Laws of Logic – Where Do They All Come From?

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Abstract: Everybody would probably agree that there are various laws of logic, such as the law of (non-)contradiction, the law of the excluded middle, *modus ponens*, *ex falso quodlibet* and so on. It is however unlikely that everybody would agree on which of these laws are the *genuine* laws, in that they are nonnegotiable. But first and foremost there is almost no agreement with respect to *the nature* of the laws, what exactly the laws are about, which domain they regulate and what is the source of their authority. This is quite surprising, for the laws appear to lie within the very foundations of logic. In this paper, we summarize a very down-to-earth and naturalistic explanation of the nature of logical laws that stems from the account of their constitution that we presented in our recent book.

Keywords: Logic, Logical laws, Logical forms, Reflective equilibrium

1 Arguments and their correctness

The term "law" is ambiguous. We surely mean something different when we speak about *laws* in legal discourse, when we speak about the *laws* of physics and when we speak about *logical laws*. And while talking about laws in jurisprudence and in the natural sciences is quite common, talking about the laws of logic within modern philosophy and specifically within modern logic may sound somewhat obsolete. Philosophers used to conceive of the laws of logic as the cornerstones of (rational) thought, but when logic, thanks to Frege and others, became independent of psychology the laws ceased to be associated with processes that occur (or ideally should occur) in the minds of thinkers. In modern logic, there is no single law that has remained unchallenged;[2] logicians may not even use the term "logical

[1]Work on this paper has been supported by Research Grant No. 17-15645S of the Czech Science Foundation.

[2]The rejection of the most solid cornerstone of traditional logic – the law of (non-)contradiction – is constitutive of the currently flourishing enterprise of paraconsistent logic.

laws" anymore (preferring terms like "principles", "rules", etc.), and they are also less prone to talk about *thinking* but more prone to talk about *reasoning* (which is not necessarily understood as a mental process), *arguments* or *proofs*. But independently of how we choose to call the principles that logic is after, the question of the nature and origins of these items seems important. In this paper, we will try to provide an answer.

Let us start our inquiry with some preliminary observations that shouldn't be controversial:
1. *Logic's primary business is with arguments/reasoning.*This is not to say that logicians aren't studying other issues, indeed they are; but studying arguments is the most substantial *raison d'être* of logic, and the other issues logicians deal with unfold from this.
2. *Arguments consist of meaningful sentences* (*or perhaps meanings of the sentences*). Steps like

(A1) *If it rains, the streets are wet*
It rains
The streets are wet

or

(A2) $(1+1=2)\vee(1+1=3)$
$\underline{\neg(1+1=3)}$
$1+1=2$

are arguments. Of course we can have "arguments" like

(AF1) $A\rightarrow B$
$\underline{A}$
B,

but insofar as A and B are not mere shortcuts for particular meaningful sentences, this is just an argument *scheme*.
3. *Logic is concerned with the correctness of arguments/reasoning*[3]. Indeed, logic is concerned with telling us that (A1) is a correct argument, while

[3]In this article, similarly as in Peregrin and Svoboda (2017), we are going to follow a terminological convention – we will speak about *correctness* and *incorrectness* in the case of (full-fledged) arguments and about *validity* and *invalidity* in the case of argument schemes (forms).

(A3) *If it rains, the streets are wet*
The streets are wet
It rains

is not.[4]

4. *Especially, logic is supposed to identify arguments which are correct merely due to their "logical form".*[5] (Though there is no general agreement w.r.t. what a logical form is.)

These points all sound quite plausible; nevertheless, they constitute the point of departure for the—*prima facie* perhaps not so plausible—story about the nature of logical laws that we are going to tell. Let us add one more point which might seem slightly more controversial, namely:

5. *Not all correct arguments are logically correct.* What we mean by this is that besides logically correct arguments, like (A1) or (A2), we can also have arguments that are analytically correct, such as, for example

(A4) *Tom is a bachelor*
Tom is male

or

(A5) *It is Monday today*
It will be Tuesday tomorrow

and even arguments that are correct in a somewhat less definite sense, like

(A6) *Tom is in Slovakia*
Tom is in Europe

or

(A7) *Bolzano was born in 1781*
Bolzano was born earlier than Frege

We suggest calling arguments like (A6) and (A7), which are commonly taken as correct but in whose case it is perhaps thinkable that they could

[4] It is worth noting that showing which arguments are logically *incorrect* is a process incomparably more tortuous than showing which ones are *correct*; see Svoboda and Peregrin (2016).

[5] This task is sometimes alternatively formulated as the task of identifying logical truths but this alternative formulation is, in our view, potentially misleading.

be incorrect (providing the world were quite different from what it is), *status quo* correct.[6] It is quite clear that logic is not supposed to demonstrate the correctness of arguments which are (only) analytically or *status quo* correct.[7] Let us now consider some more examples of arguments with which we can be confronted in real life communication:

(A8) *All transcendental numbers smaller than* 1 *are irrational*
Some transcendental numbers smaller than 1 *are irrational*

(A9) *Tom knows that gold is necessarily heavier than aluminum*
Aluminum can't be heavier than gold

(A10) *Amundsen flew to the North Pole in his airplane*
Amundsen flew somewhere

(A11) *Tom ought to learn Russian*
Tom ought to learn German or Russian

Obviously, in the case of each of these arguments we can ask whether it is *logically correct*. Clearly answering this question can be a worthy task – we often need to decide whose argumentation is conclusive, or at the very least we need to secure a mutual understanding among communicating people. And we will want to know whether, in a particular case, we can do this with the help of logic. But who is supposed to be qualified to decide whether arguments like (A8) – (A11) are logically correct?

The natural assumption seems to be that it is the job of logicians. We, however, presume that this answer wouldn't be generally adopted. Somebody, having in mind what present day logicians actually do most of the time, might maintain that logicians are not really supposed to answer questions like this. They operate within the realm of the formal (they especially work with artificial languages that can be treated purely mathematically). How the forms relate to real languages and real arguments is perhaps a matter to be left to some of the more applied scientists (perhaps linguists or specialists in communication studies?). The trouble, as we see it, is that if logicians are supposed to dwell in the realm of the formal, there is nothing that distinguishes them from mathematicians; especially, there is nothing

[6] Of course, we do not claim that in natural languages the boundary lines between different kinds of correct arguments are sharp.

[7] For more about the classification of different kinds of correct arguments, see Peregrin and Svoboda (2017, chapter 2).

that makes them prone to address argumentation/reasoning as it actually takes place.

According to us, however, it is the predicament of logicians to deal with real arguments – for verdicts on them are *constitutive* of their business. Of course, logicians often use mathematical methods and they concentrate on the *logical forms* of arguments. But the ultimate topic of their study is the correctness of fully-fledged argumentation/reasoning.

2 Logical forms and logical laws

One of our initial observations was that logicians concentrate on logical forms. Prima facie this claim sounds perspicuous and uncontroversial, but it is fully intelligible only provided the concept of logical form is clear. Again, we believe that there are some quite uncontroversial points on which a vast majority of logicians would agree:

1. *Logical forms can be ascribed to meaningful sentences*. If logicians are to fulfill the task of deciding which arguments are correct they have to determine the logical forms of the involved sentences; for example, to ascribe a logical form to sentences like *The king of France is bald*, *Tom knows that gold is necessarily heavier than aluminum* or *Amundsen flew to the North Pole in his airplane.*

2. *Logical forms are articulated in various artificial languages employed by logicians.* To say what the logical form of a sentence is we employ a formal language. (Logicians usually present the logical form of *The king of France is bald* or *Amundsen flew to the North Pole in his airplane* in the language of predicate calculus, while that of *Tom knows that gold is necessarily heavier than aluminum* in the language of modal/epistemic propositional logic.)

However, when it comes to the *nature* of logical forms, controversies start.

A possible position is that logical forms are real – independent of any languages used by humans. They may be said to display the logical (or more broadly formal) structure of the world (which we are able to recognize/view/ recollect), or they may be seen as principles underlying thought as such. In both cases, logical languages are only our (imperfect) tools used to bring them to light (hence, there must be something that is *the* objective formal structure of the world or thought and consequently something like *the* logical language that captures the forms precisely).

We are skeptical about such a picture of logical forms. We, of course, don't think that such a position can be refuted by empirical research or deci-

sive argumentation; but we are on the other hand also convinced that there is no evidence that would support such an "absolutist" stance. Thus, we suggest that we should base our discussion of logical forms on somewhat less arcane grounds – on the reflection of how logicians actually proceed when they are asked to identify the logical form of a sentence or an argument.

We are convinced that a kind of *instrumentalist* and *relativist* view is more plausible: the concept of a logical form of a natural language sentence makes, in our view, sense only relative to a logical language. The logical form of a sentence is the "best" way of capturing the inferential properties of a natural language sentence in a given logical language. It follows that *to have logical forms, we must have logical languages*. Now, as logical laws, as we have suggested, are also a matter of logical forms, fully-fledged and definite logical laws are unthinkable without artificial logical languages.[8]

An obvious objection is that logical laws should be inherent to argumentation/reasoning (even constitutive of it) rather than emerging only within the tools invented by logicians. Our reply is that relevant laws (which may be called proto-logical) do govern our practices of argumentation and reasoning (and thereby get sedimented within our natural languages), but in the form of merely *implicit* rules, which take the shape of definite principles only when they are fixed within a logical language. Only then do they become *logical laws* worth the name.

Now, it seems, we must pause and say something more about the nature of logical languages. We are convinced that artificial languages formed by logicians can be plausibly seen as specific *models* of *de facto* argumentation/reasoning. Similarly as mathematical models that physicists employ when they want to understand (e.g.) fluid mechanics, logical languages act as models that we use to get a better grasp of *de facto* argumentation/reasoning (and possibly to enhance it). In both cases, if we have "good" models (which proved useful/adequate enough) we can carry out lots of investigations inside the models.[9]

Artificial languages of logic are similar to scientific models of natural phenomena in that, in both cases, the ultimate end is to project the results achieved by the mathematical study of the models back on the original subject matter of the investigation – real cases of moving fluids or real argu-

[8] See Peregrin (2010b).

[9] Seeing the artificial languages of logic as such kinds of models is in no way unprecedented. It has already been suggested by Burgess (1992) or Shapiro (2001). The authors, however, see them primarily as models of mathematical practices, whereas we are convinced that if logic is to live up to its task, it should reflect our argumentative practices from a more general perspective.

mentation/reasoning. But, unsurprisingly, there is a vital difference. While in case of physics this projection in no way changes the original subject matter, in case of logic it may – there may be a feedback from logical theories on our practices of reasoning. Logical models are not purely descriptive or predictive, they are expected to *fix the rules* governing real argumentation.

It follows that the laws of logic(s) are not discovered – they are more in the nature of artifacts than excavations. Of course they are not created arbitrarily, they are rooted in natural languages, which are the fount of all meaning (worth the name). Laws like *modus ponens*, *excluded middle* and possibly more complex and less pronounced principles governing the 'serious discourse' (within which people are aiming at a consensus based on mutual understanding and at the extending of their knowledge) emerge as a *reflective equilibrium* (Brun, 2014; Peregrin & Svoboda, 2017). They result from our back and forth movement between the "data" (facts of argumentation/reasoning) and a "theory" (tentative articulations of rules constitutive of an artificial language that aims to attain the status of a logical language).

It is worth mentioning that the term "logical language" is somewhat ambiguous.One possibility is to view a logical language as defined by its formation rules. (Then we will say that, for example, classical propositional logic and intuitionistic propositional logic, or modal logic S2 and S5, share the same language.)[10] But we can also view logical language as not only defined by formation rules but also by transformation rules – by the axioms which establish inferential relations among its formulas/sentences or by means of a formal semantics. In this second and third sense, delineation of a logical language delineates a logic (the terms "logical language" and "logical system" or "logical calculus" are synonymous). This is also the sense in which we use the term here.

But there is another ambiguity that enters the picture when we use the term "logical" in this way. We can conceive of logical languages as being purely formal, their extralogical terms being contentless parameters and the formulas containing them thus being unable to express sentences with full-fledged meaning, or we can conceive of them as "fully-interpreted" languages whose extra-logical terms are meaningful constants whose formulas are (or at least can be) meaningful sentences.[11] This ambiguity is not so important for us here.

[10] We should note that even this "syntactical" concept of logical language is not purely formal. We naturally assume that in two logical languages which generate the same formulas the same symbols (logical constants) are associated with the same logical expressions of natural languages – for example, English expressions like "and", "every", "not" or "possibly". If they are not we wouldn't speak about the same language.

[11] For more about this, see Peregrin and Svoboda (2017, chapter 4).

3 Reflective equilibrium

Let us consider, once more, the argument

(A1) *If it rains, the streets are wet*
It rains
The streets are wet

We readily classify it as correct. Why? In our view it is because we were taught to use expressions which form it, in particular "if", in a certain way. (In fact, we would see assenting to inferences of the kind of (A1) as *touchstones* of understanding "if".) We are also sure that it would be accepted as correct by (the great majority of) the speakers of English.[12] Also, we don't doubt that the speakers would also accept as correct similar arguments like

(A12) *If you don't have any money, you cannot buy the cake*
You don't have any money
You cannot buy the cake.

Why are these arguments similar? The answer is not difficult – they are similar because they have the same form, namely

If A, B
A
B.

Now if we introduce a convention and decide to write "⇒" instead of "if" we get

⇒ *A, B*
A
B.

This form of (English) arguments is valid. Why? Because, we assume, all of its instances are arguments which would be considered as correct by English speakers who understand "if" and read "⇒" as its shortcut. (Well, in fact *almost* all, for some of the instances may sound weird, and there may

[12]This is not to say that for *any* arguments the acceptance by the majority would be equal to its correctness. However, this holds for simple and perspicuous arguments of this kind.

perhaps even be instances which would be classified as incorrect by a substantial portion of the speakers.[13]) If we now introduce one more convention and decide to write "⇒" in between two clauses instead of in front of them, we have the familiar scheme[14]

(MP) $A \Rightarrow B$
$\underline{A\quad\quad}$
$B.$

Let us now consider a similar form articulated in the language of a logical system S:

(MP*) $A \to B$
$\underline{A\quad\quad}$
B

The question whether it is valid cannot of course be answered unless we are acquainted with the system S. But, influenced by our expectations, we will tend to assume that it is *valid* (we would assume that "→" is an implication sign and we know that this scheme is valid in classical logic, as well as in many other logical calculi). What is important to keep in mind is that if the scheme of (the language of) S is valid, it is because of the definition of "→" in S – its validity is in this sense a (trivial) consequence of certain given postulates.

Now, can either (MP) or (MP*) be identified with the law of *modus ponens*? Let us first consider the second possibility. In such a case we would, as it seems, have to admit that *modus ponens* is just a trivial consequence of mathematical definitions which govern, within S, the use of "→". If, on the other hand, *modus ponens* is to be identified with (MP), then the validity of this law would depend on empirical facts – facts that have been brought about by the contingent development of English (plus our contingent convention concerning the use of "⇒") and that might come to be contravened by its further development.[15] Neither possibility looks quite plausible.

Doesn't this (admittedly sketchy) reflection on the two options suggest that it might be, after all, most reasonable to retreat to the view which places *modus ponens* somewhere beyond any (natural or artificial) languages, in

[13] In the literature, we can come across arguments which seem to shake overly bold claims about the validity of all arguments of this form (see, e.g., McGee 1985).

[14] The previous scheme would perhaps look more familiar to some Polish logicians.

[15] Cf. Peregrin (2010a).

some realm of forms of the world or of thought? Aren't principles like *modus ponens* something that resides in the ideal world of the purely formal? Maybe we, humans, were designed as rational beings or we have developed into beings with principles like *modus ponens* imprinted into our minds (and so we can recognize them *a priori*).

Though contemplations of this kind may sound tempting to many ears, we don't think that they are promising if we are searching for an understanding of the foundations of logic. The "solution" they offer is, in our view, illusory – not only because it relies on tricky metaphysical assumptions (which are sometimes hardly distinguishable from a wishful thinking), but especially because the assumption that the genuine implication is an abstract, ideal object does not solve anything. The Platonist heaven, if we admit its existence, abounds in all kinds of objects (they certainly contain classical implication, intuitionist implication, various relevance implications and many others which are not incorporated in any logical system that we have created so far), and it is quite unclear how to answer the question of which one is the mythical *genuine implication* (unless we want to say that it is the one that is expressed by a natural or a distinguished artificial language, in which case we are back at one of the kinds of answers we rejected above).

What we suggest is that the "genuine implication" is not a kind of esoteric object beyond any languages, but rather a result of a complex interplay between elements of natural and formal languages. Consider how logical systems – like our generic system S – get formed. Certainly not every language and every calculus we can put together deserves to be called *logical*. Systems that deserve such a designation emerge from a process of the complex, mutual adaptation of an artificial language (not necessarily the rich artificial language like those we know from modern logic, but possibly also a proto-artificial language, such as that of Aristotelian syllogistics) and a natural one; they result from our pre-theoretical language meeting with its systematic theoretical reflection. As we have suggested (Peregrin & Svoboda, 2017), the process of formation of such logical languages/systems (which in practice can take different shapes) has the character of a *reflective equilibrium*.[16]

[16]The term "reflective equilibrium", which is now commonly used as a name of the method/process as well as the name of its outcome, became widely used after the publication of Rawls' influential book *Theory of Justice* (Rawls, 1971). The idea (conceived more broadly than in the case of Rawls, who focuses on ethics) goes back (at least) to Goodman (1955). A formal analysis of the process is presented by Brun (to appear).

The idea is that the development of our languages was intertwined with the development of our argumentation/reasoning. Similarly as in ontogenesis, the process of learning to reason is inseparable from the process of mastering a language; the two processes, we may plausibly assume, were also inseparably connected in human phylogenesis. Natural languages have developed as governed by certain implicit rules, rules that are fuzzy and open-ended. Our theoretical reflection on these rules (which is often motivated by the need to make the language less fuzzy and more exact) resulted in our positing their explicit, crisp and closed explications. These posits get confronted with their natural counterparts and get amended where this confrontation yields overly large discrepancies. However, it is not only the posited explicit rules but also the underlying implicit ones that may get amended by the confrontation; hence, what occurs is a back-and-forth movement between the tentative theoretical generalizations and our "intuitions" underlying them. And this movement proceeds towards the kind of equilibrium which yields a theoretical tool that will serve our purposes.

As there is no one "true" logical system, there is no one "true" implication and no one "true" *modus ponens*. We have, strictly speaking, distinctive versions of *modus ponens* as articulated in different languages which have qualified as *logical*[17] (and which, at the same time, use a junction designed for straightforward formalization of conditional sentences).[18] However, there is clearly a sense in which the different versions of implication can still be seen as different species of the same kind[19] and hence we have also a general notion of *modus ponens*: it is the rule which takes us from an indicative conditional plus its antecedent to its consequent.

What is remarkable is that if we have an artificial language which has stood the test of reflective equilibrium considerations (and hence deserves the name *logical*), we can make use of its constants in a specific way. We can formulate full-fledged arguments which combine such constants (whose meaning is precisely determined) with expressions of natural languages such as English. Thus, we can identify instances of logical laws which are in a way very special. They belong neither to a logical language nor to a nat-

[17]Logicality, within this picture, is not a "yes or no" matter, it may come in degrees. For example, systems with a very limited expressive power can still count as logical if they are useful for some purposes, though generally we expect that logicality presupposes a kind of versatility.

[18]We wouldn't call the argument form $\neg A \vee B$, *A hence B* a specific version of *modus ponens* though it is in classical logic indistinguishable from $A \rightarrow B$, *A hence B*.

[19]Perhaps the situation can be compared to law – though there is nothing like one, *single* law prohibiting rape, it sounds plausible to say that we can find, in different countries, specific versions of such a law.

ural one, but they are formulated in a *hybrid language* resulting from their crossover. Thus we can, for example, "crossbreed" the language of classical propositional logic with its connective "$\rightarrow$" and English and formulate the following argument:

(AH3) *It rains* $\rightarrow$ *the streets are wet*
It rains
The streets are wet

This "hybrid" argument is clearly an instance of (a specific version of) *modus ponens*. It can be seen as both *full fledged* (meaningful, fully understandable) and *undeniably correct* (logicians who "talk the language" of classical propositional logic are the ultimate arbiters concerning the correctness).[20] In this (and not only in this) way logic(s) allow us to move the expressive potential of our language to a new level and assure (at least to some extent) firm common grounds for our discussions.

4 Conclusion

Laws of logic interconnect logical forms, while logical forms are our theoretical reconstructions of the inferential properties of the sentences we use to reason/argue, *viz.* typically declarative sentences of our natural language. While our practices of argumentation/reasoning are rule-governed in the sense that we do correct each other and thus reinforce what can be seen as "implicit rules" inherent to the enterprise, genuine, explicit rules originate only from our theoretical reflection of the practice. The rules are formed by a process in which our attempts at fixing the rules in an explicitly articulated form get accommodated to the "intuitions" which underlie our "implicit rules", while the "intuitions are amended by the emerging rules – the process of *reflective equilibrium*. Thus, logical laws are neither rules concerning directly our natural languages nor precepts embodied in definitions of a certain artificial language. But neither do they concern some *a priori* given forms beyond any language. They result from a delicate interplay between natural languages with their implicit rules and artificial languages with their stipulated explicit rules, which results in rules that are rooted in our natural languages (and the practices of argumentation/reasoning which they are the vehicle of) but which are, however, explicit and open to view.

[20] If we used (certain) strict implication or relevant implication in place of the material implication we would, of course, receive a somewhat different argument as the meaning of the first premise would be different.

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