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## **Antikaon-nucleon interaction and different properties of the $\bar{K}NN$ and $\bar{K}\bar{K}N$ systems**

Komise pro obhajoby doktorských disertací v oboru:  
”Jaderná, subjaderná a matematická fyzika”

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# 1 General description of the work

The thesis is devoted to three-body antikaon-nucleon systems, which are exotic objects containing nucleon(s) and antikaon(s). An interest to such exotic systems rose recently after the statement [21, 22] that deep and narrow quasi-bound states can exist in the  $\bar{K}NN$  and  $\bar{K}NNN$  systems. We performed a series of calculations devoted to different properties of the  $\bar{K}NN$  and  $\bar{K}\bar{K}N$  systems [1]–[20]. Namely, quasi-bound states in  $K^-pp$  and  $K^-K^-p$  were investigated, the low-energy elastic  $K^-d$  scattering amplitudes and the lowest level of the kaonic deuterium were evaluated. Three-body Faddeev-type equations, exactly describing three-body dynamics, were solved with accurate two-body potentials. A special attention was given to the ”main” antikaon-nucleon interaction, several models of the interaction were constructed.

Dynamically exact Faddeev-type Alt-Grassberger-Sandhas (AGS) equations [23] were solved for the states and processes mainly caused by strong interactions. Since the  $\bar{K}N$  interaction is coupled to the  $\pi\Sigma$  channel, the three-body system of the coupled  $\bar{K}NN$  and  $\pi\Sigma N$  channels was investigated, and the original AGS formalism was extended accordingly. Two methods of looking for a pole position in the complex energy plane were used. The first one is the obvious direct search in the complex plane. One more, a  $1/|\text{Det}(z)|^2$  method was suggested and used in [8, 10]. It was demonstrated that the second method is accurate enough for relatively narrow and clearly pronounced resonances. The exotic atom, kaonic deuterium, was studied using some version of Faddeev equations [24], which allows to include Coulomb interaction into the consideration.

We constructed models of all two-body interactions being the input for the three-body equations, except the  $NN$  potential. The reason is the fact that existing models of  $\bar{K}N$  and  $\Sigma N$  interactions were too complicated to be used in few-body calculations or not accurate in reproducing two-body experimental data. Several versions

of the potentials were constructed. In particular, we chose different forms of the antikaon-nucleon potential and fitted their parameters to the experimental data: the accurately measured threshold branching ratios, elastic and inelastic low-energy  $K^-p$  cross-sections, and 1s level shift and width of kaonic hydrogen (which, strictly speaking, is the “antikaonic hydrogen”).

A shift and a width of the 1s level of the atom are caused by the strong interaction being additional to the Coulomb one. The first versions of our  $\bar{K}N$  potentials, constructed and used in [3, 13, 14, 15] were fitted to the KEK data on kaonic hydrogen [25]. After appearance of more accurate data from the SIDDHARTA collaboration [26], the parameters were refitted. Most authors of the antikaon-nucleon potentials do not reproduce the shift and the width directly, but through a simple Deser-like formula [27]. The formula connects the characteristics of a mesonic atom with the scattering length of the strong interaction, leading to the shift. It was demonstrated by several authors (e.g. by us in [3]) that the formula, which was derived for the  $\pi N$  system, gives error  $\sim 10\%$  for the  $\bar{K}N$  interaction. Our  $\bar{K}N$  potentials reproduce the data on kaonic hydrogen directly.

An opened question of the antikaon-nucleon interaction is the nature of the  $\Lambda(1405)$  resonance, which couples the higher  $\bar{K}N$  channel with the lower  $\pi\Sigma$  channel. There are two versions of the structure of the resonance: the first says that it is a single pole, corresponding to a resonance in  $\pi\Sigma$  and a quasi-bound state in  $\bar{K}N$ . Another one came from chirally based models, which have two such poles. Due to this we constructed three models of the antikaon-nucleon interaction: phenomenological  $\bar{K}N - \pi\Sigma$  potentials with one- and two-pole structure of the  $\Lambda(1405)$  resonance and a chirally motivated  $\bar{K}N - \pi\Sigma - \pi\Lambda$  potential, which has the two-pole  $\Lambda(1405)$  resonance structure by construction. All three models describe the low-energy antikaon-nucleon experimental data with equally high level of accuracy.

The remained potentials, specially constructed for the three-body

calculations, are: the  $\Sigma N$  potential, which is coupled to the  $\Lambda N$  channel in the  $I = 1/2$  isospin state, and the  $\bar{K}\bar{K}$  potential. Experimental information on the  $\Sigma N$  interactions is more scanty than on the antikaon-nucleon one, and is simply absent in the case of the interaction of two antikaons. Due to this the parameters of the  $I = 1/2$   $\Sigma N - \Lambda N$  and  $I = 3/2$   $\Sigma N$  potentials were fitted to the existing experimental data, while the phenomenological  $\bar{K}\bar{K}$  potential was constructed using phase shifts provided by some theoretical model. The nucleon-nucleon potential is known very well, it was taken from other papers. Several versions of all potentials were used in our three-body calculations.

Our calculations of the  $K^-pp$  quasi-bound state [1, 2, 12], being among the first ones, were repeated later [8] with more advanced antikaon-nucleon potentials. The binding energy and the width of the state were predicted for the three  $\bar{K}N$  potentials. Investigations of the quasi-bound state in the  $K^-pp$  system were also performed by other authors. All theoretical results agree on the fact that the quasi-bound state really exists. But since different methods were used for description of the three-body dynamics with various two-body inputs, the obtained theoretical results differ significantly between themselves.

The question of the quasi-bound state in the  $K^-pp$  system is far from being settled also from experimental point of view. The first experimental evidence of the quasi-bound state existence occurred in the FINUDA experiment at the DAΦNE  $e^+e^-$  collider [28]. Recently performed new analyses of old experiments: OBELIX at CERN [29] and DISTO at SATURNE [30], - also claimed the observation of the state. However, there are doubts, whether the observed structures really correspond to the quasi-bound  $K^-pp$  state. The experimental results differ each from other in the same way as the theoretical ones. Moreover, all the experimentally predicted binding energies and widths of the  $K^-pp$  quasi-bound state are far from all theoretical predictions.

Keeping in mind unclear experimental situation with the  $K^-pp$  state, we also investigated another state of the lightest  $\bar{K}NN$  system, namely  $K^-d$ . In comparison with  $K^-pp$ , the  $K^-d$  system has an advantage since a process of two-body scattering of an antikaon on a deuteron can be calculated and measured. Moreover, a kaonic deuterium (the “antikaonic” deuterium), an exotic atom caused by Coulomb interaction, exists, and its  $1s$  level shift and width can be measured.

First we checked, whether a quasi-bound state caused purely by strong antikaon-nucleon interaction, similar to that one in  $\bar{K}^-pp$ , exists in the  $K^-d$  system. The result was negative for all three  $\bar{K}N$  potentials. Then near-threshold  $K^-d$  scattering amplitudes, including the scattering length, were calculated. Unfortunately, experimental data at such low energies hardly can be accurate enough, so comparison of the obtained results with eventual experiment is not possible. Due to this we used the evaluated  $K^-d$  amplitudes for approximate calculations of the characteristics of kaonic deuterium, which can be measured directly. Namely, two-body antikaon - deuteron potentials with parameters fitted to the  $K^-d$  amplitudes were used in the Lippmann-Schwinger equation together with Coulomb potential. In such a way the shift and width of the  $1s$  level of kaonic deuterium were predicted.

The study of the  $K^-d$  system was performed with the phenomenological  $\bar{K}N$  potentials reproducing the KEK data on kaonic hydrogen first [4, 16]. After publication of the new data on the atom by SIDDHARTA collaboration, we refitted our potentials and repeated the three-body calculations [5, 6]. Finally, the chirally motivated  $\bar{K}N$  potential was constructed [17], and the properties of the  $K^-d$  system were re-investigated with the three actual antikaon-nucleon potentials [7, 18, 19].

At the next step we performed dynamically exact calculations of the characteristics of kaonic deuterium [9]. Some version of Faddeev equations with Coulomb and strong interactions [24] was solved with

simple complex antikaon-nucleon potentials. The dynamically exact  $1s$  level shift and width, predicted in such a way, turned out to be quite close to those, obtained using the two-body approximation with the same input interactions.

Finally, we studied a three-body system, consisting of a nucleon and two antikaons,  $\bar{K}\bar{K}N$  [10]. The AGS equations with coupled  $\bar{K}\bar{K}N$  and  $\bar{K}\pi\Sigma$  channels were solved with the three actual antikaon-nucleon potentials: one-pole, two-pole phenomenological potentials and the chirally motivated one. The quasi-bound state was found in the  $K^-K^-p$  system with smaller binding energy and larger width than that one in the  $K^-pp$  system.

Since all three-body calculations [11, 20] were performed with different versions of two-body interactions, we studied influence of the two-body input on the three-body results. We also performed several approximate calculations, in addition to the exact ones, in order to investigate an accuracy of the usually used approximate approaches.

Our calculations are connected with the actual experiments. In particular, the question of the possible existence of the quasi-bound state in the  $K^-pp$  system is still highly uncertain, therefore, new experiments are being planned and performed by HADES [31] and LEPS Collaborations [32], and in J-PARC [33]. Characteristics of kaonic deuterium hopefully will be measured by the SIDDHARTA-2 experiment at DAΦNE [34].

## 2 Structure of the thesis

The thesis consists of the original papers (included into Section 8) preceded by a structured description (Sections 1-7). The descriptive part is necessary especially for the papers devoted to the  $\bar{K}NN$  system since during our work theoretical and experimental knowledge on  $\bar{K}N$  interaction had been changed. Due to this several versions of the antikaon-nucleon interaction models were constructed and used



in the three-body calculations. However, such repeated calculations, namely of the quasi-bound states in the  $K^-pp$  system or of the  $K^-d$  elastic scattering, does not mean simple usage of a new version of potentials only, but also something new.

The first Section of the thesis is introductory. The second Section is devoted to the two-body interactions, being the input for the three-body calculations. First, the experimental data on the antikaon-nucleon interaction are presented. Then the three versions of the  $\bar{K}N$  potentials, constructed for the three-body calculations and reproducing the experimental data, are described. Finally, several constructed models of the  $\Sigma N(-\Lambda N)$  and  $\bar{K}\bar{K}$  interactions are presented together with two versions of the  $NN$  interaction, used in the calculations.

Faddeev-type Alt-Grassberger-Sandhas equations with coupled channels, which were used for three-body calculations with purely strong interactions, are described in Section 3. The  $\bar{K}NN - \pi\Sigma N$  and  $\bar{K}\bar{K}N - \bar{K}\pi\Sigma$  systems with different quantum numbers are considered.

Section 4 is devoted to the quasi-bound states in the  $K^-pp$ ,  $K^-d$ , and  $K^-K^-p$  systems, caused by the strong interactions. First, two ways of the quasi-bound state search are described: the direct search in the complex energy plane and the  $1/|\text{Det}(z)|^2$  method of searching for an under-threshold resonance in a system with coupled channels.

The exact results for the  $K^-pp$  system are discussed and compared with the approximate ones in Section 4.2. The search of a quasi-bound states in the  $K^-d$  system gave negative results, the reason is explained in Section 4.3. Finally, the characteristics of the quasi-bound state in the  $\bar{K}\bar{K}N$  system ( $K^-K^-p$ ) are compared with those for the  $\bar{K}NN$  system ( $K^-pp$ ). Possibility of associating the predicted  $\bar{K}\bar{K}N$  quasi-bound state with experimental data on the  $\Xi(1950)$  state [35] is discussed.

The near-threshold  $K^-d$  scattering is considered in Section 5. First, dynamically exact calculations of the low-energy amplitudes,

including the  $K^-d$  scattering length, are described. Second, several approximate methods, usually used by other authors, are checked.

Section 6 starts with the description of approximate calculations of the kaonic deuterium. These calculations of the  $1s$  level shift and width of the atom were performed with a two-body  $K^- - d$  potentials, which reproduces the elastic three-body  $K^-d$  amplitudes, discussed in Section 5. Section 6.2 contains information on the dynamically exact calculations of the atom. The Faddeev-type equations with Coulomb plus strong interactions are described there. The obtained exact results are compared with the approximate ones, and an accuracy of the approximation is demonstrated.

Conclusions are given in Section 7, and the original papers are included in the last Section.

### 3 Resumé

The series of our exact or accurate calculations reported in [1]–[20] was devoted to the three-body antikaon nucleon systems, interesting exotic objects which could provide an important information about the antikaon nucleon interaction. Dynamically exact three-body Faddeev-type AGS equations with coupled channels were solved for investigation of different states and processes in the  $\bar{K}NN$  and  $\bar{K}\bar{K}N$  systems. Namely, quasi-bound states were predicted in  $K^-pp$  and  $K^-K^-p$ , their binding energies and widths were calculated; no quasi-bound state caused by strong interactions was found in the  $K^-d$  system. The scattering lengths of the  $K^-d$  system, the effective ranges and elastic near-threshold  $K^-d$  amplitudes were also evaluated. Finally, the  $1s$  level shift and width of kaonic deuterium were predicted using two methods: the accurate two-body approximation and the exact Faddeev calculation with Coulomb interaction.

Different versions of the  $\bar{K}N$ ,  $\Sigma N$ , and  $\bar{K}\bar{K}$  potentials were constructed and used in the three-body calculations. It allowed investigation of the dependence of the three-body results on two-body

input. In particular, three models of the antikaon-nucleon interaction were constructed: phenomenological  $\bar{K}N - \pi\Sigma$  potentials with one and two-pole structure of the  $\Lambda(1405)$  resonance and the chirally motivated  $\bar{K}N - \pi\Sigma - \pi\Lambda$  potential. All three potentials reproduce low-energy experimental data on  $K^-p$  scattering (the cross-sections and the threshold branching ratios) and the kaonic hydrogen with equally high accuracy.

It was found that while the quasi-bound state position strongly depends on the model of the  $\bar{K}N$  interaction, the near-threshold observables ( $K^-d$  scattering length,  $1s$  level shift and width of kaonic deuterium) are almost insensitive to it. Therefore, some conclusions on the number of poles of the  $\Lambda(1405)$  resonance and on the energy dependence of the  $\bar{K}N$  interaction could be done only if a high accuracy measurement of  $K^-pp$  binding energy and width will be done. While dependence of the three-body results on the  $\bar{K}N$  potentials is different for the different systems and processes, dependence of the three-body results on the  $NN$  and  $\Sigma N$  interactions is weak in all cases.

Additionally performed approximate calculations demonstrate an accuracy of commonly used approaches. Comparison of the exact results with some approximate ones revealed the most accurate approximations. Namely, the one-channel Faddeev calculations give for the  $K^-pp$  quasi-bound state results, which are very close to the coupled-channel calculations if the exact optical  $\bar{K}N$  potential is used. This fact gives a hope for four-body calculations, which are already very complicated without additional coupled-channel structure. It is necessary to note that the "exact optical" potential here is defined as the energy dependent potential, which exactly reproduces the elastic amplitudes of the corresponding potential with coupled channels.

As for the kaonic deuterium, influenced mainly by Coulomb interaction, the shift and width of its  $1s$  level, caused by the strong interactions, is described quite accurately by our two-body approx-

imation. The strong  $K^- - d$  complex potential should herewith reproduce the exact elastic three-body  $K^-d$  amplitudes, and the Lippmann-Schwinger equation must be solved exactly with Coulomb plus the strong potentials.

The suggested  $1/|\text{Det}(z)|^2$  method of theoretical evaluation of an underthreshold resonance position and width is accurate for rather narrow and well pronounced resonances. It works well for the  $K^-pp$  quasi-bound state. The method could supplement the direct search of the pole, providing the first estimation and working as a control. It is free from the uncertainties, connected with the calculations on the complex plane, but needs to treat the logarithmic singularities in the kernels of the integral equations.

## 4 Presentation of the results

The results were published in 20 scientific papers, listed below. Two of them have 100+ citations, another two are cited by Particle Data Group. Nine papers are conference contributions published in refereed journals, one more is a review.

The results were also presented at 13 international conferences and 11 workshops, such as:

- 18th International IUPAP Conference on Few-Body Problems in Physics (Santos, São Paulo, Brazil, August 21–26, 2006),
  - IX International Conference on Hypernuclear and Strange Particle Physics (Johannes Gutenberg-Universität Mainz, Germany, October 10–14, 2006),
  - International Conference on Exotic Atoms (EXA 2008) (Vienna, Austria, September 15–18, 2008),
  - The 10 International Conference on Hypernuclear and Strange Particle Physics (“Hyp-X”) (“RICOTTI” in Tokai, Ibaraki, Japan, September 14–18, 2009),
- and others.

# List of papers, on which the thesis is based

- [1] N.V. Shevchenko, A. Gal, J. Mareš:  
*Faddeev calculation of a  $K^-pp$  quasi-bound state.*  
Phys. Rev. Lett. 98, 082301 (2007)  
• 179 citations
- [2] N.V. Shevchenko, A. Gal, J. Mareš, J. Révai:  
 *$\bar{K}NN$  quasi-bound state and the  $\bar{K}N$  interaction: coupled-channel Faddeev calculations of the  $\bar{K}NN - \pi\Sigma N$  system.*  
Phys. Rev. C 76, 044004 (2007)  
• 160 citations
- [3] J. Révai, N.V. Shevchenko:  
*Isospin mixing effects in low-energy  $\bar{K}N - \pi\Sigma$  interaction.*  
Phys. Rev. C 79, 035202 (2009)  
• cited by PDG
- [4] N.V. Shevchenko:  
*One- versus two-pole  $\bar{K}N - \pi\Sigma$  potential:  $K^-d$  scattering length.*  
Phys. Rev. C 85, 034001 (2012)  
• cited by PDG
- [5] N.V. Shevchenko:  
*Near-threshold  $K^-d$  scattering and properties of kaonic deuterium.*  
Nucl. Phys. A 890-891, 50-61 (2012)
- [6] N.V. Shevchenko:  
*Scattering and bound states in the  $K^-d$  system.*  
Phys. Atom. Nucl. 77, 496-503 (2014)
- [7] N.V. Shevchenko, J. Révai:  
*Faddeev calculations of the  $\bar{K}NN$  system with chirally-motivated  $\bar{K}N$  interaction. I. Low-energy  $K^-d$  scattering and*

*antikaonic deuterium.*

Phys. Rev. C 90, 034003 (2014)

[8] J. Révai, N.V. Shevchenko:

*Faddeev calculations of the  $\bar{K}NN$  system with chirally-motivated  $\bar{K}N$  interaction. II. The  $K^-pp$  quasi-bound state.*

Phys. Rev. C 90, 034004 (2014)

[9] P. Doleschall, J. Révai, N.V. Shevchenko:

*Three-body calculation of the  $1s$  level shift in kaonic deuterium.*

Phys. Lett. B 744, 105-108 (2015)

[10] N.V. Shevchenko, J. Haidenbauer:

*Exact calculations of a quasi-bound state in the  $\bar{K}\bar{K}N$  system.*

Phys. Rev. C 92, 044001 (2015)

[11] N.V. Shevchenko:

Three-body antikaon-nucleon systems.

Few Body Syst. 58, 6 (2017)

• a review

[12] N.V. Shevchenko, J. Mareš, A. Gal:

Search for a  $K^-pp$  bound state.

Nucl. Phys. A 790, 659c-662c (2007)

[13] N.V. Shevchenko, J. Révai:

Isospin mixing  $\bar{K}N - \pi\Sigma$  interaction and  $\bar{K}NN - \pi\Sigma N$  quasi-bound state.

Few Body Syst. 44, 187-189 (2008)

[14] N.V. Shevchenko, J. Révai:

Phenomenological  $\bar{K}N$  interaction with isospin-breaking effects.

Hyperfine Interact. 193, 229-235 (2009)

- [15] N.V. Shevchenko, J. Révai:  
Phenomenological  $\bar{K}N$  interaction with isospin-breaking effects and  $\bar{K}NN$  system.  
Int. J. Mod. Phys. A 24, 572-575 (2009)
- [16] N.V. Shevchenko:  
Coupled-Channel Faddeev calculations of  $K^-d$  scattering length.  
Few-Body Syst., 50, 335-338 (2011)
- [17] N.V. Shevchenko:  
Faddeev Treatment of the Quasi-Bound and Scattering States in the  $\bar{K}NN - \pi\Sigma N$  System.  
Few Body Syst. 54, 1187-1189 (2013)
- [18] N.V. Shevchenko:  
Accurate treatment of the scattering and atomic states in the  $K^-d$  system.  
Nucl. Phys. A 914, 321-325 (2013)
- [19] N.V. Shevchenko:  
Faddeev treatment of the quasi-bound and scattering states in the  $\bar{K}NN - \pi\Sigma N$  system: new results.  
Few Body Syst. 55, 745-748 (2014)
- [20] N.V. Shevchenko:  
Different properties of the  $\bar{K}NN$  and  $\bar{K}\bar{K}N$  systems.  
Few-Body Syst. 58, 63 (2017)

## Other references

- [21] Y. Akaishi, T. Yamazaki: Nuclear  $\bar{K}$  bound states in light nuclei. Phys. Rev. C 65, 044005 (2002)

- [22] T. Yamazaki, Y. Akaishi:  $(K^-, \pi^-)$  production of nuclear  $\bar{K}$  bound states in proton-rich systems via  $\Lambda^*$  doorways. Phys. Lett. B 535, 70 (2002)
- [23] E.O. Alt, P. Grassberger, W. Sandhas: Reduction of the three-particle collision problem to multi-channel two-particle Lippmann-Schwinger equations. Nucl. Phys. B 2, 167 (1967)
- [24] Z. Papp, W. Plessas: Coulomb-Sturmian separable expansion approach: Three-body Faddeev calculations or Coulomb-like interactions. Phys. Rev. C 54, 50 (1996)
- [25] M. Iwasaki et al.: Observation of kaonic hydrogen  $K_\alpha$  X rays. Phys. Rev. Lett. 78, 3067 (1997)
- [26] M. Bazzi et al. (SIDDHARTA Collaboration): A new measurement of kaonic hydrogen X-rays. Phys. Lett. B 704, 113 (2011)
- [27] U.-G. Meißner, U. Raha, A. Rusetsky: Spectrum and decays of kaonic hydrogen. Eur. Phys. J. C 35, 349 (2004)
- [28] M. Agnello et al.: Evidence for a kaon-bound State  $K^-pp$  produced in  $K^-$  absorption reactions at rest. Phys. Rev. Lett. 94, 212303 (2005)
- [29] G. Bendiscioli et al.: Search for signals of bound  $\bar{K}$  nuclear states in antiproton  ${}^4\text{He}$  annihilations at rest. Nucl. Phys. A 789, 222 (2007)
- [30] T. Yamazaki et al.: Indication of a deeply bound and compact  $K^-pp$  state formed in the  $pp \rightarrow p\Lambda K^+$  reaction at 2.85 GeV. Phys. Rev. Lett. 104, 132502 (2010)
- [31] L. Fabbietti et al.:  $p\Lambda K^+$  final state: towards the extraction of the  $ppK^-$  contribution. Nucl. Phys. A 914, 60 (2013)



- [32] A.O. Tokiyasu et al.: Search for the  $K^-pp$  bound state via  $\gamma d \rightarrow K^+\pi^-X$  reaction at  $E_\gamma = 1.5 - 2.4$  GeV. Phys. Lett. B 728, 616 (2014)
- [33] S. Ajimura et al.: A search for deeply-bound kaonic nuclear state at the J-PARC E15 experiment. Nucl. Phys. A 914, 315 (2013)
- [34] C. Curceanu et al.: Unlocking the secrets of the kaon-nucleon/nuclei interactions at low-energies: The SIDDHARTA(-2) and the AMADEUS experiments at the DAΦNE collider. Nucl. Phys. A 914, 251 (2013)
- [35] K.A. Olive et al. (Particle Data Group): The Review of Particle Physics (2015). Chin. Phys. C, 38, 090001 (2014) and 2015 update
- [36] M. Sakitt et al.: Low-energy  $K^-$ -meson interactions in hydrogen. Phys. Rev. 139, B 719 (1965)
- [37] J.K. Kim: Low-energy  $K^-$ -p interaction and interpretation of the 1405-MeV  $Y_0^*$  resonance as a  $\bar{K}N$  bound state. Phys. Rev. Lett. 14, 29 (1965)
- [38] J.K. Kim: Multichannel phase-shift analysis of  $\bar{K}N$  interaction in the region 0 to 550 MeV/c. Phys. Rev. Lett. 19, 1074 (1967)
- [39] W. Kittel, G. Otter, I. Wacek: The  $K^-$  proton charge exchange interactions at low energies and scattering lengths determination. Phys. Lett. 21, 349 (1966)
- [40] J. Ciborowski et al.: Kaon scattering and charged Sigma hyperon production in  $K^-p$  interactions below 300 MeV/c. J. Phys. G 8, 13 (1982)
- [41] D. Evans et al.: Charge-exchange scattering in  $K^-p$  interactions below 300 MeV/c. J. Phys. G 9, 885 (1983)

- [42] D.N. Tovee et al.: Some properties of the charged  $\Sigma$  hyperons. Nucl. Phys. B 33, 493 (1971)
- [43] R.J. Nowak et al.: Charged  $\Sigma$  hyperon production by  $K^-$  meson interactions at rest. Nucl. Phys. B 139, 61 (1978)
- [44] G. Beer et al.: Measurement of the kaonic hydrogen X-ray spectrum. Phys. Rev. Lett. 94, 212302 (2005)
- [45] B. Borasoy, R. Nißler, W. Weise: Kaonic hydrogen and  $K^-p$  scattering. Phys. Rev. Lett. 94, 213401 (2005)
- [46] S. Deser et. al.: Energy level displacements in pi-mesonic atoms. Phys. Rev. 96, 774 (1954)
- [47] J. Révai, N.V. Shevchenko: On extracting information about hadron-nuclear interaction from hadronic atom level shifts. Few Body Syst. 42, 83 (2008)
- [48] J.A. Oller, U.-G. Meißner: Chiral dynamics in the presence of bound states: kaon-nucleon interactions revisited. Phys. Lett. B 500, 263 (2001)
- [49] K. Moriya (for the CLAS Collaboration): Measurement of the  $\Sigma\pi$  photoproduction line shapes near the  $\Lambda(1405)$ . Phys. Rev. C 87, 035206 (2013)
- [50] G. Alexander et al.: Study of the  $\Lambda - N$  system in low-energy  $\Lambda - p$  elastic scattering. Phys. Rev. 173, 1452 (1968)
- [51] B. Sechi-Zorn, B. Kehoe, J. Twitty, R.A. Burnstein: Low-energy  $\Lambda$ -proton elastic scattering. Phys. Rev. 175, 1735 (1968)
- [52] F. Eisele et al.: Elastic  $\Sigma^\pm p$  scattering at low energies. Phys. Lett. B 37, 204 (1971)

- [53] R. Engelmann, H. Filthuth, V. Hepp, E. Kluge: Inelastic  $\Sigma^-p$ -interactions at low momenta. Phys. Lett. 21, 587 (1966)
- [54] V. Hepp, M. Schleich: A new determination of the capture ratio  $r_c = \frac{\Sigma^-p \rightarrow \Sigma^0 n}{(\Sigma^-p \rightarrow \Sigma^0 n) + (\Sigma^-p \rightarrow \Lambda^0 n)}$ , the  $\Lambda^0$ -lifetime and the  $\Sigma^- - \Lambda^0$  mass difference. Z. Phys. 214, 71 (1968)
- [55] D. Lohse, J.W. Durso, K. Holinde, J. Speth: Meson exchange model for pseudoscalar meson-meson scattering. Nucl. Phys. A 516, 513 (1990)
- [56] G. Janssen, B.C. Pearce, K. Holinde, J. Speth: Structure of the scalar mesons  $f_0(980)$  and  $a_0(980)$ . Phys. Rev. D 52, 2690 (1995)
- [57] S.R. Beane et al. (NPLQCD Collaboration): The  $K^+K^+$  scattering length from lattice QCD. Phys. Rev. D 77, 094507 (2008)
- [58] K. Sasaki et al. (PACS-CS Collaboration): Scattering lengths for two pseudoscalar meson systems. Phys. Rev. D 89, 054502 (2014)
- [59] H. Zankel, W. Plessas, J. Haidenbauer: Sensitivity of N-d polarization observables on the off-shell behavior of the N-N interaction. Phys. Rev. C 28, 538 (1983)
- [60] P. Doleschall: *private communication*
- [61] R.B. Wiringa, V.G.J. Stoks, R. Schiavilla: Accurate nucleon-nucleon potential with charge-independence breaking. Phys. Rev. C 51, 38 (1995)
- [62] J. Révai: Signature of the  $\Lambda(1405)$  resonance in neutron spectra from the  $K^- + d$  reaction. Few Body Syst. 54, 1865-1876 (2013); Few Body Syst. 54, 1877 (2013)

- [63] Y. Ikeda, T. Sato: Energy Dependence of  $\bar{K}NN$  Interactions and Resonance Pole of Strange Dibaryons. Prog. Theor. Phys. 124, 533 (2010)
- [64] A. Doté, T. Hyodo, W. Weise: Variational calculation of the  $ppK^-$  system based on chiral SU(3) dynamics. Phys. Rev. C 79, 014003 (2009)
- [65] N. Barnea, A. Gal, E.Z. Liverts: Realistic calculations of  $\bar{K}NN$ ,  $\bar{K}NNN$ , and  $\bar{K}\bar{K}NN$  quasibound states. Phys. Lett. B 712, 132 (2012)
- [66] Y. Kanada-En'yo, D. Jido:  $\bar{K}\bar{K}N$  molecular state in three-body calculation. Phys. Rev. C 78, 025212 (2008)
- [67] P.M. Dauber et al.: Production and decay of cascade hyperons. Phys. Rev. 179, 1262 (1969)
- [68] S.S. Kamalov, E. Oset, A. Ramos: Chiral unitary approach to the  $K^-$ -deuteron scattering length. Nucl. Phys. A 690, 494 (2001)
- [69] J.V. Noble: Three-body problem with charged particles. Phys. Rev. 161, 945 (1967)
- [70] J. Révai: Three-body calculation of the  $1s$  level shift in kaonic deuterium with realistic  $\bar{K}N$  potentials. Phys. Rev. C 94, 054001 (2016)