# Dirichlet Laplacian in asymptotically flat triangles

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**Abstract** The Dirichlet realisation of the Laplacian in a bounded domain $\Omega \subset \mathbb{R}^n$ attracted a lot of attention in the past few years. More precisely, finding the eigenvalues and eigenfunctions of such operators is a natural physical issue and the eigenvalue problem can be formulated as follows: We want to find a pair $(\lambda, \psi)$ such that

$$\begin{cases} -\Delta\psi = \lambda\psi & \text{in } \Omega, \\ \psi = 0 & \text{on } \partial\Omega. \end{cases} \tag{1}$$

Appart from tensored domains, computing the eigenpairs $(\lambda, \psi)$ is not an easy task, mainly because the geometry of the domain $\Omega$ influences the structure of the spectrum.

However there is a class of domains that can be deal with: The asymptotically flat domains. In dimension two, they correspond to domains of the type:

$$\Omega(h) := \{(x_1, x_2) \in \mathbb{R}^2 : a < x_1 < b, 0 < x_2 < hf(x)\},$$

where $h > 0$, $a, b \in \mathbb{R}$ with $a < b$ and $f \in \mathcal{C}([a, b], \mathbb{R}_+)$ such that $f(a) = f(b) = 0$ (see Figure 1). The aim is to understand Problem (1) when $h \to 0$.

These domains have been extensively studied and in [1], Borisov and Freitas give an asymptotic expansion at any order of the eigenpair when the function $f$ has a unique smooth maximum. In [2], Friedlander and Solomyak, get rid off the smoothness hypothesis but they only obtain a finite term asymptotics for the expansion of the eigenpairs.

In this talk, I will be interested in the specific case where $\Omega(h)$ is a triangle. Using the philosophy of the Born-Oppenheimer approximation, one can reduce the problem to a one dimensional model. I will give an asymptotic expansion at any order of the eigenvalues and eigenfunctions of the Dirichlet Laplacian on triangles in the small altitude limit $h \to 0$. This talk is based on the work [3].

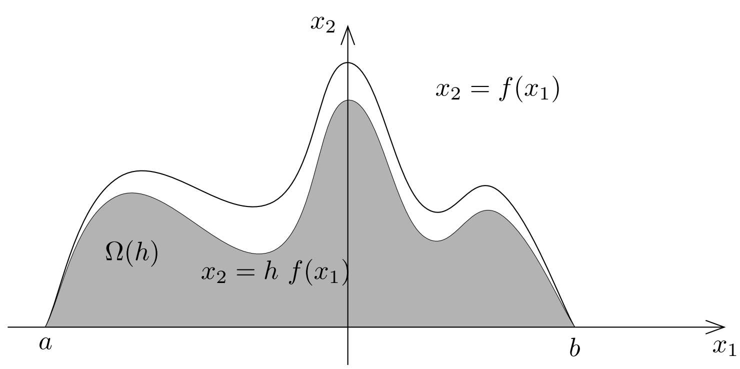


Figure 1: Asymptotically flat domain

## References

[1] D. Borisov and P. Freitas. Singular asymptotic expansions for Dirichlet eigenvalues and eigenfunctions of the Laplacian on thin planar domains. *Ann. Inst. Henri Poincaré, Anal. Non Linéaire*, 26(2):547–560, 2009.

[2] L. Friedlander and M. Solomyak. On the spectrum of the Dirichlet Laplacian in a narrow strip. *Isr. J. Math.*, 170:337–354, 2009.

[3] T. Ourmières-Bonafos. Dirichlet eigenvalues of asymptotically flat triangles. *Asymptotic Anal.*, 92(3-4):279–312, 2015.