Numerical Modeling of Pump Absorption in Coiled and Twisted Double-Clad Fibers

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***Abstract*— We report the analysis of multimode pump absorption in double-clad rare-earth-doped fibers under realistic bending conditions. The finite-element beam-propagation method is used for the analysis. The fiber bending is approximated by the transformation of the refractive index profile of the fiber. Double-clad fibers of hexagonal, circular and stadium-like cross-sections are studied as examples. In addition, the double-clad waveguide structure of a two-fiber bundle is investigated. Simulations show that the bending effects cannot be neglected in double-clad fiber multimode pump propagation analysis and optimization. The reported rigorous numerical model opens new way to design double-clad fibers and to optimize the pump absorption efficiency. We show that high pump-absorption efficiency better than the ideal (ergodic) limit, can be achieved by simultaneous coiling and twisting of the double-clad fiber.**

***Index Terms*—double-clad optical fibers, optical pumping, fiber lasers, fiber amplifiers, finite element method, beam propagation method**

I. INTRODUCTION

HIGH-POWER fiber lasers are gaining success in many research and commercial fields including industrial, medical, metrological and directed energy applications [1]. The high-power operation of fiber lasers was enabled mainly by the invention of cladding pumping within a double-clad (DC) fiber structure. Such a fiber serves as an efficient transformer of the low-brightness, high-power radiation of the laser diodes (coupled into the large area inner cladding of the double-clad fiber) into a high-brightness, high-power laser beam coming out of the rare-earth-doped, narrow fiber core. This form of cladding pumping was first proposed by Robert Maurer in the seventies [2] and later demonstrated by Elias Snitzer [3]. Even Elias Snitzer was aware of the fact that the circular shape of an optical fiber cross-section significantly deteriorates the absorption of a multimode pump in a rare-earth doped core, primarily because, from a geometrical ray optics point of view, a large family of the so-called skew rays of the circularly symmetrical fiber does not interact with the doped core; therefore, E. Snitzer fabricated his fiber with an offset-core. Since then, various cross-sectional shapes of double-clad fibers have been investigated both experimentally and theoretically in order to enhance the absorption of the multimode-pump. These shapes include a D-shaped, hexagon, octagon, flower, stadium, air-clad, stress-elements inclusion, spiral-cladding, air-hole inclusion and several other shapes having broken-circular symmetry [4-10]. The key argument for such variety in the shape of the pump guide relies on optimizing the overlap of the pump field with the doped core in order to increase the pump power absorbed [11]. For circular inner cladding, pump power absorption is limited by the contribution from the $LP_{l,m}$ modes with $l \neq 0$ that present poor overlap with the central doped core. In the geometrical limit of rays, valid as soon as the transverse dimension of the inner cladding is large compared with the wavelength, these modes are associated to skew rays. To maximize the interaction of the pump with the doped core, rays should be homogeneously spread over the whole inner cladding. Chaotic ray dynamics that results from complex reflection on the boundary of the inner cladding with non-regular shapes ensures fundamentally such a scattering of rays. Assuming a random process for the ray evolution, the fraction of rays that encounter the core gives an estimation of pump absorption efficiency [4] that roughly depends on the ratio A_{core}/A_{clad}, where A_{core} and A_{clad} are the areas of the core and inner cladding, respectively.

Ergodic modes with a speckle-like spatial field distribution are a well-known consequence of the underlying chaotic ray dynamics induced by the geometry [12]. As soon as the limit of quasi-homogeneous field spatial repartition is assumed, the overlap of the pump field with the doped core is maximized and the absorption efficiency scales to the optimal value A_{core}/A_{clad}. This optimal limit has been evocated by Kouznetsov [6] and Mortensen [9] in spiral-shaped fibers and air-clad fibers respectively for which ray dynamics is chaotic and modes are ergodic. In the spiral-cladding case, an offset-core in a vicinity of the deformation

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warrants a good overlap of the pump modes with the core. In the air-clad fibers studied in ref. [9], only high order modes present an ergodic behavior that can justified the statistical approach used. In both cases, the limit of ideal absorption efficiency is governed by an ergodic individual behavior for all of the pump modes.

However, the theoretical analyses presented so far have been almost exclusively limited to the case of a straight fiber. From an experimental point of view, it has been generally recognized that a small-loop coiling would improve the pump absorption [10, 13]. However, only little experimental studies were published in open literature about optimization of the coiling. Figure-eight- [14, 15] and kidney-shape [16, 17] coiling was experimentally proved to increase the pump absorption, see illustrative photo in Fig. 1a. The high importance of involving the effect of coiling on the signal propagation in the single-mode core of fiber lasers and amplifiers was pointed out by numerical model of John M. Fini [18]. Nowadays, these bending effects are considered indispensable for modeling, especially regarding large-mode area fibers [19, 20]. Unfortunately, neither of these models account for the effect of bending on the pump absorption. To our knowledge, the effect of coiling on the pump absorption has been theoretically studied for the first time in a seminar paper [21], where a rigorous beam-propagation numerical method for the simulation of the speckle pattern of the multimode pump transversal distribution was combined with a heuristic ray-optics approach for a description of the coiling. Rigorous approach for the description of the coiling and its effect for the multimode pump absorption was for the first time presented in a conference paper [22] and in improved version in [23, 24].

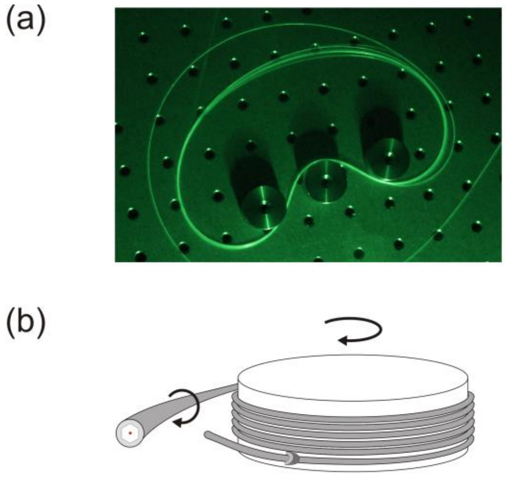

Fig. 1. Examples of unconventional coiling methods: Rare-earth-doped double-clad fiber coiled in a kidney shape (a) and a method for simultaneous coiling and twisting the fiber on a round spool (b).

In this paper we present a rigorous numerical modeling of multimode pump absorption in double-clad fibers of selected shapes of inner-cladding cross-sections. We consider the effect of coiling and twisting and we take into account realistic speckle pattern of the transversal field distribution of the pump. We shortly review results for pump absorption in fibers with circular and stadium-like shape of inner cladding [22-24]. Different diameters of coiling and pump beam positions are investigated. We show that, provided special launching and coiling conditions are fulfilled, even fibers with circular cross section can achieve high pump absorption. Emphasis is then given on analysis of pump absorption in hexagonal fiber that is often used in practical applications (preliminary results were mentioned in a conference paper [25]) and two-fiber bundle. Physical explanations of the phenomenon of pump absorption enhancement by unconventional coiling methods are discussed, particularly the effect of simultaneous coiling and twisting, see Fig. 1b.

II. THEORETICAL APPROACH

Four examples of double-clad fiber structures are considered: one with a circular symmetry and other with a broken circular symmetry: stadium-like cross section (125×250 µm), hexagonal shape (flat-to-flat distance 119 µm) and two-fiber bundle where both the pump and signal fibers have 125 µm diameter. The structures are illustrated in Fig. 2. The realistic geometrical size of fibers and doping concentrations are used in the modelling despite the fact that the computational requirements are high. Modelling of smaller structures with unrealistically high doping concentrations would largely speed up the calculations but on the other hand it may provide misleading results and decrease their impact for practice. Unless otherwise stated, the cores are identical, they have a diameter of 10 µm, with the same refractive index differences between the core and inner cladding (Δn_{core}=0.01) and inner cladding and outer cladding (Δn_{clad}=0.048). Loss-less outer coating is assumed. The absorption of the cores is modeled by the imaginary part of the core refractive index. The value of the imaginary refractive index part $\mathrm{Im}(n_{core})=\kappa=75\times10^{-6}$ corresponds to the absorption of a core homogeneously doped by 16250 ppm of Yb^{3+} at a wavelength of λ=976 nm. The relation of the value of the imaginary part of the refractive index κ and the absorption coefficient α is as follows:

$$\kappa = \mathrm{Im}(n_{core}) = \frac{\lambda}{4\pi}\alpha \quad . \tag{1}$$

The ideal limit of pump absorption [6] or ergodic limit of pump absorption efficiency [9] for the double clad fiber with neglecting the fiber bending is given by the following expression of the pump attenuation $\mathscr{A}$ in terms of the core and cladding area ratio:

$$\mathscr{A}(z) = 10\log\frac{P_{pump}(0)}{P_{pump}(z)} = 10\log\left[\exp\left(\alpha\frac{A_{core}}{A_{clad}}z\right)\right], \tag{2}$$

$$\eta(z) = 1 - \exp\left(\alpha\frac{A_{core}}{A_{clad}}z\right), \tag{3}$$

where $\mathscr{A}$ is the pump absorption in decibels and η is the pump absorption efficiency [6, 9]. The parameters of the fiber structures under study are summarized in Table I.

The stadium and circular structures were excited eccentrically, as shown in Fig. 2. Such a position of excitation should simulate pumping from spliced multimode pump delivery fiber [26]. Since the case of stadium fiber is compared with circular fiber in section III.A, the circular fiber has the same cladding area and the pump fiber is also attached asymmetrically to the input fiber end-face. For excitation, a speckle pattern was used that to some extent mimics the output of a multimode pump delivery fiber. The speckle pattern was synthesized as follows. A fundamental mode of a multimode fiber with a 105 µm core diameter and NA=0.22 was excited, i. e., it corresponds to excitation by a nearly Gaussian field profile of about 73 µm mode-field diameter. The field was then propagated through 10 cm of curved fiber having a radius of 3 cm, after which the propagation continued through 10 cm of a straight fiber, then through 10 cm of a fiber curved with a radius of 3 cm in the plane orthogonal to the previous plane of curvature, and finally propagated through 10 cm of straight fiber. At the end of this propagation and mode-mixing we obtained a speckle pattern simulating the excitation of the double clad fiber from a multimode pump delivery fiber, see the most left speckles patterns in Fig. 8 in Section III. Since the NA of the pump-delivery fiber is much lower than that of the DC fiber, no pump leakage due to NA steepening in bent fiber was observed. However, such pump loss may occur in the DC fiber for input fibers with higher NA, e. g., the output fiber of pump-signal combiners. Using a high NA pump fiber will lead to excitation of a greater number of pump modes than using a reduced numerical aperture. Ergodicity of the pump field should be then more efficient when a large number of modes are excited. However, study of the effect of pump fiber NA is beyond the scope of this paper.

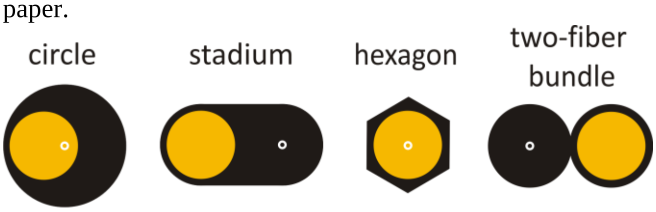

Fig. 2. Cross-sections of the double-clad fiber structures under study. Yellow circle depict input pump beam position

We performed the analysis of field propagation and absorption using the finite element beam propagation method [27, 28]. This method solves the Helmholtz equation via the slowly varying envelope approximation and is well established for similar problems [29]. The most important and new component of our analysis is modeling of the inner cladding of double-clad fiber curved by coiling. The modeling of the curved fiber is based on the modified refractive index profile [30, 31]. The model arises from a conformal transformation between a curved waveguide and a straight waveguide having a modified refractive index profile. In the case of an optical fiber having a waveguide diameter that is significantly smaller than the curvature diameter, the modified refractive index profile can be approximated by a linearly increasing refractive index profile, as follows:

$$n_{eff}(\vec{r}) = n_0(\vec{r})\cdot\left(1+\frac{\vec{l}\cdot\vec{r}}{R}\right) = = n_0(x,y)\cdot\left(1+\frac{x\cos\varphi+y\sin\varphi}{R}\right) \tag{4}$$

TABLE I
PARAMETERS OF FIBER STRUCTURES UNDER STUDY

parameter name	circle	stadium	hexagon	two-fiber bundle
Yb^{3+} concentration [ppm]	16 250	16 250	3100	16 250
Imaginary refractive index in the core	75×10^{-6}	75×10^{-6}	14.3×10^{-6}	75×10^{-6}
Δn_{core}	0.01	0.01	0.01	0.01
Core diameter [µm]	10	10	6	12
$\Delta n_{cladding}$	0.048	0.048	0.048	0.048
Cladding outer dimensions [µm]	188.4	125 (height) 250 (width)	119 (flat-to-flat)	125 & 125
Core diameter of the pump fiber [µm]	105	105	105	105
Pump wavelength [nm]	976	976	976	976

The quantity n_0 represents the original refractive index profile, n_{eff} is the modified refractive index profile, x and y are the transverse coordinates in the plane perpendicular to the fiber axis, **r**=[x,y] is the radius vector and ***l*** is unity vector characterizing the orientation of the twist in the cross-sectional plane, i.e., for the angle of rotation of the fiber with respect to spool φ=0 the vector ***l*** has the same direction as the x-axis, see Fig. 3. It should be noted that the modeling of the curved fiber based on the modified refractive index profile and conformal transformation of refractive index derived by M. Heiblum and J. Harris [30], and D. Marcuse [31] does not include fiber twist. Since we assume that the geometry of the waveguide cross-section preserves during bending and twisting, the actual refractive index profile is projected to the x-z plane (the untwisted case) by the scalar product of the vectors that define the twist. Possible birefringence effects induced by rotation of the drawn (cold) fiber are neglected. From the practical point of view, the large extent of inner cladding with respect to the pump wavelength poses significant requirements on computational resources and machine time. Therefore, most of the calculations were limited to maximum 3 m long fibers. Typical 3 m long simulation run for coiled and twisted fiber takes more than a week, e. g., 12 days for the two-fiber bundle, by using computer with Intel i7 3930k processor.

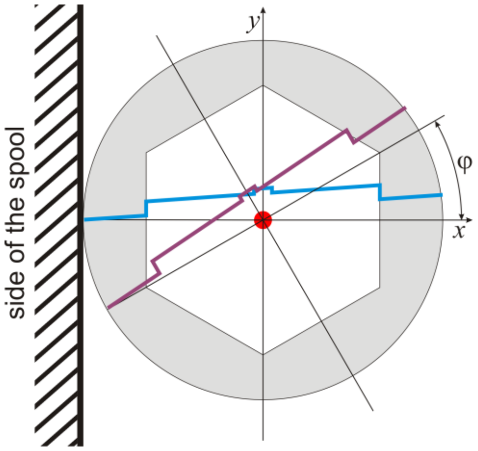

Fig. 3 Transformation of the refractive index profile for the coiled fiber of hexagonal shape (blue line) and for the hexagonal fiber simultaneously coiled and twisted by the angle φ (violet line).

III. RESULTS AND DISCUSSION

A. Effect of launching conditions for conventional coiling

We analyzed the dependence of the pump absorption in the stadium and circular double-clad fiber structures on both the curvature radius as well as the orientation of the double-clad structure (including the pump injection position) with respect to the fiber spool. The angle characterizing the orientation of the structure and the pump injection with respect to the fiber spool is illustrated in Fig. 4. The two fibers have the same area of the inner cladding and they are off-center spliced to the multimode pump fiber of 105 μm core diameter, so that the results can be as much as possible comparable. The resulting dependence of the pump absorption on the orientation of the structures is shown in Fig. 5(a) for the stadium structure and in Fig. 5(b) for the circular structure. The radius of curvature was 3 cm in both cases. From the figures it is seen that for both structures the absorption increases with increasing launch angle as it is defined in Fig. 4. This is very interesting and important result. Primarily, it shows that the regular circular structure can achieve similar absorption efficiency as the stadium-like structure for specific launching conditions. On the other hand, it shows that the absorption efficiency in a stadium-like structure can be significantly impaired when pumped improperly. Note that the highest absorption (and the overlap of the pump with the signal core) in stadium fiber is achieved for 135° launching condition and not for the 180° position where the pump is squeezed to the outer part of the fiber reel where the absorbing core is also located. The same trend was observed for different speckle pattern of the pump beam and for the flat-top pump beam [23,24].

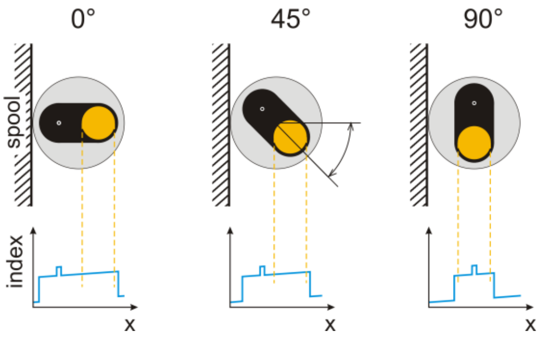

Fig. 4. Illustration of different pump launching conditions. The respective modified refractive index is schematically shown in the bottom.

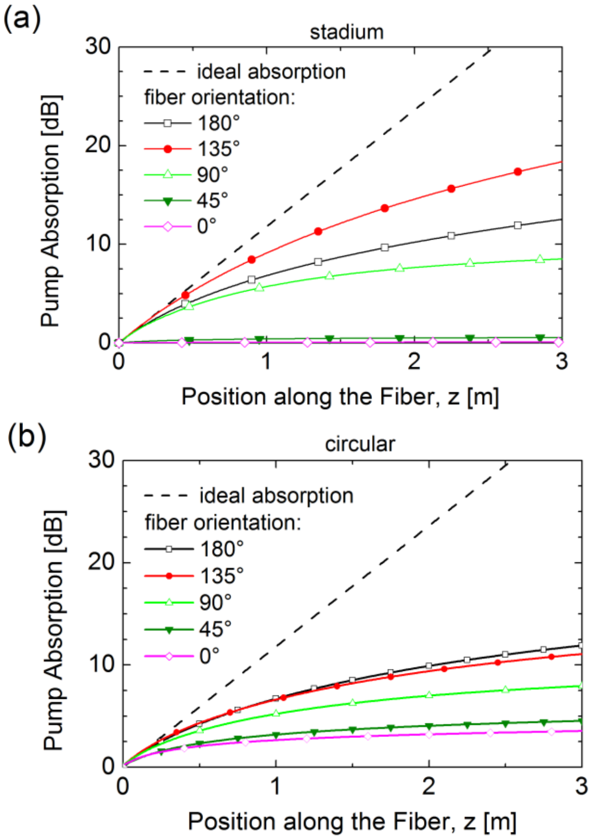

Fig. 5. Power absorption in the fiber with stadium (a) and circular (b) cross sections for different launching conditions of the pump. The spool radius was 3 cm.

The dependence of the pump absorption efficiency on the curvature radius is shown in Fig. 6. All simulations of this dependence were performed for an orientation of 180°. It can be seen that the absorption in the circular structure rapidly decreases with increasing curvature radius. In the case of the stadium structure the absorption decreases only slightly. These trends agree with results obtained using theory of chaotic billiards in the case of straight fibers and with previous experimental observations [4,5,7,10] and [15]. As the curvature radius increases, the fibers behave closer to straight fibers. In this case the absorption efficiency is better in the stadium structure than that in the circular structure.

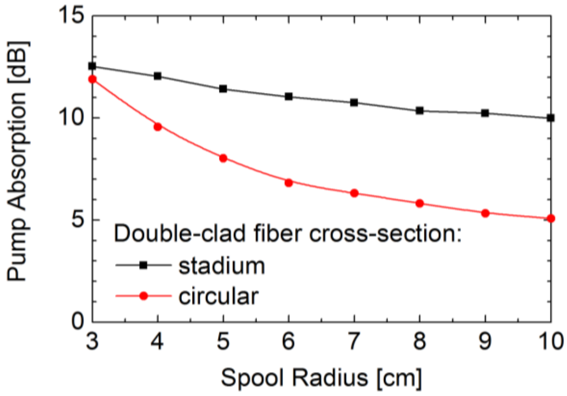

Fig. 6. The dependence of pump absorption on the radius of the spool.

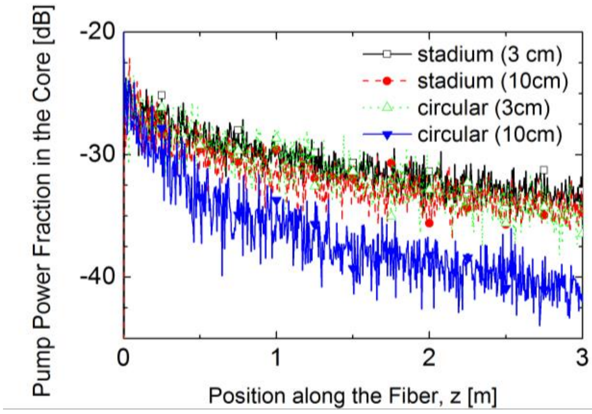

Fig. 7. Longitudinal dependence of the pump overlap with the doped core.

The reason of this effect is evident from a comparison of the pump field power portion inside the absorption core. The evolution of the ratio of pump power inside the core to total pump power is shown in Fig. 7. The calculated curves are smoothed by averaging to reveal the trends because the pump power ratio in the core oscillate fast along the fiber due to speckle-like pump field. The plots compare the behavior of both a circular and stadium structure having curvature radii of 3 cm and 10 cm and initial fiber orientation of 180°. The graphs show that the pump power depletes with a similar rate in the stadium structure for both curvature radii. The pump power depletes rapidly from the absorbing core in the circular structure having a 10 cm radius of curvature. This explains the saturation of absorption and less efficient absorption that is shown in Fig. 6. Opposed to this case, the depletion rate in the circular structure with a 3 cm radius of curvature is close to the depletion rate of the stadium structure. It can be seen that the absorption in the circular structure with a 3 cm radius of curvature is comparable to the absorption in the stadium structure.

B. Effect of twisting

In the previous section we have seen that the orientation of the fiber and input pump beam with respect to the spool affect the overall pump absorption. Similar effect as the orientation of the input pump beam and double-clad fiber with respect to the spool can occur if we change the orientation of the double-clad fiber with respect to the spool on the spool itself. There are several ways how to change the orientation in practice. One of them is changing convex and concave type of curving [16, 17], i. e., orientation of the fiber with respect to the spool in Fig 4. is changed by 180°. Similar effect can be achieved by changing the curving from convex to straight fiber [32]. The second method is based on gradual change of the fiber orientation with respect to the spool, e. g., by twisting the fiber during the coiling on the spool as shown in Fig. 1b. The twisting may be achieved also by drawing the fiber and rotating it simultaneously, so that the twist is frozen along the drawn fiber and then preserved also on the spool, which actually may happen even unintentionally and randomly during the drawing process.

The second method, i. e., the effect of twisting, is studied in this section for all four structures shown in figure 2. The hexagonal fiber and two-fiber bundle have geometry and arrangement corresponding to fibers often used in applications, including input pump fiber splicing to the center of the respective fiber of the active waveguide structure. It should be pointed out that the two-fiber bundle double-clad structure is principally different from the former three. Firstly, the two-fiber bundle belongs to the group of side-pumping schemes, i. e., the pump is coupled to the double-clad signal fiber from its side. On the other hand the other three structures belong to the end-pumping schemes where the pump is coupled to the double-clad fiber from its end, see Fig. 7 in references [1, 33]. Secondly, the two-fiber bundle offers the unique advantage of possibility to separate the pump and signal fibers, while the other structures are formed from a single fiber. The plurality of fibers in, e. g., two-fiber bundle is tightly touching thanks to a heat-shrunk tube over the fibers [34] or thanks to twisting the fiber bundle [35, 36]. Despite principal differences of the structures, the same numerical model described in the Section II can be applied for numerical modeling of the pump absorption in all four structures. The particular hexagonal fiber under study has the 6-μm core diameter and concentration of Yb^{3+} is 3100 mol ppm [25] and the core-radius of the signal fiber in the two-fiber bundle equals to 6 μm in accordance with [37]. The pump field transversal distribution of the hexagonal fiber is shown in Fig. 8 for straight fiber, coiled fiber and simultaneous coiled and twisted fiber. The speckle pattern of the input field corresponds to the figures for z=0 m. The overlap of the pump field with the core of the hexagonal fiber for the three coiling arrangements shown in Fig. 8 is plotted in Fig. 9. Due to fast change of the speckle pattern along the fiber the adjacent data points are averaged to reveal the trends. For the first 0.3 m the straight fiber exhibits highest pump power content in the core because no squeezing occurs. However, the pump power content in the core is decreasing quickly as the highly core-overlapping modes are depleted. On the other hand, the overlap with the core for the case of coiled and twisted hexagonal fiber is decreasing very slowly and stays higher than the ideal overlap for the straight fiber along the 3 m long fiber section.

The pump absorptions of the four fiber structures are compared in normalized longitudinal coordinate that accounts for different core and cladding area ratios and absorption coefficients. The normalized coordinate Z along the fiber is defined as follows [9]:

$$Z = \alpha \frac{A_{core}}{A_{clad}} z \,. \tag{4}$$

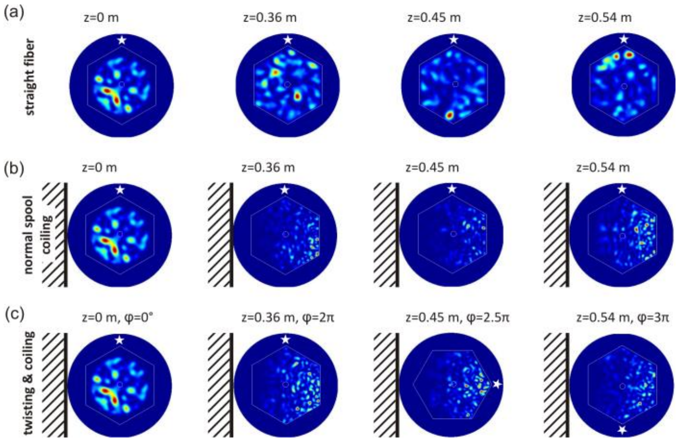

Fig. 8. Pump-field distribution at several positions along hexagonal fiber laid straight (a), coiled on 3-cm radius spool (b) and simultaneously coiled and twisted with rate dφ/dz=1°/mm on a 3-cm radius spool (c). One particular corner is labeled with a white star. The rotating horns of the hexagon smash-up the pump field and leads to effective mode-scrambling and improved pump absorption.

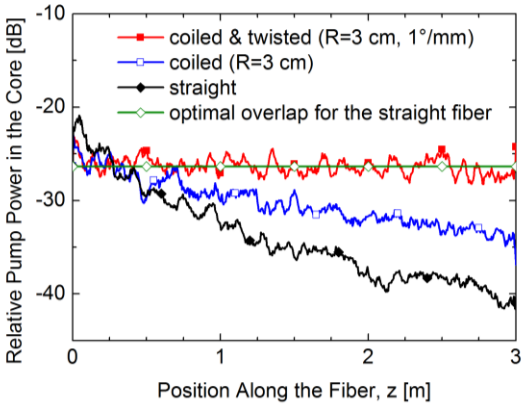

Fig. 9. Longitudinal dependence of the pump overlap with the doped core of the hexagonal fiber.

The calculated pump absorptions are summarized in Fig. 10. Since the maximum propagation distance of 3 m was used in simulations (except the hexagon fiber that was 9 m long), the extension of calculated results differs for different types of fibers in the graphs with normalized coordinate Z. The lowest pump absorption was obtained for all structures when the fiber was laid straight, see Fig. 10a. For the straight fiber case, the hexagonal fiber cross section performs best among the four DC structures. Bending of the DC fibers using standard coiling method on a spool improves absorption to some extent as shown in Fig. 10b. For the circular and stadium structures the pump beam orientation with the best and worst absorption are plotted in the graph. For the DC fibers with broken symmetry, the most efficient pump absorption was achieved when the DC fiber was coiled and twisted simultaneously, see Fig. 10c. Although the twist has almost no effect on the absorption of the DC fiber with circular cross section, the pump absorption in the other structures is significantly improved. For hexagonal shape the absorption better than the ideal limit can be achieved for significant fiber length. The improvement of pump absorption by coiling and twisting can be revealed while looking at Fig. 8. In the case of the straight fiber in Fig. 8a, the transversal distribution of the pump extends over the whole area of the inner cladding. In the case of bent fiber in Fig. 8b, the squeeze of the pump radiation outwards from the spool center is apparent. It means that the effective area of pump field radiation is decreased. The same squeezing effect can be seen in Fig. 8c for coiled and twisted fiber but in these cases additional advantageous effect of significantly enhanced mode mixing is added. For simultaneously coiled and twisted fiber the rotating horns of the hexagon are smashing-up the pump field and led to effective mode-mixing and improved pump absorption. The two combined effects, squeezing of the effective area of the pump field and efficient mode mixing by twisting can result in more efficient absorption than the ideal case shown in Fig. 10. The above mentioned unique property of the two-fiber bundle structure, i. e., the separability of the pump and signal fibers, invokes study of the pump power evolution separately in the pump and signal fibers that is shown in Fig. 11. Power evolution in pump and signal fibers was studied already in [37] by using coupled mode theory approach and in [38-39] by the beam-propagation method and assuming straight fibers in tight contact. The approach described in our work is different, we do not investigate distribution of power among the modes (realistic input-pump-speckle pattern is synthesized instead) and the fiber bundle is allowed to be bent and twisted. The results of simulations shown in Fig. 11 reveal that the absorption in straight two-fiber bundle without any externally induced mode-mixing is low. In the contrary, the twist serves not only to keep the plurality of fibers in tight contact [35, 36] but it may also improve the pump absorption.

IV. CONCLUSIONS

We have presented the rigorous numerical analysis of multimode pump absorption in double-clad fibers under realistic bending conditions, for example, when a fiber is coiled on a spool. The method is based on finite-element method and beam propagation method. The fiber bending and twisting are simulated by the transformation of the refractive index profile of the double-clad structure. Four different double-clad structures were investigated and their absorptions were compared: three double-clad fibers (with circular, stadium and hexagonal cross section) with the end-pumping scheme and the two-fiber bundle with the side-pumping scheme.

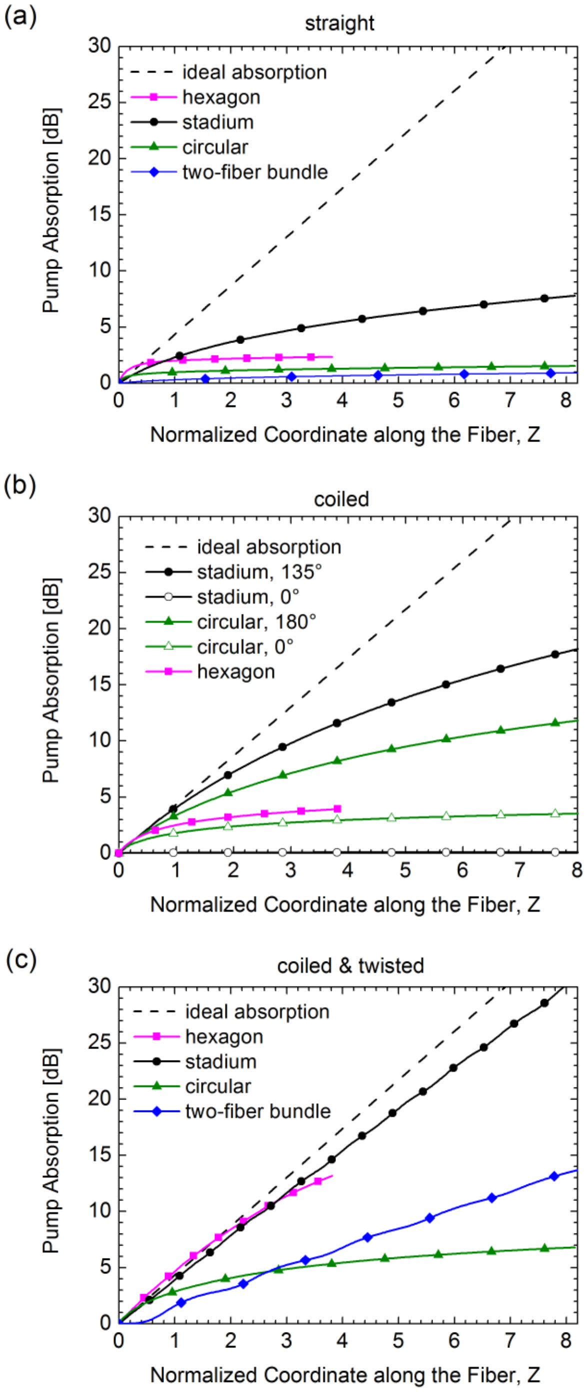

Fig. 10. Numerical modelling of pump absorption in hexagonal, circular and stadium fiber and two-fiber bundle for various arrangements of the fiber structure: (a) straight, (b) coiled, and (c) coiled and simultaneously twisted fiber structures with the rate dφ/dz=1°/mm.

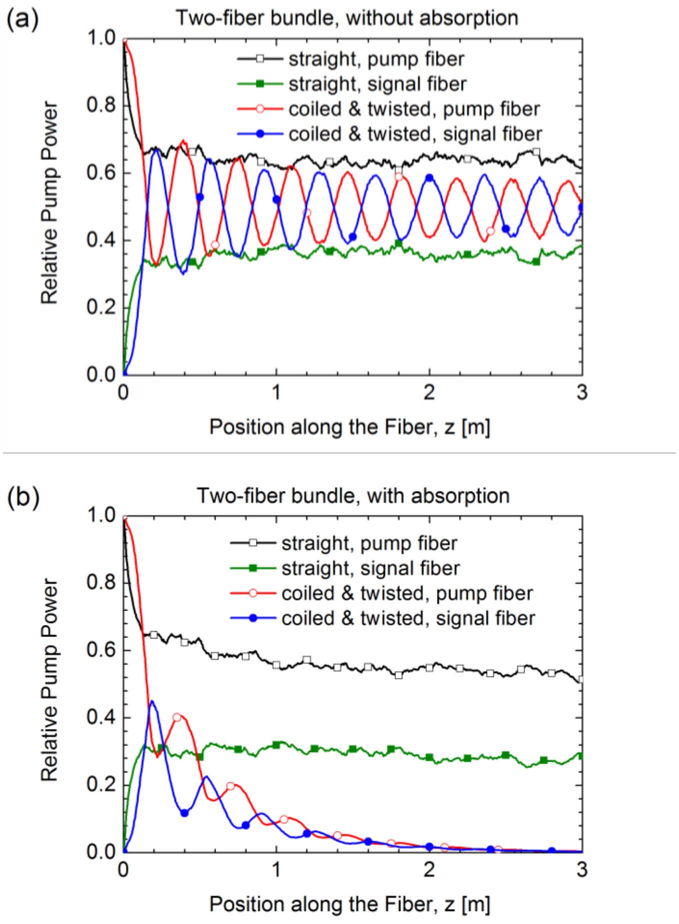

Fig. 11. Pump power distribution between the two fibers in two-fiber bundle for the case of straight arrangement and coiled and twisted fiber bundle. Two cases of signal fiber are considered, (a) without rare-earth dopants and (b) with the core doped with ytterbium.

The absorption in double-clad fibers of circular and stadium cross sections with respect to the diameter of the spool were investigated for the first time using the rigorous method. In accordance with the previous experimental observations it was found that for given pump launching scheme the absorption is only little dependent on the spool radius for the fiber with broken circular symmetry (the stadium) but for the fiber with circular cross section the pump absorption increases significantly with decreasing radius of the spool. The improvement of absorption is mainly thanks to enhanced mode mixing and also thanks to squeezing of the effective area of the inner cladding. The squeezing effect is even more pronounced in the case of improved mode mixing by coiling and twisting of the fibers with broken circular symmetry. In the case of hexagonal fiber it was shown that the absorption efficiency may exceed the ideal (ergodic) limit of absorption given by the core/cladding area ratio for the straight double-clad fibers along significant length of the DC fiber. To our knowledge, the importance of the squeezing effect for the pump absorption in double-clad fibers was recognized only by the described rigorous method of pump absorption modeling. Key role in the pump absorption improvement plays also effective mode mixing by rotating the cross section with broken circular symmetry with respect to the spool. For the first time, the pump absorption was studied by using the rigorous method in the double-clad structure of two-fiber bundle. The case of straight and simultaneously coiled and twisted fiber bundle were studied and the pump absorptions were compared. The twisting not only enables tight contact of the pump and signal fibers but it plays important role for mode mixing in the two-fiber double-clad structures.

The reported rigorous numerical model opens new way to design double-clad fibers and to optimize the pump absorption efficiency. Several examples of applications of the rigorous method demonstrated capability of the method to optimize both the fiber and the coiling method. With optimized pump absorption efficiency, the double-clad fiber of shorter length can be used in the fiber devices and in such a way the unwanted effects of background losses and nonlinear effects, e. g., stimulated Brillouin scattering and stimulated Raman scattering can be mitigated. Active fibers for moderate power fiber laser devices will particularly profit form the presented rigorous numerical modelling technique as they can be coiled and twisted. Absorption in rod-type active fibers for extremely high-peak powers may be improved through effective mode mixing by rotating the cross section with broken circular symmetry with respect to the fiber axis. Such a rotation can be achieved in controllable manner during the fiber drawing. Theoretical investigation of multimode pump propagation in bent double-clad fiber structures is still in its beginning. The prospects of future research are in the optimization of coiling and twisting parameters of different fiber designs. e. g., the coil radius, twist or rocking rate, doped-core area and pump launch distribution. Another topic would be detailed analysis of modal structure of bent fibers. Such analysis would certainly significantly contribute to proper understanding of physics involved in the operation of double-clad waveguide structures in realistic coiling conditions.

REFERENCES

[1] M. N. Zervas, C. A. Codemard, “High Power Fiber Lasers: A Review,” *IEEE J. Selected Topics in Quantum Electronics*, vol. 20, no. 5, pp. 219-241, Sept.-Oct. 2014.

[2] R. Maurer, “Optical waveguide light source,” U.S. Patent 3 808 549, Apr. 30, 1974.

[3] E. Snitzer, H. Po, F.Hakimi, R. Tuminelli, and B. C. McCollumn, “Double-clad, offset core Nd fiber laser,” in *Proc. Optical Fiber Sensors*, New Orleans, USA, postdeadline paper PD5, Jan 27-29, 1988, pp. 533-535.

[4] V. Doya, O. Legrand, and F. Mortessagne, “Optimized absorption in a chaotic double-clad fiber amplifier,” *Opt. Lett.*, vol. 26, no. 12, pp. 872-874, June 2001.

[5] P. Leproux, S. Fevrier, V. Doya, P. Roy, and D. Pagnoux, “Modeling and optimization of double-clad fiber amplifiers using chaotic propagation of the pump,” *Opt. Fiber Technol.*, vol. 6, no. 4, pp. 324-339, Oct. 2001.

[6] D. Kouznetsov and J. V. Moloney, “Efficiency of pump absorption in double-clad fiber amplifiers. II. Broken circular symmetry,” *J. Opt. Soc. Am. B*, vol. 19, no. 6, pp. 1259-1263, June 2002.

[7] P. Leproux, V. Doya, P. Roy, D. Pagnoux, F. Mortessagne, and O. Legrand, "Experimental study of pump power absorption along rare-earth-doped double clad optical fibres," *Opt. Comm.*, vol. 218, no. 4-6, pp. 249-254, Apr. 2003.

[8] I. Dritsas, T. Sun, K. T. V. Grattan, “Stochastic optimization of conventional and holey double-clad fibres,” *J. Opt. A: Pure Appl. Opt.*, vol. 9, no. 4, pp. 405-421, Apr. 2007.

[9] N. A. Mortensen, “Air-clad fibers: pump absorption assisted by chaotic wave dynamics ?,” *Opt. Express*, vol. 15, no. 14, pp. 8988-8996, July 2007.

[10] A. Martínez-Rios, A. N. Starodumov, H. Po, Y. Wang, A. A. Demidov, and X. Li, "Efficient operation of double-clad Yb3+-doped fiber lasers with a novel circular cladding geometry," *Opt. Lett.*, vol. 28, no. 18, pp. 1642-1644, Sept. 2003.

[11] M. A. Rebolledo and S. Jarabo, “Erbium-doped silica fiber modeling with overlapping factors,” *Appl. Opt.*, vol. 33, no. 24, pp. 5585-5593 Aug. 1994.

[12] V. Doya, O. Legrand, F. Mortessagne and C. Miniatura “Speckle statistics in a chaotic multimode fiber,” *Phys. Rev. E*, vol. 65, no. 5, pp. 056223-1-15, May 2002.

[13] P. Peterka, I. Kasik, V. Kubecek, V. Matejec, P. Honzatko, A. Zavadilova and P. Dvoracek, "Optimization of erbium-ytterbium fibre laser with simple double-clad structure," In Proc. SPIE 6180, pp. 618010-1-6, 2006.

[14] J. Nilsson, S. Alam, J. A. Alvarez-Chavez, P. W. Turner, W. A. Clarkson, A. B. Grudinin, "High-power and tunable operation of erbium-ytterbium Co-doped cladding-pumped fiber lasers," *IEEE J. Quantum Electronics*, vol. 39, no. 8, pp. 987-994, Aug. 2003.

[15] A. S. Kurkov, et al.,, "Yb^{3+}-doped double-clad fibers and lasers," In Proc. SPIE 4083, Advances in Fiber Optics, pp. 118-126, May 2000.

[16] Y. Li, S. D. Jackson, and S. Fleming, “High absorption and low splice loss properties of hexagonal double-clad fiber,” *IEEE Photonics Technol. Lett*., vol 16, no. 11, pp. 2502-2504, Nov. 2004.

[17] A. Tünnermann, T. Schreiber, and J. Limpert, "Fiber lasers and amplifiers: an ultrafast performance evolution," *Appl. Opt*., vol. 49, no. 25, pp. F71-F78, July 2010.

[18] J. M. Fini, “Bend-resistant design of conventional and microstructure fibers with very large mode area,” *Opt. Express*, vol. 14, no. 1, pp. 69-81, Jan. 2006.

[19] D. Jain, C, Baskiotis, and J. K. Sahu, “Bending performance of large mode area multi-trench fibers,” *Opt. Express*, vol. 21, no. 22, pp. 26663-26670, Nov. 2013.

[20] M. Hotoleanu, J. Koponen, and T. Kokki, “Spatial distribution of photodarkening in large mode area ytterbium doped fibers,” in *Proc. Advanced Solid-State Photonics*, *ASSP*, Nara, Japan, 27-30 January 2008, p. WE23.

[21] P. Peterka and V. Doya, “Double-clad fiber amplifier with direct coupling of the pump and signal into the active fiber: Modelling of multimode pump propagation,” presented at the Workshop of the COST Action 299 Optical fibers for new challenges facing the information society, Berlin, Germany, Feb. 5-7, 2008.

[22] P. Koska, P. Peterka, I. Kasik, V. Matejec, and O. Podrazky, "Double-clad rare-earth-doped fiber with cross-section tailored for splicing to the pump and signal fibers: analysis of pump propagation", In Proc. SPIE. 8775, pp. 87750V-1-6, 2013. Presented in SPIE Optics+Optoelectronics, 15-18 April 2013, Prague.

[23] P. Koška and P. Peterka, “Numerical analysis of pump propagation and absorption in specially tailored double-clad rare-earth doped fiber,” In Book of abstracts of the XXII International Workshop on Optical Waveguide Theory and Numerical Modelling (OWTNM 2014), Nice, France, June 27-28, 2014, p. O-1.1.

[24] P. Koška and P. Peterka, “Numerical analysis of pump propagation and absorption in specially tailored double-clad rare-earth doped fiber,” *Opt. Quant. Electron*., vol. 47, no. 9, pp. 3181-3191, Sep. 2015.

[25] P. Peterka, P. Honzátko, P. Koška, O. Podrazký, and I. Kašík, “Transient-fiber-Bragg grating spectra in self-swept Fabry-Perot fiber lasers,” In Proc. SPIE 9344, pp. 934423-1-6, 2015. Presented in SPIE Photonics West, San Francisco, USA, 7-12 February 2015.

[26] P. Peterka, I. Kasik, V. Matejec, V. Kubecek, and P. Dvoracek, “Experimental demonstration of novel end-pumping method for double-clad fiber devices,” *Opt. Lett*., vol 31, no. 22, pp. 3240-3242, Nov. 2006.

[27] D. Schulz, C. Glingener, M. Bludszuweit, and E. Voge, “Mixed Finite Element Beam Propagation Method,” *J. Lightwave Technol*., vol. 16, no. 7, pp. 1336-1342, July 1998.

[28] C. Geuzaine and J. F. Remacle, “Gmsh: a three-dimensional finite element mesh generator with built-in pre- and post-processing facilities,” *Int. J. Numer. Methods Eng*., vol. 79, no. 11, pp. 1309-1331, Sept. 2009.

[29] T. Muaro, K. Saitoh, and M. Koshiba, “Detailed theoretical investigation of bending properties in solid-core photonic bandgap fibers,” *Opt. Express*, vol. 17, no. 9, pp. 7615-7629, Apr. 2009

[30] M. Heiblum and J. Harris, “Analysis of curved optical waveguides by conformal transformation,” *IEEE J. Quantum Electron*., vol. 11, no. 2, pp. 75-83, Feb. 1975.

[31] Dietrich Marcuse, "Influence of curvature on the losses of doubly clad fibers," *Appl. Opt*., vol. 21, no. 23, pp. 4208-4213, Dec. 1982.

[32] Xu Mengrong, Cao Jianqiu, Huang Zhihe, Guo Shaofeng, "Study of fiber coiling methods on pump light absorption," (in Chinese), Guangxue Xuebao/Acta Optica Sinica, vol. 34, pp. s106008-1-8, Sept. 2014.

[33] M. N. Zervas, "High power ytterbium-doped fiber lasers - fundamentals and applications," *Int. J. Mod. Phys. B*, vol. 28, no. 12, pp. 1442009-1-35, May 2014.

[34] P. Polynkin, V. Temyanko, M. Mansuripur, and N. Peyghambarian, “Efficient and scalable side pumping scheme for short high-power optical fiber lasers and amplifiers,” *IEEE Photon. Technol. Lett*. vol. 16, no. 9, pp. 2024 – 2026, sept. 2004.

[35] A. B. Grudinin, J. Nilsson, P. W. Turner, C. C. Renaud, W. A. Clarkson, and D. N. Payne, “Single clad coiled optical fibre for high power lasers and amplifiers,” in Conference on Lasers and Electro-Optics, OSA Technical Digest (Optical Society of America, 1999), paper CPD26.

[36] A. B. Grudinin, D. N. Payne, P. W. Turner, J. Nilsson, M. N. Zervas, M. Ibsen, and M. K. Durkin, "Multi-fibre arrangements for high power fibre lasers and amplifiers," U.S. patent 6 826 335, November 30, 2004.

[37] M. N. Zervas, A. Marshall, and J. Kim, “Effective Absorption in Cladding-Pumped Fibers,” In Proc. SPIE 7914, Fiber Lasers VIII: Technology, Systems, and Applications, pp. 79141T-1-14 , Feb 2011.

[38] X. Qirong, R. Haicui, Y. Ping, Ch. Xiao, and G. Mali, "Theoretical study of pumping absorption in a co-linear side-pumping coupler," Opt. Comm., vol. 300, pp. 220-224, July 2013.

[39] X. Qirong, R. Haicui, Ch. Xiao, Y. Ping, and G. Mali, "Research on characteristics of co-linear side pumping couplers with special double-clad fibers," J. of Modern Optics, vol. 61, no. 4, pp. 307-314, Feb 2014.

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