TYING AND ENTRY DETERRENCE IN VERTICALLY DIFFERENTIATED MARKETS

Eugen Kováč

CERGE-EI

Charles University
Center for Economic Research and Graduate Education
Academy of Sciences of the Czech Republic
Economics Institute

WORKING PAPER SERIES (ISSN 1211-3298)
Electronic Version

266

Working Paper Series 266
(ISSN 1211-3298)

Tying and Entry Deterrence in Vertically Differentiated Markets

Eugen Kováč

CERGE-EI
Prague, August 2005

ISBN 80-7343-059-2 (Univerzita Karlova v Praze, CERGE)
ISBN 80-7344-048-2 (Národohospodářský ústav AV ČR, Praha)

Tying and entry deterrence in vertically differentiated markets*

EUGEN KOVÁČ †
(CERGE–EI‡)

August 2005

Abstract

This paper analyzes tying and bundling as an entry deterrence tool. It shows that a multi-product firm can defend its monopoly position in one market via tying even when it does not have market power in another market. This is shown on a model with two complementary goods, each of which is vertically differentiated and in which consumers' preferences for the goods are positively correlated. Some possible ways of defending against entry deterrence, and implications for competition policy, are discussed.

Abstrakt

Tento článek analyzuje svazování jako nástroj pro zabránění vstupu na trh. Ukazuje, že firma vyrábějící více produktů může ubránit svou pozici monopolisty na jednom trhu pomocí svazování i když nemá monopolistickou sílu na jiném trhu. To je ukázano na modelu s dvěma komplementárními produkty, z nichž každý je vertikálně diferencovaný a preference spotřebitelů jsou pozitivně korelovány. Některé možné způsoby obrany proti zabránení vstupu a důsledky pro ochranu hospodářské soutěže jsou diskutovnány.

Keywords: industrial organization, vertical differentiation, anti-trust policy, entry deterrence, foreclosure, tying, bundling
JEL classification: L11, L12, L13, L41

*The author would like to thank Avner Shaked, Dirk Engelmann, and Kresimir Zigic for consultations and valuable suggestions and Ekaterina Goldfain, Patrick Rey, Jakub Steiner, and Jean Tirole for helpful comments. All errors are mine.

†Address: CERGE-EI, P.O.BOX 882, Politických vězňů 7, 11121 Prague 1, Czech Republic; URL: `home.cerge-ei.cz/kovac`; e-mail: `eugen.kovac@cerge-ei.cz`.

‡A joint workplace of the Center for Economic Research and Graduate Education, Charles University, Prague, and the Economics Institute of the Academy of Sciences of the Czech Republic. Address: CERGE–EI, P.O. Box 882, Politických vězňů 7, Prague 1, 111 21, Czech Republic

1 Introduction

Tying refers to the situation in which a firm makes the purchase of one of its products conditional on the purchase of another of its products. According to leverage theory, tying "provides a mechanism whereby a firm with monopoly power in one market can use the leverage provided by this power to foreclose sales in, and thereby monopolize, a second market" (Whinston 1990). Therefore, tying is one of the basic concepts in anti-trust laws and policies dealing with monopolization.[1] The most prominent example is probably Microsoft's tying of Internet Explorer to the operating system Windows. In this case, by virtue of having a strongly dominant position in the operating system market,[2] Microsoft has been found guilty of leveraging this market power to foreclose Netscape's sales in the web browser market.

Early anti-trust cases involving monopolization by tying required proof of monopoly power in the first market. However, following the argument by Posner (1976), "...how could a tie-in be imposed unless such power existed?" (p. 176), such proof was later omitted. In Kováč (2004), I show that, in contrast to Posner's argument, tying can indeed be profitable for a multi-product firm which faces an equal competitor (a specialized single-product firm) on each market.[3] In this paper I take a further step and provide a theoretical example in which a multi-product firm without monopoly power in the first market uses tying to deter entry in the second market. Instead of assuming that firms are symmetric, I require their products to be differentiated vertically. Moreover, I assume that the multi-product firm

[1] In the U.S. anti-trust laws, monopolization is prohibited by the Sherman Act (1890), Section 2. Moreover, the Clayton Act (1914), Section 3, deals specifically with tying contracts and exclusive dealing.

[2] According to market researcher OneStat.com, in the year 2003, Windows operated more than 97% of personal computers.

[3] This means that at each separate market the firms can be "renamed" (interchanged) without any effect. In Kováč (2004), I consider two markets for symmetric products: one for a horizontally differentiated product (Hotelling model), the other for a homogeneous product.

has a weaker position (it produces a low-quality product) in the first market. The possibility of entry deterrence in such a case contrasts strongly with the current understanding of tying as an entry deterrence tool.

The theoretical literature on entry deterrence and foreclosure by tying falls in line with the argument by Posner (1976). In his seminal paper, Whinston (1990) introduces a theoretical model to support the leverage hypothesis. He considers a multi-product firm which has monopoly power in one market and competes in price with a rival on another market. The author examines the implications of tying and concludes that it may lead to the foreclosure of the rival's sales in the tied good market. It is necessary to point out that monopolistic position in the first market is a crucial assumption for this result. On the other hand, Whinston (1990) claims that the monopolist will engage in tying only if he can commit itself to doing so, which will consequently drive its rival out of the market. Moreover, tying is profitable for the monopolist precisely because of the "exclusionary effect on the market structure." The main difference of the current paper from Whinston (1990) is that I do not require the multi-product firm to have a monopoly on the first market. Moreover, I consider the case where it has a weaker position than its competitor.

Protection of monopolized markets and entry deterrence are concepts closely related to foreclosure. Recently it has been studied particularly with applications to the Microsoft case. Carlton and Waldman (2002) argue that a dominant firm can use bundling[4] to remain dominant in an industry with rapid technological change. Applying the analysis to the Microsoft case, the authors claim that Microsoft's tying and deterrence of Netscape's entry into the market for internet browsers could have increased social welfare. On the other hand, Choi and Stefanadis (2001) show that

[4]Bundling is a more general concept than tying and refers to a situation in which a package containing at least two different products is offered. The practice in which the firm offers only the bundle is called pure bundling. The practice in which the firm offers the bundle as well as some of the products separately is called mixed bundling. See also Table 2, p. 11.

if an incumbent monopolist faces simultaneous entry in several markets, it may employ tying to discourage potential rivals from entry and innovation. Such action then has negative welfare effects.

Rey and Tirole (2005) provide an excellent survey of the literature on foreclosure and incorporate all surveyed models into one framework. However, they analyze foreclosure only in cases when a monopoly power in another market is present. The authors also discuss a critique of the leverage hypothesis made by the Chicago School who argue that for complementary products there is only one monopoly rent to extract and hence a monopolist has no incentives to monopolize a second market.[5] Rey and Tirole (2005) conclude that the goods must be relatively independent so that the monopolist finds monopolization of the second market profitable.

Another survey of tying and bundling is provided by Nalebuff (2003a, 2003b) in a report for the Department of Trade and Industry. In the first part of the report, Nalebuff (2003a) surveys different motives for bundling and tying, analyzes their consequences, in particular, anti-competitive effects, and provides policy recommendations. In the second part of the report, Nalebuff (2003b) applies his conclusion to particular anti-trust cases. However, his analysis implicitly assumes that monopolization and entry deterrence by tying are possible only when the multi-product firm has monopoly power.

The purpose of this paper is therefore to provide an example which shows that foreclosure is also possible without monopoly power when the goods are complements. I show this specifically in the situation where a multi-product firm competes against a superior rival in one market and has monopoly power in another market, where it faces an inferior potential entrant. Moreover, I show that entry deterrence cannot be avoided by an entrant's cooperation with the incumbent rival (in the

[5]See Director and Levi (1956) and Posner (1976).

first market). Thus in order for entry to occur the entrant needs either to enter with a higher quality, or to enter in both markets simultaneously.

The remainder of the paper is organized as follows. In Section 2, I describe the setup of the model and explain the basic intuition. In Sections 3, 4, and 5, I analyze the multi-product firm's strategies when selling separate products, using pure bundling and mixed bundling, respectively. In Section 6, I investigate when the entry deterrence strategy is profitable for the multi-product firm and discuss ways of defending against it. Section 7 concludes, and I discuss the relevance of my results for anti-trust policies. Appendix A contains proofs of all lemmas and propositions.

2 The model

2.1 Informal description and intuition

There are two markets for two indivisible complementary goods X_D and X_E, two incumbent firms M and D, and one potential entrant firm E. Firm M (for multi-product) operates on both markets and firm D (for duopoly) operates only on the market for good X_D. Firm E (for entrant) plans to enter the market for good X_E. I assume that each of the goods is differentiated vertically and that consumers' preferences for them are positively correlated. Furthermore, I consider a situation in which the multi-product firm M offers a low-quality product in the market for good X_D (where it faces a high-quality competitor) and has a high-quality product in the market for good X_E (where it faces a low-quality potential entrant); see Table 1.[6] Under the above assumptions, I show that the multi-product firm M can protect its monopoly position in the second market (for good X_E) against low-quality entrants. This is achieved by mixed bundling when the multi-product firm

[6]In the absence of any form of bundling, there are four combinations available.

offers the monopolistic good X_E and the bundle. In other words, it ties its first good X_D to the second (monopolistic) good X_E.

Quality	First market (X_D)	Second market (X_E)
high	incumbent firm (D)	multi-product firm (M)
low	multi-product firm (M)	*potential entrant (E)*

Table 1: Markets structure

Note that the incentive for tying is different from what is found in the literature. Usually, tying is used either as a mechanism to leverage the market power (in order to deter entry) or as a tool for price discrimination between consumers with a high and low willingness to pay for the tied product. On the other hand, in this paper, although tying serves the purpose of deterring entry, it does not leverage any market power. Rather, it is used as a price discrimination tool between consumers with a high appreciation for quality and consumers with a low appreciation for quality.

The intuition for entry deterrence is the following. By selling only its monopolistic good X_E and the bundle consisting of both its goods, the multi-product firm M makes its low-quality first good unavailable separately. When firm E enters, the combinations available to consumers are:

1. firm D's (incumbent specialist firm) good X_D together with firm M's good X_E (as the highest-quality combination);
2. the bundle by multi-product firm M;
3. firm D's good X_D together with firm E's (entrant) good X_E, as the lowest-quality combination.

Thus, the entrant's good X_E can be purchased only in combination with the high-quality first good X_D produced by firm D. If the total quality[7] of this combination

[7]Here it just means the sum of qualities of separate products.

is lower than the quality of the bundle, it may be purchased only by consumers with a low appreciation for quality. However, if the market is sufficiently narrow,[8] there may be not enough "place" for that all available combinations to have a positive market share. This will foreclose the entrant's sales and will make entry unprofitable.

2.2 Formal description and assumptions

Consumers are indexed by their taste for quality (appreciation for quality) θ which is uniformly distributed over the interval $[\underline{\theta}, \overline{\theta}]$. I analyze the case in which consumers' tastes for quality for both goods are positively correlated.[9] In particular, I assume that each consumer has the same taste for quality for both goods.[10] Moreover, consumers have a positive marginal utility only from the first unit of both goods, i.e., they buy either one or zero units. Formally, I consider the following utility function (in reduced form):

$$U_\theta = \begin{cases} \theta(s_d + s_e) - p, & \text{if he buys goods } X_D,\ X_E \text{ with} \\ & \text{qualities } s_d,\ s_e \text{ by spending } p, \\ & \text{where } d \in \{D, MD\},\ e \in \{E, ME\}, \\ -p, & \text{if he buys only one good for price } p, \\ 0, & \text{if he does not buy.} \end{cases} \tag{1}$$

The parameter θ can be interpreted as taste for quality for a "package" containing goods X_D, X_E with qualities s_d, s_e. The fact that the utility is $-p$ when the consumer buys only one good means that the "direct" utility from consumption is zero. It reflects the complementarity of the goods. Note that whenever prices are positive, each consumer prefers not to buy at all to buy only one good.

[8] In particular, I assume that the markets are narrow enough in the sense that each of them cannot accommodate more than two firms in equilibrium. Such markets were studied by Shaked and Sutton (1982) who called them natural oligopolies.

[9] This assumption is reasonable for complementary goods. Note that complementarity could also be captured by using a non-additive form of utility from a package containing both goods.

[10] This assumption is technical so as to simplify the analysis and to make the model tractable. The logic is still correct when the taste for quality is not the same, but highly positively correlated.

Remark 1. Instead of characterizing consumers according to their taste for quality, they can be characterized by their income, as in Shaked and Sutton (1982). This is a more realistic approach, as income is an economically measurable and clearly defined variable. However, Tirole (1992, pp. 96–97) shows that both approaches are equivalent. When consumers differ by income, the parameter $1/\theta$ can be interpreted as the marginal rate of substitution between income and quality. Wealthier consumers correspond to higher values of θ, because they have a lower marginal utility of income. The assumption that the taste for quality is the same for both goods means that the marginal rate of substitution between income and quality is the same for both goods.

A similar approach can be found in Carbajo, de Meza and Seidmann (1990), who assume that consumers' valuations for two goods are the same. They argue that a high positive correlation is likely to occur when the goods are normal and when consumers are differentiated according to income.

I assume that production of each good is costless, but that entry exhibits certain fixed costs. Goods (varieties) X_D produced by firms M and D are differentiated vertically. Similarly, if entry occurs, goods X_E produced by firms M and E will be differentiated vertically. The structure of the markets is shown in Table 1. For $i \in \{D, E\}$ denote s_{Mi} the quality of good X_i produced by firm M, and s_i the quality of the good produced by its rival i. Furthermore, I assume that all qualities are given exogenously and that the following inequalities hold (see also previous subsection):[11]

$$0 < s_{MD} < s_D, \qquad 0 < s_E < s_{ME}, \qquad s_D + s_E < s_{MD} + s_{ME}. \tag{2}$$

These assumptions yield the following ranking of qualities for potentially available

[11] I also have explored the remaining cases but this is the only one which leads to entry deterrence. Hence the other cases are not analyzed in this paper.

combinations of goods X_D and X_E:

$$s_{MD} + s_E < s_D + s_E < s_{MD} + s_{ME} < s_D + s_{ME}. \tag{3}$$

In addition, I assume that

$$\begin{aligned} 2\underline{\theta} &< \overline{\theta}, & (4)\\ \overline{\theta}(s_D - s_{MD}) &\leq \underline{\theta}(2s_D + s_{MD}), & (5)\\ \overline{\theta}(s_{ME} - s_E) &\leq \underline{\theta}(2s_{ME} + s_E). & (6) \end{aligned}$$

The first assumption means that the market should be wide enough to accommodate two firms. The other assumptions ensure that the markets are covered in equilibrium, when the goods are sold separately; see Tirole (1992, p. 296) for more details. I will call values of parameters *admissible* if they satisfy conditions (2)–(6). To simplify the analysis I denote

$$\tau = \underline{\theta}/\overline{\theta}, \qquad \Delta_i = s_{Mi} - s_i \ (i \in \{D, E\}), \qquad \rho = \Delta_D/\Delta_E. \tag{7}$$

Using this new notation, the above conditions can be rewritten as following:

$$\Delta_D < 0, \qquad \Delta_E > 0, \qquad \Delta_D + \Delta_E > 0, \tag{8}$$

$$\max\left\{\frac{s_D - s_{MD}}{2s_D + s_{MD}}, \frac{s_{ME} - s_E}{2s_{ME} + s_E}\right\} \leq \tau \leq \frac{1}{2}. \tag{9}$$

Note that it can be easily shown that $(s_D - s_{MD})/(2s_D + s_{MD}) < (s_{ME} - s_E)/(2s_{ME} + s_E)$ if and only if $s_D s_E < s_{MD} s_{ME}$.

Obviously, τ can be interpreted as a measure for narrowness of the market. The market is narrow when τ is high and wide when τ is low. Hence an upper bound on τ means that the market should not be too narrow (i.e., wide enough), whereas a lower bound on τ means that the market should not be too wide (i.e., narrow enough); see interpretation of conditions (4), (5), and (6).

On the other hand, ρ can be interpreted as a measure for toughness or softness of competition in one market relative to the other. The competition on one market is softer when the difference in qualities (here Δ_D and Δ_E) is higher in absolute value.[12] Note that (8) implies $\rho \in (-1, 0)$. Hence ρ close to 0 means that competition on the market for X_E is softer than competition on the market for X_D, whereas ρ close to -1 means that competition on both markets is equally soft.

The whole situation can be modelled as a three-stage game. In the first stage, firm M decides which combination of goods X_D and X_E it will sell. Its options are listed in Table 2. In the second stage, firm E decides whether to enter the market for good X_E by incurring a fixed cost C.[13] In the third stage all firms compete in prices.

Remark 2. Following Whinston (1990), I assume that firm M can precommit itself not to change its bundling strategy in a later stage (e.g., not to sell one of the goods separately if it previously decided otherwise). This precommitment can be achieved, for example, by a technological setting, and may involve sunk costs, e.g., design, advertising, etc.[14] In such cases it is reasonable to assume that the bundling strategy is chosen before the pricing decisions. On the other hand if precommitment is not possible, the introduced timing is irrelevant. In this paper I show that the announcement of tying in the first stage can make firm E's second-stage entry unprofitable.

I analyze the pure-strategy equilibria of each subgame and look for a subgame perfect equilibrium of the whole game. To simplify the analysis, I consider equi-

[12]See the results in Section 3 or, for example, Tirole (1992).

[13]In general, the value of C may depend on the entrant's quality. However, this is not relevant since I assume that the qualities are given exogenously. On the other hand, note that the whole analysis can be easily accommodated to the case in which firm E's quality choice is taken in the second stage jointly with the entry choice (with C depending on the quality).

[14]See Whinston (1990) and Nalebuff (2003a) for a more extensive discussion.

libria where combinations of goods with a higher quality also have a higher price (otherwise nobody purchases the lower quality combination).

Strategy	Products offered by M
no bundling	X_D and X_E
pure bundling	bundle $\mathcal{M} = \{X_D, X_E\}$
mixed bundling	$\mathcal{M}$ and X_D; $\mathcal{M}$ and X_E; $\mathcal{M}$, X_D and X_E

Table 2: Strategies of firm M in the first stage

3 No bundling

Consider first the benchmark case where firm M decides to sell its products separately and firm E enters the market. Let p_{MD}, p_{ME}, p_D, and p_E be the prices of the respective goods offered by the firms. This notation will also be used in the following sections. In the absence of bundling, each consumer has four choices available (two for each good).

If all customers are served with both goods, the two markets are independent. On the market for good X_E, a consumer with taste for quality θ buys product X_E from firm M if and only if $\theta > \theta^*$, where $\theta^* = (p_{ME} - p_E)/\Delta_E$ represents the indifferent consumer.[15] Hence the firms' profits are $\Pi_{ME} = p_{ME}(\overline{\theta} - \theta^*)$ and $\Pi_E = p_E(\theta^* - \underline{\theta})$. Their maximization yields the following equilibrium prices and profits

$$\begin{aligned} p_{ME} &= \tfrac{1}{3}\Delta_E(2\overline{\theta} - \underline{\theta}), & p_E &= \tfrac{1}{3}\Delta_E(\overline{\theta} - 2\underline{\theta}), \\ \Pi_{ME} &= \tfrac{1}{9}\Delta_E(2\overline{\theta} - \underline{\theta})^2, & \Pi_E^{\text{noB}} &= \tfrac{1}{9}\Delta_E(\overline{\theta} - 2\underline{\theta})^2, \end{aligned} \tag{10}$$

and indifferent consumer $\theta^* = \frac{1}{3}(\underline{\theta} + \overline{\theta})$. Obviously, $p_{ME} > p_E$ and $\Pi_{ME} > \Pi_E^{\text{noB}}$. Furthermore, condition (4) implies that $\underline{\theta} < \theta^*$ and (6) assures that the $\underline{\theta}$ consumer

[15] I ignore the case of equality since it corresponds to a set of consumers with measure zero.

has a non-negative utility. Hence the market is in equilibrium covered by two firms.

The situation on the market for good X_D is reversed in the sense that now firm M has a low-quality good. Hence the equilibrium prices and profits are (note that $\Delta_D < 0$)

$$\begin{aligned} p_{MD} &= -\tfrac{1}{3}\Delta_D(\overline{\theta} - 2\underline{\theta}), & p_D &= -\tfrac{1}{3}\Delta_D(2\overline{\theta} - \underline{\theta}), \\ \Pi_{MD} &= -\tfrac{1}{9}\Delta_D(\overline{\theta} - 2\underline{\theta})^2, & \Pi_D^{\text{noB}} &= -\tfrac{1}{9}\Delta_D(2\overline{\theta} - \underline{\theta})^2. \end{aligned} \tag{11}$$

This yields firm M's profit

$$\begin{aligned} \Pi_M^{\text{noB}} &= \Pi_{MD} + \Pi_{ME} = \\ &= \tfrac{1}{9}\big((-4\Delta_D + \Delta_E)\underline{\theta}^2 + 4(\Delta_D - \Delta_E)\underline{\theta}\overline{\theta} + (-\Delta_D + 4\Delta_E)\overline{\theta}^2\big). \end{aligned} \tag{12}$$

Each firm's profit is homogeneous of degree 1 in (Δ_D, Δ_E) and homogeneous of degree 2 in $(\underline{\theta}, \overline{\theta})$. Hence, in certain cases, I will use a more convenient form of firm M's profit

$$\Pi_M^{\text{noB}} = \tfrac{1}{9}\big((4-\rho) + 4(1-\rho)\tau + (4\rho - 1)\tau^2\big) \cdot \Delta_E\overline{\theta}^2.$$

Firm E enters the market whenever $\Pi_E^{\text{noB}} \geq C$. If I denote $c = C/(\Delta_E\overline{\theta}^2)$, the "normalized" condition $\Pi_E^{\text{noB}} \geq C$ is equivalent to $(1-2\tau)^2 \geq 9c$, or $\tau \leq \frac{1}{2}(1-3\sqrt{c})$. When $c = 0$ firm E enters for all values of τ satisfying (9). As c increases, the range of values of τ where firm E enters becomes smaller.

4 Pure bundling

In the case of pure bundling, the consumer has only two options: he either can buy the products from firms D and E separately (with qualities s_D and s_E), or he can buy them in the bundle $\mathcal{M}$ from firm M. I assume that "unbundling" of goods in the bundle is impossible (or excessively costly) so that consumers cannot buy the bundle, abandon one product and buy it from another firm. This is also related to the notion of compatibility. From the market perspective, pure bundling is equivalent to making the products incompatible with rival ones; see, for example, Matutes and Regibeau (1992).[16] The unbundling assumption then means that use of incompatible products is impossible or excessively costly.

Let p_M denote the price of the bundle $\mathcal{M}$ offered by firm M. The current situation can be described as one vertically differentiated market with two products: the bundle $\mathcal{M}$ offered by firm M (with quality $s_{MD} + s_{ME}$ and price p_M) and the combination[17] of products X_D and X_E (with quality $s_D + s_E$ and price $p_D + p_E$). According to assumption (2), the former has a higher quality.

A consumer with taste for quality θ buys the bundle $\mathcal{M}$ if and only if $\theta > \theta^*$, where $\theta^* = (p_M - p_D - p_E)/(\Delta_D + \Delta_E)$ represents the marginal (indifferent) consumer. Maximization of profits yields the following equilibrium prices

$$p_D = p_E = \tfrac{1}{4}(\Delta_D + \Delta_E)(\overline{\theta} - 2\underline{\theta}), \quad p_M = \tfrac{1}{4}(\Delta_D + \Delta_E)(3\overline{\theta} - 2\underline{\theta}), \qquad (13)$$

and equilibrium profits

$$\begin{aligned} \Pi_D^{\text{pureB}} = \Pi_E^{\text{pureB}} &= \tfrac{1}{16}(\Delta_D + \Delta_E)(\overline{\theta} - 2\underline{\theta})^2 = \tfrac{1}{16}(1+\rho)(1-2\tau)^2 \cdot \Delta_E \overline{\theta}^2, \\ \Pi_M^{\text{pureB}} &= \tfrac{1}{16}(\Delta_D + \Delta_E)(3\overline{\theta} - 2\underline{\theta})^2 = \tfrac{1}{16}(1+\rho)(3-2\tau)^2 \cdot \Delta_E \overline{\theta}^2. \end{aligned} \qquad (14)$$

Moreover, $\theta^* = \frac{1}{4}(2\underline{\theta} + \overline{\theta})$. Just as for separate markets, it is necessary to check whether the conditions for market coverage are satisfied in equilibrium. It is easy to verify that (4) implies $\underline{\theta} < \theta^*$ and (5) and (6) imply that the $\underline{\theta}$ consumer has a non-negative utility. Therefore, the market will be covered in equilibrium.

Firm E enters the market if and only if $\frac{1}{16}(1+\rho)(1-2\tau)^2 \geq c$. As firm E's profit

[16] Printers and cartridges can serve as an example.

[17] In traditional economic literature a bundle means in general a combination of goods. To avoid misunderstandings I will refer to the "bundle" only as the result of bundling, i.e., a package of goods X_D and X_E sold together by firm M.

is lower than in the no bundling subgame, entry is less likely to occur. However, for $c = 0$, it occurs for all admissible values of parameters.

5 Mixed bundling

5.1 Firm M offers the bundle and good X_E — the case of entry

By offering good X_E and the bundle firm M makes the combination with the lowest quality $(s_{MD} + s_E)$ unavailable (assuming that firm E enters). Hence, consumers are left with three options with the following ranking of qualities

$$s_D + s_E < s_{MD} + s_{ME} < s_D + s_{ME}.$$

The marginal consumer who is indifferent between buying the goods from firms D and E, and buying the bundle is characterized by $\theta_2^* = (p_M - p_D - p_E)/(\Delta_D + \Delta_E)$. The marginal consumer who is indifferent between buying the bundle and highest-quality combination is characterized by $\theta_3^* = (p_M - p_D - p_{ME})/\Delta_D$, where $\underline{\theta} < \theta_2^* < \theta_3^* < \overline{\theta}$. These yield the following profits

$$\begin{aligned} \Pi_M &= p_M(\theta_3^* - \theta_2^*) + p_{ME}(\overline{\theta} - \theta_3^*), \\ \Pi_D &= p_D(\theta_2^* - \underline{\theta} + \overline{\theta} - \theta_3^*), \\ \Pi_E &= p_E(\theta_2^* - \underline{\theta}). \end{aligned}$$

The following proposition shows that under certain conditions, there may be no place for firm E on the market.

Proposition 1. *Assume that*

$$\tau \geq \frac{3 + 2\rho}{2(3 + \rho)} \tag{15}$$

and firm M decides to sell good X_E as well as the bundle. If firm E enters the market, it cannot obtain a positive market share whenever firms D and M maximize their profits.

Proof. See Appendix A.

The phrase "whenever firms D and M maximize their profits" means that firms D and M play a best response to rivals' prices. The above proposition then means that for any price firm E sets, it cannot obtain a positive profit whenever firms D and M play a best response to both rivals' prices.

The above proposition implies that under (15), entry is not profitable regardless of the entry costs C. Hence, firm M can use this form of mixed bundling to deter firm E's entry. Condition (15) means that the market should be narrow enough to leave no place for the entrant. This means it is not possible that all three available combinations have a positive market share. Hence, the lowest-quality combination will be the one which cannot have a positive market share.

Several points are worth noting: First, that the converse holds. Whenever (15) is not true, firm E can have a positive market share when p_E is small enough. This is directly observable from the expression for θ_2^* (see the proof of the proposition).

Second, this can occur only if the packages including firm E's product X_E have the lowest and the second-lowest quality. An alternative ordering of qualities with such property would be $s_D + s_E < s_{MD} + s_E < s_D + s_{ME} < s_{MD} + s_{ME}$. However, it is possible to show that this ordering allows firm E to enter the market. The main reason for this difference is the behavior of firm D. In the original ordering, firm D's market share has two margins. As its product X_D is part of the highest-quality combination, firm D sets a high price in order to earn a high profit from the highest-quality combination. In this way it sacrifices market share from the low-quality combination with firm E's product. On the other hand, in the ordering introduced above, firm D's product has only one margin and is not part of the highest-quality combination. Hence, its price is lower, allowing all three available combinations to have a positive market share. Moreover, as indicated earlier, it is possible to show that among all possible orderings, only (3) allows for entry deterrence.

Third, it is easy to recognize that the right-hand side from (15) is increasing in ρ on the interval $(-1, 0)$, with infimum $\frac{1}{4}$ (as $\rho \to -1^+$) and supremum $\frac{1}{2}$ (as $\rho \to 0^-$). Hence for any $\rho \in (-1, 0)$ there exists $\tau \leq \frac{1}{2}$ such that (15) holds.

To analyze the effect of firm E's quality, assume that firm E enters with a lower quality. Then the value of Δ_E will be higher and hence ρ will be higher (closer to 0). Therefore, (15) is more restrictive, in the sense that it holds for a smaller set of values of τ. This means that the lower the quality firm E enters with, the less likely entry deterrence is. In other words, there should be enough differentiation in order for entry to occur.

5.2 Firm M offers the bundle and good X_E — the case of no entry

In this subsection I analyze the case in which firm E decides not to enter. As mentioned in Remark 2, firm M commits to its bundling strategy in the first stage. Hence, although firm E does not enter, firm M has to sell the bundle and good X_E.[18] In this case the consumer has only two options: he can buy either the bundle, or buy good X_D from firm D together with good X_E from firm M. Denote $\theta_3^* = (p_M - p_D - p_{ME})/\Delta_D$ the consumer who is indifferent between them. Further denote $\theta_0^* = (p_D + p_{ME})/(s_D + s_{ME})$ the consumer who is indifferent between buying the latter combination and not buying at all. The following lemmas specify certain situations which do not occur in equilibrium.

[18]Firm M is not allowed to change its strategy ex-post. However, it may charge such a high price for some of its products that nobody will buy them.

Lemma 1. *Assume that firm M offers the bundle and good X_E and that firm E does not enter the market. If firm M maximizes its profit, then it is not possible that both combinations have a positive market share*[19] *and that the market is overcovered, at the same time.*

Lemma 2. *Assume that firm M offers the bundle and good X_E and that firm E does not enter the market. If firm M maximizes its profit, then it is not possible that both combinations have a positive market share and the market is exactly covered, at the same time.*

The following proposition specifies the equilibrium in this subgame.

Proposition 2. *Assume that firm M offers the bundle and good X_E and that firm E does not enter the market. Then the equilibrium prices are*

$$\begin{aligned} p_D &= -\tfrac{1}{3}\Delta_D\overline{\theta}, \\ p_M &= \tfrac{1}{2}(s_{MD}+s_{ME})\overline{\theta}, \\ p_{ME} &= \tfrac{1}{2}(s_{MD}+s_{ME})\overline{\theta}+\tfrac{1}{3}\Delta_D\overline{\theta}, \end{aligned}$$

yielding an undercovered market and profits

$$\begin{aligned} \Pi_D^{\text{deter}} &= -\tfrac{1}{9}\Delta_D\overline{\theta}^2, \\ \Pi_M^{\text{deter}} &= -\tfrac{1}{9}\Delta_D\overline{\theta}^2+\tfrac{1}{4}(s_{MD}+s_{ME})\overline{\theta}^2. \end{aligned} \tag{16}$$

Note that the price of the bundle is lower than the price of good X_E when sold separately. The crucial assumption for this is the impossibility of unbundling. This form of pricing indeed occurs in reality (although the motives may be different). Nalebuff (2003a, pp. 31–32) provides an example of cars and radios, where cars with radios are cheaper than cars without radios. On one hand it may appear

[19] I.e., they are purchased by positive measures of consumers.

counterintuitive. On the other hand it means that consumers with a high appreciation for quality are charged a higher price. This can be interpreted as a form of price discrimination between consumers with a high appreciation for quality (who want to buy good X_D from firm D) and consumers with a low appreciation for quality (who buy it from firm M).

An undercovered market indicates a possibility of entry. As was shown in the previous section, there can be no low-quality entrants in the market for good X_E. However, there could still be a potential entrant in the market for good X_D. In this case it would be necessary to analyze this firm's entry decision in the second stage simultaneously with firm E's decision. However, the narrowness of the market (assumption (5)) implies that such a firm would not be active in the no bundling subgame (and likewise for the pure bundling subgame). Hence, I will omit the possibility of additional entry in the market for good X_D.

5.3 Firm M offers the bundle and good X_D

By offering good X_D and the bundle, firm M makes the combination with the highest quality ($s_D + s_{ME}$) unavailable. Consumers are left with three options with the following ranking of qualities

$$s_{MD} + s_E < s_D + s_E < s_{MD} + s_{ME}.$$

The marginal consumers are characterized by $\theta_1^* = (p_{MD} - p_D)/\Delta_D$ and $\theta_2^* = (p_M - p_D - p_E)/(\Delta_D + \Delta_E)$.

Proposition 3. *Assume that*

$$\tau \geq \frac{1+\rho}{5+\rho}, \tag{17}$$

firm E enters the market, and firm M offers the bundle and good X_D. Then there is no equilibrium of this subgame, such that the lowest-quality combination has a *positive market share.*

The above proposition implies that under condition (17), although firm M decides to offer (in addition to the bundle) product X_D, it prefers that nobody buys it. Hence any equilibrium in this subgame will be *outcome equivalent*[20] to an equilibrium of the pure bundling subgame. The equilibrium prices and profits are then given by (13) and (14), where p_{MD} is high enough that nobody buys firm M's good X_D together firm E's good X_E.

On the other hand, from the proof of Proposition 3, it is clear that $\theta_1^* > \underline{\theta}$ whenever (17) does not hold. Moreover, it can be easily shown that $\theta_1^* < \theta_2^* < \overline{\theta}$ for all feasible values of parameters. Hence, all combinations of goods considered in this subgame have a positive market share in equilibrium.

5.4 Firm M offers the bundle and goods X_D, X_E

In this case, the consumer has all combinations of goods X_D and X_E available, with ranking of qualities given by (3). Moreover, he can buy the products from firm M either in the bundle (for the price p_M) or separately (for the price $p_{MD} + p_{ME}$). Offering a bundle makes sense only if $p_M < p_{MD} + p_{ME}$. Otherwise nobody buys it and the situation is equivalent to selling separate products.

The marginal consumers are characterized by $\theta_1^* = (p_{MD} - p_D)/\Delta_D$, $\theta_2^* = (p_M - p_D - p_E)/(\Delta_D + \Delta_E)$, and $\theta_3^* = (p_M - p_D - p_{ME})/\Delta_D$, where $\underline{\theta} < \theta_1^* < \theta_2^* < \theta_3^* < \overline{\theta}$.

Proposition 4. *Assume that*

$$\tau \geq \frac{2(1+\rho)}{7+4\rho}, \tag{18}$$

firm E enters the market, and firm M offers the bundle and both goods X_D and X_E. Then the lowest-quality combination cannot have a positive market share, whenever firms M and D maximize their profits.

[20]Two equilibria are outcome equivalent when they yield the same profits to each firm and the same utility to each consumer.

The above proposition implies that under condition (18), although firm M decides to offer product X_D too, it prefers that nobody buys it. Hence any equilibrium in this subgame will be outcome equivalent to an equilibrium of the subgame where firm M offers the bundle and good X_E, causing no entry by firm E. The appropriate equilibrium prices are specified in Proposition 2.

On the other hand, if condition (18) does not hold, firm E may obtain a positive market share with sufficiently low price p_E. A detailed analysis of equilibrium in this case requires discussion of several cases and is technically complicated. Moreover, it is not relevant for the main argument and thus will be omitted.

6 Entry deterrence

6.1 Entry deterrence as subgame perfect equilibrium

In this section I compare entry deterrence equilibrium with other equilibrium outcomes. In order for entry deterrence to occur in equilibrium, it needs to be preferred by firm M to other outcomes.

It can be easily established that condition (15) implies conditions (17) and (18); see also Figure 1 in Appendix C. This means that under condition (15), any equilibrium of the third stage after entry occurs is outcome equivalent either to the equilibrium of the pure bundling subgame, or to the equilibrium of the no bundling subgame. Hence, entry deterrence is a subgame perfect equilibrium if and only if it is more profitable for firm M than the pure bundling and no bundling equilibria.

To specify the subgame perfect equilibrium it is necessary to compare firm M's equilibrium profits Π_M^{noB}, Π_M^{pureB}, and Π_M^{deter}, given by (12), (14), and (16), respectively, under assumption (15).

Lemma 3. *For all admissible values of parameters such that condition* (15) *holds, the equilibrium profit in the pure bundling subgame is higher than in the no bundling* *subgame.*

Remark 3. This result conforms to the results in Kováč (2004) in the sense that a multi-product firm without monopoly power can also find tying profitable. There, tying yields to softer competition, which allows the firms to relax prices.

Now, what remains is to compare profit Π_M^{pure} in the pure bundling subgame to the entry deterrence profit Π_M^{deter}. The following proposition and its corollary show that profit Π_M^{deter} is higher than Π_M^{pureB} for a large set of admissible values of parameters.

Proposition 5. *For any values s_D, s_E, s_{MD}, and s_{ME} satisfying conditions* (2) *and for any $\tau \leq \frac{1}{2}$, the inequality $\Pi_M^{\text{deter}} > \Pi_M^{\text{pureB}}$ is equivalent to $\tau > \tau^*$, where*

$$\tau^* = \frac{3}{2} - \frac{1}{3}\sqrt{\frac{5s_{MD} + 9s_{ME} + 4s_D}{s_{MD} + s_{ME} - s_D - s_E}}. \tag{19}$$

In addition, $\tau^* < \frac{1}{2}$ for all admissible values of parameters.

Corollary 1. *For any values s_D, s_E, s_{MD}, and s_{ME} satisfying conditions* (2)*, there exists an open set of admissible values of τ such that entry deterrence is a subgame perfect strategy.*

As the result of the previous proposition depends on five parameters, it cannot be visualized easily. For a better illustration I will analyze the extreme case where the inequality $\Pi_M^{\text{deter}} > \Pi_M^{\text{pureB}}$ holds for all admissible values of τ that satisfy (15). This occurs when at least one of the following three conditions is satisfied:

$$\tau^* < \frac{s_D - s_{MD}}{2s_D + s_{MD}}, \quad \text{or} \quad \tau^* < \frac{s_{ME} - s_E}{2s_{ME} + s_E}, \quad \text{or} \quad \tau^* < \frac{3 + 2\rho}{2(3 + \rho)}. \tag{20}$$

I conjecture that for all admissible values of parameters, at least one of the above conditions is satisfied.

Conjecture 1. *Assume that the values of parameters are admissible and satisfy condition* (15)*. Then in the subgame perfect equilibrium, firm M uses tying to deter entry of firm E.*

A partial proof, results of numerical simulations and visualization of Conjecture 1 are presented in Appendices B and C.

Note that condition (15) is necessary for entry deterrence. Hence, if (15) does not hold, firm M cannot deter firm E's entry provided that entry costs are sufficiently low. In this case, firm E enters the market in equilibrium and in the first stage firm M simply compares the equilibrium profits from the subgames following its bundling decision. However, a detailed examination of all cases would significantly extend the analysis.

6.2 Defence against entry deterrence

When firm M sells the bundle and product X_E separately, it excludes the package containing its good X_D together with the entrant's X_E from consumption. As a possible defence against entry deterrence firm D could cooperate with firm E in order make the entry possible. I assume that such a decision would be made after firm M's bundling decision but before firm E's entry decision, since it simultaneously should be a response to M's bundling strategy and it should enable E's entry. In particular, I consider two ways of cooperation.

First, firm D can decide to sell its product X_D only in a bundle with firm E's product. This is a form of inter-firm bundling where two products produced by different firms are offered in a bundle. Bundling of hardware and software may be considered an example (e.g., CD-writers are usually purchased jointly with the appropriate application software). In this case, there are only two packages available for consumption: the bundle by firm M and the package consisting of firm D's good X_D and firm E's good X_E. However, both firms D and E behave as profit-maximizing individuals. Hence, in the last stage, this form of defence is equivalent to a pure bundling subgame. On the other hand, the cooperation may involve side payments from firm E to firm D so that it would be profitable for firm D to cooperate.

Second, firms D and E merge and bundle their products together. In this case, the packages available in the market are the same as in the previous case. However, firms D and E do not behave as two profit-maximizing individual firms, but as one firm. Therefore, this situation is equivalent to a simple vertically differentiated market with two products of qualities $s_{MD} + s_{ME}$ (offered by firm M) and $s_D + s_E$ (offered by the merged firms D and E).

Remark 4. Note that the second case suggests an additional motive for mergers as found in the literature. Traditional economic literature mainly considers mergers among firms that produce substitutable goods. A higher market power is then the main motive for mergers. Nalebuff (2002) describes the possibility of bundling as a motive for the merger of firms producing complements. His analysis is mainly meant to explain the GE-Honeywell merger. In this paper, the motive would be similar to the one by Nalebuff (2002). However, here it is used as a defence against entry deterrence. Unfortunately, it turns out not to be profitable for firm D in this model.

In order for firm D to cooperate with firm E, such cooperation needs to be profitable for firm D. Hence, it needs to yield a higher profit than in the entry deterrence subgame. As shown in Section 5.2, this profit is $\Pi_D^{\text{deter}} = -\frac{1}{9}\Delta_D\overline{\theta}^2$. Firm D's profit in the first case (without side payments) is the same as in the pure bundling subgame equilibrium, which is $\Pi_D^{\text{pureB}} = \frac{1}{16}(\Delta_D + \Delta_E)(\overline{\theta} - 2\underline{\theta})^2$. However, as a side payment, firm E may transfer part of its profit to firm E. Hence, firm E may earn up to $\Pi_D^{\text{pureB}} + \Pi_E^{\text{pureB}} = 2\Pi_D^{\text{pureB}}$. On the other hand, the joint profit in the second case is $\Pi_{DE}^{\text{merge}} = \frac{1}{9}(\Delta_D + \Delta_E)(\overline{\theta} - 2\underline{\theta})^2$. It follows that

$$2\Pi_D^{\text{pureB}} > \Pi_{DE}^{\text{merge}} > \Pi_D^{\text{pureB}}. \tag{21}$$

Hence a merger is not profitable compared to a separate profit maximization. Moreover, it can be easily shown that firm M's equilibrium profit when competing against two separate firms is higher than the profit when competing against the merged multi-product firm. The reason for this is that a merger leads to more aggressive behavior and hence all firms experience lower profits. A similar result is obtained by Nalebuff (2000) for horizontally differentiated complements.

The following proposition suggests that none of these cooperation strategies is profitable for firm D when condition (15) holds.

Proposition 6. *If condition* (15) *holds, firm D will not cooperate with firm E in the subgame perfect equilibrium.*

The above proposition implies that the suggested tools are not sufficient as a defence against entry deterrence. The main reason for this is that by cooperation with firm E, firm D would lose high profits from selling its X_D together with firm M's good X_E as the highest-quality combination. Therefore, firm D's cooperation with firm E is not profitable and hence entry deterrence cannot be prevented. As suggested in the Introduction, a successful entrant would need either to enter with a higher quality, or to enter in both markets simultaneously (with a sufficiently high total quality).

7 Conclusion

In this paper I analyze a situation in which a multi-product firm competes against a specialist firm in one market and faces a potential entrant in a second market. In the present model, I consider two markets for complementary products, produced by the multi-product firm. Each of them is produced by another single-product (specialist) firm, where one of them is incumbent and the other is a potential entrant. Within each market, the goods produced by different firms are differentiated vertically, where the multi-product firm has a lower quality than the incumbent (in the first market), but a higher quality than the potential entrant (in the second market). I show that the multi-product firm can use tying to deter entry in the second market even when it does not have monopoly power in the first market. In addition, the entry deterrence will in equilibrium not be prevented by cooperation among the specialist firms. These results are relevant for anti-trust policies since they illustrate an anti-competitive practice which was until now not taken into account.

The understanding of this issue could be in the future extended in several directions.

- First, in the paper I consider the qualities of goods given exogenously. It would be interesting to analyze an extension of the present model, where qualities are determined endogenously.
- Second, the markets considered in this paper are narrow in the sense that each of them cannot accommodate more than two firms in equilibrium. A relevant question is how the results will change for wider markets, which are still natural oligopolies in the sense of Shaked and Sutton (1982), or even for different market structures.
- Third, I considered only cooperation among specialist firms as a way to prevent entry deterrence. Some other ways could be suggested and analyzed in the introduced framework.

References

Carbajo, Jose, David de Meza, and Daniel J. Seidmann, "A Strategic Motivation for Commodity Bundling," *Journal of Industrial Economics*, 1990, *38* (3), 283–298.

Carlton, Dennis W. and Michael Waldman, "The Strategic Use of Tying to Preserve and Create Market Power in Evolving Industries," *The RAND Journal of Economics*, 2002, *33* (2), 194–220.

Choi, Jay Pil and Christodoulos Stefanadis, "Tying, Investment, and the Dynamic Leverage Theory," *The RAND Journal of Economics*, 2001, *32* (1), 52–71.

Director, Aaron and Edward Levi, "Law and Future: Trade Regulation," *Northwestern University Law Review*, 1956, *51*, 281–296.

Kováč, Eugen, "Tying by a Non-monopolist," *CERGE-EI Working Paper No. 225*, 2004.

Matutes, Carmen and Pierre Regibeau, "Compatibility and Bundling of Complementary Goods in a Duopoly," *Journal of Industrial Economics*, 1992, *40* (1), 37–54.

Nalebuff, Barry, "Competing against Bundles," *Yale School of Management Working Paper No. 7*, 2000.

———, "Bundling and the GE-Honeywell Merger," *Yale School of Management Working Paper No. 22*, 2002.

———, "Bundling, Tying, and Portfolio Effects, Part 1 — Conceptual Issues," *DTI Economics Paper No. 1*, 2003.

——, "Bundling, Tying, and Portfolio Effects, Part 2 — Case Studies," *DTI Economics Paper No. 2*, 2003.

Posner, Richard A., *Antitrust Law: An Economic Perspective*, Chicago: University of Chicago Press, 1976.

Rey, Patric and Jean Tirole, "A Primer on Foreclosure," in Mark Armstrong and Rob Porter, eds., *Handbook of Industrial Organization, vol. III*, (forthcoming), 2005.

Shaked, Avner and John Sutton, "Natural oligopolies," *Econometrica*, 1982, *51* (1), 1469–1483.

Tirole, Jean, *The Theory of Industrial Organization*, 5th ed., Cambridge, Massachusetts: The MIT Press, 1992.

Whinston, Michael D., "Tying, Foreclosure, and Exclusion," *The American Economic Review*, 1990, *80* (4), 837–859.

A Appendix: Proofs

Proof of Proposition 1. Assuming (15), I will show that $\theta_2^* < \underline{\theta}$ for any positive p_E whenever firms M and D maximize their profits. The first order conditions for maximization of firm M's and D's profits are[21]

$$\begin{aligned} 2\Delta_E p_M - \Delta_E p_D - 2(\Delta_D + \Delta_E)p_{ME} + \Delta_D p_E &= 0, \\ 2p_M - p_D - 2p_{ME} &= \Delta_D \overline{\theta}, \\ \Delta_E p_M - 2\Delta_E p_D - (\Delta_D + \Delta_E)p_{ME} + \Delta_D p_E &= \Delta_D(\Delta_D + \Delta_E)(\overline{\theta} - \underline{\theta}). \end{aligned}$$

Solution of this system (with p_E as parameter) is the following

$$\begin{aligned} p_M &= \frac{1}{6\Delta_E}\big((\Delta_D + 3\Delta_E)p_E + 2\Delta_D(\Delta_D + \Delta_E)\underline{\theta} + \\ &\quad +(-2\Delta_D^2 + \Delta_D\Delta_E + 3\Delta_E^2)\overline{\theta}\big), \\ p_{ME} &= \frac{1}{2}(p_E + \Delta_E\overline{\theta}), \\ p_D &= \frac{\Delta_D}{3\Delta_E}\big(p_E - 2(\Delta_D + \Delta_E)(\overline{\theta} - \underline{\theta})\big). \end{aligned}$$

Hence

$$\theta_2^* - \underline{\theta} = -\frac{\Delta_D + 3\Delta_E}{6\Delta_E(\Delta_D + \Delta_E)}p_E + \frac{1}{6\Delta_E}\big(-2(\Delta_D + 3\Delta_E)\underline{\theta} + (2\Delta_D + 3\Delta_E)\overline{\theta}\big).$$

It is easy to observe that the coefficient at p_E is negative. Moreover, the last term is non-positive if and only if (15) holds. In this case $\theta_2^* < \underline{\theta}$ for any positive p_E. Hence firm E cannot achieve a positive market share. □

Proof of Lemma 1. If both combinations are purchased by a positive measure of consumers and the market is overcovered, then $\theta_0^* < \underline{\theta} < \theta_3^* < \overline{\theta}$. In this case

$$\Pi_M = p_{ME}(\overline{\theta} - \theta_3^*) + p_M(\theta_3^* - \underline{\theta}). \tag{22}$$

[21] Because the profits are quadratic in prices, it is easy to check that second-order conditions also hold for any prices.

A direct computation yields

$$\begin{aligned}\frac{\partial \Pi_M}{\partial p_M} &= \frac{2p_M - p_D - 2p_{ME}}{\Delta_D} - \underline{\theta}, \\ \frac{\partial \Pi_M}{\partial p_{ME}} &= -\frac{2p_M - p_D - 2p_{ME}}{\Delta_D} + \overline{\theta},\end{aligned}$$

which implies

$$\frac{\partial \Pi_M}{\partial p_M} + \frac{\partial \Pi_M}{\partial p_{ME}} = \overline{\theta} - \underline{\theta} > 0.$$

Hence at least one of the derivatives is positive. This means that Π_M has no interior maximum, i.e., such that $\theta_0 < \underline{\theta} < \theta_3^* < \overline{\theta}$. □

Proof of Lemma 2. If the market is exactly covered, then $\theta_0^* = \underline{\theta} < \theta_3^* < \overline{\theta}$. Hence, $p_M = (s_{MD} + s_{ME})\underline{\theta}$. If $p_M < (s_{MD} + s_{ME})\underline{\theta}$, the market is overcovered (i.e., $\theta_0^* < \underline{\theta}$) and firm M's profit is given by (22). On the other hand, if $p_M > (s_{MD} + s_{ME})\underline{\theta}$, the market is undercovered (i.e., $\theta_0^* > \underline{\theta}$) and firm M's profit is

$$\Pi_M = p_{ME}(\overline{\theta} - \theta_3^*) + p_M(\theta_3^* - \theta_0^*). \tag{23}$$

When firm M maximizes its profit, the following conditions hold: $\partial \Pi_M / \partial p_{ME} = 0$ and $\partial \Pi_M / \partial p_M |^{+}_{p_M = (s_{MD} + s_{ME})\underline{\theta}} \leq 0$. Substitution yields $(2p_M - 2p_{ME} - p_D)/\Delta_D = \overline{\theta}$ and $(2p_M - 2p_{ME} - p_D)/\Delta_D \leq 2p_M/(s_{MD} + s_{ME})$. Hence $\overline{\theta} \leq 2\underline{\theta}$, which contradicts (4). □

Proof of Proposition 2. According to Lemmas 1 and 2, the market needs to be undercovered in equilibrium. In this case, firm M's profit is specified by (23) and firm D's profit is $\Pi_D = p_D(\overline{\theta} - \theta_3^*)$. Their maximization yields the following first order conditions

$$\begin{aligned}2p_M - 2p_{ME} - p_D &= 2p_M \Delta_D/(s_{MD} + s_{ME}), \\ 2p_M - 2p_{ME} - p_D &= \Delta_D \overline{\theta}, \\ p_M - p_{ME} - 2p_D &= \Delta_D \overline{\theta}.\end{aligned}$$

Obviously, the prices specified in the proposition form the unique solution of the above system. The profits are obtained after substitution. Note that $\theta_0^* = p_M/(s_{MD} + s_{ME}) =$

$\frac{1}{2}\overline{\theta} > \underline{\theta}$, hence the market is indeed undercovered. □

Proof of Proposition 3. Each of the three combinations available is purchased by consumers with a positive measure if and only if $\underline{\theta} < \theta_1^* < \theta_2^* < \overline{\theta}$. I will show that condition (17) implies $\theta_1^* < \underline{\theta}$.

Firms' profits can be written as follows: $\Pi_M = p_M(\overline{\theta} - \theta_2^*) + p_{MD}(\theta_1^* - \underline{\theta})$, $\Pi_D = p_D(\theta_2^* - \theta_1^*)$, and $\Pi_E = p_E(\theta_2^* - \underline{\theta})$. Maximization of these profits yields the following equilibrium prices:

$$
\begin{aligned}
p_M &= \frac{3\Delta_E(\Delta_D + \Delta_E)}{\Delta_D + 9\Delta_E}(2\overline{\theta} - \underline{\theta}),\\
p_{MD} &= -\frac{\Delta_D}{\Delta_D + 9\Delta_E}\big((\Delta_D + \Delta_E)\overline{\theta} - (\Delta_D + 5\Delta_E)\underline{\theta}\big),\\
p_D &= -\frac{\Delta_D(\Delta_D + \Delta_E)}{\Delta_D + 9\Delta_E}(2\overline{\theta} - \underline{\theta}),\\
p_E &= -\frac{\Delta_D}{\Delta_D + 9\Delta_E}\big((\Delta_D + 3\Delta_E)\overline{\theta} - (\Delta_D + 6\Delta_E)\underline{\theta}\big).
\end{aligned}
$$

Under these prices

$$\theta_1^* = \frac{1}{\Delta_D + 9\Delta_E}\big((\Delta_D + \Delta_E)\overline{\theta} + 4\Delta_E\underline{\theta}\big).$$

Condition (17) is equivalent to $\theta_1^* \leq \underline{\theta}$. □

Proof of Proposition 4. Each of the four combinations available is purchased by consumers with a positive measure if and only if $\underline{\theta} < \theta_1^* < \theta_2^* < \theta_3^* < \overline{\theta}$. Just as in the Proof of Proposition 1, I will consider p_E as parameter and show that condition (18) implies $\theta_1^* < \underline{\theta}$ for any $p_E > 0$.

Firms' profits can be written as follows: $\Pi_M = p_M(\theta_3^* - \theta_2^*) + p_{MD}(\theta_1^* - \underline{\theta}) + p_{ME}(\overline{\theta} - \theta_3^*)$, $\Pi_D = p_D(\theta_2^* - \theta_1^* + \overline{\theta} - \theta_3^*)$, and $\Pi_E = p_E(\theta_2^* - \underline{\theta})$. Maximization of firm M's and firm

D's profits yields:

$$\begin{aligned} p_M &= \frac{1}{6(\Delta_D + 2\Delta_E)}\Big(2(2\Delta_D + 3\Delta_E)p_E + (D_1 + \Delta_E)\big(\Delta_D\underline{\theta} + (\Delta_D + 6\Delta_E)\overline{\theta}\big)\Big), \\ p_{MD} &= \frac{\Delta_D}{6(\Delta_D + 2\Delta_E)}\big(p_E + (4\Delta_D + 7\Delta_E)\underline{\theta} - 2(\Delta_D + \Delta_E)\overline{\theta}\big), \\ p_{ME} &= \frac{1}{2}(p_E + \Delta_E\overline{\theta}), \\ p_D &= \frac{\Delta_D}{3(\Delta_D + 2\Delta_E)}\big(p_E - (\Delta_D + \Delta_E)(2\overline{\theta} - \underline{\theta}\big). \end{aligned}$$

Under these prices

$$\theta_1^* - \underline{\theta} = -\frac{1}{6(\Delta_D + 2\Delta_E)}p_E + \frac{1}{6(\Delta_D + 2\Delta_E)}\big(2(\Delta_D + \Delta_E)\overline{\theta} - (4\Delta_D + 7\Delta_E)\underline{\theta}\big).$$

Obviously, $\theta_1^* < \underline{\theta}$ for any $p_E > 0$, whenever condition (18) holds.[22] □

Proof of Lemma 3. It can be easily established that for $\rho \in (-1, 0)$ and $\tau \in (0, \frac{1}{2})$ profit Π_M^{pureB} from the pure bundling subgame is higher than profit Π_M^{noB} from the no bundling subgame if and only if

$$\tau > \frac{1}{2(13 - 7\rho)}\left(43 + 11\rho - 2\sqrt{407 - 49\rho + 200\rho^2}\right).$$

Furthermore, for any $\rho \in (-1, 0)$ the following inequalities hold:

$$\frac{1}{2(13 - 7\rho)}\left(43 + 11\rho - 2\sqrt{407 - 49\rho + 200\rho^2}\right) < \frac{1}{4} < \frac{3 + 2\rho}{2(3 + \rho)}.$$

This directly implies that $\Pi_M^{\text{pureB}} > \Pi_M^{\text{noB}}$. Figure 2 in Appendix C shows the graph of the right-hand side of the above expression together with the right-hand side of (15). □

Proof of Proposition 5. Consider s_D, s_E, s_{MD}, and s_{ME} fixed and denote

$$f(\tau) = \overline{\theta}^2(\Pi_M^{\text{pureB}} - \Pi_M^{\text{deter}}).$$

[22] Note that $\theta_1^* - \underline{\theta} = -p_{MD}/\Delta_D$.

Function f is quadratic in τ and condition (2) implies that

$$f'(\tau) = \tfrac{1}{4}(s_D + s_E - s_{MD} - s_{ME})(3 - 2\tau) < 0,$$

whenever $\tau < \frac{3}{2}$, and $f''(\tau) > 0$. Hence f is convex, attains a maximum for $\tau = \frac{3}{2}$, and is decreasing for all $\tau \leq \frac{1}{2}$. It can be easily computed that the equation $f(\tau) = 0$ has solutions

$$\tau_{1,2} = \frac{3}{2} \pm \frac{1}{3}\sqrt{\frac{5s_{MD} + 9s_{ME} + 4s_D}{s_{MD} + s_{ME} - s_D - s_E}}.$$

Let $\tau_1 > \tau_2$. Obviously $\tau_1 > \frac{3}{2}$ and the inequality $\tau_2 < \frac{1}{2}$ is equivalent to $-13s_D - 9s_E + 4s_{MD} < 0$, which holds as $s_{MD} < s_D$.

Hence, I have shown that for $\tau \leq \frac{1}{2}$, the inequality $f(\tau) < 0$ is equivalent to $\tau > \tau_2$. □

Proof of Proposition 6. According to (21), it is sufficient to show that $\Pi_D^{\text{deter}} > 2\Pi_D^{\text{pureB}}$, whenever (15) holds. In terms of ρ and τ (under the feasibility condition), this inequality is equivalent to $-\frac{1}{9}\rho > -\frac{1}{8}(1+\rho)(1-2\tau)^2$, which can be rewritten as $-8\rho/(1+\rho) > 9(1-2\tau)^2$, or

$$\tau > \frac{1}{2} - \frac{\sqrt{3}}{2}\sqrt{\frac{-\rho}{1+\rho}}.$$

To complete the proof, I will show that condition (15) implies the above condition. A direct computation shows that for $\rho \in (-1, 0)$, the inequality[23]

$$\frac{3+2\rho}{2(3+\rho)} > \frac{1}{2} - \frac{\sqrt{3}}{2}\sqrt{\frac{-\rho}{1+\rho}}$$

is equivalent to $17\rho^2 + 57\rho + 72 > 0$, which holds for any $\rho \in \mathbb{R}$. □

[23]Note that for $\rho = 0$, equality holds.

B Appendix: Conjecture 1

It can be easily established that none of the conditions (20) implies the other two.[24] The first two right-hand-side expressions have infimum equal to zero on the set of all admissible values of parameters. A direct computation shows that for admissible values parameters, the inequality $\tau^* < 0$ is equivalent to

$$97s_D + 81s_E > 61s_{MD} + 45s_{ME}. \tag{24}$$

On the other hand, the third expression from (20) has infimum equal to $\frac{1}{4}$ when $\rho \in (-1, 0)$; see also Figure 1 in Appendix C. A direct computation shows that for admissible values parameters, the inequality $\tau^* < \frac{1}{4}$ is equivalent to

$$289s_D + 225s_E > 145s_{MD} + 81s_{ME}. \tag{25}$$

As all relevant conditions, i.e., (2)–(6), (15), and (19), are homogeneous of degree 1 in $(s_D, s_E, s_{MD}, s_{ME})$, I can use a normalization $s_{MD} + s_{ME} = 1$, which represents the total quality of the bundle. Note that under this normalization the set of all admissible values of parameters is bounded (from below by 0, from above by 1). Then the set of parameters (s_D, s_E, s_{MD}) where the admissibility conditions (2) hold is the interior of a tetrahedron with vertices $(0, 0, 0)$, $(0, 1, 0)$, $(1, 0, 0)$, and $(1, 0, 1)$ in the (s_D, s_E, s_{MD})-space. Moreover, after normalization, condition (25) becomes

$$289s_D + 225s_E > 81 + 64s_{MD},$$

which holds whenever $s_D + s_E > \frac{9}{25}$, as $s_{MD} < s_D$. Figure 3 in Appendix C shows the tetrahedron representing the set of admissible values of parameters. Condition (25) is represented by the open half-space $Z_1Z_2Z_3$ which does not contain point $(0, 0, 0)$.

To verify the third condition from (20) for the rest of admissible values of parameters

[24]For example, for $s_D = 0.2$, $s_E = 0.2$, $s_{MD} = 0.1$, $s_{ME} = 0.8$ the right-hand sides are 0.35, 0.2, and 0.475, respectively. Taking $s_{MD} = 0.01$ instead yields values 0.3624, 0.4634, and 0.4564.

I used numerical simulations.[25] From them I conjecture that it holds for all admissible values of parameters.[26] This means entry deterrence is profitable for firm M for all admissible values of parameters such that condition (15) holds.

[25]I used a grid of $100 \times 100 \times 100$ on the set $[0,1] \times [0,1] \times [0,1]$.

[26]I also checked numerically the first two conditions from (20). After normalization (24) reduces to $97s_D + 81s_E > 45 + 16s_{MD}$, which holds whenever $s_D + s_E > \frac{5}{9}$. Using a grid of $1000 \times 100 \times 1000$ on the set $[0, \frac{5}{9}] \times [0, \frac{5}{9}] \times [0, \frac{5}{9}]$, the simulations indicate that the first or the second condition is satisfied whenever $s_D > 0.04$, or $s_E > 0.31$, or $s_{MD} > 0.04$.

C Appendix: Figures

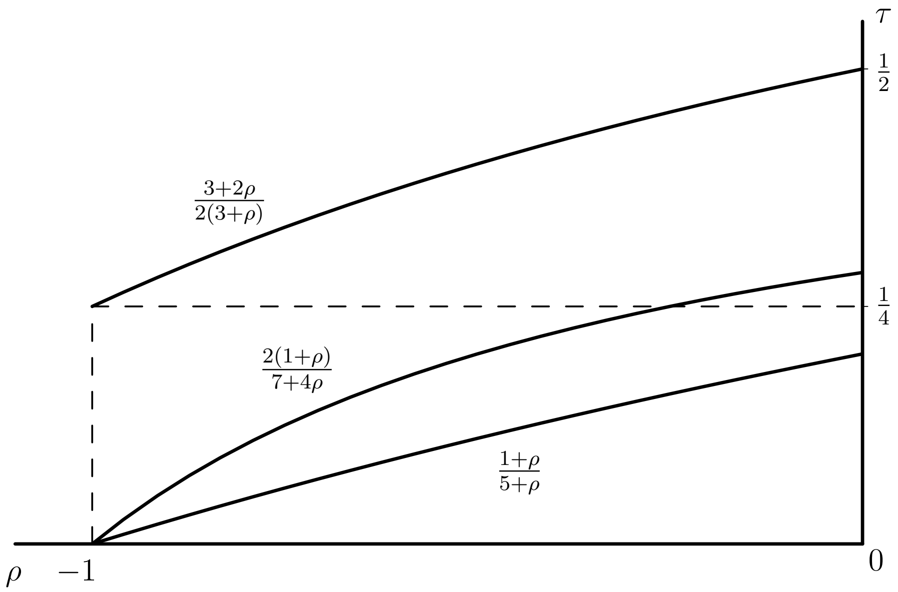

Figure 1: Conditions (15), (17), and (18)

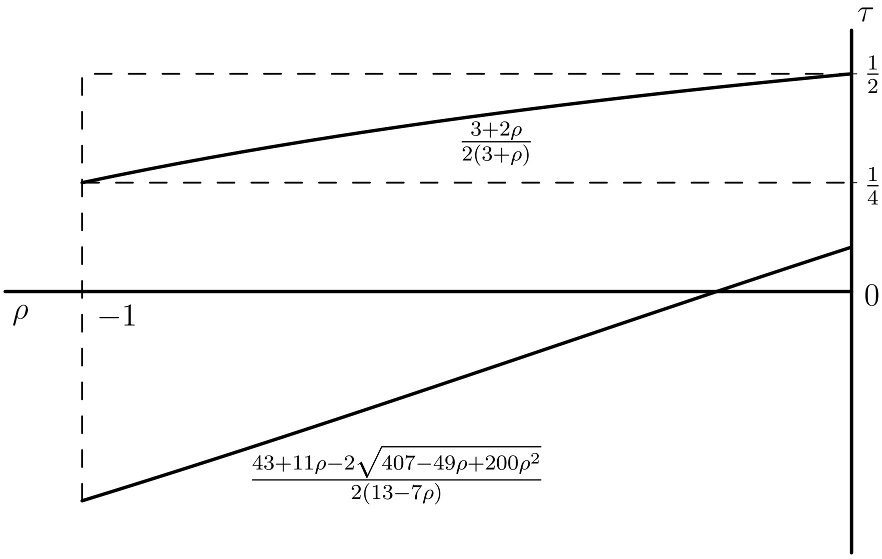

Figure 2: Graphs for the Proof of Lemma 3

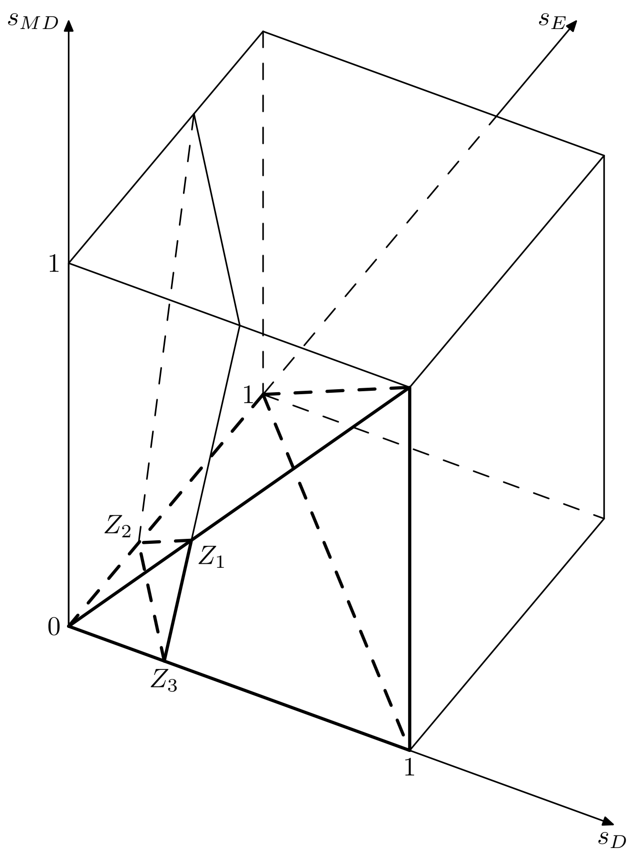

Figure 3: Admissible values of parameters when $s_{MD} + s_{ME} = 1$

Individual researchers, as well as the on-line and printed versions of the CERGE-EI Working Papers (including their dissemination) were supported from the following institutional grants:

- Center of Advanced Political Economy Research [Centrum pro pokročilá politicko-ekonomická studia], No. LC542, (2005-2009),
- Economic Aspects of EU and EMU Entry [Ekonomické aspekty vstupu do Evropské unie a Evropské měnové unie], No. AVOZ70850503, (2005-2010);
- Economic Impact of European Integration on the Czech Republic [Ekonomické dopady evropské integrace na ČR], No. MSM0021620846, (2005-2011);

Specific research support and/or other grants the researchers/publications benefited from are acknowledged at the beginning of the Paper.

Published by
Charles University in Prague, Center for Economic Research and Graduate Education (CERGE)
and
Economics Institute (EI), Academy of Sciences of the Czech Republic
CERGE-EI, Politických vězňů 7, 111 21 Prague 1, tel.: +420 224 005 153, Czech Republic.
Printed by CERGE-EI, Prague
Subscription: CERGE-EI homepage: http://www.cerge-ei.cz

Editors: Directors of CERGE and EI
Managing editors: Deputy Directors for Research of CERGE and EI

ISSN 1211-3298
ISBN 80-7343-059-2 (Univerzita Karlova v Praze, CERGE)
ISBN 80-7344-048-2 (Národohospodářský ústav AV ČR, Praha)

CERGE-EI
P.O.BOX 882
Politických vězňů 7
111 21 Praha 1
Czech Republic
http://www.cerge-ei.cz