On the Hilbert formulas on the unit circle for α-hyperholomorphic function theory

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Let us denote by $\mathbb{S}$ the unit circle in the complex plane $\mathbb{C}$ and given a limit function f $\mathbb{S}$. set $g(\theta) := f(e^{i\theta})$, $0 \leq \theta < 2\pi$, and $g = g_1 + ig_2$. Then the real components g_1 and g_2 of g are related by the following formulas known as the Hilbert formulas for the unit disc:

$$\begin{aligned} \mathcal{M}[g_1] + \mathcal{H}[g_2] =& g_1, \\ \mathcal{M}[g_2] - \mathcal{H}[g_1] =& g_2, \end{aligned}$$

where M and $\mathcal{H}$ are given by

$$\begin{aligned} \mathcal{H}[g](\theta) :=& \frac{1}{2\pi}\int_0^{2\pi} \cot\frac{\tau-\theta}{2} g(\tau)d\tau, \quad \theta \in [0, 2\pi), \\ M[g] :=& \frac{1}{2\pi}\int_0^{2\pi} g(\tau)d\tau, \end{aligned} \tag{0.1}$$

which are both defined on the linear space of real valued Hölder continuous functions $C^{0,\mu}(\mathbb{S}, \mathbb{R})$, $\mu \in (0, 1]$.

The integral $\mathcal{H}[g]$ (well-defined on $C^{0,\mu}(\mathbb{S}, \mathbb{R})$, $\mu \in (0, 1]$) is understood in the sense of the Cauchy principal value, generating the so-called Hilbert operator with (real) kernel $\frac{1}{2\pi}\cot\frac{\tau-\theta}{2}$.

The Hilbert operator (0.1) is well-known transformation in mathematics and in signal processing; for example, in geophysics and astrophysics it deals with input signals. Examples of this type of signals are seismic, satellite and gravitational data; and the Hilbert operator proves to be useful for a local analysis of them, providing a set of rotation-invariant local properties: the local amplitude, local orientation and local phase.

Various analogues of the Hilbert formulas on the unit sphere keep interest until our days.

In this talk we give some analogues of the Hilbert formulas on the unit circle for α–hyperholomorphic function theory when α is an arbitrary complex quaternion number.