A Lewisian taxonomy for deontic logic

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Georg Henrik von Wright noted in 1991 that deontic logic, which originated in its modern form in the early 1950s, "has remained something of a problem child in the family of logical theories". In 1999 he added that this field of logic still seems to him "a most problematic branch of logico-philosophical study".[1] These comments, made by the philosopher whose seminal paper "Deontic Logic" (von Wright 1951) in fact introduced the 'child' into the 'family' surely deserve to be taken seriously, the more so that they could be meaningfully repeated even in the second decade of the 21st century. Though deontic logic has taken significant steps forward since the time when von Wright voiced his objections and a number of interesting foundational contributions have been published,[2] the problems pointed out by von Wright haven't really been resolved or removed.

In this paper, I will address some foundational issues of deontic logic which are related to von Wright's complaints. My thesis will be that – if we stay within the 'problem child' simile – the child has a kind of identity problem and won't become fully mature unless she develops a proper 'sense of self', i.e. unless the child becomes aware of her roots and sorts out her ambitions and preferences. In particular, I want to argue that some problems that have troubled deontic logic for decades will appear more tractable if we look at this area of research from the perspective of language games that David Lewis introduced in the late 1970s.[3] Adoption of this perspective will open the way to a natural division of the domain of logical studies that are traditionally subsumed under the term "deontic logic" and to setting an agenda for the individual sub-theories.

What kind of insights the proposed perspective provides will be briefly illustrated in an analysis of the so-called Ross paradox - a problem that has troubled deontic logic for decades and, though it was many times pronounced solved, still keeps coming back 'alive and kicking'. I

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[1] See von Wright (1991, p. 256) and (1999, p. 15).

[2] Among the contributions which address the foundational issues, we can mention Makinson (1999), Vranas (2010) and (2011), Hansen-Pigozzi-van der Torre (2007), Broersen-van der Torre (2012) or Charlow (2014).

[3] See Lewis (1979a) and (1979b).

will argue that there is no 'one and only' solution to the paradox; if, however, we bring to light the confusions surrounding the issue, the paradox gets 'dissolved'.

The concept of Deontic Logic

Before I move on to the central theme of the paper, it would be worthwhile to devote some time to certain basic historical considerations and philosophical problems concerning the grounds of deontic logic.[4] Though I suggested that deontic logic entered the family of logical theories in the beginning of the 1950s, this is only partially true.[5] The topics studied within the discipline were addressed earlier by several modern philosophers who have been deprived of the status of founding fathers mostly because their theories turned out to be seriously flawed and hence didn't find enough supporters and elaborators.

Among these philosophers it is Ernst Mally who, in this regard, figures most prominently. This pupil of Meinong was the first to come up with the idea of building an axiomatic system capturing the logical laws of 'the ought' (das Sollen).[6] The system that he proposed in 1926 is usually classified as the first system of deontic logic. Unfortunately, Mally's theory was afflicted by a problem similar to the one that subverted the very first system of strict implication proposed by C.I. Lewis in 1918 – the axioms were set in such a way that they made the contribution of the new 'modal' symbol - here "!" - trivial. The formula $!p \leftrightarrow p$ was a theorem of the system and thus sentences like "It ought to be the case that p" and "(It is the case that) p" came out as synonymous.[7] Similar problems proved fatal for other early attempts to build logical theories of prescriptive language such as those made by Kurt Grelling, Karl Menger and Albert Hofstadter & J.C.C. McKinsey.

Even a brief look at these early attempts reveals one of the problems that have affected deontic logic through its whole history – the indistinctness of the delineation of the studies. Mally speaks about *the logic of will* or *the logic of ought* and is primarily interested in ethical judgements. Grelling (1939) talks about *the logic of ought sentences* (Sollsaetze), Menger (1939) speaks on *imperative logic*, Hofstadter and McKinsey (1939) speak about *the logic of*

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[4] Those who are well oriented in the area of deontic logic can skip this section.

[5] In fact, precursors of deontic logic can be found among medieval logicians (cf. Knuuttila 1981).

[6] The title of Mally's monograph is *Grundgesetze des Sollens. Elemente der Logik des Willens* (Mally 1926). It is worth noting that Mally's main goal was to provide an exact system of pure ethics, so he was primarily interested in the elucidation of the concept of the ethical *Ought*.

[7] This was pointed out by Karl Menger, see Menger (1939). Unlike Lewis, who swiftly corrected his system (cf. Lewis (1920)), Mally never reacted to the challenge.

imperatives. As mentioned above, the term *deontic logic,* introduced by von Wright,[8] has become the most common denomination of this area of logical theories but we can come across a number of other terms designating studies falling into the same group – *the logic of norms* (von Wright 1963), *the logic of normative systems* (Stenius 1963), *normative logic* or *logic of norm propositions* (Alchourrón 1969, Beirlaen-Straßer 2013), *the logic of commands* (Rescher 1966) or *the logic of commitment and obligation* (Segerberg 1971).

The variety of terms indicates that their proponents conceive their subject matters differently or at least adopt somewhat different perspectives, but it would still clearly be improper to suppose that each of the different terms designates a separate logical theory. It seems that it should be possible (and would be desirable) to integrate the studies into a common theoretical framework. It goes without saying that any attempt to squeeze theories that vary with respect to their philosophical assumptions and ambitions into a Procrustean bed of a single framework will be controversial, but it is still, I believe, a project worth pursuing.

Another, related, problem that has affected deontic logic throughout its history may be called foundational or philosophical. Up to now there is still not a full consensus as to whether the logical study of prescriptive language is a legitimate research project. Logic traditionally focuses on studying inferential relations among entities that are truth-bearers - indicative sentences, statements or propositions. Turning attention to prescriptive language seems to lead logical studies outside of the domain of truth and falsity and thus sets a serious challenge to the traditional conception of logic.[9]

There are, of course, deontic studies whose feasibility as logical theories is relatively uncontroversial. Sentences claiming that some way of acting is obligatory, forbidden or permitted by a certain valid normative system[10] are commonly taken to be true or false and though some logicians may be sceptical about the worthwhileness of studies that treat sentential modifiers like *it is obligatory that* (O), *it is forbidden that* (F) and *it is permitted that* (P) as logical constants, such a project surely is not doomed to failure. The crucial question is thus whether deontic logic can be conceived as a more ambitious theoretical project focused (also) on studying logical relations among prescriptions, norms or imperatives.

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[8] Von Wright gives credit to his colleague C.D. Broad, who proposed the choice of the term (see von Wright (1951, p. 1)).

[9] This challenge was clearly articulated by Jørgen Jørgensen (1937–38) and, thanks to A. Ross, it became known by the name *Jørgensen's dilemma* (cf. Ross (1941)).

[10] Systems of legal norms of a particular country are the paradigmatic example here.

Stenius' thesis

When looking back at his intellectual journey "through the landscape of deontic logic", von Wright (1991) points out that, in the beginning, the philosophic problem of the "possibility" of a logic of norms did not trouble him. In the early fifties he was simply intrigued by formal analogies between deontic and modal concepts. Only later did he appreciate the significance of the fact that common deontic sentences (e.g., sentences like "You ought to pay your debts" or "You may park in front of the house") admit to two interpretations, viz. a prescriptive and a descriptive one. Prescriptively interpreted they express norms, descriptively interpreted they express (true or false) propositions to the effect that certain norms exist.

Von Wright's compatriot and contemporary, Erik Stenius, presented the same observation (in somewhat different terminology) and called it *the method of double interpretation*:

> "If we say 'No smoking' or 'Smoking is prohibited' or even 'You ought not to smoke here' what we say can be interpreted in at least two ways: *either* as the direct expression of a prohibition *or* as a statement informing the listener about the existence of such a prohibition among certain – for the most part unspecified – regulations. According to the first interpretation 'you ought not to smoke here' is a sentence in the mood of *prohibition*, that is, in the mood of negative *obligation*, and thus expresses a genuine *norm*. According to the second interpretation it is a factual statement *about* a norm, which is true or false, as the case may be. This distinction has, by the way, been made a corner-stone in the legal philosophy of Professor Hedenius of Uppsala. I incorporate it in the formalized logic of normative systems by distinguishing between a ('genuine') *modal* interpretation of a norm sentence 'Op' and a *factual* interpretation of it, which may be expressed by the formula O$p \in$ S ('p is obligatory in S'), where S denotes a (possibly unspecified) *system of norms*." (Stenius 1963: 250)

The distinction has a central role in Stenius' formulation of one of the crucial principles of his logic of normative systems (Stenius 1963, p. 251):

PRINCIPLE III   *The logical relations between normative sentences in their modal sense are the same as the logical relations between the same sentences in their factual sense.*

In his book, *Norm and Action*, von Wright adopted a similar view. Although, according to him, "fully developed" deontic logic concerns descriptively interpreted deontic sentences, its laws hold for the respective norms as well. This allowed him to admit that logic "has a wider reach

than truth" but at the same time to not depart too far from 'traditional' modal logic (see von Wright 1957 Preface and 1963, chapter VIII).

Von Wright's development had several twists and turns prompted by his intellectual openness and genuineness, but the general idea that the logical properties of descriptively and prescriptively interpreted deontic sentences are, so to say, two sides of the same coin and that deontic logic can study them together - 'at one fling' - appealed to him for most of his career. This idea is (usually less explicitly) present in most 'classical' theories of deontic logic.[11] It is difficult to decide whether this is due to von Wright's enormous influence or instead to the fact that the view is naturally appealing; it has, however, become so common that, in my view, it deserves a specific name - *Stenius' thesis*.[12]

When David Makinson points to a long-lasting "tension between the philosophy of norms and the formal work of deontic logicians," he essentially has the same kind of problem in mind. He observes that it is widely accepted that there is a distinction between norms on the one hand and declarative statements or propositions (including those concerning norms) on the other, but that, on the formal level, "work goes on as if such a distinction had never been heard of." When we go through the literature we see that

> "...most deontic logicians accept that there is a fundamental distinction to be made, and find themselves in the uncomfortable position − indeed the unprincipled and close to inconsistent one − of conceding that norms do not bear truth-values, but hoping that for the purposes of logic they may, for some mysterious reason, be treated as if they did." (Makinson 1999, p. 29)

We will demonstrate problems that stem from the 'schizophrenic' conception of the language of deontic logic by a simple illustrative example. Let us suppose that we decide to employ formulas like Op, Pp, Fp both as norm formulation (as interpreted 'modally') and as deontic statements (norm propositions). If we stick to the choice, we, according to von Wright,

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[11] Rarely, however, does it get clearly articulated. In this disguised form it also appears in alternative approaches based e.g. on dynamic logic (see, for example, Meyer (1988), Dignum et al. (1996) or Pérez-Ramírez - Fox (2003)). Philosophers who deny the possibility that norms (prescriptions, imperatives) might be bound by logical relations don't, of course, share this idea (see e.g. Williams (1963), Wedeking (1970) or Hansen (2013)). Explicit defence of the idea that there is no distinction between the logic of norms and the logic of normative propositions can be found in Hilpinen (2006).

[12] Someone might suggest the name *von Wright's thesis*, but this would not be completely appropriate as von Wright revised his views several times (in von Wright 1999 he, for example, explicitly says that "classical deontic logic, descriptively interpreted, cannot claim to be *the* (correct) logic of norm-propositions") and he would surely have had provisos against the blunt formulation that can be found in Stenius' Principle III. On the other hand, von Wright's contribution to the spreading of this kind of approach to deontic logic is beyond doubt.

don't need *two symbolisms*, but *two interpretations* of the same symbolism. Von Wright, of course, is not naive and notices that "the question open to debate is, whether truth-connectives can be used for forming molecular complexes of *prescriptively interpreted O-* and *P-*expressions." He, however, decides not to duplicate the symbolism for the connectives, to stick with their traditional interpretation and to build deontic logic as a theory of descriptively interpreted expressions. He, nevertheless, immediately points out that "the laws (principles, rules), which are peculiar to this logic, concern logical properties of the *norms* themselves, which are then reflected in logical properties of norm-propositions. Thus, in a sense, the 'basis' of Deontic Logic is a logical theory of prescriptively interpreted *O-* and *P-*expressions."[13]

Let us see what would have happened if von Wright had made a different choice and decided to keep the ambiguous interpretation of the elementary formulas O*p*, P*p*, F*p* and duplicate the connectives. We could then have, for example "¬", "∧", "∨", "→" as standard propositional connectives and "~", "&", "∪", "⇒" as nonstandard connectives suited for making molecular expressions of *prescriptively* interpreted O, F, and P expressions, i.e. expressions that don't have truth values. We can now compare a sample set of the formulas of the language of deontic propositions, e.g.:

   1. ¬O*p*,   2. O($p \land q$),   3. O*p* ∧ P*q*,   4. O($p \lor q$),

   5. O*p*→O*q*,   6. O(p→q),   7. $p \to$ O*q*,   8. $p \lor$ (O*p*→O*q*)

with their straightforward counterparts in the language of norms:

   1'. ~O*p*,   2'. O($p \land q$),   3'. O*p* & P*q*,   4'. O($p \lor q$),

   5'. O*p* ⇒ O*q*,   6'. O(p→q),   7'. p ? O*q*,   8'. p ? (O*p*⇒O*q*)

We can immediately see that the difference between the languages does not consist merely in the fact that in the second set the original truth-functional connectives are (at all occurrences) substituted by their counterparts. It is obvious that we cannot avoid using the traditional

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[13] All quotations here are from von Wright (1963, p. 134). It should be stressed that von Wright later changed his views and opted for an independent logic of norms (see von Wright (1999)). He was also aware that his attitude might have influenced the development of deontic logic in a problematic way: "In my first paper I thought the mere fact that one can construct a formal calculus with plausible-sounding axioms was all that was needed to satisfy the demands of logic. And I think it is right to say that this attitude still implicitly underlies much of the work that is being done in deontic logic. Its problematic nature, however, has to this day remained a thorn in my logical flesh, if I may use this metaphor" (von Wright 1999, p. 19).

connectives in the prescriptive language. This, however, need not be a serious problem - we can admit that the language will be, as concerns the set of logical constants, richer. What is more serious is that some formulas, in our case formulas like 1', 5', 7' and 8', seem quite problematic.

Let us start with the simpler cases represented by 'formulas' 7' and 8'. I placed question marks at the two spots where symbols for classical implication and disjunction occur in the formulas from the first set. It is not difficult to see that the 'formulas' could become meaningful only if we had a third set of connectives, namely connectives that are able to connect a proposition and a prescription (norm) into meaningful 'hybrid' sentences.[14]

Serious questions also arise in connection with formulas like 1'. It seems that "~" works as a kind of negation but if we read Op as a prescription, i.e. if we take, e.g., the sentence "You ought to see to it that p" as close to synonymous with "Make sure (see to it) that p is the case!" then the attempt at the 'negating' of the sentence yields "Do not make sure that p is the case!", i.e. a sentence whose (prescriptive?) meaning is difficult to decipher. Does the person using such a sentence require that the addressee stays passive with respect to the occurrence of p? Or does she rather require that the addressee makes sure that non-p is the case?

On the first sight it might seem natural to view ~Op as a form of sentences which inform us about an "absence of prescription" but under closer inspection it turns out that expressions suited for articulation of such claims have no place in language whose sentences are to be interpreted prescriptively – they *describe* what is the case rather than *prescribe* what should be the case. Another option might be to take the formula ~Op as an expression of a 'derogative' sentence, i.e. interpret it as the logical form of a sentence by which we take back some previously issued prescription ("I hereby take back my prescription 'You ought to p'"). Proclamations of this kind are sometimes employed in law for introducing the so called derogative norms (cf., e.g., Kelsen (1991)), but norms of this kind are quite specific – they don't require any course of action and cannot be followed or disobeyed. Hence, it is unclear whether they should be classified as prescriptive expressions or rather as expressions belonging to a deontic meta-language.[15]

Even more problematic are formulas like 5'. It is quite unclear how they could be understood. Sentences of this form are clearly not prescriptive in the sense that they could serve as formulations of certain action-guiding principles. This gets rather clear if we compare

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[14] We have, no doubt, such connectives in natural language. E.g., "*if...(then)...*" in the sentence "If it rains, (you should) keep the window closed!", "*...or...*" in "(You should) shut up or I will shut you up" or "*...and...*" in "Stay here and I will go home" are connectives of this sort. Some linguistically oriented accounts of semantics - e.g., Starr (t.a.) - employ such connectives suitable for combining imperatives and declaratives, but I don't know about a logical theory that introduces 'versatile' connectives of this kind as proper logical constants.

[15] An interesting analysis of the different forms that derogation can take can be found in Cornides (1969).

descriptively interpreted sentence "If you should do *p*, then you should do *q*" (which makes good sense) with the clearly prescriptive sentence "If do *p* then do *q!*" which fails to make any sense.

The often overlooked disparity of the two languages provides quite a strong argument against the 'parallelism assumption'. Other convincing reasons have been provided by Carlos Alchourrón and Eugenio Bulygin, who repeatedly warned against conflating the logic of norms with the logic of normative propositions (see, e.g., Alchourrón and Bulygin (1971, p. 121nn) or (1993, p. 285nn)).[16] I will return to the issue later but, before then, let me introduce a framework within which the difference between the two kinds of linguistic means is clearly manifested. The framework builds on the idea of a prescriptive language game proposed by David Lewis (cf. Lewis (1979a), Lewis (1979b)).

Lewisian deontic language games

My version of the Lewisian language game involves three players: *the Prescriber*, *the Doer* and *the Kibitzer*.[17] The Prescriber's moves consist in issuing commands and permissions to the Doer, whose moves consist in making what the Prescriber requires. (Though the moves of the Doer are non-linguistic, they may still in some cases modify the normative situation.) The Kibitzer's moves are his descriptions of the normative situation.[18] The steps of the game are indexed by the order in which the Prescriber issues the prescriptions.[19] Possible worlds conforming to the Prescriber's commands and permissions together create *the Sphere of Permissibility* (SP). At the beginning of the game, the Sphere of Permissibility amounts to *the Sphere of Accessibility* (SA), i.e. the set of all possible worlds that are the alternatives to the actual world in the sense that they conform both to the natural laws of the actual world and to its past history.[20]

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[16] It is worth noting that pointing to precariousness of the 'parallelism assumption' doesn't amount to claiming that a logic based on a prescriptive language cannot be in principle mirrored in a logic based on a descriptive language. It just suggests that the 'double interpretation' strategy is much trickier than it seems and hasn't yielded satisfactory results.

[17] Lewis speaks about the Master, the Slave and the Kibitzer. ("Kibitzer" was originally a Yiddish term designating a bystander offering (often unwanted) advice or commentary.) Though his terminology has an advantage of transparency, the associations it evokes may be misleading. It should be noted that the role of the Prescriber is quite similar to the role that the sovereign called Rex has in Hart's model of legislation (Hart 1961).

[18] We can, of course, also assume that the players are capable of uttering common indicative sentences describing the factual world. Within the present discussion I will, however, keep things as simple as possible and leave the (no doubt very interesting) problems arising from the consideration of mixed language games combining prescriptions and/or deontic statements with factual statements aside.

[19] When considering simple oral games I will normally assume (together with Lewis) that at each step of the game the Prescriber issues just one prescription, but it, of course, makes sense to also consider the more general (and complex) case in which at each step the Prescriber issues a set of prescriptions.

[20] My conception of the SP and SA diverges slightly from the one outlined by Lewis, but they both presuppose that the actual world always belongs to SA (not necessarily to SP).

It is quite clear that the roles of the players determine what kind of moves are in their respective repertoires. The Prescriber addresses the Doer only with sentences that are to be interpreted *prescriptively*. To manifest that clearly and prevent potential confusion, I will suppose that he only uses sentences in the imperative mood and permissive sentences that (typically) employ the phrases like "you (from now) may...", "(from now) you needn't…".[21] On the formal level, I will represent the sentences in the Prescribers' repertoire by the formulas of a simple dyadic language.[22] The formulas of the shape !α/β will represent the logical form of sentences like "If (under condition) β, do (see to it that) α!" and the formula ¡α/β will represent the logical form of sentences of the following type: "If (under condition) β, you may do (see to it that) α!". In these sentences (formulas), α and β are parameters representing formulas of classical propositional calculus formed by means of the connectives ¬, ∧ and ∨. Unconditional (categorical) prescriptions are standardly represented by formulas with an arbitrary tautology in the condition, but normally I will employ 'shortcut formulae' of the form !α and ¡α.

The Kibitzer's repertoire contains first and foremost sentences suitable for the *describing* of the normative situation, i.e. statements like "You are obliged (must) ...", "The Doer is forbidden to ..." or "The Doer is allowed to ...". We can, of course, quite naturally presuppose that all sentences by which the Kibitzer describes the situation implicitly concern the Doer. In such a case, the Kibitzer can employ formulations like "It is forbidden to smoke in the house" or "Drinking wine inside of the house is permitted", which sound more familiar to ears of deontic logicians. Thus, I will use the standard dyadic formulas of the shape O(α/β) (F(α/β), P(α/β)) to represent the logical form of sentences like "If (under condition) β, you are obliged (forbidden, permitted) to do (see to it that) α" or "It is obligatory (forbidden, permitted) to do (see to it that) α if (under condition that) β". Similarly to the case of the Prescriber's language, I will use the shortcut formulae - Oα (Pα, Fα) - for unconditional deontic statements. It is perhaps worth stressing that the Kibitzer is not in a position to issue any commands, so sentences in the imperative form are not in his repertoire.

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[21] It is important to stress that nothing hinges on specifically using the imperative mood. Unlike authors focused on linguistically oriented semantic studies (e.g., Kaufmann (2012) or Starr (t.a.)), I am here indifferent as concerns the formulation of the prescriptions. They might be expressed all the way through by sentences like "it is obligatory that (you) ....", "(from now) you must (not) ..." or "I hereby command (permit) you to.... " as soon as it is clear that the sentences are understood as shaping (and not just describing) the SP. An interesting argumentation showing that if permissions are taken to have this prescriptive (action guiding) role then no possible worlds account of their truth-conditions can succeed, can be found in Fine (2014).

[22] For the employment of a version of dyadic language (in which conditional prescriptions are primitive) and arguments in favour of this choice, see, e.g., von Wright (1961) or van Fraassen (1972).

One important difference between the Prescriber's language and the Kibitzer's language has already been mentioned. We have seen that the specific expressions of the Kibitzer's language merge naturally with expressions of other 'descriptive' languages. It is thus quite convenient to adopt mixed formulas like, for example, $q \lor O(p/r)$, $(q \land O(p/q)) \rightarrow Op$, $(Op \land \Box(p \rightarrow r)) \rightarrow Or$ into the formal language whose core is formed by the dyadic formulas expressing norm-propositions. Similar merging appears quite inappropriate in the case of the Prescriber's language. Though, as I mentioned, in natural language certain 'mixed constructions' can make sense (e.g., "Go home and I will wait here"), introducing corresponding constructions into a formal language looks like a severe complication that lacks a serious motivation. It is also worth noting that a widely conceived Kibitzer's language can be used as its own meta-language in the sense that the expressive resources needed for formulating reflections on the language fit into it. But if we want to theorize about the Prescriber's language - to say, for example, that some prescription is entailed by a certain set of prescriptions - we have to make use of a descriptive language.[23]

It is also important to realize that while interdefinability of O and P - the central operators of the Kibitzer's language - is (under some assumptions) uncontroversial,[24] the attempts to introduce similar interdefinability between the operators "!" and "¡" seem futile. I have already suggested that the meaning of sentences like "Do not make sure that p is the case!" or "It is not the case that do p" (and 'formulas' like $\sim !p$ or $\neg !p$) is unclear (to say the least). The sentence "Don't do p!" seems like a straightforward 'negation' of "Do p!", but it surely does not articulate any permission (though it implicitly permits $\neg p$).

I should perhaps stress that, when I endorse the perspective of the language game which clearly separates the language of the Prescriber from the language of the Kibitzer, I don't want to deny that common sentences saying what ought (not) to be done or what somebody should (not) do are in fact open to different interpretations and that their ambiguous nature is often intentionally used or exploited. I only want to argue that it is unwise to try to mimic this ambiguity within a formalized language. In my view, it doesn't make good sense to treat the formula Op as having a truth value and at the same time ask whether it is obeyed, respected or satisfied by an agent (in a possible world). What can be obeyed or disobeyed is the prescription

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[23] Portner (2012) suggests that talking about entailment in case of sentences to which we don't attribute truth or falsity is not felicitous; he instead speaks about the relation of warranting. I, however, don't see any serious reason for avoiding the term "entailment" in this context.

[24] In most systems of deontic logic, the operator O is taken as primitive and the other is introduced by the following definition $P\alpha =_{df} \neg O\neg\alpha$. Acceptance of the definition amounts to adoption of the weak (or negative) concept of permission - permission as a 'lack of prohibition'.

whose validity is the truth maker of the formula (statement) O*p*: only statements (propositions) can be true and only prescriptions or imperatives can be obeyed.

It is also perhaps worth noting that we needn't assume that common prescriptive language games are well arranged and perspicuous. Their participants can switch their roles (i.e. from our perspective to make a step from one game to another) even within one speech act. There is, for example, nothing striking about a person saying to her mate "You are obliged to do *p*, but don't do it!" In the first part of the sentence the speaker obviously adopts the role of the Kibitzer, while in the second she takes the role of the Prescriber. Thus, she is not contradicting herself. It shouldn't be surprising that I will disregard such complexities of common communication within the present simplified model.

The Lewisian language games obviously have their logical aspect. It is natural to suppose that both the language of the Prescriber and the language of the Kibitzer are governed by certain logical rules. It should, for example, be determinable whether prescriptions issued by the Prescriber are coherent or whether she, at some point of the game, (logically) contradicts herself.

The list below consisting of the following two prescriptions can serve as an example of a record of a sample language game:

Game (1)
1. *Keep out of the cellar!*
2. *Don't smoke and don't whistle if you are inside of the house!*

It is quite clear that the two prescriptions issued by the Prescriber shape the relevant deontic situation. Without going into details, we can say that after the second step of the game the Doer clearly is not allowed to smoke inside of the house, go into the cellar, etc. So if the Kibitzer (partially) describes the situation with the sentence: "It is forbidden that the Doer smokes inside of the house" or – while talking to the Doer – "You must not smoke when you are in the house", she makes true claims, while if she says "You may whistle inside of the house", she makes a false claim.

We can, of course, also be interested in finding out which prescriptions (if any) are entailed by those issued (and hence valid) within Game (1). It seems obvious that the prescription "Don't smoke if you are inside of the house!" logically follows from the prescriptions that have been explicitly laid down (and hence is implicitly valid), while the prescription "Keep out of the kitchen!" is not valid – if the Prescriber issued it, she would change the deontic situation (the SP).

Let us now consider a somewhat more complex example:

Game (2)

1. *Always wear your anklet!*

2. *Don't smoke!*

3. *If you are in your bedroom or in the bathroom, don't wear your anklet!*

Once again it is quite clear that the three prescriptions issued by the Prescriber shape the relevant deontic state of affairs, but this time the situation is a bit more complex. The prescriptions 1 and 2 obviously shrink the SP while the last one seems to reshape it. But does it really? We can interpret the game in (at least) three different ways. The first of them is the most natural one – it views the game as a sequence of prescriptions that gradually shape the SP and assumes that the new prescriptions may supersede the previous ones. Under such an assumption, it is obvious that the prescription issued in the third move introduces an exception to the general requirement concerning the wearing of the anklet and replaces the original requirement by another one.[25]

We can, however, consider another interpretation that favours those prescriptions that are 'established' – if a new prescription collides with those that were introduced before it gets 'ignored'. This is not very plausible for games played during common communication but may be justified in specific, e.g., legal, contexts.[26] The third possible interpretation of the game neglects the order of the prescriptions. This seems unnatural in the case of oral language games but we should remember that the framework can (and should) be conceived quite generally: the role of the Prescriber can be played, for example, by an institution that sets - by a single 'normative act' - guidelines (e.g., house rules or library rules) for an addressee which is in the position of the Doer.[27] If Game (2) is conceived in this way it is indistinguishable from the game:

Game (2)*

1. *Don't smoke!*

2. *If you are in your bedroom or in the bathroom, don't wear your anklet.*

3. *Always wear your anklet!*

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[25] David Lewis straightforwardly adopts this conception of the game, see Lewis (1979b).

[26] Such a strategy is sometimes applied in law in cases where priority to the earlier acts over the later ones is granted for certain reasons (*lex prior derogat legi posteriori*).

[27] Typically, in such cases, we will have one Prescriber that addresses many Doers. We may either suppose that many 'parallel' games are being played in such a situation or that we are dealing with one game with a collective - distributively conceived - Doer. Classification of 'games' with various types of "sources and recipients" can be found in Rescher (1966, Chapter 2).

The numbers indexing the sentences are, in such a case, purely arbitrary and it would be better to remove them altogether - what we, in fact, are confronted with is a mere set of prescriptions.

If we appreciate the point of this observation, it is easy to notice that the 'static' interpretation of the game yields a problem - the prescriptions introduced during the (one step) game seem incoherent. At least *prima facie*, it is not clear what the Doer is supposed to do while staying in the bedroom or in the bathroom.[28] We may take it as an indication that as a whole the prescriptions in effect implicitly disallow entering the bedroom and the bathroom - a wise Doer should choose this strategy to avoid troubles that can stem from his failing to comply with all the prescriptions. Alternatively, we may insist that the prescriptions are in conflict with each other and search for a kind of resolution to the conflict. I will not try here to defend any position as concerns these issues and leave for later the discussion of how logical analysis should cope with (apparent?) incoherencies that may emerge in prescriptive codes.

Taxonomy of deontic logic

We are now approaching the central point of this article. I want to claim that if we adopt the perspective suggested by the language game we can distinguish different theories that can (and should) be developed within deontic logic. The theories will differ:

a) in their focus on the different kinds of linguistic means employed within the language game,

b) in their conceiving the language game either as static (all prescriptions are seen as issued simultaneously) or as dynamic (typically one prescription per step and new prescriptions 'surpass' old ones),

c) in the character of questions they are meant to answer.

Classifications of the theories studying the language games can, of course, be more or less fine-grained. Here I propose a classification that divides the theories that generally fall within the scope of deontic logic into six different categories:

StLoPr   The logical study aimed at recognizing which prescriptions are (implicitly) laid down (in a given factual state-of-affairs) if a Prescriber issues a *set* of prescriptions within a static language game.[29]

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[28] Of course, in the case of specific games some conflict-solving principles can be presupposed, e.g. adoption of the principle that more specific regulation is to be preferred against the general one (a version of the principle *lex specialis derogat legi generali*).

[29] Being (explicitly or implicitly) laid down is thus a status ('value') of a prescription within a language game. Lewis boldly treats prescriptions as having truth values. In my view, this is problematic not only because it is at

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DyLoPr   The logical study aimed at recognizing which prescriptions are (implicitly) laid down (in a given factual state-of-affairs) if a Prescriber issues a *sequence* of prescriptions within a dynamic language game.

StLoDo   The logical study aimed at recognizing how a *set* of prescriptions laid down by the Prescriber (in a given factual state-of-affairs) shapes the SP described in the Kibitzer's language.

DyLoDo   The logical study aimed at recognizing how a *sequence* of prescriptions laid down by the Prescriber (in a given factual state-of-affairs) shapes the SP described in the Kibitzer's language.

StLoKi   The logical study aiming at recognizing which sentences belonging to the Kibitzer's language are entailed by a *set* of sentences of the Kibitzer's language (in a given factual state-of-affairs).

DyLoKi   The logical study aiming at recognizing which sentences belonging to the Kibitzer's language are entailed by a *sequence* of sentences of the Kibitzer's language (in a given factual state-of-affairs).

Let us first explain the acronyms that I have just introduced as denominations of the individual kinds of theories. *StLoPr* stands for *static deontic logic of the Prescriber's language*, *DyLoPr* for *dynamic deontic logic of the Prescriber's language* and *StLoDo* for *static deontic logic for the Doer*.[30] Analogously, *DyLoDo* stands for *dynamic deontic logic for the Doer*, *StLoKi* for *static deontic logic of the Kibitzer's language* and *DyLoKi* for *dynamic deontic logic of the Kibitzer's language*.

What kind of problems do the particular theories address? Let us use the following game as a departure point for an illustration:

Game (3)

1. *Keep out of the cellar!*

2. *Don't smoke and don't whistle if you are inside of the house!*

3. *You may go to the cellar if a tornado is coming.*

4. *Stay inside of the house!*

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odds with common practice but, especially, because it obscures the crucial distinction between the descriptive language and the prescriptive language.

[30] The denomination suggests that this logic should yield answers for the Doer who lost track of the situation and wants to know whether he is allowed (obliged, forbidden) to take some course of action in a given stage of the game. It is natural to assume that the Kibitzer is the player who should be competent to employ the logic and provide the answers.

StLoPr theories view the game as static and they are supposed to provide answers to questions like: Are prescriptions like *Don't whistle if you are inside of the house!*, *Don't smoke!*, *You may stay inside of the house* or *You may go to the cellar* (implicitly) laid down by the set of prescriptions issued by the Prescriber? Are the individual prescriptions entailed by the 'premises' of Game (3)?

DyLoPr theories view the game as dynamic and they are expected to answer questions like: Are prescriptions like *Don't whistle if you are inside of the house!*, *Keep out of the cellar!* or *If you smoke, stay in the house!* (implicitly) laid down by the Prescriber after her last move? Are they, in this sense, entailed by the sequence of prescriptions that forms Game (3)?

StLoDo theories view the game as static and they are supposed to provide answers to questions like: Are the Kibitzer's statements such as *The Doer is obliged to stay in the house if a tornado is coming*, *It is forbidden that the Doer enters the cellar* or *If it is forbidden that the Doer whistles then it is forbidden that the Doer smokes* true (adequate) or false (inadequate) in the deontic situation formed by the set of prescriptions laid down in Game (3)?[31]

DyLoDo theories view the game as dynamic and they are supposed to give answers to questions like: Are the Kibitzer's statements such as *The Doer is obliged to stay in the house if a tornado is coming*, *It is not forbidden that the Doer smokes* or *If it is forbidden that the Doer whistles then it is forbidden that the Doer smokes* true (adequate) or false (inadequate) in the deontic situation shaped by the sequence of moves constituting Game (3).[32]

StLoKi theories view statements of the Kibitzer describing the game as static (forming a set) and they are designed to provide answers to questions like: Is the statement *The Doer is forbidden to smoke or the Doer is permitted to stay in the house* entailed by the (set of) statements *The Doer*

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[31] It is worth noting that the question of what the Doer is obliged to do can also be interpreted in a more specific way, namely as asking what the Doer is at a given moment and situation described by a set of factual 'premises' obliged (permitted) to do. In such a case, reports about different conditional obligations as well as other deontic statements that don't go to the point are irrelevant. The Kibitzer at the end of Game (3) is expected to just give 'direct advice' like "You are obliged to stay in the house", "You may go to the cellar" (in a situation where a tornado is coming) or "You may smoke" (in the - inadmissible - situation where the Doer is outside of the house).

[32] The logics for the Doer are obviously of a quite specific nature - they interconnect two different languages. Their 'inputs' are formulated in the language of the Prescriber while their 'outputs' are sentences belonging to the language of the Kibitzer.

is forbidden to go to the cellar, *The Doer is obliged to stay in the house*, *The Doer is forbidden to smoke if he is in the house*? We should, however, remember that the Kibitzer's language is quite rich and so also that arguments whose premises and conclusions don't have the character of straightforward reports about a deontic language game are legitimate subjects of inquiry. Thus a fully developed StLoKi should be, for example, able to provide a verdict concerning the validity of an argument with premises whose logical structure can be represented by the formulas $O(p \land q) \rightarrow Or$, $s \leftrightarrow \neg Or$ and the conclusion of the form $s \rightarrow (\neg Pq \rightarrow Pp)$.

DyLoKi theories view statements of the Kibitzer as describing a dynamic game (new statements bring new information reflecting the course of the game). They are supposed to provide answers to questions like: Is the statement *It is permitted that the Doer smokes or goes to the cellar* to be regarded as true in an information state shaped by the sequence of 'updating' statements 1) *The Doer is obliged to stay in the house*, 2) *The Doer is forbidden to smoke or to whistle if he is in the house* and 3) *It is permitted that the Doer goes into the cellar if a tornado is coming*?

At the end of this part in which I outlined the agenda of the individual theories it is perhaps worth devoting some attention to the question how the theories are - or should be - interrelated and how they are represented in the literature.[33] One thing appears rather obvious: any static language game can be seen as a limit case of a dynamic game. It might therefore seem that the problems addressed by StLoPr, StLoDo and StLoKi are in fact (to be) addressed by their dynamic counterparts and there is no need to present the static theories as specific items in the classification.[34] It is, however, easy to see that this kind of simplification wouldn't be beneficial - the static theories are both interesting and useful as such and the project of their formation and refining has a respectable history.

The question of the relation between StLoPr, StLoDo and StLoKi on the one hand, and DyLoPr, DyLoDo and DyLoKi on the other, is a more delicate one. As we have seen the differences between the languages (moves) of the Prescriber and the Kibitzer are so serious that they preclude any straightforward parallelism between the respective logical theories. Putting it somewhat crudely, we can say that the choice of LoPr does not determine the choice of LoKi and

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[33] Of course, due to the literature's richness the survey will inevitably be sketchy.

[34] Things, however, are not so simple. Dynamic games can be designed so that they don't allow for making more moves 'at once'. Also, it may be reasonable to conceive the scope of DyLoKi so that statements which are not suitable as straightforward descriptions of a deontic situation (such as $(Op \lor Oq) \leftrightarrow O\neg r$ or $Op \rightarrow \neg s$) don't figure in the updating sequences. In such a case, the problems addressed by StLoKi are not automatically 'covered' by DyLoKi.

vice versa.[35] The situation, however, gets different if the theories which I call StLoDo and DyLoDo (and which might also be called theories of *the SP shaping* or, in the dynamic case, theories of *normative kinematics*[36]) enter the scene. A triplet of logical theories belonging to the StLoPr, StLoDo and StLoKi categories can surely be harmonious or disparate. They are harmonious if StLoDo appropriately 'brings together' the other two theories and we thus receive a coherent 'full picture' of the logical side of our prescriptive language games. We might say that deontic logic should aim at forming such an ideal complex theory but ideals, as we know, might be delusive and it is often better to focus on manageable, practically-oriented projects first.

In scholarly literature we can find an abundance of well-developed logical systems that fall under our StLoKi label.[37] Logical studies that might be characterized as belonging to the category of DyLoKi are scarce in the literature.[38] To develop them would mean extending the area of the so called *belief revision theories* in such a way that they would capture the specific problems connected with the dynamics of the 'knowledge bases' of agents who process the updated information concerning development of normative language games formulated in the Kibitzer's language. Problems of this kind are admittedly somewhat marginal, though studies aimed at updating the knowledge by modal sentences of this specific sort might yield interesting results.

Unsurprisingly, the literature discussing logical issues which fall into the StLoPr category is extensive. Hare (1949), Sosa (1966) and Rescher (1966) can be mentioned as representatives of the less formal and more philosophical approach to the field of study. Recent important contributions in this area have been presented, for example, by Peter Vranas (2010, 2011) and Nate Charlow (2014). The contributions mentioned above have one common feature - they disregard permission. This, however, is not to be seen as their essential shortcoming. The role of permissions within static language games is limited.[39] With some simplification, we can say that the role of permissions appearing in a set of prescriptions issued by the Prescriber is either close to trivial or 'destructive'. If a permission issued by the Prescriber is not in conflict with any 'strong' prescription, then it is idle in the sense that it allows - thanks to the fact that in the

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[35] Someone can, for example, conclude that while StLoKi should employ a *monadic* language, StLoPr requires a *dyadic* language.

[36] This term originates, as far as I know, with Lewis (1979b).

[37] Though, as I suggested, quite a large number of the theories had bigger ambitions since their creators often wanted to constitute a 'universal' deontic logic capturing logical properties of "ought sentences" all at once in both their descriptive and prescriptive interpretations.

[38] Alchourrón-Makinson (1981), Meyer (1988), Brown (2004), Lindström-Segerberg (2006), or Stolpe (2010) can be seen as contributions to this area.

[39] This doesn't hold if we consider complex systems with hierarchized prescriptions. For discussion of such systems compare, e.g., Bulygin (1986), Hansen (2006b, 2006c).

beginning of the game the SP is identical with SA - what is allowed anyway.[40]

The role of permissions within the dynamic language games is much more important. As far as I know, no developed theory that would fall into the DyLoPr category has been presented.[41] This might be due to the fact that the problems are too difficult to capture or, perhaps, they just haven't attracted enough attention yet.

Logical theories of the StLoDo and DyLoDo types are close to non-existent. The most prominent theory that can be classified as belonging to the StLoDo category is Makinson's (1999) account of the iterative development of the output of an explicitly presented normative code. Makinson elaborates on ideas presented in Alchourrón-Bulygin (1981) and Alchourrón (1993) and defines the concept of the *output* of a normative code (under certain conditions). A normative statement of the form $O(\alpha/\beta)$, i.e. a statement formulated in the Kibitzer's language, is then classified as true with respect to a given normative code C if and only if α belongs to the output of the code given the condition β ($\alpha \in out(C,\beta)$). The theory yields interesting and plausible results. However, it would seem that Makinson didn't fully appreciate the specific qualities of the theory that he had created.[42]

As concerns DyLoDo - the classic Lewis article pointing to the problems of the kinematics of prescriptions represents a ground breaking publication which clearly shows that we are lacking a good (or we might even say any) theoretical grasp of the logical principles governing very important kinds of language games. In particular, Lewis notes that the import of permissions in such games is quite elusive and we lack a reasonable account of the 'kinematics' of prescriptive language. This might seem like a marginal problem but, in fact, it is a problem as urgent as a logical problem can be. In spite of the valuable insights that Lewis' article has provided, its results are to a large extent only negative. As far as I know, the papers by Belzer (1985a) and (1985b), and Childers – Svoboda (1999) and (2003) are the only attempts at a systematic solution to (a part of) the problems articulated by Lewis.[43]

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[40] We might, of course, consider other backgrounds of the game than the liberal one introduced by Lewis. For basic considerations concerning this possibility see Svoboda (2003).

[41] Some of these issues are, in a specific form, discussed from the perspective of jurisprudence (recently, e.g., Governatori-Rotolo (2010). Other relevant ideas can be found in Boella-van der Torre (2008) or Broersen-Gabbay-van der Torre (2012).

[42] This gets manifested in the further development of his work. In articles by Makinson and van der Torre on Input-Output logic they conceive the logic as "the logic of conditional norms" (see Makinson-van der Torre (2003a, p. 163nn)). As the norms are not truth-evaluable it seems proper to classify their logic as StLoPr. The logic, however, has features that are problematic. The Ross paradox, for example, turns out to be a valid inference pattern of the logic, and weak (negative) permissions are treated as kinds of implicit norms (see Makinson-van der Torre (2003b)).

[43] Belzer bases his solution on the assumption that possible worlds are (weakly) ordered - some are more highly

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Ross paradox

I will now try to demonstrate that the perspective opened by the taxonomy which I have just presented yields illuminating insights by providing an analysis of one of the most prominent controversial issues that has been discussed in 'deontic literature' for many decades - the *Ross paradox*. In the early forties, Alf Ross pointed at a 'paradoxical' consequence of the so called *Dubislav trick* or *Dubislav convention*. The 'trick' took its name from the German logician and philosopher Walter Dubislav, who defended the idea that there are logical relations among imperatives. As a part of his argumentation, he suggested that logical relations among imperatives are parallel to logical relations among those indicatives that describe the states of affairs that obtain if the imperatives are obeyed (see Dubislav (1937)). Thus, anybody who can identify classical logical relations can indirectly pinpoint logical relations among imperatives.

Ross spotted a controversial consequence of Dubislav's proposal. In modern literature, the problem pointed out by Ross is usually presented as a question as to whether the following inference pattern (valid by the Dubislav convention) should be accepted as valid within a reasonable system of deontic logic:

RP $\quad \underline{O\alpha}$

$\quad\quad O(\alpha \vee \beta)$

Though logicians typically admit that some inferences of the form, e.g., the following notorious instance ascribed to Ross

RP* $\quad$ *You should mail the letter!*

$\quad\quad$ *You should mail the letter or burn it!*

sound peculiar or even paradoxical, in general most of them adopt the inference pattern as valid.[44] The disputes concerning the (alleged) validity of RP and (alleged) paradoxicality of RP* seem endless.[45] Let us now look at the problem from the perspective of our model language game and consider the following two very simple cases:

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ranked (more permissible).

[44] See, e.g., von Wright (1967), Føllesdal- Hilpinen (1971), Åqvist (2002).

[45] Cf., e.g., Ross (1941), Hare (1967), Meyer-Dignum-Wieringa (1994), Danielsson (2005), Hansen (2006a), Straßer –Beirlaen (2012).

Game (4)

1. *Mail the letter and the postcard!*   (!$(m \wedge p)$)

2. *Mail the letter!*   (!m)

Game (5)

1. *Mail the letter!*   (!m)

2. *Mail the letter or burn it!*   (!$(m \vee b)$)

If we conceive the games as being dynamic, it is natural to ask whether anything substantial happened in their second steps. In the case of Game (4), it is quite clear that the SP shaped by the first prescription was not affected by the prescription issued in the second step. Thus, it is natural to conclude that the second prescription was implicitly contained in the previous prescription (and reasonable systems of DyLoPr should allow us to recognize the logical relation). In the case of Game (5), the situation is different – if the Prescriber decided to issue prescription 2 after 1, she obviously must have changed her intentions.[46] The new prescription clearly opens a choice for the Doer and thus implicitly authorizes him to burn the letter. (The Prescriber who would, at the end of the game, accuse the Doer, who burned the letter, of violating her commands would sound quite unreasonable.)[47]

It is thus clear that the prescription !$(m \vee b)$ is not (implicitly) laid down by the Prescriber who has issued the single prescription !m. This observation from the dynamic game can also be naturally adopted as a guideline for StLoPr. It is quite obvious that to adopt the inference pattern

RPP $\underline{!\alpha}$

     !$(\alpha \vee \beta)$

as valid in logical theories belonging to the DyLoPr category is a bad idea.[48]

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[46] It is presumable that at the moment of issuing the second prescription she knows or assumes that the Doer has not managed to fulfil the original order.

[47] More controversial is the question whether all cases when a Prescriber issues a weaker prescription in the situation when she previously issued a stronger one (for example *Don't speak loudly!* after *Don't speak!* or *Stay in Ireland until you finish your exams!* after *Stay in Dublin until you finish your exams!*) should be generally interpreted as moves expanding the SP (and whether such moves should be seen as testifying that the Prescriber must have changed her intentions).

[48] It is perhaps worth noting that Rescher (1966) admits that the inferential step might be admissible if the form !$(p \vee q)$ represents what he calls *alternative-indicating command* (and not, as is standard, a *choice-presenting*

Let us now turn our attention to the inference pattern RP which - in the disambiguated notation - represents the formulation of the problematic inference in the Kibitzer's language. If we want to make the situation clear, it is recommendable to avoid the somewhat ambiguous natural language instances like RP* and focus on an unequivocal formulation like:

RP$^{\#}$ *(According to the Prescriber's instructions) you are obliged to mail the letter.*
  (According to the Prescriber's instructions) you are obliged to mail the letter or burn it.

This disambiguation, however, does not yield any univocal answer to the question whether RP should be accepted as a valid inference pattern in the systems belonging to StLoKi. As far as I know there hasn't been any research done that concerns common speakers' opinions concerning the validity of inferences like RP$^{\#}$ but, *prima facie*, it seems likely that the intuitions would turn out to be rather weak and inconclusive.

We can, however, get at least some intuitive grasp of the logical features of the expressions similar to those occurring in RP$^{\#}$ if we consider simple situations arising within the model language games. Let us consider an extremely simple language game in which the Prescriber addresses the Doer with a single command:

Game (6)
1. *Walk the dog!*   ($!w$)

It is easy to imagine a Doer who was for some reason distracted and hasn't comprehended what he was asked to do. Fortunately, the Kibitzer is around so the Doer can ask her what it is he is supposed to do. Let us now consider four alternative variants of the possible answers given by the Kibitzer:

a) *You are obliged to clean the kitchen,*   (Oc)
b) *You are obliged to walk the dog,*   (Ow)
c) *You are obliged to clean the kitchen or you are obliged to walk the dog,* ($Oc \lor Ow$)
d) *You are obliged to clean the kitchen or walk the dog.*   ($O(c \lor w)$)

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command). This is not, however, a serious proviso as his alternative-indicating commands present rather marginal cases in which "or" is used to introduce a 'punishing alternative' (e.g. "Shut up or get out of here!").

How do these answers fare as concerns their correctness and adequacy? I have suggested that theories of the StLoDo type are those which should offer an authoritative answer to this question. Though I haven't presented any particular theory of this kind, it is in my view obvious that the first answer is incorrect and similarly obvious that the second one is correct.[49] But what about the other answers? It is quite clear that if the Kibitzer presents c) as her answer she does not tell a lie. In this sense, the answer can be accepted as true. We, however, would not expect this kind of reaction from a reasonable and trustworthy Kibitzer as it does not provide the Doer with the required information. So in this sense what she says is true but inadequate.[50]

What about the last answer? It is patently inadequate and not very helpful, but is the claim of the Kibitzer false? Let us mark this question, i.e. the question: "Is the Kibitzer who answers the Doer's question concerning Game (6) by saying *You are obliged to clean the kitchen or walk the dog* making a false claim?" as question **Q1**. Before we will try to respond to it let us ponder a somewhat different situation that arises if the only prescription issued by the Prescriber is the following:

Game (7)

1. *Clean the kitchen or walk the dog!*   ($!(c \vee w$)[51]

As previously, we may suppose that the Doer has been absent-minded and asks the Kibitzer what he should do. Let us, once again, consider the same answers a) - d). How do they fare now? The answers a) and b) are quite clearly incorrect (false). Though they provide information that is 'safe' in the sense that the Doer who will trust the Kibitzer is sure not to disappoint the Prescriber, they present a wrong picture of the deontic situation.[52]

What about answer c)? Here the situation is perhaps not crystal clear, but still our intuitions are (I suggest) rather convincing – the answer is not only inadequate in the sense that it violates common conversational maxims, but what the Kibitzer says is false. It suggests that the Prescriber requires that the Doer fulfils a certain, specific task, which is not the case - the Doer has a choice. What about the answer d)? It is, no doubt, the best of the four options under

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[49] We will surely require that any respectable StLoDo or DyLoDo theory yields this result.

[50] Her answer breaches conversational maxims discussed by Paul Grice (see Grice (1975)).

[51] It is perhaps worth stressing once again that 'formula' $!c \vee !w$ does not come into consideration as an expression of the Prescriber's language even though it seems to faithfully reflect the form of the sentence.

[52] This would be even more obvious if the two actions in question were clearly incompatible (e.g., *Stay in London or go to New York*). Those who would defend the two answers as admissible would be forced to claim that two incompatible statements can be true at the same time.

consideration. But is it the appropriate answer that a perfect Kibitzer should choose? Let us mark this question as question **Q2**.

It is not difficult to see that the answer to this question is intertwined with the answer to question **Q1**. If we adopt the negative answer to **Q1** we should choose the negative answer to **Q2** and vice versa. Why? Adhering to the negative answer to **Q1** (i.e. admitting that the answer is *not* false) indicates that we interpret the sentence "You are obliged to clean the kitchen or walk the dog" (and the formula $O(c \lor w)$) as bearing information that is strictly weaker than the information provided by the sentence "You are obliged to walk the dog" (Ow). Thus the logical relation between Ow and $O(c \lor w)$ is analogous to the relation between 'normal' formulas q and $(p \lor q)$. This, however, is not the only plausible reading of the 'disjunctive' sentence of the Kibitzer's language. We may read it as synonymous with the sentence "You are obliged to clean the kitchen or walk the dog, the choice is yours". If we adhere to this stronger reading of the sentence (and of the formula $O(c \lor w)$, then the answer to **Q1** obviously must be positive - the statement d) is then false in the context of Game (6). The Prescriber who requires that the Doer walks the dog surely does not ask the Doer to choose (at least) one of the two courses of action. On the other hand, the answer to **Q2** should clearly be positive too – the 'disjunctive' claim of the Kibitzer then manifestly provides an adequate description of the deontic situation formed within Game (7).

What is the moral of the deliberations that have just been presented? The moral is that the adherence to a logical system belonging to StLoKi (or DyLoKi) always implies a choice. If the RP inference scheme is adopted as valid within the system we have no other choice but to adopt the weak interpretation of sentences like "You are obliged to clean the kitchen or walk the dog" (and of formulas like $O(c \lor w)$). The obvious drawback of this choice is that the strong reading seems somewhat more natural (especially if we presume that the person using the sentence (the Kibitzer) is well informed and respects conversational maxims). But is it a serious problem?

The choice can perhaps be compared to the dilemma that we face when choosing between classical logic and the logic of strict implication. It is hardly debatable that the common meaning of sentences like "If it rains the streets are wet" is more adequately captured by the formula $p \prec q$ of the logic of strict implication than by the 'classical' formula $p \rightarrow q$. But this does not mean that we should automatically go for the logic that offers the formalization that is more faithful to common understanding of natural language conditionals. When choosing between logical theories we may (and very often do) swallow some deviations from natural language if the

shortcoming is compensated by something else, e.g., by the simplicity and elegance of the system.

We may thus conclude that adoption of a system of StLoKi in which RP is valid isn't more controversial than adopting classical logic with its 'paradoxes' that concern material implication. If we look at the relevant literature it is obvious that most of the systems of deontic logic that fit into the StLoKi category accept RP as valid. Doesn't this choice substantially limit the expressive force of the artificial (formal) language? In particular – can the Kibitzer who makes use of the language presupposing the weak reading of $O(p \vee q)$ describe the deontic situation which arises within Game (7) adequately? The answer appears to be positive – the formula

KFC $O(c \vee w) \wedge Pc \wedge Pw \wedge P\neg c \wedge P\neg w$

captures the deontic state of affairs satisfactorily. We can say that this formula fairly accurately represents the logical form of the Kibitzer's sentence "You are obliged to clean the kitchen or walk the dog, the choice is yours".[53] At this point it is perhaps worth stressing that, by saying this, I don't mean to suggest that we should introduce a specific operator suited for articulation of sentences about 'free-choice obligations' into the formal language of StLoKi.[54] We should be satisfied with the fact that the Kibitzer employing the language of standard deontic logic can adequately (although somewhat clumsily) describe the deontic state of affairs which arises when the Prescriber issues a free-choice command.

What happens if we adhere to a language presupposing the strong reading of formulas like $O(p \vee q)$ and $P(p \vee q)$? The consequences are going to be quite serious. First - the Kibitzer won't be able (unless her language is enriched by another 'disjunctive junction') to articulate statements that correspond to the weak reading of the formula. Second - we will have to give up on the extensionality of the language of StLoKi (e.g., the language of standard deontic logic). The reason why is simple: If the formulas $O(p \vee q)$ and $P(p \vee q)$ are to be interpreted as statements suited to describing a free-choice command situation, resp. a free-choice permission

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[53] The meaning of this sentence is not adequately captured by the formula $O(p \vee q)$ employing Brown's Type 2 interpretation of the operator O, which is introduced with the intention to avoid the Ross paradox. Brown's Type 2 formula $O(p \vee q)$ can be true even if there is no freedom of choice (cf. Brown (1996)).

[54] Hansson (2013) shows that the straightforward introduction of a free-choice operator P_c into the language of standard deontic logic by the definition $P_c(p \vee q) \leftrightarrow Pp \wedge Pq \wedge P\neg p \wedge P\neg q$ proposed in Woleński (1980) has clearly unacceptable consequences, and it is obvious that introduction of a free-choice obligation operator by the definition $O_c(p \vee q) \leftrightarrow O(p \vee q) \wedge Pp \wedge Pq \wedge P\neg p \wedge P\neg q$ would yield similar problems. Generally Hansson convincingly demonstrates that the free choice interpretation of formulas like $P(p \vee q)$ is untenable under standard extensional interpretation of disjunction.

situation, then the same must of course hold for formulas O$(p \vee \neg p)$ and P$(p \vee \neg p)$. As $(p \vee \neg p)$ and $(q \vee \neg q)$ are logically equivalent, the pairs of formulas O$(p \vee \neg p)$, O$(q \vee \neg q)$ and P$(p \vee \neg p)$, P$(q \vee \neg q)$ must be too if we assume that logically equivalent formulas appearing in the scope of the deontic operators are intersubstitable. But that would mean that whenever it is true that the Doer is allowed to choose between doing and not doing a certain action (e.g., the freedom of wearing socks or nor wearing socks is granted to him), then it should be true that any other action is similarly indifferent (e.g., the freedom of killing neighbours or not killing neighbours is granted). Clearly, giving up on extensionality is the only reasonable way out of this problem. These deliberations clearly suggest that giving preference to the strong reading has far reaching consequences. Thus, I conclude that accepting RP as valid within systems of StLoKi (i.e. adhering to the weak reading of O$(p \vee q)$) is *prima facie* a reasonable choice.

It is not difficult to see that the framework of the deontic language games can provide useful insights into other problems that have vexed deontic logic for a long time, such as the problem of free choice permission and the problem of the distinction between permission as an absence of prohibition (negative permission) and strong (positive) permission. Let us first briefly consider the second topic.

Can the Prescriber express a negative permission in her language? The answer is clearly negative. Claims to the effect that a certain action is not prohibited (under given conditions) belong only to the Kibitzer's repertoire and thus negative permissions don't play a part in either StLoPr or DyLoPr. We should, however, notice that a kind of weak permitting is present in the deontic game. It is hidden in the assumption that, in the beginning of the game, the SP is identical with SA - this assumption establishes what might be called a *liberal background* of the game. Presupposing this kind of background may seem natural but it is not inevitable. In fact, common deontic games typically take for granted a different kind of background which might be called *pragmatic* or *ethico-legal*.[55] Even if the Prescriber doesn't explicitly require "Don't kill me!" or "Don't cut my hair while I am asleep!" it is normally assumed that the Doer is not permitted (not even tacitly or 'negatively') to murder the Prescriber or cut her hair. Games with this kind of realistic background, however, have so many 'unknowns' that their systematic logical analysis by means of StLoDo or DyLoDo is inherently problematic. Thus, the common practice of assuming a liberal background within logical considerations is reasonable.

It is, however, worth noting that no particular background needs to be assumed. An analogous setting can be, if needed, established within the game. For example, a specific version

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[55] For more about the possible backgrounds of the game see Svoboda (2003).

of the liberal 'background' is established if, during the game, the Prescriber issues a permission like "You may do anything (you want), unless I will tell you otherwise". Clearly, all actions of the Doer which haven't been prohibited by other of the Prescriber's moves are implicitly (positively) permitted by the move. Negative (tacit) permissions then completely disappear from the picture - not only at the level of StLoPr but also on the level of StLoDo and StLoKi.

Let us now say a few words about free choice permissions. It is hardly contentious that 'disjunctive' permissive sentences belonging to the language of the Prescriber (i.e. sentences of the form ¡$(p \vee q)$ or ¡$(p \vee q)/r$) as, for example, "(If you promise to be careful) you may borrow either the big or the small crowbar") are always a choice offering.[56] It is also quite clear that they are - together with free-choice commands - very useful (if not indispensable) tools of the Prescriber's language. Pairs of formulas ¡$(p \vee q)$ and !$\neg p$ as well as ¡$(p \vee q)$ and !p are thus to be treated as inconsistent by any reasonable system of StLoPr (they should not be simultaneously in force in a static game in which prescriptions are not hierarchically ordered).[57]

The situation is quite different as it regards the language of the Kibitzer. The sentences "You may borrow either the big or the small crowbar, but I don't know which" or "You may borrow either the big or the small crowbar, try to guess" seem perfectly sensible if the speaker is in the position of the Kibitzer and not in the position of the Prescriber. Thus, when we are building systems of the StLoKi category there is no serious reason why we should treat the pair of formulas P$(p \vee q)$ and O$\neg p$ as inconsistent.[58] Whether it makes sense to enrich the formal version of the Kibitzer's language so that it allows for a straightforward formulation of 'disjunctive' sentences that are choice guarantying is an open question. I am strongly inclined to think that such a project is not worth the trouble.

To sum up: It is a clear desideratum for any reasonable logic of the Prescriber's language that RPP *is not* a valid inference scheme. In the case of logical theories examining the language of the Kibitzer, everything suggests that RP *should* be adopted as a valid inference scheme. This observation (desideratum) concerning reasonable StLoKi theories can also be schematically

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[56] For a detailed linguistic substantiation of this claim, see Portner (2012).

[57] Admittedly inconsistencies arising between a strong prescription (an order) and a weak prescription (a permission) are not as 'urgent' as inconsistencies between two strong prescriptions. (Normally we tend to presume that orders 'trump' permissions.) This, however, surely doesn't mean that we should conclude that any permission is consistent with any order.

[58] We can imagine a somewhat mischievous Kibitzer saying "You may borrow either the big or the small crowbar, try to guess which one." And then, a little later: "I will give you a hint - you must not borrow the big one". Any reasonable Doer would, of course, quickly conclude that he is allowed to borrow the small crowbar.

represented by the following schemes pertaining to StLoDo in which "▼" represents a correct 'inference' from a prescription to a deontic statement:[59]

RP# $!p$ $!q$ $!(p \vee q)$ $!(p \vee q)$

    ▼   ▼   ▼   ▼

  $O(p \vee q)$ $O(p \vee q)$ $O(p \vee q)$ $Pp \wedge Pq$

Clearly, the sets of formulas $\{O(p \vee q), Op\}$ and $\{O(p \vee q), O\neg p\}$ are consistent under the weak reading of $O(p \vee q)$ and inconsistent under the strong reading. Sets of prescriptions $\{!(p \vee q), !p\}$ and $\{!(p \vee q), !\neg p\}$ are, I suggest, to be treated as inconsistent within respectable systems of StLoPr. Any Prescriber who simultaneously offers a choice and excludes it by either enforcing or banning one of the alternatives is incoherent.[60]

Conclusion

I believe that the arguments presented in this article convincingly show that: i) The approach to deontic logic promoted by Stenius is inappropriate and potentially confusing. Though it is not, perhaps, strictly impossible to build a logical system whose sentences can be seen as double-faced, such a strategy is far from reasonable – the point of logical analysis is to remove confusions, not to replicate and reinforce them.[61] ii) Viewing deontic logic through the prism of Lewisian language games is illuminating as it allows us to see some old problems -- like the Ross paradox or the free-choice permissions/commands -- in a new light and opens up a way toward setting an agenda for different branches of studies that are naturally subsumed under the heading "deontic logic". iii) This proposed taxonomy of deontic logic (though it is not, of course, the only possible one) can provide a firmer ground for a confrontation of different theories and stimulate new research. In particular, it points to the need to develop theories of the StLoDo and DyLoDo types that are missing from the literature in spite of their obvious crucial relevance from a practical point of view.

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[59] Roughly - if the prescription in the 'premise' is laid down in a context, the statement in the 'conclusion' is true in the context.

[60] Of course, in the case of a specific game (meta)rules might be introduced that would 'instantly' remove the conflict.

[61] Someone might suggest that moral judgements are, by their very nature, double-faced (both descriptive and prescriptive) and we should try to develop a specific logic that respects this. I don't find such a view convincing. First, I think that it is important to clearly distinguish when one is making a claim about morals or about some particular ethical code and when one articulates a moral prescription. Second, I believe that ethics (resp. metaethics) as well as law (resp. jurisprudence) are areas in which one can profit from making use of the analytic tools developed within deontic logic. They, however, should be viewed as areas in which logic is applied rather than as areas that are governed by their own logics.

One of the most interesting topics for future study is also the question of mutual relationships among the different kinds of logical theories, both on a general level and among the individual theories which fall into the above-mentioned categories. The most interesting issue here is how the different theories 'fit together' and to what extent they can, together, provide a comprehensive picture of the logical aspects of common kinds of deontic language games.

REFERENCES

Ågotnes, T. – Broersen, J. – Elgesem, D. (eds) (2012). *Proceedings of the 11th International Conference on Deontic Logic in Computer Science (DEON 2012)*. Berlin, Springer.

Alchourrón C. (1969). Logic of Norms and Logic of Normative Propositions. *Logique & Analyse*, *12*, 242-268.

Alchourrón C. - Bulygin, E. (1971). *Normative Systems*. Vienna-New York, Springer-Verlag.

Alchourrón C. - Bulygin, E. (1981). The Expressive Conception of Norms. In Hilpinen (ed.) (1981), 95-124.

Alchourrón, C. (1993). Philosophical Foundations of Deontic Logic and the Logic of Defeasible Conditionals. In Meyer & Wieringa (1993), 43–84.

Alchourrón C. - Bulygin, E. (1993). On the Logic of Normative Systems. In H. Stachowiak (ed.), *Pragmatik: Handbuch pragmatischen Denkens*. Band IV. *Sprachphilosophie, Sprachpragmatik und formative Pragmatik*. Hamburg, Felix Meiner Verlag, 273-94.

Alchourrón C. - Makinson, D. (1981). Hierarchies of Regulations and their Logic. In Hilpinen (ed.) (1981), 95-124.

Åqvist, L. (2002) Deontic Logic. In D. Gabbay and F. Guenthner (eds) *Handbook of Philosophical Logic*. 2nd ed., Dordrecht, Kluwer, 147-264.

Beirlaen, M. - Straßer, Ch. (2013). Two Adaptive Logics of Norm-Propositions, *Journal of Applied Logic*, *11*, 147-168.

Belzer, M. (1985a). Normative Kinematics (I): A Solution to a Problem about Permission, *Law and Philosophy*, *4*, 257-287.

Belzer, M. (1985b). Normative Kinematics (II): the Introduction of Imperatives. *Law and Philosophy*, *4*, 377- 403.

Boella, G. - van der Torre, L. (2008). Institutions with a Hierarchy of Authorities in Distributed Dynamic Environments. *Artificial Intelligence in Law*, *16*, 53-71.

Broersen, J. - van der Torre, L. (2012). Ten Problems of Deontic Logic and Normative Reasoning in Computer Science. In *Lectures on Logic and Computation Vol. 7388*, Springer, 55-88.

Broersen, J. – Gabbay, D. – van der Torre, L. (2012). *Changing Norms is Changing Obligation Change*. In T. Ågotnes – J. Broersen – D. Elgesem (eds), 199-214.

Brown, M. (1996). Doing As We Ought: Towards A Logic of Simply Dischargeable Obligations. In M. Brown & J. Carmo (eds) *Deontic Logic, Agency and Normative Systems Workshops in Computing*, Springer, 47-65.

Brown, M. (2004). Rich Deontic Logic: A Preliminary Study. *Journal of Applied Logic*, *2*, 19–37.

Bulygin, E. (1986). Permissive Norms and Normative Concepts. In A.A. Martino, F. Socci Natali (eds) *Automated Analysis of Legal Texts*, Amsterdam, North Holland, 211–218.

Charlow, N. (2014). Logic and Semantics for Imperatives. *Journal of Philosophical Logic*, *43*, 617-664.

Childers, T. - Svoboda,V. (1999). On the Kinematics of Permission. In T. Childers (ed.) *Logica Yearbook 1998*, Prague, Filosofia, 185-203.

Childers, T. - Svoboda, V. (2003). On the Meaning of Prescriptions. In J. Peregrin (ed.) *Meaning: The Dynamic Turn*, Oxford, Elsevier, 185-200.

Cornides T. (1969). Der Widerruf von Befehlen. *Studium Generale*, *22*, 1215-1263.

Danielsson, S. (2005). Taking Ross's Paradox Seriously. A Note on the Original Problems of Deontic Logic. *Theoria*, *71*, 20–28.

Dignum F. - Meyer J.-J.Ch. - Wieringa R.J. (1996). Free Choice and Contextually Permitted Actions. *Studia Logica*, 57, 193-220.

Dubislav, W. (1937). Zur Unbegründbarkeit der Forderungssatze. *Theoria*, *3*, 330-342.

Fine, K. (2014). Permission and Possible Worlds. *Dialectica*, *68*, 317–336.

Føllesdal, D. - Hilpinen, R. (1971). Deontic Logic: An Introduction. In Hilpinen (ed.) (1971), 1–35.

Gabbay D. et al. (eds) (2013). *Handbook of Deontic Logic and Normative systems*, London, College Publications.

Governatori, G.- Rotolo, A. (2010). Changing Legal Systems: Legal Abrogations and Annulments in Defeasible Logic. *Logic Journal of the IGPL*, *18*, 157–194.

Grice, P. (1975). Logic and Conversation. In P. Cole, J. Morgan (eds) *Syntax and Semantics. 3: Speech Acts*, New York, Academic Press, 41–58.

Grelling, K. (1939). Zur Logik der Sollsaetze. *Unity of Science Forum*, 44-47.

Hansen, J. (2006a). The Paradoxes of Deontic Logic: Alive and Kicking. *Theoria*, *72*, 221-232.

Hansen, J. (2006b). Deontic Logics for Prioritized Imperatives. *Artificial Intelligence and Law*, *14*, 1-34.

Hansen, J. (2006c). Prioritized Conditional Imperatives. *Autonomous Agents and Multi-Agent Systems*, *17*, 11-35.

Hansen, J. (2013). Imperative Logic and Its Problems. In Gabbay D. et al. (eds) (2013), 137-191.

Hansen, J. - Pigozzi, G. - van der Torre, L. (2007). Ten Philosophical Problems in Deontic Logic. In G. Boella, L. van der Torre, H. Verhagen, (eds), *Normative Multi-Agent Systems*, Schloss Dagstuhl, 55-88.

Hansson, S.-O. (2013). The Varieties of Permission. In D. Gabbay et al. (eds) (2013), 195-240.

Hare, R. M. (1949). Imperative Sentences. *Mind*, *58*, 21-39.

Hare, R. M. (1967). Some alleged differences between imperatives and indicatives. *Mind, 76,* 309-326.

Hart, H. L. A. (1961). *The Concept of Law,* Oxford, Oxford University Press.

Hilpinen, R. (ed.) (1971). *Deontic Logic: Introductory and Systematic Readings*, Dordrecht, D. Reidel.

Hilpinen, R. (ed.) (1981). *New Studies in Deontic Logic*, Dordrecht, D. Reidel.

Hilpinen, R. (2006). Norms, Normative Utterances and Normative Propositions. *Análisis Filosófico*, *26*, 229–41.

Hofstadter, A. - McKinsey, J.C.C. (1939). On the Logic of Imperatives. *Philosophy of Science*, *6*, 446-457.

Jørgensen, J. (1937–38). Imperatives and Logic. *Erkenntnis*, *7*, 288–296.

Kaufmann, M. (2012). *Interpreting Imperatives*, Dordrecht, Springer.

Kelsen, H. (1991). *General Theory of Norms*, Oxford, Clarendon Press.

Knuuttila, S. (1981). The Emergence of Deontic Logic in the Fourteenth Century. In Hilpinen, R. ed. (1981), 225–248.

Lewis, C.I. (1918). *A Survey of Symbolic Logic*, Berkeley, University of California Press.

Lewis, C.I. (1920). Strict Implication – an Emendation. *The Journal of Philosophy and Scientific Method*, *17*, 300-302.

Lewis, D. (1979a). Scorekeeping in a Language Game. *Journal of Philosophical Logic*, *8*, 339-359.

Lewis, D. (1979b). A Problem about Permission. In E. Saarinen, R. Hilpinen, M. Provence–Hintikka (eds), *Essays in Honour of Jaakko Hintikka*, Dordrecht, D. Reidel, 163-175.

Lindström, S. - Segerberg, K. (2006). Modal Logic and Philosophy. In P. Blackburn, J. van Benthem, F. Wolter (eds) *Handbook of Modal Logic* (*Studies in Logic and Practical Reasoning: 3*), Amsterdam, Elsevier, 1153–1218.

Makinson, D. (1999). On a Fundamental Problem of Deontic Logic. In McNamara- Prakken (eds) (1999), 29-53.

Makinson, D. - van der Torre, L. (2003a). What is Input/Output Logic? In B. Löwe, W. Malzkorn, T. Räsch (eds) *Foundations of the Formal Sciences II, Applications of Mathematical Logic in Philosophy and Linguistics*, Dordrecht, Kluwer, 163-174.

Makinson, D. - van der Torre, L. (2003b). Permission from an Input/Output Perspective. *Journal of Philosophical Logic*, *32*, 391-416.

Mally, E. (1926). *Grundgesetze des Sollens. Elemente der Logik des Willens*, Graz, Leuschner & Leubensky.

McNamara, P.- Prakken, H. (eds) (1999). *Norms, Logics and Information Systems: New Studies in Deontic Logic and Computer Science*, Amsterdam, IOS Press.

Menger, K. (1939). A Logic of the Doubtful. On Optative and Imperative Logic. *Reports of a Mathematical Colloquium* 2, Indiana University Press, 53-64.

Meyer, J.-J. Ch. (1988). A Different Approach to Deontic Logic: Deontic Logic Viewed as a Variant of Dynamic Logic. *Notre Dame Journal of Formal Logic*, *29*, 109–136.

Meyer, J.-J. Ch. and Wieringa, R.J. (1993). *Deontic Logic in Computer Science*, Chichester, John Wiley and Sons.

Meyer, J.-J. Ch. - Dignum, F.P.M. - Wieringa, R.J. (eds) (1994). *The Paradoxes of Deontic Logic Revisited: A Computer Science Perspective*, Technical Report, Utrecht, University of Utrecht.

Pérez-Ramírez, M. - Fox, Ch. (2003). Imperatives as Obligatory and Permitted Actions. In A. Gelbukh (ed.) *Computational Linguistics and Intelligent Text Processing, LNCS*, vol. 2588, 52-64.

Portner, P. (2012). Permission and Choice. In G. Grewendorf, E. Zimmermann (eds) *Discourse and Grammar, From Sentence Types to Lexical Categories*, Berlin, de Gruyter.

Rescher, N. (1966). *The Logic of Commands*, London, Routledge & Kegan Paul.

Ross, A. (1941). Imperatives and Logic. *Theoria*, *7*, 53-71 (reprinted in A. Ross: Imperatives and Logic, *Philosophy of Science*, *11*, 1944).

Sosa, E. (1966). On Practical Inference and the Logic of Imperatives. *Theoria*, *32*, 211–223.

Segerberg, K. (1971). Some Logics of Commitment and Obligation. In Hilpinen, R. (ed.) (1971).

Starr, W. (t.a.) A Preference Semantics for Imperatives, to appear in *Semantics and Pragmatics*.

Stenius, E. (1963). The Principles of a Logic of Normative Systems. *Acta Philosophica Fennica*, *16*, 247-260.

Stolpe, A. (2010). A Theory of Permission Based on the Concept of Derogation. *Journal of Applied Logic*, *8*, 97-113.

Straßer, Ch.- Beirlaen, M. (2012). An Andersonian Deontic Logic with Contextualized Sanctions. In Ågotnes, T. – Broersen, J. – Elgesem, D. (eds) (2012), 151-169.

Svoboda, V. (2003). Modeling Prescriptive Discourse. *The Journal of Models and Modeling*, *1*, 23-46.

van Fraassen, B. (1972). The Logic of Conditional Obligation. *Journal of Philosophical Logic*, *1*, 417–438.

von Wright, G. H. (1951). Deontic Logic. *Mind*, *60*, 1-15.

von Wright, G. H. (1957). *Logical Studies*, London, Routledge & Kegan Paul.

von Wright, G. H. (1961). A New System of Deontic Logic. In *Danish Yearbook of Philosophy* 1, 173–182 (reprinted with an amendment in Hilpinen R. (ed.) (1971)).

von Wright, G. H. (1963). *Norm and Action*, London, Routledge & Kegan Paul.

von Wright, G. H. (1991). Is there a Logic of Norms? *Ratio Juris*, *4*, 265-283.

von Wright, G. H. (1999). Deontic logic - as I see it. In McNamara, P – Prakken, H. (eds) (1999), 15-25.

Vranas, P. B. M. (2010). In Defense of Imperative Inference. *Journal of Philosophical Logic*, *39*, 59-71.

Vranas, P. B. M. (2011). New Foundations for Imperative Logic: Pure Imperative Inference. *Mind*, *120*, 369-446.

Wedeking, G. A. (1970). Are There Command Arguments. *Analysis*, *30*, 161-166.

Williams, B. A. O. (1963). Imperative inference. *Analysis*, *23* (Supplement), 30-36.

Woleński, J. (1980). A note on Free Choice Permissions. *Archiv für Rechts- and Socialphilosophie*, *66*, 507-510.