2-D SCHRÖDINGER OPERATORS WITH SINGULAR INTERACTIONS ON CLOSED AND NON-CLOSED CURVES

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Two-dimensional Schrödinger operators with δ and δ'-interactions of constant strengths $\alpha > 0$ and $\beta > 0$, respectively, supported on a compact sufficiently regular closed or non-closed curve $\Sigma \subset \mathbb{R}^2$ are considered. The essential spectra of these operators coincide with the positive semi-axis.

We prove that the operator with δ-interaction on Σ (no matter if Σ is closed or non-closed) has at least one negative eigenvalue and for sufficiently small $\alpha > 0$ there exists exactly one such eigenvalue. We compute its asymptotic expansion and the expansion of the corresponding eigenstate in the limit $\alpha \to 0+$. Results on δ-interactions are obtained jointly with Sylwia Kondej.

In contrast to the δ-case in the case of δ'-interactions closedness of Σ starts to play a crucial role. It is known that for any closed Σ negative discrete spectrum is always non-empty for any $\beta > 0$. We show that for any curve Σ from a wide class of non-closed curves with two free ends there exists the critical coupling constant $\beta_{\mathrm{cr}}(\Sigma) > 0$ such that for $\beta \geq \beta_{\mathrm{cr}}(\Sigma)$ negative discrete spectrum is empty and for $0 < \beta < \beta_{\mathrm{cr}}(\Sigma)$ at least one negative eigenvalue exists. Estimates of $\beta_{\mathrm{cr}}(\Sigma)$ are derived. Results on δ'-interactions are obtained jointly with Michal Jex.