POLICY UNCERTAINTY AND LONG-RUN INVESTMENT AND OUTPUT ACROSS COUNTRIES

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ABSTRACT

I explore how and and to what extent policy uncertainty can account for the observed long-run cross-country differences in capital price and levels of aggregate investment and output. I present a model economy where the industry-level policy-related investment cost is uncertain. Holding the average one-period investment cost constant, policy uncertainty leads to a higher capital price and subsequently to lower levels of long-run aggregate investment and output. Policy uncertainty also makes industry-level investment more volatile. In an investigation of industry-level investment across countries, I find that, consistent with the model, industry-level investment in lower-income countries is more volatile. I calibrate the model so that the simulated investment sequences from the model mimic the actual investment sequences. I find that policy uncertainty can account for large differences in long-run capital price, investment, and output across countries. Between the lowest-income and the highest-income countries, policy uncertainty can account for the capital price difference by a factor of about three, and can account for investment and output level differences of a similar magnitude.

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1. Introduction

There are large and persistent differences in output levels across countries. Although some developing countries have experienced fast output growth or decline over the decades, overall the output gap between low-income and high-income countries has remained stagnant. Relatedly, in low-income countries the capital price relative to the consumption good price is higher and the investment share of output is lower when measured under internationally common prices of capital and consumption goods.[1] These observations are consistent with neoclassical theory, which predicts that a higher capital price reduces investment and a reduction in investment leads to reduced output.[2] One important question is why the capital price is higher in low-income countries, as explaning the high capital price would also in part explain the output gap. In this paper I focus on one reason, namely, uncertainty in policy-related investment cost (which I will call "policy uncertainty" in the remainder of the paper). The policy-related investment cost is meant to capture a broad spectrum of costs; it includes not only direct taxes and subsidies on investment, but also indirect costs such as bribes and the costs of following regulations.[3] Thus policy uncertainty results not only from uncertain taxes and subsidies, but also from (unexpected) changes in regulations, the level of enforcement, officials, and their demands. I construct a model where long-term policy uncertainty increases the capital price and thereby decreases the investment and output over that period. I assess the quantitative significance of policy uncertainty in accounting for the long-run cross-country differences in these variables.

One can casually observe indications of policy uncertainty in low-income countries, e.g., frequent regime changes in many African countries and the recent policy reversals in Russia.[4] More formally, empirical studies of long-run cross-country comparisons showed

[1] These observations are from the data set of Summers and Heston (1993), as noted by, for example, Parente and Prescott(1993) and Chari, Kehoe, and McGrattan (1997).

[2] Chari, Kehoe, and McGrattan (1997) found that the variations of capital price relative to consumption good price explain 4/5 of income variations across countries assuming the physical and organizational capital share of output to be 2/3. In endogenous growth theory, a higher capital price, by reducing investment, would lead to a lower growth rate of output. De Long and Summers (1991) found that the equipment investment has a strong positive association with growth. Jones (1994) found that a unit increase in the machinery price relative to consumption good price reduces the growth rate by one percent.

[3] De Soto (1989) shows that these indirect investment costs are large in the Peruvian economy. Apparently motivated by this and other similar observations, Parente and Prescott (1994) propose that the low-income countries have a greater 'barrier' to technology adoption, which effectively increases the cost of producing 'technology' capital. This paper, however, focuses on the uncertainty in, not the level of, these costs in capital production.

[4] A less visible but telling example is that under the regime of Mobutu in Congo (formally Zaire), there were cabinet shuffles about every three months; "In 1988, a typically transient year for higher-ups in Zaire, there were four full-scale cabinet shuffles." (Harden 1993).

that low-income countries in fact have greater policy uncertainty. especially if we interpret socio-political instability as an indication of policy uncertainty. For example, Aizenman and Marion (1993) and Serven (1998) found that macroeconomic policy uncertainty is negatively related to private investment. Ramey and Ramey (1995) found that countries with higher volatility of output have lower growth. Barro (1991) found that political instability is negatively related to output growth. Similarly, Alesina and Perotti (1996) found that socio-political instability is negatively related to investment and output. These authors generally conclude that uncertainty or instability leads to lower investment and output levels.[5]

On the other hand, theoretical studies on the effect of uncertainty on long-run investment are scarce, and the existing ones have found a neutral or mixed effect. The theory of investment under irreversibility has shown that uncertainty delays investment since investors gain by waiting for more information.[6] This delaying effect of uncertainty may be a significant reason why developing countries have difficulty in increasing investment after policy reform (Rodrik 1991). Aside from the effect on the timing of investment, however, irreversible investment does not imply a long-run negative relationship between uncertainty and investment. A neutral result can be found in Lucas and Prescott (1971). In a model of irreversible investment under output demand uncertainty, the authors show that the long-run investment is determined by the average demand irrespective of the level of uncertainty. Abel and Eberly (1995) show in a model of capacity investment by a firm facing demand uncertainty that uncertainty tends to increase the long-run investment both under reversibility and under irreversibility. Hopenhayn and Muniagurria (1996) derive mixed results in an aggregate model under irreversibility and uncertain investment subsidy: either greater dispersion between the possible subsidy rates (greater uncertainty) or greater persistence in the subsidy rates (less uncertainty) are likely to increase the long-run investment and output growth.

In the models demonstrating a non-neutral effect of uncertainty on long-run investment, the results depend on a common, but questionable, assumption for modeling uncertainty. That is, in these models the *time-weighted* average value of the uncertain variable (weighing each possible value by the duration under which that value is likely to prevail regardless of investment responses) is fixed under various levels of uncertainty. Although this assumption is simple, it may not be the best way of fixing the average value. For

[5] Serven (1996) also arrives at this conclusion after a survey of studies with similar results. However, Campos and Nugent (1998a, 1998b) came to the opposite conclusion.

[6] See, for example, Pindyck (1991) for an introduction and Dixit and Pindyck (1994) for an extensive exposition.

example, in the model of Hopenhayn and Muniagurria's model, the time-weighted average subsidy rate is fixed under various levels of uncertainty. However, uncertainty affects the *investment-weighted* average subsidy rate (weighing each rate by the amount of investment carried out under that rate) by affecting the investment responses under various shocks. The notion of the average subsidy rate, I think, corresponds more to the investment-weighted subsidy rate than to the time-weighted subsidy rate.[7] Under the notion of the investment-weighted average subsidy rate, the reported differences in investment are due not only to different levels of uncertainty in, but also to different levels of, the average subsidy rate. To isolate the effect of uncertainty, it is necessary to fix the investment-weighted average subsidy rate under various levels of uncertainty by appropriately scaling up or down the distribution of subsidy rates.

It appears that irreversibility alone is not enough to generate the negative effect of uncertainty on long-run investment,[8] and we need to consider some other channel through which uncertainty affects investment. In Sections 2 and 3, I present a model of investment under uncertainty that does generate a negative effect of uncertainty on long-run investment and output. In Section 2, I present a micro model of an investment environment where at each period the cost of managing an investment project is composed of a constant technical cost and an uncertain policy-related cost. A crucial feature of the model is that the investors choose among various types of investment projects that are differentiated by the duration from the start to completion, and by the technical rate of return, i.e., the return per unit of technical cost. In this environment, policy uncertainty makes investors prefer shorter-term projects even though longer-term projects have higher technical rates of return. Also, a project may be abandoned before completion if the cost becomes high. Through these two channels, policy uncertainty increases the cost of producing capital, or equivalently the capital price, holding the investment-weighted average policy-related cost constant. The model is related to those which addressed the sequential investment decision problem in Roberts and Weitzman (1981), Grossman and Shapiro (1986), and Pindyck (1993). The main difference is that these previous studies characterized the intensity or speed of investment under uncertainty for a given project, whereas the focus

[7] More generally, which notion of average cost is more appropriate depends on the source of the uncertainty. For example, if the uncertainty is in the forces of nature (e.g., rainfall) the time-weighted average cost would be more appropriate. On the other hand, if the uncertainty is in policy that is presumably chosen with the consideration of the investors' responses (e.g., tax collection), investment-weighted average cost would be more appropriate.

[8] Pindyck and Solimano (1993) and Serven (1996) state this view.

in this paper is on the problem of choosing among various types of projects as a way of deriving the effect of policy uncertainty on the competitive equilibrium capital price.

In Section 3, I present an aggregate model of investment and output, built on the model in Section 2. There are many industries and the uncertain policy-related cost is industry-specific. Policy uncertainty, by increasing the capital price through the channels described in Section 2, decreases the long-run aggregate investment and output. This result is consistent with the afore-mentioned empirical finding that low-income countries have greater policy uncertainty, higher capital prices, and lower investment. In the model, aside from the effect on the capital price and the levels of investment and output, policy uncertainty also affects the industry investment dynamics, i.e., statistics on investment sequence at the industry-level. Generally speaking, policy uncertainty increases the 'volatility' of investment. This effect of uncertainty on investment dynamics is useful in two ways. First, it implies a negative relationship between the output level and the volatility of industry investment across countries, which can be tested in a set of cross-country investment data. Second, we can calibrate the model by relating the model-generated investment dynamics to the actual investment dynamics. This is useful since the uncertainty parameter values cannot be directly inferred from data given the broad and abstract nature of policy uncertainty in the model.

In Section 4, I investigate the industry-level investment sequences from 1967 to 1988 across 27 manufacturing industries and across 16 countries of diverse output levels, and find that the investment in lower-income countries is indeed more volatile. In Section 5, I calibrate the model to assess the quantitative significance of policy uncertainty in accounting for the differences in the long-run capital price, investment, and output across countries. For the calibration, I simulate the industry investment sequences for various model parameter values and select the ones that generate the actual investment dynamics. I find that policy uncertainty can account for a large portion of the observed cross-country differences: between the lowest-income and the highest-income countries, policy uncertainty can account for the capital price difference by a factor of about 3 and the output difference of a similar magnitude. Section 6 concludes.

2. Investment under Policy Uncertainty

In this section, I first present an investor's problem under the uncertainty of policy-related investment costs. The investor chooses a new project among various types of projects that are differentiated by their duration, and decides whether to continue the project as it ages. A poject, if finished, yields new capital, whose quantity depends on the type of project. I characterize the optimal investment rule. Next, I show that under competition, the effect of uncertainty is to increase the equilibrium capital price, holding the investment-weighted average policy-related cost as constant. This happens because uncertainty causes investors to prefer shorter-term projects and abandon some projects, increasing the expected cost of producing capital.

2.1 The Investor's Problem

Consider an investor who is risk-neutral and discounts the future costs and returns by one period discount rate $\beta < 1$. At any date, the investor can choose one of three activities: start an investment project, continue an unfinished project if there is one, or remain idle. There are many types of projects. A project of type $j \in \{1, 2, \ldots\}$ requires j consecutive periods of investment to be completed. The one-period cost of investment, denoted as ϕ, is composed of 1 unit of technical cost and $\phi - 1$ units of policy-related cost. The policy-related cost is project-specific and its future values are uncertain. Thus ϕ is project-specific and uncertain as well. The value of ϕ can be either ϕ_1 or ϕ_2, where $\phi_1 < \phi_2$, and evolves according to a Markov Chain: if the current period cost is ϕ_q, $q = 1, 2$, the probability that the next period cost will be ϕ_q is π_q. I assume that $\pi_1 + \pi_2 > 1$, which implies some persistence in the one-period cost: for $q = 1, 2$ the probability for the next period cost to be ϕ_q is greater if the current period cost is ϕ_q. I also assume that at each date and for each type of project there are some new projects with the low current cost ϕ_1. This implies that there is no value of waiting to invest, which is an important feature of models with irreversible investment. As mentioned earlier, the value of waiting to invest, aside from its effect on the timing of investment, is unlikely to have a robust effect on the long-run investment, which is the concern of this paper. A project of type j, once completed, yields $h(j)$ units of new capital. I call $h(j)$ the capital return function, and assume that a unique finite j solves the problem $\max_j\{(\beta^{j-1}h(j))/\sum_{n=0}^{j-1}\beta^n\}$. This assumption ensures that the investor's problem is well defined and has a unique solution in the absence of policy uncertainty (see Proposition 2). Let P denote the capital price, which is assumed to be fixed for the moment. The value of the completed project is then $Ph(j)$. An unfinished project can be abandoned, and once abandoned, the project has no value: the costs of investment already incurred cannot be recovered and the project cannot be continued later.

In this environment, the investor's decision problem at each date is as follows. If he has an unfinished project carried over from the previous date, he decides whether to continue the project or to abandon it, given the current cost of the project. If he does not have an unfinished project or if he decides to abandon an unfinished project, he decides whether to start a project and, if he decides to, which type of project with which current cost to start. To formalize this problem, let $v(j, n, q)$ denote the value of managing (i.e., starting or continuing) a project of type j, age n, and the current cost ϕ_q. Then,

$$v(j, n, q) = -\phi_q + \pi_q \beta \tilde{v}(j, n+1, q) + (1 - \pi_q)\beta \tilde{v}(j, n+1, q') \tag{2.1}$$

for $n \in \{0, 1, \ldots, j-2\}$;

$$v(j, j-1, q) = -\phi_q + Ph(j); \tag{2.2}$$

and

$$\tilde{v}(j, n, q) = \max\{v(j, n, q) - \xi, 0\}, \tag{2.3}$$

where in each equation $q' = 1$ if $q = 2$ and $q' = 2$ if $q = 1$. The term $\tilde{v}(\cdot)$ is the value of the option to manage the project. The term ξ is the opportunity cost of managing the project, that is, the value of the best alternative to managing the project. Since the alternatives are starting a new project or being idle, we have

$$\xi = \max\{\max_{j,q}\{v(j, 0, q)\}, 0\}. \tag{2.4}$$

If the value of managing a project is greater than the opportunity cost, the investor will choose to manage the project. If it is less than the opportunity cost, he will not manage the project and instead opt for the best alternative.

If the values of all new projects are negative ($\max_{j,q}\{v(j, 0, q)\} < 0$), the investor will be idle at all dates. On the other hand, if the values of some new projects are positive ($\max_{j,q}\{v(j, 0, q)\} > 0$), the investor will manage projects at all dates. In the middle case ($\max_{j,q}\{v(j, 0, q)\} = 0$), the investor will be indifferent to managing projects or being idle. The following proposition characterizes the investor's behavior when he manages projects ($\xi \geq 0$).

Proposition 1: If there is investment ($\max_{j,q}\{v(j, 0, q)\} \geq 0$)*, the investor's behavior is characterized by optimal investment rule* (J, N)*: start a project of the type* J *with the low current cost; continue the project as long as its cost stays low; and if the cost becomes high, abandon the project if it is younger than* N *periods, and continue the project if it is as old or older than* N *periods.*
Proof: see the Appendix.

This proposition can be intuitively explained as follows: given the minimum persistence in the cost, the value of a project with a low current cost is greater than that of an identical project except for its high current cost, and so the investor would start a project with a low current cost. As for the type of project, he will select the one that has the maximum value: $J = \arg\max_j v(j, 0, 1)$.[9] As for the continuation rule, the value of an older project is greater than that of an identical project except for its younger age since an older project requires less investment to be completed and the return is sooner. Given this monotonecity of the value of a project over its age, once the investor has started a project, he will continue it as long as the cost stays low. If the cost becomes high, he will continue the project only if it is old enough to compensate for the loss of its value due to the increase in cost, and abandon the project if it is not old enough.

2.2 Capital Price in Competitive Equilibrium

In the previous subsection, the capital price was assumed to be fixed. Now let's consider how the capital price would be determined under competitive investment. In particular, consider the following environment. There are many investors who are identical to the one we considered, and who competitively produce capital by managing investment projects. Further, the underlying demand for capital is price-elastic so that the capital price responds to the changes in supply of capital. In this environment, if the value of an optimal project $v(J, 0, 1)$ is positive, more projects will be started and capital production will increase. This will in turn decrease the capital price, which will then decrease the value of the project. On the other hand, if the value of the optimal project is negative, no projects would be undertaken, thereby increasing the capital price and the value of the project. Considering these scenarios, we can conclude that there is an equilibrium price P under which the value of any optimal project is zero:

$$v(J, 0, 1) = \xi = 0. \tag{2.5}$$

Formally, given ϕ_1, ϕ_2, π_1, and π_2, an equilibrium is a captal price P and an optimal investment rule (J, N) such that under P, (J, N) is a solution to the investors problem of equations 2.1 to 2.4 and equation 2.5 is satisfied.

2.3 The Effect of Policy Uncertainty on Capital Price

In the investment environment considered in the previous subsections, there is uncertainty in investment cost due to uncertain policy-related cost, and this uncertainty is

[9] There is such finite J given the assumption on h on page 5 (see Proposition 2). However, J may not be unique.

governed by the possible one-period costs, ϕ_1 and ϕ_2, and the transition probabilities, π_1 and π_2. To consider the effect of policy uncertainty on investment in a meaningful context, I will first sharpen the meaning of uncertainty in this environment by transform the parameters $(\tau_1, \tau_2, \pi_1, \pi_2)$ to parameters that have clearer economic meanings. First, the scale parameter $\bar{\phi}$ is defined as the mean of the two costs: $\bar{\phi} \equiv (\phi_1 + \phi_2)/2$. This parameter measures the scale of the costs with a higher value meaning greater scale. Second, the dispersion parameter d is defined as $d \equiv \phi_2/\phi_1$. This parameter measures the dispersion between the two costs with a higher value meaning more dispersion. Next, define $\pi(t;q)$ as the probability that the cost t periods later will be ϕ_q given that the current cost is ϕ_q. We have $\pi(0;q) = 1$ and $\pi(t+1;q) = \pi_q \pi(t;q) + (1 - \pi_{q'})(1 - \pi(t;q))$ for all $t \geq 0$, where in each equation $q' = 1$ if $q = 2$ and $q' = 2$ if $q = 1$. From these equations, we can derive

$$\begin{aligned} \pi(t;1) &= a + (1-a)b^t, \quad \text{and} \\ \pi(t;2) &= 1 - a + ab^t, \end{aligned} \tag{2.6}$$

where $a \equiv (1-\pi_2)/(2-\pi_1-\pi_2)$ and $b \equiv \pi_1 + \pi_2 - 1$. As t increases, $\pi(t;1)$ converges to a and $\pi(t;2)$ converges to $1-a$. Thus the parameter a is the long-run probability that the cost will be ϕ_1. I will call this parameter the frequency parameter with a higher value meaning greater occurence of the low cost in the long-run. The parameter b determines how quickly $\pi(t;1)$ and $\pi(t;2)$ converge to a and $1-a$, respectively. For instance, if $b = .9$, each period $\pi(t;p)$ approaches a by 10% of the gap between the two. I will call this parameter the persistence parameter with a higher value meaning greater persistence of the cost.

Now I have defined the four new parameters $(\bar{\phi}, d, a, b)$. A combination of these four parameter values has a unique corresponding combination of the primitive parameter values $(\phi_1, \phi_2, \pi_1, \pi_2)$. Of the four new parameters, the dispersion parameter d, the frequency parameter a, and the persistence parameter b characterize the uncertainty of investment costs, whereas the scale parameter $\bar{\phi}$ determines only the scale of taxes without affecting uncertainty. The scale parameter, nonetheless, does affect the capital price. In examining the relationship between the uncertainty parameters (d, a, b) and the capital price, then we need to make an assumption as to what value of $\bar{\phi}$ accompanies each combination of (d, a, b). One simple assumption is to set $\bar{\phi}$ so that the *time-weighted* average cost, i.e., $\phi_1 a + \phi_2(1-a)$, is fixed for all sets of values of (d, a, b). This is in fact the default assumption in most studies of uncertainty. For the purpose of this paper, however, this assumption has an undesirable implication: the shares of investment that takes place under low vs. high cost vary depending on the uncertainty parameter values (d, a, b), and are not proportional to a. Under this assumption on $\bar{\phi}$, then the *investment-weighted* average cost (weighing each rate by the amount of investment carried out under that rate) varies. As was mentioned earlier, the notion of the average policy-related investment cost corresponds more to the investment-weighted average cost than to the time-weighted average cost. Given this notion of the average cost, the implication of variable average cost is undesirable since the purpose of this paper is to assess the effect of *uncertainty* in, not the *level* of, policy-related cost on investment.

For this reason, I adopt the alternative assumption on the scale parameter $\bar{\phi}$: the value of $\bar{\phi}$ is chosen so that the investment-weighted average one-period cost is constant for all (d, a, b). To be precise, I will define the investment-weighted average one-period cost, denoted as $\tilde{\phi}$, as follows. First, let a combination of parameter values, $\{\bar{\phi}, d, a, b\}$ the corresponding primitive parameter values $\{\phi_1, \phi_2, \pi_1, \pi_2\}$, and the corresponding equilibrium capital price P and optimal investment rule (J, N) be given. Given this investment rule and using the equations 2.1 to 2.5, the equilibrium value of an optimal project can be written as

$$v(J, 0, 1) = -\omega + \beta^{J-1}\pi_1^{N-1}Ph(J) \tag{2.7}$$

where the first term ω is the expected discounted sum of investment costs; the second term is the expected discounted return from investment; and

$$\omega = \sum_{n=0}^{N-1} \beta^n \pi_1^n \phi_1 + \sum_{n=N}^{J-1} \beta^n \pi_1^{N-1} \phi(n - N + 1). \tag{2.8}$$

To understand this expression intuitively, observe that the probability that a project will be continued at age n is π^n for $n = 0, 1, \ldots, N-1$, and π_1^{N-1} for $n = N, N+1, \ldots, J-1$, and that if the project is continued at age n, the one-period investment cost at that date is ϕ_1 for $t = 0, 1, \ldots, N-1$, and $\phi(n-N+1)$ for $n = N, N+1, \ldots, J-1$, where $\phi(t)$ is the expected one-period cost conditional that the cost is ϕ_1 at $t = 0$: $\phi(t) = \phi_1\pi(t; 1) + \phi_2(1 - \pi(t; 1))$ (see equation 2.6).

Now suppose that at any date the one-period investment cost is a constant value ϕ instead of ϕ_1 or ϕ_2, while the other parameter values and the investor's behavior remain the same. Let $\omega(\phi)$ denote the expected discounted sum of investment costs of a new project in this case. We have

$$\tilde{\omega}(\phi) = \sum_{n=0}^{N-1} \beta^n \pi_1^n \phi + \sum_{n=N}^{J-1} \beta^n \pi_1^{N-1} \phi. \tag{2.9}$$

Now the average one-period cost $\tilde{\phi}$ is defined as the one-period cost under which the expected discounted sum of investment costs is equal to the expected discounted sum of investment costs under the variable one-period cost:

$$\omega = \tilde{\omega}(\tilde{\phi}). \tag{2.10}$$

To equate the values of $\tilde{\phi}$ for all (d, a, b), we want to make sure that for each (d, a, b), there exists $\bar{\phi}$ under which $\tilde{\phi}$ is equal to a given constant. In Proposition A in the Appendix, I show that changes in $\bar{\phi}$ do not change J, N, or P, and simply changes $\tilde{\phi}$ proportionately. Thus, for any given (d, a, b), we can make $\tilde{\phi}$ to be any value by scaling $\bar{\phi}$ up or down: for each (d, a, b), there indeed exists $\bar{\phi}$ under which $\tilde{\phi}$ is equal to a given constant.

With the assumption on $\bar{\phi}$, now we are ready to assess the effect of policy uncertainty on the investment behavior and the capital price. The following proposition characterizes the effect:

Proposition 2: In the absence of policy uncertainty (i.e., $d = 1$, $a = 1$, or $b = 1$), the optimal type of project is unique and of the longest duration, there is no abandonment of projects and, fixing $\tilde{\phi}$, the equilibrium capital price is the lowest.
Proof: see the appendix.

This proposition can be intuitively explained as follows. There will be no policy uncertainty in future one-period costs if $d = 1$ (i.e., the two one-period costs are the same), if $a = 1$ (i.e., the one-period cost is always a low cost), or if $b = 1$ (i.e., the one-period cost never changes). With no negative shock to the cost, no projects will be abandoned. If there is policy uncertainty (i.e., $d > 1$, $a < 1$, or $b < 1$), the one-period cost, which is equal to the low cost at the beginning of the project, may change to a high cost later. The further into the future, the likelihood that the cost will be high increases: the expected one-period cost increases over the age of the project. Therefore, given the scale of the one-period cost $\bar{\phi}$ investors will lower the average one-period cost $\tilde{\phi}$ by managing a project of shorter duration. Note that each investor takes $\bar{\phi}$ as given, although holding $\tilde{\phi}$ *in equilibrium* implies that $\bar{\phi}$ is dependent on the aggregate behavior of investors. Thus uncertainty makes investors favor shorter-term projects. Uncertainty may also make investors abandon projects when the one-period cost changes to a high cost if it is preferable to start a new project with a low cost rather than to continue the old project and to pay the high costs. Holding the average one-period cost constant, the shorter-term projects yield lower expected production of capital per unit cost and the abandonment of projects increase the expected cost of producing a unit of capital. For these two reasons, uncertainty increases the cost of producing a unit of capital or, equivalently, decreases capital production per unit cost, and this, under competitive investment, leads to a higher capital price.

As for the comparison of investment behavior and the capital price across positive but different levels of uncertainty, numerical exercises show that the effect of uncertainty is not simply generalized,[10] but that overall greater policy uncertainty tends to amplify the distortionary effect on investment behavior and thus to increase the capital price. The quantitative exercises in Sections 4 and 5 will show that this relationship between policy uncertainty *and* the investment behavior and capital price holds in a sample of countries.

3. Long-run Aggregate Investment and Output under Policy Uncertainty

In this Section, I present a model of aggregate investment and output. The model is a standard neoclassical one except in the investment environment, that is essentially the same as the environment described in Section 2. There are two non-essential but useful differences between the two investment environments. First, in the following model there are many identical industries where capital is produced and the one-period investment cost is industry-specific. Modeling cost as industry-specific is perhaps more plausible than modeling it as project-specific although the resulting investment problem is essentially the same. Also, by modeling industry-specific costs, I could relate the results of the model to the industry investment data in Sections 4 and 5. Second, in this section there is a single person who manages multiple projects of various sizes across industries. Since there is no externality in the model that can be internalized by the single person, modeling multiple people and various markets and prices would complicate notations without altering the results.

In equilibrium, the person diversify investment across industries. To simplify the analysis, I assume that there are a large number of industries so that, in approximation, there is no aggregate uncertainty.[11] The person then behaves as if he were risk-neutral as in Section 2, and the exactly same results as in that section follow as well. In particular, policy uncertainty increases the capital price, and in this way sustained policy uncertainty leads to lower long-run aggregate investment and output. In the model, in addition to affecting

[10] Numerical exercises show the following. As the dispersion parameter d increases or as the frequency parameter a decreases, J tends to decrease and N tends to increase resulting in higher P. As for the persistent parameter b, for the high range of b, an increase in its value tends to increase J and to decrease N resulting in a lower P. For the low range of b, however, the effect is the opposite.

[11] More generally, the effects of idiosyncratic micro uncertainty would not be entirely cancelled after aggregation, and would affect the dynamics of aggregate variables. See Bertola and Caballero (1994) for such a model. The model in this paper abstracts away from the aggregate dynamics since the focus is on the *long-run levels* of investment and output.

the capital price and the levels of investment and output, policy uncertainty also affects the industry investment dynamics, i.e., statistics on investment sequence at the industry-level. Generally speaking, policy uncertainty increases the 'volatility' of investment. I will use this implication of uncertainty on investment dynamics in the quantitative exercises in Sections 4 and 5.

3.1 The Environment

There is one person whose one-period utility is $u(c)$, where c is consumption and u is increasing and concave, and who discounts future utility by the one-period discount rate $\beta < 1$. At each date, the person produces output using his capital, invests a part of the output to produce new capital, and consumes the rest of the output. The output production function is

$$y = k^{\alpha}, \tag{3.1}$$

where y is output, k is capital, and $0 < \alpha < 1$. The one-period resource constraint is

$$c + x \leq y, \tag{3.2}$$

where x is investment. The capital evolves according to the rule

$$k' = k(1 - \delta) + \tilde{k}, \tag{3.3}$$

where k' is the next period's capital, δ is the depreciation rate, and $\tilde{k}$ is new capital.

Capital is produced in an investment environment that is identical to that described in Section 2, except for the following two differences. First, I assume that there are many identical industries, indexed by $i \in \{1, 2, \ldots, I\}$, where capital is produced and that the one-period investment cost is industry-specific. Second, projects can be of various sizes and the person can manage multiple projects across industries at the same time. Let $z \in (0, \infty)$ denote the size of a project. A project of type j and size z requires j consecutive periods of investment, and yields $zh(j)$ units of new capital in the jth period. The one-period investment cost is ϕz units of output, where ϕ is the one-period cost for a project of size 1 in the industry to which the project belongs. As in Section 2, ϕ is composed of 1 unit of technical cost and $\phi - 1$ units of policy-related cost; the possible values of ϕ are ϕ_1 and ϕ_2, and ϕ evolves according to a Markov Chain, that is common across industries, and whose transition probabilities are π_1 and π_2. If the transition probabilities are independent across countries, the composition of industries with low vs. high costs would fluctuate over time and the long-run average fraction of industries with low costs would be equal to the frequency parameter a defined in Section 2. To simplify the investment decision, however, I assume that the transition probabilities are dependent on each other across industries so that the portion of industries with a low cost remains constant over time at the value of a; the portion of industries with a high cost then remains constant at $1 - a$.

Let $z(j, n, i)$ denote the size of the project of type j, aged n, in industry i. At each date, the representative person chooses the sizes of new projects $\{z(j, 0, i)\}$. He also decides whether to continue old projects. Let $\{g(j, n, i)\}$ denote these decisions: if $g(\cdot) = 1$, the project is continued and if $g(\cdot) = 0$, the project is abandoned. The sizes of projects then evolve according to the rule

$$z'(j, n+1, i) = g(j, n, i)z(j, n, i) \tag{3.4}$$

for $n = 0, 1, 2, \ldots, j-2$, and the quantity of new capital produced is determined by the rule

$$\tilde{k} = \sum_{i=1}^{I} \sum_{j=1}^{\infty} g(j, j-1, i)z(j, j-1, i)h(j). \tag{3.5}$$

Let $\phi(i)$ denote the one-period cost of industry i. The quantity of investment is then determined by the rule

$$x = \sum_{i=1}^{I} \sum_{j=1}^{\infty} \phi(i)z(j, 0, i) + \sum_{i=1}^{I} \sum_{j=1}^{\infty} \sum_{n=1}^{j-1} \phi(i)g(j, n, i)z(j, n, i). \tag{3.6}$$

3.2 Capital Price, Investment, and Output in Steady State

In this economy, there is uncertainty at two levels: uncertainty in the one-period cost at the industry level and uncertainty in the aggregate variables $\tilde{k}$, k, y, c, and x. The aggregate uncertainty arises because the share of a cohort of investment projects whose one-period cost changes from one date to the next is not certain and so the investment decision is adjusted to the realizations of these shares. Although aggregate uncertainty is an interesting issue in its own right, in this paper, for simplicity, I will concentrate on industry-level uncertainty and eliminate aggregate uncertainty by fixing these shares by approximation under the assumption that there are a large number of industries. The following two paragraphs explain the approximation.

I will first rewrite the constraints regarding investment (equations 3.4 to 3.6) in terms of projects indexed by current one-period cost instead of by industry. Let $\tilde{z}(j, n, q)$ denote the sum of the sizes of projects across industries that are of type j, aged n, and with the current one-period cost equal to ϕ_q:

$$\tilde{z}(j,n,q) = \sum_{i=1}^{I} e(i,q)z(j,n,i), \tag{3.7}$$

where $e(i,q)$ is an indicator function and is equal to 1 if $\phi(i,q) = \phi_q$ and equal to 0 otherwise. Let $\tilde{g}(j,n,q)$ denote the fraction of the sum $\tilde{z}(j,n,q)$ that is continued:

$$\tilde{g}(j,n,q) = \frac{\sum_{i=1}^{I} e(i,q)g(j,n,i)z(j,n,i)}{\sum_{i=1}^{I} e(i,q)z(j,n,i)}, \tag{3.8}$$

and $\tilde{\pi}(j,n,q)$ the fraction of the sum of the continued projects $\tilde{g}(j,n,q)\tilde{z}(j,n,q)$ that faces the same cost ϕ_q the next period:

$$\tilde{\pi}(j,n,q) = \frac{\sum_{i=1}^{I} e'(i,q)e(i,q)g(j,n,i)z(j,n,i)}{\sum_{i=1}^{I} e(i,q)g(j,n,i)z(j,n,i)}, \tag{3.9}$$

where $e'(\cdot)$ is the indicator function for the next period. From equations 3.4 to 3.9, we can show that the sums $\{\tilde{z}(\cdot)\}$ evolve according to

$$\tilde{z}'(j,n+1,q) = \tilde{\pi}(j,n,q)\tilde{g}(j,n,q)\tilde{z}(j,n,q) + (1-\tilde{\pi}(j,n,q'))\tilde{g}(j,n,q')\tilde{z}(j,n,q') \tag{3.10}$$

for $n = 0,1,2,\ldots,j-2$, where in each equation $q' = 1$ if $q = 2$ and $q' = 2$ if $q = 1$; the quantity of new capital is determined by

$$\tilde{k} = \sum_{q=1}^{2}\sum_{j=1}^{\infty} \tilde{g}(j,j-1,q)\tilde{z}(j,j-1,q)h(j); \tag{3.11}$$

and the quantity of investment is determined by

$$x = \sum_{q=1}^{2}\sum_{j=1}^{\infty} \phi_q \tilde{z}(j,0,q) + \sum_{q=1}^{2}\sum_{j=1}^{\infty}\sum_{n=1}^{j-1} \phi_q \tilde{g}(j,n,p)\tilde{z}(j,n,p). \tag{3.12}$$

Equations 18, 19, and 20 correspond to equations 12, 13, and 14; the difference is that they are written in terms of projects indexed by current one-period cost instead of by industry.

The values of the fractions $\{\tilde{\pi}(\cdot)\}$ are not certain and depend on the distribution of continued projects across industries $\{g(\cdot)z(\cdot)\}$ and the realizations of next-period costs across industries. The uncertainty in $\{\tilde{\pi}(\cdot)\}$ implies uncertainty in $\{\tilde{z}'(\cdot)\}$ and therefore uncertainty in future values of x and $\tilde{k}$, and this in turn implies uncertainty in future values of k, y, and c. Since the person's utility is concave, he prefers less uncertainty in the aggregate variables and, for this reason, the number of industries I is important to him. First, if I is larger, the fraction of industries whose one-period cost changes, $\sum_{i=1}^{I} e'(i,q)e(i,q)/\sum_{i=1}^{I} e(i,q)$, is more certain. Second, if I is larger, the person can spread his projects more across industries and thereby decrease the investment in one industry relative to the aggregate investment, $z(\cdot)/x$. For these two reasons, a greater number of industries allows the person to reduce uncertainty in $\{\tilde{\pi}(\cdot)\}$ and thereby in aggregate variables. As the number of industries approaches infinity, the aggregate uncertainty disappears altogether: as $I \to \infty$, $\tilde{\pi}(j,n,q) \to \pi_q$. In order to dispense with aggregate uncertainty, then I assume that the number of industries is large enough that the person's decision problem can be considered with the approximations

$$\tilde{\pi}(j,n,q) = \pi_q \tag{3.13}$$

and

$$\tilde{z}'(j,n+1,q) = \pi_q \tilde{g}(j,n,q)\tilde{z}(j,n,q) + (1-\pi_q')\tilde{g}(j,n,q')\tilde{z}(j,n,q'). \tag{3.14}$$

With no aggregate uncertainty, the person's decision problem simplifies. At each date, the person takes as given the quantity of capital k and the sizes of various on-going projects $\{\tilde{z}(j,n,q)\}_{n\geq 1}$. Let the state, denoted by μ, be the set of these variables: $\mu = \{k\} \cup \{\tilde{z}(j,n,q)\}_{n\geq 1}$. The person's decision problem is to maximize his expected discounted utility by choosing the output y, the consumption c, the investment x, the new capital $\tilde{k}$, the sizes of new projects $\{\tilde{z}(j,0,i)\}$, whether to continue on-going projects $\{\tilde{g}(j,n,q)\}_{n\geq 1}$, subject to constraints 3.1 to 3.3, 3.11, 3.12, and 3.14, and taking as given the state μ. In general, the state of the economy will change over time. In particular, given the neoclassical utility and production functions, the state will converge to the steady state, that is, the state which, once arrived at, is maintained throughout all subsequent dates.[12] Since the objective of this paper is to assess the effect of uncertainty on long-run investment and output, I will focus on the the steady state of the economy.

In steady state, the consumption stays constant so that the marginal utility stays constant. Then the person discounts future output by the one-period discount rate β. The value of a unit of capital is the sum of discounted marginal products that accrue to it.

[12] In the steady state, although the aggregate variables stay constant, the industry-level investment fluctuates over time.

This value is the implicit capital price. In steady state, the capital also stays constant so that the marginal product stays constant. We have

$$P = \sum_{t=1}^{\infty} \beta^t (1-\delta)^{t-1} \alpha k^{\alpha-1}. \tag{3.15}$$

Thus the capital price stays constant in steady state. With the constant discount rate and the constant capital price, the investment environment in the steady state is the same as in Section 2, except that here one person manages all projects, whereas in the environment in Section 2 each competitive investor manages one project. Since there are no externality or other distortionary elements in the environment in Section 2 that can be internalized by the person in this section, this difference between the two environments is not consequential in determining investment allocation and capital price: given the model parameter values regarding investment, the projects selected and continued are the same between the two environments and the implicit capital price here is the same as the equilibrium capital price in Section 2.[13] Thus the steady state investment behavior is characterized by an optimal investment rule (J, N) as in Section 2: $\tilde{z}(J,0,1) > 0$; $\tilde{z}(j,0,p) = 0$ for $j \neq J$ or $q = 2$; $\tilde{g}(J,n,1) = 1$ for $n = 1,2,\ldots,J-1$; $\tilde{g}(J,n,2) = 0$ for $n = 1,2,\ldots,N-1$; and $\tilde{g}(J,n,2) = 1$ for $n = N, N+1, \ldots, J-1$, and policy uncertainty increases the steady state capital price.[14]

Now let's consider the effect of policy uncertainty on investment and output levels. We can derive from equations 3.1 and 3.15

$$\frac{y_s}{y_{s'}} = \left(\frac{P_{s'}}{P_s}\right)^{\frac{\alpha}{1-\alpha}}, \tag{3.16}$$

where s and s' denote two countries. From this equation, we can see that a higher capital price leads to a lower output level. Thus policy uncertainty decreases the long-run output by increasing the capital price. We can also derive from equations 3.1, 3.3, and 3.15

$$\frac{\tilde{k}_s/y_s}{\tilde{k}_{s'}/y_{s'}} = \frac{P_{s'}}{P_s}. \tag{3.17}$$

[13] To be precise, the decision problem in Section 2 is to maximize the expected discounted return *per period* whereas the decision problem in this section is to maximize the expected discounted return *per unit-cost*. These two problems are not the same since the cost for a period could be high or low. Under the equilibrium capital price, however, the expected return net of the expected cost for an optimal project is zero and so an optimal type of project maximizes both the return per period and the return per unit-cost. With the opportunity cost of continuing a project equal to zero, a continuation rule also maximizes both the return per period and the return per unit-cost.

[14] An implicit assumption of the model is that there is no cross-country trade (due to immobility, restiction etc.) where a country with a high capital price could buy capital goods and sell consumption goods.

From this equation, we can see that a higher capital price leads to lower capital production *as a share of output* or, in other words, a lower investment share of output when investment is measured in units of investment output. We can also derive from equations 2.7, 2.8, 3.11, 3.12, 3.14, and 3.17, if $P_s < P_{s'}$, $x_s/y_s < x_{s'}/y_{s'}$ in general, and for β close to 1

$$\frac{x_s}{y_s} \cong \frac{x_{s'}}{y_{s'}}. \tag{3.18}$$

From this equation, we can see that if investment is measured in units of investment cost, the capital price has little effect on the investment share of output. Thus, policy uncertainty has little effect on the investment share of output in terms of cost, but reduces the share in terms of capital production by making capital production more costly. These effects of policy uncertainty on investment and output levels are consistent with the empirical findings mentioned earlier.

3.3 Industry Investment Dynamics

In steady state, although the aggregate variables stay constant, investment at the industry level fluctuates. When an industry faces a low one-period cost, new projects will be started and the old unfinished projects, if there are any, will be continued in that industry. On the other hand, when an industry faces a high one-period cost, no new projects will be started and some old unfinished projects may be abandoned. To be precise, let $\tilde{x}(i)$ denote the investment in industry i:

$$\tilde{x}(i) = z(J,0,i) + \sum_{n=1}^{J-1} g(J,n,i)z(J,n,i). \tag{3.19}$$

Recall that the person diversifies investment across industries to minimize the aggregate uncertainty. The sizes of new projects across industries will then be equal. Further, in steady state, the sizes of new projects across dates will be equal as well. Let $\bar{z}$ denote the size of new project, and $\{\hat{\phi}(i,t)\}_{0\leq t\leq J-1}$ the history of one-period costs where $\phi(i,t)$ is the cost t periods ago in industry i. The industry investment $\tilde{x}(i)$ is determined by the sizes of projects $\{z(J,n,i)\}_{0\geq n\geq J-1}$ and the continuation decisions $\{g(J,n,i)\}_{1\geq n\geq J-1}$, which are in turn determined by the size of new project $\bar{z}$, the optimal investment rule (J,N), and the history $\{\hat{\phi}(i,t)\}_{0\leq t\leq J-1}$ in the following way: for $1 \leq n \leq J-1$, $z(J,n,i) = 0$ if $\hat{\phi}(i,s) = \phi_2$ for any $s \in \{\max(1, n-N+1), \max(1, n-N+1)+1, \ldots, n\}$, and $z(J,n,i) = \bar{z}$ otherwise; $z(J,0,i) = 0$ if $\hat{\phi}(i,0) = \phi_2$ and $z(J,0,i) = \bar{z}$ if $\hat{\phi}(i,0) = \phi_1$; $g(J,n,i) = 0$ if $\hat{\phi}(i,0) = \phi_2$ or if $n < N$, and $g(J,n,i) = 1$ otherwise. In this way, as the history $\{\hat{\phi}(i,t)\}_{0\leq t\leq J-1}$ evolves over time, so does the corresponding industry investment $\tilde{x}(i)$.

Since the evolution of the history $\{\hat{\phi}(i,t)\}_{0\leq t\leq J-1}$ is governed by the parameters a and b, the industry investment dynamics (i.e., statistics on investment sequence at the industry-level) is determined by (J, N, a, b), excluding $\bar{z}$ which determines nothing more than the scale of the investment sequence. Since J and N are in turn determined by the uncertainty parameters (d, a, b), policy uncertainty affects not only the capital price and output, but also the industry investment dynamics. In particular, if there is no uncertainty (i.e., $d = 1$, $a = 1$, or $b = 1$), the industry investment $\tilde{x}(\cdot)$ will be constant over time and equal to $J\bar{z}$. Under uncertainty (i.e., $d > 1$, $a < 1$, and $b < 1$), on the other hand, the industry investment $\tilde{x}(\cdot)$ will fluctuate over time. Further, numerical exercises show that greater policy uncertainty (i.e., greater d, smaller a, or smaller b) tends to make the industry investment more 'volatile', in a sense to be made precise in Section 4. The quantitative exercises in Sections 4 and 5 will show that this positive relationship between policy uncertainty and industry investment volatility holds in a sample of countries.

4. Analysis of Industry Investment Data across Countries

In the model of Sections 2 and 3, we saw that holding the average one-period cost, policy uncertainty, by increasing the capital price, decreases the long-run aggregate investment and output, and that policy uncertainty also makes the industry-level investment more volatile. Then the model implies a negative relationship between the output level and the industry investment volatility across countries, and this can be tested in a set of cross-country investment data. To this end, in this section I investigate the industry-level investment sequences across countries of diverse output levels, and find that the investment in lower-income countries is indeed more volatile. The results in this section will also be used in Section 5 to calibrate the model and to assess the quantitative significance of policy uncertainty.

4.1 The Data

The data come from the various issues of the Industrial Statistics Yearbook published by The United Nations Industrial Development Organization (UNIDO). The data comprise the annual gross investment for 27 3-digit ISIC manufacturing industries and cover the period from 1967 to 1988 for 16 countries. I chose this period because the data before 1967 are organized by a different industrial classification system, and because the data after 1988 were not available for most countries. For many developing countries, especially those in Sub-Saharan Africa, the data are incomplete, missing for many years and industries. This incompleteness of data limits the number of countries whose data we can use. I selected

16 countries that have reasonably extensive data in terms of both years and industries and that represent various output levels.[15] Table 1 reports the list of 16 countries and the coverage of years and industries for each country. The first column of Table 2 reports the per-capita output for these countries from the data set of Summers and Heston (1993).

4.2 Industry Investment Dynamics in the Data

The raw investment data are not comparable across years and countries and require some adjustments. First, the measures of investment are in units of the current currencies of the respective countries. To compare investment across years, I converted each measure of investment into US dollars of the respective year using the Purchasing Power Parities of composite capital[16] from Summers and Heston (1993), and then converted the US dollars of various years into 1985 US dollars using the US GDP deflator. Second, the level of investment is different across industries and countries. Also, the investment sequences for some industries in some countries would exhibit long-run growth or contraction. Since the theoretical analysis in this paper is not of the differences in the size or the evolution of industry investment, I abstracted from these features of the data in the following way. Let z_{sit} denote the investment of country s, industry i, and year t in the data. Let $\bar{z}_{sit}$ be the trend investment and λ_{sit} the percentage deviation from the trend:

$$\begin{aligned} z_{sit} &= \bar{z}_{sit}(1 + \lambda_{sit}) \\ &= \rho_{si} e^{\sigma_{si}}(1 + \lambda_{sit}). \end{aligned} \tag{4.1}$$

The parameters ρ_{si} and σ_{si} are the level and the growth rate of investment for the industry i in country s, and are chosen by minimizing the sum over t of squared deviations.

I consider the investment dynamics to be three country-specific statistics defined as follows. Let $\{\lambda_{sit}\}$ be the investment sequence with respect to t holding s and i. For country s, the dispersion statistic $disp_s$ is the average over i of the standard deviations of the sequences $\{\lambda_{sit}\}$; the persistence statistic $pers_s$ is the average over i of standard deviations of the first-order differences of the sequences $\{\lambda_{sit}\}$; and the frequency statistic $freq_s$ is the average over i of the percentages of positive elements in the sequences $\{\lambda_{sit}\}$.

[15] The theoretical analysis was carried out in terms of steady state. By assuming exogenous productivity growth, I can modify the analysis to be in terms of balanced growth with no changes in the results regarding uncertainty, capital price, and output. Thus the analysis is of countries in balanced growth whose relative output levels remain constant over time. For this reason, I selected countries whose whose per capita output growth rates over the period are not exceptionally high or low. The exceptions are Brazil and Korea.

[16] Purchasing Power Parities of capital for individual industries are not available.

Columns 2 to 4 of Table 2 report these statistics. Plots 2 to 4 plot $\{disp_s\}$, $\{diff_s\}$, and $\{frac_s\}$ against output level. We can see the pattern that the investment sequences in low-income countries exhibit greater volatility: greater deviation, less persistence, and lower frequency of above-the-trend investment.[17]

5. Quantitative Adaptation of the Model Economy

The objective of this section is to assess the extent to which uncertainty can account for the differences in the long-run capital price, investment, and output across countries. To this end, it is necessary to calibrate the model (i.e., assign uncertainty parameter values to countries across various output levels as well as parameter values that are common across countries). Since in the model the policy-related cost is meant to capture a broad spectrum of costs, there is no obvious set of data that can be directly mapped to the uncertainty parameter values. As an alternative, I calibrate the model by simulation and using the results from the industry data analysis in Section 4: I simulate the industry investment sequences for various model parameter values and select the ones that generate the actual investment dynamics.[18] Once the model is calibrated, I derive the differences in the capital price, investment, and output differences implied by these parameter values. I find that uncertainty can account for a large portion of the observed cross-country differences.

In calibration, among the model parameters, the discount parameter β, the capital share parameter α, the depreciation parameter δ, and the capital return function h are assumed to be common to all countries. I consider the length of a period to be a quarter and set $\beta = .99$. This implies an annual real interest rate of about 4% in the steady state. The parameter α is important in determining the extent to which uncertainty can account for investment and output differences, and will be discussed later. The value of parameter δ is inconsequential in terms of the effect of uncertainty on the capital price, investment, and output, and so I do not assign any particular value to it. The uncertainty parameters d, a, and b are country-specific. I will explain in the following three subsections

[17] I also examined the investment dynamics at more aggregate levels, namely at the levels of manufacturing investment as a whole and the economy-wide investment. The same patterns of investment dynamics with respect to the output level are observed. Across countries, however, investment at more aggregate levels is much less volatile than at the industry level, indicating that much of the industry-level volatility and the underlying causes are idiosyncratic.

[18] Implicit in this exercise is the assumption that all industry investment dynamics is due to policy uncertainty. This assumption is rather crude, and ignores other potential sources of investment dynamics such as terms-of-trade shocks, technological innovation shocks, and shocks from nature. However, under a loose interpretation of the model, some of these other shocks can be considered as policy uncertainty. For example, monetary policy shocks are not explicitly modeled but can be considered an element of policy uncertainty since they lead to uncertainty in the investment cost in a generic sense.

the procedure that I used to derive these parameter values across countries as well as the function h.

5.1 Summarizing the Patterns of Investment Dynamics

I first summarized the patterns of investment dynamics with respect to the output level in the data by a line $\bar{l}$ in the space of $disp$, $pers$, and $freq$. Let s denote the country and $\{\eta_s\} = \{(disp_s, pers_s, freq_s)\}$, $s = 1, 2, \ldots, 16$, the investment dynamics found in the data. We can think of each η_s as a point in the space where the axes are $disp$, $pers$, and $freq$. Let $l = l(\gamma_1, \gamma_2, \gamma_3, \gamma_4)$ denote a line in that space:

$$\begin{aligned} diff &= \gamma_1 + \gamma_2 \times disp \quad and \\ frac &= \gamma_3 + \gamma_4 \times disp. \end{aligned} \tag{5.1}$$

Given a line l, the distance between two points $\eta = (disp, diff, frac)$ and $\eta' = (disp', diff',$ $frac')$ in the space is defined as

$$\Delta(\eta, \eta'; l) = \left[(disp - disp')^2 + (\frac{diff - diff'}{\gamma_2})^2 + (\frac{frac - frac'}{\gamma_4})^2 \right]^{\frac{1}{2}}. \tag{5.2}$$

The distance between a point $\eta = (disp, diff, frac)$ and a line $l = (\gamma_1, \gamma_2, \gamma_3, \gamma_4)$ is defined as

$$\Delta(\eta, l) = \min\{\Delta(\eta, \eta'; l) : \quad \eta' \in l \ \}. \tag{5.3}$$

The line that represents the relationships among the three variables in the data is defined as

$$\bar{l} = \arg\min \left\{ \sum_{s=1}^{16} \Delta(\eta_s, l)^2 \right\}. \tag{5.4}$$

In this way I derived $\bar{l} = (-.2882, 2.0135, .6204, -.8677)$. Column 5 of Table 2 reports the distances between the line and each of the country points $\{\eta_s\}$. The distances are relatively small, indicating that the line $\bar{l}$ is a good summary of the patterns of investment dynamics in the data.

5.2 Selecting $\{(J_s, N_s, a_s, b_s)\}$

Next, I simulated investment sequences for various values of the optimal investment rule (J, N) and transition probabilities, a and b, and selected ones that are "close" to $\bar{l}$. The segment of the line $\bar{l}$ that is relevant to the data is from about the point where $disp = .2$ to the point where $disp = .5$. The first point roughly corresponds to the US, and the latter to Ethiopia. To simplify the analysis, I selected 7 points on this segment as the 'target' points to be simulated. They are the points where $disp = .20, .25, .30, .35, .40, .45, .50$. Let these point be denoted by $\{\bar{\eta}_s\}$, $s = 1, 2, \ldots, 7$. What we need to find is the set of uncertainty parameters values $\{(d_s, a_s, b_s)\}$, $s = 1, 2, \ldots, 7$, and the function h that generate investment dynamics 'close' to $\{\bar{\eta}_s\}$. There is no guarantee that there are such parameter values and function. On the other hand, there may be many sets of such parameter values and functions.

The details of the simulation method that I used are as follows. Recall from Section 3.3 that the variables of direct importance for simulation are (J, N, a, b): although the value of the parameter b and the function h determine J and N, we do not need to know them for the purpose of simulation. I selected some candidate values of J, N, a and b. They are $J, N = 1, 2, \ldots, 28$, $a = .1, .2, .3, \ldots, .9$, and $b = 0, .1, .2, \ldots, .9$. For each combination of (J, N, a, b), I set the initial investment at zero and simulated 2876 periods of investment sequence. Then I discarded the first 500 elements of the sequence, leaving 2376 periods of the investment sequence. Since we consider the length of a period for the model to be a quarter and the data are annual, I annualized the simulated sequence by summing up each of four consecutive periods' investments, resulting in 594 years of the investment sequence. Then I divided the sequence into 27 of 22 year sequences, in accordance with the number of industries and the length of the sequences in the data. I calculated the investment dynamics for the simulated sequences the same way as I did for the data: I calculated $(disp, pers, freq)$ for each of the 27 sequences and then averaged them over the sequences. This average is an estimate of the expected point in the space of $disp$, $pers$, and $freq$ for the relevant combination of (J, N, a, b).

For each combination of (J, N, a, b), I calculated the distance between the point derived by simulation and each of the 7 target points $\{\bar{\eta}_s\}$ on line $\bar{l}$. Now the question is how small the distance should be in order to judge that a combination simulate a target point. One reasonable standard is that the distance should not be much larger than the average distance between the line $\bar{l}$ and the country points $\{\eta_s\}$, which is .0346. For each target point, I found many combinations that simulate it by this standard. Due to the computational burden, however, I did not carry all successful combinations to the next step of analysis: for each target point, I ranked the combinations in the order of increasing distance and selected about the first ten.[19] By permutating these selected com-

[19] It is easier to find combinations that match the target points with low-range $disp$ (high-income countries) or with high-range $disp$ (low-income countries) than with middle-range $disp$ (middle-income countries). Therefore, I select somewhat higher number of combinations for the target points with low- or high-range $disp$ than for the target points with middle-range $disp$.

binations across target points, I derived multiple sets of combinations $\{(J_s, N_s, a_s, b_s)\}$, $s = 1, 2, \ldots, 7$, that simulate the target points.

5.3 Deriving $\{d_s\}$ and h

Finally, I derived $\{d_s\}$, $s = 1, 2, \ldots, 7$, and function h that are consistent with each selected set of combinations $\{(J_s, N_s, a_s, b_s)\}$: I derived $\{d_s\}$ and h so that for each s, given (d_s, a_s, b_s) and h, (J_s, N_s) is the optimal investment rule. We can show that given a and b, if J and N are the optimal investment rule, there is a range where d must belong, and that this range is independent of h. I discretized d by setting its values to be $1.1, 1.2, 1.3, \ldots$. For each selected combination (J_s, N_s, a_s, b_s), I numerically derived the range of d, and took the lowest value of the range to be d_s that is consistent with the combination. Now we have sets of combinations $\{(J_s, N_s, d_s, a_s, b_s)\}$, $s = 1, 2, \ldots, 7$, where the elements of each combination are consistent with each other.

Now we need to derive for each set of combinations $\{(J_s, N_s, d_s, a_s, b_s)\}$, function h that is consistent with the set. Given d_s, a_s, b_s, and some function h, for J_s to be optimal, it must be that $v(J_s, 0, 1) = 0$ and $v(j, 0, 1) \leq 0$ for all j (see equations 2.4 and 2.5). Using equations 2.1 to 2.5, we can rewrite the first condition as

$$P_s h(J_s) = \psi_s(J_s) \tag{5.5}$$

and the second condition as

$$P_s h(j) \leq \psi_s(j), \tag{5.6}$$

for all j where P_s is the capital price for country s and $\psi_s(\cdot)$ is an expression implicitly defined by these conditions. The expression $\psi_s(\cdot)$ is free of P. From conditions 5.5 and 5.6, we have

$$\frac{h(J_s)}{h(j)} \geq \frac{\psi_s(j)}{\psi_s(J_s)}. \tag{5.7}$$

If the function h satisfies this condition, conditions 5.5 and 5.6 will be satisfied under the capital price P_s that solves condition 5.5, and therefore J_s will be optimal. Now for a function h to be consistent with the set $\{(J_s, N_s, d_s, a_s, b_s)\}$, condition 5.7 should be satisfied for all $s = 1, 2, \ldots, 7$. In particular, we have

$$\frac{\psi_s(J_u)}{\psi_s(J_s)} \leq \frac{h(J_s)}{h(J_u)} \leq \frac{\psi_u(J_u)}{\psi_u(J_s)} \tag{5.8}$$

for any $s, u = 1, 2, \ldots, 7$. This condition is sufficient, as well as necessary, for a function h to be consistent with the set in the sense that we can simply assume that $h(j)$ is small enough to satisfy condition 5.7 for all $j \notin \{J_s\}$.

There is no guarantee that there is a function h that satisfies condition 5.8 but, if there is such a function, there will be in general many of them. For our purposes, we do not need to characterize these functions completely. Using condition 5.8, we can derive the ranges of factor differences among $\{h(J_s)\}$ and from these differences and using condition 5.5 and equation 3.16, we can derive the ranges of factor differences in the capital price and output across countries that are consistent with the set $\{(J_s, N_s, d_s, a_s, b_s)\}$. A majority of the selected sets $\{(J_s, N_s, d_s, a_s, b_s)\}$ had no function h satisfying condition 5.8, and therefore were discarded. For each of the selected sets that had such function h, I derived the ranges of factor differences in the capital price and output across countries.

5.4 Findings and Interpretations

From the quantitative analysis in previous three subsections, I found that differences in policy uncertainty across countries can substantially account for capital price differences. Between the lowest-income and the highest-income countries, the difference in uncertainty can account for a capital price difference by a factor of about 3,[20] which is comparable to their actual price difference as reported by Summers and Heston (1993). This result, however, is a somewhat rough generalization: there are many sets of parameter values $\{(J_s, N_s, d_s, a_s, b_s)\}$ and many capital return functions h that are consistent with the model and the data, and that lead to different estimates of the effect of uncertainty on capital price. At one end, there are parameter values and functions that lead to the estimate that uncertainty accounts for capital price difference between the lowest-income and the highest-income countries by a factor of less than 2. The top half of Table 3 illustrates one such case: normalizing the capital price for a country with its *disp* equal to .2 (roughly equivalent to the US) to be 1, the capital price is 1.06 for a country with its *disp* equal to .3 (roughly equivalent to Spain); 1.19 for a country with its *disp* equal to .4 (roughly equivalent to Colombia); and 1.72 for a country with its *disp* equal to .5 (roughly equivalent to Ethiopia). At the other end, there are parameter values and functions that lead to the capital price difference between the lowest-income and the highest-income countries by a factor of more than 4. The bottom half of Table 3 illustrates one such case: they imply the corresponding numbers to be 1.16, 2.00, and 4.17, respectively. These are, however,

[20] This implies that between the lowest-income and the highest-income countries, policy uncertainty can account for their difference in the investment share of output, when investment is measured in units of investment output, by a factor of about 3 also (see equation 3.17).

extreme cases and most of the cases lead to the capital price difference between the lowest-income and the-highest income countries by a factor between 2 and 4, the average being about 3.

To what extent can policy uncertainty account for cross-country output differences? This depends not only on the capital price differences due to policy uncertainty, but also on the capital share parameter α in the production function (see equation 3.16). A value used widely in the literature is 1/3, based on the share of physical capital income in national accounts. Assuming this value, the capital price difference of a factor of 3 between the lowest-income and the highest-income countries translates to an output difference of a factor of 1.7.[21] However, if we include in capital not only physical capital but also other types such as human, organizational, and technological capital, the capital share is about 2/3 (see Mankiew, Romer, and Weil 1992, Parente and Prescott 1994, and Chari, Kehoe, and McGrattan 1997). If we make a rough assumption that in each country the investment environments for these other types of capital are the same as that for physical capital, policy uncertainty can account for output difference of a factor of 9 between the lowest-income and the highest-income countries. From these suggestive numbers, I conclude that policy uncertainty in the investment environment is an important factor in accounting for output differences across countries.

6. Conclusion

In this paper, I explored how and to what extent policy uncertainty can account for the observed cross-countries differences in long-run capital price, investment and output. I presented a model economy where policy uncertainty causes investors to favor shorter-term projects, and abandon some unfinished projects under negative shocks. Holding the average one-period investment cost, this leads to a higher capital price and subsequently to lower levels of long-run aggregate investment and output. In the model, policy uncertainty also makes industry-level investment more volatile. I investigated the industry-level investment in a sample of countries of diverse output levels, and found that, consistent with the model, the investment in the lower-income countries is more volatile. Simulated investment sequences from the model can also quantitatively mimic actual investment sequences across

[21] Note that in the model investment portion of the output is measured in terms of the investment cost whereas in practice it is in large part measured in terms of the investment output, that is, capital produced. For example, Summers and Heston (1993) measure investment using a weighted (across countries) capital price common to all countries. If we measure investment this way in the model, the investment will be adjusted downward for low-income countries and upward for high-income countries, leading to a larger output difference between them. In this sense, the output differences reported here are underestimates.

countries. For a quantitative assessment of policy uncertainty, then I calibrated the model by a simulation method, and found that policy uncertainty can account for large differences in the long-run capital price, investment, and output across countries. Between the lowest-income and the highest-income countries, policy uncertainty can account for the difference in capital price and the investment share of output by a factor of about 3, and the output difference of a similar magnitude. I emphasize that these results are attributable to cross-country differences in uncertainty, and not to the level of policy-related investment cost. It is plausible that low-income countries also have a greater level of policy-related investment cost, which would further account for the cross-country differences in the capital price, investment, and output.

APPENDIX

Proof of Proposition 1: The proof is based on the following two Lemmas.

Lemma 1: the investor prefers to manage a project with the low cost: for all $j \in \{1, 2, \ldots\}$ and all $n \in \{0, 1, \ldots, j-1\}$, $v(j, n, 1) > v(j, n, 2)$.

Proof of Lemma 1: For any given j, equation 2.2 implies that $v(j, j-1, 1) > v(j, j-1, 2)$. This condition and equation 2.3 imply that $\tilde{v}(j, j-1, 1) \geq \tilde{v}(j, j-1, 2)$. This condition and equation 2.1 imply that $v(j, j-2, 1) > v(j, j-2, 2)$ since $\pi_1 > 1 - \pi_2$ by assumption. This condition and equation 2.3 imply that $\tilde{v}(j, j-2, 1) > \tilde{v}(j, j-2, 2)$. By repeating these steps (i.e., using the established condition and the equations 2.1 and 2.3 in turns), we have $v(j, n, 1) > v(j, n, 2)$ for all $n \in \{0, 1, \ldots, j-1\}$.

Lemma 2: the investor prefers to manage an older project: for all $j \in \{1, 2, \ldots\}$, all $n \in \{0, 1, \ldots, j-2\}$, and $q = 1, 2$, $v(j, n, q) \leq v(j, n+1, q)$ and if $v(j, n, q) \geq \xi$ in addition, $v(j, n, q) < v(j, n+1, q)$.

Proof of Lemma 2: For any given j and $q = 1, 2$, equations 2.2, 2.3, and 2.4 imply that $\tilde{v}(j, j-1, q) = \max\{Ph(j) - \phi_q - \xi, 0\} < Ph(j)$. This condition and equations 2.1 and 2.2 imply that $v(j, j-2, q) < -\phi_q + \beta Ph(j) < v(j, j-1, q)$. This condition and equation 2.3 imply that $\tilde{v}(j, j-2, q) \leq \tilde{v}(j, j-1, q)$. This condition and equation 2.1 imply that $v(j, j-3, q) \leq v(j, j-2, q)$. This condition and equation 2.3 imply that $\tilde{v}(j, j-3, q) \leq \tilde{v}(j, j-2, q)$. By repeating these steps (i.e., using the established inequality and the equations 2.1 and 2.3 in turns), we have $v(j, n, q) \leq v(j, n+1, q)$ and $\tilde{v}(j, n, q) \leq \tilde{v}(j, n+1, q)$ for all $n \in \{0, 1, \ldots, j-2\}$. Now suppose $v(j, n, q) \geq \xi$ for some $n \in \{0, 1, \ldots, j-3\}$ and some $q \in \{1, 2\}$. Then for all $m \in \{n, n+1, \ldots, j-1\}$, we have $v(j, m, q) \geq \xi$ and thus $\tilde{v}(j, m, q) = v(j, m, q) - \xi$. Then for all $m \in \{n, n+1, \ldots, j-3\}$, we have $v(j, m, q) - v(j, m+1, q) = \pi_q \beta(\tilde{v}(j, m+1, q) - \tilde{v}(j, m+2, q)) + (1 - \pi_q)\beta(\tilde{v}(j, m+1, q') - \tilde{v}(j, m+2, q')) \leq \pi_q \beta(\tilde{v}(j, m+1, q) - \tilde{v}(j, m+2, q)) = \pi_q \beta(v(j, m+1, q) - v(j, m+2, q))$, where $q' = 1$ if $q = 2$ and $q' = 2$ if $q = 1$. Then we have $v(j, n, q) - v(j, n+1, q) \leq \pi_q \beta(v(j, n+1, q) - v(j, n+2, q)) \leq (\pi_q \beta)^2(v(j, n+2, q) - v(j, n+3, q)) \leq \ldots \leq (\pi_q \beta)^{j-n-2}(v(j, j-2, q) - v(j, j-1, q))$. Since $v(j, j-2, q) < v(j, j-1, q)$, we have $v(j, n, q) < v(j, n+1, q)$.

Proof of Proposition 1 (continued): Now from Lemma 1, we have $v(j, 0, 1) > v(j, 0, 2)$ and thus it is not optimal to start any project with the high cost: a project, if started, must have the low cost. Let $J = \arg\max_j \{v(j, 0, 1)\}$. There is such finite J given the assumption on h on page 5 (see Proposition 2). However, J may not be unique. Fix J for the following.

Since $\max_{j,q}\{v(j,0,q)\} \geq 0$ by assumption, we have $v(J,0,1) = \max_{j,q}\{v(j,0,q)\} \geq 0$ and, from equation 2.4, $\xi = v(J,0,1)$. Thus it is optimal to start a project of type J and with the low cost. Also, from Lemma 2, we have $v(J,n,1) > 0$ for all $n \geq 1$ and thus the unique optimal rule on continuing/abandoning the project under the low cost is to continue the project at all ages. It also follows from Lemma 2 that there is a unique M such that $v(J,n,2) < \xi$ for $n < M$, $v(J,n,2) \geq \xi$ for $n = M$, and $v(J,n,2) > \xi$ for $n \geq M$. Then, if $v(J,M,2) > \xi$, the unique optimal rule on continuing/abandoning the project under the high cost is to abandon the project if it is younger than M, and to continue the project if it is older than or as old as M. If $v(J,M,2) = \xi$, there are only two optimal rules on continuing/abandoning the project under the high cost: one is the same as if $v(J,M,2) > \xi$, and the other is the same as if $v(J,M,2) > \xi$ except that the cut-off age is $M+1$. Thus any optimal rule on continuing/abandoning the project under the high cost is characterized by a cut-off age N, where $N = M$ or $N = M+1$.

Proposition A: For any given (d,a,b), $\bar{\phi}$, and $\bar{\phi}'$, let P and (J,N) be any equilibrium associated with $\bar{\phi}$ and $\tilde{\phi}$ the corresponding average one-period cost. Let $P' \equiv (\bar{\phi}'/\bar{\phi})P$ and $\tilde{\phi}' \equiv (\bar{\phi}'/\bar{\phi})\tilde{\phi}$. Then P' and $\tilde{\phi}'$ are an equilibrium associated with $\bar{\phi}'$ and $\tilde{\phi}'$ the corresponding average one-period cost.

Proof of Proposition A: Let variables and functions without prime be associated with $\bar{\phi}$, and those with prime associated with $\bar{\phi}'$. That P and (J,N) are an equilibrium under $\bar{\phi}$ means that under P, $v(J,0,1) \geq v(j,0,1)$ for all j; $v(J,n,2) \geq \xi$ for all $n \geq N$; $v(J,n,2) < \xi$ for all $n < N$; and $v(J,0,1) = \xi = 0$. Let $\theta \equiv \bar{\phi}'/\bar{\phi}$. Consider the investor's problem under $\bar{\phi}'$ and P'. From the definitions of $\bar{\phi}$ and d (page 8), we have $\phi_1' = \theta\phi_1$ and $\phi_2' = \theta\phi_2$. Then for any given j and $q = 1,2$, we have $v'(j,j-1,q) = \theta v(j,j-1,q)$ from equation 2.2. Then, assuming $\xi' = 0$ for the moment, we have $\tilde{v}'(j,j-1,q) = \theta\tilde{v}(j,j-1,q)$ from equation 2.3. Then we have $v'(j,j-2,q) = \theta v(j,j-2,q)$ from equation 2.1, and thus $\tilde{v}'(j,j-2,q) = \theta\tilde{v}(j,j-2,q)$ from equation 2.3. By repeating these steps (using the established condition and the equations 2.1 and 2.3 in turns), we have $v'(j,n,q) = \theta v(j,n,q)$ for all j, n, and q. Then we have $v'(J,0,1) \geq v'(j,0,1)$ for all j; $v'(J,n,2) \geq \xi'$ for all $n \geq N$; $v'(J,n,2) < \xi'$ for all $n < N$; and $v'(J,0,1) = \xi' = 0$. Thus P' and (J,N) are an equilibirum under $\bar{\phi}'$. Now, since $\phi_1' = \theta\phi_1$ and $\phi_2' = \theta\phi_2$, we have $\phi'(t) = \theta\phi(t)$ from the definition of $\phi(t)$ (page 9). Then it is straight forward from equations 2.8, 2.9, and 2.10 that $\tilde{\phi}'$ is the average one-period cost under $\bar{\phi}'$.

Proof of Proposition 2: Consider the investment environment under $d = 1$, we have $\phi_1 = \phi_2$ from the definitions of d (page 8). Then it is straight forward from equations 2.1 to 2.3 that

$v(j,n,1) = v(j,n,2)$ for all j and n. Since $v(J,n,1) \geq \xi$ for all n (see Proposition 1), we then have $v(J,n,2) \geq \xi$. Then we have $N = 1$: there is no abandonment of project. Then we have, from equations 2.5 and 2.7, $v(J,0,1) = -\sum_{n=0}^{J-1} \beta^n \phi_1 + \beta^{J-1} Ph(J) = 0$ (A.1) in equilibrium. To see that the equilibrium optimal type J is unique, we can rewrite this equation as $v(J,0,1) = \sum_{n=0}^{J-1} \beta^n ((\beta^{J-1} h(J) / \sum_{n=0}^{J-1} \beta^n) - \phi_1/P) = 0$. The optimal type J must solve $\max_j \{\beta^{J-1} h(J) / \sum_{n=0}^{J-1} \beta^n\}$: if not, there is j such that $v(j,0,1) > v(J,0,1)$. Since there is a unique solution to this maximization problem by assumption (page 5), the equilibrium optimal type J is unique. Now consider the investment environment under $a = 1$ or $b = 1$. We have, from the definitions of a, b, and $\phi(t)$ (pages 8 and 9), $\pi_1 = \phi(t) = 1$ for all t. Then the one-period cost never changes from ϕ_1 to ϕ_2, and thus there is no abondonment of project. We also have, from equations 2.5 and 2.7, $v(J,0,1) = \sum_{n=0}^{J-1} \beta^n \phi_1 + \beta^{J-1} Ph(J) = 0$ in equilibrium. This equation is identical to the equation A.1. Then, following the same reasoning, we can see that the equilibrium optimal type J is unique and identical to that under $d = 1$.

Now let variables or functions without prime be associated with no uncertainty ((d,a,b) where $d = 1$, $a = 1$, or $b = 1$), and those with prime be associated with uncertainty (any given (d,a,b) where $d \neq 1$, $a \neq 1$, and $b \neq 1$). To show that $J \geq J'$, suppose $J < J'$ to the contrary. We have seen above that $v(J,0,1) = -\sum_{n=0}^{J-1} \beta^n \phi_1 + \beta^{J-1} Ph(J) = 0$ in equilibrium. Since J is unique, we have $v(J,0,1) > v(J',0,1) \geq -\sum_{n=0}^{J'-1} \beta^n \phi_1 + \beta^{J'-1} Ph(J')$. From these two conditions, we have $(\beta^{J-1} h(J))/(\beta^{J'-1} h(J')) > (\sum_{n=0}^{J-1} \beta^n)/(\sum_{n=0}^{J'-1} \beta^n)$ (A.2). We also have in equilibrium $v'(J',0,1) = -\sum_{n=0}^{N'-1} \beta^n {\pi'}_1^n \phi'_1 - \sum_{n=N'}^{J'-1} \beta^n {\pi'}_1^{N'-1} \phi'(n - N' + 1) + \beta^{J'-1} {\pi'}_1^{N'-1} P' h(J') = 0$ and, for any $m \in \{1,2,\ldots,J\}$, $0 \geq v'(J,0,1) \geq -\sum_{n=0}^{m-1} \beta^n {\pi'}_1^n \phi'_1 - \sum_{n=m}^{J-1} \beta^n {\pi'}_1^{m-1} \phi'(n-m+1) + \beta^{J-1} {\pi'}_1^{m-1} P' h(J)$. From these two conditions, we have, for any $m \in \{1,2,\ldots,J\}$, $(\beta^{J-1} h(J))/(\beta^{J'-1} h(J')) < (\sum_{n=0}^{m-1} \beta^n {\pi'}_1^{n-m+1} \phi'_1 + \sum_{n=m}^{J-1} \beta^n \phi'(n-m+1))/(\sum_{n=0}^{N'-1} \beta^n {\pi'}_1^{n-N'+1} \phi'_1 + \sum_{n=N'}^{J'-1} \beta^n \phi'(n - N' + 1))$ (A.3). From conditions A.2 and A.3, we can derive $(\sum_{n=0}^{N'-1} \beta^n {\pi'}_1^{n+1-N'} \phi'_1 + \sum_{n=N'}^{J'-1} \beta^n \phi'(n - N' + 1))/\sum_{n=0}^{J'-1} \beta^n < (\sum_{n=0}^{m-1} \beta^n {\pi'}_1^{n-m+1} \phi'_1 + \sum_{n=m}^{J-1} \beta^n \phi'(n - m + 1))/\sum_{n=0}^{J-1} \beta^n$ (A.4). The left-hand side of condition A.4 is the average of the elements of the sequence $({\pi'}_1^{1-N'}, {\pi'}_1^{2-N'}, \ldots, \pi'_1, 1, \phi'(1), \phi'(2), \ldots, \phi'(J' - N'))$ weighted by the weights $(1/\sum_{n=0}^{J'-1} \beta^n, \beta/\sum_{n=0}^{J'-1} \beta^n, \beta^2/\sum_{n=0}^{J'-1} \beta^n, \ldots, \beta^{J'-1}/\sum_{n=0}^{J'-1} \beta^n)$, and the right-hand side of condition A.4 is the average of the elements of the sequence $({\pi'}_1^{1-m}, {\pi'}_1^{2-m}, \ldots, \pi'_1, 1, \phi'(1), \phi('2), \ldots, \phi'(J - m))$ weighted by the weights $(1/\sum_{n=0}^{J-1} \beta^n, \beta/\sum_{n=0}^{J-1} \beta^n, \beta^2/\sum_{n=0}^{J-1} \beta^n, \ldots, \beta^{J-1}/\sum_{n=0}^{J-1} \beta^n)$. Comparison of the two sequences, for the two sides of condition A.4, show that the two sequences are an indentical expansion from the element 1 although possibly with different lengths in either directions, and that for the both sequences the elements are ordered in the decreasing order up to the element 1 and then in the increasing order thereafter. Since $J < J'$, these characteristics of the two sequences imply that by choosing an appropriate m we can choose the sequence for the right-hand side to be equivalent to a (middle) chunk of the sequence for the left-hand side with the property that any element of the sequence for the right-hand side is smaller than or equal to any element of the sequence for the left-hand side not belonging to that chunk. Let M be such an appropriately chosen m, and let's consider condition A.4 given M. The sequence for the left-hand side can then be broken into (up to) three subsequences. The first subsequence is $({\pi'}_1^{1-N'}, {\pi'}_1^{2-N'}, \ldots, {\pi'}_1^{N'-M})$; the middle sequence is $({\pi'}_1^{1-M}, {\pi'}_1^{2-M}, \ldots, \pi'_1, 1, \phi'(1), \phi'(2), \ldots, \phi'(J-M))$, which is equivalent to the sequence for the right-hand side; and the last sequence is $(\phi'(J+1-M), \phi'(J+2-M), \ldots, \phi'(J'-N'))$. Let Γ_1 denote the average of the elements for the first subsequence weighted by the weights $(1/\sum_{n=0}^{N'-M-1}\beta^n, \beta/\sum_{n=0}^{N'-M-1}\beta^n, \beta^2/\sum_{n=0}^{N'-M-1}\beta^n, \ldots, \beta^{N'-M-1}/\sum_{n=0}^{N'-M-1}\beta^n)$; Γ_2 the average of the elements for the middle subsequence weighted by the same weights as for the sequence for the right-hand side, which makes Γ_2 equivalent to the right-hand side; and Γ_3 the average of the elements for the last subsequence weighted by the weights $(1/\sum_{n=0}^{J'-J-N'+M-1}\beta^n, \beta/\sum_{n=0}^{J'-J-N'+M-1}\beta^n, \beta^2/\sum_{n=0}^{J'-J-N'+M-1}\beta^n, \ldots, \beta^{J'-J-N'+M-1}/\sum_{n=0}^{J'-J-N'+M-1}\beta^n)$. Since any element of the middle subsequence is smaller than or equal to any element for the first or the last subsequences, we have $\Gamma_2 \leq \Gamma_1$ and $\Gamma_2 \leq \Gamma_3$. Further we can show that the left-hand side is equivalent to the average of $(\Gamma_1, \Gamma_2, \Gamma_3)$ weighted by the weights $(\sum_{n=0}^{N'-M-1}\beta^n/\sum_{n=0}^{J'-1}\beta^n, \sum_{n=N'-M}^{J+N'-M-1}\beta^n/\sum_{n=0}^{J'-1}\beta^n, \sum_{n=J+N'-M}^{J'-1}\beta^n/\sum_{n=0}^{J'-1}\beta^n)$. Then the left-hand side is greater than or equal to Γ_2 which is equivalent to the right-hand side. Therefore, we have shown that for some m, the condition A.4 is violated. By this contradition, we conclude that $J \geq J'$.

Now let the average one-period cost $\tilde{\phi}$ be given. Under no uncertainty, $\phi_1 = \tilde{\phi}$ from equations 2.7 to 2.10 and condition A.1. Then we have $0 \geq v(J', 0, 1) \geq -\sum_{n=0}^{J'-1}\beta^n\phi_1 + \beta^{J'-1}Ph(J') = -\sum_{n=0}^{J'-1}\beta^n\tilde{\phi} + \beta^{J'-1}Ph(J)$ in equilibrium, which implies $P \leq \sum_{n=0}^{J'-1}\beta^n\tilde{\phi}/(\beta^{J'-1}h(J'))$ (A.5). From equations 2.7 to 2.10, we also have $v'(J', 0, 1) = -\sum_{n=0}^{N'-1}\beta^n{\pi'}_1^n\phi'_1 - \sum_{n=N'}^{J'-1}\beta^n{\pi'}_1^{N'-1}\phi'(n-N'+1) + \beta^{J'-1}{\pi'}_1^{N'-1}P'h(J') = -\sum_{n=0}^{N'-1}\beta^n{\pi'}_1^n\tilde{\phi} - \sum_{n=N'}^{J'-1}\beta^n{\pi'}_1^{N'-1}\tilde{\phi} + \beta^{J'-1}{\pi'}_1^{N'-1}P'h(J') = 0$ in equilibrium, which implies $P' = (\sum_{n=0}^{N'-1}\beta^n{\pi'}_1^{n-N'+1}\tilde{\phi} - \sum_{n=N'}^{J'-1}\beta^n\tilde{\phi})/(\beta^{J'-1}h(J'))$ (A.6). The right-hand side of A.5 is smaller than or equal to the right-hand side of A.6, and thus we have $P \leq P'$.

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Table 1: Years and Industries of Available Data

Country	**Years**	**Industries**
Brazil	67-69, 71-74, 76-78, 80	1-6, 8-11, 16-17, 24-26
Canada	67-88	1-17, 19-27
Colombia	67-70, 72-88	1-12, 14, 16, 18-20, 22-27
Denmark	67-88	1-6, 8-13, 17, 20, 23-27
Ecuador	67-88	1-6, 8-14, 16, 20, 23, 25-26
Ethiopia	67, 70-86	1-4, 11, 12
India	77-87	1-17, 19-26
Korea	67-88	1-17, 19-26
Malawi	69-75, 80-86	1-4, 8-10, 12, 20, 23
Philippines	68-75, 77, 79-88	1, 3-6, 8-12, 14, 16-26
Spain	67-76, 79-88	1-2, 4-7, 10-13, 16-20, 22
Sri Lanka	79, 81-86, 88	1-5, 7-10, 12-13, 16-18, 20, 23-26
Tanzania	67-74, 81-85	1-13, 16, 23-24, 26
UK	70-88	1-27
US	67-88	1-27
Zimbabwe	73-88	1-5, 7-13, 16-17, 25-26

Index of Industries

1: food products
2: beverages
3: tobacco
4: textiles
5: wearing apparel
6: leather and products
7: footwear
8: wood products
9: furniture, fixtures
10: paper and products
11: printing, publishing
12: industrial chemicals
13: other chemical products
14: petroleum refineries
15: petroleum, coal products
16: rubber products
17: plastic products
18: pottery, china etc.
19: glass and products
20: non-metal products
21: iron and steel
22: non-ferrous metals
23: metal products
24: machinery
25: electrical machinery
26: transport equipment
27: professional goods

Table o n S a i ic

Country	pout	disp	pers	freq	dist
azil	0	. 1	. 1 0	.	.01
anada	1 1	. 0	. 1	. 1	.00
olombia	00	. 1	. 1	. 1	.010
Denma k	110	. 0	.	.	.0
Ec ado		. 0	.	. 0	.0 1
E hiopia		. 0	.	.1	.0
India		. 1	.	.	.0
Ko ea		. 11	.	.	.0
Mala i		.	. 1	. 1	.0
Philippine	1 0	.	.	.	.0 1
Spain	0	.	. 0	. 0	.0
S i Lanka	0	.	.	.	.0 1
Tanzania		.	. 0	.	.0
UK	100	. 01	.	.	.0
US	1 1	.1	.1	. 1	.0
Zimbab e	1 11	. 0	. 1	. 0	.0

- po pe -capi a o p in 1 US dolla a e aged o e he pe iod
- di p di pe ion a i ic, i.e., a e age anda d de ia ion of man fac ing ind ie ' in e men in f ac ion of end in e men
- pe pe i ence a i ic, i.e., a e age anda d de ia ion of fi -o de diffe ence of man fac ing ind ie ' in e men in f ac ion of end in e men
- f e f e enc a i ic, i.e., a e age f ac ion of man fac ing ind ie ' in e men abo e end in e men
- di di ance be een (di p, pe , f e) and (di p′, pe ′, f e ′)

Table 3: Illustrations of Cross-Country Differences

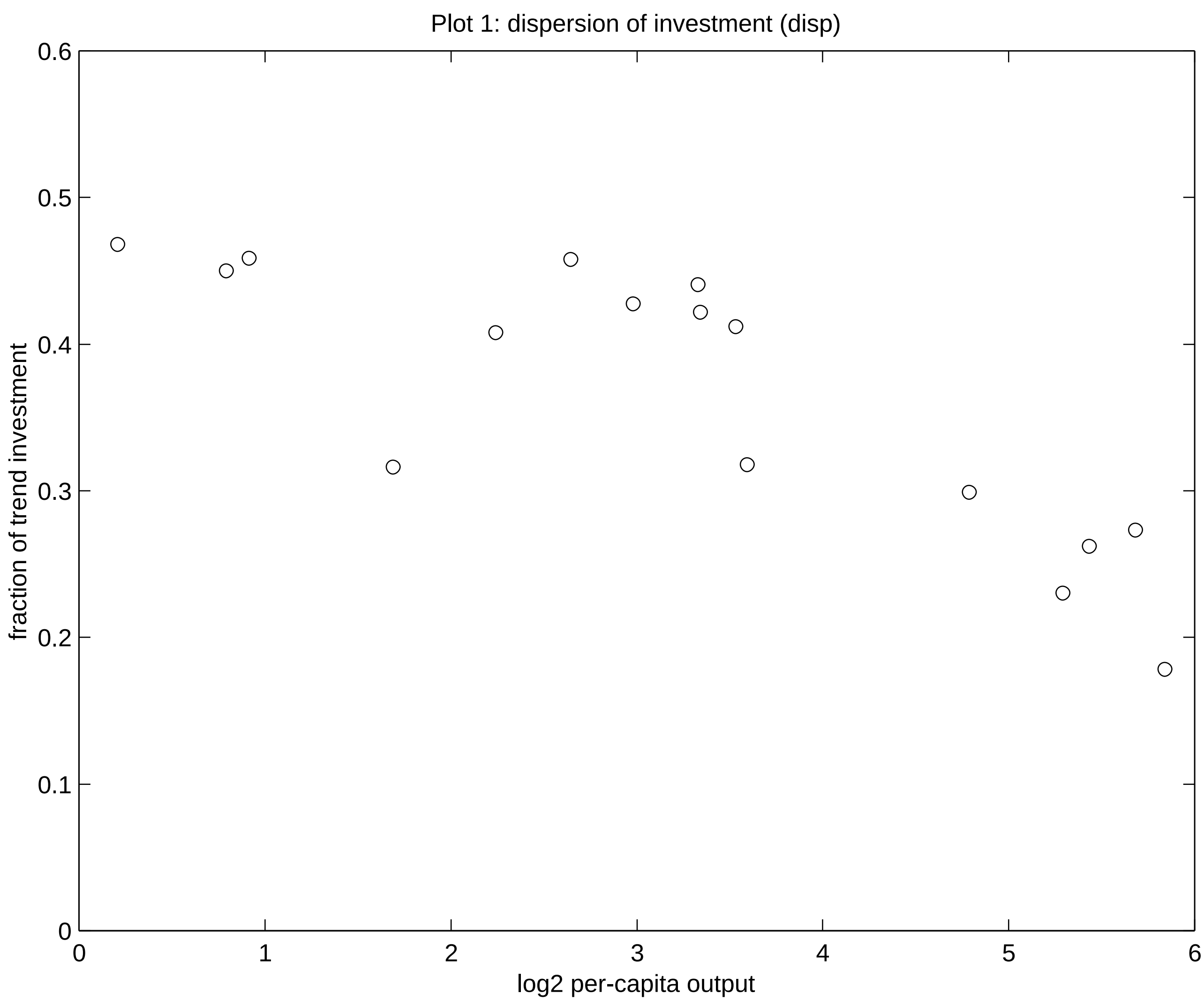
Illustration 1

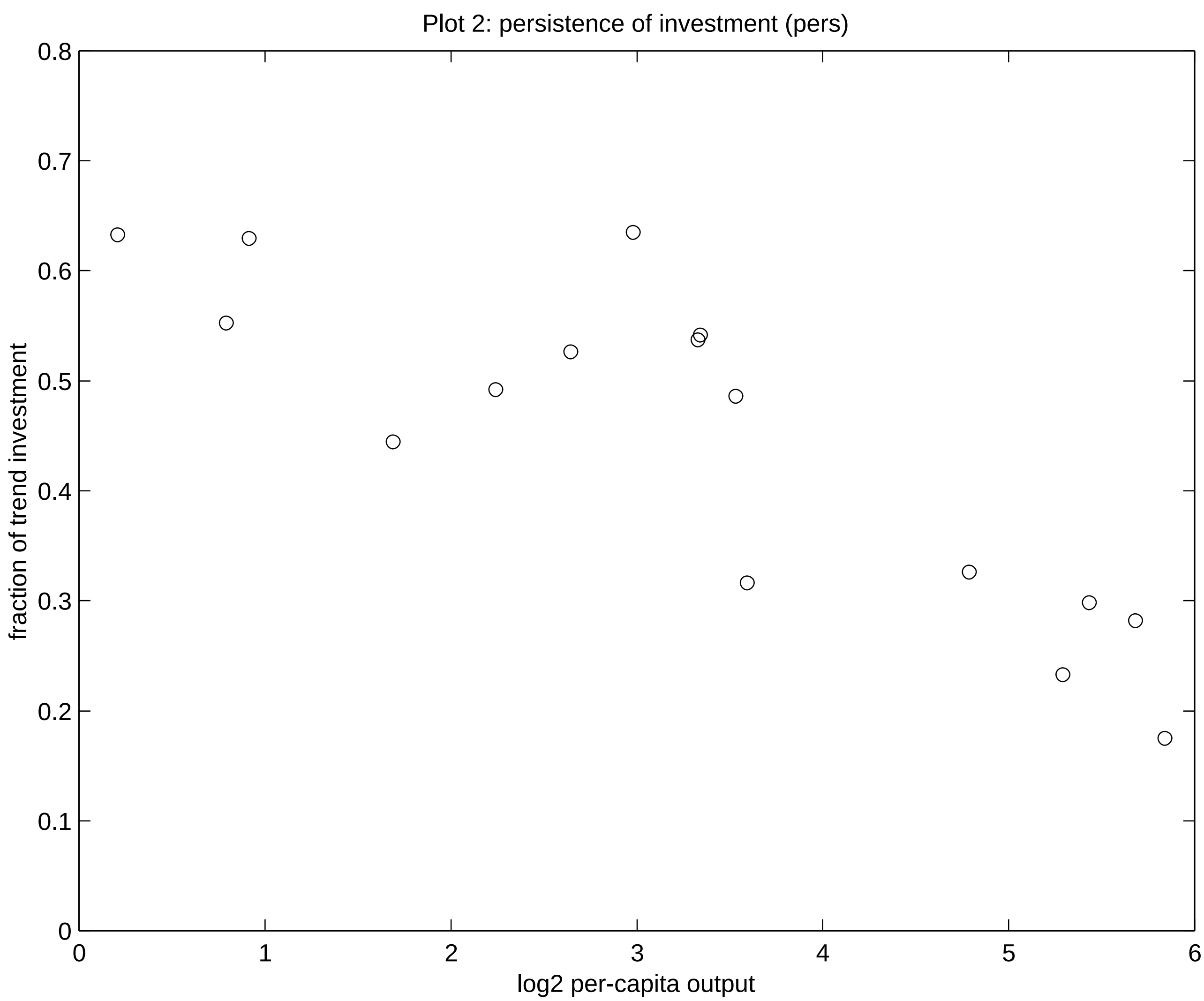
disp	**d**	**a**	**b**	**J**	**N**	**dist**	**P**
.20	1.9	.8	.3	20	2	.0084	1.00
.25	2.3	.7	.1	14	2	.0087	1.03
.30	1.9	.7	.3	10	2	.0212	1.06
.35	2.7	.6	.1	7	2	.0421	1.12
.40	2.7	.6	.0	5	2	.0412	1.19
.45	2.5	.5	.2	3	2	.0376	1.41
.50	3.0	.3	.3	2	2	.0045	1.72

Illustration 2

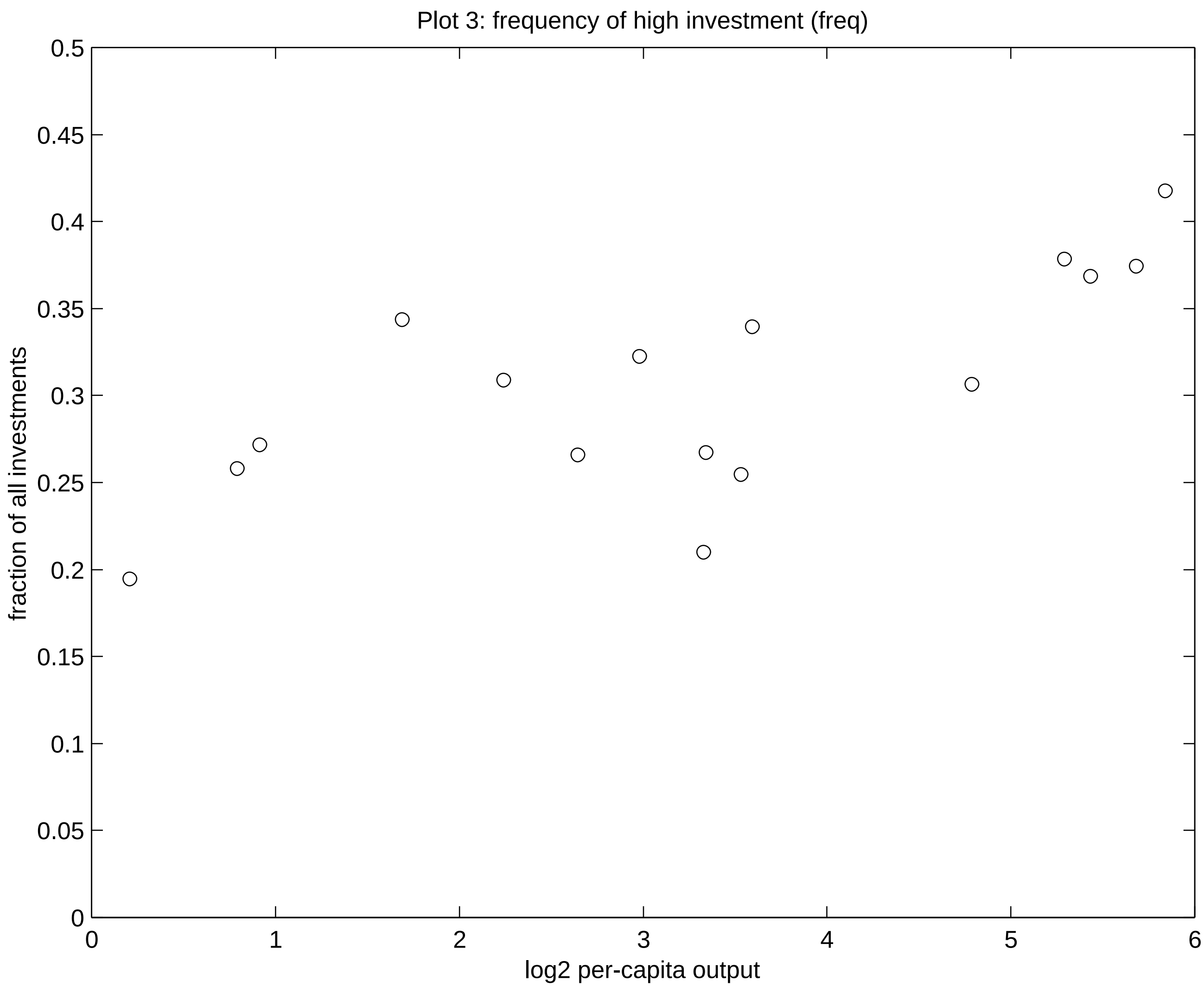
disp	**d**	**a**	**b**	**J**	**N**	**dist**	**P**
.20	1.1	.7	.6	20	1	.0093	1.00
.25	2.3	.7	.1	14	2	.0087	1.10
.30	1.9	.7	.3	10	2	.0212	1.14
.35	2.2	.6	.2	9	2	.0428	1.16
.40	4.6	.7	.0	7	3	.0429	1.82
.45	5.6	.6	.0	6	3	.0361	2.00
.50	8.1	.4	.1	3	3	.0060	4.17

- disp: dispersion value of point on line $\bar{l}$ representing an output level
- d: dispersion parameter
- a: frequency parameter
- b: persistence parameter
- J: type (i.e., duration) of investment project
- N: continuation age
- dist: distance between the point on the line $\bar{l}$ and the point corresponding to (d,a,b) from simulation
- P: capital price


Plot 1: dispersion of investment (disp)
0.6
0.5
0.4
0.3
0.2
0.1
0
fraction of trend investment
0
1
2
3
4
5
6
log2 per-capita output



Plot 2: persistence of investment (pers)
fraction of trend investment
log2 per-capita output
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
4
5
6


Plot 3: frequency of high investment (freq)
fraction of all investments
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
1
2
3
4
5
6
log2 per-capita output