Change Detection in Nonstationary Time Series in Linear Regression Framework ¹ Software Support and Applications

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Introduction

This Technical Report represents the second phase of the grant entitled "Change Detection in Nonstationary Time Series in Linear Regression Framework", received in 2002 from CERGE-EI Foundation under a program of Global Development Network.

The main goal of this project was to give a unified framework for the design and performance evaluation of some algorithms and methods for solving change detection problem in time series with application in econometrics. The following objectives have been taken into account:

- 1. To establish a methodological approach to deal with change detection in time series with application in the field of economics.
- 2. To evaluate the performances of some algorithms and methods for change detection in time series, presented in the literature, and to develop new methods and algorithms.
- 3. To design an integrated software support, implementing the best methods and algorithms for change detection in time series.
- 4. To prove the implemented methods and algorithms on case studies in the field of economics.

The Technical Report representing the first phase of the grand (Popescu 2002) had as subject some of the best algorithms and methods for change detection in nonstationary time series. Also, some numerical results for the discussed methods have been studied by simulation, to rank the methods under consideration.

The second phase of the grant is dedicated to a software support implementing the best methods and algorithms for change detection in nonstationary time series. Performance evaluation of some methods, by simulation, to investigate their robustness, constitutes a distinct part of this phase. Also, some case studies in the fields of economics, using real data are reported.

The report includes three main chapters, Chapter 2 has as objective to give an overview on the CHANGE program package for detection of changes, implemented on personal computers compatible IBM/PC. Chapter 3 is devoted to some experimental results, obtained via simulation, for the test statistics, when the change detection methods based on quadratic forms are used. Also, the robustness of the methods, as to the assumption of autoregressive data and to the model structure is discussed. Chapter 4 presents the results obtained in segmentation of some nonstationary financial and economic time series using the algorithms based on "distance" measure, quadratic forms and Kitagawa-Akaike method.

Software support for change detection

The objective of this chapter is to give an overview on the CHANGE program package for detection of changes, implemented on personal computers compatible IBM/PC. The capabilities of CHANGE package make it a powerful, user-friendly and computationally efficient software package. Computational algorithms have been selected so that the user can have complete confidence in the results of its use. All CHANGE data processing is controlled through a fully integrated menu-driven environment. The package is file oriented; it enables the user to create a large collection of numerical data and descriptive information which can be easily maintained, modified, copied and stored. It is compatible with the IDPACK/PC program package (Popescu, 1991) for system identification and time series analysis. So these packages can import and export data between them and sometimes they are used together in solving of change detection in different applications.

2.1 Software structure

CHANGE software package includes three main components:

- MES Multilevel Exploitation Subsystem
- DMS Data Management Subsystem
- APS Application Programs Subsystem

that function together to provide a full range of capabilities for handling, processing of data and presenting the results.

MES offers the frame in which may be loaded and executed data management and application programs, with a menu-driven environment that eliminates the need to memorize or look up a series of special commands for each program or function the user want to perform.

One of the most important aspects of the package is its file orientation. All the application programs that require data read it, generally, from data files created by **DMS**. The data may be listed, checked and, if necessary edited prior to use. Also, the data can be analysed by several programs and new files can be created by partitioning and merging existing data files. These options are given in **Database Manager Menu**.

APS contains a comprehensive collection of procedures for change detection in signals and systems, in time and frequency domain, but and other programs for preliminary analysis and investigation of the signals and systems (data filtering, spectral analysis, parameter estimation, simulation, etc.). All these functions are given in **CHANGE Master Menu**.

2.2 CHANGE functions

In the following we present the main functions of the package CHANGE by the options implemented in the subsystems MES, DMS and APS.

2.2.1 Multilevel Exploitation Subsystem

The MES offers the user the following facilities:

- data base and application programs starting and selection.
- advancing to a new menu.
- returning to the previous menu.
- displaying a "Help" file associated to the current menu.
- exit from the current menu and returning in the operating system.

Thus, starting from a **Master Menu**, the user has the possibility, specifying a figure or a letter to execute any data base or application program, to obtain information about system or program functions, or to return in any point of the menu tree.

CHANGE Master Menu

1. Database Manager	2. Data Generation
3. Discrete Simulation	4. Data Filtering
5. Spectral Analysis	6. Parameter Estimation
7. Change Detection in Data	8. Change Detection in Systems
9. Graphics	

H - Help Ctrl/C - Exit

R - Return to previous menu

Choice —>

The system can be used by following instructions and prompts displayed on the monitor, having the feature of a user friendly software package.

2.2.2 Data Management Subsystem

CHANGE was designed to be a file oriented program package. **DMS** creates all the data files for subsequent use in the application programs. Each data file, created by **DMS**, produces two data files. The first is called header file and contains information about the number of variables and cases, variable names, a file label and the name of the associated file containing numerical data. The second file is a standard random access file that contains the actual

numeric data. A numeric data file can be thought as a table or matrix of numbers. Each column is defined as a variable and each row represent one case, or observation, expressed in some unit or measurement. The menu for **DMS** is presented below:

Database Master Menu

1. Enter Data	2. List Data
3. Edit Data	4. Edit File Header
5. Delete File	6. Move/Merge/Transform
7. Delete Samples	8. Vertical Augment

H - Help Ctrl/C - Exit

R - Return to previous menu

Choice —>

The options mentioned above have the following functions:

- Enter Data permits the user to create a new file or to add data to an existing file.
- List Data is to list all variables or just a subset of the variables, all of cases or a subset of cases.
- Edit Data allows changes to be made in individual cases.
- Edit File Header serves two purposes: first, it allows display of the file header (i.e. number of variables, number of cases, variables names and file label); second, the program may be used to change the variable names and/or the file label.
- **Delete Files** is simply a convenience feature in that it allows files to be destroyed while in the program mode.
- Move/Merge/Transform performs three different functions. Move simply refers to transferring data from one file to another. Merge is the same as Move except that the variables are selected from two different files. New variables may be created by performing Transform function on variables from input file(s).
- Vertical Augment allows some cases from one file to be appended to another file and to create a new file.

All the programs included in CHANGE read the header file first, when a data file is need, and display its content.

2.2.3 Application Programs Subsystem

This subsystem of the package consists of about 40 main programs which implements various procedures for data generation, discrete simulation, data filtering, spectral analysis, parameter estimation, change detection in data and systems (more than 20 programs). These modules are grouped in 7 options of the **CHANGE Master Menu**, positions 2-8. For each such option correspond one or more programs, so that by selecting a certain option another menu will be displayed and the desired application program can be chosen.

All programs and subprograms are written in standard Fortran 77 language. A great attention was paid to obtain efficient and reliable codes and to implement the best known numerical algorithms.

CHANGE programs permit the user to select the variables/or cases and confirm the selection made. The output is normally listed on the terminal, but the user can require to obtain for many programs the results on the printer. A title for each problem solved may by optionally entered. The programs present to user lists of options. Generally, the options are indicated by a figure. Default are provided.

Data generation

This function of the package assures the generation of different data to be used in simulation:

- Deterministic data (pulse, step, ramp, frequential, etc.)
- Stochastic data (pseudo-random binary sequence, univariate gaussian data with given mean and variance, univariate data with given spectrum).

Discrete simulation

It is performed simulation of ARX and ARMAX models, for MISO systems, in deterministic or stochastic conditions.

Data filtering

This function includes options for lowpass, highpass and bandpass data sets filtering, after designing of a corresponding Butterworth sinus filter (Othnes and Enochson, 1978).

Spectral analysis

These functions are used for dynamic properties evaluation of data and systems. Functional dependence between input-output data can be determined once the input-output data properties are known. Options are provided for power and cross spectrum computation using parametric models (Akaike, 1978).

Parameter estimation

This option implements the on-line parameter estimation for the following types of models: AR, ARMA, ARX and ARMAX, investigated for change detection in their behaviour. The methods used are least squares method and prediction error method, in recursive form (Tertisco, Stoica and Popescu, 1987). These programs can be used in the preliminary analysis of data, to obtain a priori information, before the change detection analysis.

Change detection in data

The menu associated to this option is presented below:

Change Detection in Data Menu

- 1. Change Detection in Mean
- 3. Change Detection in Spectrum
- 5. Change Detection with Cusum Tests
- 7. Change Detection in AR Parameters
- 2. Change Detection in Amplitude
- 4. Change Detection with Cepstrum Distance
- 6. Change Detection with Quadratic Forms
- H Help Ctrl/C Exit
- R Return to previous menu

Choice —>

All these options are implemented in the sequential detection mode (on-line), and for the great part of these are implemented all the approaches, described in the section dedicated to the implementation aspects (Popescu, 2002). The off-line detection of changes can be direct applied as a particular case of the on-line detection. All the test statistics evaluated are graphically displayed.

The option Change Detection in Mean performs the change detection in the mean value of a signal using Hinkley test (Hinkley, 1971; Basseville and Benveniste, 1983).

The second option, for Change Detection in Amplitude, implements three detection techniques based on Kullback information, Kullback divergence and Bhattacharyya distance (Ishii, Iwata and Suzumura, 1979).

Change Detection in Spectrum performs discrimination of two spectral densities and uses Kullback information. This is evaluated by the correlations between the regressive coefficients of AR model and data sequences analysed (Ishii, Iwata and Suzumura, 1979).

Change Detection with Cepstral Distance implements change detection in frequency contents of a signal using cepstral distance (Gray and Markel, 1976; Markel and Gray, 1977); three approaches concerning data selection for reference and current model are implemented.

Change Detection with Cusum Tests performs implementation of three methods for change detection using one model and two models (Basseville and Benveniste, 1983). The last two methods, based on two models, make use of logarithmic likelihood ratio and of mutual entropy between the conditional probability laws. The decision concerning change occurring is taken on the Hinkley test. Only the A3 approach is implemented for all three methods.

Change Detection with Quadratic Forms makes use of quadratic forms of some stochastic Gaussian variables (model parameters, serial and partial correlations of residuals, etc.), which have a χ^2 distribution in the absence of a change (Stoica, 1990); three approaches concerning data selection for reference and current model are implemented.

The option Change Detection in AR Parameters performs change detection in these parameters of an ARMA model with MA coefficients unknown and strong nonstationary (Basseville, Benveniste and Moustakides, 1984)

Change detection in systems

The following menu is associated to this option:

Change Detection in Systems Menu

1. Change Detection in Time	2. Change Detection in Frequency
H - Help R - Return to previous menu	Ctrl/C - Exit
Choice —>	

The both implemented options work in on-line mode, A1 approach. The off-line version can be obtained as a particular case of the on-line operation mode.

Change Detection in Time performs change detection in the dynamics of a system SISO, described by an ARX model (Carlsson, 1988; Popescu 1995). After the estimation of parameter and covariance matrix of these, using input-output data, three test variables (T1, T2, T3) are computed and used in decision concerning the presence or absence of a change. The evolutions of these statistics are graphically displayed, the decision concerning the presence of a change being made using T3 statistics.

Change Detection in Frequency implements the statistical tests (T5, T6, T7, T8) suggested by Wahlberg (1989), based on the previous results obtained by Ljung (1987). These test variables are more robust to the experimental conditions than the test variables in time domain, depending also on the spectral density functions of the input and noise. The model used is an ARMAX model for a SISO system. The evaluation of the spectral density functions for the input and for the noise is based on a parametric method (Akaike, 1978). As in the case of change detection in time domain, the evolutions of the computed test variables are displayed, the decision concerning the presence of a change being made using T5 statistics.

Graphics

The package offers, also, some graphic facilities for the data and for the results, presented in the following menu:

Graphics Menu

1. Data Plot (2D)	2. Data Plot (3D)
3. Mesh Plot	4. Bode Diagram

H - Help Ctrl/C - Exit

R - Return to previous menu

Choice —>

These functions are performed using DISPLAY graphic processor (Netoiu, 1990).

Performance evaluation of some change detection methods

This section is devoted to some experimental results, obtained via simulation, for the test statistics when the change detection methods based on quadratic forms (Popescu, 2002), described in Section II. Also, the robustness of the methods, as to the assumption of autoregressive data and to the model structure is discussed.

The methods have been applied to the cases shown in Table 3.1. In each case it was generated one realization of $\{y_t^{(1)}\}$ and 100 independent realizations of $\{y_t^{(2)}\}$, of 500 sample points each. Using the multiple simulation runs, we can evaluate the probability of accepting H_1 under H_0 (first type of risk), which is also called "false alarm", and the probability of accepting H_0 under H_1 (second type of risk) for the testing methods under consideration. Note that the studied cases are grouped into two classes: for the first 3 cases in Table 3.1, the assumption concerning the autoregressive data are satisfied, while for the last 3 cases are not.

In all cases, in the beginning, only the filter which identifies the model AR_1 is activated, and after 200 sample points the second filter (sliding block) and the test are activated. If the size of the window used for identifying model AR_1 is too small, false alarms may occur due to poor estimation of AR coefficients. For this reason the window size has been chosen of 200 samples. Because the number of sample points used for the second filter is 200, it results that two successive changes which occur within less than 200 sample points could not be detected by the investigated methods. For all the methods, the critical probability value α was set to $\alpha = 0.05$.

3.1 Test statistics comparison

The results obtained for C1,C2 and C3 are given in Table 3.2. It can be noted that the combination MIII-A3 has no sense. The model order used was: p = 1 for C1, p = 2 for C2 and p = 4 for C3.

Remark 1. It can be noted that the first type of risk for MI is greater (for A1 and A2 approaches) than that of MII and MIII. At the same time MI leads to the smallest second type of risk in all cases considered.

Remark 2. Initially, the data window for the reference model will contain only data from $\{y_t^{(1)}\}$. When the data window used for the current model includes enough data from $\{y_t^{(2)}\}$, the change is detected. Afterwards, the data window for the reference model will contain data

Case	Generation of $\{y_t^{(1)}\}$ and $\{y_t^{(2)}\}$
C1	$y_t^{(1)} = 0.6y_{t-1}^{(1)} + \epsilon_t; \sigma^2 = 1.$
	$y_t^{(2)} = 0.1 y_{t-1}^{(2)} + \epsilon_t; \sigma^2 = 1.$
C2	$y_{t-1}^{(1)} = 0.3y_{t-1}^{(1)} + 0.5y_{t-2}^{(1)} + \epsilon_t; \sigma^2 = 4.$
	$y_t^{(2)} = 0.3y_{t-1}^{(2)} + 0.5y_{t-2}^{(2)} + \epsilon_t; \sigma^2 = 0.25$
С3	$y_{t-1}^{(1)} = 0.3y_{t-1}^{(1)} + 0.5y_{t-2}^{(1)} + \epsilon_t; \sigma^2 = 0.09$
	$y_t^{(2)} = 0.5y_{t-1}^{(2)} - 0.3y_{t-2}^{(2)} + 0.6y_{t-3}^{(2)} -$
	$-0.5y_{t-4}^{(2)} + \epsilon_t; \sigma^2 = 0.16$
C4	$y_t^{(1)} = \sqrt{2}\sin(0.2\pi t) + \epsilon_t; \sigma^2 = 0.64$
	$y_t^{(2)} = 0.7y_{t-1}^{(2)} + 0.5y_{t-2}^{(2)} - 0.56y_{t-3}^{(2)} +$
	$+\epsilon_t; \sigma^2 = 1.$
C5	$y_t^{(1)} = \sqrt{2}\sin(0.2\pi t) + \epsilon_t; \sigma^2 = 0.64$
	$y_t^{(2)} = \sqrt{2}\sin(0.23\pi t) + \epsilon_t; \sigma^2 = 0.64$
С6	$y_t^{(1)} = \sqrt{2}\sin(0.2\pi t) + \epsilon_t; \sigma^2 = 0.64$
	$y_t^{(2)} = \sqrt{2}\sin(0.23\pi t) + \epsilon_t; \sigma^2 = 1.$

Table 3.1: The cases considered in simulation

from $\{y_t^{(1)}\}$ and $\{y_t^{(2)}\}$ and the data window for current model will include only data from $\{y_t^{(2)}\}$. Sometimes, in this case a second change is detected. This depends on the number of data samples from $\{y_t^{(1)}\}$ for which the reference model is computed. Thus, the real change instant will be included between two successive change detection instants. Table 3.3 presents for C1, the number of cases with a single and double change, in the analysed realizations. It can be noted that the number of double change detections reduces for A3 approach, in comparison with the A1 and A2 approaches, for MI and MII. It results that for MI-A3 and MII-A3 the change detection instant will be very close to the real change instant.

Remark 3. MII and MIII are not sensitive to a scaling of data. More exactly, MIII is completely insensitive to scaling (it is based on correlations that are not affected by scaling) and MII is only slightly sensitive (due to a slight modification of the AR model fitted to the concatenated set $\{y_t^{(1)}, y_t^{(2)}\}$, produced by a "reasonable" scaling of $\{y_t^{(2)}\}$).

Remark 4. Concerning the computational burden involved, MI is comparable to MIII.

3.2 Assumption of autoregressive data

For the C4,C5 and C6 the assumption of autoregressive data is not satisfied. The results obtained in these cases are given in Table 4, in the same manner as for the cases C1,C2 and C3. The model order was chosen in all cases, p = 3.

Remark 5. The results obtained for C4, where $\{y_t^{(2)}\}$ data are generated by an AR process, are similar to the previous results. For C5, where there appears only a small change (the angular frequency jumps from 0.2π to 0.23π) all the methods and approaches indicate a great second type of risk. In C5, where this insignificant change is accompanied by an increase of variance, the second type of risk will decrease, especially for MI.

Case	Testing	Estim. first	Estim. second
	method	type of risk	type of risk
C1	MI-A1	0.10	0.00
	MI-A2	0.08	0.00
	MI-A3	0.00	0.00
	MII-A1	0.03	0.00
	MII-A2	0.04	0.00
	MII-A3	0.00	0.00
	MIII-A1	0.02	0.00
	MIII-A2	0.05	0.00
C2	MI-A1	0.06	0.00
	MI-A2	0.00	0.00
	MI-A3	0.01	0.00
	MII-A1	0.00	0.18
	MII-A2	0.01	0.04
	MII-A3	0.00	0.00
	MIII-A1	0.02	0.24
	MIII-A2	0.02	0.00
С3	MI-A1	0.22	0.00
	MI-A2	0.23	0.00
	MI-A3	0.01	0.00
	MII-A1	0.10	0.00
	MII-A2	0.15	0.00
	MII-A3	0.00	0.00
	MIII-A1	0.10	0.00
	MIII-A2	0.14	0.00

Table 3.2: Results for C1,C2,C3 cases

Testing	No. cases with	No. of cases with
method	single change	double change
MI-A1	0	100
MI-A2	3	97
MI-A3	83	17
MII-A1	3	97
MII-A2	7	93
MII-A3	88	12
MIII-A1	15	85
MIII-A2	10	90

Table 3.3: No. of cases with single and double change for $\mathrm{C}1$

Case	Testing	Estim. first	Estim. second
	method	type of risk	type of risk
C4	MI-A1	0.07	0.00
	MI-A2	0.13	0.00
	MI-A3	0.00	0.00
	MII-A1	0.11	0.00
	MII-A2	0.12	0.00
	MII-A3	0.00	0.00
	MIII-A1	0.14	0.00
	MIII-A2	0.18	0.00
C5	MI-A1	0.00	0.87
	MI-A2	0.00	0.40
	MI-A3	0.00	0.67
	MII-A1	0.00	0.53
	MII-A2	0.00	0.49
	MII-A3	0.00	0.97
	MIII-A1	0.00	0.54
	MIII-A2	0.00	0.45
С6	MI-A1	0.00	0.10
	MI-A2	0.00	0.08
	MI-A3	0.00	0.68
	MII-A1	0.00	0.14
	MII-A2	0.00	0.22
	MII-A3	0.00	0.92
	MIII-A1	0.00	0.29
	MIII-A2	0.00	0.33

Table 3.4: Results for C4,C5,C6 cases

3.3 Importance of model order

In the cases where the order of the AR model is not known, for the investigated methods, the underestimation of this order can cause poor detection. The results obtained for C3 case with a filter of order 3 and respective 2, instead of real order 4, are given in Table 3.5.

Remark 6. It can be noted that the behaviour of the detector, especially for the second type of risk, is not affected by the underestimation of the model order. It seems that the practice of identifying AR filters in lattice form may prevent this fact (see A2, A3 for all methods). The first type of risk will be affected by an underestimation of the order.

Remark 7.A strong improvement of the change detection for the second type risk can be noted for the C5 (non autoregressive data), when the model order increases from p = 3 to p = 5 and respectively p = 10. The results are given in Table 3.6. This improvement is accompanied by a slight increase of the first type of risk.

р	Testing	Estim. first	Estim. second
	method	type of risk	type of risk
3	MI-A1	0.25	0.00
	MI-A2	0.37	0.00
	MI-A3	0.03	0.00
	MII-A1	0.34	0.00
	MII-A2	0.39	0.00
	MII-A3	0.05	0.00
	MIII-A1	0.32	0.00
	MIII-A2	0.35	0.00
2	MI-A1	0.37	0.00
	MI-A2	0.50	0.00
	MI-A3	0.07	0.00
	MII-A1	0.25	0.18
	MII-A2	0.37	0.04
	MII-A3	0.02	0.00
	MIII-A1	0.26	0.24
	MIII-A2	0.34	0.00

Table 3.5: Results for C3 case: p=3, p=2

р	Testing	Estim. first	Estim. second
	method	type of risk	type of risk
5	MI-A1	0.00	0.02
	MI-A2	0.01	0.00
	MI-A3	0.00	0.06
	MII-A1	0.03	0.62
	MII-A2	0.07	0.12
	MII-A3	0.01	0.76
	MIII-A1	0.06	0.09
	MIII-A2	0.21	0.00
10	MI-A1	0.01	0.00
	MI-A2	0.02	0.00
	MI-A3	0.00	0.00
	MII-A1	0.02	0.25
	MII-A2	0.01	0.01
	MII-A3	0.00	0.79
	MIII-A1	0.02	0.00
	MIII-A2	0.02	0.00

Table 3.6: Results for C5 case: p=5, p=10

3.4 Conclusions

The performance evaluation problem of some methods for sequential detection of changes in non-stationary time series has been addressed. The detection algorithms, considered in the paper, are based on quadratic forms of a Gaussian random variable (estimated AR parameters, estimated residual variance and sample serial and partial residual correlations). The robustness of these algorithms is also investigated. The final conclusion is that, of the methods studied in the paper, MI and the approaches A2 and A3, should be preferred in most practical applications.

Experimental results

The objective of this section is to compare the results obtained in segmentation of some nonstationary financial and economic time series using the algorithms based on "distance" measure, quadratic forms and Kitagawa-Akaike method (Popescu 2002). The following data have been used:

- US bond yield daily 1 April 29 December 1989.
- UK bond yield daily 1 April 29 December 1989.
- West Germany bond yield daily 1 April 29 December 1989.
- Japan bond yield daily 1 April 29 December 1989.
- 1 month tbill monthly 30.01.1926 30.12.1996.
- US treasury bill 2nd market middle rate, daily 11.06.1986 1.12.1995.

The source of the first four data sets is the book "The Econometrics Modelling of Financial Time Series" Terence C. Mills, Cambridge University Press, 1993, and the source of the last two data sets is US Federal Reserve web site at the St. Louis Fed. FRED.

4.1 US bond yield daily 1 April - 29 December 1989

The results obtained for this data set are represented in figure 4.1: BONDUS and statistics U, U1, U2 (Popescu 2002) when cusum tests were applied, figure 4.2: BONDUS and statistics X, X1 and X2 for method I, II and III when quadratic form based tests were applied (Popescu, 2002), and in figure 4.3 under the form of the BONDUS and resulting segmentation (vertical lines) when three AR models and evaluation of Akaike Information Criterion (AIC) - Kitagawa - Akaike method was applied.

For the statistics used in figure 4.1 it was used A1 approach (Popescu, 2002) and Hinkley test with drift parameter $\nu=1$, and the threshold h=1. The sliding window size was L=50 and the model order p=3. It can be noted strong similarities when cusum tests, based on "distance" measures were used.

For the statistics represented in figure 4.2 it was used A1 approach and $\chi^2_{0.05}(4)$ and $\chi^2_{0.05}(6)$ as thresholds for the statistics X, X1 and X2 when method I and respectively methods II and III were used. The sliding window size was L=50 and the model order p=3. It can be

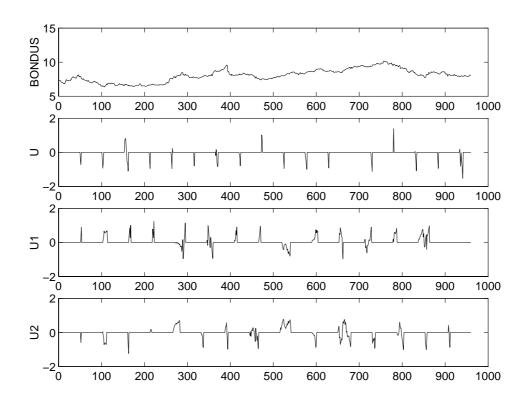


Figure 4.1: US bond yield daily 1 April - 29 December 1989 - Cusum Tests

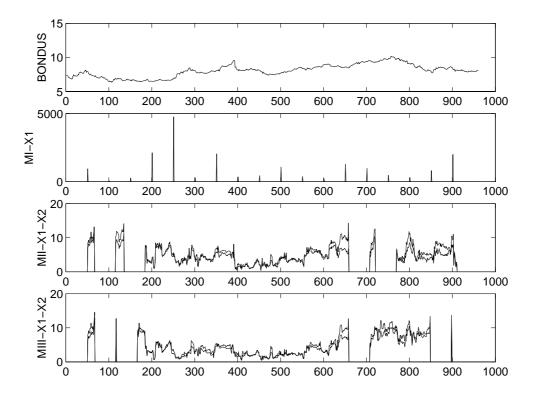


Figure 4.2: US bond yield daily 1 April - 29 December 1989 - Quadratic Form Tests

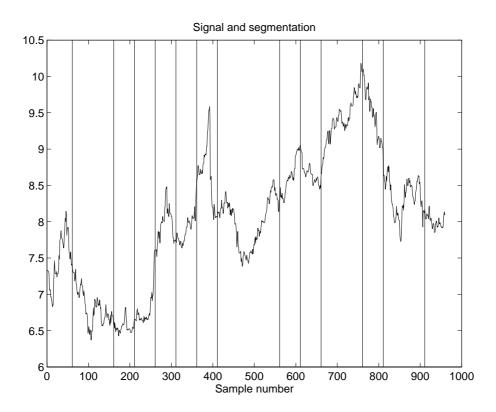


Figure 4.3: US bond yield daily 1 April - 29 December 1989 - Kitagawa - Akaike Method

noted that the methods II and III are less sensitive to possible changes in data dynamics in the interval 150 - 600. For the rest of the data the results are similar.

The results represented in figure 4.3 were obtained for a sliding window size of L=50. The model maximum order was p=10. The detailed results obtained in this case are given in Appendix A.

4.2 UK bond yield daily 1 April - 29 December 1989

The results obtained for this data set are represented in figure 4.4: BONDUK and statistics U, U1, U2 when cusum tests were applied, figure 4.5: BONDUK and statistics X, X1 and X2 for method I, II and III when quadratic form tests were applied, and in figure 4.6 under the form of the BONDUK and resulting segmentation (vertical lines) when three AR models and evaluation of Akaike Information Criterion (AIC) - Kitagawa - Akaike method was applied.

For the statistics used in figure 4.4 it was used A1 approach (Popescu, 2002) and Hinkley test with drift parameter $\nu=1$, and the threshold h=1. The sliding window size was L=50 and the model order p=3. It can be noted strong similarities when cusum tests, based on "distance" measures were used.

For the statistics represented in figure 4.5 it was used A1 approach and $\chi^2_{0.05}(4)$ and $\chi^2_{0.05}(6)$ as thresholds for the statistics X, X1 and X2 when method I and respectively methods II and III were used. The sliding window size was L=50 and the model order p=3. It can be noted that the methods II and III are less sensitive to possible changes in data dynamics in the interval 300 - 900. For the rest of the data the results are similar.

The results represented in figure 4.6 were obtained for a sliding window size of L = 50.

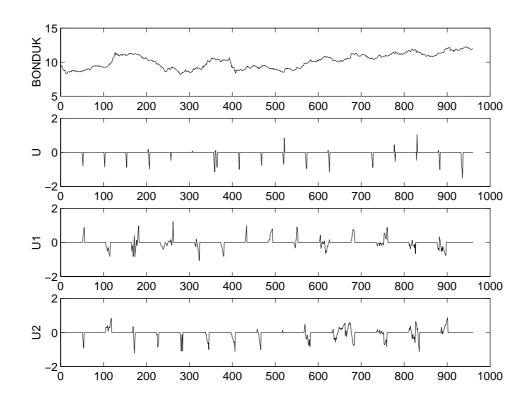


Figure 4.4: UK bond yield daily 1 April - 29 December 1989 - Cusum Tests

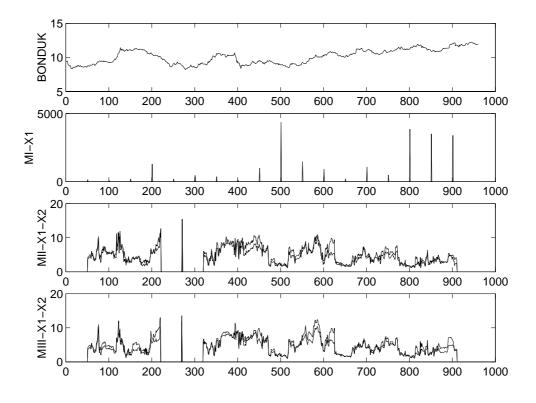


Figure 4.5: UK bond yield daily 1 April - 29 December 1989 - Quadratic Form Tests

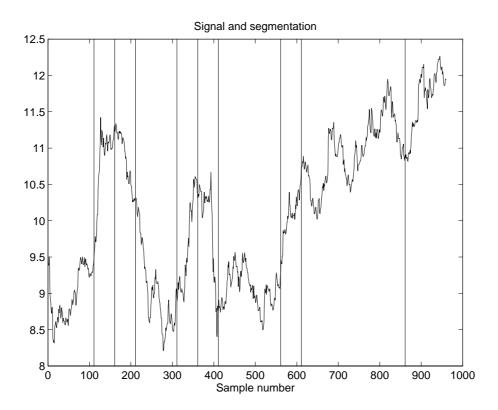


Figure 4.6: UK bond yield daily 1 April - 29 December 1989 - Kitagawa - Akaike Method

The model maximum order was p = 10. The detailed results obtained in this case are given in Appendix A.

4.3 West Germany bond yield daily 1 April - 29 December 1989

The results obtained for this data set are represented in figure 4.7: BONDWG and statistics U, U1, U2, when cusum tests were applied, figure 4.8: BONDWG and statistics X, X1 and X2 for method I, II and III when quadratic form tests were applied, and in figure 4.9 under the form of the BONDWG and resulting segmentation (vertical lines) when three AR models and evaluation of Akaike Information Criterion (AIC) - Kitagawa - Akaike method was applied.

For the statistics used in figure 4.7 it was used A1 approach (Popescu, 2002) and Hinkley test with drift parameter $\nu = 1$, and the threshold h = 1. The sliding window size was L = 50 and the model order p = 3. It can be noted strong similarities when cusum tests, based on "distance" measures were used.

For the statistics represented in figure 4.8 it was used A1 approach and $\chi^2_{0.05}(4)$ and $\chi^2_{0.05}(6)$ as thresholds for the statistics X, X1 and X2 when method I and respectively methods II and III were used. The sliding window size was L=50 and the model order p=3.

The results represented in figure 4.9 were obtained for a sliding window size of L=50. The model maximum order was p=10. The detailed results obtained in this case are given in Appendix A.

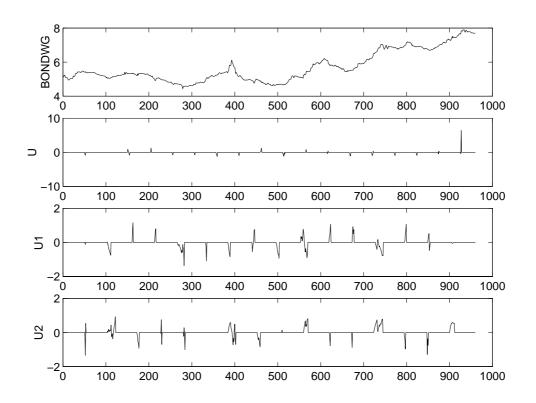


Figure 4.7: West Germany bond yield daily 1 April - 29 December 1989 - Cusum Tests

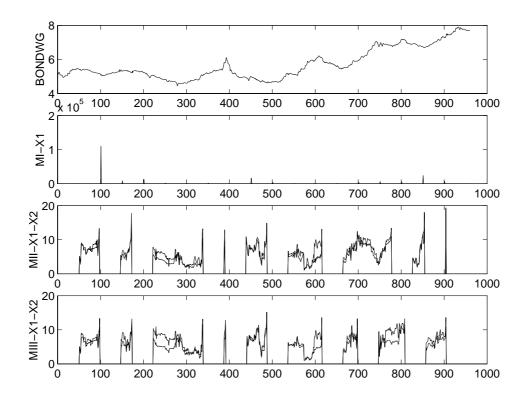


Figure 4.8: West Germany bond yield daily 1 April - 29 December 1989 - Quadratic Form Tests

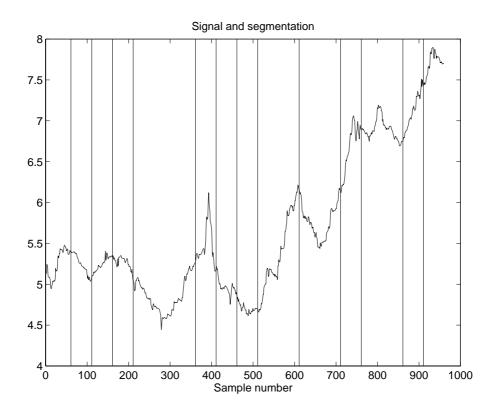


Figure 4.9: West Germany bond yield daily 1 April - 29 December 1989 - Kitagawa - Akaike Method

4.4 Japan bond yield daily 1 April - 29 December 1989

The results obtained for this data set are represented in figure 4.10: BONDJP and statistics U, U1, U2, when cusum tests were applied, figure 4.11: BONDJP and statistics X, X1 and X2 for method I, II and III when quadratic form tests were applied, and in figure 4.12 under the form of the BONDJP and resulting segmentation (vertical lines) when three AR models and evaluation of Akaike Information Criterion (AIC) - Kitagawa - Akaike method was applied.

For the statistics used in figure 4.10 it was used A1 approach (Popescu, 2002) and Hinkley test with drift parameter $\nu=1$, and the threshold h=1. The sliding window size was L=50 and the model order p=3. It can be noted strong similarities when cusum tests, based on "distance" measures were used.

For the statistics represented in figure 4.11 it was used A1 approach and $\chi^2_{0.05}(4)$ and $\chi^2_{0.05}(6)$ as thresholds for the statistics X, X1 and X2 when method I and respectively methods II and III were used. The sliding window size was L=50 and the model order p=3. It can be noted that the methods II and III are less sensitive to possible changes in data dynamics in the interval 300 - 750. For the rest of the data the results are similar.

The results represented in figure 4.12 were obtained for a sliding window size of L = 50. The model maximum order was p = 10. The detailed results obtained in this case are given in Appendix A.

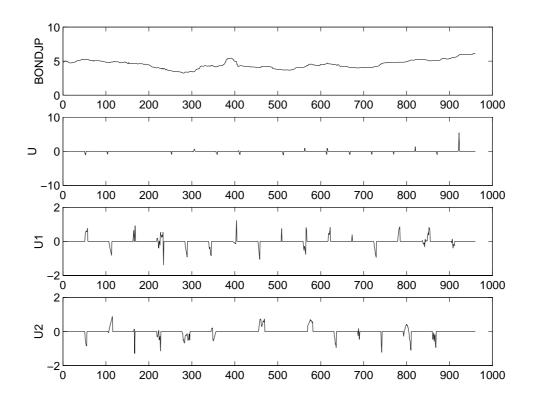


Figure 4.10: Japan bond yield daily 1 April - 29 December 1989 - Cusum Tests

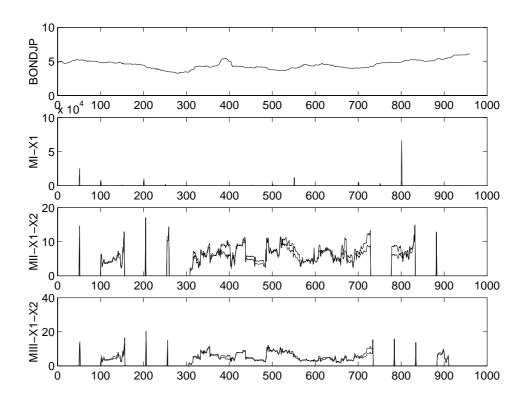


Figure 4.11: Japan bond yield daily 1 April - 29 December 1989 - Quadratic Form Tests

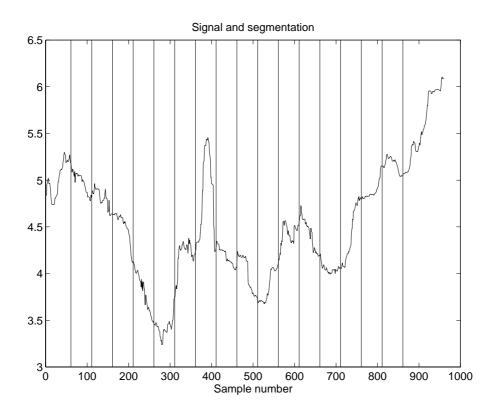


Figure 4.12: Japan bond yield daily 1 April - 29 December 1989 - Kitagawa - Akaike Method

4.5 1 month - tbill monthly 30.01.1926 - 30.12.1996.

The results obtained for this data set are represented in figure 4.13: TBILL and statistics U, U1, U2, when cusum tests were applied, figure 4.14: TBILL and statistics X, X1 and X2 for method I, II and III when quadratic form tests were applied, and in figure 4.15 under the form of the TBILL and resulting segmentation (vertical lines) when three AR models and evaluation of Akaike Information Criterion (AIC) - Kitagawa - Akaike method was applied.

For the statistics used in figure 4.13 it was used A1 approach (Popescu, 2002) and Hinkley test with drift parameter $\nu = 1$, and the threshold h = 1. The sliding window size was L = 50 and the model order p = 3. It can be noted strong similarities when cusum tests, based on "distance" measures were used.

For the statistics represented in figure 4.14 it was used A1 approach and $\chi^2_{0.05}(4)$ and $\chi^2_{0.05}(6)$ as thresholds for the statistics X, X1 and X2 when method I and respectively methods II and III were used. The sliding window size was L=50 and the model order p=3.

The results represented in figure 4.15 were obtained for a sliding window size of L = 50. The model maximum order was p = 10. The detailed results obtained in this case are given in Appendix A.

4.6 US treasury bill 2nd market - middle rate, daily 11.06.1986 - 1.12.1995

The results obtained for this data set are represented in figure 4.16: US-TR and statistics U, U1, U2, when cusum tests were applied, figure 4.17: US-TR and statistics X, X1 and X2 for

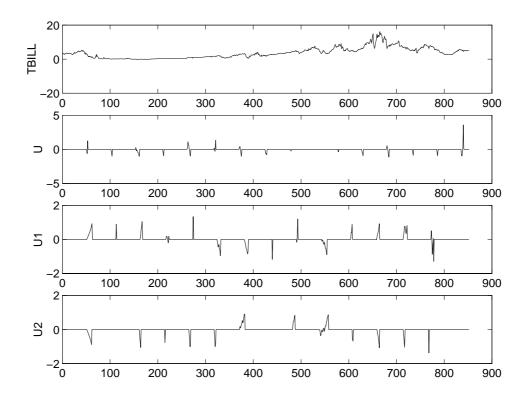


Figure 4.13: 1 month - tbill monthly 30.01.1926 - 30.12.1996 - Cusum Tests

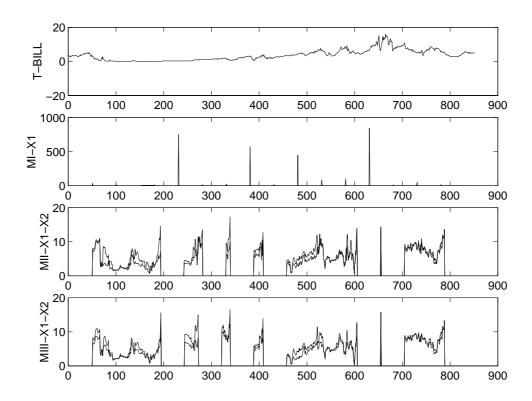


Figure 4.14: 1 month - tbill monthly 30.01.1926 - 30.12.1996 - Quadratic Form Tests

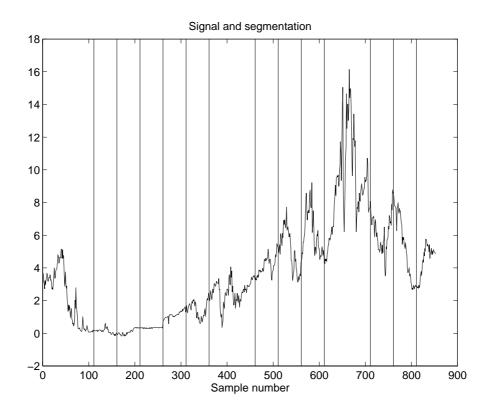


Figure 4.15: 1 month - tbill monthly 30.01.1926 - 30.12.1996 - Kitagawa - Akaike Method

method I, II and III when quadratic form tests were applied, and in figure 4.18 under the form of the US-TR and resulting segmentation (vertical lines) when three AR models and evaluation of Akaike Information Criterion (AIC) - Kitagawa - Akaike method was applied.

For the statistics used in figure 4.16 it was used A1 approach (Popescu, 2002) and Hinkley test with drift parameter $\nu = 1$, and the threshold h = 1. The sliding window size was L = 50 and the model order p = 3. It can be noted strong similarities when cusum tests, based on "distance" measures were used.

For the statistics represented in figure 4.17 it was used A1 approach and $\chi^2_{0.05}(4)$ and $\chi^2_{0.05}(6)$ as thresholds for the statistics X, X1 and X2 when method I and respectively methods II and III were used. The sliding window size was L=50 and the model order p=3. It can be noted that the methods II and III are less sensitive to possible changes in data dynamics.

The results represented in figure 4.18 were obtained for a sliding window size of L = 50. The model maximum order was p = 10. The detailed results obtained in this case are given in Appendix A.

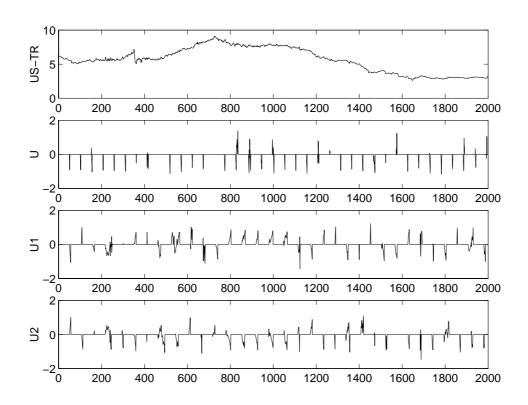


Figure 4.16: US treasury bill 2nd market - middle rate, daily 11.06.1986 - 1.12.1995 - Cusum Tests

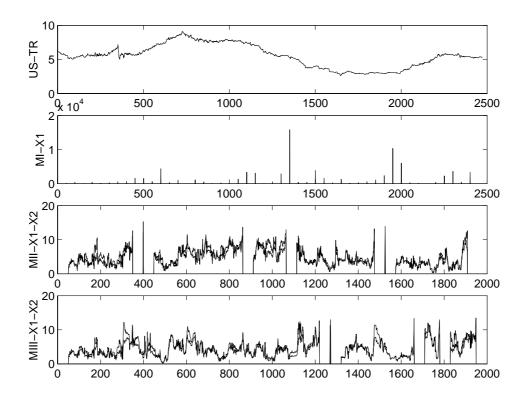


Figure 4.17: US treasury bill 2nd market - middle rate, daily 11.06.1986 - 1.12.1995 - Quadratic Form Tests

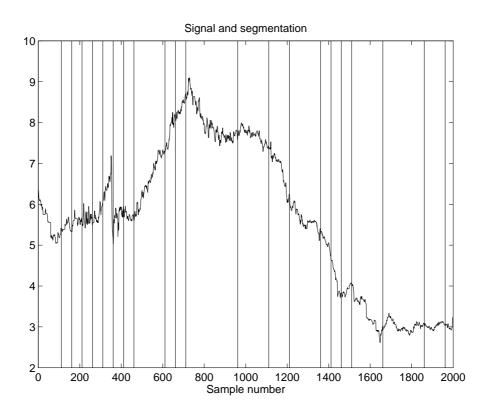


Figure 4.18: US treasury bill 2nd market - middle rate, daily 11.06.1986 - 1.12.1995 - Kitagawa - Akaike Method

Conclusions

The main goal of this project was to give a unified framework for the design and performance evaluation of some algorithms for solving change detection problem in time series with application in econometrics. The following objectives have been taken into account:

- 1. To establish a methodological approach to deal with change detection in time series with application in the field of economics.
- 2. To evaluate the performances of some algorithms and methods for change detection in time series, presented in the literature, and to develop new methods and algorithms.
- 3. To design an integrated software support, implementing the best methods and algorithms for change detection in time series.
- 4. To prove the implemented methods and algorithms on case studies in the field of economics.

The proposed problem in this project assumes off-line or bach-wise data processing, although the solution in data and an on-line data processing can be used. The segmentation model is the simplest possible extension of linear regression models to series with abruptly changing properties, or piece-wise linearizations of non-linear models. It is assumed that the time series can be described by one linear regression within each segment with distinct parameter vector and noise variance.

The significance of the research can be considered from two points of view: From methodological point of view:

- To establish a unified and integrated approach for change detection in time series to be used in economics.
- To promote advances solutions (methods and algorithms) to problems in the field of analysis of economical processes.

From practical point of view:

• To propose a set of recommendations, based on the performance evaluation of the methods and on the case studies.

• To build an integrated software, implementing the best methods and algorithms for change detection problem solving in nonstationary time series analysis.

In conclusion, concerning the problem making the object of this grant, we can mention the following remarks:

- Although the problem of change detection reached the maturity, there is a gap between theory and practice.
- The effort is now directed to robust change detection and diagnosis methods using reduced order models and adequate distance measures.
- These methods can not be reduced to repeated identification. Out purpose isn't to determine a good model, we use the model only like a tool in change detection schemes. Good and precise models offer high performance in change detection schemes, but also biased parametric models can be used for change detection and isolation. This bias decreases, but does not annihilate the performance of the detection procedure.

In our opinion, a coherent methodology is now available to the designer, together with the corresponding set of tools, which enables him to solve a large variety of change detection problem in dynamical systems. The general opinion of the scientific community with preoccupations in this field is that there is a gap between theory and practice and that the model based methods have many more possibilities in the real practical problems than they so far have proved to have. The topics of change detection are of increasing practical importance and therefore theoretical as well as applied research is a challenge for the future.

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APPENDIX - Analysis results for Kitagawa - Akaike method

US bond yield daily 1 April - 29 December 1989 Series

```
Title ---> BONDUS
No. of data used ---> 960
Maximum order of AR model ---> 10
Length of basic local span ---> 50
Parameter KSW --->
Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
      Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
OInitial local model: NS = 50 MS = 1 SDS = .12022D-01 AICS = -217.051
0 ..... Current Model .......
                 Coefficients
                                      Innovation Variance
            Т
                      A(I)
                                      SD = .1202195852D-01
                     .9647011268
       This model was fitted to the data (X(11), ..., X(60))
  ............
Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
      Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
```

```
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .4355D-02 AICS = -484.869
                            MP = 1 SDP = .8534D-02 AICP = -472.374
Constant model: (NP = 100)
0************
****
        NEW MODEL ADOPTED
****
***********
0 ..... Current Model ........
               Coefficients
                                  Innovation Variance
                                   SD = .4355342029D-02
           Ι
                    A(I)
                  1.0120927316
           1
      This model was fitted to the data (X(61), ..., X(110)).
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
      Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .5413D-02 AICS = -524.765
Constant model: (NP = 100) MP = 1 SDP = .4974D-02 AICP = -526.359
0***** Constant model adopted *****
```

```
0 ..... Current Model ......
              Coefficients
                               Innovation Variance
                  A(I)
                               SD = .4973728178D-02
          Ι
          1
                1.0034315481
      This model was fitted to the data (X(61), ..., X(160))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 100, NS = 50) MS = 1 SDS = .2220D-02 AICS = -827.874
Constant model: (NP = 150)
                    MP = 1 SDP = .4062D-02 AICP = -821.927
0************
****
****
       NEW MODEL ADOPTED
***********
0 ..... Current Model ......
            Coefficients
                               Innovation Variance
                               SD = .2219856836D-02
          Ι
                  A(I)
                 .9997866692
     This model was fitted to the data (X(161), ..., X(210)).
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
```

Where

M: Order of the model

E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared ---

OMoving model: (NF = 50, NS = 50) MS = 2 SDS = .3907D-02 AICS = -572.763 Constant model: (NP = 100) MP = 2 SDP = .3081D-02 AICP = -572.254

0**********

***** NEW MODEL ADOPTED ****

0 Current Model

. Coefficients Innovation Variance . I A(I) SD = .3907163636D-02

1 1.2818947339 . 2 -.2907320183 .

.

Basic Autoregressive Model

$$X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)$$

Where

M: Order of the model

E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared ---

OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .9121D-02 AICS = -502.107 Constant model: <math>(NP = 100) MP = 1 SDP = .6883D-02 AICP = -493.865

0************* NEW MODEL ADOPTED **** *********** 0 Current Model Coefficients Innovation Variance SD = .9120822507D-02Ι A(I)1 .9143075460 This model was fitted to the data ($X(261), \ldots, X(310)$) Basic Autoregressive Model X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)Where Order of the model M: E(I): Gaussian white noise with 0 mean and SD(M) variance 0--- The following two models are compared ---OMoving model: (NF = 50, NS = 50) MS = 3 SDS = .4429D-02 AICS = -493.841Constant model: (NP = 100) MP = 1 SDP = .7222D-02 AICP = -489.057()************* **** **** NEW MODEL ADOPTED **** ****

```
0 ..... Current Model .......
              Coefficients
                                  Innovation Variance
                                  SD = .4428769764D-02
           Ι
                   A(I)
                  .9948522896
           1
                  -.5377818677
           3
                  .5066074625
      This model was fitted to the data (X(311), ..., X(360))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .2201D-01 AICS = -449.804
                      MP = 4 SDP = .1229D-01 AICP = -429.889
Constant model: (NP = 100)
0************
****
        NEW MODEL ADOPTED
***********
0 ..... Current Model ...............
              Coefficients
                                  Innovation Variance
           Ι
                                  SD = .2200565868D-01
                    A(I)
                  .9775618090
     This model was fitted to the data ( X( 361), ..., X( 410) )
```

Basic Autoregressive Model

$$X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)$$

Where

M: Order of the model

E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared ---

OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .4510D-02 AICS = -452.893 Constant model: (NP = 100) MP = 4 SDP = .1213D-01 AICP = -431.187

0************

***** NEW MODEL ADOPTED *****

0 Current Model

. Coefficients Innovation Variance

.

Basic Autoregressive Model

$$X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)$$

Where

M: Order of the model

E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared ---

OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .3415D-02 AICS = -546.046 Constant model: (NP = 100) MP = 1 SDP = .4013D-02 AICP = -547.810

```
0 ..... Current Model ........
             Coefficients
                                 Innovation Variance
                                  SD = .4013470944D-02
           Ι
                    A(I)
                   .9856813910
      This model was fitted to the data (X(411), \ldots, X(510)).
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
М:
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 100, NS = 50) MS = 1 SDS = .3150D-02 AICS = -831.822
Constant model: (NP = 150)
                            MP = 1 SDP = .3726D-02 AICP = -834.858
0**** Constant model adopted ****
0 ..... Current Model ........
              Coefficients
                                  Innovation Variance
                                  SD = .3726129391D-02
           Ι
                    A(I)
                  .9847138189
      This model was fitted to the data (X(411), ..., X(560))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
```

0**** Constant model adopted ****

```
0--- The following two models are compared ---
OMoving model: (NF = 150, NS = 50) MS = 6 SDS = .2206D-02 AICS =-1126.695
                              MP = 4 SDP = .3421D-02 AICP = -1125.586
Constant model: (NP = 200)
0************
****
        NEW MODEL ADOPTED
****
***********
0 ..... Current Model .......
                Coefficients
                                    Innovation Variance
                                    SD = .2205635344D-02
            Ι
                     A(I)
                    .8675696810
            1
            2
                    .3558243791
            3
                   -.0454105580
            4
                   -.3029665955
            5
                   -.1064942638
            6
                    . 2607045555
       This model was fitted to the data (X(561), ..., X(610))
  .......
Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
      Order of the model
M:
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .2219D-02 AICS = -593.375
Constant model: (NP = 100)
                              MP = 1 SDP = .2658D-02 AICP = -589.027
```

0************* NEW MODEL ADOPTED **** *********** 0 Current Model Coefficients Innovation Variance SD = .2218880803D-02Ι A(I)1 .9767738533 This model was fitted to the data ($X(611), \ldots, X(660)$) Basic Autoregressive Model X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)Where Order of the model M: E(I): Gaussian white noise with 0 mean and SD(M) variance 0--- The following two models are compared ---OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .4677D-02 AICS = -565.791Constant model: (NP = 100) MP = 1 SDP = .3634D-02 AICP = -557.749()************* **** **** NEW MODEL ADOPTED **** ****

```
0 ..... Current Model .......
               Coefficients
                                 Innovation Variance
                    A(I)
                                  SD = .4677116497D-02
           Ι
           1
                  1.0115146157
      This model was fitted to the data ( X(661), ..., X(710))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
     Order of the model
M:
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 3 SDS = .3305D-02 AICS = -541.862
Constant model: (NP = 100) MP = 3 SDP = .3975D-02 AICP = -544.777
0**** Constant model adopted ****
0 ..... Current Model ........
             Coefficients
                                  Innovation Variance
           T
                    A(I)
                                  SD = .3974842718D-02
                  1.0945675756
           1
                  -.3202662468
           3
                  .2367242031
     This model was fitted to the data ( X( 661), ..., X( 760) )
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
```

```
0--- The following two models are compared ---
             (NF = 100, NS = 50) MS = 5 SDS = .6751D-02 AICS = -782.677
OMoving model:
Constant model: (NP = 150)
                              MP = 1 SDP = .5881D-02 AICP = -766.416
0************
****
        NEW MODEL ADOPTED
****
**********
0 ..... Current Model .......
               Coefficients
                                   Innovation Variance
           Ι
                     A(I)
                                    SD = .6751476253D-02
                   .8089839600
           1
           2
                   -.0230434007
           3
                   -.1240133528
                   .0201699770
           5
                   .2906134387
       This model was fitted to the data ( X(761), ..., X(810))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
      Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .7967D-02 AICS = -475.521
Constant model: (NP = 100)
                          MP = 1 SDP = .8541D-02 AICP = -472.291
```

```
()*************
****
       NEW MODEL ADOPTED
***********
0 ..... Current Model .......
             Coefficients
                               Innovation Variance
          Ι
                               SD = .7967155081D-02
                  A(I)
                 .9107487291
          1
     This model was fitted to the data ( X( 811), ..., X( 860) )
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
М:
    Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 10 SDS = .5032D-02 AICS = -480.213
                          MP = 1 SDP = .7868D-02 AICP = -480.498
Constant model: (NP = 100)
0**** Constant model adopted ****
0 ..... Current Model .......
              Coefficients
                               Innovation Variance
                                SD = .7867754870D-02
                  A(I)
          Ι
                 .9415110814
          1
      This model was fitted to the data ( X(811), ..., X(910))
  Basic Autoregressive Model
```

```
Where
M:
      Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 100, NS = 50) MS = 3 SDS = .2447D-02 AICS = -773.152
Constant model: (NP = 150)
                          MP = 1 SDP = .6179D-02 AICP = -758.990
0***********
****
        NEW MODEL ADOPTED
****
***********
0 ..... Current Model ......
               Coefficients
                                   Innovation Variance
                     A(I)
                                   SD = .2446556769D-02
           Ι
                   1.0806702410
           1
           2
                   -.4640445237
                   . 2478232619
      This model was fitted to the data ( X(911), ..., X(960) )
```

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

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```
Title ---> BONDUK
No. of data used ---> 960
Maximum order of AR model ---> 10
Length of basic local span ---> 50
Parameter KSW --->
Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
      Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
OInitial local model: NS = 50 MS = 1 SDS = .67356D-02 AICS = -246.017
0 ..... Current Model .......
                Coefficients
                                     Innovation Variance
            Ι
                      A(I)
                                     SD = .6735632579D-02
            1
                     .9980234744
       This model was fitted to the data (X(11), ..., X(60))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
      Order of the model
M:
E(I): Gaussian white noise with 0 mean and SD(M) variance
```

```
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .4241D-02 AICS = -515.160
Constant model: (NP = 100)
                             MP = 1 SDP = .5555D-02 AICP = -515.299
0***** Constant model adopted *****
0 ...... Current Model ........
              Coefficients
                                   Innovation Variance
           Ι
                     A(I)
                                   SD = .5555377255D-02
                   .9939921151
           1
     This model was fitted to the data (X(11), ..., X(110)).
  ......
Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
      Order of the model
M:
E(I): Gaussian white noise with 0 mean and SD(M) variance
O--- The following two models are compared --- OMoving model:
(NF = 100, NS = 50) MS = 5 SDS = .1282D-01 AICS = -721.117
                             MP = 8 SDP = .7760D-02 AICP = -710.825
Constant model: (NP = 150)
0************
****
        NEW MODEL ADOPTED
****
***********
```

```
0 ..... Current Model.........
              Coefficients
                                Innovation Variance
                   A(I)
                                SD = .1282494656D-01
          Ι
          1
                 1.1133009149
          2
                 -.1739047004
                 . 2372752394
                 .1271409361
          5
                 -.3135832495
      This model was fitted to the data (X(111), ..., X(160)).
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .4476D-02 AICS = -472.270
Constant model: (NP = 100)
                          MP = 6 SDP = .8674D-02 AICP = -460.743
0***********
****
       NEW MODEL ADOPTED
****
***********
0 ..... Current Model ........
              Coefficients
                                Innovation Variance
          Ι
                                SD = .4475980942D-02
                   A(I)
          1
                 .9766990573
      This model was fitted to the data (X(161), ..., X(210))
  Basic Autoregressive Model
```

```
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
```

Where

M: Order of the model

E(I): Gaussian white noise with 0 mean and SD(M) variance

O--- The following two models are compared --
OMoving model: (NF = 50, NS = 50) MS = 10 SDS = .6558D-02 AICS = -495.807

Constant model: (NP = 100) MP = 3 SDP = .7019D-02 AICP = -487.920

0 Current Model Coefficients Innovation Variance SD = .6557776025D-02Ι A(I)1 1.3520613156 -.3910405942 2 3 .3685640625 4 -.71853300995 .7007781741 6 -.6916839127 7 .7582497334 8 -.6190429380 9 . 5253373025 10 -.2968507686 This model was fitted to the data (X(211), ..., X(260))

Basic Autoregressive Model

$$X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)$$

Where

M: Order of the model

E(I): Gaussian white noise with 0 mean and SD(M) variance

```
0--- The following two models are compared ---
           (NF = 50, NS = 50) MS = 2 SDS = .8541D-02 AICS = -461.497
OMoving model:
                             MP = 2 SDP = .9107D-02 AICP = -463.867
Constant model: (NP = 100)
0**** Constant model adopted ****
 Coefficients
                                  Innovation Variance
           Ι
                    A(I)
                                  SD = .9107415758D-02
           1
                  1.1966703563
                  -.1975618333
      This model was fitted to the data (X(211), \ldots, X(310))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 100, NS = 50) MS = 1 SDS = .1425D-01 AICS = -672.421
                            MP = 2 SDP = .1100D-01 AICP = -670.423
Constant model: (NP = 150)
0************
****
****
        NEW MODEL ADOPTED
**********
```

```
0 ..... Current Model .......
             Coefficients
                               Innovation Variance
                               SD = .1424875424D-01
          Ι
                  A(I)
                 .9710230618
          1
     This model was fitted to the data ( X(311), ..., X(360) )
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .2702D-01 AICS = -385.117
                         MP = 1 SDP = .2082D-01 AICP = -383.170
Constant model: (NP = 100)
0************
       NEW MODEL ADOPTED
****
***********
0 ...... Current Model ........
             Coefficients
                               Innovation Variance
          Ι
                  A(I)
                               SD = .2701767717D-01
          1
                1.0112190602
     This model was fitted to the data (X(361), ..., X(410))
  Basic Autoregressive Model
```

```
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
      Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .6710D-02 AICS = -422.768
Constant model: (NP = 100)
                              MP = 6 SDP = .1505D-01 AICP = -405.620
0************
       NEW MODEL ADOPTED
****
***********
0 ..... Current Model .......
               Coefficients
                                    Innovation Variance
                     A(I)
                                    SD = .6710438360D-02
            Ι
            1
                    . 9873663597
      This model was fitted to the data (X(411), ..., X(460))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
      Order of the model
M:
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 3 \text{ SDS} = .5485D-02 \text{ AICS} = -498.488
Constant model: (NP = 100)
                               MP = 1 SDP = .6456D-02 AICP = -500.272
```

0**** Constant model adopted ****

```
0 ..... Current Model .......
               Coefficients
                                 Innovation Variance
                    A(I)
                                  SD = .6456160042D-02
           Ι
           1
                   .9964605484
      This model was fitted to the data (X(411), \ldots, X(510))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 100, NS = 50) MS = 2 SDS = .6440D-02 AICS = -746.532
Constant model: (NP = 150) MP = 1 SDP = .6549D-02 AICP = -750.264
0**** Constant model adopted ****
0 ..... Current Model ........
             Coefficients
                                 Innovation Variance
           Τ
                    A(I)
                                  SD = .6549115009D-02
                   .9945920739
           1
      This model was fitted to the data (X(411), ..., X(560))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
```

```
0--- The following two models are compared ---
OMoving model:
           (NF = 150, NS = 50) MS = 1 SDS = .8217D-02 AICS = -986.339
Constant model: (NP = 200)
                            MP = 1 SDP = .7230D-02 AICP = -981.903
0*************
****
****
        NEW MODEL ADOPTED
**********
0 ..... Current Model .......
             Coefficients
                                  Innovation Variance
           Ι
                    A(I)
                                  SD = .8217342290D-02
           1
                   .8948249396
      This model was fitted to the data (X(561), ..., X(610))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
     Order of the model
Μ:
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 2 SDS = .5091D-02 AICS = -494.088
                            MP = 1 SDP = .7011D-02 AICP = -492.022
Constant model: (NP = 100)
0************
****
        NEW MODEL ADOPTED
****
                         ****
***********
```

```
0 ..... Current Model ...............
               Coefficients
                                   Innovation Variance
                                   SD = .5091147733D-02
           Ι
                    A(I)
           1
                  1.2067549224
                  -. 2359478897
      This model was fitted to the data (X(611), ..., X(660))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
      Order of the model
M:
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
            (NF = 50, NS = 50) MS = 2 SDS = .7747D-02 AICS = -495.034
OMoving model:
Constant model: (NP = 100) MP = 2 SDP = .6488D-02 AICP = -497.783
0**** Constant model adopted ****
0 ...... Current Model ........
               Coefficients
                                  Innovation Variance
                                   SD = .6487807469D-02
           Т
                    A(I)
                  1.2748744410
           1
                  -.2803526764
      This model was fitted to the data ( X(611), \ldots, X(710) )
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
      Order of the model
M:
E(I): Gaussian white noise with 0 mean and SD(M) variance
```

```
0--- The following two models are compared ---
OMoving model: (NF = 100, NS = 50) MS = 1 SDS = .6117D-02 AICS = -748.615
Constant model: (NP = 150)
                             MP = 2 SDP = .6419D-02 AICP = -751.285
0**** Constant model adopted ****
 ..... Current Model ..........
               Coefficients
                                   Innovation Variance
                                   SD = .6418515869D-02
           Ι
                    A(I)
                  1.2126915027
           1
                  -.2203504876
      This model was fitted to the data (X(611), \ldots, X(760)).
  ......
Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
      Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
O--- The following two models are compared --- OMoving model:
(NF = 150, NS = 50) MS = 1 SDS = .7963D-02 AICS = -988.931
Constant model: (NP = 200)
                             MP = 2 SDP = .6861D-02 AICP = -990.383
0**** Constant model adopted ****
0 ..... Current Model ........
               Coefficients
                                   Innovation Variance
           Ι
                     A(I)
                                   SD = .6860916360D-02
                  1.1813645389
           1
                  -.1811811690
      This model was fitted to the data (X(611), ..., X(810))
  Basic Autoregressive Model
```

```
Where
M:
      Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 200, NS = 50) MS = 3 SDS = .7045D-02 AICS =-1230.155
                             MP = 2 SDP = .7121D-02 AICP = -1230.170
Constant model: (NP = 250)
0**** Constant model adopted ****
0 ..... Current Model ........
               Coefficients
                                    Innovation Variance
                     A(I)
                                    SD = .7121182891D-02
            Ι
            1
                   1.1836115083
                   -.1882209690
      This model was fitted to the data (X(611), ..., X(860)).
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
      Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 250, NS = 50) MS = 1 SDS = .9314D-02 AICS = -1459.981
Constant model: (NP = 300)
                             MP = 2 SDP = .7582D-02 AICP = -1458.583
0************
       NEW MODEL ADOPTED
****
```

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

```
0 ..... Current Model .......
             Coefficients
                               Innovation Variance
                               SD = .9314261067D-02
          Ι
                  A(I)
                1.0106588265
          1
     This model was fitted to the data (X(861), ..., X(910)).
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
    Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .5741D-02 AICS = -483.817
                         MP = 1 SDP = .7591D-02 AICP = -484.085
Constant model: (NP = 100)
0**** Constant model adopted ****
0 ..... Current Model .......
              Coefficients
                               Innovation Variance
                               SD = .7590550030D-02
                  A(I)
          Ι
                1.0039752537
      This model was fitted to the data (X(861), ..., X(960)).
```

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```
Title ---> BONDWG
No. of data used ---> 960
Maximum order of AR model ---> 10
Length of basic local span ---> 50
Parameter KSW --->
Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
      Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
OInitial local model: NS = 50 MS = 1 SDS = .11843D-02 AICS = -332.931
0 ..... Current Model ........
                Coefficients
                                     Innovation Variance
            Ι
                      A(I)
                                     SD = .1184275481D-02
            1
                     .9834050874
       This model was fitted to the data (X(11), ..., X(60))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
      Order of the model
M:
E(I): Gaussian white noise with 0 mean and SD(M) variance
```

```
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .2463D-03 AICS = -744.373
Constant model: (NP = 100)
                            MP = 7 SDP = .6443D-03 AICP = -718.733
0************
****
****
        NEW MODEL ADOPTED
                         ****
****
***********
0 ..... Current Model ...............
               Coefficients
                                  Innovation Variance
           Ι
                    A(I)
                                  SD = .2463315733D-03
                  1.0144169954
      This model was fitted to the data (X(61), ..., X(110))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 3 SDS = .5847D-03 AICS = -775.662
                        MP = 6 SDP = .4818D-03 AICP = -749.802
Constant model: (NP = 100)
()**************
****
****
       NEW MODEL ADOPTED
****
**********
```

```
0 ..... Current Model .......
               Coefficients
                                  Innovation Variance
                    A(I)
                                  SD = .5847050198D-03
           Ι
           1
                   .6287258746
           2
                  -.1058545691
                   .4519342190
      This model was fitted to the data (X(111), ..., X(160))
Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
      Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 2 SDS = .8770D-03 AICS = -710.172
                            MP = 3 SDP = .8448D-03 AICP = -699.640
Constant model: (NP = 100)
0************
****
        NEW MODEL ADOPTED
****
***********
0 ..... Current Model ......
               Coefficients
                                  Innovation Variance
           Ι
                    A(I)
                                  SD = .8769738156D-03
                   .7166081950
           1
           2
                  .3005653564
      This model was fitted to the data (X(161), ..., X(210))
  Basic Autoregressive Model
```

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```
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
      Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 2 SDS = .6172D-03 AICS = -709.469
Constant model: (NP = 100)
                             MP = 1 SDP = .8175D-03 AICP = -706.924
0************
       NEW MODEL ADOPTED
****
***********
0 ..... Current Model .......
               Coefficients
                                    Innovation Variance
                                    SD = .6171808644D-03
            Ι
                     A(I)
           1
                   1.2877193008
                   -. 2833314395
            2
       This model was fitted to the data ( X(211), \ldots, X(260) )
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
      Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 3 SDS = .6848D-03 AICS = -719.838
```

MP = 3 SDP = .6746D-03 AICP = -722.142

Constant model: (NP = 100)

0**** Constant model adopted ****

```
0 ..... Current Model .......
               Coefficients
                                  Innovation Variance
                                  SD = .6745806098D-03
           Ι
                    A(I)
                  1.3551472794
           1
                  -.4967597080
           3
                  . 1430579047
      This model was fitted to the data ( X(211), \ldots, X(310))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 100, NS = 50) MS = 2 SDS = .4591D-03 AICS =-1100.451
                            MP = 2 SDP = .6234D-03 AICP = -1101.042
Constant model: (NP = 150)
0**** Constant model adopted ****
0 ..... Current Model ........
              Coefficients
                                  Innovation Variance
           T
                                  SD = .6234261030D-03
                    A(I)
                  1.3435457615
           1
                  -.3448985578
      This model was fitted to the data (X(211), ..., X(360))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
```

```
M: Order of the model
```

E(I): Gaussian white noise with 0 mean and SD(M) variance

```
0--- The following two models are compared ---  

0Moving model: (NF = 150, NS = 50) MS = 10 SDS = .2315D-02 AICS =-1382.457  

Constant model: (NP = 200) MP = 10 SDP = .1246D-02 AICP =-1315.514
```

0 Current Model Coefficients Innovation Variance SD = .2315107491D-02Ι A(I) 1 1.4643241027 2 -.7576640551 3 . 4993084323 4 -.2293942647 5 .3238658328 6 -.3702638391 7 . 0984628629 8 .1550519331 9 -.7996350070 10 .5798835348 This model was fitted to the data (X(361), ..., X(410))

Basic Autoregressive Model

$$X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)$$

Where

M: Order of the model

E(I): Gaussian white noise with 0 mean and SD(M) variance

```
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 2 SDS = .7614D-03 AICS = -634.435
Constant model: (NP = 100)
                            MP = 10 SDP = .1852D-02 AICP = -607.173
0************
****
        NEW MODEL ADOPTED
****
****
***********
0 ..... Current Model ..............
               Coefficients
                                  Innovation Variance
           Ι
                    A(I)
                                  SD = .7613598406D-03
           1
                  1.3437469987
           2
                  -.3406983869
      This model was fitted to the data (X(411), ..., X(460))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
      Order of the model
Μ:
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .6149D-03 AICS = -718.725
                             MP = 1 SDP = .7374D-03 AICP = -717.234
Constant model: (NP = 100)
0************
****
****
        NEW MODEL ADOPTED
                         ****
***********
```

```
0 ..... Current Model ...............
             Coefficients
                                Innovation Variance
                                SD = .6148780173D-03
          Ι
                   A(I)
          1
                 1.0032922526
      This model was fitted to the data (X(461), \ldots, X(510))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .1623D-02 AICS = -682.889
Constant model: (NP = 100)
                           MP = 1 SDP = .1182D-02 AICP = -670.034
0***********
       NEW MODEL ADOPTED
****
***********
0 ..... Current Model ........
              Coefficients
                                Innovation Variance
                  A(I)
                                SD = .1622664722D-02
          Ι
                  .9841365294
          1
      This model was fitted to the data (X(511), ..., X(560))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
```

Where

M: Order of the model

E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared ---

OMoving model: (NF = 50, NS = 50) MS = 3 SDS = .1916D-02 AICS = -622.069 Constant model: (NP = 100) MP = 1 SDP = .1853D-02 AICP = -625.117

0***** Constant model adopted *****

0 Current Model

.

. Coefficients Innovation Variance . I A(I) SD = .1852598010D-02 . 1 .9848339495

. This model was fitted to the data (X(511), ...,X(610)) .

Basic Autoregressive Model

$$X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)$$

Where

M: Order of the model

E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared ---

OMoving model: (NF = 100, NS = 50) MS = 1 SDS = .1100D-02 AICS = -961.738

Constant model: (NP = 150) MP = 2 SDP = .1592D-02 AICP = -960.393

0*************

***** NEW MODEL ADOPTED *****

```
0 ..... Current Model ........
              Coefficients
                                 Innovation Variance
                                  SD = .1100013699D-02
           Ι
                    A(I)
                   .9443502949
      This model was fitted to the data (X(611), \ldots, X(660)).
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 2 SDS = .9120D-03 AICS = -680.615
Constant model: (NP = 100)
                            MP = 2 SDP = .1028D-02 AICP = -682.053
0**** Constant model adopted ****
0 ..... Current Model ........
                                  Innovation Variance
               Coefficients
                                  SD = .1027602730D-02
           Ι
                    A(I)
                  1.2904473838
                  -.3071816898
     This model was fitted to the data (X(611), \ldots, X(710)).
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
     Order of the model
M:
```

E(I): Gaussian white noise with 0 mean and SD(M) variance

```
0--- The following two models are compared ---
OMoving model: (NF = 100, NS = 50) MS = 8 SDS = .1503D-02 AICS = -989.072
Constant model: (NP = 150)
                         MP = 8 SDP = .1281D-02 AICP = -981.031
0************
****
****
       NEW MODEL ADOPTED
                          ****
****
                          ****
***********
0 ...... Current Model ........
              Coefficients
                                    Innovation Variance
                                    SD = .1502847926D-02
           Ι
                     A(I)
                   1.6207701926
           1
           2
                  -.7285185127
           3
                   . 1314636948
           4
                  -.5241078468
           5
                  1.1058045429
           6
                  -.9397992070
           7
                   . 6064463859
                   -.2704506142
      This model was fitted to the data (X(711), ..., X(760)).
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
      Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .6615D-03 AICS = -669.072
Constant model: (NP = 100)
                             MP = 6 SDP = .1303D-02 AICP = -650.282
```

```
()*************
****
       NEW MODEL ADOPTED
***********
0 ..... Current Model ........
              Coefficients
                                 Innovation Variance
          Ι
                                 SD = .6614708593D-03
                   A(I)
                 1.0036801016
          1
     This model was fitted to the data ( X(761), ..., X(810))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
    Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .4668D-03 AICS = -741.533
Constant model: (NP = 100)
                           MP = 4 SDP = .5311D-03 AICP = -744.052
0**** Constant model adopted ****
0 ..... Current Model .......
              Coefficients
                                 Innovation Variance
                   A(I)
                                 SD = .5311195171D-03
          Ι
                 1.0797625809
          1
          2
                  .0493674008
          3
                 . 1413924465
                 -.2713690251
      This model was fitted to the data ( X(761), ..., X(860) )
  .................
Basic Autoregressive Model
```

```
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
```

M: Order of the model

E(I): Gaussian white noise with 0 mean and SD(M) variance

```
0--- The following two models are compared ---
```

OMoving model: (NF = 100, NS = 50) MS = 8 SDS = .8387D-03 AICS = -1080.238 Constant model: (NP = 150) MP = 4 SDP = .8349D-03 AICP = -1053.227

0************

***** NEW MODEL ADOPTED *****

. Coefficients Innovation Variance . I A(I) SD = .8386617285D-03 . .6224985571 . .2060069357

. 3 -.0983751496 . 4 .1166707613 . 5 -.3172579043

. 6 -.0550672288 . 2350280058 . 3244465187

This model was fitted to the data (X(861), ..., X(910))

Basic Autoregressive Model

$$X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)$$

Where

M: Order of the model

E(I): Gaussian white noise with 0 mean and SD(M) variance

```
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .1475D-02 AICS = -658.149
Constant model: (NP = 100) MP = 1 SDP = .1412D-02 AICP = -652.248
0***********
       NEW MODEL ADOPTED
****
****
***********
0 ..... Current Model .......
              Coefficients
                                 Innovation Variance
                    A(I)
                                 SD = .1474723240D-02
           Ι
           1
                 1.0021982427
      This model was fitted to the data (X(911), \ldots, X(960))
```

Japan bond yield daily 1 April - 29 December 1989 Series

```
Title ---> BONDJP
No. of data used ---> 960
Maximum order of AR model ---> 10
Length of basic local span ---> 50
Parameter KSW --->
Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
      Order of the model
М:
E(I): Gaussian white noise with 0 mean and SD(M) variance
OInitial local model: NS = 50 MS = 2 SDS = .72540D-03 AICS = -355.439
0 ...... Current Model ........
                Coefficients
                                     Innovation Variance
                                     SD = .7254000779D-03
            Ι
                      A(I)
            1
                   1.4898167733
                    -.4886664694
       This model was fitted to the data (X(11), ..., X(60))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
      Order of the model
M:
E(I): Gaussian white noise with 0 mean and SD(M) variance
```

```
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 2 SDS = .7592D-03 AICS = -708.601
Constant model: (NP = 100)
                     MP = 3 SDP = .8707D-03 AICP = -696.625
0***********
****
****
       NEW MODEL ADOPTED
                        ****
****
***********
0 ..... Current Model ........
             Coefficients
                                 Innovation Variance
           Ι
                  A(I)
                                 SD = .7592170885D-03
                  .6753691505
           1
           2
                  .3025671994
     This model was fitted to the data (X(61), ..., X(110)).
  ......
Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .1724D-02 AICS = -667.315
Constant model: (NP = 100) MP = 2 SDP = .1228D-02 AICP = -664.205
0***********
****
      NEW MODEL ADOPTED
****
****
***********
```

```
0 ..... Current Model .......
              Coefficients
                               Innovation Variance
          Ι
                  A(I)
                               SD = .1724037498D-02
                 .9792729084
          1
      This model was fitted to the data ( X(111), \ldots, X(160) )
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
     Order of the model
M:
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
           (NF = 50, NS = 50) MS = 2 SDS = .3255D-03 AICS = -709.661
OMoving model:
Constant model: (NP = 100)
                      MP = 1 SDP = .1132D-02 AICP = -674.371
0***********
****
****
       NEW MODEL ADOPTED
***********
0 ..... Current Model ......
             Coefficients
                               Innovation Variance
          Ι
                  A(I)
                               SD = .3255050749D-03
                1.5096599871
          1
          2
                -.4825622027
      This model was fitted to the data (X(161), ..., X(210))
  Basic Autoregressive Model
```

```
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
```

M: Order of the model

E(I): Gaussian white noise with 0 mean and SD(M) variance

O--- The following two models are compared --
OMoving model: (NF = 50, NS = 50) MS = 5 SDS = .1298D-02 AICS = -715.842

Constant model: (NP = 100) MP = 2 SDP = .1056D-02 AICP = -679.344

0 Current Model

. Coefficients Innovetion Variance

•		Coefficients		Innovation variance	
	I	A(I)	SD =	.1298280204D-02	
	1	. 5425737917			
	2	. 2986217667			
	3	1395631686			
	4	.0538871910			
	5	. 2839923318			

. This model was fitted to the data (X(211), ..., X(260)) .

Basic Autoregressive Model

$$X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)$$

Where

M: Order of the model

E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared ---

```
OMoving model: (NF = 50, NS = 50) MS = 2 SDS = .1170D-02 AICS = -651.893
Constant model: (NP = 100) MP = 1 SDP = .1571D-02 AICP = -641.612
0***********
****
        NEW MODEL ADOPTED
****
                         ****
***********
0 ..... Current Model ...............
              Coefficients
                                 Innovation Variance
           Ι
                    A(I)
                                  SD = .1169530509D-02
           1
                  1.2859301480
                  -.2891679558
      This model was fitted to the data (X(261), \ldots, X(310))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .2244D-02 AICS = -632.534
                           MP = 3 SDP = .1761D-02 AICP = -626.167
Constant model: (NP = 100)
0***********
****
        NEW MODEL ADOPTED
****
****
***********
```

```
0 ...... Current Model ........
              Coefficients
                                Innovation Variance
                                 SD = .2243929721D-02
          Ι
                   A(I)
           1
                  .9579703050
      This model was fitted to the data (X(311), ..., X(360))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 8 SDS = .6083D-02 AICS = -538.086
                          MP = 4 SDP = .5253D-02 AICP = -514.901
Constant model: (NP = 100)
0***********
****
       NEW MODEL ADOPTED
***********
0 ..... Current Model ................
              Coefficients
                                 Innovation Variance
          Τ
                   A(I)
                                 SD = .6083408498D-02
                 .9272812516
          1
                  .4219626062
          3
                 -.1921564440
          4
                 -.4313933505
                 .1544657128
          5
          6
                 .1599322419
          7
                  .5455013413
                 -.6428171162
      This model was fitted to the data (X(361), ..., X(410))
  Basic Autoregressive Model
```

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```
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
```

M: Order of the model

E(I): Gaussian white noise with 0 mean and SD(M) variance

O--- The following two models are compared --
OMoving model: (NF = 50, NS = 50) MS = 8 SDS = .1934D-03 AICS = -646.656

Constant model: (NP = 100) MP = 8 SDP = .3775D-02 AICP = -539.929

Coefficients Innovation Variance

A(I) SD = .1933661209D-03

1 1.1224340817

2 -.3124072805

. 3 .0477740883 . 4 .1070076141 . 5 .0222366888 . 6 -.0351555439

. 7 -.0126970020 . 8 .0825312850

This model was fitted to the data (X(411), ..., X(460))

Basic Autoregressive Model

$$X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)$$

Where

M: Order of the model

E(I): Gaussian white noise with 0 mean and SD(M) variance

```
0--- The following two models are compared ---
OMoving model:
           (NF = 50, NS = 50) MS = 1 SDS = .1678D-02 AICS = -725.048
Constant model: (NP = 100)
                            MP = 1 SDP = .9871D-03 AICP = -688.072
0*************
****
****
        NEW MODEL ADOPTED
***********
0 ..... Current Model ........
              Coefficients
                                  Innovation Variance
           Ι
                    A(I)
                                  SD = .1678213507D-02
           1
                  1.0108015295
      This model was fitted to the data (X(461), ..., X(510))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
     Order of the model
Μ:
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 2 SDS = .2839D-03 AICS = -717.851
                            MP = 2 SDP = .1037D-02 AICP = -681.174
Constant model: (NP = 100)
0************
****
        NEW MODEL ADOPTED
****
                         ****
***********
```

```
0 ..... Current Model ......
             Coefficients
                               Innovation Variance
                               SD = .2838687468D-03
          Ι
                  A(I)
                1.5001070959
          1
                -.5054635193
      This model was fitted to the data (X(511), ..., X(560))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
     Order of the model
M:
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .1825D-02 AICS = -713.654
Constant model: (NP = 100)
                    MP = 1 SDP = .1206D-02 AICP = -668.048
0************
****
****
       NEW MODEL ADOPTED
***********
0 ..... Current Model ......
            Coefficients
                               Innovation Variance
                               SD = .1825154463D-02
          Ι
                  A(I)
                 .9117581309
     This model was fitted to the data (X(561), ..., X(610)).
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
```

M: Order of the model

E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared ---

OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .1404D-02 AICS = -635.736 Constant model: (NP = 100) MP = 1 SDP = .1707D-02 AICP = -633.284

0***********

***** NEW MODEL ADOPTED ****

0 Current Model

· .

. Coefficients Innovation Variance . I A(I) SD = .1403712963D-02

. 1 1.0141031230 .

Basic Autoregressive Model

$$X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)$$

Where

M: Order of the model

E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared ---

OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .3192D-03 AICS = -722.918

Constant model: (NP = 100) MP = 1 SDP = .8632D-03 AICP = -701.492

0************ **** NEW MODEL ADOPTED *********** 0 Current Model Coefficients Innovation Variance Ι SD = .3191888529D-03A(I) 1.0037259143 1 This model was fitted to the data (X(661), ..., X(710)) Basic Autoregressive Model X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)Where М: Order of the model E(I): Gaussian white noise with 0 mean and SD(M) variance 0--- The following two models are compared ---OMoving model: (NF = 50, NS = 50) MS = 3 SDS = .8431D-03 AICS = -744.406MP = 6 SDP = .5684D-03 AICP = -733.273Constant model: (NP = 100) 0************ **** **** NEW MODEL ADOPTED

```
0 ..... Current Model ...............
              Coefficients
                                Innovation Variance
                                SD = .8431351199D-03
          Ι
                   A(I)
                1.0935788532
          1
                 . 2799304014
          3
                 -.3895814826
     This model was fitted to the data (X(711), ..., X(760))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared --- OMoving model:
(NF = 50, NS = 50) MS = 6 SDS = .1399D-03 AICS = -775.652
                         MP = 6 SDP = .5033D-03 AICP = -745.436
Constant model: (NP = 100)
()*************
****
***** NEW MODEL ADOPTED
                       ****
***********
0 ..... Current Model ........
                                Innovation Variance
              Coefficients
                                SD = .1398907373D-03
          Ι
                   A(I)
                 .9640741939
          2
                 -.0376125072
          3
                 .1257255023
          4
                 .7668361222
                 -.4581815884
                 -.3533310149
      This model was fitted to the data (X(761), ..., X(810))
```

84

Basic Autoregressive Model

```
Where
M:
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 3 SDS = .1786D-03 AICS = -853.256
Constant model: (NP = 100)
                            MP = 3 SDP = .2186D-03 AICP = -834.843
0************
      NEW MODEL ADOPTED
****
***********
0 ..... Current Model .......
              Coefficients
                                  Innovation Variance
           Ι
                    A(I)
                                  SD = .1785795961D-03
                 1.2292336061
          1
           2
                  .0971009491
           3
                  -.3282494335
    This model was fitted to the data ( X(811), \ldots, X(860) )
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
     Order of the model
M:
E(I): Gaussian white noise with 0 mean and SD(M) variance
```

OMoving model: (NF = 50, NS = 50) MS = 2 SDS = .5480D-03 AICS = -792.982

MP = 3 SDP = .3760D-03 AICP = -780.593

0--- The following two models are compared ---

Constant model: (NP = 100)

```
()*************
****
        NEW MODEL ADOPTED
***********
0 ..... Current Model ........
              Coefficients
                                  Innovation Variance
           Ι
                    A(I)
                                  SD = .5480348919D-03
                  1.3060049544
           1
                  -.2968285390
      This model was fitted to the data (X(861), ..., X(910))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
Μ:
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 2 SDS = .4132D-03 AICS = -753.037
Constant model: (NP = 100)
                            MP = 2 SDP = .4878D-03 AICP = -756.563
0**** Constant model adopted ****
0 ...... Current Model ........
              Coefficients
                                  Innovation Variance
           Ι
                    A(I)
                                  SD = .4877893866D-03
                  1.3915793141
           1
                  -.3865144088
      This model was fitted to the data (X(861), ..., X(960))
```

1 month - tbill monthly 30.01.1926 - 30.12.1996 Series

```
Title ---> TBILL
No. of data used ---> 2000
Maximum order of AR model ---> 10
Length of basic local span ---> 50
Parameter KSW --->
Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
      Order of the model
M:
E(I): Gaussian white noise with 0 mean and SD(M) variance
OInitial local model: NS = 50 MS = 1 SDS = .42011D-02 AICS = -269.621
0 ...... Current Model ........
               Coefficients
                                     Innovation Variance
            Ι
                                     SD = .4201070362D-02
                     A(I)
            1
                    .9479417157
       This model was fitted to the data (X(11), ..., X(60))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
      Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
```

0--- The following two models are compared ---

```
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .2603D-02 AICS = -563.179
Constant model: (NP = 100) MP = 1 SDP = .3426D-02 AICP = -563.634
0**** Constant model adopted ****
 ...... Current Model .........
              Coefficients
                                Innovation Variance
          Ι
                A(I)
                                SD = .3426080540D-02
          1
                  .9804021510
      This model was fitted to the data (X(11), ..., X(110)).
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 100, NS = 50) MS = 1 SDS = .1600D-02 AICS = -881.532
Constant model: (NP = 150)
                        MP = 1 SDP = .2819D-02 AICP = -876.699
0************
       NEW MODEL ADOPTED
****
***********
0 ..... Current Model .......
              Coefficients
                                Innovation Variance
                                 SD = .1599655720D-02
                   A(I)
          Ι
                  .9926502236
           1
      This model was fitted to the data ( X(111), ..., X(160) )
  ......
Basic Autoregressive Model
```

```
Where
M:
      Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .2965D-02 AICS = -604.947
Constant model: (NP = 100)
                          MP = 1 SDP = .2451D-02 AICP = -597.128
0************
****
        NEW MODEL ADOPTED
****
***********
0 ..... Current Model ........
               Coefficients
                                  Innovation Variance
                    A(I)
                                   SD = .2964711313D-02
           Ι
                   .8177201100
           1
      This model was fitted to the data (X(161), ..., X(210))
  .............
Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
      Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
```

```
0--- The following two models are compared --- OMoving model: (NF = 50, NS = 50) MS = 6 SDS = .7829D-02 AICS = -515.546 Constant model: (NP = 100) MP = 2 SDP = .6707D-02 AICP = -494.461
```

```
0***********
****
         NEW MODEL ADOPTED
****
                           ****
***********
0 ..... Current Model ...............
                Coefficients
                                     Innovation Variance
            Ι
                      A(I)
                                     SD = .7828794350D-02
            1
                    .7303433506
            2
                   -.2006145382
            3
                   -.3283505685
            4
                   .1161868098
            5
                    .0041268626
                   -.3256361923
      This model was fitted to the data (X(211), ..., X(260))
Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
      Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .4265D-02 AICS = -497.357
Constant model: (NP = 100)
                              MP = 2 SDP = .8060D-02 AICP = -476.079
0*************
         NEW MODEL ADOPTED
****
```

```
0 ...... Current Model ........
             Coefficients
                               Innovation Variance
                               SD = .4265490095D-02
          Ι
                  A(I)
                 .9726952078
          1
     This model was fitted to the data (X(261), ..., X(310)).
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 2 SDS = .1647D-01 AICS = -468.170
                         MP = 4 SDP = .9897D-02 AICP = -451.557
Constant model: (NP = 100)
0************
       NEW MODEL ADOPTED
****
***********
0 ...... Current Model .......
             Coefficients
                               Innovation Variance
          Ι
                               SD = .1647020230D-01
                  A(I)
                1.4914838828
          1
                -.5181329165
      This model was fitted to the data ( X(311), ..., X(360) )
  ......
Basic Autoregressive Model
```

```
Where
M:
      Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .1636D-01 AICS = -400.966
Constant model: (NP = 100)
                          MP = 2 SDP = .1849D-01 AICP = -393.027
0************
****
        NEW MODEL ADOPTED
****
***********
0 ..... Current Model ........
               Coefficients
                                   Innovation Variance
                     A(I)
                                    SD = .1635665439D-01
           Ι
                    .8009757592
           1
      This model was fitted to the data ( X(361), \ldots, X(410) )
  .............
Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
      Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
```

OMoving model: (NF = 50, NS = 50) MS = 2 SDS = .2820D-02 AICS = -489.204

MP = 1 SDP = .9685D-02 AICP = -459.716

Constant model: (NP = 100)

```
0************
****
        NEW MODEL ADOPTED
****
***********
0 ..... Current Model ......
              Coefficients
                                 Innovation Variance
                   A(I)
                                 SD = .2820141750D-02
           Ι
                  .6200490247
           1
           2
                  . 2328659984
     This model was fitted to the data ( X(411), \ldots, X(460) )
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .4185D-02 AICS = -557.355
Constant model: (NP = 100)
                          MP = 1 SDP = .3828D-02 AICP = -552.550
0************
****
***** NEW MODEL ADOPTED
```

```
0 ...... Current Model ........
               Coefficients
                                  Innovation Variance
                    A(I)
                                  SD = .4185448895D-02
           Ι
           1
                  1.0165495104
      This model was fitted to the data (X(461), \ldots, X(510))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 4 SDS = .2494D-02 AICS = -559.491
Constant model: (NP = 100) MP = 1 SDP = .3503D-02 AICP = -561.421
0**** Constant model adopted ****
0 ...... Current Model ........
             Coefficients
                                  Innovation Variance
           Т
                    A(I)
                                  SD = .3502734841D-02
           1
                  1.0118262010
      This model was fitted to the data (X(461), ..., X(560))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
```

```
0--- The following two models are compared ---
             (NF = 100, NS = 50) MS = 3 SDS = .2553D-02 AICS = -851.946
OMoving model:
Constant model: (NP = 150)
                              MP = 8 SDP = .2981D-02 AICP = -854.305
0**** Constant model adopted ****
  Current Model .....
                Coefficients
                                    Innovation Variance
                                     SD = .2981379768D-02
            Ι
                     A(I)
                    .9854575446
            1
            2
                    .1608721015
            3
                   -.3275966153
            4
                    .1333461090
            5
                    .0956518662
            6
                   -.2188706840
            7
                    .0443924516
            8
                    .1372545462
       This model was fitted to the data (X(461), ..., X(610))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
      Order of the model
M:
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
             (NF = 150, NS = 50) MS = 3 SDS = .5432D-02 AICS = -1107.078
OMoving model:
Constant model: (NP = 200)
                             MP = 1 SDP = .3952D-02 AICP = -1102.726
()**************
****
        NEW MODEL ADOPTED
***********
```

```
0 ..... Current Model .......
              Coefficients
                                Innovation Variance
                   A(I)
                                SD = .5431931617D-02
          Ι
          1
                 .8912814546
          2
                 -.1587836332
                 .2787328738
      This model was fitted to the data (X(611), \ldots, X(660)).
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
М:
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .2692D-02 AICS = -544.654
Constant model: (NP = 100)
                       MP = 3 SDP = .4110D-02 AICP = -541.439
0************
****
       NEW MODEL ADOPTED
***********
0 ..... Current Model .......
              Coefficients
                                Innovation Variance
                                SD = .2691598426D-02
                   A(I)
          Ι
                 1.0039010099
          1
      This model was fitted to the data ( X(661), ..., X(710))
  ......
Basic Autoregressive Model
```

```
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
      Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .5011D-02 AICS = -552.691
Constant model: (NP = 100)
                         MP = 1 SDP = .3913D-02 AICP = -550.353
0************
****
        NEW MODEL ADOPTED
****
***********
 Coefficients
                                  Innovation Variance
                    A(I)
                                   SD = .5010561222D-02
           Ι
                   .9984327540
           1
      This model was fitted to the data (X(711), ..., X(760))
  ......
Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
      Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .5663D-02 AICS = -515.503
```

Constant model: (NP = 100)

0**** Constant model adopted ****

MP = 4 SDP = .5032D-02 AICP = -519.199

```
0 ...... Current Model ........
              Coefficients
                                 Innovation Variance
                                 SD = .5031715643D-02
           Ι
                    A(I)
                 1.1344982850
           1
                 -.1041378307
           3
                 -. 2433655905
                  .2098990017
      This model was fitted to the data (X(711), \ldots, X(810)).
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 100, NS = 50) MS = 2 SDS = .5065D-02 AICS = -777.472
                           MP = 8 SDP = .4735D-02 AICP = -784.928
Constant model: (NP = 150)
0**** Constant model adopted ****
 Coefficients
                                 Innovation Variance
                                 SD = .4734634666D-02
                    A(I)
           Ι
           1
                 1.1752762032
           2.
                 -.1153391958
           3
                 -. 2681733382
                  .1011783030
           4
                  . 2982587152
          5
           6
                 -.2481928878
           7
                 -.0975806183
                  .1517708905
      This model was fitted to the data ( X(711), ..., X(860) )
  ......
Basic Autoregressive Model
```

```
Where
M:
      Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 150, NS = 50) MS = 4 SDS = .3972D-02 AICS =-1051.358
Constant model: (NP = 200)
                                MP = 6 SDP = .4659D-02 AICP = -1059.800
0**** Constant model adopted ****
0 ..... Current Model ........
                Coefficients
                                      Innovation Variance
                                      SD = .4658797625D-02
            Ι
                      A(I)
                    1.1647501844
            1
            2
                    -.1719389254
            3
                    -.1775309544
            4
                    .1056589687
            5
                     . 2465816368
            6
                    -.1700998212
       This model was fitted to the data (X(711), ..., X(910))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
      Order of the model
M:
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 200, NS = 50) MS = 1 SDS = .3392D-02 AICS = -1340.119
Constant model: (NP = 250)
                                MP = 6 SDP = .4356D-02 AICP = -1345.036
```

0**** Constant model adopted ****

```
0 ..... Current Model .......
              Coefficients
                                 Innovation Variance
                    A(I)
                                 SD = .4356248907D-02
           Ι
           1
                 1.1495396983
           2
                 -.1456618724
           3
                 -.2040601471
                  .1209593453
          4
           5
                  . 2381570361
                 -.1612078556
      This model was fitted to the data (X(711), ..., X(960))
  ......
Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
     Order of the model
М:
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 250, NS = 50) MS = 3 SDS = .1666D-02 AICS =-1656.909
Constant model: (NP = 300)
                           MP = 6 SDP = .4015D-02 AICP = -1641.318
0************
****
        NEW MODEL ADOPTED
****
***********
0 ..... Current Model ......
              Coefficients
                                 Innovation Variance
           Ι
                    A(I)
                                 SD = .1665776305D-02
                  .8996612945
           1
           2
                 -.1709091464
                  .2725156792
      This model was fitted to the data ( X(961), ..., X(1010))
  ......
Basic Autoregressive Model
```

```
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
```

M: Order of the model

E(I): Gaussian white noise with 0 mean and SD(M) variance

```
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .1809D-02 AICS = -623.612
Constant model: (NP = 100)
                                   MP = 3 SDP = .1757D-02 AICP = -626.417
```

0**** Constant model adopted ****

```
0 ..... Current Model ........
```

Coefficients Innovation Variance SD = .1756965423D-02Ι A(I). 9415292936 1 -.1295319572 3 .1883656762

This model was fitted to the data (X(961), ..., X(1060))

Basic Autoregressive Model

$$X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)$$

Where

Order of the model М:

E(I): Gaussian white noise with 0 mean and SD(M) variance

```
0--- The following two models are compared ---
```

OMoving model: (NF = 100, NS = 50) MS = 1 SDS = .1917D-02 AICS = -935.269MP = 1 SDP = .1874D-02 AICP = -937.954Constant model: (NP = 150)

0**** Constant model adopted ****

```
0 ..... Current Model .............
               Coefficients
                                  Innovation Variance
                    A(I)
                                  SD = .1873975120D-02
           Ι
           1
                   .9993060562
      This model was fitted to the data (X(961), ..., X(1110))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
      Order of the model
M:
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 150, NS = 50) MS = 4 SDS = .3342D-02 AICS =-1213.010
Constant model: (NP = 200) MP = 4 SDP = .2281D-02 AICP = -1206.615
0************
****
****
        NEW MODEL ADOPTED
 ************
0 ..... Current Model ......
              Coefficients
                                  Innovation Variance
           Ι
                    A(I)
                                  SD = .3342246862D-02
                  .7593613394
           1
           2
                  -.0013999676
           3
                  -.0014993162
                  . 2358494097
     This model was fitted to the data ( X( 1111), ..., X( 1160) )
```

```
Basic Autoregressive Model
```

```
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
```

M: Order of the model

E(I): Gaussian white noise with 0 mean and SD(M) variance

```
O--- The following two models are compared ---

OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .2658D-02 AICS = -567.571

Constant model: (NP = 100) MP = 1 SDP = .3284D-02 AICP = -567.876

O***** Constant model adopted *****
```

Coefficients Innovation Variance

A(I) SD = .3283775146D-02

. 9907699211

. This model was fitted to the data (X(1111), ..., X(1210)) .

Basic Autoregressive Model

$$X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)$$

Where

M: Order of the model

E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared ---

OMoving model: (NF = 100, NS = 50) MS = 2 SDS = .1787D-02 AICS =
$$-878.250$$
 Constant model: (NP = 150) MP = 1 SDP = .2868D-02 AICP = -874.108

```
0 ..... Current Model .......
               Coefficients
                                  Innovation Variance
                    A(I)
                                  SD = .1786525166D-02
           Ι
           1
                  1.1950209393
                  -.2369123443
     This model was fitted to the data (X(1211), ..., X(1260)).
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .1782D-02 AICS = -622.889
                           MP = 5 SDP = .1671D-02 AICP = -627.459
Constant model: (NP = 100)
0**** Constant model adopted ****
0 ..... Current Model .......
               Coefficients
                                  Innovation Variance
                                  SD = .1670571205D-02
                    A(I)
           Ι
           1
                  1.1244038145
          2.
                 -.2215894201
                  . 2769580175
                  -.4003080129
           5
                  .1752364068
      This model was fitted to the data (X(1211), \ldots, X(1310)).
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
```

M: Order of the model

E(I): Gaussian white noise with 0 mean and SD(M) variance

```
O--- The following two models are compared ---
```

OMoving model: (NF = 100, NS = 50) MS = 1 SDS = .1890D-02 AICS = -937.008 Constant model: (NP = 150) MP = 2 SDP = .1846D-02 AICP = -938.194

0***** Constant model adopted *****

0 Current Model

Coefficients Innovation Variance

I A(I) SD = .1846198112D-02

1 1.0858834220

2 -.1226891946

.

Basic Autoregressive Model

$$X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)$$

Where

M: Order of the model

E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared ---

OMoving model: (NF = 150, NS = 50) MS = 1 SDS = .8895D-03 AICS = -1285.434 Constant model: (NP = 200) MP = 2 SDP = .1681D-02 AICP = -1271.635

0************

***** NEW MODEL ADOPTED *****

```
0 ..... Current Model ...............
             Coefficients
                                Innovation Variance
                                SD = .8895381666D-03
          Ι
                   A(I)
          1
                 1.0210187466
      This model was fitted to the data ( X(1361), ..., X(1410))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .3047D-02 AICS = -632.928
Constant model: (NP = 100)
                           MP = 1 SDP = .1982D-02 AICP = -618.350
0***********
       NEW MODEL ADOPTED
****
***********
0 ..... Current Model .......
              Coefficients
                                Innovation Variance
                   A(I)
                                SD = .3046558348D-02
          Ι
                 1.0108262528
          1
      This model was fitted to the data ( X(1411), ..., X(1460))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
```

Where

M: Order of the model

E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared ---

OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .7202D-03 AICS = -643.488 Constant model: <math>(NP = 100) MP = 1 SDP = .2000D-02 AICP = -617.453

0************

***** NEW MODEL ADOPTED *****

0 Current Model

. Coefficients Innovation Variance . I A(I) SD = .7201706675D-03

Basic Autoregressive Model

X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)

Where

M: Order of the model

E(I): Gaussian white noise with 0 mean and SD(M) variance

0--- The following two models are compared ---

OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .1916D-02 AICS = -666.690 Constant model: (NP = 100) MP = 3 SDP = .1281D-02 AICP = -657.984

```
()*************
****
       NEW MODEL ADOPTED
***********
0 ..... Current Model .......
             Coefficients
                               Innovation Variance
          Ι
                                SD = .1915503690D-02
                   A(I)
                 1.0026858287
          1
     This model was fitted to the data ( X( 1511), ..., X( 1560) )
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
М:
    Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .2100D-02 AICS = -613.177
                          MP = 1 SDP = .2014D-02 AICP = -616.778
Constant model: (NP = 100)
0**** Constant model adopted ****
0 ..... Current Model .......
              Coefficients
                               Innovation Variance
                                SD = .2013698405D-02
                  A(I)
          Ι
                1.0040522764
          1
      This model was fitted to the data ( X(1511), ..., X(1610))
  Basic Autoregressive Model
```

```
Where
M:
      Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 100, NS = 50) MS = 1 SDS = .2327D-02 AICS = -915.932
Constant model: (NP = 150)
                               MP = 1 SDP = .2134D-02 AICP = -918.467
0**** Constant model adopted ****
0 ..... Current Model ........
               Coefficients
                                    Innovation Variance
                                    SD = .2133945333D-02
            Ι
                     A(I)
                   1.0024936061
            1
      This model was fitted to the data (X(1511), ..., X(1660)).
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
      Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 150, NS = 50) MS = 2 SDS = .9173D-03 AICS = -1262.172
Constant model: (NP = 200)
                             MP = 1 SDP = .1858D-02 AICP = -1253.644
0***********
****
        NEW MODEL ADOPTED
```

```
0 ...... Current Model ........
                                  Innovation Variance
               Coefficients
                                  SD = .9172940519D-03
           Ι
                    A(I)
                   .7978437591
           1
                   .2002976118
      This model was fitted to the data (X(1661), \ldots, X(1710))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
М:
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .8150D-03 AICS = -695.318
Constant model: (NP = 100)
                            MP = 2 SDP = .8813D-03 AICP = -697.413
0**** Constant model adopted ****
0 ..... Current Model ........
                                  Innovation Variance
               Coefficients
                                  SD = .8812835049D-03
           Ι
                    A(I)
                   .8541138389
                   .1459299097
      This model was fitted to the data (X(1661), ..., X(1760)).
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
     Order of the model
M:
```

E(I): Gaussian white noise with 0 mean and SD(M) variance

```
0--- The following two models are compared ---
OMoving model: (NF = 100, NS = 50) MS = 1 SDS = .6877D-03 AICS =-1057.518
Constant model: (NP = 150) MP = 1 SDP = .8300D-03 AICP = -1060.114
0**** Constant model adopted ****
 Coefficients
                                 Innovation Variance
                    A(I)
           Ι
                                  SD = .8299915492D-03
           1
                  .9999655304
     This model was fitted to the data ( X( 1661), ..., X( 1810) )
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 150, NS = 50) MS = 1 SDS = .7622D-03 AICS =-1415.078
Constant model: (NP = 200)
                            MP = 1 SDP = .8137D-03 AICP = -1418.784
0**** Constant model adopted ****
 ...... Current Model .........
              Coefficients
                                 Innovation Variance
                                  SD = .8136991383D-03
                   A(I)
           Ι
                  .9997913478
           1
      This model was fitted to the data ( X(1661), ..., X(1860))
  .................
Basic Autoregressive Model
```

```
Where
M:
      Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 200, NS = 50) MS = 3 SDS = .4064D-03 AICS =-1801.190
Constant model: (NP = 250)
                          MP = 1 SDP = .7432D-03 AICP = -1797.138
0***********
****
        NEW MODEL ADOPTED
****
***********
0 ..... Current Model .......
               Coefficients
                                  Innovation Variance
                    A(I)
                                   SD = .4064199119D-03
           Ι
                  1.1522670490
           1
           2
                  -.5031986610
                   .3519184456
     This model was fitted to the data ( X( 1861), ..., X( 1910) )
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
```

OMoving model: (NF = 50, NS = 50) MS = 8 SDS = .2833D-03 AICS = -772.853

0--- The following two models are compared ---

```
MP = 3 SDP = .4002D-03 AICP = -774.361
Constant model: (NP = 100)
0**** Constant model adopted ****
0 ..... Current Model .......
              Coefficients
                                Innovation Variance
                   A(I)
                                SD = .4001736290D-03
          Ι
          1
                1.0743705113
          2
                 -.3876565459
                 .3135270338
     This model was fitted to the data (X(1861), \ldots, X(1960))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
     Order of the model
М:
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared --- OMoving model:
(NF = 100, NS = 40) MS = 2 SDS = .6870D-03 AICS = -1059.686
Constant model: (NP = 140)
                          MP = 2 SDP = .5396D-03 AICP = -1047.461
0************
****
       NEW MODEL ADOPTED
****
***********
0 ..... Current Model .......
             Coefficients
                                Innovation Variance
          Ι
                   A(I)
                                SD = .6870355958D-03
                 1.4502860384
          1
                 -.4515822692
      This model was fitted to the data (X(1961), ..., X(2000))
```

US treasury bill 2nd market - middle rate, daily 11.06.1986 - 1.12.1995

```
Title ---> USTR
No. of data used ---> 852
Maximum order of AR model ---> 10
Length of basic local span ---> 50
Parameter KSW --->
Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
      Order of the model
M:
E(I): Gaussian white noise with 0 mean and SD(M) variance
OInitial local model: NS = 50 MS = 10 SDS = .10739D+00 AICS = -89.564
0 ...... Current Model ........
                Coefficients
                                     Innovation Variance
                                     SD = .1073918978D+00
            Ι
                      A(I)
                    .8142619274
            1
                   -.0243569474
            3
                    .1628427160
            4
                    .0673123556
           5
                   -.0842926555
            6
                    . 1585937023
           7
                    .0551659674
           8
                   -.1093448476
            9
                    . 2547753917
           10
                   -.5130654729
       This model was fitted to the data ( X(11), ..., X(60))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
```

```
Where
М:
      Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
             (NF = 50, NS = 50) MS = 2 SDS = .1274D+00 AICS = -186.601
OMoving model:
Constant model: (NP = 100)
                             MP = 2 SDP = .1422D+00 AICP = -189.028
0**** Constant model adopted ****
 Coefficients
                                   Innovation Variance
           Ι
                     A(I)
                                   SD = .1422342933D+00
                   .8546922009
           1
           2
                   .1465067912
      This model was fitted to the data (X(11), ..., X(110))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
      Order of the model
M:
E(I): Gaussian white noise with 0 mean and SD(M) variance
```

```
O--- The following two models are compared --- OMoving model: (NF = 100, NS = 50) MS = 2 SDS = .7303D-02 AICS = -429.002 Constant model: (NP = 150) MP = 2 SDP = .9727D-01 AICP = -343.535
```

```
0 ..... Current Model ......
               Coefficients
                                  Innovation Variance
                    A(I)
                                  SD = .7302964645D-02
           Ι
           1
                   .7751441340
                   .2257251631
     This model was fitted to the data ( X(111), \ldots, X(160) )
Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared --- OMoving model:
(NF = 50, NS = 50) MS = 4 SDS = .3381D-02 AICS = -514.448
                           MP = 4 SDP = .5386D-02 AICP = -512.389
Constant model: (NP = 100)
()*************
****
***** NEW MODEL ADOPTED
***********
0 ..... Current Model .......
              Coefficients
                                  Innovation Variance
          Т
                                  SD = .3381302356D-02
                   A(I)
                  .8875893560
           1
                 -.1513561224
           2
                  .0020621675
           3
                  . 2583617842
     This model was fitted to the data ( X(161), \ldots, X(210) )
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
```

```
Where
```

M: Order of the model

E(I): Gaussian white noise with 0 mean and SD(M) variance

```
O--- The following two models are compared ---
```

OMoving model: (NF = 50, NS = 50) MS = 8 SDS = .2710D-03 AICS = -667.147 Constant model: (NP = 100) MP = 10 SDP = .1872D-02 AICP = -606.065

0************

NEW MODEL ADOPTED

0 Current Model

.

	Coefficients		Innovation Variance		
I	A(I)	SD =	. 2709854070D-03		
1	0634244337				
2	. 1858700878				
3	.5131659912				
4	. 2876769606				
5	2986671268				

. 5 . 2986671268 . 6 . 2396089525 . 7 - . 2492092033

. 8 -.2125375850

. This model was fitted to the data (X(211), ..., X(260)) .

Basic Autoregressive Model

$$X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)$$

Where

M: Order of the model

E(I): Gaussian white noise with 0 mean and SD(M) variance

```
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 2 SDS = .1437D-01 AICS = -598.806
Constant model: (NP = 100)
                            MP = 3 SDP = .7683D-02 AICP = -478.873
0************
****
        NEW MODEL ADOPTED
****
****
***********
0 ..... Current Model ........
               Coefficients
                                  Innovation Variance
           Ι
                    A(I)
                                  SD = .1436915400D-01
           1
                   .6257819454
           2
                  .3607041739
      This model was fitted to the data (X(261), ..., X(310))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
      Order of the model
Μ:
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 6 SDS = .3711D-01 AICS = -356.826
Constant model: (NP = 100)
                             MP = 6 SDP = .2798D-01 AICP = -343.618
0************
****
****
        NEW MODEL ADOPTED
                         ****
***********
```

```
0 ..... Current Model ......
               Coefficients
                                   Innovation Variance
                                   SD = .3711092225D-01
           Ι
                     A(I)
                  1.0557506460
           1
                  -.4281592214
           3
                   . 6516979156
           4
                  -.2196969060
           5
                   . 2181749561
                  -.2900330934
      This model was fitted to the data ( X(311), ..., X(360) )
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
      Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
O--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 8 SDS = .1242D+00 AICS = -237.000
                            MP = 8 SDP = .9025D-01 AICP = -222.518
Constant model: (NP = 100)
0************
****
        NEW MODEL ADOPTED
****
```

```
0 ..... Current Model .......
               Coefficients
                                   Innovation Variance
                                   SD = .1241631874D+00
           Ι
                    A(I)
                   .7221252841
           1
                   .2530333371
           3
                   .1503961570
           4
                   .2738013687
           5
                  -.3532092769
           6
                  -.2063205405
           7
                  -.3516248412
           8
                   .4651069241
      This model was fitted to the data ( X(361), \ldots, X(410) )
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
      Order of the model
M:
E(I): Gaussian white noise with 0 mean and SD(M) variance
O--- The following two models are compared --- OMoving model:
(NF = 50, NS = 50) MS = 2 SDS = .9775D-01 AICS = -196.573
Constant model: (NP = 100) MP = 8 SDP = .1159D+00 AICP = -197.501
0**** Constant model adopted ****
0 ..... Current Model ........
               Coefficients
                                   Innovation Variance
                                   SD = .1159016034D+00
           Ι
                    A(I)
           1
                   . 6879043088
           2.
                   . 2825946839
                   .1314164623
           3
           4
                   .1560521411
           5
                  -.1951437268
                  -.2802317859
           7
                  -.1091228165
                   .2871934769
      This model was fitted to the data ( X(361), ..., X(460))
  ......
Basic Autoregressive Model
```

```
Where
M:
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 100, NS = 50) MS = 1 SDS = .6236D-01 AICS = -332.243
                           MP = 8 SDP = .1028D+00 AICP = -323.191
Constant model: (NP = 150)
0************
****
        NEW MODEL ADOPTED
****
***********
 Coefficients
                                  Innovation Variance
                    A(I)
                                  SD = .6236033434D-01
           Ι
                   .9927226049
           1
      This model was fitted to the data (X(461), ..., X(510))
  .................
Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
```

OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .2433D+00 AICS = -201.408

MP = 1 SDP = .1531D+00 AICP = -183.680

Constant model: (NP = 100)

```
0************
       NEW MODEL ADOPTED
****
************
 Coefficients
                              Innovation Variance
                              SD = .2433325292D+00
          Ι
                  A(I)
          1
                 .9615332505
     This model was fitted to the data ( X(511), \ldots, X(560) )
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared --- OMoving model:
(NF = 50, NS = 50) MS = 1 SDS = .4979D+00 AICS = -97.532
                         MP = 3 SDP = .3559D + 00 AICP = -95.319
Constant model: (NP = 100)
0************
****
       NEW MODEL ADOPTED
****
                      ****
***********
0 ..... Current Model .......
             Coefficients
                              Innovation Variance
                              SD = .4979180959D+00
                  A(I)
          Ι
                 .9699178392
          1
      This model was fitted to the data ( X(561), ..., X(610))
  ............
Basic Autoregressive Model
```

```
Where
M:
      Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model:
             (NF = 50, NS = 50) MS = 8 SDS = .1176D+01 AICS =
                                                          -4.748
Constant model: (NP = 100)
                              MP = 9 SDP = .9200D+00 AICP =
                                                          11.666
0************
****
        NEW MODEL ADOPTED
***********
  ..... Current Model ..........
               Coefficients
                                    Innovation Variance
                     A(I)
                                    SD = .1176272069D+01
           Ι
                    .9673324650
           1
           2
                   -.0834132028
           3
                   -.4435569852
           4
                   . 2724508868
           5
                    .1565939501
           6
                   -.1426076554
           7
                   -.1031673917
                   .4242229164
       This model was fitted to the data ( X(611), \ldots, X(660) )
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
```

E(I): Gaussian white noise with 0 mean and SD(M) variance

Order of the model

Μ:

```
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 50) MS = 1 SDS = .1124D+01 AICS =
                                                           35.943
Constant model: (NP = 100)
                         MP = 9 SDP = .1171D+01 AICP =
                                                           35.776
0**** Constant model adopted ****
0 ..... Current Model ........
               Coefficients
                                    Innovation Variance
            Ι
                                    SD = .1170888238D+01
                     A(I)
                   1.1070490875
           1
                   -. 2285442687
            3
                   -.0466392163
           4
                   .0189499919
            5
                   . 1121471468
            6
                   .0724526218
            7
                   -.3378385263
            8
                   . 5124858283
                   -.2186610934
      This model was fitted to the data (X(611), ..., X(710))
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
Μ:
      Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 100, NS = 50) MS = 4 \text{ SDS} = .3420D+00 \text{ AICS} =
                                                           -7.875
Constant model: (NP = 150)
                          MP = 9 SDP = .9406D+00 AICP =
                                                           10.809
0*************
****
        NEW MODEL ADOPTED
***********
```

```
0 ..... Current Model .......
              Coefficients
                                Innovation Variance
                                SD = .3419755844D+00
          Ι
                   A(I)
                 .8519317821
          1
                 .1205712707
                 -.3326560785
                 .3591311411
      This model was fitted to the data (X(711), \ldots, X(760)).
  Basic Autoregressive Model
X(I) = A(1)*X(I-1) + A(2)*X(I-2) + ... + A(M)*X(I-M) + E(I)
Where
M:
     Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
O--- The following two models are compared --- OMoving model:
(NF = 50, NS = 50) MS = 1 SDS = .1405D+00 AICS = -137.790
                          MP = 4 SDP = .2454D+00 AICP = -130.497
Constant model: (NP = 100)
0************
       NEW MODEL ADOPTED
****
***********
0 ..... Current Model ........
              Coefficients
                                Innovation Variance
          Ι
                                SD = .1404674898D+00
                  A(I)
          1
                 .9561232626
      This model was fitted to the data (X(761), ..., X(810))
  Basic Autoregressive Model
```

```
Where
M:
      Order of the model
E(I): Gaussian white noise with 0 mean and SD(M) variance
0--- The following two models are compared ---
OMoving model: (NF = 50, NS = 42) MS = 5 SDS = .5965D-01 AICS = -200.551
Constant model: (NP = 92)
                            MP = 2 SDP = .1116D+00 AICP = -195.778
0************
       NEW MODEL ADOPTED
****
***********
0 ..... Current Model ......
               Coefficients
                                   Innovation Variance
           Ι
                                   SD = .5964624286D-01
                    A(I)
                   .7530995438
           1
           2
                   .3741982381
           3
                  -.0830894482
                   .3396315855
                  -.4103577267
```