Risk and Revelation: Changing the Value of Information

Anthony Creane*

Michigan State University
and
CERGE-EI

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Abstract

A different approach is introduced to determine the value of an information sharing agreement: the measure of risk aversion. (An information sharing agreement is a commitment to reveal information that you are *going* to receive. These agreements are common within industries.) This approach is natural as the work in risk is partly based on Blackwell's work in *experimentation*, a measure of information and its value. This different approach has four main benefits. First, the relationship between information sharing models and the extensive literature in risk is made explicit. Second, the approach here is more general than previous models of information sharing as there are fewer restrictions on the distribution of the unknown parameter and the information. Third, this generality enables me to extend the model to include uncertainty regarding the slope of the demand which previous work eschews. Finally, I find that the incentive to reveal information may be more prevalent than previously thought: firms prefer to commit to reveal private valued information in both quantity and price competition. This result differs from previous work where the incentive to commit depended on the type of competition.

Key words: Blackwell, Experimentation, Risk, Information Sharing. JEL: D80, L13

Abstrakt

K určování hodnoty dohody o sdílení informace je použit odlišný přístup - míra averze k riziku. (Dohoda o sdílení informace je závazek uveřejnit informaci, kterou obdržím. Tyto dohody jsou obvyklé v rámci průmyslových odvětví.) Tento přístup je přirozený v tom, že práce s rizikem je částečně založena na Blackwellově práci o experimentování, míře informace a její hodnotě. Přístup má čtyři hlavní důsledky. Za prvé, vztah mezi modely sdílení informace a rozsáhlou literaturou zabývající se rizikem je definován explicitně. Za druhé, tento přístup je obecnější než předchozí modely sdílení informace tím, že je zde méně

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Anthony Creane: Department of Economics, Michigan State University, East Lansing, MI 48824-1038; tel. 1-517-355-4667, fax 432-1068, e-mail 22728bit@msu.edu.

podmínek na rozdělení neznámého parametru a informace. Za třetí, tato obecnost nám umožňuje rozšířit model tak, aby zahrnoval nejistotu o sklonu poptávky, která nebyla obsažena v předchozích modelech. Závěrem, je ukázáno, že popud k zveřejnění informace může být více rozšířený, než se dříve předpokládalo: podniky dávají přednost závazku zveřejnit privátní hodnotnou informaci jak při soutěži v kvantitě, tak i při soutěži v ceně. Tento výsledek je odlišný od předchozí práce, kde popud k závazku záleží na typu soutěže.

Klíčová slova: Blackwell, experimentování, riziko, sdílení informace. JEL: D80, LI3.

1. Introduction

An expanding body of research has been exploring the non-collusive reasons for industrial disclosure associations, or information sharing agreements (see [38]). These information sharing agreements are enforceable contracts, *commitments*, compelling each firm to reveal the information it receives in the future. The agreements must be enforceable as a firm may receive information that it does not want to reveal. In previous work, a firm's value of a commitment is determined by comparing the expected profits under the commitment, to the profits under no commitment. In these models, the information each firm receives is modeled as a signal with known precision.

This paper considers these information sharing agreements, but with information modeled in a more general manner by using Blackwell's [2, 3] definition of information. The conditions for more information to be valuable [2, 3] are equivalent to the conditions for an increase in risk to be utility enhancing. Thus, to determine the value of information sharing, I use techniques that are also used in the risk literature [9, 29, 30] which is based on [2, 3]. This approach offers an alternative method of assessing the incentive to reveal information. Further, this technique establishes a correspondence between the information sharing literature and the risk literature.

Using this approach, I replicate earlier results in information sharing [e.g., 10, 33] and discriminate between conflicting inferences in [32] and [10]. I also use this more general approach to determine the value of information sharing when the *slope* of the firm-specific demand is unknown. Earlier work has not examined this case because of the difficulties inherent in evaluating non-linear expectations.[1] Moreover, while earlier results are sensitive to the type of competition (price versus quantity competition), I find that if the firm specific slope is unknown, then a firm would enter an enforceable contract to reveal the information when it competes in prices or quantities. This and the results in [24] suggest that the sensitivity of previous results to the type of competition may ensue from assuming an unknown linear term of the profit function. Finally, this paper's application of results in the risk literature to an information

[1] Previous work examines unknown constant parameters (unknown demand intercept or constant marginal cost). Concurrent to this work, the authors in [24] develop a model from another perspective to examine separate issues including the incentive to share information regarding an unknown *common* slope.

sharing model provides a first step to further generalizations of the information sharing literature.[2]

2. Previous Results in Information Sharing

It might be supposed that it is the benefit from obtaining the rival's information that induces a firm to share information with its rival. Instead, an information sharing equilibrium usually exists because a firm *wants* its rival to learn what it will learn. Roughly, a firm wants to enter an information sharing agreement because the possible advantage it gains from its rival learning what it learns is greater than the possible loss from its rival learning what it learns. Thus, most work considers whether a firm agrees to an enforceable contract of non-exclusionary disclosure, i.e., disclosure independent of the rival's disclosure decision.

Many variations to the standard information sharing model have been explored (see references in [38]). In Table 1, I characterize the incentive for information sharing with consumption substitutes and uncorrelated information. I restrict attention to these cases to make the distinctions in the main results clear.

***Table 1**: Incentive for Information Sharing*

Unknown variable / Type of competition	Quantity competition	Price competition
firm-specific cost [10]	reveal	do not reveal

[2] The concept of risk aversion and its relationship to the value of information should not be confused with the use of risk aversion in models that examine how having risk averse agents affects the incentive to share information [17, 18]. All other information sharing models assume risk neutral agents.

[3] The author in [10] argues that *not reveal* is the actual outcome.

[4] The author in [5] contends. Though the authors in [32] explore costs uncertainty, they assume that firms already know their own costs, which obviously does not apply in the common cost case.

There is some disagreement between [10] and [31] regarding the incentive to reveal information about a firm-specific intercept in the price competition case. The assertion in [10], however, is in the conclusion and, thus might be supposition. On the other hand, if [10] is correct, then there is something systematic to the incentive to reveal information. First, the incentive to reveal information in price competition would always be the reverse of the incentive to reveal in quantity competition. The results, then, would consistently depend on whether the goods compete as strategic substitutes or strategic complements. Second, if [10] is correct, then the incentives to reveal firm-specific linear costs and firm-specific demand intercept are the same in price competition. (In quantity competition, cost and intercept uncertainty are mathematically equivalent.) I find that contrary to [10], the conclusions in [31] are correct.

In the next section the basic model is described using the standard strategic framework of the information sharing literature and drawing on [2, 3, 21] to define learning. The fourth section uses this model and the work in [9] to establish information sharing results. Finally, since the work here draws on the risk literature, and work that the risk literature is based on, I compare my results to research in the risk literature regarding increases in the value of information.

3. The Model

For basis of comparison, consider the standard information sharing framework [37, 38]: a duopoly comprised of risk neutral, profit maximizing firms, each producing a single good (q^a, q^b). Firm *i* (=a,b) faces a linear demand function of $q^i=a^i-b^ip^i+dp^j$, $i\neq j$, $a>0$, $b>d>0$ if firms compete in prices, and with p and q transposed and -d if firms compete in quantities. Firms have constant marginal cost c^i. The parameters c, b and a are chosen such that in equilibrium $q^i>0$.

The structure of the game is the same as the standard information sharing models. There are three stages or periods. Let subscripts denote periods. In period 0, each firm chooses whether to make a commitment (enforceable contract) to non-exclusionary disclosure of the verifiable information it will obtain.[5] In period 1 a firm obtains the (fixed amount of) information and meets all commitments. Finally, in period 2 the firms compete in output or prices.

Let the same firm-specific parameter (a, b, or c) be unknown for each firm. There is a common belief regarding the parameter for each firm, the prior. Let

[5] For issues of verifiability of information, see [27].

the expected value of the unknown parameter for firm i be θ^i (which is defined precisely below). There is common knowledge in period 0: all firms have the same belief that each parameter has the value $\theta^{i,0}$. In period 1, each firm receives private, independent information regarding its private parameter. The characteristics of the information (as defined in the next paragraphs) are known to both firms. The firms update their beliefs using Bayes' rule into the period 1 belief, $\theta^{i,1}$, i.e., the posterior of $\theta^{i,0}$. Current beliefs are rational and the firms do not expect the information to change the belief of the parameter: $E^0[\theta^{i,1}]=\theta^{i,0}$. Similarly, each firm expects that its rival's beliefs will not change.

Information is defined following [2, 3] with clarification from [21] (see also [6, 8, 12, 19, 20]). Let a parameter of the linear demand function be unknown (call it θ). There are n possible values of θ ({θ

In period 1, the firms obtain the information (run the experiments) and meet all commitments. In period 2, the firms, based on their beliefs of the unknown parameters, choose output (q) to maximize profits in quantity competition (and similarly in price competition): $\pi^i = [a^i - b^i q^i - dq^j - c^i]q^i$. From the first order conditions, the firm's best response function in quantity competition is,

$$q^i = \frac{(a^i - c^i - dq^j)}{2b^i} = \frac{(a^i - c^i)}{2b^i} - \frac{d}{2b^i}q^j \qquad)$$

In price competition, the firm's best response function is

$$p^i = \frac{a^i + c^i b^i + dp^j}{2b^i} = \frac{a^i + c^i b^i}{2b^i} + \frac{d}{2b^i}p^j \qquad)$$

Substituting the first order condition in quantity competition (1) into the profit expression yields profits in terms of q: $b^i(q^i)^2$. Similarly, substituting the first order condition in price competition (2) into the profit expression yields profits as a function of p: $b^i(p^i-c^i)$. As profits are quasiconcave and continuous, there exists a pure strategy Nash equilibrium.[6]

The experiment provides information regarding the firm's unknown parameter θ^i, changing the firm's belief regarding θ^i. Information, through its affect on $\dot{\theta}_1$, affects a firm's output. If the firm does not reveal its information, then the output the rival expects the firm to set *does not change* after the firm receives its information since the rival's information regarding the firm has not changed. That is, the rival's (expected) choice (q^j) in (1) is fixed with respect to the information the firm obtains regarding the firm's unknown, firm-specific parameter (recall that the information is independent). In this case, equations (1) and (2) are used to examine the effect a change in beliefs has on the firm's choice and hence its value (or decision [2, 3]) function, $\pi^{i,n}(q^i(\theta^i))$. The expected value v_k and V are defined following earlier notation [2, 3].

Committing to reveal information changes the way information affects the choice variable. Revealing information to a rival indirectly affects the firm's

[6] See, e.g., [7] and especially the reference to and discussion of Glicksberg's Theorem.

output by affecting the rival's output. The rival incorporates the information into its choice q^j, and the firm incorporates the fact that the rival incorporates the information, etc. In this case a firm's choice can be expressed in the beliefs that all firms share. Using equation (1) for each firm yields the equilibrium quantities

$$q^i = \frac{(a^i - c^i - dq^j)}{2b^i} = \frac{(a^i - c^i)2b^j - (a^j - c^j)d^i}{4b^i b^j - d^2}. \qquad)$$

Similarly using equation (2) yields the equilibrium prices

$$p^i = \frac{a^i + c^i b^i + dp^j}{2b^i} = \frac{(a^i + c^i b^i)2b^j + (a^j + c^j b^j)d}{4b^i b^j - d^2}. \qquad)$$

Equations (3) and (4) are used to examine the effect a change in beliefs has on the firm's choice and hence the effect on its value function, $\pi^{i,r}(q(\theta^i, q^j(\theta^i)))$.

4. The Value of Sharing Agreements

By agreeing to share information a firm changes the convexity of its value function. Moreover, an increase in convexity, normalized by the slope, implies an increase in expected profits. In terms of the risk literature, to determine a firm's value of revealing information, I compare the measure of a firm's 'risk aversion' (loving, actually) [1, 28] when it commits to reveal information to its measure of risk loving when it does not commit to reveal information. Comparing a firm's risk aversion measures for these two cases follows the work of Diamond and Stiglitz [9]. The choice with the greater measure of risk loving has the greater expected value.

Diamond and Stiglitz [9] show that when mean utility is held constant, an increase in risk aversion is equivalent to an increase in the risk premium or a decrease in expected utility. The risky parameter of [9] can be interpreted here as the posterior probability distribution of the possible states the firm may face. When the experiment has no information, the expected values are the same whether a firm reveals information or not; this corresponds to mean utility being held constant when comparing risk loving in [9]. Following [9], then, a choice (revealing or not) yields greater risk loving if and only if the choice has an increase in value to the firm. To ascertain, then, if a firm prefers to commit to reveal information only requires comparing the magnitudes of the measures of risk loving under the two options (revealing and not revealing).

Determining first that firms value information (the value function is convex in beliefs) provides the basis for comparing the magnitudes of the measure of risk loving. Showing that information is valuable also puts the problem formally into the context of Blackwell's definitions.

LEMMA: Private information is valuable.

PROOF: By the definition of information [2, 3], convexity is sufficient. Simple calculation shows that the value function (π^i) is convex for unknown, privately valued a,b,or c: $\pi_{\theta\theta} > 0$.

For ease in notation, let r^n be the risk discount for a firm when it does not commit to reveal information, i.e., $r^n = \pi^n_{\theta\theta}/\pi^n_{\theta}$. Let r^r be the risk discount for the firm when it does commit to reveal information. The proofs to the following propositions are left to the appendix.

To see if this model replicates previous results, I first determine whether a firm reveals information for the cases that previous information sharing models are in agreement, but without the restrictions previous information sharing models place on the signal and learning.

PROPOSITION 1: A) In quantity competition, if the firm-specific demand intercept or cost is unknown ($\theta^i = a^i$ or c^i), then a non-exclusionary contract to reveal information raises expected profits; B) In price competition, if the firm-specific cost is unknown ($\theta^i = c^i$), then a non-exclusionary contract to reveal information lowers expected profits.

For each case, the choice to reveal or not to reveal is the dominant strategy. The results of Proposition 1 correspond exactly to the literature (see table 1).

Next, I examine the incentive to reveal information for a case for which previous work provides contradictory results: firm-specific demand intercept when the firms compete in prices. The approach here obtains the same result as in [31].

PROPOSITION 2: In price competition, if the firm-specific demand intercept is unknown ($\theta^i = a^i$), then a non-exclusionary contract to reveal information increases expected profits.

The type of competition, then, does not always determine the incentive to reveal information (see table 1). As a result, deriving intuition regarding the conditions under which firms reveal information becomes more difficult. Further, proposition 2 shows that for price competition there is no equivalence between uncertainty regarding the demand intercept and uncertainty regarding constant marginal costs. Proposition 2, however, does provide a set of conditions under which a firm prefers to reveal information. The generality of the conditions under which firms share information is important for at least two reasons. First, a general set of conditions for information sharing could be applied beyond the setting of strategic competition between firms, e.g., suppliers and buyers. Second, the general conditions for firms to reveal information would establish how widespread is the incentive to reveal information for non-collusive reasons. For this reason, the evaluation of industrial information disclosure associations by economists and policy makers partly depends on the generality of the incentive to reveal information.

The last case I study is an extension of the information sharing literature: the firm's firm-specific slope is unknown. Unknown slope can be examined here because comparison of the risk measures, in contrast to the comparison of profit levels, is more straightforward.

PROPOSITION 3: In both price and quantity competition, if the firm-specific slope is unknown ($\theta^i = b^i$), then a non-exclusionary contract to reveal information raises expected profits.

The most interesting aspect of Proposition 3 is that when the unknown term enters profits non-linearly, the incentive to reveal is the same in both types of competition. The model dependent incentive toward revealing information found in previous work may be partly a result of examining uncertainty regarding the linear terms in the objective function. Propositions 2 and 3 also suggest that the strategic relationship between the two goods in determining the incentive to reveal information is not as important as previous work intimated. Finally, Propositions 2 and 3 provide to the information sharing literature additional cases in which firms prefer to reveal information. These additional cases strengthen the argument that firms may reveal information simply for the value of sharing information *per se* and not for other reasons (e.g., to abet collusion). Concurrently, work [24] that approaches the information sharing from yet another perspective, also finds that for unknown *common* slope firms can benefit from revealing information in both price and quantity competition.

5. Risk, Experiments and Information

The changes in the value of information explored in this paper are similar to questions explored in the risk literature. To show the distinction between them, their relationship needs to be delineated. Rothschild and Stiglitz [29, 30] interpret Blackwell's theorem [2, 3] as an increase in risk, putting Blackwell's work in the context of the Arrow-Pratt measure of risk aversion [1, 28]. Since [29], much work [4, 11, 13, 15, 23, 34, 35, 36, 39] has explored the conditions for information to become more valuable as risk increases using the definitions in [29, 30].

In most risk papers that explore the value of information [4, 11, 15, 23, 34, 35], the value of information is defined as a comparison between two cases. For both cases, there is a distribution over a set of n possible states. In case one, the agent learns first what the true state is (obtains full information), and then makes its choice. Following [11], let V^L equal the expected value of the game in case one (e.g., expected profits based on setting price after the state is revealed). In case two, the agent must make its choice before learning the true state. Let V^{NL} equal the expected value of the game in case two. The value of information is, then, $I \equiv V^L - V^{NL}$. The question that these papers nominally pose is, does an increase in risk increase the value of information? In [4, 11, 15, 23], V^{NL} is held constant, so that the real question posed is, does V^L increase with risk? This increase in risk can be interpreted as a "more informative" or a "sufficient" experiment in [2, 3].[7]

The information question that I ask differs from that in the risk literature both in the setting (in information sharing contracts) and the manner (by exploring changes in the value function) that I use. The question that I pose, in terms of the risk literature, is do information sharing agreements increase risk loving? That is, do information sharing agreements change the value of V^L without changing the amount of risk? At a general level, this is what is explored in [9]. In related work, the authors in [8, 12, 19, 20] apply Blackwell's Theorem to model the demand for information. These papers differ from the my work as the authors in these papers consider changes in the level of information/riskiness

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[7] In [11] it is shown that a mean preserving increase in risk (equivalent to a mean preserving stochastic transformation [2, 3]) does <u>not</u> increase the value of information in general. Though true, in [2] it is shown that for every continuous convex function $\phi(p)$, where p is the vector of densities of the possible states, ϕ increases with information. Conditions similar to [2, 3], but in an economic setting are established in [15].

(as in [4, 11, 15, 23, 34, 35]) and not changes in the value of a given level of information. Closer to my model, [13, 39] examine the connection between risk aversion and the demand for information. However, in terms of my model, since risk aversion is part of an individual's preferences and is exogenously given, comparisons between levels of risk aversion for a given individual are not explored in [13, 39].

Calculations & Proofs

Substituting the first order condition into the profit function yields the following value function $\pi^{i}(\bullet)$ for when the firms compete in prices and quantities. The first and second derivatives with respect to the unknown parameter are then calculated.

$$\pi^{i}(p^{i}(\theta^{i},p^{j}(\theta^{i}))) = b^{i}(p^{i}-c^{i})^{2}$$

$$\pi_{\theta}(p^{i}(\theta^{i},p^{j}(\theta^{i}))) = 2b^{i}(p^{i}-c^{i})(\frac{\partial p^{i}}{\partial\theta^{i}}-\frac{\partial c^{i}}{\partial\theta^{i}}) + \frac{\partial b^{i}}{\partial\theta^{i}}(p^{i}-c^{i})^{2}$$

$$\pi_{\theta\theta}(p^{i}(\theta^{i},p^{j}(\theta^{i}))) = 4\frac{\partial b^{i}}{\partial\theta^{i}}(p^{i}-c^{i})(\frac{\partial p^{i}}{\partial\theta^{i}}-\frac{\partial c^{i}}{\partial\theta^{i}}) + 2b^{i}(\frac{\partial p^{i}}{\partial\theta^{i}}-\frac{\partial c^{i}}{\partial\theta^{i}})^{2} - 2b^{i}(p^{i}-c^{i})\frac{\partial^{2}p^{i}}{(\partial\theta^{i})^{2}}$$

$$\pi^{i}(q^{i}(\theta^{i},q^{j}(\theta^{i}))) = b^{i}(q^{i})^{2}$$

$$\pi_{\theta}(q^{i}(\theta^{i},q^{j}(\theta^{i}))) = 2b^{i}q^{i}\frac{\partial q^{i}}{\partial\theta^{i}} - \frac{\partial b^{i}}{\partial\theta^{i}}(q^{i})^{2}$$

$$\pi_{\theta\theta}(q(\theta^{i},q^{j}(\theta^{i}))) = 4\frac{\partial b^{i}}{\partial\theta^{i}}q^{i}\frac{\partial qi}{\partial\theta^{i}} + 2b^{i}(\frac{\partial q^{i}}{\partial\theta^{i}})^{2} + 2b^{i}q^{i}\frac{\partial^{2}q^{i}}{(\partial b^{i})^{2}}$$

$$r = |\frac{\pi_{\theta\theta}}{\pi_{\theta}}| \qquad D = 4b^{i}b^{j} - d^{2}$$

PROPOSITION 1: A) If $\theta^{i}=a^{i}$ or c^{i}, a firm commits to reveal information in quantity competition; B) If $\theta=c$, a firm does not want to reveal information in price competition.

PROOF: First I need the following lemma.

LEMMA 2: a) If $\theta=a$, then $dq^{r}/d\theta > dq^{n}/d\theta$

b) If $\theta=c$, then $1 > dp^{r}/d\theta > dp^{n}/d\theta > 0$.

PROOF:

$$a: \quad \frac{\partial q^{r}}{\partial a^{i}} = [\ \frac{1}{2b} - \frac{d}{2b}(-\frac{d}{2b}\frac{\partial q^{j}}{\partial a}) =]\ \frac{2b^{j}}{4b^{i}b^{j}-d^{2}} = \frac{1}{2b^{i}-\frac{d^{2}}{2b^{j}}} > \frac{1}{2b} = \frac{\partial q^{n}}{\partial a^{i}}$$

$$b: \quad 1 > \frac{2b^{i}b^{j}}{4b^{i}b^{j}-d^{2}} = \frac{\partial p^{r}}{\partial c^{i}} = \frac{1}{2-\frac{d^{2}}{b^{i}b^{j}}} > \frac{1}{2} = \frac{\partial p^{n}}{\partial c^{i}}.//$$

Thus, from lemma 2, it follows that

A) $r^r = (dq^r/da)/q > (dq^n/da)/q = r^n$;

B) $r^r = \{(dp^r/dc)-1\}/(p-c) < \{(dp^n/dc)-1\}/(p-c) = r^n$.//

PROPOSITION 2: If $\theta^i = a^i$, then a firm wants to commit to reveal information in price competition.

PROOF: First I need the following lemma.

LEMMA 3: If $\theta = a$, then $dp^r/d\theta > dp^n/d\theta$.

PROOF:

$$\frac{\partial p^r}{\partial a^i} = \frac{2b^j}{4b^i b^j - d^2} = \frac{1}{2b^i - \frac{d^2}{2b^j}} > \frac{1}{2b^i} = \frac{\partial p^n}{\partial a^i}.//$$

Calculating r, $r^r = (dp^r/da)/(p-c) > (dp^n/da)/(p-c) = r^n$ where the inequality follows from lemma 3.//

PROPOSITION 3: Let $\theta^i = b^i$, then a firm wants to commit to reveal information.

PROOF: I will show this first for price competition and then for quantity competition. First the r calculations for the price case:

$$r^r = \frac{4(p-c)\frac{\partial p^r}{\partial b^i} - 2b^i[\frac{2b^j}{D}(2p-c)]\frac{\partial p^r}{\partial b^i} - 2b^i(p-c)\frac{8b^j}{D}\frac{\partial p^r}{\partial b^i}}{(p-c)(p-c + 2b^i\frac{2b^j}{D}(2p-c)} = \frac{\frac{\partial p^r}{\partial b^i}[(2p-c)b^i b^j + (p}{(p-c)[pb^i b^j + (p-c}$$

$$r^n = \frac{4(p-c)\frac{\partial p^n}{\partial b^i} - 2b^i\frac{1}{b^i}(2p-c)\frac{\partial p^n}{\partial b^i} - 2b^i(p-c)\frac{2}{b^i}\frac{\partial p^n}{\partial b^i}}{(p-c)(p-c - 2b^i\frac{1}{2b^i}(2p-c))} = \frac{\frac{\partial p^n}{\partial b^i}(2p-c)}{p(p-c)}$$

First I will show that $dp^r/db^i > dp^n/db^i$, and then that the rest of r^r is greater than the rest of r^n.

LEMMA 4: If $\theta = b$, then $dp^n/db < dp^r/db$.

PROOF:

$$\left|\frac{\partial p^n}{\partial b^i}\right| = \frac{1}{2b^i}(2p-c^i) < \frac{1}{2b^i - \frac{d^2}{2b^j}}(2p-c) = \frac{2b^j}{b^i b^j - d^2}(2p-c) = \left|\frac{\partial p^r}{\partial b^i}\right| .//$$

Next, I show that the remaining terms of r^r is greater than the remaining terms of r^n.

$$4p > 2p-c \Leftrightarrow pd^2(p-c) > \frac{(2p-c)}{4}d^2(p-c) \Leftrightarrow$$

$$pd^2(p-c) + p(2p-c)b^i b^j > \frac{(2p-c)}{4}d^2(p-c) + p(2p-c)b^i b^j \Leftrightarrow$$

$$((2p-c)b^i b^j + d^2(p-c))p > \left(\frac{d^2(p-c)}{4} + pb^i b^j\right)(2p-c) \Leftrightarrow$$

$$\frac{(2p-c)b^i b^j + d^2(p-c)}{pb^i b^j - d^2\frac{(p-c)}{4}} > \frac{(2p-c)}{p}$$

Therefore, by lemma 4 and the last inequality: $r^r > r^n$ with prices. For the case of quantity competition, calculating the risk aversion measure yields

$$r^n = \frac{-4q\frac{1}{b^i}q + 2b^i\frac{1}{(b^i)^2}q^2 + 2b^i q\frac{2}{(b^i)^2}q}{q^2 - 2b^i q\frac{1}{b^i}q} = \frac{2}{b^i}$$

$$r^r = \frac{\frac{q^2}{D^2}16(2b^i b^j + d^2)b^j}{\frac{q^2}{D}(4b^i b^j + d^2)} = \frac{16b^j(2b^i b^j + d^2)}{16(b^i b^j)^2 - d^4} > \frac{2}{b^i} = r^n$$

The last inequality follows because $r^r = r^n$ if d=0 and r^r is increasing in d.//

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