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Operator norm resolvent approximations in homogenization of composites and similar structures

It is widely acknowledged that novel artificial material structures acquire desired properties as a result of a carefully designed microstructure. Very often these are periodic *composites* with a sufficiently small periodicity cell. In the simplest case, one can obtain a two-phase composite, distributing impurity grain in a host material. If any holes, hollows or cavities are instead of grains here, there appear *perforated media*. Similar structure is intrinsic to *porous media* either. The mathematical apparatus involved to study or to model such type heterogeneous media is the theory of homogenisation which is destined, first, to find their effective(or homogenized) description and, second, to evaluate the error of homogenization.

An elliptic equation with ε-periodic rapidly oscillating, as ε tends to zero, coefficients is one of model examples in homogenization theory. It may describe various physical processes, say, in small-period composites. We are concerned with approximations for the solutions to this equation with remainder term of order ε and ε^2. These approximations are to be constructed in a regular way with a controlled error bound, besides,

(i) under minimal regularity assumptions on data of the equation;
(ii) with majorants in the estimates that admit operator presentation of the approximations and estimates, namely, in terms of the resolvents of the original and homogenized equations and maybe some correcting operators;
(iii) following more or less standard way of two-scale expansions depending on a slow variable x and a fast variable x/ε.

In the talk we are going to discuss results on operator-type approximations and corresponding estimates in homogenization of elliptic equations; part of them are known, e.g., from the overview [1], others are new.

References

[1] V. V. Zhikov and S. E. Pastukhova, *Operator estimates in homogenization theory*, Russian Math. Surveys 71 (2016), 417511.