R&D IN DUOPOLY WITH SPILLOVERS: EVOLUTION AND ASPIRATION LEARNING *

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The paper applies the evolutionary concept to an analysis of the role of intellectual property rights protection in the model of two countries North and South (and two firms) where only the Northern firm conducts innovative activity. The concept of social evolution and learning in oligopolistic industries (an aspiration-based model) is developed and the general algorithm of social evolution and aspiration learning for asymmetric duopoly is presented. The evolutionary equilibrium in R&D duopoly with spillovers is presented and analyzed. The results show that strengthening intellectual property rights protection always has negative welfare effects. In particular, it decreases not only the profit (producer surplus) of both the Northern and the Southern firms, but also the consumer surplus in both countries, and, consequently, it lowers social welfare in both countries.

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1. Introduction

It has long been argued (see, e.g., Marshall, 1948, or Nelson 1995) that economic science should aim to understand not simply the sustaining configuration of the set of variables in economic equilibrium but economic change as a whole. To explain the nature of this change many economists have become interested in the application of *biological concepts* based on the Darwinian theories of natural selection and survival. The rationale behind this development is the common agreement that real firms do not maximize profit (as is claimed in most economic textbooks) but instead use “rules of thumb” for solving problems and making decisions (see, for example, Hall and Hitch, 1939; Cyert and March, 1963, or Simon, 1947). It has also been suggested that firm's rules of thumb might evolve over time in a manner analogous to Darwinian evolution (Alchian, 1950; Nelson and Winter, 1982; Winter, 1971; Moss, 1991).

To model such behavior economists have made the connection between learning and evolutionary models (Binmore, Gale and Samuelson, 1993; Binmore and Samuelson, 1992, 1993a, 1993b; Canning, 1992; Kandori, Mailath and Rob, 1993; Young, 1993; Selten, 1991).[1] The central notion of this approach is very simple, and is closely related to the notion of bounded rationality (Simon, 1947, 1955, 1959, 1976, 1978, 1979, 1987). In the complex and uncertain world in which we live, economic agents follow *strategies*, or simple rules/heuristics, which tell them what to do. Different agents try out different strategies (and/or the same agents try out different strategies). Over time, strategies which are more successful than others become more common (either through a form of propagation or imitation) and, finally, strategies that make firms more profitable will tend to predominate over time. These strategies are usually explained as being the result of a process of social evolution.

To date a number of evolutionary models have been developed and results from the biology literature have been generalized (see, e.g., Weibull, 1997). In recent years, evolutionary concepts in economics (based on evolutionary theory in biology and sociology[2] or game theory[3]) have been employed increasingly in formal evolutionary theorizing (see, e.g., Nelson and Winter 1982; Dosi et al. 1988; Saviotti and Metcalfe 1991, Day and Eliasson, 1986). An elegant analytical history of evolutionary theoretical models

[1] The key popularizer of this idea in the general social sciences in recent years has been Axelrod (1984).

[2] See, for example, Hirshleifer and Martinez-Coll (1988).

[3] See, e.g., Young (1993), Kandori, Mailath and Rob (1993) or Friedman (1991).

in economics has been provided by Hodgson (1993), and later by Weibull (1997). It should be stressed that the development of the analytical tools and formal models has always been supported by simulation experiments (simulations were considered useful in that they point the way to theoretical results). For the purpose of modeling evolutionary systems, special computer simulation procedures (genetic algorithms), based on the Darwinian theories of natural selection, have been developed (see, Koza 1992; Goldberg 1989; Holland 1970; Bauer, 1994).[4] These algorithms describe the evolution of rules, representing different beliefs, in response to experience. Rules whose application has been more successful are likely to become more frequently represented in the population through a process similar to natural selection in population genetics. Random mutations also create new rules by changing certain features of rules previously represented in the population, thus allowing new ideas to be tried out. Genetic algorithms and other computer-based adaptive algorithms have been used in a variety of economic environments (see, e.g., Miller, 1989; Mariman, McGrattan and Sargent, 1990; Binmore and Samuelson, 1990; Dixon, Moss and Wallis, 1994; Dixon, 1995).[5]

While genetic algorithms are appropriate in a biological context, it is not that clear whether or not they are suitable for the modeling of economic behavior in a duopoly or oligopoly context (where firms are located in a particular industry and face the same competitors over time)[6]. In modeling oligopolistic markets, the features specific to the firms' behavior (learning and imitation instead of random matching) need to be introduced. In particular, the evolutionary process beyond random matching needs to be developed, to explore the ways in which firms move about and change partners (the issue of learning has already been the subject of papers by Moss and Edmonds, 1994; Moss, Dixon and Wallis, 1993; and also Dixon, 1995, 1997). Therefore, in order to analyse the evolution of oligopolistic markets, more sophisticated adaptive algorithms of social evolution and learning based on models of biological replication have been developed (see, e.g., Selten, 1991; Kandori at al, 1993, Young, 1993; and Dixon, 1995).

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[4] The first application of genetic algorithms to economics was described in Miller (1989).

[5] See Holland and Miller (1991) for an overview of the applications of computer-based adaptive algorithms in economics.

[6] For the survey of the existing literature on adaptive learning in oligopoly and strategic environments see, for example, Marimon (1995).

In the this paper the concept of social evolution and learning in oligopolistic industries (an aspiration based model), proposed by Dixon (1995, 1997), is developed and applied to an analysis of R&D in duopoly with spillovers (see, e.g., Chin and Grossman, 1990; and Zigic, 1998a) where Northern and Southern firms compete in quantities in a common world market and where only the Northern firm conducts innovative activity. In particular, the paper seeks to investigate the question: what types of behavior would we expect from R&D duopolists operating in a Cournot environment, and how does the evolutionary equilibrium differ from the static one?

The paper is organized as follows. Section 2 presents the general algorithm of social evolution and aspiration learning for asymmetric duopoly. In Section 3 an evolutionary equilibrium is characterized and the convergence of the evolutionary process to the equilibrium is proven. The model of R&D in asymmetric duopoly with spillovers and the main results of its game-theoretical analysis are presented in Section 4. In Section 5 the evolutionary equilibrium for the model of R&D in asymmetric duopoly with spillovers is characterized and the evolutionary process is illustrated by simulation experiments. Finally, in Section 6 the effects of different policies concerning intellectual property rights protection on the characteristics of market equilibrium are analyzed and discussed. Section 7 concludes.

2. Aspiration Learning in Asymmetric Duopoly—a Model

Following the model proposed by Dixon (1995, 1997) consider an economy consisting of a large number of identical markets (N). In each market there are two firms that operate in discrete time t (t=0,1,2,…) and play a game which is assumed to be the same in all markets in each period of time. Assume that the firms do not know the cost structures of their competitors, and, consequently, they are not able to find the Nash equilibrium strategies. The firms must earn at least normal profits to survive in the market in the long run.[7] The failure to achieve this activates a market mechanism such as bankruptcy or takeover and the firm is replaced by another one. Thus, in this model we assume that in the long-run each firm's profit aspiration should be at least as great as the normal profit. If, in a given period, the firm earns a profit below its aspiration level, then in the next period it tries a new strategy, i.e., it changes the volume of the output produced (it experiments). If the firm's profit is at or above the

[7] See, e.g., Dixon (1997) for details.

aspiration level, then in the next period the firm does not experiment (i.e., it produces the same volume of output). Furthermore, since firms are never perfectly rational in reality, we assume that they can make mistakes, i.e., firms which do not reach their aspiration levels may, by making a mistake, not experiment, while firms which perform at or above the aspiration level may, unintentionally, experiment.[8]

Similar models were considered by Lewin (1936), Simon (1947, 1987), Siegel (1957) or Lant (1992). In the present paper, however, following Dixon (1995, 1997), the aspiration level (linked to the normal profit in the economy) is endogenous[9] and depends on profits of all firms in the whole economy. Since the Northern and the Southern firms can have different cost structures, the normal profit in the economy is defined as the average profit of the lower-profit firms from each pair (in each market in the economy). In particular, we make the following hypothesis about the aspiration level $a^{(t)}$:

$$a^{(t)} = \frac{1}{N}\sum_{j=1}^{N} \min\{p_{s,j}^{(t)}, p_{n,j}^{(t)}\}, \qquad (1)$$

where N is a number of markets, and $p_{n,j}^{(t)}$ and $p_{s,j}^{(t)}$ denote the profits of the Northern and the Southern firms operating in market j (j=1,2,…,N) in period t (t=0,1,2,…), respectively.[10] We believe that this assumption is reasonable, since firms can always get information about the normal level of profits in the economy (from the capital market or the stock market, for example) and use this information to judge its own performance.

Since the profits of all firms (and in particular, the profits of the lower-profit firms in each particular market) may vary over time (due to the new strategies applied), the value of the aspiration level of firms changes from period to period reflecting the past experience of firms and the current profitability of all firms in the economy.

The formal description of the evolution of duopolistic markets and the characteristics of the equilibrium are presented in the next section.

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[8] We assume that the probability of error in both cases is identical and that the firm with unstable profit is more likely to make errors, i.e., the probability of error is inversely proportional to the variability of the profit (see Section 3).

[9] See also Borgers and Sarin (1994), for similar approach.

[10] Note that in each duopolistic market, either the profit of one firm is higher and the profit of the other one is lower or the profits of both firms are the same. Thus, the only available aggregate information about the profitability of firms in the whole economy is: the average profit of all firms, the average profit of the higher-profit firms, and the average profit of lower-profit firms.

3. Evolutionary equilibrium

Consider an economy with N identical single commodity markets. For the sake of simplicity suppose that inverse demand in each market is linear, given as

$$P(Q) = \begin{cases} A - Q & \text{if} \quad 0 \le Q \le A \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

where $P(Q)$ is the market price if the volume Q is supplied to the market (A is a positive constant). Assume that there are two firms operating in each market: the Northern firm and the Southern firm. The profit of the Northern firm is given as

$$p_n \ (q_s, q_n) = \max\{P(q_n + q_s)q_n - c_n q_n - b_n, 0\}, \tag{3}$$

where q_n is the output of the Northern firm, q_s is a quantity supplied to the market by a Southern firm, c_n is a marginal cost and b_n is a fixed cost. The profit of the Southern firm is determined as

$$p_s(q_s, q_n) = \max\{P(q_n + q_s)q_s - c_s q_n - b_s, 0\} \tag{4}$$

where q_s is the output of the Southern firm, c_s is a marginal cost and b_s is a fixed cost of the Southern firm.

The firms perform in discrete time t (t=0,1,2,…) and in any period of time each firm selects its output level by taking the output of its competitor in the market as given. Since firms do not have perfect information about the cost structure of their competitors, they are not able to find Nash equilibrium strategies. Instead they try out various strategies, and, finally, the strategies that allow the firms to perform not worse than other firms in the economy tend to predominate over time in all markets.

To simplify the analysis, suppose that each firm in the economy chooses its output (its strategy) from a finite set of possible values. In particular, assume that the Northern firm chooses its strategy from the set

$$\boldsymbol{S_n} \equiv \{q_n^1, q_n^2, \ldots, q_n^{K_n}\} \subset [0, A], \tag{5}$$

and the Southern firm from the set

$$\boldsymbol{S_s} \equiv \{q_s^1, q_s^2, \ldots, q_s^{K_s}\} \subset [0, A]. \tag{6}$$

Furthermore, suppose that the solution to the following maximization problem (joint profit maximization under the equal profit condition):

$$\max_{\substack{q_s \in [0,A] \\ q_n \in [0,A] \\ q_s + q_n \in [0,A]}} \{p_s(q_s, q_n) + p_n(q_s, q_n)\}, \tag{7}$$

$$s.t.\ p_s(q_s,q_n)=p_n(q_s,q_n)$$

is included in the sets of possible strategies $S_s \times S_n$.

Note that solution to *(7)* cannot be obtained by the Lagrange multiplier method since functions $p_s(q_s,q_n)$, $p_n(q_s,q_n)$ are not differentiable on the whole domain specified by the constraints. It is quite clear, however, that if (i) c_n, c_s are close enough and (ii) b_n, b_s are close enough and small, then the solution to *(7)* must coincide with $(\tilde{q}_s,\tilde{q}_n)$ that solves the following maximization problem:

$$\max_{\substack{q_s\in[0,A]\\ q_n\in[0,A]}} \left\{(A-q_s-q_n)q_s-c_sq_s-b_s+(A-q_s-q_n)q_n-c_nq_n-b_n\right\} \tag{7b}$$

$$\text{s.t. } (A-q_s-q_n)q_s-c_sq_s-b_s=(A-q_s-q_n)q_n-c_nq_n-b_n.$$

Moreover, it must hold that $0<\tilde{q}_s+\tilde{q}_n<A$[11].

The solution to maximization problem *(7b)* can be represented as[12, 13]

$$\tilde{q}_n=\frac{(1+l)\left(d_s-(1-l)d_n\right)}{2l^2}\quad , \tag{8}$$

$$\tilde{q}_s=\frac{(1-l)\left(d_n-(1+l)d_s\right)}{2l^2}\quad , \tag{9}$$

where $d_n=A-c_n$, $d_s=A-c_s$, $l=-\dfrac{D_1}{D_3}+\dfrac{D_3}{3D_2\sqrt[3]{2}}$,

$$D_1=2^{\frac{1}{3}}(d_s^2-d_n^2),\ D_2=4b_n-4b_s-d_n^2+d_s^2,$$

$$D_3=\left(-54D_2^2(d_n-d_s)^2+\sqrt{108(-d_n^2+d_s^2)^3D_2^3+2916D_2^4(d_n-d_s)^4}\right)^{\frac{1}{3}}.$$

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[11] This is because in the case of exact equalities $c_s=c_n$ and $b_s=b_n=b$, where b is small, the constraint in *(7b)* effectively disappears, and the unconstrained problem will boil down to a monopolist's maximization, leading to $\tilde{q}_s^{Mon}=\tilde{q}_n^{Mon}=\frac{1}{2}q_{Mon}$, $0<\tilde{q}_s^{Mon}+\tilde{q}_n^{Mon}<A$. The continuity of profit functions in *(7b)* then assures the existence of a solution, even for small deviations in c_s, c_n, b_s and b_n, as well as its "closeness" to $(\tilde{q}_s^{Mon},\tilde{q}_n^{Mon})$.

[12] Note that the solution is not feasible if $l=0$ (this can happen only if $c_s=c_n$ and $b_s=b_n$). In this case the solution is given as $q_s=q_n=\frac{1}{4}d$, where $d=A-c_s=A-c_n$.

[13] If one of the expressions *(8)* or *(9)* is not defined, then the interior solution to *(7b)* does not exist. In this case the solution to *(7)* is not unique, and it is given by any pair $(q_s,q_n)\in[0,A]\times[0,A]$ such that $p_s(q_s,q_n)=p_n(q_s,q_n)=0$.

Assume that the number of duopolistic markets in the economy N is large, i.e., $N > K_s K_n$, and suppose that in any period t (t=0,1,2,...) each firm's aspiration level is determined according to the following expression

$$a^{(t)} = \sum_{i=1}^{K_n} \sum_{j=1}^{K_s} p_{ij}^{(t)} \min\left\{p_s(q_s^i, q_n^j), p_n(q_s^i, q_n^j)\right\} \tag{10}$$

where $p_{ij}^{(t)}$ denotes the probability with which the strategy pair $\left(q_s^i, q_n^j\right)$ is played in the economy ($p_{ij}^{(t)} = N_{ij}^{(t)} / N$, where $N_{ij}^{(t)}$ denotes number of markets in the economy in which $\left(q_n^i, q_s^j\right)$ is being played in period t)[14].

In any period of time t all firms compare their profits with the aspiration level $a^{(t)}$. If the profit of a particular firm is lower than the aspiration level, then in the next period (t+1) the firm intends to experiment[15]. Otherwise, in period t+1 the firm does not intend to experiment, i.e., it intends to follow the same strategy (to produce the same output as in period t).

Moreover, taking into account that firms are never perfectly rational in reality we assume that, with the probability $e_k^{(t)}$ ($k \in \{s,n\}$) they can make mistakes, i.e., with this probability each firm experiments even if it earns a profit greater than or equal to $a^{(t)}$, and does not experiment otherwise. Since firms learn during the evolutionary process, we assume that the probability of making a mistake decreases with the number of subsequent periods in which the firm does not experience any change in its profit. In particular, in the simplest case, we assume that the firm can make a mistake only if its profit has been changed in a given period, i.e.,

$$e_k^{(t)} = \begin{cases} g_k \left| p_k(q_s^{(t)}, q_n^{(t)}) - p_k(q_s^{(t-1)}, q_n^{(t-1)}) \right|, & if \quad t > 0 \\ 0 \quad , & if \quad t = 0, \end{cases} \tag{11}$$

where $k \in \{s,n\}$, $q_k^{(t)}, q_k^{(t-1)}$ denote the strategies of the firm under consideration in periods t and (t-1), respectively, and $g \in (0, ß_k)$, is an exogenously given constant ($ß_k$ is chosen so that the probability of mistake is bounded away from 1).

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[14] Note that by definition $\sum_{i=1}^{K_n} \sum_{j=1}^{K_s} p_{ij}^{(t)} = 1$, for any t=0,1,2,… .

[15] By experimenting we understand a random choice of the quantity produced. The randomness is determined by the uniform probability measure over the set of feasible strategies S_s (S_n).

Let N_t denote the number of markets in the economy where the pair of strategies $\left(q_s^{i*}, q_n^{j*}\right) \equiv \left(\tilde{q}_s, \tilde{q}_n\right)$ is being played at time t, and with probability one will be played in period t+1 (t=0,1,2,…)[16].

In the analysis which follows we will show that, if all firms follow the set of rules described above, then there exists an evolutionary equilibrium $\left(q_s^{i*}, q_n^{j*}\right) \equiv \left(\tilde{q}_s, \tilde{q}_n\right)$, and, in particular, if at the certain period of time t_0 at least in one market a firm plays the pair of equilibrium strategies, then this is the only absorbing pair of strategies in the whole economy (i.e., an evolutionary process converges to this equilibrium with probability one). This result is characterized formally by the theorem below.

Theorem 1: If all firms in the economy follow the set of rules described above, and

(i) $K_s > 2$, $K_n > 2$,

(ii) $\left(\tilde{q}_s, \tilde{q}_n\right)$ is a unique interior solution to maximization problem *(7b)*, such that it coincides with the solution to *(7)*, and $p_s(\tilde{q}_s, \tilde{q}_n) = p_n(\tilde{q}_s, \tilde{q}_n) = \tilde{p}$ >0.

(iii) the probability of making a mistake is bounded away from 1,

(iv) for a certain t_0, $N_{t_0} > 0$,

then, as $t \to \infty$, the pair of strategies $\left(q_s^{i*}, q_n^{j*}\right) \equiv \left(\tilde{q}_s, \tilde{q}_n\right)$, prevails on all markets, i.e.,

$$\lim_{t \to +\infty} p_{i^* j^*}^{(t)} = 1.$$

<u>Proof:</u> By point (i) of *Lemma 1* (see the Appendix), it holds for any t=1,2,… that $a^{(t)} \le \tilde{p} = p_s\left(q_s^{i*}, q_n^{j*}\right) = p_n\left(q_s^{i*}, q_n^{j*}\right)$. Consequently, firms playing the equilibrium strategies $\left(q_s^{i*}, q_n^{j*}\right) = \left(\tilde{q}_s, \tilde{q}_n\right)$ never experiment intentionally. This means that, if the pair of strategies $\left(q_s^{i*}, q_n^{j*}\right)$ has been played in a given market twice in a row (thereby excluding the possibility of future mistakes), this pair of strategies $\left(q_s^{i*}, q_n^{j*}\right)$ will be played there forever. Thus, N_t (where N_t specifies the number of markets in which firms have played the pair of

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[16] Due to the specific form of the probability of the error assumed in the paper, N_t can be expressed as: $N_t = \sum_{m=1}^{N} v_m^{(t)}$, where

$$v_m^{(t)} = \begin{cases} 1, \text{ if } q_s^{(t-1)} = q_s^{(t)} = q_s^{i^*} \text{ and } q_n^{(t-1)} = q_n^{(t)} = q_n^{j^*} \\ 0, \text{ otherwise} \end{cases}.$$

strategies $\left(q_s^{i*}, q_n^{j*}\right)$ in both period t and (t-1)) is a non-decreasing function of time. This, together with assumption (*iv*), implies that $N_t > 0$ for any $t \geq t_0$ (note that a situation when all firms in the economy earn identical profits smaller than $p_s\left(q_s^{i*}, q_n^{j*}\right) = p_n\left(q_s^{i*}, q_n^{j*}\right)$ and yet reach their aspiration level is excluded).

Suppose now that for a certain $t \geq t_0$: $N_t < N$ (where N is the total number of markets in the economy). It further follows from *Lemma 1* (see the Appendix) that in period t at least one market must exist (henceforth referred to as M) in which firms play strategies $\left(q_s^i, q_n^j\right) \neq \left(q_s^{i*}, q_n^{j*}\right)$, and the profit earned by at least one firm is smaller than $\tilde{p} = p_s\left(q_s^{i*}, q_n^{j*}\right) = p_n\left(q_s^{i*}, q_n^{j*}\right)$. Hence, in period t the following three situations can occur in market M:

$(a)\, p_s^{(t)}\left(q_s^i, q_n^j\right) < a^{(t)}$ and $p_n^{(t)}\left(q_s^i, q_n^j\right) < a^{(t)}$: both firms intend to experiment,

$(b)\, p_s^{(t)}\left(q_s^i, q_n^j\right) < a^{(t)} \leq p_n^{(t)}\left(q_s^i, q_n^j\right)$: only the Southern firm intends to experiment,

$(c)\, p_n^{(t)}\left(q_s^i, q_n^j\right) < a^{(t)} \leq p_s^{(t)}\left(q_s^i, q_n^j\right)$: only the Northern firm intends to experiment.

In the analysis which follows we will show that for all three situations listed above, the probability that in period $t+3$ the firms in market M will play the pair of strategies $\left(q_s^{i*}, q_n^{j*}\right)$ is bounded away from zero, i.e., $P(N_{t+3}=N_t+1) > d > 0$. Note that to show this it is suffices to examine only the lower bound to the probability $P(N_{t+3}=N_t+1)$. Therefore, for each possible situation specified above it is enough to identify only one possible "time path" that brings market M into the equilibrium, and evaluate the probability that such a path is followed.

Consider situation (*a*), when both firms intend to experiment, and look at the following possible "time path":

(*i*) In period t both firms experiment (change their strategies), and, as a result, in period t+1 the pair of strategies $\left(q_s^{i*}, q_n^{j*}\right)$ is played in the market. The probability of this event can be computed as a product of the probability that both firms do not make mistakes, and the probability that the pair of strategies $\left(q_s^{i*}, q_n^{j*}\right)$ will be chosen as result of experimenting, i.e.,

$$P(i) = (1-e_s^{(t)})(1-e_n^{(t)})\frac{1}{K_s K_n} \geq (1-w)^2 \frac{1}{K_s K_n}, \qquad (12)$$

where $w = \min\{w_s, w_n\}$ and $w_k = \max_{\substack{q_s,q_n \\ q_s',q_n'}} \{ g_k |p_k(q_s,q_n) - p_k(q_s',q_n')| \}$.[17]

(ii) In period t+1 (when the pair of strategies $\left(q_s^{i*}, q_n^{j*}\right)$ is played in market M, and thus no firm intends to experiment) neither firm makes a mistake. The probability of this event is equal to the probability that both firms do not make mistakes, i.e.,

$$P(ii/i) = \left(1 - e_s^{(t+1)}\right)\left(1 - e_n^{(t+1)}\right) \geq (1 - ?\,)^2 . \qquad (13)$$

(iii) In period t+2 no firm intends to experiment (see *Lemma 1*), and it cannot make a mistake (the pair of strategies $\left(q_s^{i*}, q_n^{j*}\right)$ is played in the market already the second time in a row). Thus, the probability of this event is equal to one, i.e.,

$$P(iii / ii) = 1 . \qquad (14)$$

Consequently, the probability of the whole "time path" can be estimated from below as

$$P(a) = P(i)P(ii / i)P(iii / ii) \geq (1 - w)^4 \frac{1}{K_s K_n} . \qquad (15)$$

Now consider situation (b), when only the Southern firm intends to experiment, and look at the following time path:

(i) In period t no firm makes a mistake, i.e., only the Southern firm experiments, while the Northern firm, in the next period, adheres to its current strategy. As a result, in time period t+1 a pair of strategies $\left(q_s^h, q_n^j\right)$, where $h \in \{1,2,...,K_s\}$ and $h \neq i$, is played in the market, such that the profits of both firms change[18]. The probability of this event is equal to the probability that neither firm makes a mistake, i.e.,

$$P(i) = \left(1 - e_s^{(t)}\right)\left(1 - e_n^{(t)}\right) \geq (1 - ?\,)^2 . \qquad (16)$$

(ii) At time period t+1 the following three situations are possible:

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[17] Note that, as the probability of making errors is bounded away from 1, $?_k$ is strictly smaller than 1, and so is $?$.

[18] Note that such h always can be chosen. $\frac{¶p_n}{¶q_s} < 0$ implies that p_n changes whenever $h ¹ i$. p_s is a quadratic function of q_s,; therefore, for fixed q_n^j at most two distinct $q_s^i, \bar{q}_s^i$ exist such that $p_s(q_s^i, q_n^j) = p_s(\bar{q}_s^i, q_n^j)$. But, as $K_s > 2$, in addition to q_s^i and $\bar{q}_s^i$, there must be a third q_s^h included in the grid.

(*ii.a*) Both firms truly experiment and, as a result, in period t+1 the pair of strategies $\left(q_s^{i*}, q_n^{j*}\right)$ is played in the market. The probability of this event is equal to[19]:

$$P(ii.a/i) \geq (1-?)^2 \frac{1}{K_s K_n} . \qquad (17)$$

(*ii.b*) Only the Southern firm intends to experiment and does not make a mistake. The Northern firm does not intend to experiment, yet makes a mistake and experiments as well. As a result, in period t+1 the pair of strategies $\left(q_s^{i*}, q_n^{j*}\right)$ is played in the market. The probability of this event can be estimated from below as (recall that the profit of both firms changed in period t):

$$P(ii.b/i) = \left(1 - e_s^{(t+1)}\right) e_n^{(t+1)} \frac{1}{K_s K_n} \geq (1-?)?_n \frac{1}{K_s K_n} , \qquad (18)$$

where $\Delta_n = \min\limits_{\substack{q_s, q_n, \\ q_s', q_n'}} g_n \left| p_n(q_s, q_n) - p_n(q_s', q_n') \right|$,

$$s.t. \qquad p_n(q_s, q_n) \neq p_n(q_s', q_n')$$

(*ii.c*) Only the Northern firm intends to experiment and does not make a mistake. The Southern firm, although it does not intend to experiment, makes a mistake and experiments as well. As the result in the next period (t+1) the pair of strategies $\left(q_s^{i*}, q_n^{j*}\right)$ is played in the market. The probability of this event can be estimated from below as

$$P(ii.c/i) = \left(1 - e_n^{(t+1)}\right) e_s^{(t+1)} \frac{1}{K_s K_n} \geq (1-?)?_s \frac{1}{K_s K_n} , \qquad (19)$$

where $\Delta_s = \min\limits_{\substack{q_s, q_n, \\ q_s', q_n'}} g_s \left| p_s(q_s, q_n) - p_s(q_s', q_n') \right|$,

$$\text{s.t.} \qquad p_s(q_s, q_n) \neq p_s(q_s', q_n') \ .$$

Whatever the situation is, the pair of strategies $\left(q_s^{i*}, q_n^{j*}\right)$ is played in the market in period t+1.

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[19] See (*i*) in the analysis of situation *a*.

(*iii*) In time period t+2 no firm makes a mistake. The probability of this event is equal to

$$P(iii/ii) = \left(1-e_s^{(t+2)}\right)\left(1-e_n^{(t+2)}\right) \geq (1-?)^2. \tag{20}$$

Although any sequence of events considered is possible, i.e., (*i*)(*ii.a/i*)(*iii/ii*), (*i*)(*ii.b/i*)(*iii/ii*) or (*i*)(*ii.c/i*)(*iii/ii*), the probability of the path (*i*)–(*iii*) can be estimated from below as[20]

$$P(b) > (1-?)^4 ?, \tag{21}$$

$$\text{where } q = \min\left\{(1-w)^2 \frac{1}{K_s K_n}, (1-w)\Delta_n \frac{1}{K_s K_n}, (1-w)\Delta_s \frac{1}{K_s K_n}\right\}. \tag{22}$$

In situation (*c*), when only the Northern firm intends to experiment, the probability of the corresponding path can be estimated from below by the expression identical to *(21)*.

Putting together (*a*), (*b*) and (*c*) we get

$$P(N_{t+3} = N_t + 1) \geq (1-w)^4 \min\left\{q, \frac{1}{K_s K_n}\right\} = d > 0. \tag{23}$$

Therefore, if there exists a certain t_0 ($t_0 \geq 0$) such that $N_{t_0} > 0$, then for any t ($t=t_0$, t_0+1, $t_0+2, \ldots, t^*-1$, where t^* is such that $N_{t^*}=N$ and $N_{t^*-1}<N$) :

$$N_{t+3} = \begin{cases} N_t + 1, \text{ with probability at least } d > 0. \\ N_t, \text{ with probability at most } (1-d) > 0. \end{cases}$$

Consequently, the probability that starting from t_0 after a finite number of periods $3l$, $l > N - N_{t_0}$, the pair of strategies $\left(q_s^{i*}, q_n^{j*}\right)$ will be played in all markets can be estimated by the following expression

$$P(N_{t=3l+t_0} = N) \geq \sum_{j=N-N_0}^{l} \binom{l}{j} d^j (1-d)^{l-j}, \tag{24}$$

where the right hand side of *(24)* describes the probability that for all t=t_0 the unrestricted process

$$N_{t+3} = \begin{cases} N_t + 1, \text{ with probability at least } d > 0. \\ N_t, \text{ with probability at most } (1-d) > 0. \end{cases}$$

reaches N from N_0 within 3l periods.

The right hand side in the expression *(24)* can be represented as

---

[20] Note that $P(ii/i)=P(ii.a/i) + P(ii.b/i) + P(ii.c/i) \geq \min\{P(ii.a), P(ii.b), P(ii.c)\}$.

$$\sum_{j=N-N_0}^{l}\binom{l}{j}d^j(1-d)^{l-j}=\sum_{j=0}^{l}\binom{l}{j}d^j(1-d)^{l-j}-\sum_{j=0}^{N-N_0-1}\binom{l}{j}d^j(1-d)^{l-j}. \qquad (25)$$

Note that for any natural l

$$\sum_{j=0}^{l}\binom{l}{j}d^j(1-d)^{l-j}=1, \qquad (26)$$

and for the j-th term of the sum $\sum_{j=0}^{N-N_0-1}\binom{l}{j}d^j(1-d)^{l-j}=1$ the following holds:

$$0<\binom{l}{j}d^j(1-d)^{l-j}=\underbrace{\frac{1}{j!}\left(\frac{d}{1-d}\right)^j}_{A(j)}\left[l(l-1)\cdots(l-j+1)\right](1-d)^l\le A(j)l^j(1-d)^l. \qquad (27)$$

Hence,

$$0\le\lim_{l\to+\infty}\left\{\binom{l}{j}d^j(1-d)^{l-j}\right\}\le A(j)\lim_{l\to+\infty}\left\{l^j(1-d)^l\right\}=0, \qquad (28)$$

and,

$$\lim_{l\to+\infty}P(N_{t=3l}=N)=\lim_{l\to+\infty}\left\{\sum_{j=N}^{l}\binom{l}{j}d^j(1-d)^{l-j}\right\}=1. \qquad (29)$$

QED.

The above theorem predicts that in long-run stationary state the normal profit in the economy must be equal to the long-run average profit, but it says little about the path towards the long-run stationary state. An examination of the evolution of aspiration levels can be done for each particular model using computer simulation (see Section 5).

4. R&D in Duopoly with Spillovers - a Game Theoretical Model

Following Chin and Grossman (1990) and Zigic (1998,1998a) assume that there are two countries, a developed one—"North," and a developing one—"South." In each country there is a single firm. The firms operate in the integrated world market (which consists of Northern and Southern parts) and compete in quantities. The Northern firm is the only one which conducts R&D.[21] The Southern firm does not perform R&D but benefits through spillovers

[21] World patent statistic shows that about 99% of existing patents are held by nationals of developed countries (see Braga, 1990).

from the R&D activity of the Northern firm.[22] The effects of R&D are captured by an "R&D production function" which exhibits diminishing returns, i.e., every additional unit invested in R&D results in a smaller reduction of the unit costs.

The Northern firm chooses the value of its R&D expenditures taking into account the subsequent competition in the product market and the fact that a fraction of its R&D output goes to its Southern competitor. Furthermore, following Zigic (1998a), the initial (pre-innovative) unit costs of the North and the South are assumed to be the same (the Northern firm could be represented by a subsidiary which is physically located in the South, and it relies on the same labor force as the South).[23]

The game theoretical model of North-South interaction is two stage game. In the first stage, the Northern firm chooses its R&D expenditure *x*. In the second stage, the firms compete in quantities. The Northern firm has unit costs of production:

$$C(x) = a - \sqrt{gx}\,, \quad x \leq a^2/g\,, \tag{30}$$

where parameter g ($g \geq 0$) describes the efficiency of the R&D process and x denotes the R&D expenditures of the Northern firm. The expression $\sqrt{gx}$ is the "R&D production function," and it is assumed to have the same functional form as in Chin and Grossman (1990) and Zigic (1998a); a is a parameter which can be thought of as a pre-innovative unit cost.

The Southern firm benefits through spillovers from R&D activity carried out by the Northern firm. Its unit cost function is

$$c(x) = a - ß\sqrt{gx}\,, \tag{31}$$

where b denotes the level of spillovers. The value of b is perceived as a parameter by both firms and is assumed to be common knowledge.

There is a single world market with inverse demand function (assumed to be linear with units chosen such that the slope of the inverse demand function is equal to one)

$$P(Q) = \begin{cases} A - Q & \text{if } 0 \leq Q \leq A \\ 0 & \text{otherwise} \end{cases}, \tag{32}$$

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[22] Helpman (1993), for example, pointed out that most technological imitation takes place in developing (newly industrialized) countries.

[23] Helpman (1993) and Vishwasrao (1994) claim that the Southern unit costs are lower due to lower wages. This might be true if the production involves only a low-skilled labor force, but low wages and low unit costs are two different things. Recently, the share of low-skilled labor has fallen to only 5–15% of total production costs in developed economies, from 25% in the 1970s. Moreover, direct labor costs account for only 3% of total costs in semiconductors, 5% in the manufacturing of color televisions, and 10–15% in the car industry.

where $Q = q_s + q_n$, $A > a$. Parameter A captures the size of the market, whereas q_s and q_n denote the choice variables—the corresponding outputs—of the Southern and the Northern firms.

After the first stage, in which the Northern firm completes its R&D projects, the firms engage in Cournot-Nash competition. The Northern firm maximizes its profit, i.e.,

$$\max_{q_n}\{\max[P(q_n+q_s)q_n - C(x)q_n - x, 0]\} \tag{33}$$

given q_s. The Southern firm's optimization problem can be represented as

$$\max_{q_s}\{\max[P(q_n+q_s)q_s - c(x)q_s, 0]\}, \tag{34}$$

given q_n[24]. The Cournot outputs and price as a function of R&D investment can be represented as

$$q_n(x) = \frac{A + c(x) - 2C(x)}{3} \tag{35}$$

$$q_s(x) = \frac{A - 2c(x) + C(x)}{3} \tag{36}$$

$$P(x) = \frac{A + c(x) + C(x)}{3} \tag{37}$$

Rearranging expressions *(33)-(37)* we get the Northern profit function expressed in terms of R&D investment:

$$p_n(x) = \frac{A + c(x) - 2C(x)}{9} - x\,. \tag{38}$$

In the first stage of the game, the Northern firm selects x to maximize its profit. Substituting expressions *(30)* and *(31)* into *(38)* and maximizing with respect to R&D investment yields[25]:

$$x^* = \frac{g(A-a)(ß-2)^2}{\left[g(ß-2)^2 - 9\right]^2}\,. \tag{39}$$

The equilibrium outputs and the profits for the Northern and the Southern firms are given by the following expressions:

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[24] Zigic (1998a) considers more simple profit definitions, namely, $p_n = (A - q_n - q_s)q_n - C(x)q_n - x$, and $p_s = (A - q_n - q_s)q_s - c(x)q_s$. While these definitions (i) allow for negative profits, and (ii) ignore the break in the inverse demand function at $Q=A$, they are easier to handle arithmetically. Since both formulations are equivalent in the equilibrium for all economically "reasonable" sets of parameters A, α, g and b, the difference between both approaches is only formal. Consequently, in the rest of this section we allow ourselves, without further discussion, to use Zigic's definitions to derive equations *(35)-(43)*.

[25] The second-order condition is satisfied for all permissible values of parameters, and the optimal R&D expenditure x^* is always positive.

$$q_n^{\ *} = \frac{3(a-A)}{g(2-ß)^2-9}, \qquad p_n^{\ *} = \frac{(a-A)^2}{9-g(2-b)^2}, \qquad (40,41)$$

$$q_s^{\ *} = \frac{(A-a)\left[g(ß-2)(ß-1)-3\right]}{g(ß-2)^2-9}, \quad p_s^{\ *} = \frac{(A-a)^2\left[g(ß-2)(ß-1)-3\right]^2}{\left[g(ß-2)^2-9\right]^2}. \qquad (42,43)$$

It can be shown that the equilibrium market structure depends on the parameter of spillovers ***b***. The equilibrium structures that may appear in the equilibrium (given that there are only two firms), are duopoly, constrained monopoly and unfettered monopoly (see Zigic 1998a). Moreover, Zigic (1998a) shows that in the case of duopoly the Northern firm's equilibrium output as well as the Northern firm's equilibrium profit are monotonically decreasing with spillovers, while the Southern firm's equilibrium output, its equilibrium profit, equilibrium price and consumer surplus depend on parameters of the model. In particular, for $g<3(5-4b)/(b-2)^2$ the Southern firm's equilibrium output and its equilibrium profit decrease with b; they increase with b, otherwise. Equilibrium price increases with b for $g>3(1-2b)/(b-2)^2$, and decreases with b, otherwise, while consumer surplus in both countries decreases with b for $g>3(1-2b)/(b-2)^2$, and increases with b, otherwise.

We shall mention a few interesting results concerning social welfare. Zigic (1998a) shows that in R&D duopoly the North always benefits from lowering the parameter of the spillover (i.e., the higher the degree of protection of intellectual property rights in the South, the better off the North is). On the other hand, the South is always better off when the strength of intellectual property rights protection is relaxed, provided that the R&D efficiency is low ($g<3(1–2b)/(b – 2)^2$), and the level of spillovers is small ($b<1/2$). When the R&D efficiency of the Northern firm is high and when spillovers are large (that is, when $g>3(5–4b)/(b–2)^2$ and $b>1/2$), then the South benefits from strengthening the intellectual property rights.

The results presented above were derived from a game-theoretical approach. In the analysis which follows we will show what can be gained from the model assuming that firms do not have perfect information about the competitors, are not perfectly rational, and, consequently, instead of maximizing profit use rules of thumb for making decisions. In particular, we assume that firms in R&D duopoly perform according to the evolutionary algorithm presented in Section 2 and Section 3.

5. Social Evolution and Learning in R&D Duopoly with Spillovers

Consider an economy with a large number of identical markets and assume that in each market there are two firms: the Northern firm and the Southern firm, where only the Northern firm conducts innovative activity. The cost structures of the Northern and the Southern firms are the same, as is described in Section 4. The firms perform in discrete time t (t=0,1,2,…) according to the evolutionary algorithm presented in Sections 2 and 3, i.e., they decide whether or not to experiment according to the global aspiration level determined as

$$a^{(t)} = \sum_{i=1}^{K_n} \sum_{j=1}^{K_s} p_{ij}^{(t)} \min\left\{p_s(q_s^i, q_n^j, x), p_n(q_s^i, q_n^j, x)\right\}. \qquad (44)$$

However, unlike in Sections 2 and 3, the strategy of the Northern firm in this case involves a choice of two variables: the volume of output produced—q_n and the level of R&D expenditures—x. Thus, the set of possible strategies of the Northern firm can be represented as

$$S_n^{R\&D} \equiv \left\{(q_n^i, x^j)\right\} \subset [0, A] \times [0, \tfrac{a^2}{g}], \text{ where } i=1,...K_n, j=1, ...,K_x, \qquad (45)$$

and the set of possible strategies of the Southern firm can be represented as

$$S_s^{R\&D} \equiv \left\{q_s^1, q_s^2, \ldots, q_s^{K_s}\right\} \subset [0, A]. \qquad (46)$$

Now consider joint profit maximization under the equal profit condition

$$\max_{\substack{q_s \in [0,A[,\ q_n \in [0,A[,\\ q_s+q_n \in [0,A]\\ x \in [0,a^2/g]}} \left\{p_s(q_s, q_n, x) + p_n(q_s, q_n, x)\right\} \qquad (47)$$

$$\text{s.t., } p_s(q_s, q_n, x) = p_n(q_s, q_n, x),$$

where $p_s(q_s, q_n, x)$, $p_n(q_s, q_n, x)$ are given by *(33)* and *(34)*, respectively. Similarly to equation *(7)*, the solution to *(47)* cannot be obtained directly by the Lagrange multiplier method, because profit functions *(33)* and *(34)* are not differentiable on the whole domain specified by constraints in *(47)*. One can, however, find the solution to the following maximization problem:

$$\max_{\substack{q_s \in [0,A],\ q_n \in [0,A],\\ q_s+q_n \in [0,A]\\ x \in [0,a^2/g]}} \left\{(A-Q)q_n - C(x)q_n - x + (A-Q)q_s - c(x)q_s\right\} \qquad (47b)$$

$$\text{s.t. } (A-Q)q_n - C(x)q_n - x + (A-Q)q_s - c(x)q_s$$

The unique solution to *(47b)* can be represented as[26]

$$\tilde{q}_n = R_1 - \frac{r(1-l)(1+l-b)R_2}{2[1-b(1-l)]R_3}, \tag{48}$$

$$\tilde{q}_s = \frac{r(1+l)R_2}{2R_3}, \tag{49}$$

$$\tilde{x} = \frac{1}{4}g\left(\tilde{q}_n + b\frac{1-l}{1+l}\tilde{q}_s\right)^2, \tag{50}$$

where $r = A - a$, $R_1 = \dfrac{r(1-l)(1-b)}{2[1-b(1-l)]}$, $R_2 = 4[1-b(1-l)] - (1-b)[4-bg(1-l)]$

$$R_3 = R_2 + b(1-l)[-4l - gb[1-b(1-l)+gl]], \quad l = -\frac{1}{3R}\left(U + 4V\sqrt[3]{\frac{2}{Y}} - \sqrt[3]{\frac{Y}{2}}\right),$$

with auxiliary expressions *U, R, V* and *Y* given as

$$R = 4 - g + gß^2, \quad V = 20 - g(1+20b-21b^2) - g^2(1-5b+7b^2-3b^3),$$

$$Y = E + \sqrt{256V^3 + E^2}, \text{ where } E = -2U^3 - 27g(1-b^2)R^2 + 9UR(12-U),$$

and $U = 4 - g + 4gß - 3gß^2$.

Note that $\tilde{x}$ given by expression *(50)* may generally not satisfy the constraint $\tilde{x} \le \dfrac{a}{g^2}$. This is because the constraint is imposed somewhat "artificially"—it is not necessary for the solution *(48)-(50)* to exist; rather, it is needed for a "reasonable" economic interpretation of the cost functions.[27, 28]

Assume that the solution to the optimization problem *(47b)* coincides with that of *(47)*, and it is included in the set of possible strategies of the Northern and the Southern firms, respectively. That is, there exist $i^* \in \{1,2,\ldots,K_s\}$, $j^* \in \{1,2,\ldots,K_n\}$ and $k^* \in \{1,2,\ldots,K_x\}$ such that $q_s^{i*} = \tilde{q}_s$, ($q_s^{i*} \in S_s^{R\&D}$), $q_n^{j*} = \tilde{q}_n$, $x^{k*} = \tilde{x}$, ($\left(q_n^{j*}, x^{k*}\right) \in S_n^{R\&D}$).

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[26] Note that the solution given by *(48)-(50)* may not exist (? may not be defined, as some of the square roots in it may have negative arguments). As in Section 2, however, one can verify that *(48)-(50)* lead to the solution, provided that the difference between $c(x)$ and $C(x)$ is small (i.e., $ß$ is close to 1).

[27] As marginal costs of both firms depend negatively on *x*, the "effective" demand faced by either firm ($A - a + \sqrt{gx} - q_s - q_n$ for the Northern firm, and analogously for the Southern one) rises with *x*. A large value of *x* may therefore lead to the higher sum of profits, even though it makes the expression $a - \sqrt{gx}$ negative.

[28] For similar reasons it may happen that $\tilde{q}_s + \tilde{q}_n > A$. In what follows, we presume that neither $\tilde{x} > ag^{-2}$ nor $\tilde{q}_s + \tilde{q}_n > A$ happens.

The convergence of the evolutionary process to the equilibrium strategies given by expressions *(48)-(50)* is formally characterized by the following theorem.

Theorem 1*: If all firms in the economy follow the evolutionary algorithm described above, and

(*i*) $K_s > 2$, $K_n > 2$,

(*ii*) $(\tilde{q}_s, \tilde{q}_n, \tilde{x})$ is a unique interior solution to the maximization problem *(47b)* such that it coincides with the solution to *(47)* and $p_s(\tilde{q}_s, \tilde{q}_n, \tilde{x}) = p_n(\tilde{q}_s, \tilde{q}_n, \tilde{x}) > 0$.

(*iii*) the probability of making a mistake is bounded away from 1,

(*iv*) for certain $t_0, N_{t_0} > 0$,

then, as $t\to\infty$, the pair of strategies $[q_s^{i*}, (q_n^{j*}, x^{k*})] \equiv [\tilde{q}_s, (\tilde{q}_n, \tilde{x})]$ prevails on all markets, i.e.,

$$\lim_{t\to+\infty} p^{(t)}_{i^*(j^*,k^*)} = 1.$$

<u>Proof .</u> The proof of *Theorem 1** is analogous to the proof of the *Theorem 1*. Recall that the proof of *Theorem 1* bases on the following three properties:

a) *Lemma 1* holds. This is necessary to ascertain that, unless all markets are in equilibrium, there exists market *M* where at least one firm tends to experiment. Since *Lemma 1** (the analogue of *Lemma 1* for R&D doupoly) is proven in the Appendix, this point is satisfied here as well.

b) The probability of making a mistake is bounded away from 1. This point is satisfied here by the assumption.

c) For any $q_s^i \in \boldsymbol{S_s}$, $q_n^j \in \boldsymbol{S_n}$ there exists at least one $q_s^h \in \boldsymbol{S_s}$, such that $p_s(q_s^i, q_n^j) \neq p_s(q_s^i, q_n^h)$, and similarly for p_n.

In *Theorem 1* this is ensured by K_s>2 (or K_n>2) and the fact that p_s is a quadratic function of q_s (and analogously for p_n). Clearly, the additional variable *x*, changes nothing about the functional dependence of p_s on q_s (and p_n on q_n), and, consequently, the analogue of point (*c*) holds here as well. Therefore, *Theorem 1** can be proven analogously to *Theorem 1*. <u>QED.</u>

To illustrate the process of the evolution of profits in the economy, consider a model of R&D duopoly with spillovers specified by the following set of parameters {*A*=5, *a*=1, *ß*=0.5, *g*=1} The pair of long-run stationary strategies $[q_s^{i*}, (q_n^{j*}, x^{k*})]$, which corresponds to this set of parameters is given as [1.102,(1.256, 0.933)].

To simulate the process we allowed for K_s=10 strategies of the Southern firm, such that

$$S_s^{R\&D} \equiv \left\{ q_s^{(i)} : q_s^{(i)} = \frac{(i-1)(A-a)}{2K_s}, i = 1,2,\ldots,K_s \right\},$$

and K_n=100 strategies of the Northern firm

$$S_n^{R\&D} \equiv \left\{ (q_n^{(i)}, x^{(j)}) : \quad q_n^{(i)} = \frac{(i-1)(A-a)}{2K_n}, x^{(j)} = \frac{(j-1)\sqrt{a}}{gK_n}, \quad i, j = 1,2,\ldots,\frac{K_n}{10} \right\}.$$

Sets of possible strategies were modified by including the values of the long-run stationary strategies specified above. The probability of error e_k was chosen equal to 0.2 ($k\hat{I}\,\{s,n\}$). The simulations were initiated from the initial position with the uniform distribution over all possible pairs of strategies (i.e., at t=1 each possible pair of strategies was played in one market).

The results of the simulation experiments are depicted in Figures 1-4. In Figure 1 the evolution of the proportions of the market where firms play the pair of the long-run stationary strategies $[q_s^{i*},(q_n^{j*},x^{k*})]$ is presented. The time path of average profits of Southern and Northern Firms in the economy is shown in Figure 2. Figure 3 shows the evolution of average market profit and Figure 4 presents the dynamics of the normal profit in the economy.

From Figure 1, the proportion of markets where firms play a pair of long-run stationary strategies is monotonic but not smooth. There are jumps, which correspond to falls in firms' aspiration level (normal profit in the economy). These occur because the increase in the proportion of markets where firms play a pair of long-run stationary strategies depends on the number of experimenting firms.

Comparing Figure 2 with Figure 4 one can find that the average profits and the normal profit in the economy (an aspiration level) converge to the same value. However, the time paths are non-monotonic. At particular times large drops appear. The reason for this is actually quite intuitive. As the value of the aspiration level increases, markets where firms follow certain strategies reach a *critical level*, and firms start to experiment. As a result the profits of firms in those markets fall below the aspiration level and with further experimenting rise again.

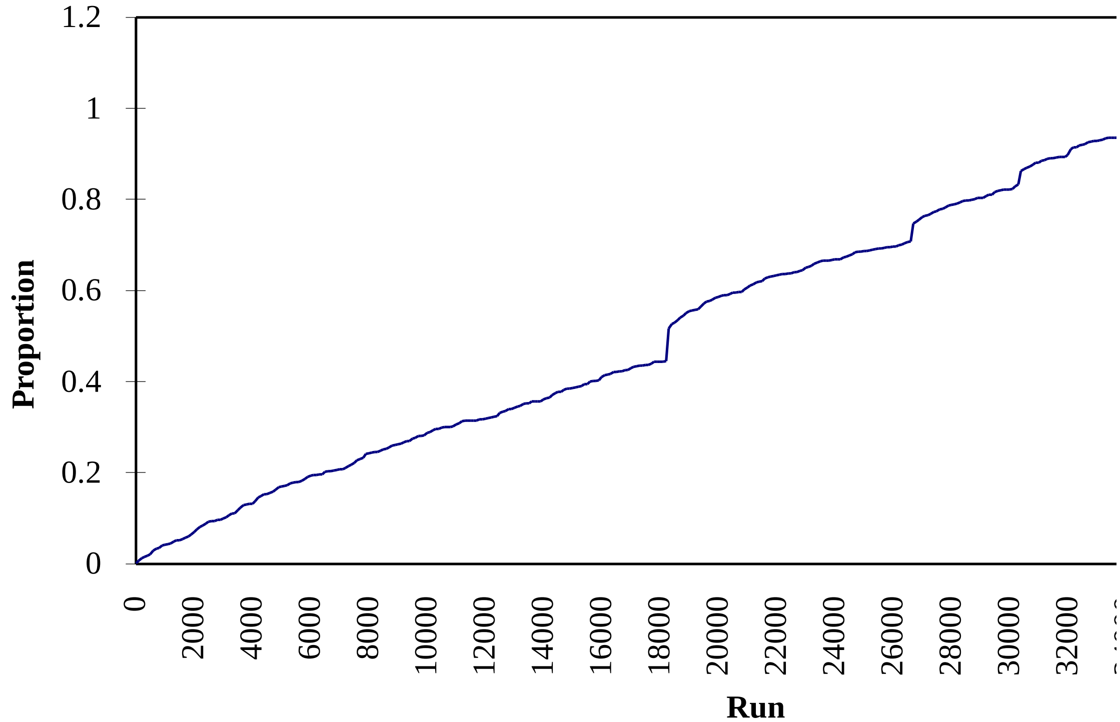

Figure 1: Proportion of firms in equilibrium

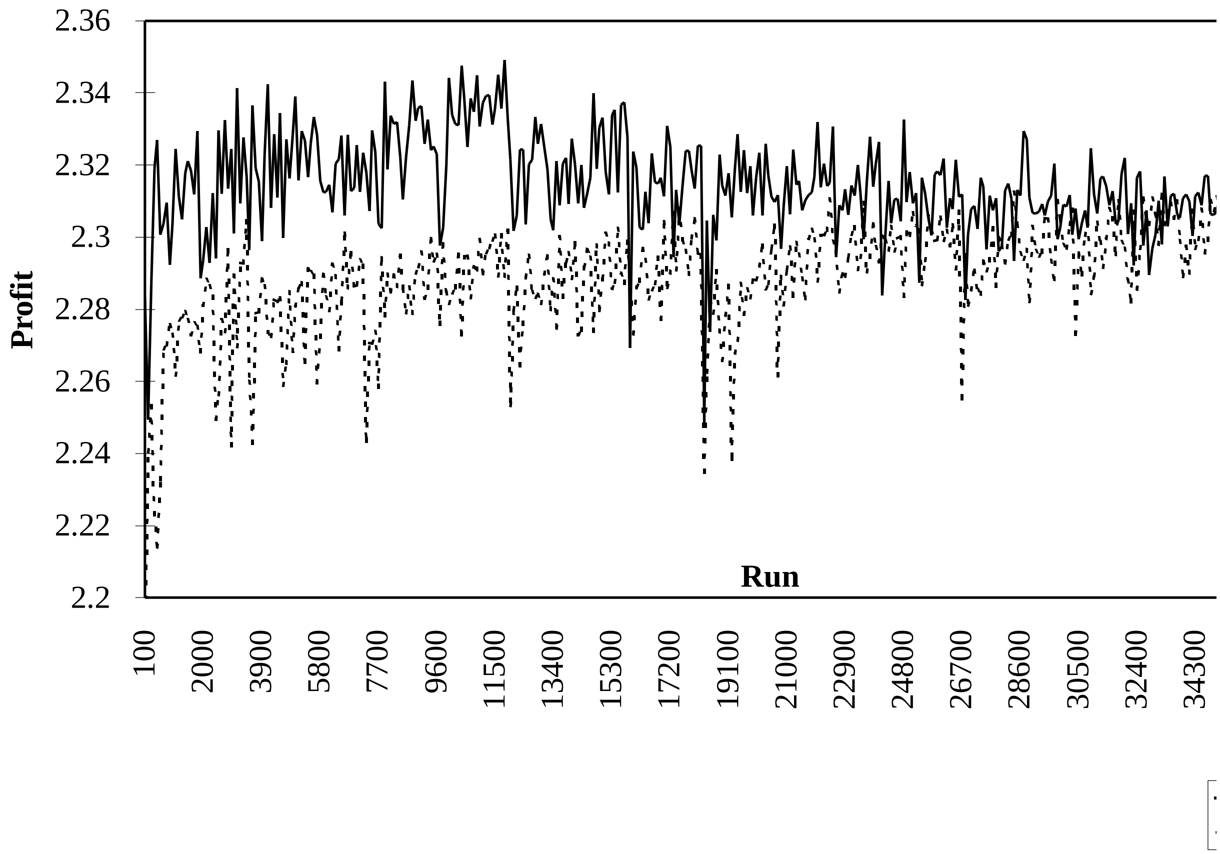

Figure 2: Evolution of firms' profits

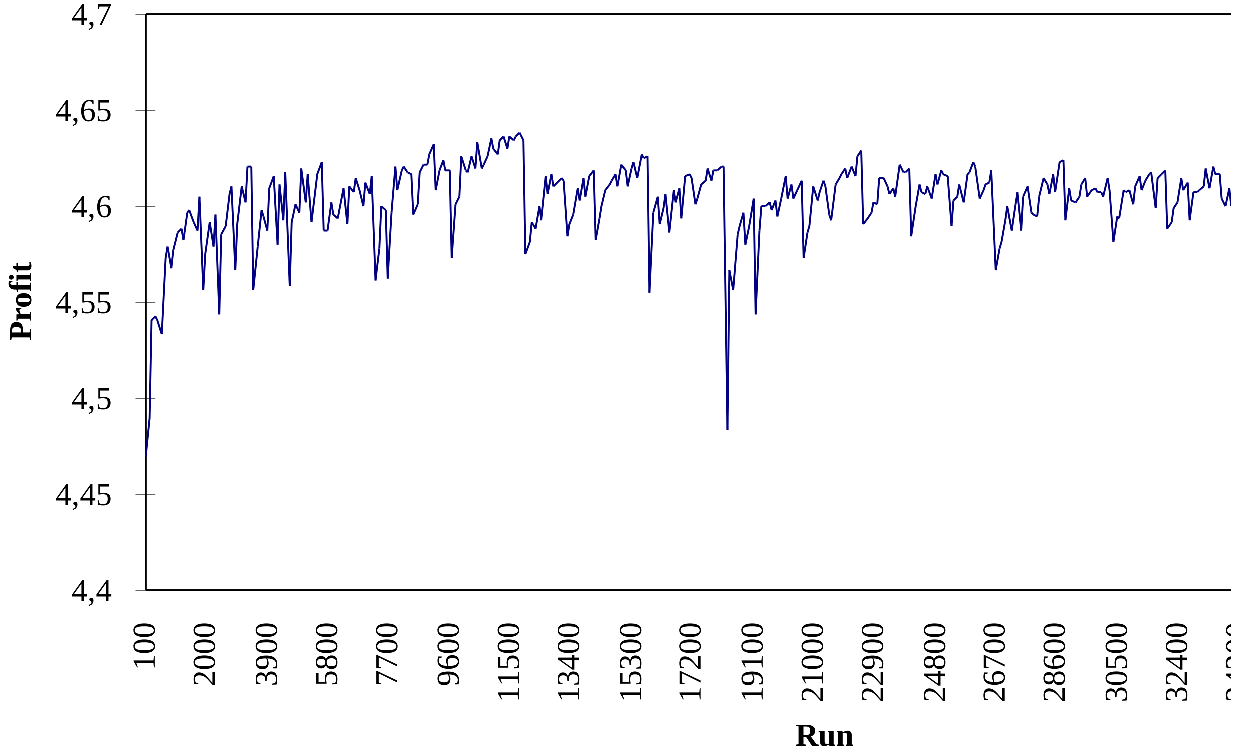

Figure 3. Evolution of average market profit

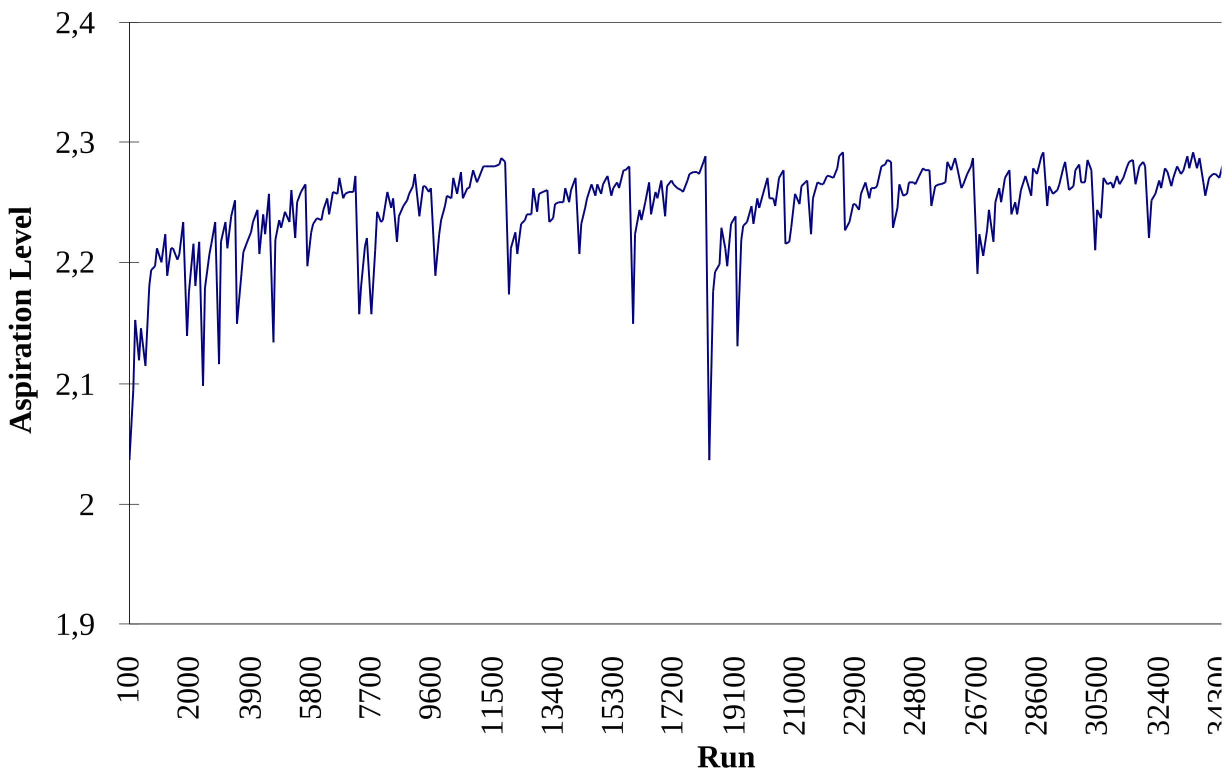

Figure 4. Evolution of the aspiration level

6. Implications for R&D duopoly with spillovers

The analysis of the evolution of the R&D duopoly with spillovers presented above shows a number of interesting results concerning the characteristics of economic equilibrium and the effects of changes in the level of spillovers (the strength of intellectual property rights protection) on equilibrium market structure, the profits of the Northern and Southern firms, and social welfare in both countries.

First of all, note that in evolutionary equilibrium all firms in the economy earn the same profit (a normal profit). Consequently, in contrast to the results derived from game theoretical analysis (see Zigic, 1998a), in evolutionary equilibrium there is always a single market structure: a duopoly. The level of equilibrium profit depends on the parameter of spillovers b (or on the strength of intellectual property rights protection, which is the same). However, unlike in the game theoretical equilibrium, the equilibrium profit of both the Southern and the Northern firms monotonically increases with the level of spillovers b (see Figure 5).

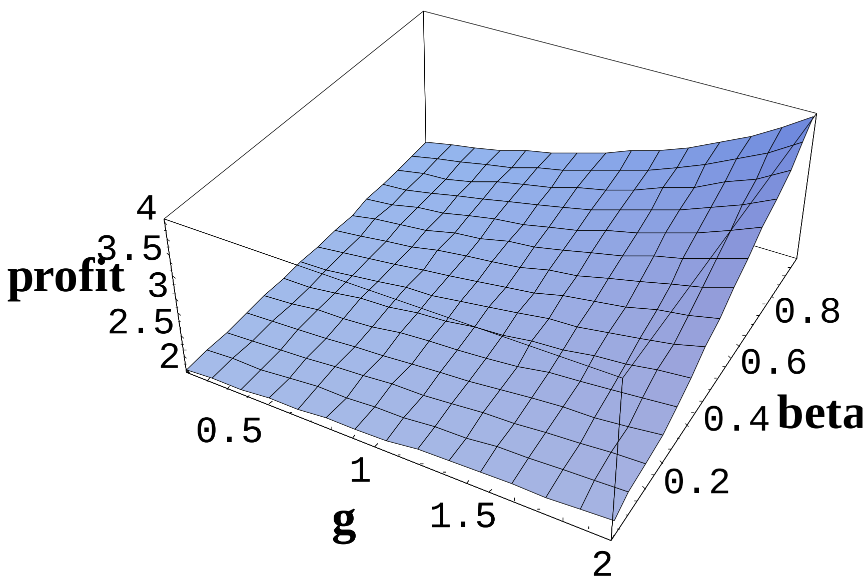

Figure 5. Equilibrium profit of the Southern and the Northern firms as a function of the level of spillovers *b* (*b*Î (0,1)) and the efficiency of the R&D process *g* (*g*Î (0,2])[29].

[29] Here and henceforth A =8 and a=4 so that, for all {b,g}∈(0,1)x(0,2] $\tilde{x} \le a g^{-2}$ holds. The different choice of A and a will only result in a rescaling of the figures (recall that $\tilde{x}, \tilde{q}_s, \tilde{q}_n$ do not depend on A and a explicitly, but only r=A-a), and in a change in the range of permissible {b,g}.

The results indicate that the equilibrium level of R&D expenditures *x* and the equilibrium volume of output of the Northern firm increase monotonically if the level of spillovers increases (see Figure 6 and Figure 7). The equilibrium output of the Southern firm for relatively small values of *b* increases with *b*, and for large values of *b* decreases with *b* (see Figure 8).

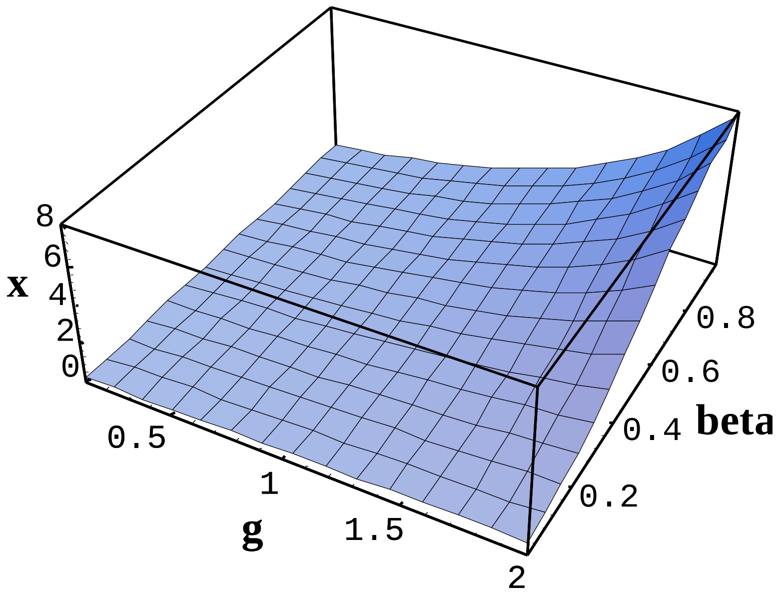

Figure 6. Equilibrium level of R&D expenditures *x* as a function of the level of spillovers *b* (*b*Î (0,1)) and the efficiency of the R&D process *g* (*g*Î (0,2]), *A*=8, *a*=4.

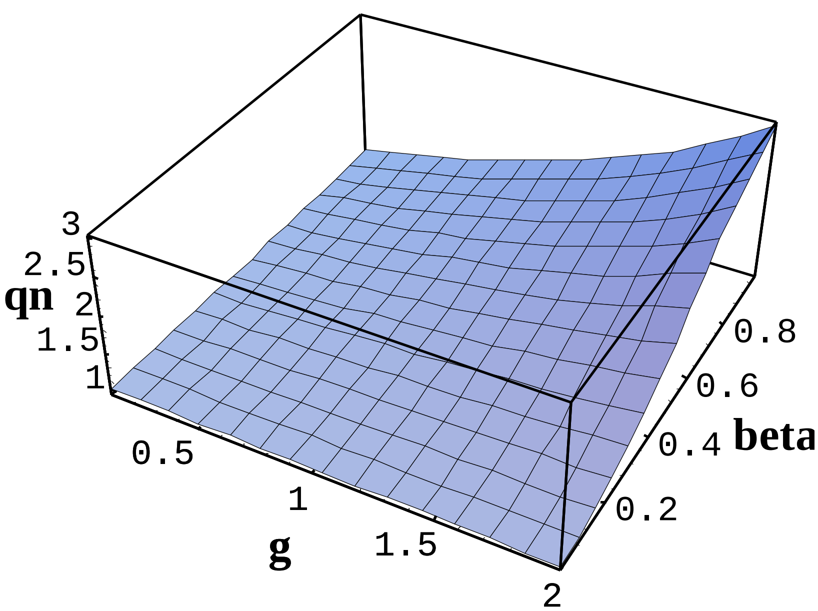

Figure 7. The equilibrium volume of output of the Northern firm as a function of spillovers *b* (*b*Î (0,1)), and the efficiency of the R&D process *g* (*g*Î (0,2]) *A*=8, *a*=4

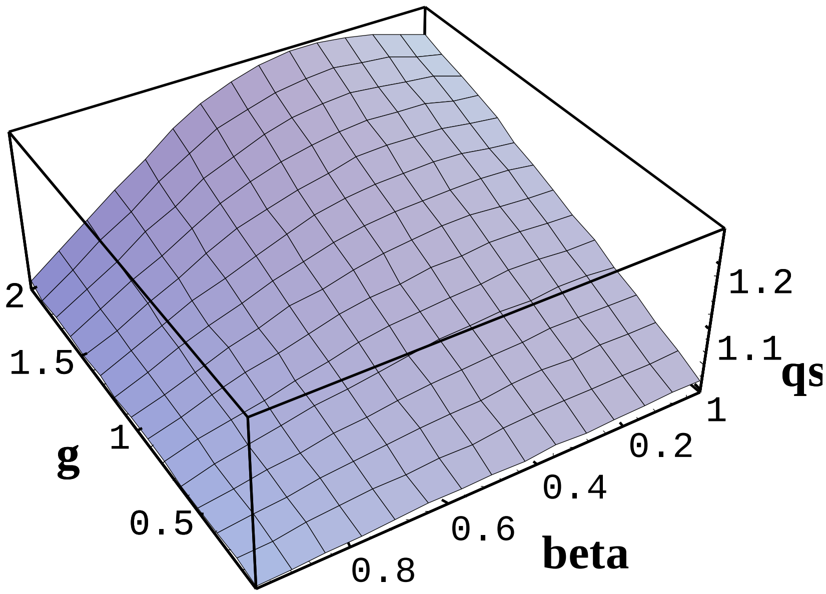

Figure 8. The equilibrium volume of output of the Southern firm as a function of the level of spillovers *b* (*b*Î (0,1)) and the efficiency of the R&D process *g* (*g*Î (0,2]) , *A*=8, *a*=4.

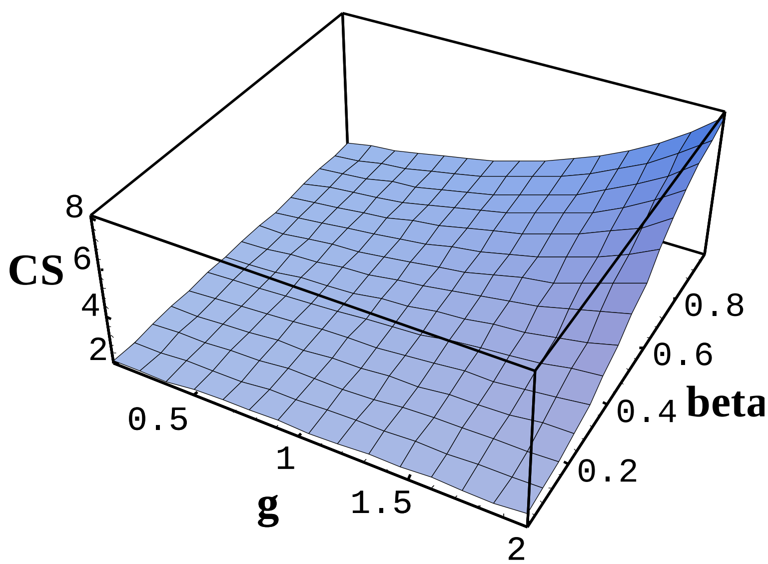

Figure 9. Consumer surplus $CS = \frac{1}{2}(q_s + q_n)^2$ **as a function of the level of spillovers *b* (*b*Î (0,1)) and the efficiency of the R&D process *g* (*g*Î (0,2]) , *A*=8, *a*=4.**

The change in the level of spillovers *b* has the following impact on social welfare in the Southern and Northern countries. The producer surplus in both countries increases with *b* since equilibrium profits of the Northern and the Southern firm are identical and are inversely related to the level of intellectual property rights protection (see Figure 5). Moreover, the consumer surplus *CS* (which in the case of linear demand equals $\frac{1}{2}(q_s + q_n)^2$)[30] also increases with the level of spillovers b (see Figure 9). Finally, the social welfare *SW* (the sum of the consumer and producer surplus in each country) increases if the level of spillovers b increases (see Figure 10).

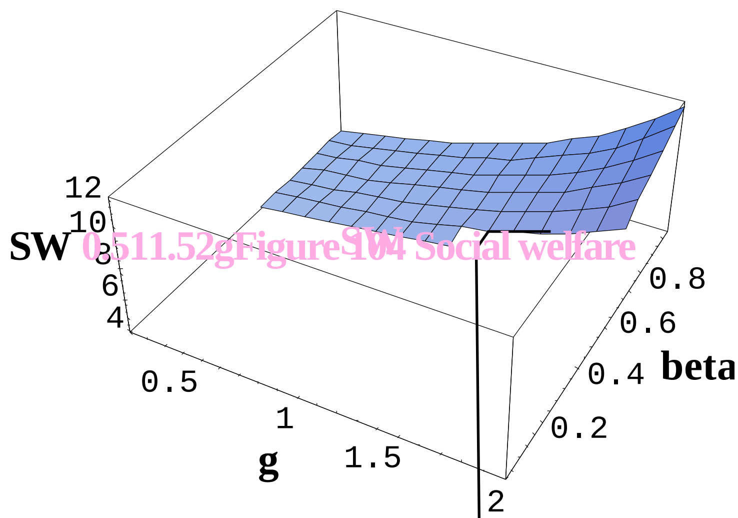

Figure 10: Social welfare

R&D (asymmetric) duopoly with spillovers. Our main findings (based on the evolutionary approach) and conclusions may be summarized as follows:

1. Characteristics of market equilibrium depend crucially on the level of intellectual property rights protection, which is the main determinant of the level of spillovers.
2. In market equilibrium the Southern and the Northern firms earn the same profit (a normal profit in the economy).
3. In market equilibrium there is always a single market structure: a duopoly.
4. The equilibrium profit of both the Southern and the Northern firms (and the normal profit in the economy) decreases monotonically if the degree of intellectual property rights protection increases (i.e., it increases with the level of spillovers).
5. The equilibrium level of R&D expenditures decreases monotonically if the degree of intellectual property rights protection increases (i.e., it increases with the level of spillovers).
6. The equilibrium volume of output of the Northern firm decreases monotonically if the degree of intellectual property rights protection increases (i.e., it increases with the level of spillovers).
7. The equilibrium volume of output of the Southern firm increases if the level of intellectual property rights protection decreases. However, if the level of intellectual property rights protection decreases beyond a certain value, the equilibrium volume of output of the Southern firm decreases as well.
8. Producer and consumer surpluses, as well as social welfare in each country increase if the level of spillovers increase.

These results differ from earlier literature on this topic because, in contrast to other studies (see Zigic 1998a, for example), they suggest that, independently on the parameters of the model, strengthening intellectual property rights protection always decreases profit the of both the Northern and the Southern firms. Moreover, strengthening intellectual property rights protection decreases consumer surplus and social welfare in both countries analyzed. Therefore, we can conclude that evolutionary analysis show that strengthening intellectual property rights protection always has negative welfare effects. Consequently, reducing property rights protection and increasing R&D spillovers will increase social welfare in both countries.

Appendix

Lemma 1: Let $\left(\tilde{q}_s, \tilde{q}_n\right)$ be a unique interior solution to problem *(7b)* such that it coincides with the solution to *(7)*, and $\tilde{p} = p_s(\tilde{q}_s, \tilde{q}_n) = p_n(\tilde{q}_s, \tilde{q}_n) = p_s\left(q_s^{i*}, q_n^{j*}\right) = p_n\left(q_s^{i*}, q_n^{j*}\right) > 0$. Then,

(*i*) $\min\left\{p_s(q_s^i, q_n^j), p_n(q_s^i, q_n^j)\right\} < \tilde{p}$, for any pair of (i,j) such that $(i,j)^{1}(i^*,j^*)$;

(*ii*) for any $t=0,1,2,\ldots$ it holds that $a^{(t)} \le \tilde{p}$, and the equality occurs if and only if all firms play $\left(q_s^{i*}, q_n^{j*}\right)$, i.e., $p_{ij}^{(t)} = 0$ for all $(i,j) \neq (i^*,j^*)$, where $i=1,2,\ldots,K_s$, and $j=1,2,\ldots,K_n$;

(iii) $\left(q_s^{i*}, q_n^{j*}\right)$ is the only pair of strategies for which (*i*) and (*ii*) are satisfied.

Proof of Lemma 1:

Maximization problem *(7b)* can equivalently be represented as

$$\max_{(q_s,q_n)\in S\cap\Omega} F(q_s,q_n), \qquad (A.1)$$

where $F(q_s,q_n) = 2\min\left\{\Pi_s(q_s,q_n), \Pi_n(q_s,q_n)\right\}$,

with $\Pi_s(q_s,q_n)$, $\Pi_n(q_s,q_n)$ defined as

$$\Pi_s(q_s,q_n) = (A - q_s - q_n)q_s - c_s q_s - b_s,$$

$$\Pi_n(q_s,q_n) = (A - q_s - q_n)q_n - c_n q_n - b_n, \text{ and}$$

$$S \equiv \underbrace{[0,A]\times[0,A]}_{\text{denote this } S(A)} \cap \left\{(q_s,q_n) \text{ s.t. } 0 \le q_s + q_n \le A\right\},$$

$$O = \left\{(q_s,q_n) \text{ s.t. } \Pi_s(q_s,q_n) = \Pi_n(q_s,q_n)\right\}.$$

As the solution to *(7b)* is, by assumption, unique and interior, so is the solution to *(A.1)*. From the condition $\tilde{p} > 0$, it follows that $F(\tilde{q}_s, \tilde{q}_n) > 0$.

Consider now the unconstrained maximization problem

$$\max_{(q_s,q_n)\in S} F(q_s,q_n) \qquad (A.2)$$

Since $F(q_s,q_n)$ is continuous in both arguments, it attains its maximum on any compact set (in particular on S). Consequently, the solution (q_s^*, q_n^*) to (*A.2*) exists. Moreover, since on the boundary of $S(A)$ function F takes non-positive values, the solution to (*A.2*) must be an element of the interior of $S(A)$ (otherwise $F(\tilde{q}_s, \tilde{q}_n) > 0 > F(q_s^*, q_n^*)$, which is a contradiction). For similar reason, it must hold that $q_s^* + q_n^* < A$. One can therefore conclude that (q_s^*, q_n^*) lies in the interior of S.

We are now going to show that the solutions to *(A.2)* and *(A.1)* coincide. Since we have already proven that $(q_s^*, q_n^*) \in S$, it remains to be shown that $(q_s^*, q_n^*) \in \Omega$ (in other words that $\Pi_s(q_s^*, q_n^*) = \Pi_n(q_s^*, q_n^*)$), and that $F(q_s^*, q_n^*) = F(\tilde{q}_s, \tilde{q}_n)$.

Assume that $\Pi_s(q_s^*, q_n^*) < \Pi_n(q_s^*, q_n^*)$. Then, by virtue of $\frac{\partial \Pi_s}{\partial q_n} < 0$, there exists e>0, such that $(q_s^*, q_n^* - e) \in S$ and $\Pi_s(q_s^*, q_n^*) < \Pi_s(q_s^*, q_n^* - e) < \Pi_n(q_s^*, q_n^* - e)$. This, however, contradicts (q_s^*, q_n^*) being a solution to (*A.2*). Since the opposite inequality $\Pi_s(q_s^*, q_n^*) > \Pi_n(q_s^*, q_n^*)$ can be ruled out analogously, it must be that $\Pi_s(q_s^*, q_n^*) = \Pi_n(q_s^*, q_n^*)$, and hence $(q_s^*, q_n^*) \in \Omega$.

Since maximization problem (*A.2*) is less constrained than (*A.1*), it must be that $F(q_s^*, q_n^*) \geq F(\tilde{q}_s, \tilde{q}_n)$. The strict inequality is, however, impossible because $(q_s^*, q_n^*) \in \Omega$. It then follows from the uniqueness of $(\tilde{q}_s, \tilde{q}_n)$ that $(q_s^*, q_n^*) \equiv (\tilde{q}_s, \tilde{q}_n)$.

By proving that $(q_s^*, q_n^*) \equiv (\tilde{q}_s, \tilde{q}_n)$, we have in fact proved that

$$\min\{\Pi_s(q_s, q_n), \Pi_n(q_s, q_n)\} < \Pi_s(\tilde{q}_s, \tilde{q}_n) = \Pi_n(\tilde{q}_s, \tilde{q}_n) = \tilde{p} \qquad (A.3)$$

for all $(q_s, q_n) \in S$, $(q_s, q_n) \neq (\tilde{q}_s, \tilde{q}_n)$.

Since (i) function $p_s(q_s, q_n)$ differs from $\Pi_s(q_s, q_n)$ only at points where $\Pi_s(q_s, q_n)$ takes negative values (at such points $p_s(q_s, q_n)$ =0),

(ii) the same relationship holds between $p_n(q_s, q_n)$ and $\Pi_n(q_s, q_n)$,

(iii) $(\tilde{q}_s, \tilde{q}_n)$ is the solution to both *(A.1)*, *(A.2)* and *(7)*, and finally,

(iv) $\Pi_s(\tilde{q}_s, \tilde{q}_n) = \Pi_n(\tilde{q}_s, \tilde{q}_n) = p_s(\tilde{q}_s, \tilde{q}_n) = p_n(\tilde{q}_s, \tilde{q}_n) = \tilde{p} > 0$,

it follows from *(A.3)* that

$$\min\{p_s(q_s^i, q_n^j), p_n(q_s^i, q_n^j)\} < \tilde{p}, \qquad (A.4)$$

for any pair of (i,j) such that $(i,j) \neq (i^*, j^*)$.

It further follows from definition $a^{(t)}$ that for any t=0,1,2,…

$$a^{(t)} = \sum_{i=1}^{K_n} \sum_{j=1}^{K_s} p_{i,j}^{(t)} \min\{p_s^{(t)}(q_s^i, q_n^j), p_n^{(t)}(q_s^i, q_n^j)\} < \tilde{p} \sum_{i=1}^{K_s} \sum_{j=1}^{K_n} p_{i,j}^{(t)} = \tilde{p} \qquad (A.5)$$

unless $p_{i,j}^{(t)} = 0$, for all $i \neq i^*$ ($i \in \{1,2,...,K_s\}$) and $j \neq j^*$ ($j \in \{1,2,...,K_n\}$), and $p_{i^*,j^*}^{(t)} = 1$.

This proves (*i*) and (*ii*). Since the optimal strategy $(\tilde{q}_s, \tilde{q}_n)$ is unique, (q_s^{i*}, q_n^{j*}) is the only strategy for which (*i*) and (*ii*) are satisfied. QED.

Lemma 1*: Let $(\tilde{q}_s, \tilde{q}_n, \tilde{x})$ be a unique interior solution to the maximization problem *(47b)* such that it coincides with the solution to *(47)*, i.e., $\tilde{q}_n \in (0,A)$, $\tilde{q}_s \in (0,A)$, $\tilde{q}_s + \tilde{q}_n \in (0,A)$ $\quad \tilde{x} \in (0, a^2/g)$. Let $\tilde{p}$ denote $p_s\left(q_s^{i*}, q_n^{j*}, x^{k*}\right) = p_n\left(q_s^{i*}, q_n^{j*}, x^{k*}\right) = p_s\left(\tilde{q}_s, \tilde{q}_n, \tilde{x}\right) = p_n\left(\tilde{q}_s, \tilde{q}_n, \tilde{x}\right)$, and let $\tilde{p}$ >0.

Then the following is true:

(i^*) $\min\left\{p_s(q_s^i, q_n^j, x^k), p_n(q_s^i, q_n^j, x^k)\right\} < \tilde{p}$, for any pair of $[i,(j,k)]$ such that $[i,(j,k)] \neq [i^*,(j^*,k^*)]$.

(ii^*) for any t (t=0,1,2,…): $a_t \le \tilde{p}$; the equality occurs if and only if all firms play $\left(q_s^{i*}, q_n^{j*}, x^{k*}\right)$, i.e., $p_{ij}^{(t)} = 0$ for all $[i,(j,k)]^{1}\,[i^*,(j^*,k^*)]$.

(iii^*) $\left(q_s^{i*}, q_n^{j*}, x^{k*}\right)$ is the only triple of strategies for which (i^*) and (ii^*) are satisfied.

<u>Proof of Lemma 1*</u>: The proof is analogous to the proof of *Lemma 1*.

Let us define the two following maximization problems:

$$\max_{\substack{(q_s, q_n) \in S \cap \Omega \\ x \in [0, a/g^2]}} F(q_s, q_n, x), \tag{$A^*.1$}$$

where $F(q_s, q_n, x) = 2\min\{\Pi_s(q_s, q_n, x), \Pi_n(q_s, q_n, x)\}$,

$$\Pi_s(q_s, q_n, x) = (A - q_s - q_n)q_s - c(x)q_s,$$

$$\Pi_n(q_s, q_n, x) = (A - q_s - q_n)q_n - C(x)q_n - x,$$

$$S \equiv \underbrace{[0,A] \times [0,A]}_{\text{denote this } S(A)} \cap \{(q_s, q_n) \text{ s.t. } 0 \le q_s + q_n \le A\},$$

$$O = \{(q_s, q_n, x) \text{ s.t. } \Pi_s(q_s, q_n, x) = \Pi_n(q_s, q_n, x)\},$$

and

$$\max_{\substack{(q_s, q_n) \in S \\ x \in [0, a/g^2]}} F(q_s, q_n, x). \tag{$A^*.2$}$$

Clearly, *(A^*.1)* coincides with *(47b)*; hence $(\tilde{q}_s, \tilde{q}_n, \tilde{x})$ is its solution. As $F(q_s, q_n, x)$ is continuous on $S \times [0, a/g^2]$, and $S \times [0, a/g^2]$ is obviously a compact set, *(A^*.2)* must have a solution too. Let us denote it as (q_s^*, q_n^*, x^*). Analogously to *Lemma 1,* it may be concluded that (q_s^*, q_n^*) lies in the interior of *S*. Assume that $(q_s^*, q_n^*) \notin \Omega$, namely that $\Pi_s(q_s^*, q_n^*, x^*) < \Pi_n(q_s^*, q_n^*, x^*)$. As $\Pi_s(q_s, q_n, x)$ depends negatively on q_n, and both Π_s and Π_n are continuous in q_n, a small negative perturbation to q_n^* (such that

$(q_s^*, q_n^* - e) \in S$) would lead to $\Pi_s(q_s^*, q_n^*, x^*) < \Pi_s(q_s^*, q_n^* - e, x^*) < \Pi_n(q_s^*, q_n^* - e, x^*)$. This would, however, contradict (q_s^*, q_n^*) being a solution to *(A*.2)*.

Replicating further the same steps as in *Lemma 1* one comes to the conclusion that $(\tilde{q}_s, \tilde{q}_n, \tilde{x}) \equiv (q_s^*, q_n^*, x^*)$. Points ($i^*$) and ($ii^*$) are then an immediate consequence of this fact and the relationships between functions Π_l and p_l, $l \in \{s, n\}$. Point (iii^*) follows from the uniqueness of $(\tilde{q}_s, \tilde{q}_n, \tilde{x})$. See *Lemma 1* for details. QED.

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