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A Theory of Paradigm Change

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A Theory of Paradigm Change

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RESUME

The notion of paradigm was introduced by Thomas S. Kuhn in 1962 in his influential book *The Structure of Scientific Revolutions* (Kuhn 1962). There Kuhn presented his famous theory of scientific revolutions and described *paradigm change* in sociological terms as a sudden change of behavior of the scientific community. I believe that it is possible to discriminate three different kinds of scientific revolutions. For each of them it is possible to formulate, besides the common sociological, also a specific cognitive and epistemological description of the dynamics of paradigm change. Kuhn's theory can be likened to a picture that arises from mixing together photographs of three different faces. Each of the original photographs is sharp and rich in specific detail. By their superimposing, however, the details will be lost, and what will remain is the gross structure of the face – the overall contours, dark spots instead of eyes and a blot instead of the mouth. Similarly, when Kuhn superimposed the “photographs” of the three types of scientific revolutions, he lost the details of *cognitive dynamics* and of *epistemological structure*, that are specific for each type, and what remained in the resulting picture were only features common to all three types of revolutions – the *social dynamics* of the response of the scientific community to change. By means of the metaphor of mixing or superimposing of three different photographs I do not want to say that Kuhn had three different notions in mind, which he intentionally mixed or superimposed. I believe that the mixing was unintentional and it was caused by Kuhn's being unaware of (or perhaps not paying attention to) the differences between the particular kinds of revolutions. The aim of the metaphor is not to criticize Kuhn, but to draw attention to the fact that Kuhn's stress of the sociological aspects of scientific revolutions may be the result of such an unintentional mixing.

The aim of the theory of paradigm change presented here is to *discriminate* the different kinds of scientific revolutions; for each of them *develop methods* for its cognitive and epistemological analysis; and then by means of particular case studies to *identify the patterns* of paradigm change

that are characteristic for each kind of scientific revolution. In order to achieve these aims, I needed some heuristics (in the sense of Lakatos).

The heuristics behind the theory of paradigm change was twofold. On the one hand the heuristics was to *turn to mathematics*, where we have a longer history of evolution that could be characterized as ‘normal science’. While physics was established as a normal science at the end of the 17th century, and so we are dealing with not more than 350 years of its history, in mathematics ‘normal science’ was established 300 B.C. and so we have 2300 years of history based on more or less articulated paradigms. Therefore, in mathematics it should be easier to find cases that would allow a classification of patterns of paradigm change. The results of that line of research were summarized in *Patterns of Change, Linguistic Innovations in the Development of Classical Mathematics* (Kvasz 2008a) and are not the subject matter of the present dissertation. The second heuristics was to *transfer the conceptual tools of the analysis of paradigm change from mathematics to science*. The identification of the particular patterns in mathematics helped much, because when it is clear what kind of pattern we are looking for, the analysis of the development of the particular scientific discipline turns out to be much easier. This made it possible to identify in the history of physics *re-codings* (in Kvasz 2011a) and *relativizations* (in Kvasz 2013), the two patterns described in (Kvasz 2008a).

Nevertheless, these two heuristics were not sufficient. *Patterns of Change* contain a lacuna – they lack the description of *idealizations*, which is the third kind of scientific revolution. The reason for the omission of idealizations was that in mathematics idealization took place between Tales and Euclid, i.e. during a period from which we lack almost any mathematical texts. Therefore a direct reconstruction of the process of idealization in mathematics is impossible. On the other hand, in physics idealization occurred between Galileo and Newton, which is one of the best documented, most thoroughly studied and well understood periods in the history of science. Therefore in science it was possible, besides a theory of *re-codings* and a theory of *relativizations*, to develop also a theory of *idealizations* (for details see Kvasz 2012b).

The *main results* of the theory of paradigm change, contained in the dissertation are:

I. *The development of a theory of idealizations.* The theory of idealizations is the main point in which the theory of paradigm change differs from the theory of linguistic innovations in mathematics. That is the reason why four of the seven papers forming the dissertation are devoted to this theory. Perhaps the most interesting aspect of it is a process that can be called a *paradigm shift*. I offer a new interpretation of the *Scientific Revolution* of the 17th century. According to the standard interpretation, the scientific revolution was a replacement of the paradigm of Aristotelian physics by the paradigm of Newtonian science, i.e. a revolutionary overturn occurring in a fixed area of knowledge. The proposed theory of idealization interprets the Scientific Revolution instead as a replacement of Euclid's *Elements* in the role of the paradigm by Newton's *Principia*, i.e. a shift of the paradigmatic discipline from mathematics to physics. The theory of idealization is the content of the 2nd, 3rd, 4th, and 5th paper of the dissertation.

II. *A new interpretation of Cartesian physics.* A further important result of the theory of idealization is a new interpretation of Cartesian physics. While most historians of science view Cartesian physics as a result of a (misguided) metaphysical project, I am showing that it is a truly physical theory that formed an important stage in the development of physics connecting Galilean theory of motion with Newton's theory of interaction. The new interpretation of Cartesian physics is the content of the 3rd paper.

III. *Extending the theory of re-codings from mathematics to physics.* This extension indicates that the notions like *expressive power*, *integrative power*, or *explanatory power* of language, that were introduced in the reconstruction of re-codings in mathematics, can be used also in the description of the development of physics. In this area I consider as the most interesting feature the introduction of the so called *Theories of Continua and Fluids* as an independent stage in the development of physics. Thus I propose to consider theories as the theory of phlogiston or the theory of caloric (together with hydrodynamics and mechanics of continua) to be an independent developmental stage with a particular linguistic framework that gives these theories a methodological, epistemological and heuristic unity. The theory of re-codings in physics is presented in the 6th paper (i.e. pages 98 – 117).

IV. *Extending the theory of relativizations from mathematics to physics.* This means that the basic notions of the theory of relativizations, such as *pictorial form*, *epistemic subject*, *horizon*, *background*, or *ideal elements* can be used in the analysis of the development of physical theories. This is shown on the example of *classical mechanics*. This is satisfactory not only because it creates a connection between geometry, algebra, and classical mechanics (and thus enables us to understand the unity of the work of mathematicians such as Lagrange, who made fundamental contributions to algebra as well as to mechanics) but also because of its relation to the work of the early Wittgenstein. The *picture theory of meaning* from the *Tractatus* was the main inspiration of the theory of *relativizations* in geometry. It is well known that Wittgenstein was in his picture theory inspired by Hertz's *Principles of Mechanics*. So I consider it as a kind of completing a circle, when it turns out that the theory of *relativizations*, that is based on the *picture theory of meaning* can be applied to mechanics, that is, to the original source of its inspiration. The theory of relativizations in classical mechanics is presented in the last, 7th paper of the dissertation.

A THEORY OF PARADIGM CHANGE

Some fifteen years after publishing *The Structure of Scientific Revolutions*, Kuhn spoke in the introduction to *The Essential Tension* of large and small revolutions (Kuhn 1977, p XVII). This may have led some of the early commentators on Kuhn's work, as for instance McMullin in his paper at the conference held at the Massachusetts Institute of Technology in 1990 to distinguish between *shallow revolutions* (the discovery of Roentgen radiation), *intermediate revolutions* (the replacement of the phlogiston theory of combustion by the oxidation theory) and *deep revolutions* (the Newtonian revolution) (see McMullin 1993, pp. 59-61). Kuhn, who was present, did not accept this distinction and in his response to McMullin wrote:

“There are only two points in his [i.e. McMullin's] presentation of my work, from which I have wanted to distance myself. The first is the distinction between deep and shallow revolutions: even though revolutions may differ in size, the epistemological problems they bring are identical for me” (Kuhn 1993, p. 337).

Thus it seems that in the philosophy of science, at least during Kuhn's lifetime, no classification of scientific revolutions could be developed.

Nevertheless, new impetus for the development of a particular theory often appears when the conceptual framework of the theory is applied to an area for which it was originally not intended. In the new area the concepts of the theory undergo shifts of meaning which open new prospects for the development of the theory. In the case of Kuhn's theory of scientific revolutions this occurred when historians tried to use Kuhn's conceptual framework to describe the development of mathematics. When Kuhn formulated his theory, he did not consider mathematics to be an area of its application, and so the question of whether the theory of scientific revolutions can be used in the history of mathematics sparked a vivid debate among historians of mathematics.

At the Workshop on the Evolution of Modern Mathematics held in Boston, Michael Crowe formulated the thesis that “*Revolutions never occur*

in mathematics” (Crowe 1975, p. 19). Some months later, at the meeting of the Society for History of Science in Norwalk, Joseph Dauben expressed the view that

“revolutions can and do occur in the history of mathematics, and the Greeks’ discovery of incommensurable magnitudes and Georg Cantor’s creation of transfinite set theory are especially appropriate examples of such revolutionary transformations” (Dauben 1984, p. 50).

A compromise view between these positions is that of Herbert Mehrtens, according to whom some concepts of Kuhn (scientific community, normal science, anomaly) have an explanatory value and offer a tool for the historical study of mathematics, while others (revolution, crisis, incommensurability) are in mathematics without an explanatory value and direct the debate to non-productive disputes (Mehrtens 1976). The debate was summarized in the anthology *Revolutions in Mathematics* (Gillies 1992).

In the introductory essay to the anthology the editor Donald Gillies sees the source of the disagreements between Crowe and Dauben in different understanding of the concept of scientific revolution. Crowe understands revolution narrowly, as changes during which “*some previously existing entity (be it king, constitution, or theory) is overthrown and irrevocably discarded*” (Crowe 1975, p. 19). In contrast, Dauben understands revolution in a wider sense, as changes during which a particular entity need not be irrevocably discarded, but is “*relegated to a significantly lesser position*” (Dauben 1984, p. 52). According to Gillies both interpretations are justified because ***there are different kinds of revolution***:

„This suggests that we may distinguish two types of revolution. In the first type, which could be called *Russian*, the strong Crowe condition is satisfied, and some previously existing entity is overthrown and irrevocably discarded. In the second type, which could be called *Franco-British*, the previously existing entity persists, but experiences a considerable loss of importance. ... It is at once clear that the Copernican and the chemical revolution were Russian revolutions, while the Einsteinian revolution was Franco-British. After the triumph of Newton, Aristotelian mechanics was indeed irrevocably discarded. It was no longer taught to budding scientists, and appeared in the

university curriculum, if at all, only in history of science courses. The situation is quite different for Newtonian mechanics, for, after the triumph of Einstein, Newtonian mechanics is still being taught, and is still applied in a wide class of cases.“ (Gillies 1992, p. 5)

These different kinds of scientific revolution can be illustrated by examples discussed by Kuhn himself. The *Newtonian revolution* is an example of revolution of the first kind, because in its course Aristotelian physics was *overthrown and irrevocably discarded* from the professional training of scientists. If today a student of physics is confronted with Aristotelian physics at all, it is only during the history of science courses. On the other hand, the *Einsteinian revolution* is, according to Gillies, a revolution of the second kind, because in its course Newtonian physics was not *irrevocably discarded*. Students are still learning Newtonian physics and it is still used in a variety of cases. It was only relegated from the position of *the* fundamental theory of the universe to *a significantly lesser position* of a useful first approximation.

It is important to realize that the difference between the total overthrow of Aristotelian physics during the *Newtonian revolution* and the relegation of Newtonian physics during the *Einsteinian revolution* concerns the behavior of the scientific community and thus it is a sociological fact that every proponent of Kuhn's theory must accept. In his essay *The Fregean revolution in Logic* (Gillies 1992b) Gillies tries to apply his discrimination of the two types of scientific revolutions to an analysis of Frege's contribution to logic. It turns out that the *Fregean revolution*, consisting in the transition from the Aristotelian syllogistic logic to the predicate calculus, satisfies neither Crowe's nor Dauben's definition. It does not satisfy *Crowe's definition*, because Aristotelian logic, with a few restrictions, is still considered valid, while Crowe's definition requires it to be irrevocably discarded. On the other hand, the Fregean revolution does not satisfy *Dauben's definition* either, because Aristotelian logic is relegated in a more fundamental way than Newtonian mechanics was during the Einsteinian revolution (which does satisfy Dauben's definition). Even if Aristotelian logic is still regarded as valid, nobody argues today in syllogisms, while engineers or architects use in their calculations Newtonian mechanics. Gillies believes that in the case of Fregean revolution we are dealing with a third type of revolutions. Nevertheless, the view that the Fregean revolution is different from the kinds of revolution described by Crowe and Dauben is

not the only possible interpretation of this example. Both the Einsteinian revolution in physics and the Fregean revolution in logic may be seen as revolutions of the same magnitude. The reason why Fregean revolution *appears* to be greater than the Einsteinian one (“*today nobody argues in syllogisms, but the engineers are still using Newton’s equations*”) is that we forget that the syllogisms of Aristotelian logic were not a paradigm of argumentation of the Ancient science either. In the Ancient world, just like today, nobody argued in syllogisms. Thus what we need is a criterion for classification of scientific revolutions.

1. ON THE LEGITIMACY OF A CLASSIFICATION OF SCIENTIFIC REVOLUTIONS

Kuhn had reasons for rejecting McMullin’s suggestion to discern revolutions of different kinds and for insisting that the epistemological problems they bring are identical. Already in *The Structure* he writes:

„Can Newtonian dynamics really be derived from relativistic dynamics? What would such a derivation look like? Imagine a set of statements, E_1, E_2, \dots, E_n , which together embody the laws of relativity theory. ...To prove the adequacy of Newtonian dynamics as a special case, we must add to the E_i ’s additional statements like $(v/c)^2 \ll 1$, restricting the range of the parameters and variables. This enlarged set of statements is then manipulated to yield a new set, N_1, N_2, \dots, N_m , which is identical in form with Newton’s laws of motion, the law of gravity, and so on. Apparently Newtonian dynamics has been derived from Einsteinian, subject to a few limiting conditions.

Yet the derivation is spurious, at least to this point. Though the N_i ’s are a special case of the laws of relativistic mechanics, they are not Newton’s Laws. Or at least they are not unless those laws are

reinterpreted in a way that would have been impossible until after Einstein's work. The variables and parameters that in Einsteinian E_i 's represented spatial position, time, mass, etc., still occur in the N_i 's; and they there still represent Einsteinian space, time, and mass. But the physical referents of these Einsteinian concepts are by no means identical with those of the Newtonian concepts that bear the same name. ...Unless we change the definitions of the variables of the N_i 's, the statements we have derived are not Newtonian. If we do change them, we cannot properly be said to have derived Newton's Laws, at least not in any sense of „derive“ now generally recognized.“ (Kuhn 1962, p. 100)

It seems that in this point we must agree with Kuhn. In the limit $(v/c)^2 \rightarrow 0$ we really obtain not Newtonian mechanics, but only a fragment of relativistic mechanics, which from the formal point of view resembles Newtonian mechanics, but on the conceptual level differs from it. Einstein defines his basic concepts in a different way than Newton did. For instance he defines the length of a moving body using a system of synchronized watches. Newton would never have come to the idea of giving a separate definition of the length of a moving body. In his conceptual system the length of a body is independent of its motion. That is a principle which he probably regarded evident. Thus even if we obtain in the limit $(v/c)^2 \rightarrow 0$ that there is no contraction of length, and so we have seemingly justified Newton's theory, we have proven this by using the Einsteinian concept of length. For the Newtonian concept there is nothing to prove. Length is constant *a priori*; the whole Newtonian mechanics is built on the supposition of its constantness. So Kuhn is right in saying that such formal reconstructions contribute nothing to the understanding of Newtonian physics. That Einsteinian length depends on the speed of light, and that in the limit case it becomes constant, what has this to do with Newton? In his mechanics Newton never mentioned the speed of light.

This agreement with Kuhn has one presupposition. Kuhn is right, as long as he speaks about *a single isolated scientific revolution*. To understand more deeply the Einsteinian revolution, formal reconstructions are really of minimal help. On the other hand, formal reconstructions can help us very much if we wish to *compare different revolutions*. Our aim is to take not one or a small number of revolutions, as Kuhn (and also

McMullin) did, but to take 20 or 30 cases discussed in the literature, and try to compare them. In such comparisons the formal analysis of the transition from one theory to the other during the revolution can serve as an *indicator of the magnitude of the revolution*. I sorted the different revolutions discussed in the literature according their apparent magnitude into few classes. Even if Kuhn could criticize every particular item of this classification, after separating the revolutions of the different magnitudes, some *remarkable regularities started to appear*.

I would like to liken this situation to the work of Mendeleev. If you take any two chemical elements and insist that they are alike and thus should belong to the same class, the opponent of the classification could with the same credibility insist that they are different. What is convincing on Mendeleev's classification appears only when you have 20 or 30 elements classified. Only then the regularities of the proposed classification start to be evident. I believe that in the case of scientific revolutions the classification works similarly. Kuhn and his followers can attack every particular case, but in spite of this, the general patterns are convincing. Of course this preliminary classification is only a heuristic. What is necessary to do next is to find for every class of scientific revolutions appropriate concepts and methods for their conceptual reconstruction.

2. METHODS OF ANALYSIS OF THE DIFFERENT KINDS OF REVOLUTIONS

As already mentioned in the preface, the reconstruction of *idealizations* is perhaps the most important result of the present dissertation. That is why there are four papers dedicated to it. The analysis of idealizations required the development of three methods of reconstruction. The first is the *method of intentional reconstruction*. Science is a human activity, therefore to understand the development of science (and of its language) at a particular level requires first of all understanding the *intentions*, the *motivations*, and

the *aims* of the particular actors of this development. This is partially an exercise in psychology, but not entirely, because these intentions have a public, social form of a *problem* (like the problem of finding the form of the ballistic curve, or the problem of determining the dynamics of the Solar system), or a *program* (like Galileo's program of the mathematization of nature). This means that several scientists can identify with the same intention, work on the solution of the same problem, cooperate on the same program. In this way the subjective dimension of science gets connected with the social one. This is particularly important for the Scientific Revolution, because scientists like Descartes or Newton, despite the fact that their theories of motion were incompatible and even contradictory, can be seen as working on the same program. The intentional analysis is thus able to disclose unity even where the logical analysis would find only incompatibility.

The next method of reconstruction of the development of the language of science is the *method of reconstruction of the linguistic innovations and deficiencies*. It turns out, that scientists finding a solution of a problem or contributing to a program often introduce some *linguistic innovations*. As an example we can take Descartes and his introduction of the notion of quantity of motion and of the law of its conservation in order to solve the problem of collision of moving bodies, or Newton's introduction of forces acting at a distance to solve the problem of interaction among bodies. We can speak about innovations and deficiencies, and not solely about changes, because the common intention enables us to compare the different solutions of the same problem. We can judge one change as *innovative* compared to another, if it helps better to fulfill the original intention. Similarly a particular aspect of the language can be judged as a *deficiency*, and not solely a characteristic feature of it, when it hindered the progress towards the fulfillment of the intention. The *reconstruction of linguistic innovations and deficiencies* is important, because it enables us to see some objective features of the particular contributions proposed by individual scientists and by means of these innovations we can often explain why a particular solution was more successful than another one and we can also understand the way how a program developed or degenerated.

Of course what we are interested in are not isolated linguistic innovations but rather the formation of a new linguistic framework (like that which characterizes the birth of classical mechanics, of field theory or of

quantum mechanics). Therefore we must turn from the analysis of the particular innovations to the *reconstruction of the process of the merging of separate linguistic innovations into a single linguistic framework*. Language is social and not private; therefore the different linguistic innovations introduced by individual scientists must undergo the process of social negotiation. Every linguistic framework is created by merging of several innovations stemming from many different authors. Thus in the case discussed in the papers I argue, that the linguistic framework of classical physics was created by merging the innovations stemming from Galileo, Descartes, and Newton. After creating the linguistic framework of Newtonian physics the process of idealization stops. It is so because due to the process that I called the “paradigm shift” the next idealization will take place in an area not connected with the area of idealization in physics.

Nevertheless, after a linguistic framework is created, the evolution of language does not stop, its dynamics just happens on a scale of smaller magnitude. Thus after the process of idealizations was completed, the development of the linguistic framework of physics occurs at the level of *re-codings*, while the level of idealizations remains stable. It is precisely this stable framework of idealization that introduces regularities and thus also patterns of change into the evolution on this lower level. This evolution has a very interesting form, which I suggest to call *bipolarity*. If we take the evolution of geometry, along the line *synthetic geometry*, *analytic geometry*, and *fractal geometry*, we discover that these developmental stages of the iconic language were separated by developmental stages of the symbolic language. Thus synthetic and analytic geometry were separated by the creation of algebra and similarly, analytic and fractal geometry were separated by the creation of the infinitesimal calculus. And it was not a mere historical coincidence. In the process of creation of analytic geometry Descartes made a substantial use of algebraic symbolism—the particular algebraic curves that he introduced were all defined by means of their equations. And similarly in the definition of the objects of fractal geometry the limit transition, which was introduced in the infinitesimal calculus, is used in a fundamental way. So we see that the evolution of the linguistic framework at the level of *re-codings* has a bipolar character. The new developmental stage in the development of the iconic language of geometry is reached by means of an intermediate symbolic stage and vice versa.¹

¹ This bipolarity is a regularity that can be identified only after we have classified a considerable number of revolutions (in this case at least 6 of them)

A similar bipolar dynamics can be found also in physics, in the development of the language of physical theories. The ***reconstruction of the bipolar process of the evolution of language*** is, after the methods of reconstruction of the *intentional structure*, of the *linguistic innovations*, and of the *process of their merging*, the fourth method of epistemological reconstruction of the language of science. It can be applied to *re-codings* and *relativizations*, while it cannot be applied to idealizations, due to a shift of the paradigmatic area.

In the bipolar dynamics of the development of language we can identify particular aspects of language. In the case of re-codings these are the *analytical power* – how complex formulas the language allows us to derive; *expressive power* – what new terms, predicates and relations can the language express, which were inexpressible at the previous stages; *explanatory power* – how the language can explain the failures which occurred at the previous stages; *integrative power* – what sort of unity and order the language enables us to conceive there, where we perceived just unrelated particular cases at the previous stages; *logical boundaries* – that are marked by occurrences of unexpected paradoxical expressions; and *expressive boundaries* – that are marked by failures of the language to describe some complex situations. I suggest calling these six objective characteristics as ***potentialities of language***.

As a fifth method of epistemological reconstruction of the language of science we can therefore introduce the ***reconstruction of the potentialities of the language***. The evolution of the language of science consists in the growth of its analytical and expressive power—the later stages of development of the language make it possible to derive more formulas and to describe a wider range of phenomena. The explanatory and the integrative power of the language also gradually increases—the later stages of development of the language enable deeper understanding of its methods and offer a more unified view of its subject. To overcome the analytical and the expressive boundaries of language, more and more sophisticated techniques are developed. It is important to realize that the potentialities of language mentioned above are objective features of the language.

After we identified the potentialities of language the question arises how are they constituted. As the potentialities are objective aspects of

language, for each potentiality there has to exist a particular structure of language, the change of which causes the increase of the corresponding potentiality. These structures cannot be connected to a particular subject matter. They must be formal, to allow the increase of the corresponding potentialities. I suggest calling them *formal aspects* of language. Thus the sixth method of epistemological reconstruction of language is the *reconstruction of the formal aspects of language*. In the case of relativizations we can introduce the following formal aspects: *the epistemic subject of the language* from the point of view of which the theory is formulated; *the horizon of the language*, i.e. the boundary of the world that can be represented by the theory; *the individua of the language*, i.e. the elementary constituents, that the language is able to distinguish; *the fundamental categories of the language*, i.e. categories, which the language does not allow to further analyze; *the ideal objects of the language* i.e. objects that are introduced in order to make the universe of discourse complete; and *the background of the language*, i.e. a neutral medium such as the space or the number system, in which all the individua are situated.

3. DESCRIPTION OF THE DIFFERENT KINDS OF REVOLUTIONS

As a result of the application of the methods of analysis presented in the previous chapter we obtain a classification of scientific revolutions into three kinds. In the following text I will characterize each kind and illustrate it on a few examples.

3.1 Idealizations

An idealization is an epistemic change of the greatest possible magnitude. It concerns the ideal objects (numbers, geometric figures, dynamical systems), by means of which we search for order in nature. An example of idealization was the epistemic change accompanying the *Scientific Revolution*, which

separates ancient science, which tried to reveal an *unchanging order* in nature, from modern physics that looks for *dynamic laws* beyond the phenomena.²

Idealization can be best explained by comparing the theories of Kepler and of Newton. Kepler was perhaps one of the last eminent scientists following the ideal of science of classical antiquity: he sought in nature for eternal, unchanging forms. His law of the elliptical shape of the planetary orbits is a typical law of this kind. From the Newtonian point of view we can say that Kepler was really lucky, because the tables left behind by Tycho Brahe were *sufficiently precise* to discriminate the (elliptical) shape of the orbit of Mars from a circular shape. On the other hand they were *sufficiently inaccurate* not to reflect the perturbations of the orbit of Mars caused by Jupiter and the other planets. Only so could Kepler in good conscience assert (in agreement with the ideal of science) that the orbit of Mars *has the form* of an ellipse. In fact this orbit, just like any other orbit in the Solar system, has no pre-given form, which could be described by means of geometry. The motion of the planets is lawful not in the sense that its trajectory would have a particular geometric shape (as ancient scientists believed), but that the motion is generated by the action of forces. The trajectories of the planets are not lawful in the sense that they would *reveal some eternal and unchangeable geometric shape* (as Kepler and before him all ancient scholars understood the role of science), but in the sense that it *fulfills the dynamic law describing their generation* (as Newton, and after him the majority of scientists, understood the role of science). The creation of a linguistic framework that enables the transition from the former (geometric, Euclidian) to the latter (dynamic, Newtonian) paradigm is the core of the process of idealization.

3.2 Re-codings

Re-codings are changes of lesser magnitude than idealizations. As an

² *Idealization* in the case of physics consisted in the *introduction* of a general linguistic pattern that is common for every (sufficiently general) physical theory. It consists of: 1. determination of the *measurable quantities*; 2. description of the *state of the physical system*; and 3. choice of an equation describing the *temporal evolution of the state*. Newton's mechanics, Maxwell's electrodynamics, Clausius' thermodynamics, Schroedinger's quantum mechanics, and many other theories have this common structure.

example of re-codings in physics we can take the creation of field theory, or the creation of quantum mechanics. All re-codings in physics take place in a common linguistic framework of idealization described above. This common framework prescribes that every re-coding must have *a particular* list of measurable quantities, *a particular* description of state, and *a particular* equation describing the temporal evolution of the state. Re-codings differ in what specific measurable quantities they use, how they describe the state, and by means of which differential equation they describe the temporal evolution of the state. But what they all have in common is the general linguistic scheme described above.³

The Newtonian paradigm of re-coding, which was the first example of that general scheme, has as measurable quantities *time, position, velocity, acceleration, and weight*; the state of a system is given by two vectors – the *vector of position* and the *vector of momentum* for each particle; and the dynamical equation is *Newton's second law*.⁴ In this respect Newtonian physics differs from *field theory*, which introduces further measurable quantities, such as *electric charge, electric current, electric field and magnetic field*; the state of a system is given by a pair of vector fields – the *electric field* $\mathbf{E}(x, y, z, t)$ and the *magnetic field* $\mathbf{B}(x, y, z, t)$; and the dynamical equations are the well known *Maxwell's equations*.

³ A *re-coding* is a *change* of the set of measurable quantities (it introduces new techniques that make it possible to measure quantities, that were hitherto immeasurable); a *change* of the description of the state of the physical system (the new quantities make it possible to incorporate into the description of the state new aspects that were until then not included into the notion of the state); and a *change* of the fundamental equation of the theory. These changes can be seen as an introduction of a new linguistic framework.

⁴ Saying that Newton was the creator of *idealization*, I meant that he created the first theory built according to this general scheme and so he actually created this scheme of the physical representation of reality. But at the same time he was the creator of a particular realization of this general scheme having the form of *Newtonian physics*. Thus the emergence of Newtonian physics was at the same time an *idealization* (fixing the type of ideal objects by means of introducing a general linguistic framework for physical theories) and also the first *re-coding* (fixing a particular set of measurable quantities, a particular description of state and a particular dynamic law).

3.3 Relativizations

Relativizations are epistemic changes of lesser magnitude than re-codings. They take place inside the framework constituted by a previous re-coding.⁵ As examples of relativizations in classical mechanics (i.e. in the framework of the Newtonian paradigm of re-coding) we can take *Newtonian*, *Lagrangian* and *Hamiltonian mechanics*. Despite their similarity there are interesting differences between these systems. For the sake of simplicity I will illustrate these differences on the example of the description of the system of the Earth with the Moon.⁶

Newtonian mechanics describes the system Earth – Moon as a motion of two bodies in a three-dimensional space, which is described by a system of six second order differential equations. *Lagrangian mechanics* describes the system Earth – Moon as a motion of a single body in a six-dimensional configuration space (whose first three coordinates determine the position of the Earth, the remaining three coordinates the positions of the Moon), while the differential equations are six equations of the second order. *Hamiltonian mechanics* describes the system Earth – Moon as a motion of a single body in a twelve-dimensional phase space (whose first three coordinates determine the position of the Earth, another three coordinates the position of the Moon, three other coordinates the momentum of the Earth and the last three coordinates the momentum of the Moon), and the differential equations are twelve equations of the first order. In these three theories we should be able to recognize not only their unity given by their common Newtonian paradigm of re-coding (they all describe the state of the Earth and the Moon using only mechanical quantities mentioned above – positions and momenta), but also their differences, which are given by the different relativizations.

⁵ Here we see a certain type of nesting in the sense that *re-codings* take place inside the framework established by an *idealization*, and *relativizations* take place inside the framework established by a *re-coding*.

⁶ A *relativization* consists in the change of the *epistemic subject* from the point of view of which the theory is constructed. In (Kvasz 2008) I showed, that the epistemic subject is closely connected with the space on the background of which the theory is constructed (space is actually the set of all possible positions of the epistemic subject). In a short description of the three examples above it is not possible to explain the connection between the epistemic subject and space, and thus I take the notion of *space* as an alternative tool for the characterization of relativizations (parallel to the notion of *epistemic subject*).

4. THE TEXTS CONTAINED IN THE DISSERTATION

There are seven papers included in this dissertation. The order, in which they appear in the dissertation, differs from the chronological order in which they were written. As already mentioned, *Patterns of Change* (Kvasz 2008) did not contain a description of idealizations in mathematics. Originally, when I was writing the book, I wanted to include in it a description of the process of idealization in physics. So until approximately 2006 in the project of the *classification of scientific revolutions* (announced in Kvasz 1999a) I did not discriminate between mathematics and physics. But during the final stage of the work on the manuscript I was advised not to include the theory of idealization in physics in the book. The rest of the book dealt exclusively with mathematics, and including a chapter on physics would probably hinder the understanding and the reception of the book. So in 2007 the project of the classification of scientific revolutions bifurcated into two different but closely related projects: *the theory of linguistic innovations in classical mathematics* that culminated in *Patterns of Change* and *the theory of paradigm change* that is the subject matter of the present dissertation. Because in mathematics it is impossible to reconstruct the process of idealization, the first project contained only the description of *re-codings* and of *relativizations*.

The first part of the second project consisted therefore in developing a theory of *idealizations*. This theory is contained in the series of three papers, written before the bifurcation of the original program, and published in the journal *Philosophia Naturalis* as (Kvasz 2002a, 2003a, and 2005b). I have returned to idealizations in the paper *What Can the Social Sciences Learn from the Process of Mathematization in the Natural Sciences* (Kvasz 2012a), where I confronted my account of idealizations with Kuhn's theory.

After the theory of idealizations, the next step in developing a theory of paradigm change was to transfer to physics the methods of analysis of *re-codings* and *relativizations*. This was achieved for relativizations in the

paper *Classical mechanics between history and philosophy* (Kvasz 2011a), and for re-codings in the paper *On boundaries of the language of physics* (Kvasz 2013). After writing these two papers the theory of paradigm change could be considered as established. The final step was to publish an overview confronting the theory of paradigm change with Kuhn's theory of scientific revolutions. A good occasion for this was the conference *The Progress of Science*, held in Tilburg from 25th to 27th April 2012 and dedicated to the 50th anniversary of the publication of Kuhn's classics *The Structure of Scientific Revolutions*. The result is the paper *Kuhn's Structure between sociology and epistemology* published in *Studies in History and Philosophy of Science* (Kvasz 2014b), that is included as the first paper of the dissertation.

4.1 Kuhn's *Structure* between sociology and epistemology (Kvasz 2014b)

As stated above, my main criticism of Kuhn's theory is that it does not differentiate between various types of scientific revolutions and covers under the concept of scientific revolution different types of change: *idealizations* represented by the *Newtonian revolution*, *re-codings* represented by the *Copernican revolution*, and *relativizations* represented by the *Einsteinian revolution*.⁷ This has the consequence that the basic categories of Kuhn's theory such as *paradigm*, *anomaly*, *crisis*, and *revolution*, which he introduced on the basis of such a heterogeneous material, allow only approximate and nonspecific characterization. Each of Kuhn's categories encompasses three different concepts. Something else forms a paradigm in the case of an *idealization*, something else in the case of a *re-coding*, and something else in the case of a *relativization*. Thus it is not surprising that some commentators criticized Kuhn for the ambiguity of his basic categories.

⁷ As this paper was read at a conference dedicated to Kuhn, I included among the scientific revolutions a fourth kind of change, which I call *re-formulations*. I did it to bring the theory as close as possible to the views expressed by Kuhn, who explicitly included the discovery of the Uranus among revolutions. This is an example of a re-formulation, as all books have to be rewritten to bring them into agreement with this discovery. A re-formulation differs from a reformulation in that the two formulations („There are 6 planets.“ and „There are 7 planets.“) are not equivalent. A further difference is that in the paper I use the term *re-presentation* instead of *re-coding*.

In order to characterize more precisely the fundamental categories of Kuhn's theory, we must distinguish different types of scientific revolutions. Then it will be possible to distinguish between three kinds of paradigms. A *paradigm of idealization* codifies the kind of ideal objects that are used. In the description of planetary motion Kepler used ideal objects of geometry to represent the form of the planetary orbits, while Newton used differential equations to represent the process of generation of the trajectory. A *paradigm of re-coding* codifies the measurable quantities, the description of state and the dynamic equation. *The Newtonian paradigm of re-coding* uses as measurable quantities position, velocity, acceleration and weight; the state is described by means of two vectors – the vector of position and the vector of momentum; and the dynamic equation is Newton's second law. *The Maxwellian paradigm of re-coding* uses as measurable quantities, besides the previously mentioned ones, also electric charge, electric current, etc.; a state is described using two vector fields; and the dynamic equations are Maxwell's equations. A *paradigm of relativization* codifies the type of space, the nature of objects and the way of the description of action. *The paradigm of Newtonian mechanics* uses the three-dimensional Euclidean space; the objects are individual material bodies placed in this three-dimensional space; and action is described by forces. *The paradigm of Lagrangian mechanics* uses for the description of n bodies the $3n$ -dimensional configuration space; the state of a system of n bodies is given by the position and the velocity of a point in this space; and action is described by Lagrange's function defined as the difference between the kinetic and the potential energy of the system.⁸

The different types of scientific revolutions consist in changes of the paradigm of the particular type. So an *idealization* changes the *paradigm of idealization*, a *re-coding* changes the *paradigm of re-coding*, and a

⁸ If we take a concrete example, as for instance Newton's derivation of the law of universal gravitation from Kepler's laws, it becomes obvious that this derivation can be seen as "paradigmatic" in different ways. First, it exemplifies the *Newtonian idealization* because it shows how it is possible to derive from dynamic laws the geometrical shape of an orbit. Secondly, it exemplifies the *Newtonian re-coding* characterized by the particular description of state and the particular type of dynamic equation. Thirdly, it exemplifies the *Newtonian relativization* because Newton makes use of the three-dimensional space, of bodies as material points and of forces acting at a distance. This multilayered structure of Newton's paradigmatic text is displayed by the fact that I mentioned Newton as an illustration of idealizations, of re-codings and of relativizations.

relativization changes the *paradigm of relativization*. It is probable that each type of scientific revolutions has not only a different kind of paradigm, but also different sorts of anomalies, and a different nature of the crisis. It is therefore possible that after we separate the various types of scientific revolutions, we will succeed for each type of revolution in describing its true cognitive dynamics.

4.2 Galilean physics in light of Husserlian phenomenology (Kvasz 2002a)

Just like the theory of *relativizations* is based on Wittgenstein's picture theory of meaning from the *Tractatus* (Wittgenstein 1921), and the theory of re-codings on Frege's description of the development of symbolic languages in mathematics in *Funktion und Begriff* (Frege 1891), as a basis for the reconstruction of *idealizations* I have taken Husserl's interpretation of Galilean physics in *Die Krisis der Europäischen Wissenschaften und die Transzendente Phänomenologie* (Husserl 1954) and tried to bring it into agreement with contemporary historical research. Husserl's book contained a criticism of the positivist philosophy of science. According to positivism, scientific theories are based on accumulation and inductive generalization of empirical statements, derived directly from neutral sense data. Husserl overthrew this picture, showing that there is nothing like neutral sense data, and that from the very beginning we are dealing with an interpreted world, which he called life-world (*Lebenswelt*). Further, Husserl showed that science does not form its theories by accumulation and inductive generalization of empirical statements, but on the contrary, the rise of science consisted in a very radical shift away from experience in the life-world. Husserl called this radical shift *idealization*, and interpreted it as *replacement* of phenomena by idealities, as *turning the world of qualitative phenomena into a universe of mathematical quantities*.

The first step on the road to a theory of idealizations was an analysis of this replacement. I tried to put into Husserl's theory as much historical detail concerning Galileo and the early history of science as possible in order to turn Husserl's schematic philosophical sketch into a comprehensive cognitive and epistemological theory of idealization.

4.3 The Mathematization of Nature and Cartesian Physics (Kvasz 2003a)

Even though Husserl's analysis of Galilean science was a criticism of positivist philosophy of science, Husserl unwittingly remained in the framework in which positivism used to discuss science. According to positivism, the central issue in philosophy of science is to explain the relation of scientific theories to experience. Husserl has overthrown the positivist philosophy of science, but he still remained within the framework of positivist philosophy reducing the discussion of scientific theories to the question of their relation to experience. A radical rejection of positivism requires a rejection of not only what positivists say about science, but also of the framework in which their theory of science is formulated. The positivist philosophy of science consists not only of all that, what positivists said about science, but also of all those aspects of science which they excluded from consideration.

Modern science is based not only on Galilean empiricism which the positivists liked to contemplate about. It is equally based on Cartesian and Newtonian metaphysics, which the positivists liked to pass by in silence, and which therefore also Husserl did not analyse. Therefore the next step in the development of a theory of idealization was a reconstruction of Cartesian physics following Husserl's analysis of Galileo. Adopting Husserl's approach I interpreted Descartes' contribution to physics as an idealization. Nevertheless, the Cartesian idealization is not an idealization of isolated phenomena of the life-world, as was the case with Galileo, but it is rather an *idealization of the ontological foundations* of the life-world. The life-world has, beside its phenomenal level, also an ontological level. We understand that the objects of our everyday experience possess an ontological unity, despite the great variety of phenomenal aspects we perceive in them. I interpreted Descartes' contribution to the rise of modern science as the *replacement* of the objects of the life-world by their mathematical representation in the form of extended bodies.

4.4 The Mathematization of Nature and Newtonian Physics (Kvasz 2005b)

Husserl interpreted idealization as a process, in which a phenomenon of life-world is replaced by a mathematical ideality. The aim of the paper was to argue that Newtonian physics can be interpreted as *idealization of action* in

this Husserlian sense. In the process of this idealization the phenomenon of action, as we know it from our experience, is replaced by the Newtonian action mediated by forces. This Newtonian replacement can be seen as a continuation of the Cartesian reduction. Even though on the ontological level Descartes abandoned the life-world and created his mathematical universe of extended bodies, his understanding of action (as pushing and pulling) remained very close to the ordinary notion of action. Pushing and pulling is precisely what we do in our everyday lives. When we write, we push the pen against the paper, and when we want to undo our shoelaces, we simply pull them. Thus Descartes transferred into his mathematical universe of extended bodies our ordinary understanding of action in the life-world. I tried to show that many aspects of Newtonian physics can be understood as a consequence of the replacement of the Cartesian notion of action based on everyday experience by a new, mathematical notion of action that is absolutely alien to any experience; the action of forces at a distance. In other words the Newtonian notion of action can be interpreted as idealization in the Husserlian sense. It was this mathematical description of action that enabled Newton to complete the process of mathematization started by Galileo.

The idealization, on which modern physics is based, has three layers. The first layer is the Galilean *idealization of phenomena*. It consists in the replacement of the phenomena of the life-world by mathematical quantities, obtained by measurement. The second layer is the Cartesian *idealization of ontology*. It consists in the replacement of the objects of the life-world by extended bodies obtained in the process of the ontological reduction of reality. The third layer is the Newtonian *idealization of action*. It consists in the replacement of the action between objects of the life-world by a mathematical representation of forces acting at a distance. By joining these three layers of idealization, that is by putting together the mathematical description of quantities, of states and of action, Newton created an idealized world, by which science replaces the world of our ordinary experience. This replacement is so successful because besides its own empirical basis the world of science has its own ontology and its own causality. The world of science is closed not only on the empirical level of facts, but also on the ontological level of objects and the causal level of action.

4.5 What Can the Social Sciences Learn from the Process of Mathematization in the Natural Sciences (Kvasz 2012a)

According to Kuhn the main difference between natural and social sciences consists in the fact that while in natural sciences we have to do with *normal science* based on a widely *accepted paradigm*, in social sciences there is nothing comparable to paradigms, and scholars again and again question the foundations of their disciplines. In contemporary science the paradigm is formed by physics and so we can call all disciplines, in which the methods of quantification and measurement lead to success, as *paradigmatic disciplines*. Further I suggested introducing the term *elusive region of the paradigm* for those disciplines where the methods and approaches of the particular paradigm cannot be employed. Besides these two kinds of disciplines I introduced two other kinds which lie between the paradigmatic region and the elusive region of the paradigm. The first are the so called *mixed disciplines*. This term is used by historians to describe a remarkable set of disciplines from late Antiquity, such as Euclidean optics, Archimedean theory of the lever, or Ptolemaic astronomy.⁹

A second category of disciplines lying between the paradigmatic and the elusive region can be called the *metaphorical region of the paradigm*. It forms a counterpart to the mixed disciplines. While in the case of the mixed disciplines the notions and methods of the paradigm are used in a precise and unambiguous way, and the problem is only that they are being used outside the area where their use can be justified by the paradigm's methodology, in the *metaphorical region* the fundamental notions of the paradigm are used with a *transferred, distorted and stretched* meaning. As a representative of the metaphorical region of the ancient paradigm we can consider *Aristotle's theory of local motions*, according to which heavy bodies fall downwards while light bodies float upwards. The Aristotelian theory of local motions can be interpreted as a geometrical theory. It is based on the image of a geometrically ordered universe and it understands motion

⁹ I suggest (in contrast to Kuhn) to consider Euclid's *Elements* as the paradigm of Ancient science. It may sound unusual to call *Elements* a paradigmatic theory. We understand paradigms as a part of science and for us mathematics does not belong to science. Nevertheless, it is problematic to use our contemporary classification of disciplines in interpreting the past. If we look at Ancient science not from our but from its own viewpoint, it is rather the *Elements* than the *Almagest* that had a paradigmatic status. Ptolemaic astronomy that Kuhn characterized as paradigmatic I suggest to include among the mixed disciplines.

as a transition between different places of this geometrical order. Nevertheless, geometry is used here in a different manner from that in the mixed disciplines. Geometry does not enter the Aristotelian view of the order of the cosmos as a set of exact notions and methods for making constructions and proving theorems (as it enters the Archimedean theory of the lever), but only as a set of metaphors, by means of which we can discern order and meaning in the phenomena.

It turns out that it were the mixed disciplines and their conflict with the metaphorical realm of the paradigm which were the driving force of the Scientific Revolution. Newtonian physics was created not inside the paradigmatic region of the old paradigm. The paradigmatic region of ancient science was mathematics. The birth of Newtonian physics stimulated the creation of several mathematical disciplines, but we cannot say that inside of mathematics there occurred some massive refutation of the previous research. It is fair to say that the scientific revolution of the 17th century took place on the contact of the mixed disciplines of the ancient paradigm (astronomy, optics, the theory of simple machines) and the metaphorical region of that paradigm (the geocentric view of the cosmos). And this is natural.

In the *paradigmatic region* the methodological standards are so strict and well founded that a refutation of the overall picture is improbable. On the other hand the *elusive region of the paradigm* is not sufficiently stable and therefore changes happen there too often to be able to cause some deeper considerations. It is in the area of the *mixed sciences*, where the methods of the paradigm offer sufficiently effective means of research so that their progress is intensive. On the other hand the application of the paradigmatic methods to unintended areas of phenomena increases the probability of the discovery of something radically new and unexpected, something that will be in sharp contrast with all that we are used expecting in the paradigmatic region. The *metaphorical region of the paradigm* is important for another reason. There the research is carried out on the fringe of what the paradigm allows to thematize and therefore the metaphorical region is often the place for the basic cultural projections with the emotional charge that accompanies such projections. The mixed disciplines alone would probably never have led to a revolution. Had Galileo accepted the suggestions of the Church and discussed the Copernican system only as a hypothesis, i.e. if he had restricted himself to the technical realm of the

mixed disciplines, it is probable that the new astronomical discoveries would remain on the periphery of interest as incomprehensible, innocuous technical hypotheses. The dynamic of the scientific revolution was driven by the conflict of the mixed disciplines with the metaphorical region when not absolutely sure results of scientific inquiry got into conflict with metaphors by means of which we articulate our place in the universe.

4.6 On boundaries of the language of physics (Kvasz 2013)

The aim of the paper was to outline a method of reconstruction of the historical development of the language of physical theories. It applied the theory presented in *Patterns of Change* to the analysis of linguistic innovations in physics. There are six aspects of the language, the changes of which accompany re-codings: 1. *Logical power* – how complex formulas can be proven in the language; 2. *Expressive power* – what new things the language can express, which were inexpressible at the previous stages; 3. *Explanatory power* – how the language can explain the failures which occurred at the previous stages; 4. *Integrative power* – shows the sort of unity and order the language enables us to see in places where we perceived just unrelated particular cases at the previous stages; 5. *Logical boundaries* – are marked by occurrence of unexpected paradoxical expressions; 6. *Expressive boundaries* – are marked by failures of the language to describe complex situations (Kvasz 2008, p. 16). The evolution of the language consists in the growth of its *logical* and *expressive power*—the later stages of development make it possible to prove more theorems and to describe a wider range of phenomena. The *explanatory* and the *integrative power* of the language also gradually increase—the later stages of development of the language provide a deeper understanding of its methods and offer a more unified view of its subject. To overcome the *logical* and *expressive boundaries*, more sophisticated and subtle techniques are developed. My aim was to introduce these aspects into the analysis of the language of physics.

To transfer the notion of *logical power* from mathematics to physics is not difficult. In physics it is more appropriate to call it *analytical power* of language, and to understand it as related not to proving of theorems, but to derivation of formulas. I characterized the analytical power of the language of a particular physical theory by the kind of formulas which it is possible to derive in the given language using the accepted postulates of the theory. As an illustration we can take Newton's derivation of Kepler's laws. For

Kepler, the elliptical form of the planetary orbits was an empirical fact. In the language of Newtonian mechanics this proposition can be derived from the law of gravity. The ability of the language to derive a particular law illustrates its analytical power.

Similarly clear is the case of the *expressive power*, which represents the ability of the language to represent some aspect of nature. In the history of physics there are many cases when a phenomenon that defied description by means of the language of the “old” theory and was therefore seen as an anomaly could be easily described by means of the language of the “new” theory. Such cases illustrate the expressive power of the language of physics.

We can find in physics also an analogy of the *explanatory power* of language. As an example we can take the explanation of stability of matter by quantum mechanics. In classical physics it was not clear why the electrons that orbit in the atoms forming for instance a chair do not disintegrate. It follows from the principles of classical physics that it is not possible to form a stable configuration of charged particles that would be maintained by electromagnetic forces only. A perturbation of the atoms of the chair would lead to large changes in the trajectories of the electrons, causing a disintegration of the whole chair. Heisenberg’s principle of uncertainty makes it possible to explain why matter is stable, i.e. why despite perturbations the electrons remain near their original locations. According to this principle, electrons can get closer to a proton (i.e. make their location in space more precise) only at the price of increasing their energy (due to the increase in the uncertainty of their momentum). This mechanism ensures the stability of the ground state of the atoms. Thus the language of quantum mechanics makes it possible to explain the stability that for classical physics was a mystery.

Illustrations of the *integrative power* of language are the great unifications, such as Newton’s unification of the terrestrial and celestial mechanics, or Maxwell’s unification of electrodynamics and optics. The paper ascribed these unifications to the integrative power of language. In addition to these “positive” aspects of the language I transferred to physics also the notions of *analytical* and *expressive boundaries* of language. In my opinion, these boundaries are one of the most interesting aspects of language of science.

4. 7 Classical Mechanics between History and Philosophy (Kvasz 2011a)

In a series of papers (Kvasz 1998, 2005a and 2006) I proposed an interpretation of the development of mathematical theories as *changes of the pictorial form* in the sense of the *Tractatus*. In the development of geometry and of algebra it was possible to identify six pictorial forms, each of which determines the way how linguistic representations are coordinated with each other as well as with the particular subject matter, represented by the language of the theory. Here I applied this approach to the epistemological interpretation of the development of classical mechanics. Thus I interpreted Newton's *Philosophiae naturalis principia mathematica* (1687) as a theory of mechanical motion based on the perspectivist form of language; Euler's *Mechanica sive motus scientia analytice exposita* (1736) as the work that introduced into mechanics the projective form of language; d'Alembert's *Traité de Dynamique* (1743) as a theory developing mechanics on the basis of the compositive form of language; and finally Lagrange's *Mécanique analytique* (1788) as the work that introduced into mechanics the interpretative form.

5. MY PAPERS DEALING WITH ASPECTS OF THE THEORY OF PARADIGM CHANGE

(entries set bold are part of the dissertation)

Kvasz, L. (1993): How do theories represent reality? *Mesotes*, 1993/2, 263-272.

Kvasz, L. (1996a): Poincaré and the Epistemological Interpretation of the Erlangen Program. *Philosophia Scientiae* **1**, 107-118.

Kvasz, L. (1996b): Was bedeutet es, ein geometrisches Bild zu verstehen? In: Dagmar Reichert (ed.), *Räumliches Denken*, vdf Hochschulverlag an der ETH Zürich, 95-123.

Kvasz, L. (1998): History of Geometry and the Development of the Form of its Language. *Synthese* **116**, 141-186.

Kvasz, L. (1999a): On classification of scientific revolutions. *Journal for the General Philosophy of Science* 1999, 201-232.

Kvasz, L. (1999b): Epistemological Foundations of Geometry in the 19th Century, *Philosophia Scientiae* **3**, 183-202.

Kvasz, L. (1999c): Tarski and Wittgenstein on Semantics of Geometrical Figures. In: J. Wolenski and E. Köhler (eds.), *Alfred Tarski and the Vienna Circle, Vienna Circle Institute Yearbook 6* (1998), Kluwer, 179-191.

Kvasz, L. (2000): Changes of Language in the Development of Mathematics. *Philosophia mathematica* **8**, 47-83.

Kvasz, L. (2001): Leibniz's criticism of the Cartesian physics. In: *Nihil sine ratione - VII. Internationaler Leibniz-Kongress*, Berlin 2001, (ed.) H. Poser, 669-676.

- Kvasz, L. (2002a): Galilean physics in light of Husserlian phenomenology. *Philosophia Naturalis* 39, 209-233.**
- Kvasz, L. (2002b): Lakatos' Methodology Between Logic and Dialectic. In: G. Kampis et al. (eds.), *Appraising Lakatos. Mathematics, Methodology and the Man*, Kluwer, 211-241.
- Kvasz, L. (2003a): The Mathematisation of Nature and Cartesian Physics. *Philosophia Naturalis* 40, 157-182.**
- Kvasz, L. (2003b): Epistemological aspects of the history of painting. In: *Proceedings of the 7th Central European Seminar on Computer Graphics*, Budmerice, Slovakia. Eds: I. Viola, Th. Theussl, a L. Szirmay-Kalos, 7-24.
- Kvasz, L. (2004): How can a falsified theory remain corroborated? In: Friedrich Stadler (ed.), *Induction and Deduction in the Sciences*. Kluwer, 263-271.
- Kvasz, L. (2005a): Similarities and differences between the development of geometry and of algebra. In: Carlo Cellucci and Donald Gillies (eds.), *Mathematical Reasoning and Heuristics*. King's College Publications London, 25-47.
- Kvasz, L. (2005b): The Mathematization of Nature and Newtonian Physics. *Philosophia Naturalis* 42, 183-211.**
- Kvasz, L. (2006): History of Algebra and the Development of the Form of its Language. *Philosophia Mathematica* 14, 287-317.
- Kvasz, L. (2008a): *Patterns of Change, Linguistic Innovations in the Development of Classical Mathematics*. Birkhäuser Verlag AG, Basel.
- Kvasz, L. (2008b): Forms of Transcendence in Science and in Religion. *Theology and Science*, 89-106.
- Kvasz, L. (2011a): Classical Mechanics between History and Philosophy. In: A. Máté, M. Rédei and F. Stadler (eds.), *The Vienna Circle in Hungary*. Springer Wien, Wien, 129-154.**
- Kvasz, L. (2011b): Kant's Philosophy of Geometry—On the Road to a Final Assessment. *Philosophia Mathematica* 19, 139-166.
- Kvasz, L. (2012a): What Can the Social Sciences Learn from the Process of Mathematization in the Natural Sciences. In: Dieks, D. et al., *Probabilities, Laws, and Structures*. Springer, Dordrecht, 379-389.**

- Kvasz, L. (2012b): Galileo, Descartes, and Newton – Founders of the Language of Physics. *Acta Physica Slovaca* **62**, 519-614.
- Kvasz, L. (2013): On boundaries of the language of physics. In: E. Barbin a R. Pisano (eds.), *The Dialectic Relation Between Physics and Mathematics in the XIXth Century*. Springer, Dordrecht, 139-158.**
- Kvasz, L. (2014a): Mathematics and Experience. In: M. C. Galavotti, E. Nemeth a F. Stadler (eds.), *European Philosophy of Science – Philosophy of Science in Europe and the Viennese Heritage, Vienna Circle Institute Yearbook 17*. Springer, Dordrecht, 117-129.
- Kvasz, L. (2014b): Kuhn’s *Structure* between sociology and epistemology. *Studies in History and Philosophy of Science* **46**, 78-84.**

MAIN REFERENCES USED IN THE DISSERTATION

- Aliseda, A. and Gillies, D. (2007): Logical, Historical and Computational Approaches. In: Kuipers 2007, 431–513.
- Ariew, R. (1992): Descartes and scholasticism: The intellectual background to Descartes' thought. In: Cottingham 1992, 58–90.
- Bechtel, W and Hamilton, A. (2007): Reduction, integration, and the unity of science: natural, behavioral, and social sciences and the humanities. In: Kuipers 2007, 377–430.
- Böhme, G. (1989): Philosophische Grundlagen der Newtonschen Mechanik. In: Hutter, 1989, 5–20.
- Clarke, D. M. (1992): Descartes' philosophy of science and the scientific revolution. In: Cottingham 1992, 258–285.
- Coelho, R. L. (2002): Zur Physik von Descartes: Naturgesetze und Stossregeln. *Philosophia Naturalis* **39**, 45–60.
- Cottingham, J. (ed. 1992): *The Cambridge Companion to Descartes*. Cambridge UP, New York.
- Crowe, M. (1975): Ten 'laws' concerning patterns of change in the history of mathematics. Reprinted in: Gillies (1992), 15–20.
- Dauben, J. (1984): Conceptual revolutions and the history of mathematics: two studies in the growth of knowledge. In: Gillies 1992, 49–71.
- Descartes, R. (1637): *Discourse on Method, Optics, Geometry, and Meteorology*. Boobs-Merrill, Indianapolis 1965.
- Descartes, R. (1644): *Principles of Philosophy*. Translated by V. R. Miller and R. P. Miller, Reidel, Dordrecht 1983.
- Drake, S. (ed. 1957): *Discoveries and opinions of Galileo*. New York.
- Drummond, J. J. (1992): Indirect Mathematization in the Physical Sciences. In: L. Hardy and L. Embree (eds.), *Phenomenology of Natural Science*. Dordrecht, 71–92.
- Dugas, R. (1955): *A History of Mechanics*, Dover, New York 1988.
- Dunmore, C. (1992): Meta-level revolutions in mathematics. In: Gillies (1992), 209–225.

- Euler, L. (1736): *Mechanica sive motus scientia analytice exposita*. Russian translation: *Osnovy dinamiki tochki*. GRTTL, Leningrad 1938, 29–262.
- Euler, L. (1750): Découvert d'un principe de Mécanique. *Opera Omnia* II, 5: 81–108.
- Fauvel, J. (ed. 1993): *Newtons Werk. Begründung der modernen Naturwissenschaft*. Birkhäuser, Basel.
- Frege, G. (1891): Funktion und Begriff. In: *Funktion, Begriff, Bedeutung*. Vandenhoeck & Ruprecht, Göttingen 1989, 17–39.
- Franklin, A. (2007): The role of experiments in the natural sciences: Examples from physics and biology. In: Kuipers 2007, 219–274.
- Gaukroger, S. (ed. 1980): *Descartes, Philosophy, Mathematics and Physics*. Harvester Press, Sussex.
- Gaukroger, S. (1980): Descartes' Project for a Mathematical Physics. In: Gaukroger 1980, 97–140.
- Gillies, D. (ed. 1992): *Revolutions in Mathematics*. Clarendon Press, Oxford.
- Gillies, D. (1992b): The Fregean revolution in Logic. In: Gillies 1992, 265–306.
- Gillies, D. (1993): *Philosophy of science in the Twentieth Century*. Blackwell, London.
- Gonzalez, W. (2007): The role of Experiments in the Social Sciences: The Case of Economics. In: Kuipers (ed. 2007), pp. 292–294.
- Grosholz, E. (1980): Descartes' unification of algebra and geometry. In: Gaukroger 1980, 156–168.
- Gueroult, M. (1980): The Metaphysics and Physics of Force in Descartes. In: Gaukroger, 196–230.
- Gurwitsch, A. (1967): Galilean physics in the light of Husserl's phenomenology. In: McMullin 1967, 388–401.
- Hall, R. (1967): The significance of Galileo's thought for the history of science. In: McMullin, 67–81.
- Heelan P. A. (1987): Husserl's later philosophy of natural science. *Philosophy of Science* **54**, 368–390.
- Herivel, J. (1965): *The Background to Newton's Principia*. The Clarendon Press, Oxford.
- Hill, D.K. (1988): Dissecting trajectories: Galileo's Early Experiments on Projectile Motion and the Law of Fall. *Isis* **79**, 646–668.
- Horwich, P. (ed. 1993): *World Changes. Thomas Kuhn and the Nature of Science*. Cambridge, Mass: The MIT Press.
- Husserl, E. (1954): *The Crisis of European Sciences and Transcendental Phenomenology*. Evanston, Northwestern University Press, 1970.
- Hutter, K. (ed. 1989): *Die Anfänge der Mechanik, Newtons Principia gedeutet aus ihrer*

- Zeit und ihrer Wirkung*. Springer Verlag, Berlin.
- Kockelmans, J. J. and Kisiel, T. J. (eds. 1970): *Phenomenology and the Natural Sciences*. Evanston.
- Koyré, A. (1939): *Galileo Studies*. The Harvest Press, Hassocks 1978.
- Koyré, A. (1957): *From the Closed World to the Infinite Universe*. The John Hopkins UP, Baltimore.
- Kuhn, T. S. (1962): *The Structure of Scientific Revolutions*. University of Chicago Press, Chicago.
- Kuhn, T. S. (1974): Second Thoughts on Paradigms. In: Kuhn 1977, 293–319.
- Kuhn, T. S. (1977): *The Essential Tension*. Chicago: The University of Chicago Press.
- Kuhn, T. S. (1993): Afterwords. In: Horwich 1993, 311–341.
- Kuhn, T. S. (2000): *The Road since Structure*, University of Chicago Press, Chicago.
- Kuipers, T. (ed. 2007): *Handbook of the Philosophy of Science: General Philosophy of Science – Focal Issues*. Elsevier, Amsterdam.
- Mach, E. (1883): *The Science of Mechanics*. Chicago: The Open Court 1902.
- Machamer, P. (ed. 1998): *The Cambridge Companion to Galileo*, New York.
- Marion, J. (1992): Cartesian metaphysics and the role of the simple natures. In: Cottingham, 115–139.
- Masterman, M. (1970): The Nature of a paradigm. In: I. Lakatos and A. Musgrave (eds.), *Criticism and the Growth of Knowledge*. Cambridge: Cambridge University Press, 59–89.
- McMullin, E. (ed. 1967): *Galileo, Man of Science*. New York.
- McMullin, E. (1993): Rationality and Paradigm Change in Science. In: Horwich 1993, 55–78.
- Mehrtens, H. (1976): T. S. Kuhn's theories and mathematics: a discussion paper on the 'new historiography' of mathematics. In: Gillies 1992, 21–41.
- Mormann, T. (1991): Husserl's philosophy of science and the semantic approach. *Philosophy of Science* **58**, 61–83.
- Naylor, R. H. (1980): Galileo's Theory of Projectile Motion. *Isis* **71**, 550–570.
- Naylor, R. H. (1990): Galileo's Method of Analysis and Synthesis. *Isis* **81**, 695–707.
- Newton, I. (1673): De gravitatione. In: Newton 1962, 89–121.
- Newton, I. (1684): De Motu Corporum in Gyrum. In: Herivel 1965, 257–303.
- Newton, I. (1687): *The Principia*, A New Translation by I. B. Cohen and A. Whitman, University of California Press, Berkeley 1999.
- Newton, I. (1962): *Unpublished scientific papers of Isaac Newton*. A. Hall and M. Hall (eds.), Cambridge University Press, Cambridge.

- Nickles, T. (ed. 2002): *Thomas Kuhn*. Cambridge: Cambridge University Press.
- Olby, R. C. et al. (eds. 1990): *Companion to the History of Modern Science*. Routledge, London.
- Ronchi, V. (1967): The influence of the early development of optics on science and philosophy. In: McMullin 1967, 195–206.
- Settle, T. (1967): Galileo's use of experiment as a tool of investigation. In: McMullin 1967, 315–337.
- Shea, W. R. (1998): Galileo's Copernicanism: The science and the rhetoric. In: Machamer 1998, 211–243.
- Soffer, G. (1990): Phenomenology and scientific realism: Husserl's critique of Galileo. *Review of Metaphysics* **44**, 67–94.
- Swerdlow, N. (1998): Galileo's discoveries with the telescope and their evidence for the Copernican theory. In: Machamer 1998, 244–270.
- Szabó, I (1977): *Geschichte der mechanischen Prinzipien und Anwendungen*. Birkhäuser, Baseel 1996.
- Wallace, W. A. (1984): *Galileo and His Sources: The Heritage of the Collegio Romano*. Princeton.
- Westfall, R. S. (1971): *Force in Newton's Physics*. Macdonald, London.
- Wisn, W. L. (1984): Galileo and the Process of Scientific Creation. *Isis* **75**, 269–286.
- Wittgenstein, L. (1921): *Tractatus Logico-philosophicus*. Suhrkamp, Frankfurt 1989.