Symmetry decomposition of functions on compact semisimple Lie groups

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The familiar demposition

$$f(x) = f_s(x) + f_a(x)\,, \qquad 0 \leq x \leq 1$$

of a given function $f(x)$ into its symmetric and antisymmetric parts,

$$f_s(x) = \tfrac{1}{2}(f(x) + f(1-x))\,, \qquad f_a(x) = \tfrac{1}{2}(f(x) - f(1-x))\,,$$

can be interpreted as the central decomposition of a class function $f(x)$ on $SU(2)$.

In the talk we describe central decomposition of class functions $f(x_1, x_2, \ldots, x_n)$ on a compact semisimple Lie group G of rank $n < \infty$ and of any type, into as many symmetry components as is the order of the center of G. Such decomposition is either continuous if $x_1, x_2, \ldots, x_n \in \mathbb{R}$, or discrete if the variables specify a point of an n-dimensional lattice L_M of symmetry compatible with G and of any density M.

Examples of central decompositions of functions on $SU(2) \times SU(2)$, $SU(3)$, $Sp(4)$, $E(6)$, and some useful properties of the component functions will be pointed out.