Formal Epistemology – the Future Synthesis

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Logic as a philosophical discipline originated in ancient Greece, at about the same time as mathematics was constituted as a deductive science. From ancient times two logical systems have survived – the Aristotelian syllogistic logic and the Stoic propositional logic. It took almost two thousand years, until in the works of Boole, Frege and Peano logic was turned into a formal discipline. The resulting synthesis, which was created by Whitehead and Russell, has brought a remarkable unification of the ideas of these authors. It seems that epistemology is recently undergoing formalization similar to the one occurring in logic in the 19th century.

Epistemology as a philosophical discipline was established during the 17th and 18th century under the influence of Descartes, Locke, Hume and Kant, at about the same time as physics was constituted as an empirical science. Since the days of the founders of epistemology sufficient time has elapsed so that the conversion of epistemology into a formal discipline could start. It is likely that, as in the case of logic, also in epistemology, the resulting theory will be a synthesis of multiple streams, and today it is probably not possible to do more than to outline the direction in which their synthesis can be expected.

In the process of transformation of logic into a formal discipline we can distinguish two phases. The first phase, initiated by Boole, consisted in the transition from the naturalistic Millian understanding of logic to an algebraic calculus. This phase is known under the name *algebra of logic* and culminated in the three-volume *Vorlesungen über die Algebra der Logik* written by Schroeder. Despite many undisputed successes, however, algebra of logic did not lead to contemporary formal logic. This was achieved only during the second phase, initiated by the works of Frege and Peano, consisting in the transition from the Boolean algebraic approach to the functional approach, known as *predicate calculus*.

In the case of epistemology the first phase of its transformation into a formal science started with the project of *naturalized epistemology* initiated by Quine and his followers. It led to formalization of epistemology by means of mathematical theories such as *epistemic logic*, *game theory*, *Bayesian induction* and *social choice theory*. This phase saw the founding of specialized centers dedicated (partially or fully) to the study of formal methods in epistemology (e.g. the *Munich Center for Mathematical Philosophy* at the Ludwig-Maximilian University in Munich, the *Institute for Logic, Language and Computation* at the University of Amsterdam, or the *Center for Formal Epistemology* at the Carnegie Mellon University), publication of special issues of journals, establishment of specialized conferences and raised a flow of innovative publications. However, it seems that the formalization of epistemology in the framework of naturalized epistemology has also its limits. It corresponds rather to Boole's algebra of logic than to Frege's predicate calculus. This first phase of formalization of epistemology will be followed by a second one. This, just like in the case of logic, will bring a radicalization of its certain aspects and create a deeper unity of its methods.

The aim of the present project is to contribute to the radicalization of the project of formalization of epistemology and to initiate a transition analogous to the transition from the Boolean to the Fregean approach in logic.

On the way to its full formalization logic had to undergo a series of changes and was forced to give up a number of deep-seated beliefs. It seems that similar beliefs that hindered the formalization of logic can be found also in contemporary epistemology. Therefore in the project I tentatively identify the factors that prevented the formalization of logic and on the basis of their analysis I try to formulate some open problems leading to a radical formalization of epistemology.

It is likely that this approach will raise doubts and dislike in the followers of *naturalized epistemology*, just like Frege's program met with resistance from the representatives of the *algebra of logic*. After all, naturalized epistemology is an attempt to put epistemology onto solid scientific foundations, so it is difficult to accept a criticism of this program. But in logic in the second half of the 19th century the situation was in many respects analogous – the effort to build logic on psychological foundations was introduced by the idea to use methods of experimental science, and so to liberate logic from philosophical speculation. Therefore logicians that understood logic in the naturalistic way could not accept Frege's antinaturalistic turn. They could not see why he went beyond their algebraic approach.

I have identified six obstacles that hindered the formalization of logic and for each of them I tried to find a parallel obstacle that hinders the formalization of epistemology. To overcome these obstacles is thus in each particular case an open problem (problems ***P1*** – ***P6***).

P1. Separation of logic from mathematics

Although Aristotle's theory of syllogisms can be considered as one of the first axiomatic theories in history, and so from the modern point of view, it is a mathematical theory, Aristotelian logic had become a part of philosophy and evolved separately from mathematics as one of the fundamental philosophical disciplines (alongside physics and ethics). Proponents of Aristotelian logic considered logic to be something fundamentally different from mathematics, and similar views were held also by mathematicians. Euclid wrote a treatise on optics, and Archimedes was the author of a treatise about the lever. However, it is not known that any ancient mathematician would write something about logic. It seems that physics was in antiquity closer to mathematics than logic. One of the main contributions of Boole was to overcome the prejudices that separated mathematics from logic and to bring logic into a *fruitful mutual contact with mathematics*. Boole came up with the idea to use algebraic notation to rewrite the syllogisms of Aristotelian logic. He expressed judgments in the form of algebraic equations and then reduced the proof of the syllogistic judgment to solving of the corresponding equations using the method of elimination of the unknown. Boole thus created the first variant of formal logic, which was a milestone in the transformation logic into a mathematical discipline.

If we look on epistemology from this perspective, we see that classical epistemology was isolated from mathematics and other exact sciences to almost the same extent as Aristotelian logic was isolated from mathematics. The creators of classical epistemology identified knowledge with empirical knowledge (which is ultimately based on sensory perception) and so they gave up the possibility to bring epistemology into connection with the practice of real mathematics—its methods of proof and conceptual analysis. Similarly, they ignored the instrumental aspect of science that is characteristic for physics, and so instead of analyzing experiments and measurements they stuck to the analysis of sensory perception also in the case of physical knowledge. Thus they gave up the possibility of bringing epistemology into connection with exact sciences. Therefore, the first problem of formalization of epistemology is the necessity to *bring epistemology into closer contact with mathematics (not as a tool but rather as the subject of study), physics, and other disciplines of exact science and to study the processes of knowledge acquisition in these disciplines*.

P2. Narrow understanding of the subject of logic

Traditional logic, as it was cultivated on the medieval and early modern universities, represents only a fragment of modern formal logic. This fragment can be characterized as *monadic logic* – the logic admitting only predicates having one argument. Before formal logic could arise, it was necessary to *radically extend the scope of logic* and first of all to incorporate the theory of Boolean operators, logic of relations, and a theory of polyadic quantification into logic. Pierce, Frege, and Peano brought substantial extension of the scope of logic. One of the main motivations for this expansion came—after overcoming the separation of logic from mathematics—from the *logical analysis of mathematical reasoning and proof*. Although Boole introduced neither logic of relations, nor polyadic quantification into his system, his calculus was much richer than traditional logic. Besides this, the use of the language of algebra for representing logical propositions and judgments provides a natural means for the development of logic of relations. Therefore we can consider Boole's algebra of logic as one of the first attempts to extend the scope of classical logic that opened the perspective for further developments.

Similarly, epistemology as it is presented in mainstream philosophical literature, represents only a fraction of what has to be incorporated into epistemology before it will be able to develop a comprehensive formalization of epistemology. It can be argued that the *experimental and theoretical practice of physics* from the time of Newton onwards substantially exceeds the understanding of physical knowledge as justified true belief, just like the patterns of reasoning and proof in classical mathematics exceeded the scope of monadic logic. Scientific theories and theoretical models are in complex relations of mutual approximation; they exhibit different kinds of limit transitions and idealization. Nevertheless, true in any absolute sense is neither of them. Therefore, the second problem of formalization of epistemology is the necessity to *expand the scope of epistemology to include epistemological analysis of the experimental and theoretical practice of physics, such as approximations, limit transitions, and idealizations*.

P3. Focusing of logic on problems formulated in ordinary language

Despite the undeniable progress initiated by Boole's algebra of logic, this approach was not sufficiently radical. Boole accepted that the subject matter of logic was given by the content and scope of traditional logic. His goal was only to rewrite the Aristotelian theory of syllogisms by means of algebra. This brought logic into a contact with mathematics, and allowed to develop a calculus that was gradually overcoming the too narrow understanding of logic in the Aristotelian tradition. But Boole did not address the most problematic aspects of traditional logic, namely its close ties with ordinary language. Aristotelian analysis of propositions and syllogisms is predetermined by the structure of ordinary language (its decomposition of sentences into the naming and verb phrase). For the creation of formal logic it was crucial to liberate logic from its dependence on ordinary language. This liberation was achieved by Frege and Peano. Although Boole used *the language of mathematics as a tool* for a more accurate representation of logical reasoning (he reduced reasoning to solving of algebraic equations), as the subject that he studied by means of these new mathematical tools, he took the syllogisms of traditional logic.

Frege and Peano fundamentally changed this situation, when they began to study not only judgments formulated in ordinary language, but mathematical proofs themselves, i.e. when they took *the language of mathematics as the subject matter of their study*. With respect to the relation between mathematics and logic it was an important step forward – Frege and Peano turned mathematics into a subject matter of logical analysis. In this way they liberated logic from the captivity of problems formulated in ordinary language and linked it with mathematics in a more substantial way than Boole. It turned out that the forms of reasoning used in mathematics since the times of Euclid go beyond Aristotelian logic. By means of syllogisms it is not possible to formalize almost any mathematical proof.

It seems that the contemporary approaches to formalize epistemology are closer to the Boolean way of using mathematics than to that of Frege and Peano. Even though they *use mathematical language* in the description of the process of knowledge acquisition and change, but this mathematical language is applied mostly to the analysis of traditional epistemological problems discussed from the time of inception of epistemology as a philosophical discipline. Therefore, the third problem of formalization of epistemology is the necessity to *replace the analysis of epistemological problems formulated in ordinary language by the reconstruction of knowledge acquisition and change in mathematics and physics*.

P4. Psychologism preventing the separation of the form from the content of reasoning

Early modern logic before Frege, Peano, and Russell was generally understood as the description of correct reasoning, as a description of how an empirical subject should actually reason. Even Boole, one of the creators of formal logic understood logic this way. This psychological approach to logic was responsible for the illusion that the Aristotelian theory of syllogisms, and the subject-predicate analysis of propositions, on which this theory is based, is an adequate basis for logic. From a psychological perspective thinking is intentional; it is always directed towards something, which is in the proposition that expresses the thought, represented as its subject. Therefore, if we remain in the realm of examples, which occur in ordinary language, the subject-predicate form of propositions seems to be adequate. It was Frege who realized that logic has to deal with the relation of entailment among propositions, i.e. an *objective relation between abstract objects,* independently of any subjective intentions. Typical examples that Frege analyzed by means of his logic were mathematical propositions and proofs.

The abandoning of the psychological understanding of logic and the replacement of the analysis of judgments made by a psychological subject by the analysis of propositions formulated in a certain exact discipline is essential for the separation of the form of logical reasoning from its content. When we look at Frege’s conceptual writing as a heuristic tool, we can see that it very sharply and clearly separated the formal, logical aspect of an inference from its content. The formal aspect of a judgment is expressed by means of geometric diagrams that Frege invented for this purpose, while the content is expressed by means of ordinary language (if the analyzed judgment is expressed in ordinary language), or by means of mathematical symbolism (if the subject matter of our analysis is a mathematical judgment). In Frege’s notational system there is a line, to the right of which we encounter symbols expressing the content of the analyzed judgment, while to the left of which there occur only geometrical diagrams of Frege’s system, expressing the logical form. The form and content are thus clearly and unambiguously separated, and the content is excluded from contaminating the form. One of the reasons why Frege so radically rejected psychologism was that in the framework of psychologism it is not possible to separate form from content, because both these aspects are interwoven in a particular act of thought and the transitions between them are continuous.

Currently there are a number of approaches that attempt to formalize epistemology by means of formalizing the activities of the epistemic subject. To achieve this goal they use tools such as *dynamic epistemic logic*, *game theory*, *Bayesian induction,* or *social choice theory*. It seems, however, that these approaches are parallels to Boolean algebra of logic. The creation of truly formal logic was based on Frege’s thesis that formal logic is not a discipline studying the laws of thought of particular psychological subjects, but a formal science studying the entailment relations between propositions. It seems therefore reasonable to require that a formalized epistemology should study not knowledge acquisition and change in particular agents, but it should be an objective science studying the epistemological changes within scientific theories. The rejection of psychologism is not a matter of taste, but it is a step necessary for the separation of the formal aspects of knowledge from its content. Epistemology must abandon the question of knowledge attribution to subjects and focus instead on the relationships among scientific theories. The fourth problem of formalization of epistemology is the necessity to *replace the understanding of knowledge as the result of the activities of an epistemic subject by its understanding as objective relations among scientific theories*.

P5. Using already existing mathematics

Boole shared with Frege the goal of mathematizing logic. His problem, however, was that he *accepted the scope* of Aristotelian logic and the subject-predicate analysis of propositions, and for its mathematization he *used already existing mathematics*, namely algebra. Boole's aim was to express by means of the language of algebra the Aristotelian logic in a more precise manner. Frege, by contrast, *rejected the Aristotelian framework* of logic. He made mathematical propositions the subject matter of logical analysis, which necessitated a radical extension of the scope of logic. Thanks to a rejection of psychologism Frege was able to separate logical form from the content. For the representation of logical form he created an entirely new mathematical language—*the predicate calculus*—that went far beyond the expressive power of the language of algebra that was used by Boole. We can say that unlike Boole, Frege ***created entirely new mathematics***.

It seems that the attempts to formalize epistemology, as we know them today, are closer to Boole's program to formalize logic by means of already existing mathematics than to Frege's approach leading to the creation of an entirely new mathematical theory. Whether we take *epistemic logic, Bayesian epistemology,* or *theory of social choice*, they are examples of application of *already existing* mathematical methods and tools to problems of *classical* epistemology. Of course, these applications of mathematical methods and tools can be, and usually also are, innovative and introduce many changes into mathematics itself. But they more resembles the work of Boole, who introduced a new type of algebraic structures—Boolean algebras—into algebra, than the work of Frege, who created a new mathematical discipline. Therefore, the fifth problem of formalization of epistemology is the necessity to *replace the use of already existing mathematics in describing knowledge acquisition by an effort to uncover the mathematical structure of knowledge acquisition and change in the exact sciences that will lead with a great likelihood to mathematical structures of new kind*.

P6. Asking already existing questions

Boole largely remained within the range of questions usually posed in classical logic. He strived to rewrite the syllogisms of Aristotelian logic in a more precise and transparent manner, but he did not bring a set of fundamentally new problems. Frege, on the other hand, thanks to the creation of the predicate calculus, formulated a new program, known as logicism. Although it turned out to be unfeasible (which is still being debated), logicism played a crucial role in the development of logic and was one of the main projects of the foundations of mathematics in the 20^{th} century.

In order to formalize epistemology, we must abandon the scope of questions asked by traditional epistemology and formulate a new program that would grow out of the inner motives of formal epistemology. Therefore, the sixth problem of formalization of epistemology is the necessity to *abandon the problem area of traditional epistemology and try to formulate a suitable program for epistemology of paradigm change, which would be able to motivate its further development*.

Project Objectives

Thus far I have formulated six open problems, the solution of which could lead to the creation of epistemology as a formal discipline. These problems were predominantly formulated negatively; they were oriented towards overcoming some of the limitations of contemporary epistemology. At the end of each paragraph, I have pointed out a possible way to solve the particular open problem. This led to a list of ***project objectives***.

o1. *bringing epistemology into closer contact with mathematics, physics, and other disciplines of exact science and studying the processes of knowledge acquisition in these disciplines;*

o2. *expanding the scope of epistemology to include epistemological analysis of the experimental and theoretical practice of physics; approximations, limit transitions, and idealizations*;

o3. *replacing the analysis of epistemological problems formulated in ordinary language by the reconstruction of knowledge acquisition and change in mathematics and physics;*

o4. *replacing the understanding of knowledge as the result of the activities of an epistemic subject by its understanding as objective relations among theories;*

o5. *replacing the use of already existing mathematics in describing knowledge acquisition by an effort to uncover the mathematical structure of knowledge acquisition and change in the exact sciences, that will lead with a great likelihood to mathematical structures of new kind;*

o6. *abandoning the problem area of traditional epistemology and try to formulate a suitable program for formal epistemology, which would be able to motivate its further development.*

As already mentioned, the first two of these objectives are to a certain degree met by contemporary attempts to formalize epistemology. Nevertheless, I include them here because it is probable that also in these areas some further innovations will occur. The remaining four objectives are the core of the program of epistemology of paradigm change.

The interdisciplinary nature of the project: Interdisciplinary work is challenging – it involves different perspectives, methodologies, vocabularies, etc. This challenge will be met by a combination of several measures: a thorough selection of team members, their expertise in different areas covered by the project (physics, history of science, philosophy of science, logic, and epistemology), regular meetings of the team.

The impact of the project: The results of the project will contribute to a radical change of paradigm in epistemology, parallel to the change induced by the creation of formal logic. It is impossible to foresee its consequences, but they may be comparable to the changes introduced by formal logic: the birth of algorithm theory leading finally to the creation of the digital computer.

For the creation of formal logic it was crucial that Frege turned from the analysis of judgments formulated in natural language to an analysis of mathematical proofs. Due to the rejection of psychologism he understood judgments not as subjective cognitive acts, but as objective relations between assumptions and conclusions. Of course, every judgment has besides its logical form also specific content that is independent from its form. The first task of logical analysis of a mathematical proof was to *separate the logical form* from the content and to verify whether the conclusion follows from the assumptions *on the basis of logical form*, without any regard to the content. In order to fulfill this task, Frege created a special *notation system* called conceptual script. The conceptual script represented the logical form of a proposition in a way in which all extra-logical components were written as inputs and placed to the right of the representation of the logical form itself. This ensured a sharp *separation of the logical form from the extra-logical content*. The notational system contained also transformation rules by means of which it was possible, after a judgment (or a mathematical proof) was transcribed into this conceptual script, to check whether the conclusion follows from the assumptions *solely on the basis of logical form*. Frege thus reached a full formalization of logic.

If we want to reach in epistemology of paradigm change a level of formalization comparable with that in logic, it may be necessary to attempt an analogous *separation of the epistemic form of a theory from the content*. It is an open question, whether this will require the development of a notational system analogous to Frege's conceptual script. Such a notational system would allow checking the correctness of attribution of epistemological status to theories similarly as Frege's conceptual script allowed to check the correctness of attribution of logical validity to judgments. In the course of developing his notational system Frege changed many basic elements of classical logic. We will discuss the most significant of them and try to identify the leading *principles that guided Frege's work*. We will attempt to transform each principle into a concrete task for our project. So we will obtain a series of task which represent the working plan of our project.

Project work plan

The entire logical tradition before Frege considered concepts as the fundamental level from which the logical analysis ought to start. Judgments were formed from concepts and arguments from judgments. Despite the fact that Frege called his notational system "conceptual script" that could lead one to suppose he proceeded in a similar fashion, he *took the level of propositions as the fundamental level for the beginning of logical analysis*. In other words, the elements, from which the edges of his notational system start, must be elements that are either true or false. From there start his diagrams representing the logical form. Concepts, as they cannot be true or false, do not enter Frege's notational system. Thus Frege shifted the starting point of logical analysis one level of complexity higher – from the level of concepts to the level of propositions.

It seems that in epistemology it will be necessary to change the fundamental level at which we start the epistemological analysis in an analogous way. Just like Frege turned from concepts to propositions, in epistemology our first task seems to be to *turn from propositions to theories as the basic units to which we attach epistemic status*.

Aristotelian logic understood judgments as the union of a subject with a predicate. This approach was upheld even by Boole. It is natural from the psychological point of view. Our thinking is intentional; when we think, our attention is focused on something, and we consider this something as the subject of the judgment. What we assert about the subject in a judgment is the predicate. After rejecting psychologism Frege declared the distinction between the subject and the predicate a *rhetorical emphasis* that had no relation to the logical form. Furthermore, Aristotle's subject-predicate analysis of judgments was natural also from the point of view of the structure of Aristotelian logic. If the basic level of analysis is the level of concepts, it is natural to assume that an elementary judgment is formed as a combination of two concepts in the form "*A is B*", where *A* and *B* are the subject and the predicate respectively. When Frege took as the basic level of logical analysis the level of propositions, he had no reason to give a privileged status to judgments formed as combinations of two concepts. For mathematical propositions, the subject-predicate form of judgment is inadequate. In a mathematical proposition (e.g. the Pythagorean Theorem) it is not clear what is the subject and what the predicate. So Frege *replaced the Aristotelian method of analysis of judgments into subject and predicate by their analysis into function and arguments*.

If we look for an epistemological analogy of the subject-predicate structure of a proposition, it seems natural to take the *analytic-synthetic structure of theories*. Like in logic, also here the opposition is of psychological origin. The reason why the analytic-synthetic distinction is so compelling is the difference in subjective degrees of certainty, already emphasized by Descartes. When Quine pointed out the vagueness of the boundary between analytic and synthetic judgments, he pointed to an important problem. Nevertheless, when Frege noticed a similarly vague distinction between the subject and the predicate, he did not stop by pointing out this problem, but he replaced the analysis of a judgment into subject and predicate by a more general analysis into function and arguments. It was this functional approach to propositions that revolutionized logic. Thus following Frege we should not be content with Quine's blurring the boundaries between analytic and synthetic knowledge, but we must *replace the traditional analysis of knowledge into analytic and synthetic judgments by a more general one*. This leads us to an epistemological analysis of the way the scientist justifies his experimental results and relates them to his theories. We will study the role of scientific instruments and of mathematical language in this justification.

Frege showed that what Aristotle considered as elementary judgments (e.g. "*Every bird is mortal*"), were composed propositions (implications $(\forall x)(B(x) \Rightarrow M(x))$). This shows that in Aristotelian logic implication was concealed in the connection of the subject and the predicate. Frege *made the Aristotelian concealed implication explicit*. This was a fundamental change. Aristotle could have only one implication in a judgment (concealed in the connection of the subject and the predicate), while Frege could have any number of them. Moreover, Frege *fundamentally changed the notion of elementary judgment*. For Aristotle, an elementary judgment was a connection of two concepts. Such a connection is, according to Frege, a composite judgment (an implication). An elementary judgment is a connection of a function (i.e. of *one concept*) and arguments (i.e. referring expressions such as names, constants, variables, or terms). A function can have many arguments; thus an elementary proposition can contain several referring expressions. In Aristotle's logic a judgment was the union of a subject and a predicate (*man is mortal*), and quantification expressed the scope, in which the predicate "*mortal*" is asserted about the subject "*man*". Judgments were divided into *universal*, *particular* and *singular* depending on whether the predicate is asserted universally (*All man are mortal*), partially (*Some man are mortal*) or individually (*Socrates is mortal*). This approach to quantification seems natural, because it copies our linguistic practice. Frege replaced the subject-predicate analysis of judgments by the function-argument analysis, and so he *freed quantification from its connection to the subject*. Quantification is according to Frege related to the arguments and not to the subject of the judgment. This has several consequences. Firstly a judgment, understood as the composition of a function with its arguments, *can contain several quantifiers*. This means that while in the Aristotelian logic quantified could be only one element of a judgment, namely its subject, a Fregean proposition can contain several quantifiers.

If we turn to epistemology and keep in mind that the basic level of epistemological analysis should be the level of theories, we find that *epistemology uncritically took over the notion of a theory from logic* where a theory is understood as a set of propositions (closed under logical consequence). Thus in epistemology the analogy of the Aristotelian notion of *judgment as a union of concepts* is the understanding of *a theory as a set of propositions*. Thus, I see as a major problem to develop an epistemological notion of a theory that would not copy the logical concept of a theory, but would include the theory acquisition and change. In epistemology the operation analogous to quantification should concern the determination of the scope of the relation between a theory and the aspect of reality that it represents. An analogous step to Frege's liberation of quantification from its ties to the Aristotelian notion of subject is to liberate the above mentioned relation from its reduction to verification or falsification. So we have to *study the complexity of relations between theories and their intended aspects of reality***.** We will turn to the history of science and follow the changes of the way a particular theory, such as classical mechanics, has been related to reality. We will study how the limitations of a particular theory's scope was discovered and how it can be epistemologically interpreted.

Frege's quantification theory, together with the logical connectives of implication and negation, became an element of generating complexity of the logical form. Frege's conceptual script contains a set of rules for connecting implications, negations, and quantification, thanks to which it can *express the logical form of any mathematical proposition*. Aristotelian logic was far from anything similar. Our aim is to introduce a similar formalism, that could express the epistemological form of scientific theories.