Social Security Reform in a Dynastic Life-Cycle Model with Endogenous Fertility

Radim Boháček and Volha Belush*
Economics Institute, Prague, Czech Republic

September 22, 2008

Abstract

This paper studies the effects of a fully funded social security reform with endogenous fertility in a detailed, general equilibrium life-cycle model with dynasties whose members differ in skills and life uncertainty. We find that as high skill households tend to save relatively more in assets than in children, models with exogenous fertility underestimate the aggregate capital stock in the PAYG steady state. These models also predict that the capital stock increases after the fully funded reform. However, because the high skill households respond to the reform by having more children and investing less in assets and intergenerational transfers, the average fertility increases and the aggregate capital stock falls. The welfare gains from the elimination of social security seem to more than compensate the agents for the lost insurance against life-span and earnings risks.

J13 Fertility; H55 Social Security and Public Pensions; E62 Fiscal Policy; Public Expenditures, Investment, and Finance; Taxation

*We are grateful to the Czech Academy of Sciences (GAAV IAA700850702) and CERGE-EI Foundation and GDN (RRC VII-55) for financial support. We received valuable suggestions from Albert Marcet, Evi Papa, Carlos Perez-Verdia, Hugo Rodriguez, Dimitrios Christelis, Tullio Jappelli, Marco Pagnozzi, Michal Myck, and seminar participants at Instituto de Análisis Económico (CSIC) at UAB, CSEF in Naples, University of Edinburgh, the SED Meetings, and the Spring Meetings of Young Economists in 2008. We are especially grateful to Luisa Fuster for providing us the parameters from her model. Serhiy Kozak provided an excellent research assistance. Address: Economics Institute, Academy of Sciences of the Czech Republic, Politickych veznu 7, 111 21 Prague 1, Czech Republic. Email: radim.bohacek@cerge-ei.cz and volha.belush@cerge-ei.cz.

1 Introduction

This paper studies the effects of a fully funded social security reform on welfare, efficiency and inequality in a dynastic, life-cycle general equilibrium model with endogenously imposed fertility decisions.

When studying a social security reform, there are several important assumptions featured in the models that are crucial and give contradictory results to the analysis. One stream of the literature is represented by the pure life-cycle models with heterogenous agents and exogenous fertility: Conesa and Krueger (1999), De Nardi et al. (1999), or Imrohoroglu et al. (1999), among others. These papers find that an elimination of social security brings large welfare, aggregate and distributional effects. They report important general equilibrium effects coming from a huge, around 30% increase of the capital stock in the fully funded steady state. In these models, agents are generally better off in the new steady state but the cost of a transition could be prohibitively high.[1]

Many papers, however, have explored a relationship between fertility rate and the social security size. In such models the motivation for having children is 'parental altruism' or 'old-age security'. In the first one, utility of children is a part of parents' utility (see the seminal paper by Barro and Becker (1989)). In the 'old age security' models (also the 'children as investment' in Boldrin and Jones (2002) or Nishimura and Zhang (1992)), consumption of parents is a part of children's utility. These two approaches have different qualitative and quantitative implications. Boldrin et al. (2005) show that 'old age security' models fit the empirical evidence much better than the ones of 'parental altruism'. That is, they explain about 50% of observed fertility decrease together with an increase in social security system across countries and over time. In their computational experiment, an increase in social security from 0 to 10% of GDP leads to an increase in the capital-output ratio from 2.2 to 2.4, a fall of consumption by 3%, and a reduction of TFR from 1.15 to 0.9.[2] Thus, to study possible reform of social security it is important to endogenize fertility. We assume, however, the more realistic assumption and the motivation for fertility in our model comes from a combination of these two approaches.

Our paper builds on the last important contribution to the social security literature, the dynastic models with two-sided altruism but with exogenous fertility based on Fuster (1999), Fuster, Imrohoroglu, and Imrohoroglu (2003) and Fuster, Imrohoroglu, and Imrohoroglu (2007). These papers emphasize an importance of intergenerational transfers within a family when studying social security reform and their results are contradictory to the majority of the literature on this issue. Fuster et al. (2007) analyze a possible social security reforms with endogenous labor supply and some of their proposals gain a majority support. In Fuster et al. (2003) labor supply decisions are exogenous and a

[1] In a related paper Conesa and Garriga (2008) propose to eliminate these transition costs by issuing bonds that will be repaid from future efficiency gains.

[2] Ehrlich and Kim (2005) in a model with altruistic parents find that an increase of social security from 0% to 10% decreases fertility by 0.1 children per woman. To obtain a simultaneous growth in per capita income and a fall in fertility, Fernandez-Villaverde (2001) studies a model with "capital-specific technological change and capital-skill complementarity".

majority of households is worth off with fully funded system. In both papers intergenerational transfers play a crucial role. In the first one, they smooth welfare losses from the reform. In the second, they influent a saving rate such that it does not change much in the steady state with fully funded system. Consequently, increase of the steady state capital stock with the reform is much smaller as compared to the non-altruistic pure life cycle models.

Our goal is to evaluate these results in a detailed, dynastic life-cycle model where fertility is endogenous. As in Fuster et al. (2003), we present a general equilibrium, overlapping generations model with two-sided altruism among individuals whose differences in skills (education) and life-time expectancy lead to heterogeneity in income, wealth, and therefore, fertility. Old age security and parental altruism, together with the social security system, are the major forces behind fertility decisions. In the PAYG system, the old age support parents receive is independent of the number of children they have. The fully funded system internalizes the fertility decision: parents finance their retirement consumption from savings or from the old age support provided by their own children. Thus children are perceived as an alternative investment good, costly in terms of time and goods. Finally, the fully funded reform eliminates the social security tax which is distortive and costly for borrowing constrained agents, possibly enabling them to have more children.

Fertility choice in a dynastic model requires two theoretical contributions: first, we adapt the transformation in Alvarez (1999) to individual dynastic households composed of three overlapping generations. Second, contrary to models with exogenous fertility, dynasties that die off cannot be replaced by artificially created new families. Rather, it is the fertility choice of other households that more than replaces the deceased dynasties and leads to a constant population growth in the steady state.

We calibrate our benchmark model to the U.S. data. As Fuster et al. (2003), we assume an exogenous labor supply, abstract from individual earnings uncertainty over the life cycle, and limit attention to steady states. We find that the effects of endogenous fertility are large and important in their direction. First, in the PAYG system, low skill (low education) agents invest relatively more in terms of children while the high skill (high education) agents relatively more in terms of assets and intergenerational transfers. These savings-fertility differences lead to a 20% higher aggregate capital stock than in an otherwise identical PAYG steady state but with exogenous fertility. Second, we find that a fully funded social security reform increases fertility by 10.3% and decreases the capital stock by 8.3%. This is because high skill individuals shift from investment in capital to investment in children (the reform reduces fertility differences across household types). Finally, as in data, the PAYG system increases the capital-output ratio.

Assumptions on agents' heterogeneity (survival probabilities and skill differences) are quantitatively important: fertility and allocation responses by different types of households significantly affect aggregate levels and equilibrium prices. To isolate these effects, we simulate four alternative cases that differ in their assumptions on survival and income uncertainty. We find that children are used relatively more for insurance against survival uncertainty while assets are used for insurance against skill risk in future generations. It seems that the PAYG system provides the high skill households with means to insure against the latter risk. Namely, their bequests are much higher than in the fully funded system, contributing to greater wealth inequality.

We also find that all newborn household and the majority of the population are better off in the fully funded steady state. Unfortunately, the complexity of the model does not allow us to simulate a transition between the two steady states. In other papers with exogenous fertility, agents usually prefer the new steady state but the transition to reach it is too costly. The main reason is that agents need to accumulate capital during initial stages of the transition (see Conesa and Krueger (1999)). However, in our endogenous fertility model, the capital stock decreases in the fully funded steady state. This deaccumulation of capital could provide for additional consumption to households who would suffer from the transition in models with exogenous fertility.

These results indicate that models with exogenous fertility contain two errors: First, they undervalue the capital stock in the PAYG steady state by forcing the high skill agents to invest in children as much as the low skill agents do. Second, these models predict the opposite direction of changes (with a huge magnitude) in the capital stock after the fully funded reform. These errors might lead to misleading conclusions about behavior of different groups of population, aggregate outcomes, welfare gains, transition dynamics and political support for the social security reform.

The next Section describes the modeling issues important for endogenous fertility. Section 3 develops the main model and equilibrium. In Section 4 presents the calibration of the benchmark model. Numerical results are shown in Section 5. Section 6 provides discusses extensions and directions for future work. The Appendix contains some analytical results.

2 Modeling Endogenous Fertility

This section develops a life-cycle model with endogenous fertility in the dynastic framework. For a simple exposition of modeling issues, we present a reduced one-period version of the dynastic model with individual households as in Fuster et al. (2003).

2.1 A Simple Dynastic Model with Exogenous Fertility

The decision unit is a household, composed of one father $f = 1$ and a fixed number of sons $\bar{s}$. In this model of two-sided altruism, two generations pool resources and maximize the same utility from a consumption per household member, $u(c/(f + \bar{s}))$. We abstract here from life uncertainty.

The father is retired without any income. His sons of the next generation work for a wage w at a stochastic productivity shock z. Effectively, the sons provide a transfer to the father as an old age security support. At the end of the period, the father dies and an exogenous number of children $\bar{n} = \bar{s}$ is born to each son. In the following period, the sons establish $\bar{s}$ new households in which each son becomes the single father with his own $\bar{n}$ sons. Savings a' is divided equally between the sons.

Households are heterogeneous in their assets, a, and skills, z. The Bellman equation for a household with a state $(a, f = 1, \bar{s}, z)$ is

$$v(a, \bar{s}, z) = \max_{c,a'} \left\{ u\left(\frac{c}{1+\bar{s}}\right) + \bar{s}^{\eta}\beta E[v(a', \bar{n}, z')|z] \right\},$$

subject to a budget constraint,

$$c + \gamma\bar{s}\bar{n} + \bar{s}a' \leq (1+r)a + \bar{s}wz,$$

where a is assets, r is the interest rate, and γ is a cost of raising children.[3] Altruism in the sense of Barro and Becker (1989) is represented by a parameter η. The skill of sons in each household is partially correlated with the father's skill through a Markov process.

The important point is that the discounted present value on the right hand side of the Bellman equation is, like in the Barro-Becker formulation, multiplied by $\bar{s}^{\eta}$. This multiplication incorporates the present discounted value of the new $\bar{s}$ households into the dynastic value function. In this way the value function covers all households that have belonged, belong, and will belong to the dynasty. Note that while these new households have the same amount of assets and composition, they might differ in their realized household idiosyncratic shock z'.

2.2 A Simple Dynastic Model with Endogenous Fertility

However, it is not possible to simply multiply the future value by s^{η} when fertility is a choice. The dynamic programming problem would not be well defined. Alvarez (1999) suggests the following transformation,

$$V(a, s, z) = \max_{c,a',n\geq 0} \left\{ u\left(\frac{c}{1+s}\right)(1+s)^{\eta} + \beta E[V(a', n, z')|z] \right\}, \tag{1}$$

subject to the same budget constraint.

This transformation does not work when the decision-making unit is an individual household rather than the economy-wide dynasty: As the future value is not multiplied by s^{η}, the dynasty does not take into account its division into the s new households. All but one of the newly established households headed by former sons, who inherit the same amount of assets, are not valued. For example, imagine two households: one with one son and the other with five sons. Assume that they consume the same amount per household member, both have the same number of children per son, n, and the same savings per son, a'. Equation (1) would correctly value only the first household. The other one with five sons would be substantially undervalued.

In order to study behavior and allocations of individual heterogeneous households, we need to incorporate the splitting households back into the model up to a limit: If all these households were fully incorporated, the dimensionality of the state space would grow geometrically. The goal is to find an abstraction where 1) the value function and

[3] In their model of exogenous fertility, Fuster, Imrohoroglu, and Imrohoroglu (2003) do not have a cost of children ($\gamma = 0$). Their altruism parameter $\eta = 1$.

the budget constraint remain related to a single household; and at the same time 2) the number of relatives towards which a household feels altruism stays manageable and realistic.

For the latter condition we choose to only keep track of the newly established (or separated) households. These households are headed by the new fathers who are brothers. Their number is b, equal to the number of sons s in the previous period. Being identical, these brothers start their own families with the same bequest a' and the same number of sons s. On the other hand, their children might draw different skills.

Thus the value of altruism is based on the fact that the head of the modeled household comes from a family that had b sons. Their number only multiplies the utility from consumption of the household members applying the same parameter of altruism η. Importantly, none of these separated sons enters the other households' period utilities nor their budget constraints.

INSERT FIGURE 1 ABOUT HERE

A timeline for a household is in Figure 1. The state of an individual household is (a, b, s, z), where b is the number of sons in the previous period. The value function is

$$V(a, b, s, z) = \max_{c,a',n\geq 0} \left\{ u\left(\frac{c}{1+s}\right)(1+s)^{\eta} b^{\eta} + \beta E[V(a', s, n, z')|z] \right\},$$

subject to the same budget constraint,

$$c + \gamma sn + sa' \leq (1+r)a + swz.$$

In other words, the economic unit is a single household. The existence of living, direct relatives has a positive externality. This externality is lost when the head of a household (father) dies. Everything else kept constant, large families have higher utility than smaller ones.

3 The Economy

This section describes the full overlapping generations model with endogenous fertility based on Fuster et al. (2003). The economy is populated by $2T$ overlapping generations. Each household consists of a father, $f \in F = \{0, 1\}$, sons $s \in S = \{0, 1, 2, 3, ...\}$, and children each son decides to have, $n \in S$. For dynastic reasons discussed above, each household also values the fact that it comes from a family with $b \in S$ sons in the previous period. We will abbreviate the household composition as $h = (f, b, s, n)$. Because the full model has uncertain lifetimes, zeros indicate persons that are not alive. We assume that children share the mortality of sons and that when the father dies, the connection to his brothers is lost and the household ceases to value other households of the dynasty. All decisions are jointly taken by the $m = f + s$ adult members in a household.

The model period is 5 years. A household lasts $2T$ periods or until all its members have died. A timeline for a household is shown in Figure 2. The timing is related to the age of sons, who are 20 in period $j = 1$ when the father is 55 (model age $j = T + 1$). The father retires at age 65 in period j_R and lives at most to age 90 which corresponds to model age $j = 2T$.

INSERT FIGURE 2 ABOUT HERE

In a life-cycle model with endogenous fertility we have to allow the sons to choose the number of children in the appropriate period of the life cycle. If the sons survive to period $j_N - 1$ (age 30), they choose a number of children born in the following period j_N. Children live in the same household in periods $j = j_N, \ldots, j_T$. After period T, the sons form s new households in which each of them becomes the single father in period $T+1$. Each son takes his n children to his new household. Therefore, conditional on survival, during the first $j_N - 1$ periods an individual's life overlaps with the life of the parent and during the remaining $j = j_N, \ldots, 2T$ periods also with the lives of his own children. Fertility decisions of all households imply an endogenous population growth rate $\bar{n}$ that an individual household takes as given.

Households are different in their assets, skills, age and composition. The skill is revealed to each son in period $j = 1$ when he is aged 20 and enters the labor market. The skill is correlated with that of his father: it can be high, H, or low, L, following a first-order Markov process

$$Q(z, z') = Prob(z' = j \mid z = i) \qquad i, j \in \{H, L\},$$

where z' and z are the labor abilities of the sons and their father, respectively. In other words, within a household, all offsprings have the same skill which might be different from that of the parent.

The skill is fixed for the whole life and determines an efficiency profile and a life-expectancy of an individual, $\{\varepsilon_j(z)\}_{j=1}^{2T}$ and $\psi_j(z)$, respectively. $\psi_j(z)$ is the probability that an individual with an ability z who is alive in period j will survive to period $j+1$ and $j = 1, \ldots, 2T$. We impose that at the terminal age $\psi_{2T}(z) = 0$. If all household members die, this branch of the dynasty disappears and their assets are distributed to all living adult persons in the economy by the government as lump sum accidental bequests.

3.1 Preferences

As in Jones and Schoonbroodt (2007), we assume that parents care about three separate objects: own consumption in the period, the number of children, and the utility of children. In particular, 1) parents like the consumption good (utility is increasing and concave in own consumption); 2) parents are altruistic (holding fixed the number of children and increasing their future utility increases (strictly) the utility of the parent); 3) parents like having children (holding children's utility fixed and increasing their number increases (strictly) the utility of the parent); and finally, 4) the increase described in 3) is subject to diminishing returns.

We assume that as only the adult members of a household have the decision power, the household jointly maximizes utility from a per-adult consumption,

$$U(c, h) = u\left(\frac{c}{f+s}\right)(f+s)^{\eta} b^{\eta},$$

where η is a parameter of altruism as in Barro and Becker (1989). The last term, b^η, incorporates the number of sons in the previous period.[4]

If the father is not alive, the link to other households in the dynasty is broken and the utility of the household with a state $h = (0, 0, s, n)$ is

$$U(c, h) = u\left(\frac{c}{s}\right) s^\eta.$$

The function u is a standard CES utility function, $u(\tilde{c}) = \frac{\tilde{c}^{1-\sigma}}{1-\sigma}$, for a per-adult consumption $\tilde{c}$. The preference parameters must satisfy the monotonicity and concavity requirements for optimization. We follow the standard assumption in the fertility literature for the Barro-Becker framework, where children and their utility are complements in the utility of parents (see also Lucas (2002)). Therefore, $u(c) \geq 0$ for all $c \geq 0$, u is strictly increasing and strictly concave and $0 < \eta < 1$. This implies $0 \leq \eta + \sigma - 1 < 1$ and $0 < 1 - \sigma < 1$. Jones and Schoonbroodt (2007) analyze these properties in detail.[5]

In terms of Boldrin and Jones (2002), this model exhibits a cooperative care for parents by all siblings. This model would be classified as an A-efficiency model by Golosov, Jones, and Tertilt (2007), where additional child's value is equivalent to the additional utility this child brings to the parents, siblings or other relatives.

3.2 Production

The aggregate technology is represented by the standard Cobb-Douglas production function,

$$F(K_t, L_t) = K_t^\alpha (A_t L_t)^{1-\alpha},$$

where K_t and L_t represent aggregate capital stock and labor (in efficiency units) in period t. The technology parameter A_t grows at an exogenous rate $g > 0$. $\delta \in (0, 1)$ is a constant rate of capital depreciation. Firms maximize their profits at competitive prices r_t and w_t, respectively.

3.3 Government

The government in the economy finances its consumption G by levying taxes on labor income τ_l, capital income τ_k, and consumption τ_c. Social security benefits B are financed by a tax on labor income τ_{ss}. Finally, the government administers the redistribution of accidental bequests. All these activities are specified in detail below.

[4] For simplicity, we apply the same parameter of altruism. Observe that the brothers might not be actually alive because they draw their survival shocks independently.

[5] The other combination of parameters has different implications for quantitative properties of the model. Children and their utility are substitutes if $u(c) \leq 0$ for all $c \geq 0$, u is strictly increasing and strictly concave and $\eta < 0$. Comparison of these two parameterizations will be a subject of our future work.

3.4 Budget Constraint

Households are heterogeneous regarding their assets holdings, age, abilities, and composition. Denote (a, h, z, z') as the individual state of an age-j household, where a represents assets, $h = (f, b, s, n)$ is the household's composition, and (z, z') are the father's and sons' skills, respectively.

The budget constraint of a household with $m = f + s$ adult members is

$$(1 + \tau_c)(c + \gamma_j^g(h; z, z')) + (1 + g)a' = [1 + r(1 - \tau_k)]a + e_j(h; z, z') + m\xi, \tag{2}$$

where c is the total household consumption, a' is savings of the whole household, ξ is the lump-sum transfer of accidental bequests, τ_c and τ_k are the consumption and capital tax rates, respectively. In the calibration section we explain in detail the expenditures on children, $\gamma_j^g(h; z, z')$, a function of household's income and number of children n in period j.

The after tax earnings of the adult members is given by

$$e_j(h; z, z') = \begin{cases} fB_{j+T}(z) + s(1 - \gamma_j^w(n))\varepsilon_j(z')(1 - \tau_{ss} - \tau_l)w & \text{if } j \geq j_R - T, \\ [f\varepsilon_{j+T}(z) + s(1 - \gamma_j^w(n))\varepsilon_j(z')](1 - \tau_{ss} - \tau_l)w & \text{otherwise,} \end{cases} \tag{3}$$

where $\gamma_j^w(n)$ represents a fraction of sons' working time devoted to n children in period j, τ_{ss} and τ_l are the social security and labor income tax rates, respectively. $B_{j+T}(z)$ are social security benefits, which depend on father's average life-time earnings and wage in the retirement period.[6]

In all optimization problems below we impose a no-borrowing constraint, $a' \geq 0$.

3.5 Value Function at Age $j = 1, 2, ..., j_N - 2$

Let $V_j(a, h, z, z')$ be a value function of an age-j household with a assets, h members, and (z, z') skills. In periods $j = 1, 2, ..., j_N - 2$ there are no children yet so $h = (f, b, s, 0)$. The maximization problem is

$$V_j(a, h, z, z') = \max_{c, a'} \left\{ U(c, h) + \beta(1 + g)^{1-\sigma} \widetilde{V}_{j+1}(a', h', z, z') \right\},$$

subject to the budget constraint (2) and the after-tax earnings defined in (3).

The transition process for the value function is, due to life uncertainty,

$$\widetilde{V}_{j+1}(a', h', z, z') = \begin{cases} \psi_{j+T}(z)\psi_j(z')V_{j+1}(a', (1, b, s, n'), z, z') \\ \quad + \psi_{j+T}(z)(1 - \psi_j(z'))V_{j+1}(a', (1, b, 0, 0), z, z') \\ \quad + (1 - \psi_{j+T}(z))\psi_j(z')V_{j+1}(a', (0, 0, s, n'), z, z') \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{if } f = 1, s > 0, \\ \psi_{j+T}(z)V_{j+1}(a', (1, b, 0, 0), z, z') \qquad \text{if } f = 1, s = 0, \\ \psi_j(z')V_{j+1}(a', (0, 0, s, n'), z, z') \qquad \text{if } f = 0, s > 0, \end{cases} \tag{4}$$

While this transition is specified for all possible ages $j < T$, with no children in periods $j = 1, 2, ..., j_N - 2$, we impose $n' = n = 0$. Note that sons share their survival uncertainty with their children and that the household stops remembering other relatives b when the father dies. Finally, if all members of the household die this branch of the dynasty disappears.

[6] For detailed definition see Fuster et al. (2003) and the calibration section.

3.6 Value Function at Age $j_N - 1$

At the age $j_N - 1$, each son chooses the number of children that will be born in the following period. Being identical, all sons choose the same number of children, $n' \geq 0$,

$$V_{j_N-1}(a, h, z, z') = \max_{c,a',n' \geq 0} \left\{ U(c, h) + \beta(1 + g)^{1-\sigma} \widetilde{V}_{j_N}(a', h', z, z') \right\}.$$

While the budget constraint (2) and after-tax earnings (3) are unchanged, the transition for the value function (4) now has $n' \geq 0$.

3.7 Value Function at Age $j = j_N, \ldots, T - 1$

Children are born in period j_N and become a state variable in $h = (f, b, s, n)$ in periods $j_N, \ldots, T - 1$. The value function is,

$$V_j(a, h, z, z') = \max_{c,a'} \left\{ U(c, h) + \beta(1 + g)^{1-\sigma} \widetilde{V}_{j+1}(a', h', z, z') \right\},$$

subject to (2), (3), and (4). The number of children per son in the next period is $n' = n$ or $n' = 0$ if the sons die. Having children is costly in terms of goods, $\gamma_j^g(h; z, z')$, and working time, $\gamma_j^w(n)$.

3.8 Value Function at Age $j = T$

At the end of period T, a household transforms itself into s new households of the next dynastic generation in period $j = 1$. The father reaches the end of his life, each of the s sons becomes a single father and the children become n sons in each of the s newly established households (conditional on survival). Therefore, at $j = T$, the value function of a household $h = (f, b, s, n)$ with assets a and skills (z, z'), is

$$V_T(a, h, z, z') = \max_{c,a'} \left\{ U(c, h) + \beta(1 + g)^{1-\sigma} \sum_{z''} \psi_T(z') V_1(a', h', z', z'') Q(z', z'') \right\},$$

subject to a special period-T budget constraint,

$$(1 + \tau_c)(c + \gamma_T^g(h; z, z')) + (1 + g)sa' = [1 + r(1 - \tau_k)]a + e_T(h; z, z') + m\xi,$$

the after-tax earnings (3), and a transformation to s new households with a composition

$$h' = (1, s, n, 0),$$

provided that the sons survive to form their own households. Otherwise, this branch of the dynasty dies off.

Note that the sons equally divide the household's assets. The skill of sons in each of the new households is z'', correlated with the ability of the father, z'. Finally, there are no children in period $j = 1$ so $n' = 0$.

3.9 Stationary Recursive Competitive Equilibrium

Let $x = (a, f, b, s, n, z, z') \in X = (A \times F \times S \times S \times S \times Z \times Z)$ is an individual household's state. Denote $\{\lambda_j\}_{j=1}^{T}$ as an age-dependent measures of households over x. Its law of motion for each $(a', f', b', s', n', z, z') \in X$ in periods $j = 1, ..., T-1$, is

$$\lambda_{j+1}(a', f', b', s', n', z, z') = \sum_{\{x:\, a'=a_j(x), n'=n_j(x)\}} \Psi_j(f', s'; f, s)\, \lambda_j(x),$$

where $\Psi_j(f', s'; f, s)$ is the probability that age j household with the household members f and s will consist of f' and s' members next period. The number of sons from the last period is remembered $b' = b$ if the father survives and zero otherwise.[7] For a simpler exposition, $n_j(x) = 0$ for $j < j_{N-1}$ and $n_j(x) = n$ for $j \geq j_N$.

The law of motion for the measure of age $j = 1$ households is

$$\lambda_1(a', f', b', s', 0, z', z'') = \sum_{\left\{x: a'_j = a'_T(x)\right\}} s\, \Psi_T(f', s'; f, s)\, Q(z', z'')\, \lambda_T(x),$$

must be adjusted for the newly formed sons' households. Now $b' = s$ if the sons survive and zero otherwise.[8]

Importantly, dynasties whose members die disappear from the economy. They are not artificially replaced by new households with zero assets and some arbitrary composition. In an equilibrium with endogenous fertility, new households established by the sons are so many that they not only replace the deceased dynasties but also deliver the desired population growth. Therefore, in the following definition there is no condition on new dynasties.

Definition 1 *Given fiscal policies $(G, B, \tau_l, \tau_k, \tau_c, \tau_{ss})$, a stationary recursive competitive equilibrium is a set of value functions $\{V_j(\cdot)\}_{j=1}^{T}$, policy functions $\{c_j(\cdot), a'_j(\cdot)\}_{j=1}^{T}$ and $n'_{j_N-1}(\cdot)$, factor prices (w, r), aggregate levels (K, L, C), lump-sum distribution of accidental bequests ξ, cost of children (γ_j^g, γ_j^w), measures $\{\lambda_j\}_{j=1}^{T}$, and a population growth rate $\overline{n}$, such that:*

[7] The transition probability matrix $\Psi_j(f', s'; f, s)$ for periods $j = 1, ..., T-1$ is

	$f' = 0, s > 0$	$f' = 1, s' = 0$	$f' = 1, s' > 0$	$f' = 0, s' = 0$
$f = 0, s > 0$	$\psi_j(z')$	0	0	$1 - \psi_j(z')$
$f = 1, s = 0$	0	$\psi_{j+T}(z)$	0	$1 - \psi_{j+T}(z)$
$f = 1, s > 0$	$(1 - \psi_{j+T}(z))\psi_j(z')$	$\psi_{j+T}(z)(1 - \psi_j(z'))$	$\psi_{j+T}(z)\psi_j(z')$	$(1 - \psi_{j+T}(z))(1 - \psi_j(z'))$
$f = 0, s = 0$	0	0	0	1

[8] For $j = T$ the probability matrix, $\Psi_T(f', s'; f, s)$, is

	$f' = 0, s' = 0$	$f' = 1, s' = n$
$s = 0, n = 0$	1	0
$s > 0, n \geq 0$	$1 - \psi_T(z')$	$\psi_T(z')$

1. *given fiscal policies, prices and lump-sum transfers, the policy functions solve each household's optimization problem;*

2. *the prices (w, r) satisfy*

$$r = F_K(K, L) - \delta \text{ and } w = F_L(K, L);$$

3. *markets clear:*

$$\begin{aligned} K &= \sum_{j,x} a\,\lambda_j(x)(1+\overline{n})^{1-j}, \\ L &= \sum_{j,x} [f\,\varepsilon_{j+T}(z) + s(1-\gamma_j^w(n))\,\varepsilon_j(z')]\,\lambda_j(x)(1+\overline{n})^{1-j}, \\ C &= \sum_{j,x} c_j(x)\,\lambda_j(x)(1+\overline{n})^{1-j}; \end{aligned}$$

4. *the measures $\{\lambda_j\}_{j=1}^{T}$ grow at the constant population growth rate, $\overline{n}$;*

5. *the lump-sum distribution of accidental bequests satisfies*

$$\xi = (1+r)\sum_{j,x} a'_j(x)\Psi_j(0,0;f,s)\,\lambda_j(x)(1+\overline{n})^{1-j};$$

6. *the government's budget is balanced*

$$G = \tau_k\, r\Big(K - \frac{\xi}{1+r}\Big) + \tau_l\, wL + \tau_c(C + C^g),$$

   where C^g is the aggregate cost of children in terms of goods;

7. *the social security budget is balanced*

$$\sum_{j=j_R}^{2T}\sum_{x} f\,B_j(z)\,\lambda_j(x)(1+\overline{n})^{1-j} = \tau_{ss} wL;$$

8. *and the aggregate feasibility constraint holds,*

$$C + C^g + (1+\overline{n})(1+g)K + G = F(K,L) + (1-\delta)K.$$

4 Calibration of the Benchmark Economy

In order to obtain results comparable to those in the previous papers on social security reforms, we use the same parameters as Boldrin and Jones (2002) and Fuster et al. (2003). In particular, we set the intertemporal elasticity of substitution to $\sigma = 0.95$ (as in Boldrin and Jones (2002)) and the annual discount factor $\beta = 0.988$ (as in Fuster et al. (2003)). We find that a parameter of altruism $\eta = 0.055$ leads to the same population growth in the steady state of the benchmark economy with a replacement rate $\theta = 0.44$ as in the U.S. data ($\bar{n} = 0.012$). These parameter values also satisfy the optimization restrictions (see Jones and Schoonbroodt (2007) for details). The resulting capital-output ratio is 2.83, similar to that in Boldrin et al. (2005). All parameters are presented in Table 1.

INSERT TABLE 1 ABOUT HERE

The production function is Cobb-Douglas with a capital share $\alpha = 0.34$ and an annual depreciation rate $\delta = 0.044$ as in Fuster et al. (2003).

The demographic structure is the same as in Fuster et al. (2003), sho follow Elo and Preston (1996) and set the survival probabilities ψ such that the life expectancy at real age 20 is five years longer for high skill individuals than that of low skill.

4.1 Earnings, Social Security and Taxation

The efficiency profiles for low and high skills as well as their transition probabilities are the same as in Fuster et al. (2003). In their model with exogenous fertility, the proportion of high skill agents equals the share of college graduates (28%) and the correlation between father's and sons' wages is the same as in the data (0.4).

Retirement benefits in the benchmark economy with a replacement rate 44% of the average earnings are calibrated according to Fuster et al. (2003). The marginal replacement rate equals 20% for earnings below the average, 33% for earnings above the average and below 125%, and 15% for earnings above 125% and below 246% of the average earnings. The benefits are further adjusted for low and high skill individuals. The social security tax $\tau_{ss} = 0.115\%$ clears the social security budget at 7.6% of GDP. In the steady state of the fully funded economy the replacement rate is set to zero.

The fiscal parameters are standard, taking values of 35% for the capital income tax and 5.5% for the consumption tax. The labor income tax clears the government budget constraint. Government consumption is set at 22.5% of the total output. As the latter does not change much across all steady states we model, tax on labor income τ_l around 0.16 clears the government budget constraint in all these steady states.

Fuster et al. (2003) find important differences depending on whether the FF reform is neutral with respect to government consuming the same *percentage* of GDP or the same *amount* of real goods. In our model with endogenous fertility, it turns out that when the government in the FF reform consumes the same percentage of GDP it also consumer almost the same amount of goods as in the PAYG steady state (the outputs in both steady states are very close). Thus in our paper the comparison of these two scenarios of government consumption neutrality is redundant.

4.2 Cost of Children

Table 3-6 in the *Report on the American Workforce* by the U.S. Department of Labor (1999) are "Average combined weekly hours at work and average combined annual hours at work for married couples by presence and age of youngest child". In 1997, the combined annual (weekly) hours were 3,686.6 (74.8) for couples with no children under age 18, 3,442.7 (70.4) with children aged 6 to 17, 3,545.0 (72.2) with children aged 3 to 5, and 3,316.5 (68.3) with children under 3 years. The labor force participation increases with time elapsed since the last birth, age of mother, education, and annual family income. It decreases only with the number of children. Table 2 presents these time costs, γ^w, adapted to this model's period structure as a fraction of a son's working time. We take the weekly measure as it is close to estimates in Boldrin and Jones (2002).

INSERT TABLE 2 ABOUT HERE

The good's costs of children are from Consumer Expenditure Survey of the Bureau of Labor Statistics for 1990-1992. Estimates of the major budgetary components are for year 1998 for 12,850 husband-wife families with average before-tax income $47,900. They suggest that expenditures on the single child are 24% more in a single-child family than in a family with two children. Consequently, family with more than two children spends 23% less on a child than the one with two children exactly. Therefore, the USDA adjusts the expenditures by multiples of 1.24 and 0.77, respectively.[9] The cost in the model γ^g is shown in the last column as a fraction of the average household before-tax income.

5 Results

The pays-as-you-go social security system provides two important insurance roles. First, it partially substitutes for missing annuity markets during retirement. Second, it partially insures individuals against their permanent labor productivity shock. On the other hand, the social security tax is distortive on the consumption-savings margin and costly for borrowing constrained agents. Finally, as any social security system it affects fertility decisions and influences the timing, direction, and amount of intergenerational transfers. Because the social security benefits are independent of the number of children inside the household, the PAYG system does not internalize the fertility decision.

To understand the importance of these effects, we compare the benchmark steady state of the PAYG social security system with a replacement rate $\theta = 0.44$, calibrated to the U.S. data, to that of the fully funded system with a zero replacement rate.

We also follow Fuster et al. (2003) and present four additional cases of the PAYG and FF steady states in order to isolate individual forces in the model. In the first case, we impose certain lifetimes ($\psi = 1$) for all household types. In the second case, we keep uncertain lifetimes but impose the same survival probabilities for both high and low skill individuals, i.e., $\psi_H = \psi_L$. The third case has no skill differences ($\varepsilon_H = \varepsilon_L$) and, therefore, the same but uncertain lifetimes ($\psi_H = \psi_L$). Finally, in the fourth case, we

[9]Expenditures include housing and education. These numbers are comparable to findings of Deaton and Muellbauer (1986) and Rothbarth (1943). They find that in an average husband-wife family 25-33% of household expenditures are attributable to one child, 35-50% to two children, and 39-60% to three children. In Boldrin and Jones (2002), children cost 3% of family time (6% of mother's) and 4.5% of goods in per capita GDP. The latter was used to match their benchmark TFR and is rather low.

compute the two steady states with exogenous fertility. In all these cases we hold all parameters constant.

Appendix A analyzes the first order conditions for the intertemporal and fertility decisions.

5.1 The Benchmark PAYG Steady State and its Fully Funded Reform

Table 3 shows the results for the benchmark calibration. The aggregate allocations and prices are on the top of the table. In the middle, there are fertility, savings, welfare and demographic outcomes for different household types. At the bottom, we show the political support for the FF reform.

INSERT TABLE 3 ABOUT HERE

In the benchmark PAYG steady state, the average fertility is 1.67 children per son, matching the U.S. population growth rate 1.24%. Among the complete households, the LL type of households have the highest fertility (1.83 children per son) and the HH type the lowest (1.11). When the sons live alone, the L types have on average 1.85 children. The FF reform eliminates the social security tax ($\tau_{ss} = .115$) and the social security benefits. It internalizes the old-age support inside the households. Consequently, the overall fertility increases by 10.2 percent.[10] All households increase fertility except for the lonely L sons. The reform substantially increases fertility of households with H father (21.2% for HL and 57.3% for HH households, respectively). These households are shifting from saving in assets to saving in children.

In the benchmark PAYG steady state, the relation of fertility by household type is $LL > LH > HL > HH$. There are several reasons why the HH households have fewer children in the PAYG system: their replacement ratio is low relative to that of low skill individuals, the opportunity cost of children is relatively high, and children's future incomes are uncertain. Fertility decreases in skills (education) of the father and then of the sons. In the 2004 Population Survey, high skill individuals have higher fertility than low skill individuals.[11] In our model, the difference for L and H lonely sons in the benchmark PAYG steady state is 13.0%. Among the complete households, fertility in LH households is 9.9% lower than in LL households, and 28.8% lower in HH than in HL households. After the FF reform, these numbers decline to 2.3% (H vs. L), 5.3% (LH vs. LL), and 8.5% (HH vs. HL). The FF reform reduces fertility differences across

[10] In the Caldwell model in Boldrin and Jones (2002), the increase in fertility is 20-25%.

[11] In their survey of the U.S. demographic history, Jones and Tertilt (2006) find that fertility is declining in education and income. The Children Ever Born measure (CEB, per woman per cohort among 40-44 year old women) with a high school diploma is 1.943, and for women with a college degree 1.672. That is, high skill individuals have 13.9% lower CEB than those with a low skill (when measured by the educational attainment of husband, it is 12%). For the 1958 cohort, the average CEB is 1.80, the CEB gap between the bottom and top half of the income distribution is 0.23, and the fertility of the bottom income decile is 2.03 compared to that of the top decile, 1.58.

different household types.[12] The relation of fertility becomes $HL > LL > LH > HH$. Note that the change is brought by the jump of HL households.

The substantial jump of fertility in HH and HL households leads to an increase in the efficiency labor supply by 5.8%. These changes in labor and capital inputs offset each other for only a small 0.8% increase in output. The average consumption decreases as higher fertility implies higher cost of children.[13]

Importantly, in the FF steady state the capital stock falls by 8.3% and the capital-output ratio falls from 2.839 to 2.582. The other Caldwell-type models[14] with endogenous fertility (Boldrin and Jones (2002) and Boldrin et al. (2005)) show a similar fall in the capital-output ratio when social security is eliminated. Again, these results fit the observed capital-output pattern in data.

The reason for the reduction of the capital stock is that in our model children and assets are substitutes. This is apparent from the next part of Table 3 which shows the change in assets held by different types of households. Households with a high skill father decrease their asset holdings by around 20 percent. On the other hand, households with a low skill father slightly increase their savings (note that their welfare increases as well). The only exception is again are the lonely sons who lower their fertility as well as assets.

These changes in the capital stock are opposite to models where fertility is exogenous. Similar dynastic models by Fuster et al. (2003) and Fuster et al. (2007) report an increase of the capital stock by 6.1% and 12.1%, respectively. Pure life-cycle models of Conesa and Krueger (1999), De Nardi et al. (1999), or Auerbach and Kotlikoff (1987), all suggest an increase of around 30%. Imrohoroglu et al. (1999) report that capital stock increases by 26% and Storesletten et al. (1999) by 10% to 25%. These large changes in the capital stock are driven by the forced savings imposed on high skill households in assets rather than children as well as by the underestimated capital stock in the original PAYG steady state. Below, we will show in our fourth case that a PAYG steady state with exogenous fertility has around 20% lower capital stock than the benchmark PAYG steady state with endogenous fertility.

5.1.1 Life-Cycle Savings

In order to document the forces important for these savings-fertility decisions, Figure 3 shows the life-cycle accumulation of assets by complete households for the PAYG and FF steady states. The accumulation of wealth culminates in period $j = 4$ when children are born. In the consequent periods, the cost of children and father's retirement drive down the average wealth for all household types. Notice that a father's skill determines the shape of life-cycle savings: Households with a low skill father save less and leave

[12] Jones and Tertilt (2006) document that over the last 150 years of the U.S. demographic history, there has been not only a decrease in fertility but also a decrease in fertility inequality, especially with respect to income. The difference from the top to the bottom of the income distribution of fertility has been falling, from around 1.6 CEB with the 1863 cohort to a quarter of a child by the 1923 birth cohort, where it has stabilized.

[13] In the PAYG steady state, the cost of raising children is 29.6% of GDP with the consumption-to-output ratio 0.634. In the FF steady state, the cost is 33.0% of GDP and the ratio is 0.645.

[14] See Caldwell (1982).

lower bequests than those with a high skill father.

INSERT FIGURE 3 ABOUT HERE

In the PAYG regime and households where the father has high skills, assets are mostly transferred to sons' new households. Especially in the HH households assets are almost fully bequest: social security benefits allow these households not to dissave at the end of the life cycle. On the contrary in the FF reform, the HH and HL households use their assets for consumption in the retirement periods. While savings in households with a low skill father remains basically the same after the FF reform, savings in households with a high skill father dramatically decline after period $j = 4$. As these types also substantially increase fertility, their bequest per son is even lower.

This means that the lower capital stock in the FF steady state comes not so much from the ability to accumulate capital during the pre-retirement periods but rather from a different usage of the capital stock during retirement. The PAYG benefits allow households with a high skill father to transfer assets across generations. In the FF system, savings are used for retirement consumption and much less for bequests. Therefore, even sons of a high skill father now start their own households with a low stock of assets. Although the after-tax income is higher, this could be costly for those who draw a low ability shock.

Consequently in the FF steady state, assets are not so persistently accumulated across generations and wealth inequality decreases. Gini coefficient of wealth inequality is 0.48 in the PAYG benchmark steady state while in the FF benchmark steady state it is 0.45. De Nardi et al. (1999) and Fuster (1999) find similar decrease of Gini coefficient.

These results suggest that assets are used, if household budget constraint permits in the PAYG steady state, as an insurance tool against a low realization of skill in future generations.

5.1.2 Intergenerational Transfers

Figure 4 shows the average intergenerational transfers as a percentage of each type of household's consumption.[15]

INSERT FIGURE 4 ABOUT HERE

In the PAYG system, fathers compensate and support their sons for social security taxes the latter pay. High skilled fathers increase their transfers in periods when children are born. The FF system preserves the direction of transfers but their size is reduced. Only in the HL households the transfers are large and always positive. In all other households the sons support their father in his last period of life.[16]

[15] For a definition of how to compute the intergenerational transfers see the Appendix in Fuster et al. (2003).

[16] For an interesting survey of intergenerational transfers in Europe, see SHARE (2005).

5.1.3 Income and Old-Age Support Effects

Our general equilibrium effects are not so large as in the pure life-cycle, exogenous fertility models. In the FF reform, the lower capital stock and the elimination of the social security tax increase both after-tax interest rate and wages (the before-tax wage is lower). Consequently, the income of all household types is higher, in the range from 7% (the HL types) to 24% (the HH types). When children and the utility of children are complements in the utility of the parents, fertility should be increasing in income.

Given these predictions and responses by different household types, we try to analyze two effects present in the FF reform: First, the income effect from the elimination of the social security tax, and second, the old-age support provided by the PAYG benefits. In order to decompose these two effects, we simulate two counterfactual reforms in which the social security budget is obviously not balanced: The first reform isolates the income effect by eliminating the social security tax while preserving the PAYG benefits. In the second reform, we keep the social security tax and eliminate the benefits to isolate the role of old-age support.

It seems that each effect contributes to around 1/2 of the total fertility increase and capital decrease between the benchmark PAYG and FF steady states. However, each provides fertility incentives to different types of households: the income effect lowers the opportunity cost of children and is important for HH and H households (60% and 80% of the total fertility difference between the benchmark PAYG and FF steady states, respectively). The old-age support is stronger for LH households (71% of the difference, low skill fathers need more support from high skill sons) and the L sons (80%, who are poor and lose benefits): these types need more children for old-age transfers. Finally, both effects have the same impact on the LL and HL households.

5.1.4 Demographics

Table 3 also shows the distribution of households across skill types, together with a fraction of high skill households and the dependency ratio. Complete households with both father and sons alive constitute around 77% of all households, those with only sons alive 21% and those with only father alive 2%. Given the transition function for the intergenerational transmission of skills, the most numerous households are of the LL type with around 45%.

As the FF reform mostly increases the fertility of high skilled agents, their fraction increases by 2.3%. Simultaneously, the aggregate labor productivity grows by 5.8% also due to the increased population growth. The increase in fertility also leads to a lower dependency ratio of the retired to working population from 18.7% to 17.5% (despite the increased longevity in the population).

Thus the FF reform improves the dependency ratio as well as the productivity of the economy through a better demographic and human capital composition.

5.1.5 Welfare Gains and Political Support

The middle part of Table 3 shows the percentage of consumption in the PAYG steady state that a particular household type would have to receive in order to be as well off as in the FF steady state. If the number is positive, the FF steady state is preferred to that with the PAYG system. We compare all newly established (newborn) households of sons at age $j = 1$ who survived their separation from their father's household as well as the average welfare for all households.

The FF reform brings large welfare gains to all types of newborn households. However, when we compute the average welfare across all generations, households with a high skill father (HL and HH) are worse off. These households sacrifice high pensions while facing the increased cost of raising more children they have chosen for providing the old-age support. On the other hand, households with a low skill father (LL and LH) are always better off: they do not increase fertility much and their numerous sons bring home more income after the elimination of the social security tax. LH households are those who benefit the most: low skilled fathers lose small pensions while high skilled sons bring home higher incomes. Naturally, the lonely sons always prefer the FF system and the lonely fathers the PAYG benefits.

The bottom of Table 3 shows the political support for the reform, i.e., the percentage of households in each cohort and overall that are better off in the FF steady state. All newly established households (generation $j = 1$) as well as the majority of households are better off. On the other hand, generations $j = 3, 4, 5$ are worse off as they raise more (costly) children and support the retired father at the same time.[17] In later periods, as both sons' earnings and the likelihood of father's death increase, the majority of households again prefers the reforms.

5.1.6 Return on PAYG Social Security

Another way to understand these results is to examine the rate of return on social security in the PAYG system with a 44% replacement rate for households with both father and sons alive.[18]

INSERT TABLE 4 ABOUT HERE

In Table 4, the rate of return on social security increases with the age of father, who is collecting social security benefits for a longer time. Also, this rate is higher the lower are contributions made and the higher are the benefits received. Hence the returns on PAYG are highest for the HL households and negative for the LH households no matter how long the father can collect the benefits (in Fuster et al. (2003) the return is positive for death at age 85). Of course, these LH households benefit the most from the reform. Households where only sons are alive have negative return on social security. Also, the return is higher for HH and HL households cause they have fewer sons and hence pay lower taxes.

In expectation with respect to the father's survival, only HL household has a social security return higher than a return on capital. This confirms our findings above that HL households prefer the PAYG system while LH households with negative return prefer the PAYG system the least.

[17]These life-cycle effects are not present in Fuster et al. (2003) who do not consider cost of children.

[18]See Fuster et al. (2003) for a definition and computation of the return.

In our paper the relation of returns is $r^{LH} < r^{LL} < r^{HH} < r^{HL}$, the same as for household wealth. In Fuster et al. (2003), the positions of HH and LL types are switched. It is because the number of children each type has is very different. For high skill fathers the PAYG is important: They have few sons and the PAYG pensions represent a larger part of their old-age income.

5.2 Alternative Model Cases

As in Fuster et al. (2003) we present four additional cases of the PAYG and FF steady states that differ in lifetime uncertainty and/or skill differentials. In all these cases we keep the parameters from the benchmark calibration (namely η, the parameter of altruism).

5.2.1 Case 1: Certain Lifetimes ($\psi = 1$)

The first of our alternative calibrations is presented in Table 5. At the cost of increased longevity (all agents live till age 90), certain lifetimes eliminate the risk of losing the old-age support because of sons' death. Note that there are only complete households.

INSERT TABLE 5 ABOUT HERE

Compared to the benchmark steady states, both assets and fertility fall. Compared to the benchmark PAYG steady states, fertility declines by 14.9% and assets by 8.6%. The removal of survival uncertainty reduces the need for the buffer stock of children (their future incomes) and savings.[19] Prolonging expected lifetimes increases the population growth rate, but fertility falls. A smaller productive population contributes to lower output.

The FF reform increases fertility by 14.1% while the capital stock falls by 6.9%. Changes in fertility depend mostly on the father's skill. Fertility differences between parents of different skills are higher than in the benchmark specification but very similar: in the PAYG steady state, they are 11.2% (LH vs. LL) and 19.7% (HH vs. HL). After the FF reform, these numbers are 11.1% (LH vs. LL) and 19.9% (HH vs. HL). The LH is the only type of households that increases its savings.

Welfare gains have the same signs as in the benchmark calibration (observe how HL households are worse off and newborns are better off). Naturally, the higher longevity increases the dependency ratio to more than 25%. Longer lives of low skill individuals substantially increase the fraction of LL types (61%). Finally, the social security tax rises to 16.9% in order to finance the retirement benefits.

As there is a larger fraction of retired agents who lose their social security benefits, the FF reform has the lowest political support (57%, still a majority). The support among generations with a recently retired father is very small.

[19]Kalemli-Ozcan (2002) stresses the role of survival uncertainty for the insurance strategy, or hoarding of children, where the actual number of children is greater than the optimal number of children for the parents.

5.2.2 Case 2: Equal Survival Probability ($\psi_H = \psi_L$)

Table 6 shows that when lifetimes are uncertain but same for both skills, fertility decreases for all household types in both PAYG and FF steady states (relative to their benchmarks).[20]

INSERT TABLE 6 ABOUT HERE

The main reason is that the survival probabilities increase on average (i.e., for the most numerous low skill individuals) while skill uncertainty remains. The children of low skill parents are now more likely to survive and support the parents in their old age. This drives the fertility of low skill agents down in the PAYG system. Overall, fertility declines by 5%. On the other hand, high skill households now face a higher mortality risk and they do not dissave as much as in the certain lifetime case. The capital stock is similar to that in the benchmark steady states.

The FF reform increases fertility by 12% and reduces the capital stock by 3.1%. Again, fertility differences between different skills are higher than in the benchmark specification and similar across steady states. Especially the low skill agents increase their fertility as their survival probability increases.[21] Welfare gains and political support are not much different from the benchmark and certain lifetimes cases.

Compared to the benchmark specification, this and the certain lifetimes cases exhibit higher average survival probabilities. The reduced fertility suggests that children are used as insurance against survival uncertainty.

5.2.3 Case 3: Limited Heterogeneity ($\psi_H = \psi_L$, $\varepsilon_H = \varepsilon_L$)

In the limited heterogeneity case in Table 7, all households are the same in their survival probability and skills. The only risk they face is the equal survival uncertainty.

INSERT TABLE 7 ABOUT HERE

Fertility is the same for all agents, close to that in the benchmark PAYG and FF steady states. As there are no high skill agents and all households have the same fertility, the capital stock decreases by 22%: there is no need for the buffer stock except for life uncertainty. Correspondingly, the output declines as well.

The FF reform increases fertility by 7.8% while not changing much the capital stock. Welfare gains from the FF reform are big (for the average household) and the limited heterogeneity case obtains almost 100% support from all generations.

The large decline in savings in this case suggests that assets are primarily used to insure against a low future realization of skills among children.

[20] The survival probability is the weighted average of ψ_H and ψ_L.

[21] In the PAYG steady state 13.8% (H vs. L), 11.0% (LH vs. LL), and 25.7% (HH vs. HL), and in the FF steady state to 9.5% (H vs. L), 10.7% (LH vs. LL), and 19.4% (HH vs. HL).

5.2.4 Case 4: Exogenous Fertility

Finally, Table 8 shows the case of exogenous fertility. Here, the fertility of all agents in the benchmark PAYG steady state is set to match the U.S. population growth rate.[22] We keep the same fertility rates in the FF steady state.

INSERT TABLE 8 ABOUT HERE

Exogenous and equal fertility across different household types implies that agents who would otherwise choose a low fertility are now forced to save in children rather than assets, and vice versa. As the high skill agents are those who have to lower their savings the most, the aggregate capital stock falls relative to the benchmark PAYG steady state by 20.6% (correspondingly, output falls too).

Importantly, the FF reform with exogenous fertility has the opposite effect on aggregate levels. Only in this case the stock of capital increases by 6.7%, together with consumption, output, and the capital-output ratio.[23] Note that in this case it is households with a low skill father that increase savings after the FF reform while assets of households with a high skill father remain almost constant. As the high skill agents are forced to have more children, the fraction of high skill agents increases to 29.7%.

Further, only in this specification all types of households and cohorts are better off in the FF steady state, and by big percentages.[24] The highest gain is to households with low-skilled fathers (13% for LL, 3.8% for L).[25] Finally, exogenous fertility has the highest political support of all cases we have studied.

Overall, the assumption of exogenous fertility dramatically affects the predictions of the model, especially those related to the capital stock.

6 Conclusions

The social security reform is one of the most important economic and political issues in the United States and other developed countries. This paper analyzes a social security reform in a general equilibrium model with altruistic dynasties and endogenous fertility. In turns out that assumptions on agents' heterogeneity (survival probabilities and skill differences) are quantitatively important: the fertility and allocation responses by different types of households lead to very different aggregate levels and equilibrium prices.

[22] In Fuster et al. (2003), the average number of children is 1.52. Our number is higher because we do not replace dynasties that die off.

[23] This is similar to Fuster et al. (2003), where the FF reform increases capital stock by 6.1%, output by 1.8%, consumption by 1.1%, and the capital-output ratio falls from 2.48 to 2.59.

[24] With the exception of lonely fathers.

[25] In Fuster et al. (2003), only the LH households prefer to be born into the FF steady state (gain +1.39%). All other households types suffer a welfare loss (most the HL types, -1.71%). However, large gains to L and H lonely sons lead to a positive average welfare gain of +0.43% of the steady state consumption. On the other hand, in Fuster et al. (2007) all newborn households prefer the FF reform. However, it is not clear that these gains come from endogenous labor or from a different assumption on government consumption.

A reduction in any type of uncertainty removes the need for precautionary savings in terms of assets and/or children.

The main actors of this model are the high skill agents. In the PAYG system, these agents save relatively more in assets than in children. Consequently, models with exogenous fertility underestimate the aggregate capital stock in the PAYG steady state. Further, the high skill households' responses to FF reform are much higher than those of the low skill households. As high skill agents switch to investing in children rather than in intergenerational transfers, fertility increases while the capital stock falls. Thus the FF reform with endogenous fertility leads to opposite aggregate outcomes than the same reform with exogenous fertility. Because the high skill agents have more children, the aggregate productivity increases together with the fraction of high skill (educated) individuals. The welfare gains from the elimination of social security tax seem to more than compensate the agents for the lost social security insurance against life time uncertainty and labor productivity shocks.

These results indicate that models assuming exogenous fertility might be misleading with respect to the behavior of different groups of the population, aggregate outcomes, welfare gains and political support for the reform. Finally, endogenous fertility is also important for the transition analysis. In the literature with exogenous fertility, agents usually prefer the FF steady state but the transition to it is too costly as they need to invest a lot during the transition. However, in our endogenous fertility model, the capital-output ratio and the capital stock are already high in the PAYG system and both fall after the FF reform. The high initial stock of capital provides an additional consumption source for households who would otherwise suffer from the transition. This is important for theoretical purposes as well as for policy recommendations. Transition costs could be lower or could even turn into gains as there is no need to accumulate a higher capital stock for the new steady state.

This life-cycle dynastic model with endogenous fertility is open to many extensions as the incorporation of endogenous labor, postponing of the retirement age or different fiscal arrangements. In many countries government policies support fertility by child allowances, maternity support, or on the other hand, try to limit fertility by restricting the number of children. Finally, an analysis of the transition between the PAYG initial steady state and the reformed steady state would evaluate the true cost of the reform. It would also enable us to study the elimination of social security benefits that is gradual or occurs only after a certain period of time.

References

Alvarez, F. (1999). Social mobility: The barro-becker children meet the laitner-loury dynasties. *Review of Economic Dynamics* (2), 65–103.

Auerbach, A. J. and L. J. Kotlikoff (1987). *Dynamic Fiscal Policy*. New York, N. Y.: Cambridge University Press.

Barro, R. J. and G. Becker (1989). Fertility choice in a model of economic growth. *Econometrica 57*(2), 481–501.

Becker, G. (1981). Altruism in the family and selfishness in the market place. *Economica 48*, 1–15.

Boldrin, M., M. De Nardi, and L. E. Jones (2005). Fertility and social security. *NBER Working Paper No. 11146*.

Boldrin, M. and L. Jones (2002). Mortality, fertility and savings decisions. *Review of Economic Dynamics 5*, 775–814.

Caldwell, J. C. (1982). *Theory of Fertility Decline*. New York: Academic Press.

Conesa, J. C. and C. Garriga (2008). Optimal fiscal policy in the design of social security reforms. *International Economic Review 49*, 291–318.

Conesa, J. C. and D. Krueger (1999). Social security reform with heterogeneous agents. *Review of Economic Dynamics 2*(4), 757–795.

De Nardi, M., S. Imrohoroglu, and T. J. Sargent (1999). Projected u.s. demographics and social security. *Review of Economic Dynamics 2*(3), 576–615.

Deaton, A. S. and J. Muellbauer (1986). On measuring child costs: With applications to poor countries. *Journal of Political Economy 94*(4), 720–744.

Elo, I. T. and S. H. Preston (1996). Educational differentials in mortality: United states, 19791985. *Social Science and Medicine 42*, 47–57.

Fernandez-Villaverde, J. (2001). Was malthus right? economic growth and population dynamics. *University of Pennsylvania Working Paper*.

Fuster, L. (1999). Is altruism important for understanding the long-run effects of social security? *Review of Economic Dynamics 2*(3), 616–637.

Fuster, L., A. Imrohoroglu, and S. Imrohoroglu (2003). A welfare analysis of social security in a dynastic framework. *International Economic Review 44*(4), 1247–1274.

Fuster, L., A. Imrohoroglu, and S. Imrohoroglu (2007). Elimination of social security in a dynastic framework. *International Economic Review 74*(1), 113–145.

Galor, O. and D. N. Weil (1996). The gender gap, fertility, and growth. *American Economic Review 86*, 374–387.

Golosov, M., L. E. Jones, and M. Tertilt (2007). Efficiency with endogenous population growth. *Econometrica 75*(4), 1039–1071.

Imrohoroglu, A., S. Imrohoroglu, and D. Joines (1999). Social security in an overlapping generations economy with land. *Review of Economic Dynamics 2*, 638–665.

Jones, L. E. and A. Schoonbroodt (2007). Complements versus substitutes and trends in fertility choice in dynastic models. *University of Minnesota Working Paper*.

Jones, L. E. and M. Tertilt (2006). An economic history of fertility in the u.s.: 1826-1960. *NBER Working Paper No.12796*.

Kalemli-Ozcan, S. (2002). A stochastic model of mortality, fertility, and human capital investment. *Working Paper*.

Lucas, Jr., R. E. (2002). The industrial revolution: Past and future. In *Lectures on Economic Growth*, pp. 109–190. Cambridge: Harvard University Press.

Nishimura, K. and J. Zhang (1992). Pay-as-you-go public pension with endogenous fertility. *Journal of Public Economics 48*, 239–258.

Rothbarth, E. (1943). Note on a method of determining equivalent income for families of different composition. In C. Madge (Ed.), *War-Time Pattern of Saving and Spending*, Cambridge. Cambridge University Press.

SHARE (2005). *Health, Aging and Retirement in Europe: First Results from the Survey of Health, Aging and Retirement in Europe*. Mannheim Research Institute for the Economics of Aging (MEA), Germany.

Storesletten, K., C. Telmer, and A. Yaron (1999). The risk sharing implications of alternative social security arrangements. *Carnegie-Rochester Conference Series on Public Policy 50*, 213–260.

7 Appendix A: The First Order Conditions

For simplification, we present the first order conditions for an economy with T periods of the life-cycle collapsed into one and with certain lifetimes. Denote

$$\begin{aligned}
\beta &\equiv (\widetilde{\beta}(1+\widetilde{g})^{1-\sigma})^T, \\
(1+g) &\equiv (1+\widetilde{g})^T, \\
U(c,h) &\equiv T\widetilde{U}(\frac{c}{T},h), \\
1+r(1-\tau_k) &\equiv (1+\widetilde{r}(1-\tau_k))^T, \\
\gamma^g(h;z,z') &\equiv \sum_{j=1}^{T}\gamma_j^g(h;z,z'), \\
B(z) &\equiv \sum_{j=j_R-T}^{T} B_{j+T}(z), \\
\varepsilon(z) &\equiv \sum_{j=1}^{j_R-T-1} \varepsilon_{j+T}(z),
\end{aligned}$$

where tilde denotes the parameters of the original life-cycle model, c is total consumption and ξ are total bequests in T periods. Denote the next T-period values with a plus sign. Prices are constant in the stationary equilibrium.

The value function for an individual household with a state (a,b,s,z,z') is,

$$V(a,b,s,z,z') = \max_{c,a',n\geq 0}\left\{U(c,h) + \beta E_{z''}V^+(a',s,n,z',z'')\right\}, \tag{5}$$

subject to a budget constraint

$$\begin{aligned}
(1+\tau_c)(c+\gamma^g(h;z,z')) &+ (1+g)sa' = (1+r(1-\tau_k))a + w(1-\tau_{ss}-\tau_l)\times \\
&\times (\varepsilon(z) + s\sum_{j=1}^{T}(1-\gamma_j^w(n))\varepsilon_j(z')) + B(z) + (1+s)\xi.
\end{aligned}$$

Let λ be the Lagrange multiplier near the budget constraint. The first order conditions are

$$\begin{aligned}
U_c(\cdot) &= (1+\tau_c)\lambda, && (6)\\
\beta E_{z''}V_a^+(\cdot) &= (1+g)s\lambda, && (7)\\
\beta E_{z''}(V_s^+(\cdot) + \beta E_{z'''}V_b^{++}(\cdot)) &= [(1+\tau_c)\gamma_n^{\prime g}(h;z,z') + \\
&+ w(1-\tau_{ss}-\tau_l)s\sum_{j=1}^{T}\gamma_j^{\prime w}(n)\varepsilon_j(z')]\lambda. && (8)
\end{aligned}$$

The envelope conditions are

$$\begin{aligned}
V_a(\cdot) &= [1+r(1-\tau_k)]\lambda, && (9)\\
V_b(\cdot) &= U_b(\cdot), && (10)\\
V_s(\cdot) &= U_s(\cdot) + \lambda[\xi - (1+g)a' - (1+\tau_c)\gamma_s^{\prime g}(h;z,z') \\
&+ (1-\tau_{ss}-\tau_l)w\sum_{j=1}^{T}(1-\gamma_j^w(n))\varepsilon_j(z'))]. && (11)
\end{aligned}$$

Substituting the envelope conditions into the first order conditions we obtain

$$\beta E_{z''}[(1+r(1-\tau_k))U_c^+(\cdot)] = s(1+g)U_c(\cdot), \tag{12}$$

a version of the standard intertemporal Euler equation. Note that because assets are split equally among the new s households, the marginal utility of one household in the next period is s times larger than the current one. As the social security benefits depend only on father's own life-time ability, their elimination brings an additional risk from the dependence on sons' uncertain future incomes. Each household has two options to smooth consumption over time: either by saving more or by increasing the number of its future members, s.

To analyze the fertility decision, express partial derivatives of the household's utility function as

$$U_c(\cdot) = \frac{1}{1+s}u'\left(\frac{c}{1+s}\right)b^{\eta}(1+s)^{\eta}, \tag{13}$$

$$U_s(\cdot) = \frac{1}{1+s}\left\{\underbrace{\eta U(\cdot)}_{E} - \underbrace{cU_c(\cdot)}_{F}\right\}, \tag{14}$$

$$U_b(\cdot) = \frac{\eta}{b}U(\cdot). \tag{15}$$

After combining equations (8) and (11) and plugging in the derivatives of the utility function (14) and (15), we get the Euler equation for the fertility choice:

$$\beta E_{z''}\left[\underbrace{\frac{\eta}{1+n}U^+(\cdot)}_{A} + \underbrace{\beta\frac{\eta}{n}E_{z'''}U^{++}(\cdot)}_{B} + \underbrace{\frac{U_c^+(\cdot)}{1+\tau_c}\cdot\Phi}_{C}\right] = \frac{U_c(\cdot)}{1+\tau_c}\cdot\Psi, \tag{16}$$

where

$$\begin{aligned}\Phi &= \xi - (1+g)a'^{+} - (1+\tau_c)\gamma_s'^{g}(h^+;z',z'') \\ &+ (1-\tau_{ss}-\tau_l)w\sum_{j=1}^{T}(1-\gamma_j^w(n^+))\varepsilon_j(z'') - \underbrace{(1+\tau_c)\frac{c^+}{1+n}}_{D}, \\ \Psi &= (1+\tau_c)\gamma_n'^{g}(h;z,z') + w(1-\tau_{ss}-\tau_l)s\sum_{j=1}^{T}\gamma_j'^{w}(n)\varepsilon_j(z'),\end{aligned}$$

where parts E and F of equation (14) are represented by terms A and D, respectively.

The fertility first order condition in equation (16) equalizes the expected discounted marginal benefit of having an additional child in the next period to the marginal cost of having this child in this period. The marginal benefit in the next period consists of two purely altruistic parts (A and B) and the change in marginal utility from consumption (C). The first arises due to having more sons in the next period while the second due to having more brothers in two periods. Consumption benefit Φ in the next period consists of extra bequests from more sons, extra wages net of extra savings from new sons, and the cost of children those new sons will have. The additional negative term D comes from the decomposed equation (14) and represents the cost of having to share the household's consumption among more adult members. The consumption cost in this period, Ψ, consists of additional child expenditures and forgone wages. The elimination of social security tax increases both marginal cost in terms of extra current goods (forgone income from having an additional child) and marginal benefit in the future (higher after-tax income from children).

Overall, the FF reform seems to result in children having relatively higher return compared to assets so that a household is willing to increase fertility and decrease asset holdings until their marginal returns are equalized. Also, it appears that the positive income effect dominates the increased opportunity cost of children (contrary to Becker (1981) or Galor and Weil (1996), for example).

Naturally, the elimination of social security provides additional incentives to save more within the T periods of the life-cycle. These incentives are not captured in the compressed formulation here. Finally, the social security reform affects equilibrium prices $(r,\, w)$. Wages decrease as higher fertility increases the efficiency of labor in the economy. Interest rate increases in fertility as children are substitute for investment.

Parameters		
	Population	
j_{2T}	$= 14$	Maximum lifetime (90 years)
j_R	$= 10$	Retirement age (65 years)
j_N	$= 4$	Fertility age (35 years)
$\bar{n}_{USA}$	$= 0.012$	Population growth rate U.S.
ψ		Survival probabilities (Fuster)
	Utility	
β	$= 0.988$	Annual discount factor
σ	$= 0.95$	Relative risk aversion
η	$= 0.055$	Altruism
	Production	
g	$= 0.014$	Annual technology growth
δ	$= 0.044$	Annual depreciation rate
α	$= 0.34$	Capital share
$\pi_{LL} = 0.83$	$\pi_{HH} = 0.57$	Transition matrix for skills
e		Earnings profiles (Fuster)
	Fiscal Policy	
τ_k	$= 0.35$	Capital income tax rate
τ_c	$= 0.055$	Consumption tax rate
G	$= 0.225$	Government purchases (% GDP)

Table 1: Parameters

Cost of Children: Working Time		
Age of	Fraction of Combined Hours (γ_w)	
(Younger) Child	Weekly	Annual
0-4	0.0441	0.0755
5-9	0.0392	0.0662
10-14	0.0392	0.0662
15-19	0.0392	0.0662

Cost of Children: Expenditures		
Age of (Younger) Child	Annual Expenditure	Fraction of Income (γ_g)
One-Child Household		
0-4	\$10,354 = \$8,350 $\cdot$ 1.24	0.22
5-9	10,540 = 8,500 $\cdot$ 1.24	0.22
10-14	11,226 = 9,054 $\cdot$ 1.24	0.24
15-19	11,408 = 9,200 $\cdot$ 1.24	0.25
Two-Child Household		
0-4	\$17,690 = \$8,350 + \$9,340	0.36
5-9	17,840 = 8,500 + 9,340	0.37
10-14	18,394 = 9,054 + 9,340	0.38
15-19	18,540 = 9,200 + 9,340	0.39
n-Child Household ($n > 2$)		($n = 3$)
0-4	(\$8,350 + \$9,340 + (n-2)$\cdot$ \$9,200) $\cdot$ 0.77	0.43
5-9	(8,500 + 9,340 + (n-2)$\cdot$ 9,200) $\cdot$ 0.77	0.43
10-14	(9,054 + 9,340 + (n-2)$\cdot$ 9,200) $\cdot$ 0.77	0.44
15-19	(9,200 + 9,340 + (n-2)$\cdot$ 9,200) $\cdot$ 0.77	0.45

Sources: Working time: *Report on the American Workforce*. 1999. U.S. Department of Labor. Table 3-6. Expenditures: Table 9 in *Expenditures on Children by Families. 1998 Annual Report*, United States Department of Agriculture. Miscellaneous Publication Number 1528.

Table 2: Cost of Children

Benchmark PAYG and FF								
	τ_{ss}	K	L	Y	C	K/Y	$r(1-\tau_k)$	$w(1-\tau_{ss}-\tau_l)$
PAYG	0.115	0.757	1.785	1.333	0.549	2.839	0.047	0.357
FF	0.00	0.694	1.889	1.344	0.536	2.582	0.053	0.394
(%)		-8.3	+5.8	+0.8	-2.4			+10.1

	Both f and s Alive				Only s Alive			
	LL	HL	LH	HH	L	H	Average	Growth
Fertility								
PAYG	1.83	1.56	1.65	1.11	1.85	1.61	1.67	1.24%
FF	1.88	1.89	1.78	1.73	1.80	1.76	1.84	1.46%
(%)	+2.7	+21.2	+7.9	+57.3	-2.7	+9.3	+10.2	
Assets (Changes in %)								
(%)	+0.5	-22.2	+3.5	-23.7	-4.1	-19.4	-8.3	
Welfare Gains from FF Reform (%)								
All	+0.09	-0.85	+0.75	-0.66	+2.35	+1.88	+0.41	
Newborns	+1.56	+0.63	+2.19	+0.88	—	—	+1.42	
Demographics (%)							H-Skill	Retired
PAYG	46.0	9.2	9.5	12.3	15.4	5.5	26.6	18.7
FF	44.8	10.1	9.2	13.5	14.8	5.6	28.9	17.5

Political Support								
Generation	1	2	3	4	5	6	7	All
(%)	100	71	10	28	42	81	89	60

Table 3: Steady State Results: Benchmark Steady States

Return on Benchmark PAYG				
Conditional on Father's Survival				
f's Age	Both f and s Alive			
at Death	LL	HL	LH	HH
65	<0	<0	<0	<0
70	0.5	3.3	<0	0.5
75	3.8	6.9	<0	3.3
80	5.2	8.4	<0	5.3
85	6.0	9.0	<0	6.1
In Expectation				
	LL	HL	LH	HH
$E[r_{SS}]$	2.7	5.7	<0	3.4
	All			
$E[r_{SS}]$	2.4			

The after-tax return on capital is 4.7%.

Table 4: Return on Social Security. Benchmark Steady State PAYG with $\theta = 0.44$.

Case 1: Certain Lifetimes $\psi = 1$								
	τ_{ss}	K	N	Y	C	K/Y	$r(1-\tau_k)$	$w(1-\tau_{ss}-\tau_l)$
PAYG	0.169	0.692	1.624	1.215	0.531	2.846	0.046	0.331
FF	0.00	0.644	1.764	1.252	0.532	2.570	0.054	0.393
(%)		-6.9	+8.6	+3.0	+0.2			+18.7

	Both f and s Alive				Only s Alive			
	LL	HL	LH	HH	L	H	Average	Growth
Fertility								
PAYG	1.52	1.37	1.35	1.10	—	—	1.42	1.06%
FF	1.71	1.66	1.52	1.33	—	—	1.62	1.44%
(%)	+12.5	+21.2	+12.6	+20.9			+14.1	
Assets (Changes in %)								
(%)	-0.0	-14.1	+7.5	-12.0	—	—	-6.9	
Welfare Gains from FF Reform (%)								
All	+0.11	-1.37	+1.85	-0.31	—	—	+0.09	
Newborns	+3.11	+1.74	+4.80	+2.90	—	—	+3.14	
Demographics (%)							H-Skill	Retired
PAYG	61.6	11.1	12.6	14.7	—	—	25.8	27.6
FF	61.3	11.3	12.5	14.9	—	—	26.2	25.0

Political Support								
Generation	1	2	3	4	5	6	7	All
(%)	100	84	3	10	23	81	99	57

Table 5: Steady State Results: Certain Lifetime Steady States

Case 2: Equal Survival Probability $\psi_H = \psi_L$								
	τ_{ss}	K	N	Y	C	K/Y	$r(1-\tau_k)$	$w(1-\tau_{ss}-\tau_l)$
PAYG	0.116	0.733	1.730	1.292	0.547	2.836	0.047	0.357
FF	0.00	0.711	1.836	1.330	0.542	2.673	0.051	0.402
(%)		-3.0	+6.1	+2.9	-0.9			+12.6

	Both f and s Alive				Only s Alive			
	LL	HL	LH	HH	L	H	Average	Growth
Fertility								
PAYG	1.72	1.48	1.53	1.10	1.67	1.44	1.58	1.09%
FF	1.87	1.80	1.67	1.45	1.79	1.62	1.77	1.38%
(%)	+8.7	+21.6	+9.2	+31.8	+8.9	+12.5	+12.0	
Assets (Changes in %)								
(%)	+0.0	-15.4	+1.9	-16.5	-4.3	-13.2	-3.0	
Welfare Gains from FF Reform (%)								
All	+0.37	-1.26	+1.13	-1.04	+2.98	+2.27	+0.63	
Newborns	+2.08	+0.75	+2.85	+1.03	—	—	+1.87	
Demographics (%)							H-Skill	Retired
PAYG	47.5	8.4	9.7	11.1	15.4	5.7	25.4	19.5
FF	47.3	8.7	9.7	11.6	14.9	5.7	26.3	18.0

Political Support								
Generation	1	2	3	4	5	6	7	All
(%)	95	68	11	27	74	82	91	63

Table 6: Steady State Results: Equal Survival Probability Steady States

Case 3: Equal Survival Probability $\psi_H = \psi_L$ and Productivity $\varepsilon_H = \varepsilon_L$								
	τ_{ss}	K	N	Y	C	K/Y	$r(1-\tau_k)$	$w(1-\tau_{ss}-\tau_l)$
PAYG	0.134	0.589	1.775	1.220	0.495	2.415	0.058	0.320
FF	0.00	0.573	1.884	1.257	0.504	2.281	0.062	0.369
(%)		-2.7	+6.1	+3.0	+1.8			+15.5

	Both f and s Alive	Only s Alive	Average	Growth
Fertility				
PAYG	1.66	1.79	1.67	1.21%
FF	1.80	1.81	1.80	1.39%
(%)	+8.4	+1.1	+7.8	
Assets (Changes in %)				
(%)	-2.2	-5.1	-2.7	
Welfare Gains from FF Reform (%)				
All	+3.33	+5.74	+3.79	
Newborns	+5.27	—	+5.27	
Demographics (%)			H-Skill	Retired
PAYG	77.1	20.9	—	18.8
FF	77.4	20.6	—	18.0

Political Support								
Generation	1	2	3	4	5	6	7	All
(%)	100	99	86	86	90	97	100	94

Table 7: Steady State Results: Limited Heterogeneity Steady States

Case 4: Exogenous Fertility

	τ_{ss}	K	N	Y	C	K/Y	$r(1-\tau_k)$	$w(1-\tau_{ss}-\tau_l)$
PAYG	0.115	0.601	1.785	1.233	0.501	2.438	0.057	0.331
FF	0.00	0.641	1.785	1.261	0.525	2.546	0.054	0.391
(%)		+6.7	0.0	+2.3	+4.8			+18.1

	Both f and s Alive				Only s Alive			
	LL	HL	LH	HH	L	H	Average	Growth
Fertility								
	—	—	—	—	—	—	1.71	1.24%
Assets (Changes in %)								
(%)	+11.6	+0.7	+9.9	+0.3	+7.3	+1.9	+6.7	
Welfare Gains (%)								
All	+4.18	+2.87	+5.20	+3.92	+6.73	+6.24	+4.57	
Newborns	+6.13	+4.93	+7.11	+5.94	—	—	+6.06	
Demographics (%)							H-Skill	Retired
	44.1	10.2	9.1	13.7	15.1	5.7	29.7	18.9

Political Support								
Generation	1	2	3	4	5	6	7	All
(%)	100	99	98	97	97	97	100	98

Table 8: Steady State Results: Exogenous Fertility Steady States

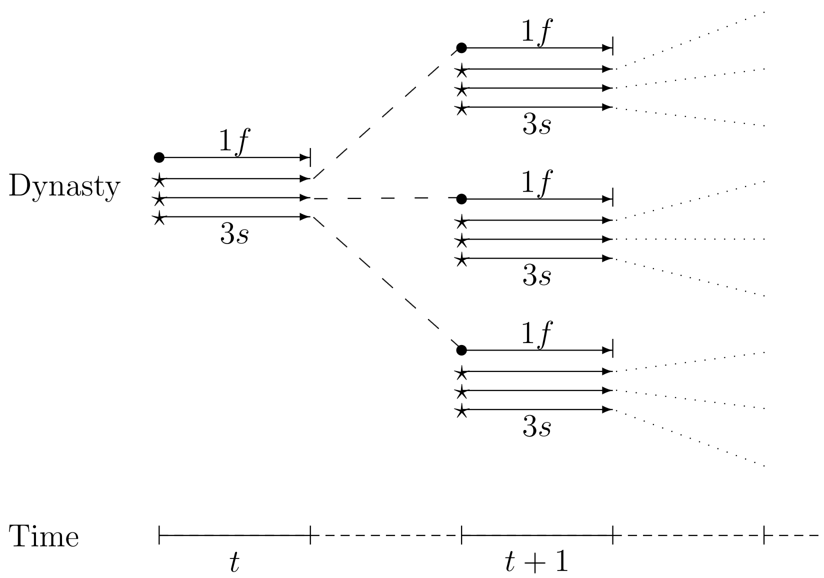

Figure 1: Timeline for Dynasties

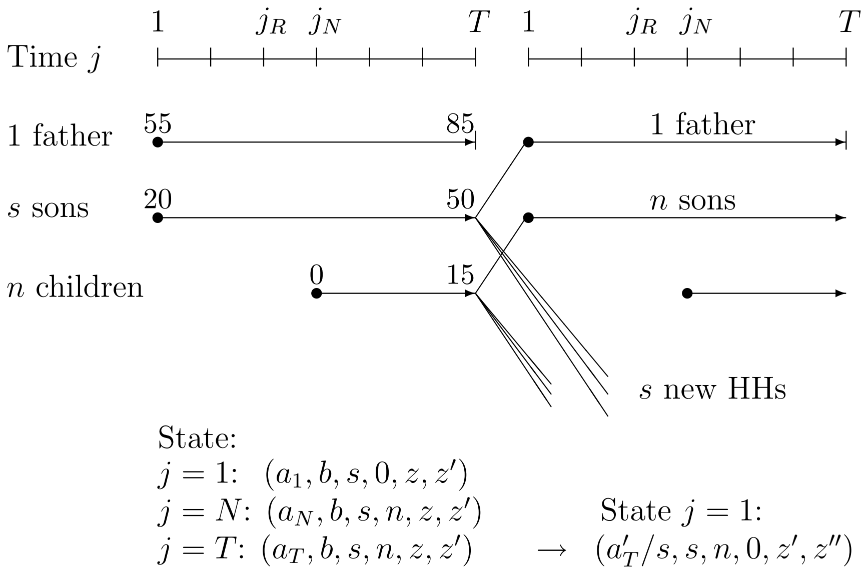

Figure 2: Timeline for Households

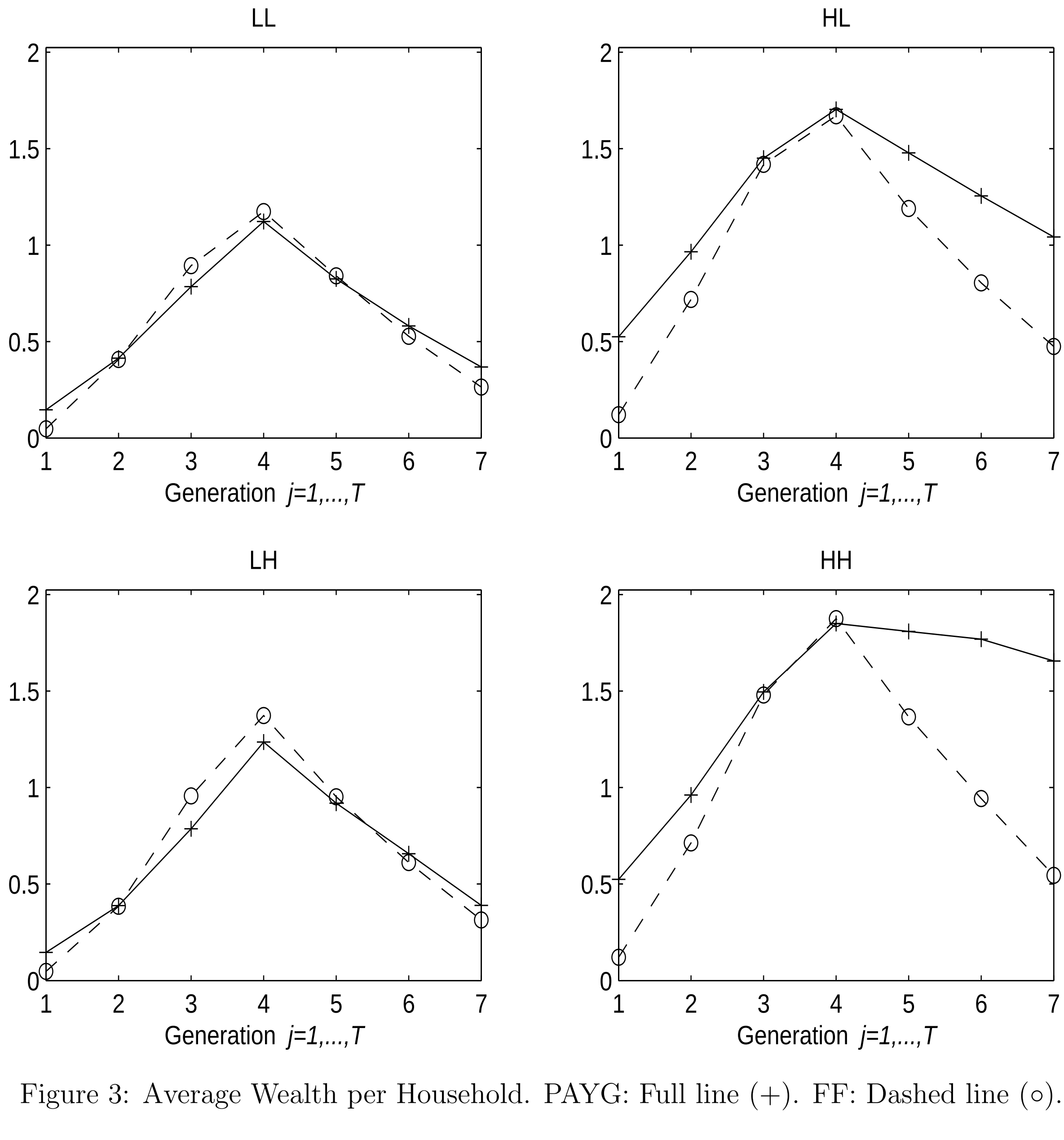

Figure 3: Average Wealth per Household. PAYG: Full line (+). FF: Dashed line (∘).

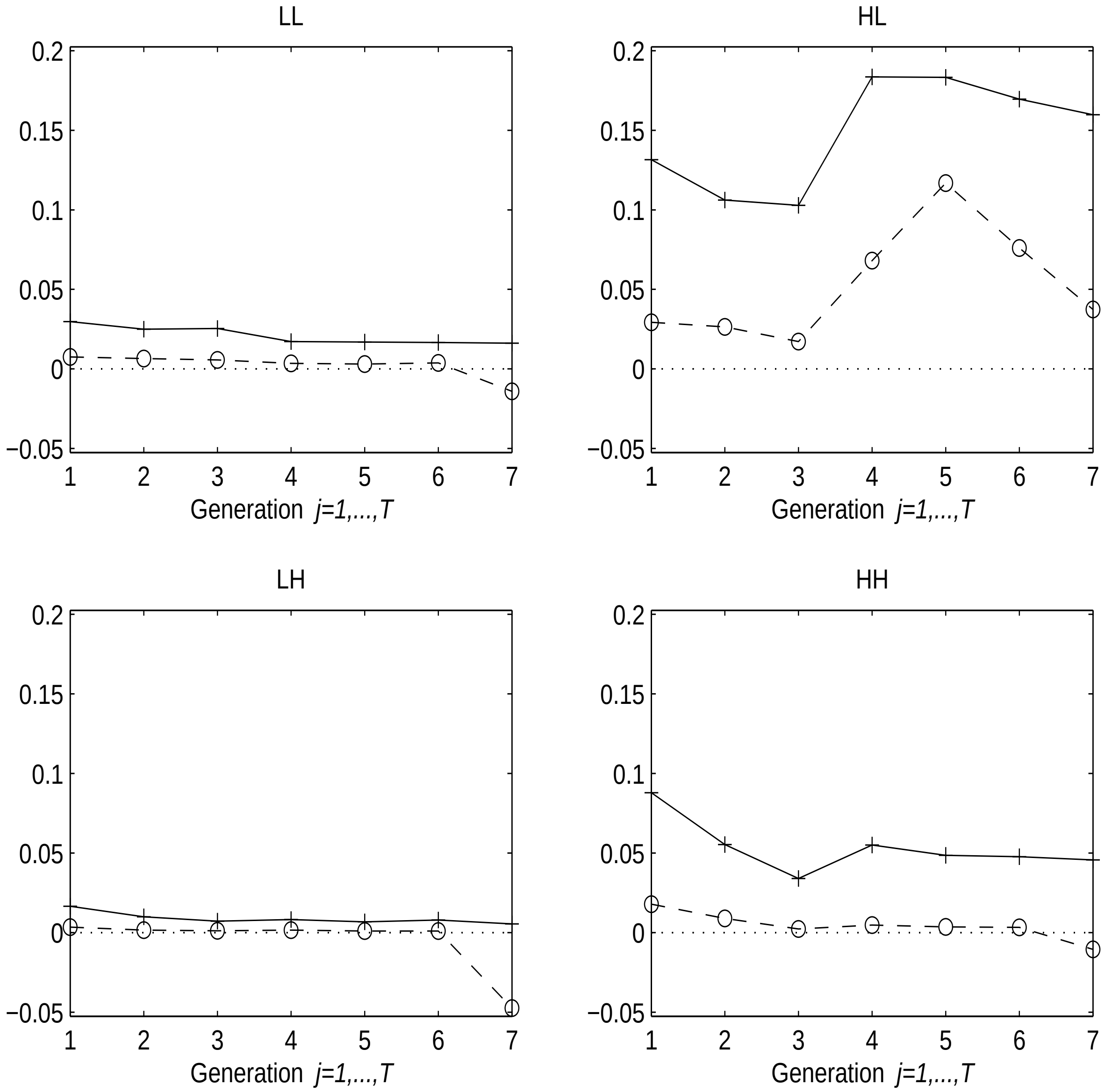

Figure 4: Average Intervivo Transfers as % of Household Consumption. Transfer from father to 1 son. PAYG: Full line (+). FF: Dashed line (∘).