real gross investment, $I(t)$ is the nominal level of gross investment spending, δ is the instantaneous rate of physical depreciation of the real capital stock $K(t)$ and $p(t)$ is the price level (price index). We assume (for simplicity) that the price index of capital goods is the same as the price index of the other input and output commodities.

Particularly we assume (for the simplicity of the solution) that the firm expects the price level to follow the trajectory

$$p(t) = p(0) e^{\pi t} \tag{6}$$

where π is the anticipated rate of inflation.

2.4. The Employment Level of the Firm and Wage

Generally we consider the amount of labor to be decision variable, but particularly we consider it to be the "exogenous" input variable of the model that is determined on the basis of strategic considerations. The reasons for this are both analytical and economic.

The analytical reason is to simplify the solutions of dynamic models into manageable forms. The economic reasons are following. The theoretical assumption that firms can adjust the amount of labor employed costlessly is not adequate for the description of the trajectory of $L(t)$. Under the assumption of competitive behavior, it leads to the result that, in equilibrium, the gross (of taxes) wage in period $t, w(t)$, must equal the marginal product of labor and then to the equation which expresses the demand for labor as a function of the level of the output and the wage or the wage as a function of the stock of capital and labor in the same year.

This seems to be rational, but Czech managers say that they cannot assume that the adjustment of the amount of labor employed is costless. When they expect the growth of demand and production in the next period, they cannot dismiss qualified workers from employment, because of the large costs of dismissal, the subsequent search for new workers and their specialized training and also the heavy loss on the firm's production in the next period. They know that their firm has accumulated the stock of human capital in its employees and that they obviously cannot deal with them as with cement or the other inputs. The reasons for this are also social, ethical and cultural.

The selection of the optimal trajectory $L(t)$ is a complicated problem, which is usually solved by the strategic management of the firm. We do not deal with it here. We particularly take the trajectory of the employment level of the firm as exogeneously given and restrict the analysis to the selection of the optimal trajectory of $K(t)$.

Particularly we assume (for simplicity) that the growth of the labor force can be described by means of the exponential function

$$L(t) = L(0)\, e^{nt} \tag{7}$$

where n is the growth rate of the labor force, which can be positive, zero or negative. If $n = 0$, we write $L(t) = L$.

Similarly, it can be assumed that the trajectory of the wage is exogenously given (the firm is a price taker on the labor market) and can be described by the exponential function

$$W(t) = W(0)e^{(\pi + \zeta)t} \tag{8}$$

where $W(t)$ is the nominal wage, ζ is the growth rate of the real wage and π is the anticipated inflation rate. Then the trajectory of the real wage is given by

$$w(t) = \frac{W(t)}{p(t)} = w(0)\, e^{\zeta t}, \tag{9}$$

where $w(t)$ is real wage.

2.5. The Objective Function

We assume that the goal of investment decision making of a joint-stock firm is to maximize the price of a share of equity (shareholder's value of the firm). According to macroeconomic theory, equities and government-issued bonds can be considered to be perfect substitutes (from the viewpoint of investment funds and the other wealth holders), which implies that the nominal interest rate on bonds is an appropriate rate for discounting the firm's cash flow and the market price of an existing share of equity is determined by the present value of the firm's future cash flow discounted at this market rate of interest (nominal rate of return). For simplicity, we assume that a firm finances all of its net investment by issuing new equities. We do not consider the possibility of selling bonds, retaining earnings and using banking credits. However, these factors could be taken into account in the interpretation of the model.

If we do not consider the influences of a corporation income tax and uncertainty and assume that the nominal rate of return on earning assets to remain constant into the indefinite future, the price of a share of equity in the representative joint-stock firm can be expressed by means of the relation

$$P_e = \frac{1}{E}\int_0^{\infty} p(t)[f(L(t),K(t),t) - w(t)L(t) - I_r(t) - C(\dot{K})]\, e^{rt} dt, \tag{10}$$

where P_e is the price of a share of equility, E is the number of existing shares of equity, r is the nominal rate of return and $C(\dot{K})$ denotes the real resource costs of installing new capital (adjustment costs), $\dot{K} = \frac{dK}{dt}$.

The level of gross real investment $I_r(t) = I(t)/p(t)$ can be expressed, by means of equation (5), in the form of equation

$$I_r(t) = \dot{K}(t) + \delta K(t). \tag{11}$$

Now let us consider the impact of taxation of the firm's income on the construction of the objective function. According to the new Czech Income Tax Act (which became effective in January 1992), corporation tax is constructed as a linear income tax with profits as the tax base (in 1994 it is 42 per cent). This act also includes some investment incentives, especially the tax allowance ("investment tax credit"), which refunds a certain fraction of the cost of an asset in the year purchased, and the acceleration of depreciation allowances, which defers tax liabilities on new investments into the future without assessing the taxpayer interest on this deferral. Tax allowances are on investment in new machines and equipment. Companies are allowed to deduct 10 per cent of the price of an asset purchased from the tax base.

Assume that the fraction z of the cost of new investment that is deducted by the company is constant in time. Then the profit tax $T(t)$ of a joint-stock company can be expressed as

$$T(t) = \tau(\Pi(t) - zI(t)), \tag{12}$$

where τ is the profit tax rate and $\Pi(t)$ is profit given by

$$\Pi(t) = p(t)\,(f(L(t),K(t),t) - w(t)L(t) - dK(t) - C(\dot{K}(t))), \tag{13}$$

where d is discounted value of depreciation allowances for tax purposes.

Substituting the profit function (13) in (12) and using relations $I(t) = p(t)\,I_r(t)$ and (11) we obtain the tax function of the form

$$\begin{aligned} T(t) \;=\; & \tau p(t)[f(L(t),K(t),t) - w(t)L(t) - \\ & - (d + z\delta)K(t) - z\dot{K}(t) - C(\dot{K})] \;. \end{aligned} \tag{14}$$

By means of this tax function it is possible to modify the cash flow in the equation (10) and we obtain the dynamic objective function of the investment process of a joint-stock firm

$$\begin{aligned} J = \frac{1}{E}\int_0^{\infty} & \{p(t)[f(L(t),K(t),t) - \\ & - w(t)L(t) - I_r(t) - C(\dot{K})] - T(t)\}e^{-rt}dt \end{aligned} \tag{15}$$

Plugging (14) and (11) into (15), the objective function may be rewritten in the form

$$\begin{aligned} J = \frac{1}{E}\int_0^{\infty} & p(t)\{(1-\tau)[f(L(t),K(t),t) - w(t)L(t) - \\ & - C(\dot{K})] - (\delta - \tau(d + z\delta))K(t) - (1 - \tau z)\dot{K}(t)\}e^{-rt}dt. \end{aligned} \tag{16}$$

If we denote

$$\bar{\delta} = (\delta - \tau(d + z\delta)),$$

$$\alpha = (1 - \tau z)$$

and the rate of change in the capital stock (real net investment) by

$$i(t) = \dot{K}(t),$$

we obtain the function (16) in the form

$$J = \frac{1}{E}\int_0^{\infty} p(t)[(1-\tau)(f(L(t),K(t),t)) - \\ - w(t)L(t) - C(i(t)) - \bar{\delta}K(t) - \alpha i(t)]e^{-rt}dt \qquad (16a)$$

This dynamic criterion describes the influences of the main factors on the price of a share of equity.

3. The Model and the Analysis of its Behavior

3.1. Model formulation

Summing up the above discussion and relations, the Czech joint-stock firm is faced with the following general optimal control problem:

$$\max_{i(t)} \frac{1}{E} \int_0^{\infty} p(t)[(1-\tau)(f(L)(t),K(t),t) - \\ - w(t)L(t) - C(i(t)) - \bar{\delta}K(t) - \alpha i(t)]e^{-rt}dt \tag{17a}$$

subject to

$$\dot{K}(t) = i, \tag{17b}$$

$$K(0) = K_0. \tag{17c}$$

This problem has one state variable K and two control variables i and L. It can be analysed by means of the Pontryagin's maximum principle. We do it in 4 steps that differ in assumptions on the exogenous parameters and the control variables.

3.2. The Analysis of the Behavior of the Model

Step 1: In the first step of the analysis we assume that there is no influence of technological progress and that the price level and real wage are constant, i.e. $p(t) = p, w(t) = w,$ and that the amount of labor is determined endogenously as the second control variable. The problem involves the selection of optimal trajectories $L(t)$ and $K(t)$.

If there is no technological change in the firm, then the production function f is invariant in time, i.e. $Y(t) = f(L(t),K(t))$. We also assume that capital adjustment costs are given by the quadratic function

$$C(i(t)) = \frac{a}{2} i(t)^2, \tag{18}$$

where a is a positive constant.

The optimization problem is therefore given by

$$\max_{\mathbf{i}(t),L(t)} \frac{p}{E}\int_0^{\infty}\{(1-\tau)[f(L(t),K(t))-wL(t)]- \\ -\bar{\delta}K(t)-\alpha i(t)-(1-\tau)\frac{a}{2}i(t)^2\}e^{-rt}dt \tag{19}$$

subject to

$$\dot{K}(t)=i,$$

$$K(0)=K_0.$$

The joint-stock firm seeks to chose trajectories of $L(t)$ and $K(t)$ that maximize the price of an existing share of equity.

The present-value Hamiltonian for this problem is given by

$$H^*=(1-\tau)[f(L(t),K(t))-wL(t)]- \\ -\bar{\delta}K(t)-\alpha i(t)-(1-\tau)\frac{a}{2}i(t)^2+mi \tag{20}$$

where m is the multiplier equal to $e^{rt}\psi(t)$ and $\psi(t)$ is the ordinary multiplier.

Necessary conditions for a maximum are given by

$$\frac{\partial H^*}{\partial L}=(1-\tau)\left(\frac{\partial f(L,K)}{\partial L}-w\right)=0 \tag{21}$$

$$\frac{\partial H^*}{\partial i}=-(\alpha+(1-\tau)ai)+m=0 \tag{22}$$

$$\frac{dm}{dt} = \dot{m} = -\frac{\partial H^*}{\partial K} + rm = -(1-\tau)\frac{\partial f(L,K)}{\partial K} + \bar{\delta} + rm = 0. \qquad (23)$$

The condition (21) leads to the standard (Walrasian) result

$$\frac{\partial f(L,K)}{\partial L} = f_L = w, \qquad (24)$$

which means that in equilibrium, the marginal product of labor must be equal the gross (of taxes) wage in the given period. This can be used for generating the labor demand function

$$L^d = L^d(w,K).$$

Differentiating the first-order condition (22) with respect to time, combining the resulting expression with the equation (23) and using the definition $i = \dot{K}$, we obtain the following second-order differential equation

$$\ddot{K} - r\dot{K} + \frac{1}{(1-\tau)a}\left[(1-\tau)\frac{\partial f}{\partial K} - \alpha r - \bar{\delta}\right] = 0 \qquad (25)$$

The steady - state solution to this differential equation we obtain by means of condition $\ddot{K} = \dot{K} = 0$. It is given by $K = K^*$, that solves the equation

$$\frac{\partial f(L^d(w,K^*),K^*)}{\partial K^*} = \frac{\alpha r + \bar{\delta}}{(1-\tau)} \qquad (26)$$

where $\alpha = (1-\tau z)$ and $\bar{\delta} = (\delta - \tau(d + z\delta))$.

This means that, in the steady - state, the optimal value of the capital stock K equates the marginal revenue product of capital with the price of capital that is given by

$$\nu = \frac{\alpha r + \bar{\delta}}{1-\tau} = \frac{(1-\tau z)r + (\delta - \tau(d + z\delta))}{1-\tau} = \frac{(1-\tau z)(r+\delta) - \tau d}{1-\tau} \qquad (27)$$

This can be interpreted as the after-tax cost of capital services (or depreciable assets).

The equation (26) can be used in investigating the influence of investment incentives, especially of the fraction of the cost of new-investment that is deducted, on the level of the firms' investment. If we assume that $f_{KK} < 0$, then $\frac{\partial f}{\partial K^*}$ is a decreasing function and the increase of z leads to a decrease of ν that causes an increase in the optimal value of the capital stock K and investment demand.

Under the absence of technological progress and the other given assumption, the optimal firm's investment behavior only gradually closes the gap between the existing stock of capital and its optimal value. Under a certain assumption about $f(L,K)$ it is possible to derive a differential equation, the solution of which describes the trajectory of $K(t)$ from a given initial state $K(0)$ to the optimal state $K^*(t)$ and then we can derive a corresponding investment demand function.

If the production function $f(L,K)$ is linearly homogeneous, i.e. there are constant returns to scale in the firm, then the capital stock $K(t)$ and the size of the firm are indeterminate, because thus

$$\frac{\partial}{\partial K} f(L,K) = \frac{\partial F(\mu L, \mu K)}{\partial \mu K}$$

and equation (26) has an infinite number of solutions (for $\mu \in (0,\infty)$).

If there are no additional constraints, there is no optimal capital stock K^* and the size of the firm is indeterminate. The contraints have to describe real contraints in the firm's activities. They exist especially in the area of employment, investment processes and financing and also in demand for the

production of the firm. We describe the contraints on the employment level by means of the exponential function $L(t) = L(0)e^{nt}$. Some contraints in the investment process can be described by means of the function of capital stock adjustment cost $C(i)$. The constraints for the output of the firm can be considered in the given model as an exogenous variable, which is predicted by strategic management of the firm and used in strategic decisions about the future level of employment.

Step 2: This step provides appropriate modifications to the analysis of the first step for the case of constant but nonzero expected inflation and the exogenously given employment level, $L(t) = L(0)e^{nt}$. The possibility of technological change is still ruled out. The real wage is therefore also assumed to be constant. We restrict our analysis to the firm's selection of an optimal trajectory of $K(t)$.

If the price level is given by $p(t) = p(0)e^{\pi t}$, and if we assume constant returns to scale, then the objective function (17a) may be transformed (under given assumptions) in the following form

$$J = \frac{p(0)}{E}\int_0^{\infty} L(t)\left\{(1-\tau)\left[f(1,\frac{K(t)}{L(t)}) - w - \frac{C(i(t))}{L(t)}\right] - \right. \\ \left. - \bar{\delta}\frac{K(t)}{L(t)} - \alpha\frac{i(t)}{L(t)}\right\}e^{-\rho t}dt. \qquad (28)$$

where $\rho = (r - \pi)$ is the real interest rate. If we use the definition of the ratio of capital to labor $k(t) = K(t)/L(t)$, the relation

$$\frac{i(t)}{L(t)} = \frac{\dot{K}(t)}{L(t)} = \dot{k}(t) + nk(t),$$

where $\dot{k}(t)$ denotes $\frac{dk}{dt}$, and the production function of the form (4), then above the criterion may be expressed as

$$J = \frac{p(0)}{E}\int_0^{\infty} L(t)\Big\{(1-\tau)\Big[F(k)-w-\frac{C(i(t))}{L(t)}\Big]- \\ -(\bar{\delta}+\alpha n)k(t)-\alpha\dot{k}(t)\Big\}e^{-\rho t}dt. \tag{29}$$

In order to use this criterion in the analytically solvable model we have to make some assumptions about the form of the capital-stock adjustment term. This term describes the fact that attempts to rapidly install a large quality of new capital may be prohibitively expensive. It is quite obvious that the particularly high rates of installation of new capital $\dot{K} = i$ may be expensive to implement. We might expect that the absolute size of adjustment costs are likely to rise as the size of the firm grows and we assume that the per capita value of these costs is the quadratic function of $\dot{k}$

$$\frac{C(i(t))}{L(t)} = \frac{1}{2}a(\dot{k}(t))^2. \tag{30}$$

If we use this function together with the function $L(t) = L(0)e^{nt}$ we obtain the criterion

$$J = \frac{p(0)L(0)}{E}\int_0^{\infty}\Big\{(1-\tau)\Big[F(k)-w-\frac{a}{2}(\dot{k})^2\Big]-\tilde{\delta}k-\alpha\dot{k}\Big\}e^{-(\rho-n)t}dt \tag{31}$$

where $\tilde{\delta} = (\bar{\delta}+\alpha n)$.

If we define the control variable as $u(t) = \dot{k}(t),$ the optimization problem is given by

$$\max_{u(t)} \int_0^{\infty}\Big\{(1-\tau)\Big[F(k)-w-\frac{a}{2}u^2\Big]-\tilde{\delta}k-\alpha u\Big\}e^{-(\rho-n)t}dt \tag{32a}$$

subject to

$$\dot{k} = u \tag{32b}$$

$$k(0) = k_0, k(\infty) \geq 0. \tag{32c}$$

The present-value Hamiltonian for this problem is given by

$$H^* = (1-\tau)(F(k) - w - \frac{a}{2}u^2) - \tilde{\delta}k - \alpha u + mu \tag{33}$$

where m is the current value multiplier given by $m = e^{(\rho - n)t}\Psi$.

Necessary conditions for a maximum are given by

$$\frac{\partial H^*}{\partial u} = -(1-\tau)au - \alpha + m = 0, \tag{34}$$

$$\dot{m} = -\frac{\partial H^*}{\partial k} + (\rho - n)m = -(1-\tau)F'(k) + \tilde{\delta} + (\rho - n)m. \tag{35}$$

The transversality conditions for this problem are given by

$$\begin{aligned} &\lim_{t\to\infty} m(t)e^{-(\rho-n)t} \geq 0 \\ &\text{and} \\ &\lim_{t\to\infty} m(t)k(t)e^{-(\rho-n)t} = 0 \end{aligned} \tag{36}$$

Differentiating the first-order condition (34) with respect to time we obtain

$$\dot{m} = -(1-\tau)a\dot{u} = (1-\tau)a\ddot{k}. \tag{37}$$

Now combining this expression as well as the expression (34) with the equation (35) we obtain

$$(1-\tau)a\ddot{k} = -(1-\tau)F'(k) + \tilde{\delta} + (\rho - n)[(1-\tau)a\dot{k} + \alpha].$$

Upon using $\tilde{\delta} = (\bar{\delta} + \alpha n)$ and rearranging, we obtain the following second-order differential equation

$$\ddot{k} - (\rho - n)\dot{k} + \frac{1}{(1-\tau)a}[(1-\tau)F'(k) - (\bar{\delta} + \alpha\rho)] = 0. \qquad (38)$$

This is a nonlinear, nonhomogenous differential equation in k. By means of conditions $\ddot{k} = \dot{k} = 0$ we obtain its steady-state solution. It is given by $k = k^*$, that solves

$$F'(k^*) = \frac{\bar{\delta} + \alpha\rho}{1-\tau}. \qquad (39)$$

Since $k = K/L$ and $F(k) = f\left(1, \frac{K}{L}\right)$, then

$$F'(k) = \frac{\partial f\left(1, \frac{K}{L}\right)}{\partial\left(\frac{K}{L}\right)} = \frac{\partial\left[Lf\left(1, \frac{K}{L}\right)\right]}{\partial K} = \frac{\partial f\ (L,K)}{\partial K}.$$

The relation (39) can be rewritten as

$$\frac{\partial f(L,K)}{\partial K} = \frac{\alpha\rho + \bar{\delta}}{1-\tau}. \qquad (39a)$$

This equation is practically only the modification of the equation (26), with respect to inflation and exogenously determined $L(t) = L(0)e^{nt}$. .
The right hand side of (39a) can be rewritten in the following manner

$$\nu = \frac{\alpha\rho + \bar{\delta}}{1-\tau} = \frac{(1-\tau z)\rho + (\delta - \tau(d + z\delta))}{1-\tau} = \frac{(1-\tau z)(\rho+\delta) - \tau d}{1-\tau}. \qquad (40)$$

The value of ν can be interpreted as the after-tax real cost of capital services (depreciable assets).

If the initial value is $k(0) = k_0 = k^*$, then the optimal trajectory of the system is

$$k(t) = k^* = const.,$$

i.e. to sustain the ratio of capital to labor $k(t) = K(t)/L(t)$ on the constant optimal level. The optimal trajectory of $K(t)$ is then

$$K(t) = L(t)k(t) = L(0)e^{nt}k^*.$$

If the initial state is $k(0) = k_0 \neq k^*$, then the optimal firm's investment behavior has to close the gap between the existing $k(t)$ and the optimal k^*.
Mathematically it is possible to analyse this transient behavior by solving the nonlinear differential equation (38).

The homogenous part of the solution for the optimal trajectory of *k* is governed by

$$\ddot{k} - (\rho - u)\dot{k} + \frac{F'(k)}{a} = 0 \qquad (41a)$$

In order to obtain an analytical solution of this equation we can locally approximate (by means of Taylor series) the nonlinear production function

$F(k)$ in the neighborhood of the steady stable solution, k^*, by a quadratic function $Q(k)$, such that

$$Q'(k) \doteq F'(k^*) + (k - k^*)F''(k^*). \tag{41b}$$

In this way we get a linear, second-order differential equation in k

$$\ddot{k} - (\rho - n)\dot{k} + (k - k^*)F''(k^*)/a + F'(k^*)/a = 0 \tag{41c}$$

where $F'(k^*)$ and $F''(k^*)$ are constants.

By solving of characteristic equation of the linear part of this equation we obtain the following characteristic roots σ_1, σ_2

$$\sigma_i = \frac{1}{2}\left\{(\rho - n) \pm \left[(\rho - n)^2 - \left(\frac{4F''(k^*)}{a}\right)\right]^{\frac{1}{2}}\right\}, \quad i = 1,2. \tag{42}$$

If we assume that $\rho - n > 0$, then the equation (42) has just one real negative root σ_1 which produces the solution satisfying the transversality conditions. For this root σ_1 we obtain nonexplosive time function

$$k(t) = k^* + (k_0 - k^*)e^{\sigma_1 t} \tag{43}$$

which can be taken for the approximate solution of (38) in a neighborhood of the steady state k^*.

The function (43) describes the transient trajectory from the initial state $k(0) = k_0$ to the stationary trajectory $k(t) = k^*$. Differentiating the optimal trajectory we obtain

$$\dot{k}(t) = \sigma_1(k_0 - k^*)e^{\sigma_1 t}. \tag{44}$$

Using approximation (41b) and the relation (39) we obtain

$$F'(k) - \frac{\bar{\delta} + \alpha\rho}{1-\tau} = F'(k^*) + (k-k^*)F''(k^*) - \frac{\bar{\delta} + \alpha\rho}{1-\tau} = (k-k^*)F''(k^*),$$

in general for any k and for $k = k_0$ in particular. We therefore obtain

$$k_0 - k^* = \frac{F'(k_0) - \dfrac{\bar{\delta} + \alpha\rho}{1-\tau}}{F''(k^*)}.$$

Evaluating $\dot{k}(t)$ at $t = 0$ we obtain

$$\dot{k}(0) = \sigma_1(k_0 - k^*)$$

or

$$\dot{k}(0) = \frac{\sigma_1}{F''(k^*)}\left[F'(k_0) - \frac{\bar{\delta} + \alpha\rho}{1-\tau}\right].$$

If we denote per capita net investment $\dot{K}/L = i/L = \dot{k} + nk \equiv \tilde{i},$ we obtain a function

$$\tilde{i} = \xi_k\left[F'(k) - \frac{\bar{\delta} + \alpha\rho}{1-\tau}\right] + nk, \; \xi_k = \frac{\sigma_1}{F''(k^*)} > 0 \qquad (45)$$

Using the definition of $\tilde{i}$ and (40) we obtain an investment function

$$i(t) = L(t)\tilde{i}(t) = L(0)e^{nt}\left\{\xi_k[F'(k(t)) - \nu] + nk(t)\right\}. \qquad (46)$$

This function relates firm's demand to accumulate newly produced capital at some finit rate per unit time directly to the gap between the marginal productivity $F'(k(t))$ and the real cost of capital ν and also to the rate of growth (or decrease) of the labor force.

According to (45), firms invest at a hig nal product of capital is higher and the real interest rat $= (\delta - \tau(d + z\delta))$, the transient investment demand of a firm d rameters τ, z, d. The increase of z and d, which descri tives, causes the increase of investment demand, because ase of $\bar{\delta}$ and then the decrease of the cost of capital ν. A nds directly on the profit tax rate τ , which affects in the

Step 3: In this step we extend the p or the case with the neutral technological change and al wage given by $w(t) = w(0)e^{\zeta t}$. We do not conside licity) adaption costs. Instead of these prohibitive costs we nts on the investment variable $u = \dot{k}$ of the form

$$\underline{u} \leq u$$

where $\underline{u}$ is the lower value and $\bar{u}$ is t of u. For the description of technology we use the Cobb-Douglas function of the form (3), i.e. $y = A_0 e^{\lambda t} k^{\beta}$.

We can obtain the optimization proble ese assumptions by means of the simple modification of the problem 2c). It has the form

$$\max_{u(t)} \int_0^{\infty} \{(1-\tau)[A_0 e^{\lambda t} k^{\beta} - w(\tilde{\delta} k - \alpha u\} e^{-(\rho - n)t} dt \qquad (47a)$$

ject to

$$\dot{k} = \qquad (47b)$$

Since $k(t) = k_\lambda(t)\, e^{\frac{\lambda}{1-\beta}t}$ and

$$\dot{k} = \frac{dk}{dt} = \frac{dk_\lambda}{dt}\, e^{\frac{\lambda}{1-\beta}t} + \frac{\lambda}{1-\beta} k_\lambda(t) e^{\frac{\lambda}{1-\beta}t},$$

the equation (47b) can be transformated into the form

$$\frac{dk_\lambda}{dt} = -\frac{\lambda}{1-\beta} k_\lambda + u\, e^{-\frac{\lambda}{1-\beta}t} = -\frac{\lambda}{1-\beta} k_\lambda + u_\lambda, \tag{51}$$

and the constraints (47c) have the form

$$\underline{u}_\lambda(t) \le u_\lambda(t) \le \bar{u}_\lambda(t) \ , \tag{52a}$$

where

$$\underline{u}_\lambda(t) = \underline{u} e^{-\frac{\lambda}{1-\beta}t} \tag{52b}$$

$$\bar{u}_\lambda(t) = \bar{u} e^{-\frac{\lambda}{1-\beta}t}. \tag{53c}$$

The formal optimization problem is therefore given by

$$\max_{u\ (t)}$$

subject to

$$k_\lambda(0) = k_0,\ k_\lambda(\infty) \geq 0. \qquad \text{(54d)}$$

This system is autonomous. The present-value Hamiltonian for this problem is given by

$$H \quad A\,k \quad k \quad u$$

The transversality conditions for this problem are given by

$$\lim_{t\to\infty} m(t)e^{-\rho' t} \geq 0 \ \textit{and} \ \lim_{t\to\infty} m(t)k_\lambda(t)e^{-\rho' t} = 0. \tag{56e}$$

First, let us consider the state of the system, when $m(t) = \alpha = const.$ Then

$$\frac{dm}{dt} = 0$$

and the equation (56b) has the form

$$\left(\rho' + \frac{\lambda}{1-\beta}\right)\alpha + \tilde{\delta} - \beta(1-\tau)A_0 k_\lambda^{\beta-1} = 0. \tag{57}$$

The solution of this equation is given by

$$k_\lambda(t) = \bar{k} = \left[\frac{\tilde{\delta} + \left(\rho' + \frac{\lambda}{1-\beta}\right)\alpha}{\beta(1-\tau)A_0}\right]^{1/\beta-1}. \tag{58a}$$

Using definitions

$$\begin{aligned} \tilde{\delta} &= \bar{\delta} + \alpha n = \delta - \tau(d + z\delta) + \alpha n, \\ \alpha &= (1-\tau z), \end{aligned}$$

$$\rho' = \rho - n - \frac{\lambda}{1-\beta},$$

we can transform equation (58a) in the form

$$\begin{aligned} k_\lambda(t) = \bar{k} &= \left[\frac{\delta - \tau(d + z\delta) + \alpha n + (\rho - n)\alpha}{(1-\tau)\beta A_0}\right]^{1/\beta-1} = \\ &= \left[\frac{(1-\tau)\beta A_0}{(1-\tau z)\rho + \delta - \tau(d + z\delta)}\right]^{1/1-\beta}. \end{aligned} \tag{58b}$$

This equation implies that k_λ is constant in time and

$$\frac{dk_\lambda}{dt} = 0.$$

This means that equation (58b)describes a stationary trajectory of $k_\lambda(t)$. After rearranging equation (57) into the form

$$\beta A_0 k_\lambda(t)^{\beta-1} = \frac{(1-\tau z)(\rho+\delta)-\tau d}{(1-\tau)}$$

and substituting

$$k_\lambda(t) = k(t)e^{-\frac{\lambda}{1-\beta}t}$$

we obtain

$$\beta A_0 e^{\lambda t} k(t)^{\beta-1} = \frac{(1-\tau z)(\rho+\delta)-\tau d}{(1-\tau)}. \qquad (58c)$$

The left hand side of this equation is the first derivate $F'(k(t))$ of the production function

$$F(k,t) = A_0 e^{\lambda t} k(t)^{\beta},$$

that we used. The right hand side of this equation is equal to the right hand side of (40). This means that it can be interpreted also as the after-tax price of capital. Consequently, the relation (58c) is practically a marginal condition for optimum.

The optimal trajectory, however, is not stationary. The labor productivity and the ratio of capital to labor are increasing. The optimal trajectory $k(t)$ is given (after the inverse transformation of (58b) by means of (48a)) by

$$k^*(t) = k_\lambda(t) e^{\frac{\lambda}{1-\beta}t} = \bar{k} e^{\frac{\lambda}{1-\beta}t}. \tag{59}$$

We obtain the optimal control function from equation (54b). Since $\frac{dk_\lambda}{dt} = 0$, it has the form

$$-\frac{\lambda}{1-\beta} k_\lambda + u_\lambda = 0 \tag{60}$$

and its solution is

$$u_\lambda(t) = \frac{\lambda}{1-\beta} k_\lambda(t). \tag{61}$$

Using (58b) in (61) we obtain

$$u_\lambda(t) = \frac{1}{1-\beta} \bar{k}. \tag{62}$$

and after the inverse transformation (by means of (48b)) of (62) we obtain the asymptotic optimal solution of the problem (47a - 47d) in the form

$$u^*(t) = u_\lambda(t) e^{\frac{\lambda}{1-\beta}t} = \frac{\lambda}{1-\beta} \bar{k} e^{\frac{\lambda}{1-\beta}t}. \tag{63}$$

We can see that $u^*(t) = \dot{k}^*(t) = \frac{\lambda}{1-\beta} k^*(t)$.

If the initial state of a firm is such that

$$k(0) = k_0 = k^*(0) = \bar{k},$$

then the trajectory $u^*(t)$ given by (63) is the optimal solution of the problem (47a - 47b).

If the system is not in this initial state, then (according to the neccesary condition for maximum) it has to be a transient process.

If it is

$$k(0) < k^*(0),$$

then $m(0) > \alpha$ and in the initial period $(0,t_1)$ it has to be applied (according to (56d)) the control function $u(t) = \bar{u}$ (corresponding to $u_\lambda(t) = \bar{u}_\lambda(t)$) up to the point t_1 in which the ratio of capital to labor is equal to its optimal level

$$k(t_1) = \bar{k}\, e^{\frac{\lambda}{1-\beta} t_1}.$$

In the following period the asymptotic trajectory of the type (63)can be applied.

If it is $k(0) > k^*(0),$ then the control function $u(t) = \underline{u}$ has to be applied up to the time t_1 for which is

$$k(t_1) = \bar{k}\, e^{\frac{\lambda}{1-\beta} t_1}.$$

Practically, it means to invest or disinvest as much as possible up to the point where the optimal ratio of capital to labor is reached.

Step 4 - An Adaptive Model: Our previous analysis of the model is based on restricting the trajectories of the processes for $w(t)$, $p(t)$ and $L(t)$. We have assumed that the firm expects changes of w, p and L at constant rates and that the structural parameters $\delta, d, \tau, z, A_0, \beta, \lambda$ are constant in time. The models cannot be expected to describe investment demand for arbitrary trajectories of $p(t), w(t)$ and $L(t)$ which the firm believes itself to be confronted with. Economic processes in the long-term horizont are not deterministic nor stochastic. Firms do not predict their trajectories by means of statistical methods. For this purpose

they use predominantly forecasting methods and models. In general, the optimal investment plan of the firm does not obey a simple mathematical solution like (46) or (63).

This plan usually depends on the predictions of the development of $p(t), w(t)$ and demand for production and also on the firm's conception of future technologies and employment. This influences the development of the future technological parameters of their models. There are also possible changes in the fiscal policy of the government and in tax rates.

In order to manage this complicated behavior we have developed an adaptive approach to modelling the dynamics of a production firm (see[ŠTACH(1991)]). In the adaptive aggregate model of the investment behavior of a firm we assume that the structural parameters of the model $A_0, \beta, \lambda, \delta, d, n, \tau, z, \rho, \zeta$ are changing in time. Some of these changes have, however, a more continuous nature than the character of discontinuous jumps.

An infinite time horizont $T = \{t \mid 0 \le t \le \infty,\ t \in \boldsymbol{R}\}$ of the basic deterministic model (17a - 17c) can be partitioned in shorter horizons $<0,t_1>,<t_1,t_2>,...,<t_n,\infty)$, such that the changes of the parameters of the model within them are not large and we can neglect them in the analysis. Then we do not investigate the complicated solution of the dynamic optimization problem with changing parameters, but we analyse only a sequence of simple dynamic problems with constant parameters $A_0^i, \beta^i, \lambda^i, \delta^i, d^i, n^i, \tau^i, z^i, \rho^i, \zeta^i$, where $i = 1,2,...,$ in the sliding horizons $<t_1,\infty), <t_2,\infty),...,<t_i,\infty),....$ It is also possible to choose a finite time horizon $T = \{t \mid 0 \le t \le t_T\}$, if it is appropriate, and to solve partial problems with constant parameters in finite sliding horizons

$$<0,t_T>,<t_1,t_1+t_T>,<t_2,t_2+t_T>,...,<t_n,t_n+t_T>.$$

The trajectories that are the solutions of these problems (respectively their initial parts) substitute approximately the single segments of the optimal trajectory of the given system. It is assumed that the strategic management of the joint-stock

firm estimates the expected values of these parameters for the first segment of the sliding horizon $<t_i,t_i+t_T>$ and in the remaining segment of this horizon we can (for simplicity) assume using the same parameters as in the first segment. These estimates can be changed in the following adjustment of parameters.

Assume that the firm solves the dynamic optimization problem of the type (47a - 47d) in the sliding horizons $<t_i,\infty), i = 1,2,....$ Let us measure time in a single optimization problem from the zero value, i.e. in the framework of the partial optimization problem it is $t\in<0,\infty)$. For simplicity, it is assumed that the firm estimates the parameters of its decision models periodically, e.g. in the framework of the annual modification of the long-term strategy. In an adaptive approach we assume that the parameter n is also an exogenous strategic decision variable as well as an investment variable

$$u = \dot{k}\left(= \frac{i}{L} - nk = \frac{\dot{K}}{L} - nk \right),$$

which is assumed to be limited by $\underline{u} \le u(t) \le \overline{u}.$ The lower value $\underline{u}$ can be negative. This means that there is possible disinvestment. In this case the rate of growth of employment can also be negative.

Adaptive solving of the sequence of the optimization problems of the given type in the sliding horizon forms an adaptive strategy, which at a given time approximates the firm's optimal strategy. The length of the sliding horizon has to be large enough, in order that it may not be the effect of the end of the time horizon. According to the maximum principle, in the last period of a finite time horizon the investment has to be minimal. From this point of view it is better to assume that $t_T \to \infty$. However, in practice only the short segment of this infinite horizon is used.

Under this assumption the strategy in the given segment of the time horizont consists of two parts. In the first period it is invested or disinvested as much as possible up to the point by which the ratio of capital to labor, $k(t)$, is equal

to its optimal level $k^*(t)$ given by the equations (58b) and (59). If it is possible, the amount of labor per unit of time $L(t)$ can be changed at the same time.

In practice managers do not determine the optimal level of the ratio of capital to labor by means of the simple equations, but on the basis of complicated economic analyses of the state of the stock of capital in the single parts of the firm. They can estimate adjustment cost and the optimal strategy of changes of the stock of capital by means of the methods of operation research.

After reaching of the optimal level of $k(t)$ it is appropriate for the firm (according to the given model) to move along the asymptotic trajectory $k^*(t)$ with the optimal value of investment parameters $u^*(t) = \dot{k}^*(t)$ (see equation (63)). That implicates the investment function

$$i(t) = L(t)\tilde{i}(t) = L(t)(\dot{k}^*(t) + nk^*(t)) = L(t)(u^*(t) + nk^*(t)) \tag{64}$$

where $\tilde{i} = \frac{i}{L} = \dot{k} + nk.$ Numerically it is possible to compute the investment strategy $i(t), t \in \langle 0, t_T \rangle$, more adequately by means of the dynamic programming techniques.

The firm can continue on this asymptotic trajectory as long as the changes of structural parameters are not so great, that the real values of $k(t)$ differs essentially from its optimal values $k^*(t)$ which corresponds to new parameters of the given production and tax system at the time t_i.

If there is a greater difference between $k(t)$ and $k^*(t)$, then the firm can seek the appropriate regulation process leading to the state $k^*(t)$ and then adapt investment $i(t)$, in order that it may reach the optimal investment parameter $u^*(t)$ determined on the basis of new predictions of the system parameters for the next segment of the time horizon. In a long-term horizon this process can be periodically (or non-periodically) repeated.

4. Interpretation of the Model

Our model is based, as the other models are, on the simplifications and assumptions which are described in Section 2. It does not describe explicitly the influence of the demand for the firm's production and the influence of risk and uncertainty on the price of a share of equity. The influence of the demand for production is implicitly involved by means of restricting the trajectory of the amount of labor $L(t)$. The uncertainty in the future development is respected by means of the adaptive approach to modelling. In this approach we use the mean values of the parameters and variables of the model.

The influence risk has on the valuation of assets can be respected by means of the certainty equivalent of a firm's cash flow, which is reduced by means of a risk discount factor, the variances of the firm's profits and their covariances with the profits of the other firms (see STIGLITZ (1972)). Within the framework of a mean-variance model, the value of a firm can be assessed by means of Lintner's formula. In our model, the influence of risk on the valuation of assets can be involved by heightening the nominal rate of return, r , regarding to the risk factor.

For the analysis of the problem formulated in Section 1, the most important results of solving the model are the equations (40), (45), (46), (63) and (64).

According to equation (40), the real after-tax cost of depreciable assets (under the new income-tax law) can be expressed by means of the formula

$$\nu = \frac{(1-\tau z)(\rho+\delta)-\tau d}{1-\tau} \quad .$$

This year, 1994, the profit tax rate is 42 %, i.e. $\tau = 0.42$. The fraction z of the cost of new investment that is deducted by the firm depends on the fraction z_m of investment in new machines and equipment with tax allowances and on fraction z_p that is allowed to deduct (in 1994, it is 10 %), so that

$$z = z_p z_m = 0.1 z_m.$$

If, for example, the average fraction z_m in the economy will be 0.5, then $z = 0.05$.

The influence the tax allowance has on the real cost of depreciable assets can be expressed by the formula

$$\Delta \nu = -\frac{\tau z}{1-\tau}(\rho + \delta) = -\frac{0.42 \times 0.05}{1-0.42}(\rho + \delta) = -0.044(\rho + \delta).$$

Where $\Delta \nu = \nu - \nu_b$ and ν_b is the after-tax real cost of capital without investment incentives. This means, that introducing this tax allowance can only influence the real cost of new capital only 4.4 % of before-tax real cost of capital $(\rho + \delta)$.

Another effective investment incentive is accelerated depreciation. This is based on the following basic rules, which use the coefficients that are prescribed by the new income-tax law. The depreciation of an asset in the first year is computed by means of the formula

$$D(1) = \frac{V_c}{k(1)} \tag{65a}$$

and in the remaining years by means of the formula

$$D(n) = \frac{2(V_c - o(n))}{k(n) - n + 1}, \quad n = 2,3,...,N, \tag{65b}$$

where $D(n)$ is depreciation in the n^{th} year, V_c is the input price of an asset, $o(n)$ is the total amount of depreciations from the beginning of depreciation i.e.

$$o(n) = \sum_{i=1}^{n-1} D(i), \; n = 2,3,...,N.$$

and $k(n)$ is the coefficient that is given by Table 4.1.

Table 4.1

Group of assets	Lifetime	$k(n)$	
		for $n = 1$	$n = 2,3,...$
1	4	4	5
2	8	8	9
3	15	15	16
4	30	30	31
5	50	50	50

For different groups of assets, with different prescribed lifetimes, the various values of $k(n)$ are prescribed and the values in the first year, $k(1)$, differ from the other values $k(2),k(3),...,k(N)$. The depreciation rates for linear depreciation that are prescribed by the income-tax law are shown in Table 4.2. A firm can choose between linear or accelerated depreciation rules in the first year of the lifetime of the given equipment or building, but it cannot change the chosen depreciation rule.

Table 4.2

Group of assets	Depreciation rate (% a year)	
	in the first year	in the remaining years
1	14.2	28.6
2	6.2	13.4
3	3.4	6.9
4	1.4	3.4
5	1.0	2.0

Accelerated depreciation allows firms to "write off" the cost of their new plant and equipment investments more rapidly than linear depreciation rules. Thus policy allows firms to postpone the taxes owed on the income generated by newly purchased investments. The exact quantification of the total future effects of accelerated depreciation on the cost of depreciable assets is difficult. This is because the rule of accelerated depreciating is complicated and the percentage of using this rule is uncertain because it depends on the firm's decisions.

The effect of postponing taxes, E_p , with regard to the given equipment or building can be computed by means of the formula

$$E_p = \sum_{n=1}^{N} \frac{(D(n) - \omega(n) V_c)\tau}{(1+i)^{n-1}} \qquad (66)$$

where $\omega(n)$ is the linear depreciation rate in the year n , i is the nominal interest rate and $D(n)$ is given by the formulas (65a) and (65b).

The formula (66) expresses the present discounted value of additional tax allowances (in the first year of lifetime) in the case where accelerated depreciation is used instead of linear depreciation. For example, if the input value of machine $V_c = 100000$, lifetime $N = 4$, the income tax rate $\tau = 0.42$ and the nominal interest rate $i = 0.15$, then $E_p = 2200$. This means, that 2.2 % of the machine's input value is refunded during its lifetime.

In our model, the influence of accelerated depreciation is contained in the parameter d , which quantifies the average rate of capital stock depreciation for tax purposes. The greater the volume of new investment goods and the larger the application of the accelerated depreciation rule on existing newer equipment and buildings will be, the greater the coefficient d and smaller the after-tax cost of depreciable assets ν will be in the given year. If the investment and application of accelerated depreciating in a certain year is small, then the parameter can decrease, because (according to formula (65)) the depreciations of the older machines and buildings are lower. This means that the average rate of depreciation for tax purposes, d, can decrease.

There are other factors that influence the after-tax real cost of capital ν , especially "useful lifetimes" that are prescribed by the tax law, inflation processes, the real interest rate ρ and the profit tax rate τ .

The redefinition of lifetimes in the tax law influences depreciation rates both in linear depreciating and accelerated depreciating. Shortening the lifetime of

various types of equipment can enable firms to gain further benefits from accelerated depreciation.

In the Czech economy lifetimes were redefined at the beginning of 1991. Depreciation rates were increased at the 61 % of machine and equipment branches. The maximal possible depreciation rate (for linear depreciating machines) was 12 %. Further shortening lifetimes were, in the framework of the tax reform, at the beginning of 1993, along with the introduction of accelerated depreciation. For example, the minimum useful lifetime of a number of machines and equipment was reduced from eight to four years.

Therefore if the depreciation of assets is based on input prices, it can be strongly influenced by inflation processes. This can negatively influence real firms' cash flows and the formation of internal funds for investment. In our model this factor is described by means of the price index $p(t)$. Capital $K(t)$ is measured by means of constant prices. The influence of an increasing price level $p(t)$ can be explained by equations (12) and (13). In (13) the nominal value of depreciation allowances for tax purposes is $p(t)dK(t)$. Thus the total value of tax allowances is $\tau p(t)dK(t)$. If the price level $p(t)$ is constant, depreciation rates and capital structure do not change, then the average depreciation rate d , which is used for tax purposes is constant. However, if (under the same assumptions) the price level $p(t)$ is increasing and depreciation rules are based on input prices (of the year of purchase), then the value of the average depreciation rate d decreases. This is because the nominal and real values of depreciations are lower for older machines than the depreciations for new machines, that are computed on the basis of higher input prices.

The decrease of d negativelly effects an firm's investment in two ways. First, there is a decrease in the total value of tax allowances $\tau p(t)dK(t)$ and after-tax profit. Second, the real value of depreciations that can be used for financing a firm's investment also decreases.

In the Czech economy in 1991, the negative effect resulting from the increase of the price level outweighed the positive effect of shortening lifetimes at the beginning of that year. The rate of inflation in 1991 (index of retail prices) reached 54 per cent annually. The price index of investment goods increased by 50 %, the industry price index increased by about 70 % and the producer price index increased by 54.8 %.

A further shortening of lifetimes in the 1993 tax law did not compensated for this negative affect and the price level increased further (more then 30 %) in the years 1992 and 1993.

The effect of real interest rate on the cost of capital is direct, which we can see in the formula for computing ν . The increase of ρ causes the increase of the real cost of capital ν .

Diferentiating the formula (40), we can show that $\frac{\partial \nu}{\partial \tau} > 0$. This means that the influence of the profit tax rate τ on the real after-tax cost of capital is also direct , i.e. the increase of τ causes the increase of ν .

The equation (46) that relates the net real investment, $i(t)$, of a representative firm to the gap between marginal productivity $F'(k(t))$ and the real cost of capital ν is based on the assumption that there is no technological change. In the adaptive model, it can be used for the approximate description of regulation processes on transient trajectories. This can be possible in a situation where there is less difference between $k(t)$ and $k^*(t)$. In a short time interval we can assume that there is no technological change and that in the production function

$$y(t) = A_0 e^{\lambda t} k(t)^{\beta} = F(k,t),$$

the term $A_0 e^{\lambda t} = A(t)$ is constant in time, i.e. $A(t) = A$.

Then we can use the production function

$$y(t) = F(k(t)) = Ak(t)^{\beta} \tag{67}$$

as in the second step of the analysis of the behavior of the model.

In relation (46) in the form

$$i(t) = L(0)e^{nt}\{\xi_k[F'(k(t)) - \nu] + nk(t)\} \tag{68}$$

ξ_k is the positive coefficient defined by the formula (see (42) and (45))

$$\xi_k = \frac{\sigma_1}{F''(k^*)} = \frac{1}{2F''(k^*)} \left\{ (\rho - n) - \left[(\rho - n)^2 - \left(4\frac{F''(k^*)}{a} \right) \right]^{1/2} \right\} .$$

By differentiating (67) we obtain

$$F' = \beta A k(t)^{\beta - 1},$$
$$F'' = \beta(\beta - 1) A k(t)^{\beta - 2} .$$

If we use a local aproximation of the production function by the quadratic function (see (4lb)), then $F''(k^*)$ is a constant. Thus in the neighborhood of the asymptotic trajectory $k^*(t)$, the coeficient ξ_k can be considered constant and independent on the tax parameters. Under these assumptions, the investment function has the following simple interpretation.

The net investment of the firm that is in the state $k(t)$, which differs from the optimal state $k^*(t)$ (on an asymptotic trajectory), depends directly on the growth rate of the labor force n (which depends on the growth of demand for production) and indirectly on the after-tax cost of capital ν. It depends directly on the gap between the marginal productivity $F'(k(t))$ and the real cost of capital ν

If a firm is in the state $k(t) = k^*$ given by (39) or $k(t) = k^*(t)$ given by (59) and (58b) (i.e. on the asymptotic optimal trajectory), therefore $F'(k^*) = \nu$ and in (67) $(F'(k(t)) - \nu) = 0$. If all the firms in the economy have the ratio of capital to labor $k(t)$ on the optimal level and parameters $\rho, \delta, d, A_0, \lambda, \beta, n$ do not change, then (according to (68)) the change of the real after-tax cost of capital ν by P per cent can temporally induce the change of the flow of the total net real investment of all joint-stock companies (and possibly also of the profit maximizing firms) by more then P per cent. The equation (68), which can be used as the investment function of a representative joint-stock company, is linear in ν. This relation can be used in analysing the impact of changes in tax policy (and also in the real interest rate ρ), especially of investment incentives, on the level of investment in the economy.

Since both $F''(k^*)$ and σ_1 are assumed to be negative, the coefficient ξ_k is positive. The percentage of the total net investment change depends linearly on the value of ξ_k, which depends indirectly on the coefficient a of the

quadratic capital adjustment cost function (30). If these costs are not too high, the change in the level of investment will be great. Thus investment activity can be stimulated by means of temporally (e.g. for one year) heightening investment incentives, especially the tax allowance percentage (z_p), which can be deducted. However, this approach can be successful if there are possibilities of growth in exports, especially the exports of machinery. It can stimulate the growth of demand for the production of machinery, the capacities of which are not utilized in the Czech economy. The increase in investment can also begin the multiplier process of shifting the aggregate expenditures schedule, which have a compound effect on output, and can enable a greater change in aggregate demand and in the output of the economy.

Once there is economic growth in the economy and a firm's ratio of capital to labor, $k(t)$, is adjusted to its optimal level $k^*(t)$, the investment function (64) can be applied. By means of this function we can analyse the development of the real net investment of a representative joint-stock firm (and also of a profit maximizing firm) in a growing economy with technological change and an increasing real wages.

By substituting the functions (7), (59) and (63) into the equation (64) we obtain the investment function of the form

$$\begin{aligned} i(t) &= L(t)(u^*(t) + nk^*(t)) = \\ &= L(0)e^{nt}\left(\frac{\lambda}{1-\beta}\bar{k}e^{\frac{\lambda}{1-\beta}t} + n\bar{k}e^{\frac{\lambda}{1-\beta}t}\right) = \\ &= L(0)\bar{k}\left(\frac{\lambda}{1-\beta} + n\right)e^{\left(\frac{\lambda}{1-\beta}+n\right)t} \end{aligned} \tag{69}$$

where (see (58b) and (40))

$$\bar{k} = \left[\frac{(1-\tau)\beta A_0}{(1-\tau z)\rho + \delta - \tau(d + z\delta)}\right]^{\frac{1}{1-\beta}} = \left[\frac{\beta A_0}{\nu}\right]^{\frac{1}{1-\beta}} \tag{70}$$

By substituting (70) into equation (69) we obtain the following form of the investment function

$$i(t) = L(0)\left[\frac{\beta A_0}{\nu}\right]^{\frac{1}{1-\beta}}\left(\frac{\lambda}{1-\beta}+n\right)e^{\left(\frac{\lambda}{1-\beta}+n\right)t} \qquad (71)$$

This function relates the net real investment of the representative firm that is moving along the optimal asymptotic trajectory, if there is technological change. We can see that the net investment level of a firm depends also indirectly on the after-tax cost of capital ν and moreover it depends directly on the parameter of technological change λ and explicitly and exponentially on time t. This means that all tax policy parameters that influence the value of ν also influence the optimal level of net investment in the case when a firm is moving on the asymptotic trajectory. In this case, a higher tax allowance z_p percentage leads also to a higher level of investment. This influence is strenghtened by technological progress. The higher value λ is the higher will be the volume of net investment $i(t)$.

The volume of gross real investment on the optimal asymptotic trajectory can be obtained by means of the equation (11)

$$I_r(t) = \dot{K}(t) + \delta K(t).$$

Substituting

$$\dot{K}(t) = i(t)$$

and

$$K(t) = L(0)e^{nt}k^*(t) = L(0)e^{nt}\bar{k}e^{\frac{\lambda}{1-\beta}t} = L(0)\left[\frac{\beta A_0}{\nu}\right]^{\frac{1}{1-\beta}}e^{\left(\frac{\lambda}{1-\beta}+n\right)t}$$

into equation (11) we obtain the investment demand function of the form

$$I_r(t) = L(0)\left[\frac{\beta A_0}{\nu}\right]^{\frac{1}{1-\beta}}\cdot\left(\frac{\lambda}{1-\beta}+n+\delta\right)e^{\left(\frac{\lambda}{1-\beta}+n\right)t}\ . \qquad (72)$$

Function (72) describes the influence of tax policy parameters on the level of investment of the representative firm and also the influence technological change has on investment. It can also be useful also in formulating hypotheses for the construction of a macroeconomic investment function and in developing the methodology for analysing investment decision - making.

At the end of this section, we have taken into account the influence of financing investment spending. Our model is based on the assumption, that all profits are distributed to equity holders, who are the sole owners of the firm, and current net investment is financed by selling new equity. We have assumed that firms retain no earnings, that they have no bonds outstanding and no banking credids. However, these assumptions could easily be relaxed.

If the retained profits are reinvested, then the firm's equility share price increases and its nominal value is also frequently heightened (or new equities are issued). From this point of view, the final effect is similar to the effect of financing by means of issuing new equity.

The influence of bonds on firm's investment can be investigated by using of Modigliani and Miller's analysis. One of the implications of the Modigliani-Miller theorem, which was proved in the context of a model that explicitly recognized the existence of uncertainty, was that in the absence of a corporate income tax, the firm's cost of capital is independent of whether the firm raises funds by retaining earnings, issuing bonds or issuing equities. Under this assumption, the total value of the firm's debt plus equity equals the present value of the firm's return (across states of nature, evaluated at the price of claims to one dollar contigent on the associated state of nature) and it is independent of the ratio of debt to equity.

If there is a corporate income tax and individual income tax with a different rate for interests income, matters become more complex. Usually it is assumed that for the firm an optimal debt-equity ratio exists. Thus the stock of bonds outstanding is constrained. The amount of long-term banking investment credits is strongly limited in the present Czech banking system. The income tax rate for interest income (25 %) in lower only by 17 % then the corporate income tax rate (42%) and interest rates are frequently higher than profit rates. Financing firms' investment by means of selling bonds is not frequently practised by the Czech joint-stock companies. Retained earnings are used more frequently. If a firm uses financing by means of bonds and banking credits, it may be assumed that the cost of this capital does not differ greatly from the cost of capital that is financed by means of issuing equities. Thus our model may be used for the aproximate analysis of the investment behavior of the all Czech joint-stock companies.

Conclusions

The analysis of the general adaptive model of a representative Czech joint-stock firm developed in this paper indicates the certain positive influence of investment incentives, which were included in the new Czech Income Tax Act, on the level of the investment activity of a joint-stock company. The tax allowances on investment in new machines and equipment and accelerated depreciation cause the decrease in the after-tax real cost of depreciable assets by several per cent.

The investment demand function, which was developed for an asymptotic trajectory, indicates, that the level of investment depends indirectly on this cost of capital and directly on the parameter of technological change and the growth rate of the labor force.

The temporal raise of investment incentives, especially the percentage of tax allowance on investment, can be used to stimulate investment activity and the growth of aggregate demand. However, this measure cannot completely solve the low level of demand (especially of demand for the production of machinery) and the problem of the low level of internally - generated firm's funds for financing investment, which is caused by the low level of the real value of the depreciation of older assets after inflation processes. These problems can be solved by means of the exports-support policy and by revaluating input prices of older assets.

Another interesting result of our analysis is the description of the effect of technological progress. This can influence not only the dynamic efficiency and the cash-flow of a firm, but also its investment demand and thus the multiplier effect of investment on aggregate demand. Firms' research activities, product innovations and process innovations can effectively contribute to the growth of output and the real wages in the Czech economy in the near future.

Table of Symbols

	Net output of the firm
	Stock of capital employed by the firm
	Amount of labor
	Time variable
	Production function
	Transformed linearly homogenous production function
$_L$	First derivatives of f
f_{KL}	Second derivatives of f

r	Nominal rate of return
ρ	Real interest rate
$C(K)$	Real resource costs of installing new capital $\left(\dot{K} = \frac{dK}{dt}\right)$
a	Positive constant in a quadratic function of capital adjustment costs
Π	Profits of the firm
T	Profit tax of a firm
τ	Profit tax rate
d	Discounted value of depreciation allowances for tax purposes
z	Fraction of the cost of new investment deducted by the firm
z_m	Fraction of investment in new machines and equipment with tax allowances
z_p	Fraction of new assets that is allowed to deduct
ν	Real cost of depreciable assets
$\bar{\delta}$	$=(\delta - \tau(d + z\delta))$
δ	$=(\delta + \alpha n)$
α	$=(1 - \tau z)$
H^*	Present-value Hamiltonian
u	Control variable $(u = k)$
σ_i	$=\frac{1}{2}\left\{(\rho - n) \pm \left[(\rho - n)^2 - \left(\frac{4F''(k^*)}{a}\right)\right]^{1/2}\right\},\ i = 1,2$
ζ_k	$=\sigma_1 / F''(k^*), \sigma_1 >$
$D(n)$	Depreciation of an asset in the n^{th} year
$\omega(n)$	Linear depreciation rate in the year n
V_c	Input price of an asset
$\bar{k}$	$= \left[\frac{\beta A_0}{\nu}\right]^{\frac{1}{1-\beta}}$

References

Auerbach, A.J. - Kotlikoff, L.J. (1987): Dynamic Fiscal Policy. Cambridge University Press, Cambridge.

Christensen, L.R. (1970): "Tax Policy and Investment Expenditures in a Model of General Equilibrium." In: *American Economic Review*, No 2, pp. 18-22.

Jorgenson, D.W. (1973): Investment and Production: A Review. Stanford, Institute for Mathematical Studies in the Social Sciences, Stanford University.

Kalous, M. (1993): "Decrease or Growth?" (in Czech). *Ekonom*, No 21, pp. 24-25.

Sargent, T.J. (1987): Macroeconomic Theory. Academic Press, New York.

Statistická ročenka České a Slovenské Federativní Republiky 1992. SEVT, Praha 1992.

Statistická ročenka České republiky 1993. Český spisovatel, Praha 1993.

Stiglitz, J.E. (1972):"On the Optimality of the Stock Market Allocation of Investment." In: *Quarterly Journal of Economics*, No 1, pp. 24-60.

Stiglitz, J.E.(1993): Principles of Macroeconomics. W.W. Norton & Company, New York.

Štach, J.(1991): "An Adaptive Model of the Dynamics of Production Enterprise (in Czech). *Ekonomicko-matematický obzor* 27, pp. 235-249.

Štach J. (1993): "The Investment Performance of Firms," Discussion Paper No. 10, CERGE-EI, Prague.

Štach J., Nedoma J.(1988): "Models of Investment Dynamism of an Enterprise." Ekonomický ústav ČSAV, Prague

Šujan, I. (1993): "The Macroeconomic Situation in the Czech Republic," Discussion Paper No. 22, CERGE-EI, Prague.

Modelování investičního chování českých akciových společností

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Souhrn
Studie se zabývá konstrukcí, analýzou a interpretací obecného modelu globálního investičního chování typické akciové společnosti, která vznikla v procesu kupónové privatizace v České republice a funguje v existujících institucionálních a tržních podmínkách. Model byl konstruován zejména pro účely zkoumání vlivu investičních stimulů, které byly zahrnuty do rámce nového zákona o dani z přijmu právnických osob, na úroveň investiční aktivity firmy a posléze i na objem investic v celé ekonomice. Model může usnadnit také analýzu reakcí akciových společnosti na změny úrokové míry i pochopení vlivu inflačních procesů a technického pokroku na investiční proces.

Studie vychází jednak z metodologických poznatků z oblasti modelování investiční dynamiky podniku, které autoři zformulovali ve svých dřívějších pracích, a dále z poznatků obsažených v různých mikroekonomických a makroekonomických teoriích investic. Zejména z hypotéz o investičním chování firem, které zformuloval a ekonometricky ověřil D.W. Jorgenson. Dále se vychází z empirických poznatků získaných z pozorování a rozboru chování českých akciových společností, investičních fondů a trhů cenných papírů v novém institucionálním prostředí.

V **první kapitole** se vymezuje modelovaný problém. Model by měl usnadňovat zejména odpověď na otázku: Jaký je vliv nových investičních stimulů na úroveň investiční aktivity akciové společnosti? Jde zejména o vliv odpočtu stanoveného procenta vstupní ceny nového hmotného majetku od základu daně z hmotného a nehmotného majetku, které umožňuje nový zákon o dani z přijmů, který vstoupil v účinnost 1.1. 1993.

Ve **druhé kapitole** se formulují základní předpoklady, na kterých je model postaven a základní relace používané v různých verzích modelu.

Předpokládá se zejména, že investiční chování typické české akciové společnosti, která vznikla v procesu kupónové privatizace a je ovládána privatizačními investičními fondy, je zaměřené na maximalizaci tržní hodnoty akcií existujících akcionářů. Pro popis technologie firmy se používá rostoucí a konkávní agregátní produkční funkce, která popisuje kauzální závislost mezi čistou (redukovanou) agregátní produkcí firmy a základními výrobními faktory (objem práce a objem výrobních fondů). Používá se zejména funkce Cobbova-Douglasova typu s konstantními výnosy z rozsahu produkce, která se ve třetí verzi modelu rozšiřuje o Solowův člen zobrazující vliv neutrálního technického pokroku. Užívá se v transformovaném tvaru, který vyjadřuje závislost produktivity práce na fondové vybavenosti práce.

Dalšími relacemi modelu jsou: diferenciální rovnice popisující změny stavů výrobních fondů a funkce popisující vývoj pracovních zdrojů a vývoj reálné a nominální mzdy.

Základním prvkem modelu je cílová funkce. Tato funkce má tvar dynamického kriteriálního funkcionálu, který vyjadřuje současnou diskontovanou hodnotu budoucího cash flow firmy,

který připadá na jednu existující akcii. Funkcionál zobrazuje i vliv daně ze zisku, daňových investičních stimulů a adjustačních nákladů, které vyvolává instalace rozvojových investic.

Ve **třetí kapitole** se model kompletuje a analyzuje se jeho chování. Analýza se provádí (pomocí principu maxima) ve čtyřech krocích, ve kterých se postupně uvolňují předpoklady modelu a získávají se stále hlubší aproximace komplikovaného chování modelovaného reálného systému.

V prvním kroku se abstrahuje od vlivu technického pokroku a inflačních procesů. Předpokládá se konstantní cenová hladina a reálná mzda. Základní řídící veličinou modelu je objem rozvojových investic. Objem práce se v prvním kroku považuje za endogenní veličinu modelu, která je určována modelem jako druhá řídící veličina. Pomocí principu maxima se odvozují podmínky, které musí splňovat stacionární řešení této verze modelu. Jde o standardní (Walrasovské) podmínky, že v rovnováze se mezní produkt práce rovná hrubé mzdě v daném období a že optimální úroveň výrobních fondů vyrovnává mezní produkt kapitálu s cenou (náklady) kapitálu. Náklady kapitálu zde zahrnují i vliv daně ze zisku a investičních stimulů.

V závěru prvního kroku se ukazuje, že za situace, kdy existují konstantní výnosy z rozsahu produkce firmy, má uvažovaná optimalizační úloha nekonečně mnoho řešení. Nejsou-li dány žádné další omezující podmínky, pak neexistuje optimální velikost výrobních fondů a velikost firmy je nedeterminovaná. Omezující podmínky se mohou týkat poptávky po produkci firmy, objemu pracovních zdrojů nebo finančních zdrojů použitelných na investice. V další analýze se pak předpokládá, že je zadáno omezení pracovních zdrojů. Objem práce se v dalších krocích považuje za vstupní ("exogenní") veličinu modelu, která je určena strategickými rozhodnutími firmy, na základě prognózy budoucího vývoje poptávky po produkci firmy.

Ve druhém kroku analýzy se opouští předpoklad, že cenová hladina je konstantní. Předpokládá se exponenciální růst cenového indexu. Objem práce se považuje za vstupní veličinu modelu, jejíž vývoj je zadán pomocí exponenciální funkce. Vliv technického pokroku se zatím neuvažuje, a předpokládá se, že reálná mzda je konstantní. Model se převádí do tvaru využívajícího transformované produkční funkce, která vyjadřuje závislost mezi produktivitou práce a fondovou vybaveností práce. Řídící veličinou je v této verzi modelu rychlost změny fondové vybavenosti práce.

Pomocí principu maxima se odvozuje jednak stacionární řešení modelu a dále i funkce rozvojových investic pro přechodové trajektorie modelu. Tato funkce, kterou lze aplikovat v určitém okolí stacionárního stavu firmy, vyjadřuje explicitně vliv daňových parametrů i reálné úrokové míry a nákladů kapitálu na přechodovou investiční poptávku firmy.

Ve třetím kroku je předcházející analýza rozšířena na situace, které budou aktuální po přechodu české ekonomiky na trajektorii intenzivního růstu. Řeší se úloha, která zahrnuje vliv neutrálního technického pokroku. Zahrnutí tohoto vlivu umožňuje také uvolnit předpoklad, že reálná mzda je konstantní. Ve třetí verzi modelu se předpokládá, že reálná mzda roste exponenciálně, při konstantním tempu růstu. Pro jednoduchost analýzy se v této verzi neuvažují adaptační náklady. Místo těchto prohibitivních nákladů se zavádějí omezující podmínky týkající se řídící veličiny investičního procesu. Později, v interpretační analýze, se však toto zjednodušení částečně nahrazuje adekvátnějším popisem přechodových procesů pomocí investiční funkce odvozené ve druhém kroku.

Optimálním řešením třetí verze modelu je exponenciální investiční funkce, která určuje optimální hodnotu rychlosti změny fondové vybavenosti práce v každém časovém okamžiku. Tomuto řešení odpovídá optimální asymptotická trajektorie fondové vybavenosti, která je také popsána pomocí exponenciální funkce.

Řešení dané úlohy se využívá ve čtvrtém kroku analýzy pro odvození investiční funkce popisující optimální úrovně rozvojových investic na asymptotické trajektorii.

Ve čtvrtém kroku se ještě více uvolňují předpoklady modelu, aby se jeho chování maximálně přiblížilo chování uvažovaného reálného systému. Uvolňují se předpoklady o trajektoriích vývoje pracovních zdrojů, reálné mzdy a cenové hladiny. Nepředpokládají se konstantní tempa růstu těchto veličin. Opouští se také předpoklad, že ostatní strukturní parametry modelu jsou konstantní v čase. Respektuje se skutečnost, že chování firmy v dlouhodobém horizontu nemá deterministickou ani stochastickou povahu. Pro zvládnutí problémů, které z toho vyplývají, rozpracovali autoři adaptivní verzi daného modelu.
V rámci adaptivního modelu se již nepovažuje tempo růstu pracovních zdrojů za "exogenně" zadanou vstupní veličinu, která je spolu s investiční proměnnou určována pomocí modelu.

Čtvrtá kapitola studie se zabývá ekonomickou interpretací nejzávažnějších relací, které byly odvozeny pomocí modelu a některými možnými aplikacemi modelových řešení v oblasti národohospodářského investičního procesu. Zejména se analyzují možnosti a meze působení investičních stimulů na úroveň investiční aktivity firem za současné situace v české ekonomice. Vychází se při tom z parametrů fiskální politiky stanovených pro rok 1994.

V této kapitole je také zkompletována poptávková investiční funkce akciové společnosti, která synteticky popisuje vliv parametrů daňové politiky i technického pokroku na objem hrubých reálných investic firmy, která se pohybuje po asymptotické optimální trajektorii.

Dále se také diskutuje vliv neurčitosti na tržní cenu akcie a vliv různých způsobů financování investičních výdajů (emise nových akcií. emise firemních obligací, reinvestování nerozděleného zisku) na modelová řešení.

Analýza modelu ukazuje určitý pozitivní vliv zavedených investičních stimulů na investiční aktivitu akciových společnosti. Tyto stimuly však mohou snížit reálné nákladu na jednotku nového hmotného majetku pouze o několik procent (méně jak 10 %). Odvozená funkce poptávky po investicích vypovídá, že na asymptotické trajektorii může toto snížení nákladů na výrobní fondy ovlivnit investiční poptávku také jen o několik procent. Závažným faktorem, který může tuto poptávku zvýšit, je také technický pokrok.

Analýza přechodových trajektorií modelu ukazuje, že dočasné (např. na jeden rok) zvýšení procenta odpočtu vstupní ceny nového hmotného majetku od základu daně z příjmů může vyvolat výrazné zvýšení úrovně investiční aktivity v ekonomice. Toto řešení by bylo možno využít v případě, že přijatá opatření k oživení investiční aktivity nevyvolají dostatečné zvýšení agregátní investiční poptávky.

Toto opatření však nemůže vyřešit celý problém nízké úrovně agregátní poptávky a problém nízké úrovně tvorby vnitřních firemních investičních zdrojů, který vznikl v důsledku nízkých odpisů ze staršího hmotného majetku firem po inflačních procesech. Zde je nutno hledat jiná řešení.