An Economic Analysis of Numerical Data Processing in the Firm[1]

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Abstract

This paper provides a coherent framework within which to understand the economics of data processing in enterprises. The computational processes in the firm are described with the help of a dynamic parallel processing model of associative computation. The basic model is extended to include the assumption that the speed of information processing in each individual computational center depends upon the capital and labor allocated to it. In such a model, the conditions for the efficient organization of numerical data processing are defined, and the architecture of efficient structures is analyzed. It is shown that, similarly as in the computer systems, the so-called 'skip-level reporting' structures are efficient for decentralized numerical computation in business firms. However, if processing elements in the skip-level reporting structures cannot be equally loaded, then their computational power has to be adjusted to the given information workload. The method of adjustment of the resources allocated to processing elements to the information workload of the one-shot skip-level reporting structures of information processing is presented, and the efficiency frontier is characterized. Furthermore, the optimal organization of numerical data processing in enterprises is analyzed on the example of predicting demand in the firm.

Keywords: Information-processing, organization of the firm, decentralization, hierarchy.
JEL Classification: D8, D2.

Abstrakt

Tento článek podává ucelený rámec pro pochopení ekonomické stránky zpracování dat v podnicích. Výpočetní procesy ve firmě jsou popsány pomocí modelu dynamického paralelního zpracování asociativního výpočtu. Základní model je rozšířen o předpoklad, že rychlost zpacování informace v každém jednotlivém výpočetním centru závisí na přiděleném kapitálu a práci. V takovém modelu jsou definovány podmínky pro efektivní organizaci zpracování číselných dat a je analyzována architektura efektivních struktur. Je ukázáno, že pro decentralizovaný numerický výpočet jsou v obchodních firmách podobně jako v počítačových systémech efektivní takzvané "skip-level reporting" struktury. Jestliže však elementy zpracování nemohou být v "skip-level reporting" strukturách vloženy stejně, musí být jejich výpočetní síla upravena na dané informační zatížení. Je prezentována metoda přizpůsobení

[1] Most of this work was done during the author's stay at the Institute for Advanced Studies (IHS) in Vienna.

zdrojů určených pro elementy zpracování informačního zatížení a je charakterizována hranice efektivnosti. Na příkladu předpokládané poptávky ve firmě je navíc analysována optimální organizace zpracování číselných dat v podnicích.

Klíčová slova: Zpracování informací, organizace firmy, decentralizace, hierarchie.
Klasifikace JEL: D8, D2.

1. Introduction

In classical microeconomic theory, the firm is usually considered as a relatively simple profit-maximizing unit. A complex organizational system, containing a number of interconnected parts, is visualized as a large ‘black box’ transforming inputs into outputs according to a rule described by a production function. The attention of economists is traditionally focused on the production process, and it is typically assumed that changes in the volume of the firm’s output also affect the size of the parts of the firm that are not directly involved in production, such as administration, managing and control, production planning, and so forth. On the other hand, in the modern firm more than one third of the employees work full time carrying out activities that are not directly connected with the production process such as, for instance, processing and communicating information, monitoring actions of other members of the firm, analyzing the market, planning, training employees, making decisions, and so on (see Radner, 1992). All these actions (called ‘managing activities’) are based on the processing of information (see Radner, 1992). Computational processes in the management of the firm need a number of economic resources (labor, computational and telecommunication equipment, offices, etc.) which can be used in many different ways producing better or worse results. Consequently, information processing in management affects profitability of the firm and therefore has to be economized.

The present paper focuses on numerical data processing in enterprises and attempts to explore the relationship between organizational aspects of computational processes, efficiency of information processing and profitability of the firm.

Section 2 presents a short overview of research related to the internal structure of the firm, the organizational forms of data processing in enterprises, and the human information-processing in decision making.

In Section 3, data processing in the firm, for the purpose of predicting demand, is considered. It is shown that the value of the computational service depends upon the delay in computation and the stochastic processes underlying demand in its sources. The costs and benefits from information processing (for the purpose of predicting demand) are formally defined, and the objective of the firm in numerical data processing is specified.

In Section 4, information processing in the firm is described in the conceptual framework of the dynamic parallel processing model of associative computation.

The original model is presented and extended to include the assumption that the computational power of the processing elements is determined by the capital and labor allocated to the information-processing structure. The relationship between the resources allocated to each individual computational center and its computational power is determined by the technology of information processing, and it is characterized by an information-processing function.

Section 5 focuses on the issue of the efficient organization of numerical data processing in the firm. The efficiency condition is defined, and it is shown that if information processing in the firm is decentralized then so-called ‘skip-level reporting’ structures are efficient for numerical computation. However, if the processing elements of decentralized structures cannot be equally loaded, then a nonequal distribution of resources among processing elements decreases the delay in information processing. The method of adjustment of the computational power of the processing elements to a given information workload is presented. Moreover, it is shown that the optimal size of the information-processing structures can be easily determined through a solution to a simple discrete optimization problem. Finally, the efficiency frontier in numerical data processing in the firm is formally described.

Section 6 illustrates the concepts presented in the paper by the mean of a numerical example of the optimal organization of information processing for the purpose of predicting demand in the business firm.

2. Related research

Although in classical microeconomic theory the firm is analyzed at a high level of abstraction (i.e. as a black-box transforming inputs into outputs), there is a prevailing opinion that the performance of the firm is influenced by its internal structure. Moreover, the structure (or the architecture) of the firm is considered as a factor which determines its profitability (see, for example, Sah and Stiglitz, 1986; Alchian and Demsetz, 1972; Milgrom and Roberts, 1990; Keren and Levhari, 1983, or Williamson, 1986).

In recent economic literature special attention is paid to the analysis of the relationship between the managing sector and the economic performance of the firm. The overview of the contributions of recent research to understanding the economic significance of the management sector in modern enterprises and large corporations is presented, for example, by Radner (1992).

A number of economic papers concentrated on the analysis of information structures in enterprises (see, for instance, Aoki, 1986; or Marschak and Radner, 1972) or different aspects of information processing in management in the firm. Marschak and Radner, for instance, considered the organization of decision making in a network of processors (see Marschak and Radner, 1972, Ch.9) and explored the implications of the delay in information processing on the value of decisions (see, Marschak, 1972; or Marschak and Radner, 1972, Ch.7). Returns to scale in information processing and its implications on the firm's size were studied by Keren and Levhari (1979, 1983), or Radner and Van Zandt (1992). Efficient organization of data processing in enterprises was investigated by Radner (1992, 1993), Radner and Van Zandt (1992, 1993), Van Zandt (1990), or Bolton and Dewatripont (1994).

Radner (1992, 1993) and Radner and Van Zandt (1992, 1993) examined information processing in the firm in the conceptual framework of the dynamic parallel processing model of associative computation adopted from computer science literature (see, Gibbons and Rytter, 1988). In this model, the processing elements of the information-processing structure of the firm are considered as the processors of an idealized parallel computer, i.e. all processing elements are identical and their computational power[2] is given. In real firms, however, unlike in computer systems, the computational power of each individual processing element is not fixed, but it depends upon the capital and labor allocated to it. The fact that in real firms computations are done with the help of the capital and labor has been emphasized, for example, by Eliasson (1990), Eveland and Bikson (1988), Keen and Scott Morton (1978), or Eerola (1990). The significance of human information-processing in decision making and the specific features of data-processing in the team of decision makers such as, for instance, capacity limitations, distribution of goals in data analysis and situational factors that affect human work, have been analyzed, for example, by Holloway (1979), Kenney and Raiffa (1976), Lindsay and Norman (1977), or O'Reilly III (1990). These studies provide the foundations for the extension of the dynamic parallel processing model of associative computation to assume that in real firms the computational power of the processing elements is determined by the amounts of the capital and labor allocated to them.

[2] The computational power of the processing element is understood as the number of operations it can perform in one unit of time.

3. The objective of the firm in numerical data processing

To present the economic significance of numerical data processing in enterprises, consider the firm which chooses a level of output in order to maximize profit defined as the difference between revenue and cost. The firm chooses the optimal level of output making periodical estimations of demand (Q_t) coming from a given number of sources (N). Demand in each individual source is described by the stochastic process $Q_{i,t}$ (i=1,2,...,N, and t is an integer number, $-\infty<t<+\infty$), such that $Q_{i,t}= \mu_i + X_{i,t}$, where μ_i is the expected value of demand from source i, $X_{i,t}$ is the deviation from the mean, which depends on the history of the process ($X_{i,t}$ can be given, for instance, by a linear first order autoregressive process)[3].

If the computation of total demand is instantaneous, then the estimation (A_t) of demand (Q_t) is perfectly accurate, i.e.

$$A_t=Q_t = \sum_{i=1}^{N} Q_{i,t} \ .$$

In this case the firm produces an efficient output $Q^*=Q_t=A_t$ and earns the maximum profit.

If total demand (Q_t) is computed with a very small delay, then the prediction (A_t) is close to Q_t, and the profit of the firm is close to its maximum. If the delay is substantial, then the expected absolute value of the error between the real demand (Q_t) and its prediction (A_t) is high, and the information produced is almost worthless (see Radner and Van Zandt, 1992). Thus, the value of the prediction and, consequently, the value of the computational service provided depend on how good the resulting prediction is compared to how good it would be without the service. It turns out that the value of the computational service (V) is inversely proportional to the absolute value of the prediction error ($E=|Q_t-A_t|$), determined by the delay in information processing (D_N). Therefore, the value of the computational service can be represented as a decreasing, continuous function of the delay in information processing, i.e.

$$V(D_N)= \Psi_{max} - \Psi(D_N), \text{ such that } dV(D_N)/dD_N<0,$$

where $\Psi(D_N)$ is the loss in the firm's profit when demand is predicted with delay D_N ($d\Psi(D_N)/dD_N>0$ and $\Psi(0)=0$), Ψ_{max} is the maximum loss in the firm's profit ($\Psi_{max} =\lim_{D_N\to\infty}\Psi(D_N)$).

[3] See section 6, for the example.

Assume that the delay in computation (D_N) is a function of the capital (K) and labor (L) allocated to information processing[4], i.e. $D_N=D_N(K,L)$, such that $\delta D_N(K,L)/\delta K<0$ and $\delta D_N(K,L)/\delta L<0$, and that it depends upon the architecture (S) of the information-processing structure (see Radner, 1992, 1993; or Radner and Van Zandt, 1992). Therefore, the value of the loss due to the prediction error should be considered as a function of capital (K) and labor (L), related to the given structure of information processing (S), and stochastic processes underlying demands in their sources:[5]

$$\Psi = \Psi_{Q_{1,t},Q_{2,t},...,Q_{N,t}}(D_{N,S}(K,L)) \quad .$$

Assuming that the cost of data items is small relative to the cost of capital and labor, and, consequently, can be neglected (see Radner and Van Zandt, 1992), the total cost of the computational service is

$$C(K,L) = rK + wL \quad ,$$

where w is the price of labor, and r is the price of capital.

Thus, the profit (Π) of the firm in which demand for its production is estimated is specified as

$$\Pi = \pi_0 + V(D_{N,S}(K,L)) - C(K,L) \quad ,$$

where π_0 is the profit of the firm when demand for its production is not estimated:

$\pi_0 = \rho Q^* - \Psi_{max}$,
ρ denotes a profit per unit of output,
Q^* is the optimal output,
Ψ_{max} is the maximum loss in the firm's profit,
$V(D_{N,S}(K,L))$ is the value of the computational service,
$V(D_{N,S}(K,L)) = \Psi_{max} - \Psi(D_{N,S}(K,L))$,
$\Psi(D_{N,S}(K,L))$ is the loss due to the error in prediction based on the computational service, in which resources K and L are used for information processing in the structure S;
C(K,L) denotes the cost of inputs to information processing.

[4] See section 4, for detailed analysis of the relationship between the resources used in computation and the delay in data processing.

[5] See Radner and Van Zandt (1992).

After rearrangement, the expression above can be represented as

$$\Pi = \rho Q^{*} - \Psi(D_{N,S}(K,L)) - C(K,L) \quad .$$

If the deviation from the highest profit due to noninstantaneous and costly information processing is

$$\Phi_{N,S}(K,L) = \Psi(D_{N,S}(K,L)) + C(K,L) \quad ,$$

then the profit of the firm can be computed as

$$\Pi = \rho Q^{*} - \Phi_{N,S}(K,L) \quad .$$

The analysis above shows that the profit of the firm can be considered as a function of the loss due prediction error (Ψ) which depends on the stochastic processes underlying demand in its sources. Thus, the objective of the firm is to maximize the expected value of the profit (i.e. $E(\Pi)$).

The latest expression implies that, the firm's expected profit maximization is equivalent to the minimization of the expected value of the deviation from the highest profit $\Phi_{N,S}(K,L)$. Consequently, information processing in the profit maximizing firm has to be organized in a way which minimizes the following expression

$$\underset{S}{Min}\left\{ \underset{K,L}{Min}\ E(\Phi_{N,S}(K,L)) \right\} ,$$

where

$$E(\Phi_{N,S}(K,L)) = E(\Psi(D_{N,S}(K,L))) + C(K,L) \quad ,$$

and $E(\Psi(D_{N,S}(K,L)))$ is the expected value of the loss caused by non-instantaneous information processing in the structure S to which the capital (K) and labor (L) are allocated, C(K,L) denotes the cost of resources used.

4. Information processing in the firm

To describe the computation of numerical data in the firm, consider the information-processing sector in which cohorts of N data items are summarized, and assume that the information-processing system works in a one-shot regime, i.e. delays between subsequent cohorts of data coming into the system are greater (or at least equal) to the time of a single cohort processing (it ensures that queues of data in the information-processing structure cannot arise).

Following Radner (1992, 1993), represent the computational process in the firm as in an idealized parallel computer, i.e. assume that each processing element is modelled as a processor which contains an infinite memory where data are stored (called a buffer), and a register where summations are made. Each processor can read a single data item from its memory and add the value to the register, setting it to the resulting sum (errors in computation are not allowed). Thus, loading and adding a single datum to the contents of the register is called an operation. The time is assumed to be the same whatever the values of data added or when a datum is added to the cleared register (i.e. to zero). A processor can send the contents of its register to an output (or to the buffer of any other processor through a communication channel) in zero time, i.e. it is assumed that communication does not need time (see Radner and Van Zandt, 1992, for details).

Assuming that processors cannot make errors, the value of the computational service[6] is determined by the quality of the result computed which is inversely related to the delay in information processing (see Radner and Van Zandt 1992). Consequently, the value of the computational service (V) can be considered as a decreasing function of the delay in information processing (D), i.e. $V=V(D)$ such that $dV(D)/dD<0$.

Each processor adds data items in a serial fashion. Thus, in order to speed up the computational process, data processing can be done in parallel using more than one processor. Processors involved in decentralized computations and communication channels form a computational structure. Assuming that communication channels and data items used in the computational process are not costly (Radner, 1993), the only scarce economic inputs to information processing in the model are processors. Therefore, the computational structure is said to be efficient for a given number of data items processed (N), if the number of processors (P) cannot be decreased without increasing the delay (D) or vice-versa (Radner, 1992, 1993).

The structures satisfying the criterion above (called ‘skip-level reporting’ structures) contain P processors (where P is a power of 2) organized in

[6] The value of the computational service in decision-making is measured as a difference between the value of the decisions based on the computational service and the value of the decisions without the service (Radner and Van Zandt, 1992).

hierarchical (multilevel) formations where each processor has one immediate subordinate at every lower level[7] of the hierarchy, and data items belonging to the cohort processed are distributed as equally as possible among all the processors (see Gibbons and Rytter, 1988; or Radner, 1992, for the proof). As an example, the skip-level reporting structure with P=8 processors designed for the summation of N=40 items of data is presented in fig. 4.1. The time diagram describing the computational process in this structure is presented in fig. 4.2..

Assuming that each individual operation takes (small) d units of time, the minimum delay (D) needed to add N data items in the one-shot skip-level reporting structure with P processors is given as

$$D_N = \{ \lfloor \frac{N}{P} \rfloor + \lceil \log_2 (P+N \bmod P) \rceil \} \, d \ ,$$

where brackets $\lfloor \ \rfloor$ and $\lceil \ \rceil$ denote rounding down and up to the nearest integer, respectively.

As mentioned in section 2, in the information processing sector, similarly to other parts of the firm, labor (i.e. managers, accountants, staff, clerks, secretaries, computer engineers, and so forth) and capital (embodied in computers, buildings, telecommunicational channels or other equipment) are involved in the computational process. Thus, the speed of the computation in each individual processing element is assumed to be a function of the capital (k) and labor (l) allocated to it.

The relationship between the resources allocated to an individual processing element and the number of operations it can compute in a unit of time is determined by the existing technology of information processing, and can be written in functional form as F(k,l)[8]: $R_+ x R_+ \rightarrow R_+$, where F(k,l) is continuous, twice differentiable and strictly concave in k and l.

[7] The processor belongs to the level

$$X = \begin{cases} 0 \ , & \text{if it does not have any subordinate processors,} \\ x+1, & \text{otherwise,} \end{cases}$$

where x denotes the highest level of the hierarchy to which one of its immediate subordinate processors belongs (see fig.4.1, for the example).

[8] F(k,l) is called an ‘information-processing function’, and is understood as a ‘production function’ in information-processing.

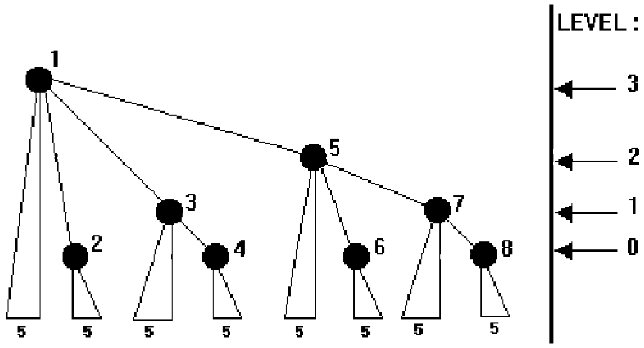

Fig. 4.1
The skip-level reporting structure of information processing (N=40, P=8).[9] (Radner 1992,1993)

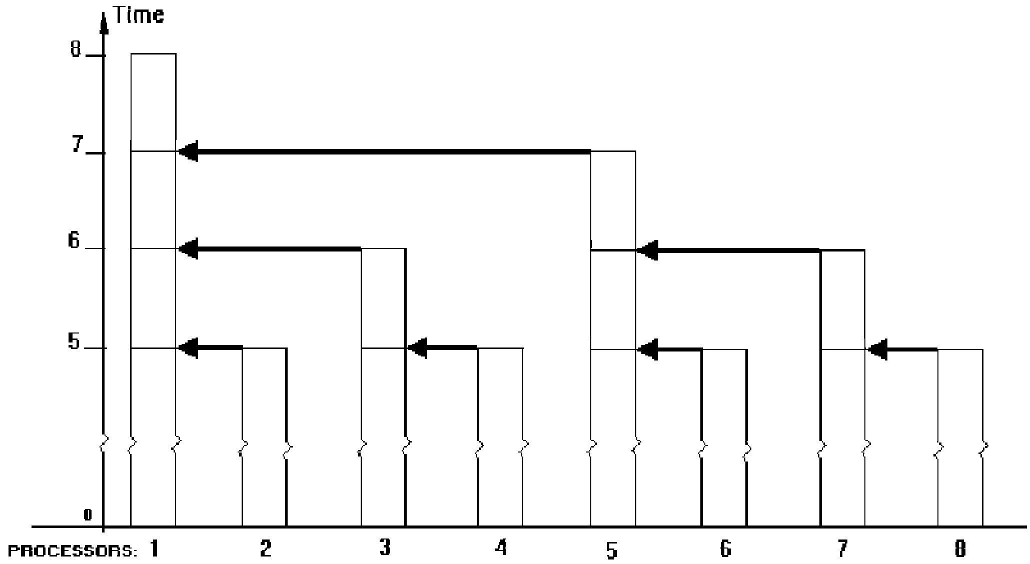

Fig. 4.2 The time diagram corresponding to the information-processing structure presented in fig. 4.1 (d=1).

[9] Every triangle denotes 5 data items.

This implies that the duration of a single operation (d) is also a function of capital (k) and labor (l) employed in the processing element ($d(k,l)=1/F(k,l)$).

If all processing elements of the structure are identical then the duration of each individual operation can be specified as

$$d\left(\frac{K}{P},\frac{L}{P}\right)=\frac{1}{F\left(\frac{K}{P},\frac{L}{P}\right)} \quad ,$$

where K and L denote capital and labor allocated to information processing, respectively, and P is the number of the processing elements in the structure considered.

In any information-processing structure, the delay in summation of N items of data (D_N) is proportional to the durations of the individual operations, and, consequently, is a decreasing function of the resources allocated to the computational sector of the firm, i.e. $\delta D_N(K,L)/\delta K<0$ and $\delta D_N(K,L)/\delta L<0$.

The considerations above show that in the firm, unlike in the computer system, capital (K) and labor (L) are considered as inputs to information processing, while the number of processing elements (P) corresponds to the size of the structure. Consequently, the issue of the efficient organization of information processing in the firm should be analyzed differently than in the model of an idealized parallel computer where the processors with fixed computational power are considered as the only scarce resources.

5. Efficient organization of numerical information processing in the firm

In the framework of the model presented above, the computational process is said to be organized in an efficient way if, for a given number of data items processed (N), it is not possible to get the same delay in information processing (D_N) using less of one input to information processing (i.e. capital or labor) and no more of the other.

The definition above implies that, similar to the computer systems, the skip-level reporting structures of information processing are efficient for decentralized numerical computation in the firm. It has been shown in the literature (see, for example, Gibbons and Rytter, 1988) that in such structures, the smallest number of processors (with fixed computational power) is needed to summarize cohorts of N data items with any given delay D_N. This means that in order to achieve the same delay in information processing in any other computational structure more processors (with the same computational power) or more powerful processors are needed. If more powerful processors are used (i.e. more resources are allocated to them) then the same (or smaller) delay can be achieved with the smaller number of the processors (i.e. using less of the resources) if they are organized as the skip-level reporting structure. Consequently, skip-level reporting structures are efficient for decentralized numerical computation in the firm.

Note that the skip-level reporting structures are efficient only when data processing is decentralized, but this implies that a centralized structure (P=1) could be efficient as well.

Thus, the number of processing elements (P) in an arbitrary efficient structure (centralized or decentralized skip-level reporting) is a power of 2 (i.e. the possible values of P increase very quickly P= 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, ...). On the other hand, P is bounded ($P \leq \lfloor N/2 \rfloor$)[10]. Consequently, for a quite big number of data items processed (N), only information-processing structures with few possible sizes should be considered (for example, if N=67 000 000, then only 25 structures of different size have to be analyzed). This implies that if all processing elements are identical then the efficiency frontier can be simply derived from the following optimization problem:

$$D_N(K,L) = \underset{P}{Min}\ \{\ (\frac{N}{P} + \log_2 P)\, d(\frac{K}{P}, \frac{L}{P})\ \},$$

where P is the number of processing elements in the efficient structure ($P=2^x$, $x=0,1,2,...,\lfloor \log_2(N/2) \rfloor$), d(K/P,L/P) is the duration of a single operation if capital K/P and labor L/P are used in each individual processing element, K and L denote the total amount of the capital and labor used in information processing, respectively.

This expression, however, does not characterize the efficiency frontier if the processing elements could not be identical, i.e. if the resources could be nonequally distributed among the processing elements. To clarify the statement above, consider the skip-level reporting structure of information processing with P=4 processing elements (fig. 5.1) working in a one-shot regime.

[10] This is so because at least two data items have to be assigned to each processing element of the structure.

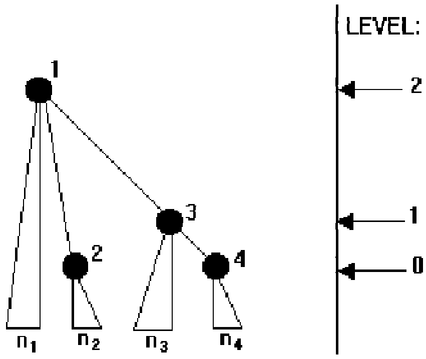

Fig. 5.1 The skip-level reporting structure with P=4 processing elements.

Assume that the information workload of the processing elements is given by the vector (n_1,n_2,n_3,n_4), such that $n_1+n_2+n_3+n_4=N$, where n_j denotes the number of data items assigned to the processing element j (j=1,2,3,4), and suppose that data items cannot be equally distributed among the processing elements of the structure, e.g., that $n_1=\lfloor N/P \rfloor+1$ and $n_2=n_3=n_4=\lfloor N/P \rfloor$.

If all processing elements of the structure considered are identical, then the partial results computed in the processing elements with numbers 2 and 3 cannot be immediately used for the remaining computations (see fig. 5.2). The waiting states can be eliminated if the computational power of the processing elements is adjusted to the given information workload. The time diagram describing the computational process in the structure with nonidentical processing elements is presented in fig. 5.3.

In order to eliminate the states in which the partial results computed are waiting for the remaining computations (in the structure under study) the following conditions have to be satisfied[11]:

$$n_1 d_1 = n_2 d_2 \ ,$$

[11] Each condition corresponds to one communication channel in the structure (or to one arrow on the time diagram).

$(n_1+1)d_1= (n_3+1)d_3$,
$n_3d_3= n_4d_4$,

where d_j denotes the duration of a single operation performed by the processing element j (j=1,2,3,4). It turns out that the operations performed at the top-level processing element have to be such that

$d_1=(n_2/n_1)d_2$,
$d_1=[(n_3+1)/(n_1+1)]d_3$,

and

$d_1=(n_4/n_3)[(n_3+1)/(n_1+1)]d_4$.

This implies that if $n_1>n_2=n_3=n_4$ then $d_1<d_2,d_3,d_4$. Consequently, $d_1<d(K/P,L/P)$, and the total delay in information processing ($D_N=(n_1+\log_2P)d_1$) is smaller than in the case where $d_1=d(K/P,L/P)$, and all processing elements are identical.

Consider now the skip-level reporting structure of information processing of the optional size (P). Assume that the vector $(n_1,n_2,...,n_P)$, such that $n_1+n_2+...+n_P=N$, describes the information workload of the processing elements enumerated according to a recursive procedure: NUMBERING(J,L)[12]. The algorithm of this procedure is presented below[13]:

Step 1. Set the level (i) of the immediate subordinate processing element equal to zero (i.e. set i=0);
Step 2. Assign the number $J+2^i$ to the immediate subordinate processing element of the computational center J, on the level i;
Step 3. If i>0 then call (recursively) the procedure NUMBERING($J+2^i$,i);
Step 4. Increase the level of the immediate subordinate processing element, i.e. set i=i+1;
Step 5. If i<L (where L is the level of the element J) then execute step 2.

[12] To enumerate the processing elements in the skip-level reporting structure, one has to assign the number 1 to the top-level processor, and call the procedure NUMBERING(J,L) with parameters J=1 and $L=\log_2P$.

[13] The processing elements in the structures presented in fig.4.1 or fig.5.1 are enumerated according to this procedure.

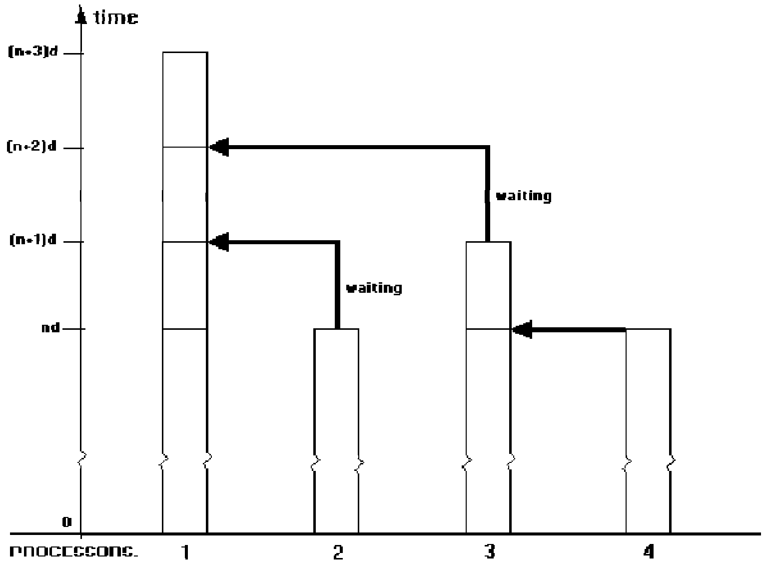

Fig. 5.2.
The time diagram of the computational process in the skip-level reporting structure with information workload (n_1,n_2,n_3,n_4), such that n_2=n_3=n_4=n and n_1=n+1, when all processing elements are identical.

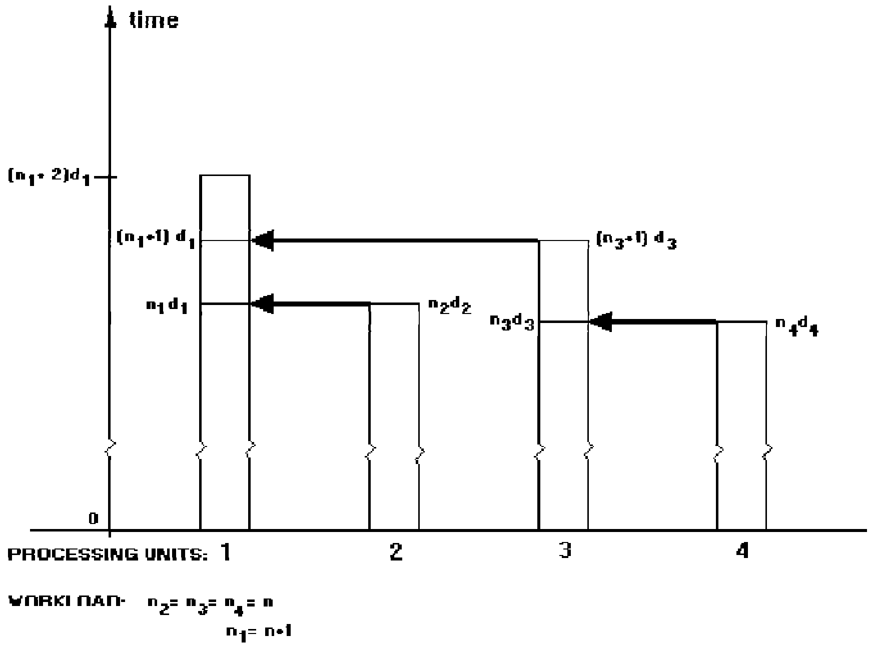

Fig. 5.3
The time diagram of the computational process in the skip-level reporting structure with information workload (n_1,n_2,n_3,n_4), such that n_2=n_3=n_4=n and n_1=n+1, when the processing elements are nonidentical.

For the optional information workload $(n_1,n_2,...,n_P)$, the waiting states are eliminated from the computational process in the skip-level structure of size P, if

$$(n_m+z)\,d_m=(n_{(m+2^z)}+z)\,d_{(m+2^z)} \quad ,$$

m=2b-1 (b=1,2,...,P/2), and z=0,1,...,Level(m)-1, where Level(m) denotes the level of processing element m in the structure considered.

Represent the duration of the individual operation performed in the processing element j (j=1,2,...,P) as

$$d_j(k_j,l_j)=\frac{1}{F(k_j,l_j)} \quad ,$$

where $k_j=\alpha_j K/P$ and $l_j=\beta_j L/P$; K and L denote capital and labor allocated to information processing, respectively; P is the number of processing elements in the structure; α_j and β_j (j=1,2,...,P) are coefficients of the adjustment of capital and labor to the given information workload, such that

$$K=\sum_{j=1}^{P}\alpha_j\frac{K}{P} \quad , \quad or \quad \left(P=\sum_{j=1}^{P}\alpha_j \right) \quad ,$$

and

$$L=\sum_{j=1}^{P}\beta_j\frac{L}{P} \quad , \quad or \quad \left(P=\sum_{j=1}^{P}\beta_j \right) \quad .$$

If the information-processing structure, S (skip-level reporting with P processing elements), and the information workload, N, are given, then the objective function of the firm in information processing (see section 3) can be represented as

$$\underset{k_j,l_j}{Min}\left\{ E(\Psi(D_{N,S}(F(k_j,l_j))) + r\frac{k_j}{\alpha_j}P + w\frac{l_j}{\beta_j}P \right\} \quad ,$$

where $K=k_jP/\alpha_j$ and $L=l_jP/\beta_j$, j=1,2,...,P. From the first order conditions one can find that

$$\frac{w}{r}=\frac{\beta_j}{\alpha_j}\frac{\frac{\delta F(k_j,l_j)}{\delta l_j}}{\frac{\delta F(k_j,l_j)}{\delta k_j}}\ ,$$

where w is the price of labor and r is the price of capital. On the other hand, the cost-minimizing input combination in each individual processing element j (j=1,2,...,P) is at the point where the ratio of the marginal productivities of the labor and capital allocated to the processing element equals to the ratio of the market prices of the corresponding resources, i.e.

$$\frac{w}{r}=\frac{\frac{\delta F(k_j,l_j)}{\delta l_j}}{\frac{\delta F(k_j,l_j)}{\delta k_j}}\ .$$

This implies that the ratio β_j/α_j equals 1 (j=1,2,...,P), and, consequently, that $\alpha_j=\beta_j$, for j=1,2,...,P.

Thus, the coefficients α_j (j=1,2,...,P), as functions of the information workload $(n_1,n_2,...,n_P)$, can be derived from the following system of equations[14]:

$$\sum_{j=1}^{P}\alpha_j=P\ ,$$

$$\frac{n_m+z}{F(\alpha_m\frac{K}{P},\alpha_m\frac{L}{P})}=\frac{n_{(m+2^z)}+z}{F(\alpha_{(m+2^z)}\frac{K}{P},\alpha_{(m+2^z)}\frac{L}{P})}\ ,$$

where m=2b-1 (b=1,2,...,P/2), and z=0,1,...,Level(m)-1.

An example of the adjustment of resources in the one-shot skip-level reporting structure of information processing with P=4 processing elements, to the information workload (n_1,n_2,n_3,n_4), when the processing elements' information-processing function has a Cobb-Douglas form, is presented in Appendix 1.

The consideration above implies that the efficiency frontier in information processing in the firm can be derived from the following optimization problem:

[14] Note that the second expression specifies P-1 equations.

$$D_N(K,L)=\underset{P}{Min}\{\frac{n_1+\log_2 P}{F(\alpha_1(n_1,n_2,...,n_P)\frac{K}{P},\alpha_1(n_1,n_2,...,n_P)\frac{L}{P})}\},$$

where P is the number of processing elements in the efficient structure ($P=2^x$, $x=0,1,2,...,\lfloor\log_2(N/2)\rfloor$); K and L denote the total amounts of capital and labor used in information processing, respectively; and $\alpha_1(n_1,n_2,...,n_P)$ is the coefficient of adjustment of the resources allocated to the top-level processing element to the information workload $(n_1,n_2,...,n_P)$.

As an example, the efficiency frontier, for N=50 data items and information-processing function $F(k,l)=k^{\alpha}l^{\beta}$ ($\alpha=\beta=0.25$), is presented in fig. 5.4.. Fig. 5.5 demonstrates the relationships between the resources used in order to summarize N=50 data items with the fixed delay ($D_N=5$) in the skip-level reporting structures with P=8 identical and nonidentical processing elements.

The curves presented in fig. 5.5 show explicitly that a given delay in information processing can be achieved with the smaller amounts of the resources if they are nonequally distributed across the processing elements of the structure. But this implies that the skip-level reporting structures with identical computational centers cannot be (in general) considered as efficient. Such structures, however, remain efficient if the information workload of the processing elements is equalized (i.e. when $\alpha_j=\beta_j=1$, for j=1,2,...,P)[15].

6. Optimal organization of information processing in the firm for the purpose of predicting demand (an example of analysis)

To illustrate the concept of the optimal organization of numerical data processing in enterprises, consider an example of the firm in which demand for its production is estimated. Assume that the technology of information processing is described by the following information-processing function:

$$F(k,l)=k^{\alpha}l^{\beta}\ ,$$

where α and β are constant coefficients.

[15] See Appendix 1, for the example.

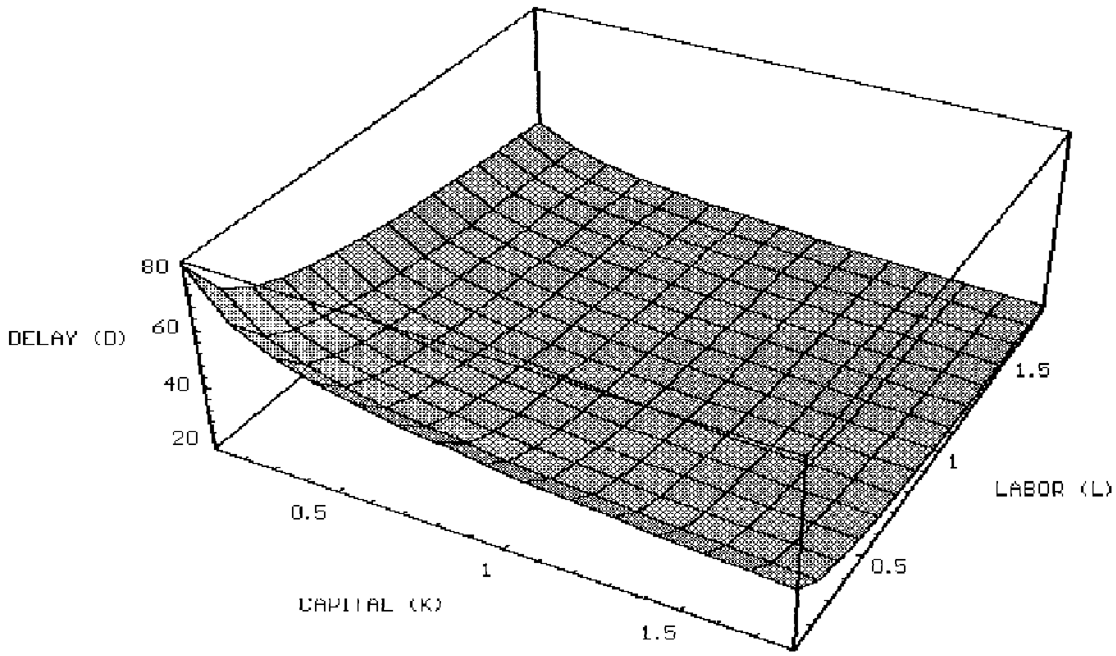

Fig. 5.4
Information-processing efficiency frontier for N=50, $F(k,l)=k^{\alpha}l^{\beta}$ (α=0.25, β=0.25).

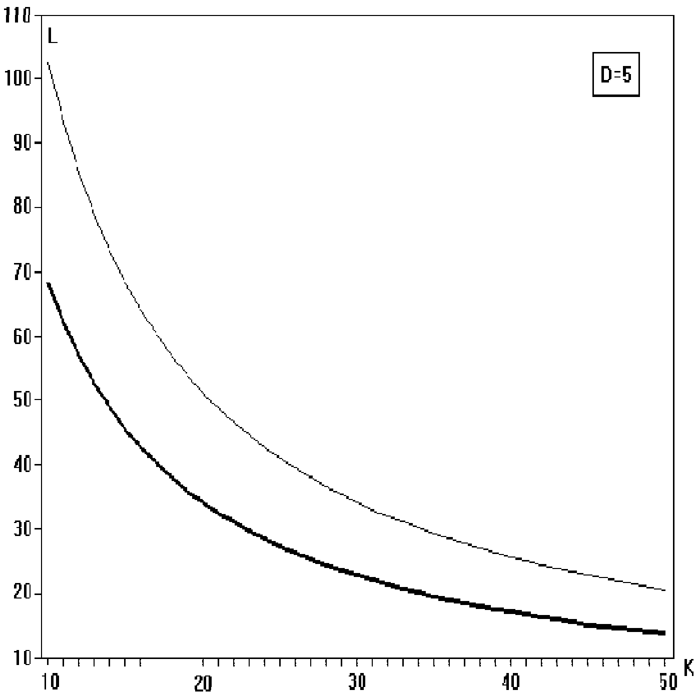

Fig. 5.5
Indifference curves (D=5, N=50) in the skip-level reporting structures with P=8 identical (thin curve) and nonidentical (thick curve) processing elements.

Suppose that the loss due to the prediction error is proportional to the square of the difference between the estimation of demand in moment t (A_t) and the real value of demand (Q_t), i.e. $\Psi=(A_t-Q_t)^2$. Assume also that the stochastic processes generating demands $Q_{i,t}$ (i=1,2,...,N, and t is an integer number, $-\infty<t<+\infty$) are independent and identically distributed, specified as follows:

$$Q_{i,t} = \mu + X_{i,t} ,$$

where μ is the mean value of demand, and $X_{i,t}$ is the difference between $Q_{i,t}$ and its mean described as a first order autoregressive process:

$$X_{i,t} = \gamma X_{i,t-1} + \varepsilon_{i,t}, \quad (|\gamma|<1),$$

where $\varepsilon_{i,t}$ are independent and identically distributed Gaussian variables with mean equal to zero and variance ω^2. The variance (ξ^2) of each individual stochastic process around its mean is

$$\xi^2 = E(X_{i,t}^2) = \frac{\omega^2}{1-\gamma^2} .$$

The demand estimation in moment t, done on the basis of the history of process $X_{i,t}$ up to date (t-s), is given as $\gamma^s X_{i,t-s}$.

The expected value of the square of the error in estimation (for each individual source of demand) is

$$E[(\gamma^s X_{i,t-s}-X_{i,t})^2]=(1-\gamma^{2s})\xi^2.$$

If demand coming from N data sources is estimated with lag s, then the expected value of the loss due to the prediction error (Ψ_N) equals

$$\Psi_N(s) = N(1 - \gamma^{2s})\xi^2 .$$

Taking $s=D_{N,S_P^*}$, where D_{N,S_P^*} is the delay in information processing in an efficient structure with P processing elements, the expected value of the loss due to prediction error is

$$\Psi(D_{N,S_P^*}(K,L)) = N(1-\gamma^{2D_{N,S_P^*}(K,L)})\xi^2 .$$

The delay in information processing in the efficient structure with P processing elements is given as follows:

$$D_{N,S_P^*}(K,L) = \frac{n_1+\log_2 P}{F(\alpha_1\frac{K}{P},\beta_1\frac{L}{P})} ,$$

where n_1 is the number of data items assigned to the top-level processing element ($n_1 = \lfloor N/P \rfloor$, if ($N \bmod P$)=0, or $n_1 = \lfloor N/P \rfloor + 1$, otherwise), α_1 and β_1 are coefficients of adjustment of resources allocated to the top-level processing element to the information workload of the structure S_P^*. The expected value of the loss due to the prediction error is therefore

$$\Psi(D_{N,S_P^*}(K,L)) = N(1-\gamma^{2\frac{n_1+\log_2 P}{F(\alpha_1 \frac{K}{P},\, \beta_1 \frac{L}{P})}})\,\xi^2 .$$

Taking into account that the information-processing function in each processing element of the structure is $F(k,l)=k^{\alpha}l^{\beta}$ (α and β are constant coefficients), and, consequently, that $\alpha_1=\beta_1$ (see section 5), the expected value of the deviation from the highest profit caused by noninstantaneous information processing in an efficient structure (S_P^*) is specified by the following expression:

$$\Phi_{N,S_P^*}(K,L) = N(1-\gamma^{2\frac{(n_1+\log_2 P)\,P^{\alpha+\beta}}{\alpha_1^{\alpha+\beta} K^{\alpha} L^{\beta}}})\,\frac{\omega^2}{1-\gamma^2} + rK + wL \; .$$

Therefore, the optimal size of the efficient information-processing structure and the optimal allocation of resources should be derived from the following optimization problem:

$$\underset{P}{Min}\,\{\,\underset{K,L}{Min}[N(1-\gamma^{2\frac{(n_1+\log_2 P)\,P^{\alpha+\beta}}{\alpha_1^{\alpha+\beta} K^{\alpha} L^{\beta}}})\,\frac{\omega^2}{1-\gamma^2} + rK + wL\,]\,\}\,,$$

where $P=2^x$, $x=0,1,2,\ldots,\lfloor \log_2(N/2) \rfloor$; $n_1 = \lfloor N/P \rfloor$, if ($N \bmod P$)=0, or $n_1 = \lfloor N/P \rfloor + 1$, otherwise.

To illustrate the concept of the optimal organization of information processing in the firm, consider the particular shape of the information-processing function specified by $\alpha=0.25$, $\beta=1-\alpha=0.75$ (a Cobb-Douglas information-processing function), and assume that $N=50$, $\gamma^2=0.5$, $\omega^2=1$, $r=0.01$ and $w=0.01$.

The total number of sources of demand ($N=50$) implies that only five possible organizational structures should be considered ($P=1,2,4,8,16$). The relationships between the loss due to noninstantaneous and costly information processing (Φ) and the resources (K,L) used in computation in the structures with $P=1,2,4,8,16$ processing elements are presented in fig. 6.1.

The optimal size of the structure is $P^*=1$ (a centralized structure), the optimal amounts of capital and labor equal $K^*=187.20$ and $L^*=561.61$, respectively. The optimal value of the objective function is $\Phi^*=15.29$.

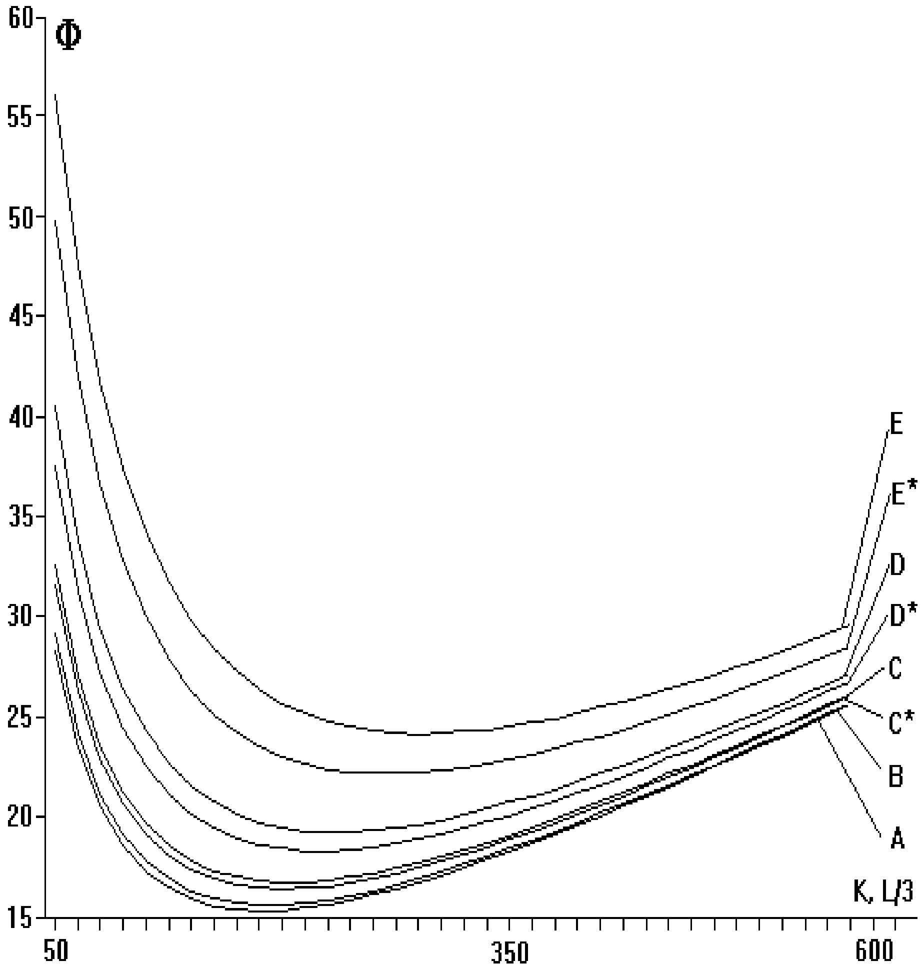

Fig. 6.1.
The relationships between the loss, caused by non-instantaneous and costly information processing (Φ), and the resources (K,L) used in computation (N=50, $\gamma^2=0.5$, $\omega^2=1$, r=w=0.01, $F(k,l)=k^{0.25}l^{0.75}$, where k=K/P, l=L/P),

A: P=1 ($K^*=L^*/3=187$, $\Phi^*=15.3$),
B: P=2 ($K^*=L^*/3=191$,$\Phi^*=15.6$)
C: P=4 ($K^*=L^*/3=204.4$, $\Phi^*=16.72$),
C^*: P=4, $(n_1,n_2,n_3,n_4)=(13,12,13,12)$, $\alpha_1=\beta_1=1.04$, ($K^*=L^*/3=200.46$, $\Phi^*=16.40$),
D: P=8 ($K^*=L^*/3=234.4$, $\Phi^*=19.23$),
D^*: P=8, $(n_1,n_2,n_3,n_4,n_5,n_6,n_7,n_8)=(7,6,6,6,7,6,6,6)$, $\alpha_1=\beta_1=1.11$, ($K^*=L^*/3=223.2$, $\Phi^*=18.29$),
E: P=16 ($K^*=L^*/3=291.84$, $\Phi^*=24.16$),
D^*: P=16, $(n_1,...,n_{16})=(4,3,3,3,3,3,3,3,4,3,3,3,3,3,3,3)$, $\alpha_1=\beta_1=1.2$, ($K^*=L^*/3=268.8$, $\Phi^*=22.15$)

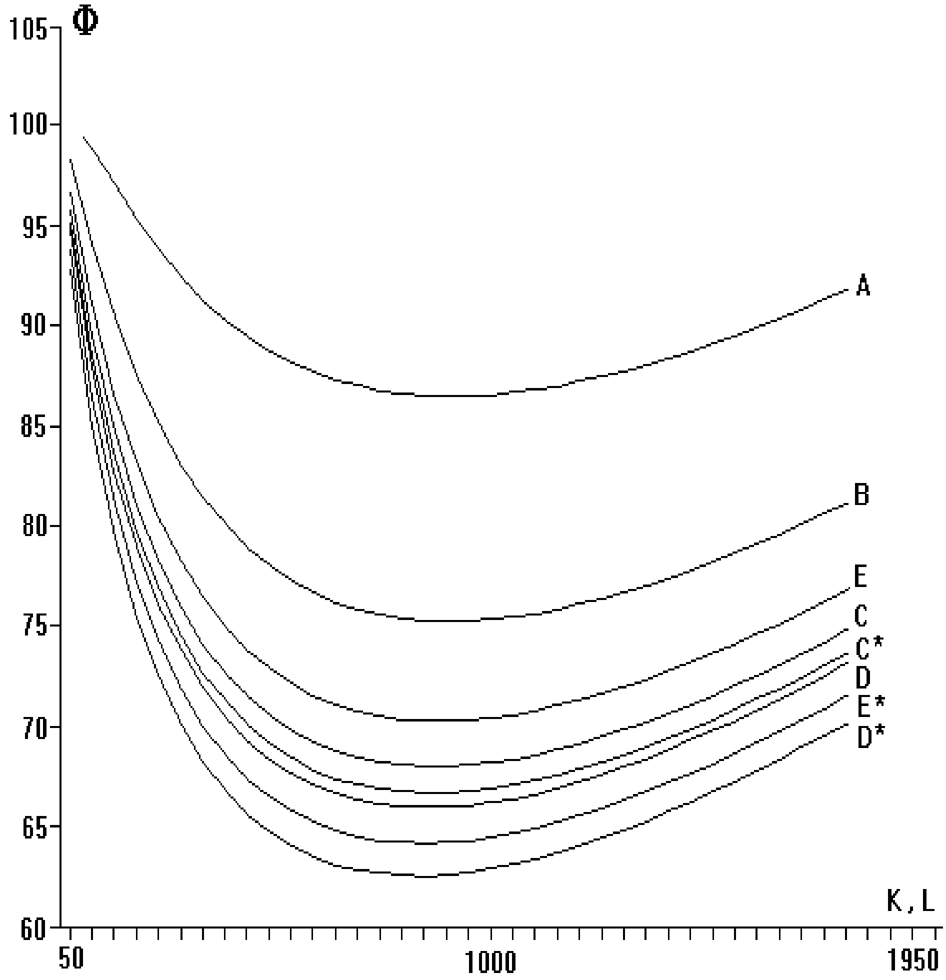

Fig. 6.2
The relationships between the loss, caused by non-instantaneous and costly information processing (Φ), and the resources (K,L) used in computation (N=50, γ^2=0.5, ω^2=1, r=w=0.01, F(k,l)=$k^{0.25}l^{0.25}$, where k=K/P, l=L/P),

A: P=1 (K^*=L^*=911, Φ^*=86.5),
B: P=2 (K^*=L^*=908, Φ^*=75.24)
C: P=4 (K^*=L^*=872, Φ^*=67.99),
C^*: P=4, (n_1,n_2,n_3,n_4)=(13,12,13,12), α_1=β_1=1.08, (K^*=L^*=862, Φ^*=66.66),
D: P=8 (K^*=L^*=856.98, Φ^*=65.95),
D^*: P=8, (n_1,n_2,n_3,n_4,n_5,n_6,n_7,n_8)=(7,6,6,6,7,6,6,6), α_1=β_1=1.22, (K^*=L^*=831.2, Φ^*=62.53),
E: P=16 (K^*=L^*=884.8, Φ^*=70.25),
D^*: P=16, (n_1,...,n_{16})=(4,3,3,3,3,3,3,3,4,3,3,3,3,3,3,3), α_1=β_1=1.42, (K^*=L^*=843.7, Φ^*=64.15)

The curves presented in fig. 6.2 correspond to N=50, γ^2=0.5, ω^2=1, r=0.01, w=0.01 and α=β=0.25 (i.e. $F(k,l)=k^{0.25}l^{0.25}$). In this case the optimal information-processing structure is decentralized (skip-level reporting) with P^*=8 nonidentical processing elements (α_1=1.22), the optimal amounts of the resources used are equal K^*= L^*=831.20, and the optimal value of the objective function is Φ^*= 62.53.

The examples considered confirm that the optimal organizational structures of numerical data processing in the firm do not necessarily have to be decentralized. Moreover, they show that the size (P^*) of the efficient information-processing structures depends upon the shape of the information-processing function, but the detailed analysis of the conditions under which decentralized structures are efficient is left for further investigation.

7. Conclusion

The analysis of information processing in the firm has appeared frequently in the economic literature. In the most recent papers, data processing in the firm has been described in terms of a dynamic parallel processing model of associative computation. This model has been directly adopted from the computer science literature, and, consequently, its conceptual framework differs from that which is usually used in microeconomic research. The present paper shows how information processing in the firm should be described and analyzed in a microeconomic framework.

The analysis focuses on numerical computations in the firm for the purpose of predicting demand. Information processing is modelled using a dynamic parallel processing model of associative computation extended to include the assumption that the speed of computation in each individual processing element is determined by the capital and labor allocated to it. To describe the relationship between the resources allocated to a single computational center and its processing power, the concept of an information-processing function is introduced. For such a model, the efficiency criterion is defined and the architecture of the efficient structures of numerical data processing is analyzed. The paper shows that, in the firm, similar to parallel computers, so-called skip-level reporting structures are efficient. However, in the case when the information workload of the processing elements cannot be equalized, the computational power, and, consequently, the resources allocated to the processing elements, have to be adjusted to the given information workload. Finally, based on the examples of numerical computation for the purpose of predicting demand in the firm, it is shown that the size of the optimal computational structures is determined by the form of the information-processing function, and, consequently, the optimal structures of numerical data processing in the firm do not necessarily have to be decentralized.

The main contribution of this paper to the current research in the theory of the firm is the introduction of the concept of the information-processing function to the dynamic parallel processing model of associative computation. This concept provides the same methodological framework for the analysis of the information processing in the management and production sectors of the firm, and allows one to employ the model presented for the study of more complex economic issues in which these parts of the firm have to be described separately but analyzed together.

APPENDIX

ADJUSTING RESOURCES TO A GIVEN INFORMATION WORKLOAD IN A SIMPLE ONE-SHOT SKIP-LEVEL REPORTING STRUCTURE

Consider a skip-level reporting structure of information processing with P=4 processing elements (as in fig. 5.1) working in the one-shot regime. Assume that cohorts of N items of data are summarized, and data items are distributed among the processing elements of the structure as (n_1,n_2,n_3,n_4), where $n_1+n_2+n_3+n_4=N$, and n_j denotes the number of data items assigned to the processing element j (j=1,2,3,4). Suppose that the processing elements' information-processing function has a Cobb-Douglas form:

$$F(k_j,l_j)= k_j^{\gamma} l_j^{1-\gamma},$$

where γ is a constant coefficient, such that $0<\gamma<1$, (j=1,2,3,4).

The delay of a single operation performed in the processing element j is specified as follows:

$$d_j=d(k_j,l_j)=\frac{1}{F(k_j,l_j)}=k_j^{-\gamma} l_j^{\gamma-1} ,$$

where $k_j=\alpha_j K/P$ and $l_j=\beta_j L/P$ denote capital and labor allocated to the processing element j (j=1,2,3,4), respectively; α_j and β_j are the coefficients of adjustment of resources to a given information workload, such that

$$P=\sum_{j=1}^{P}\alpha_j \quad and \quad P=\sum_{j=1}^{P}\beta_j , \quad (P=4).$$

Taking into account that $\alpha_j=\beta_j$ (j=1,2,3,4)[16], the duration of a single operation (d_j) can be represented as

$$d_j = (\alpha_j \frac{K}{P})^{-\gamma}(\beta_j \frac{L}{P})^{\gamma-1} = \alpha_j^{-1}(\frac{K}{P})^{-\gamma}(\frac{L}{P})^{\gamma-1}.$$

Thus, coefficients $\alpha_j=\beta_j$ (j=1,2,3,4) can be determined from the following system of equations:

[16] See section 5, for details.

$$n_1\,\alpha_1^{-1}\,(K/P)^{-\gamma}\,(L/P)^{\gamma-1} = n_2\,\alpha_2^{-1}\,(K/P)^{-\gamma}\,(L/P)^{\gamma-1}$$

$$(n_1+1)\,\alpha_1^{-1}\,(K/P)^{-\gamma}\,(L/P)^{\gamma-1} = (n_3+1)\,\alpha_3^{-1}\,(K/P)^{-\gamma}\,(L/P)^{\gamma-1}$$

$$n_3\,\alpha_3^{-1}\,(K/P)^{-\gamma}\,(L/P)^{\gamma-1} = n_4\,\alpha_4^{-1}\,(K/P)^{-\gamma}\,(L/P)^{\gamma-1}$$

$$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = P$$

The solution to the above system of equations can be represented as

$$\alpha_1(n_1,n_2,n_3,n_4) = \frac{P}{1+\frac{n_2}{n_1}+\frac{n_3+1}{n_1+1}\left(1+\frac{n_4}{n_3}\right)},$$

$$\alpha_2(n_1,n_2,n_3,n_4) = \frac{n_2}{n_1}\,\frac{P}{1+\frac{n_2}{n_1}+\frac{n_3+1}{n_1+1}\left(1+\frac{n_4}{n_3}\right)},$$

$$\alpha_3(n_1,n_2,n_3,n_4) = \frac{n_3+1}{n_1+1}\,\frac{P}{1+\frac{n_2}{n_1}+\frac{n_3+1}{n_1+1}\left(1+\frac{n_4}{n_3}\right)},$$

$$\alpha_4(n_1,n_2,n_3,n_4) = \frac{n_4}{n_3}\,\frac{n_3+1}{n_1+1}\,\frac{P}{1+\frac{n_2}{n_1}+\frac{n_3+1}{n_1+1}\left(1+\frac{n_4}{n_3}\right)}.$$

Consequently, for the summation of N items of data in the skip-level structure with P=4 computational centers, the capital (K) and labor (L) used in information processing should be distributed among the processing elements of the structure according to the following expressions:

$$k_1 = \frac{K}{1+\frac{n_2}{n_1}+\frac{n_3+1}{n_1+1}\left(1+\frac{n_4}{n_3}\right)}, \qquad l_1 = \frac{L}{1+\frac{n_2}{n_1}+\frac{n_3+1}{n_1+1}\left(1+\frac{n_4}{n_3}\right)},$$

$$k_2=\frac{n_2}{n_1}\frac{K}{1+\frac{n_2}{n_1}+\frac{n_3+1}{n_1+1}(1+\frac{n_4}{n_3})}, \qquad l_2=\frac{n_2}{n_1}\frac{L}{1+\frac{n_2}{n_1}+\frac{n_3+1}{n_1+1}(1+\frac{n_4}{n_3})},$$

$$k_3=\frac{n_3+1}{n_1+1}\frac{K}{1+\frac{n_2}{n_1}+\frac{n_3+1}{n_1+1}(1+\frac{n_4}{n_3})}, \qquad l_3=\frac{n_3+1}{n_1+1}\frac{L}{1+\frac{n_2}{n_1}+\frac{n_3+1}{n_1+1}(1+\frac{n_4}{n_3})},$$

$$k_4=\frac{n_4}{n_3}\frac{n_3+1}{n_1+1}\frac{K}{1+\frac{n_2}{n_1}+\frac{n_3+1}{n_1+1}(1+\frac{n_4}{n_3})}, \qquad l_4=\frac{n_4}{n_3}\frac{n_3+1}{n_1+1}\frac{L}{1+\frac{n_2}{n_1}+\frac{n_3+1}{n_1+1}(1+\frac{n_4}{n_3})}.$$

The example considered shows explicitly that if $n_1=n_2=n_3=n_4=n=N/P$ then $\alpha_1=\alpha_2=\alpha_3=\alpha_4$, and, consequently, the resources (i.e. capital and labor) are equally distributed among the processing elements of the structure, and all the processing elements have the same computational power.

Moreover, if $n=N/P$ is sufficiently big, then the resources are distributed among the processing elements almost equally, i.e.

$$\lim_{n\to\infty} k_1/k_2=\lim_{n\to\infty}(n+1)/n=1,$$

and

$$\lim_{n\to\infty} k_1/k_3=\lim_{n\to\infty} k_1/k_4=\lim_{n\to\infty}(n+2)/(n+1)=1.$$

It follows that adjustment of the resources allocated to the computational centers in the structure considered to a given information workload is much more important when small cohorts of data are processed than otherwise.

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