The Adiabatic Limit of the Laplacian on Thin Fibre Bundles

Jonas Lampart

We consider the Laplacian on a Riemannian submersion with compact fibres $F \to M \xrightarrow{\pi} B$ with an ε-scaled metric $g_\varepsilon = g_F + \varepsilon^{-2}\pi^* h$. We allow F to have a boundary, in which case we use the Dirichlet Laplacian. Under suitable boundedness conditions on the geometry of M and π we show that the eigenspaces of the Laplace Beltrami operators of $g|_{TF_x}$ induce a vector bundle with a Berry connection over the base B. On the sections of this bundle we construct an effective operator H_{eff} whose unitary evolution is a good approximation of that generated by the Laplacion of g_ε as $\varepsilon \to 0$.

We explain how these results can be used to obtain natural generalisations of thin tubes and other quantum wave guides.

University of Tübingen, Institute of Mathematics, Auf der Morgenstelle 10, 72076 Tübingen, Germany

E-mail address: jjlampart@maphy.uni-tuebingen.de