Wage Trap Model - Producer Behavior in Economies with Heterogeneous Labor Force

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Abstrakt

Hlavním cílem této práce je přispět k určení důvodů, proč komunistické (a částečně i transformační) ekonomiky jsou podstatně méně efektivní než ekonomiky tržní. Tyto důvody budou analyzovány na modelu maximalizace zisku (založeném na teorii efektivních mezd), který povede k existenci více ziskových optim. Neefektivita plánovaných a transformačních ekonomik pak může být brána jako důsledek neschopnosti dosáknout nejvýššího z těchto ziskových optim (tato situace bude označována jako mzdová past, protože tato produkční suboptimalita je způsobena nemožností zvolit optimální hodnotu mzdy). Výsledky analýzy budou dále použity k vysvětlení nespojitých změn v produktivitě a ziskovosti některých firem v transformačních ekonomikách, a také k vysvětlení možných výhod zahraničních firem působících na trzích transformačních ekonomik.

Abstract

The main goal of this paper is to help determine why the communist (and partly even transition) economies are substantially inefficient in comparison with their market counterparts. The reasons will be analyzed with the help of the profit maximization model (based on the efficiency wage theory) leading to the existence of multiple profit optima. The inefficiency of planned and transition economies will then be considered mainly as the consequence of the impossibility to reach the highest profit optimum (this situation will be referred to as the wage trap, because the production suboptimality was due to the lack of freedom to set the optimal wage level). The results of the analysis will also be used to explain the huge discontinuous shifts in the profitability or production of certain firms in transition economies as well as of some possible initial advantages of foreign firms acting in transition economies.

INTRODUCTION

One of the most interesting problems that appeared in economics after the changes in the former communist block countries was why the communist economies were so inefficient in comparison with their western counterparts.

The paper makes use of the profit maximization model (related to the efficiency wage theory), the solution of which leads to the existence of multiple isolated optima with unequal levels of optimal profit. According to this model, the inefficiency of the communist economies can be explained by the fact that producers were not allowed to change wages to influence workers' productivity (huge occurrence of ceiling rates, etc.). This meant that they could apparently achieve only the local profit optimum (while the global one could be much higher).[1] This situation will be referred to as the **wage trap**.

The presented model can also explain huge discontinuous shifts in profitability or production in transition economies. Because the new owners of firms in these economies already have the freedom to influence workers' productivity by wages, those changes can be easily explained as the movement between local and global optima in the wage trap model.

The model belongs to the class of efficiency wage models. Those models are based on the assumption that the wage is not exogenous, but is set by an employer to influence workers' productivity.

The standard form of production function in efficiency wage theory is F(q(W)N), where N denotes labor, W is the wage rate and q(.) is a labor quality index function (increasing with respect to W). In contrast, in the model introduced here wages will be used to affect output directly (not only through labor).

[1] The model belongs to the efficiency wage theory, therefore the wage is the decision variable here.

The Formulation of the Problem

We will assume that the firm production function is given:

$$y = F(x,w) = f(x)g(w) \qquad \textbf{(1)}$$

where x denotes labor, w is wage and y is output given (x,w). Function f(.) is a **technological production function** that has the following properties:

1) f(0)=0
2) f is increasing
3) f is strictly concave, twice continuously differentiable
4) the following Inada conditions hold:

$$\lim(x \to 0)\frac{df(x)}{dx} = \infty \qquad \lim(x \to \infty)\frac{df(x)}{dx} = 0 \qquad \textbf{(2)}$$

The technological production function states maximal technologically producible output y given labor x.

Function g(.) will be referred to as the **effort function**. This function states a share of the utilization of the maximal (technologically) producible output. This share depends on the quality of the labor force. Therefore, we will assume g(.) to be the increasing function of a wage (the higher wage implies the possibility to hire the labor of better quality), concave and with the following properties:

$0 \leq g(w) < 1$ for all $0 \leq w < w_p$
$g(w) = 1$ for all $w \geq w_p$

where w_p denotes a "commitment" wage, which can be viewed as the reservation wage of absolute professionals. When this wage is offered by an employer, only workers who are willing to produce the technological maximum will be hired.

Remark: we will assume that when w increases, more skilled workers with greater reservation wages enter the labor force. It will be further assumed that if wage $\underline{w}$ is offered by the employer, there are infinite queues of workers with reservation wages $\underline{w}$ waiting for jobs (we will assume that the demand for labor of the studied firm is small enough with respect to the size of the labor market). We will also suppose that an employer has the ability to recognize (and to hire) the best workers who apply for a job.

A Comparison of Efficiency Wage and Wage Trap Production Functions:

First, let us assume the situation when only absolutely skilled labor is used for production. Let N be the number of work units (e.g. hours worked) produced by this skilled labor force. Let F(N) be the resulting production function of a firm. Let f(N) be the technological production function. If only absolutely skilled labor is used, the wage w_0 is set in order for both $q(w_0)$ and $g(w_0)$ to be equal to one. In that case both the resulting efficiency wage and the wage trap production functions are the same, since $F(N)=f(q(w_0)N)=f(N)g(w_0)$. Therefore, if n work units are used, the amount of output is Y(n) in both cases.

Now let us consider the situation when an employer decides to use n work units produced by less skilled labor (he pays the wage w such that q(w)<1 and g(w)<1). The resulting production functions are not the same. According to the efficiency wage theory the production function stays the same, but less labor (q(w)n) is used and $Y^e(n)$ is produced , while the wage trap function is changed to (f(N)g(w)) and the amount of output is $Y^t(n)$:

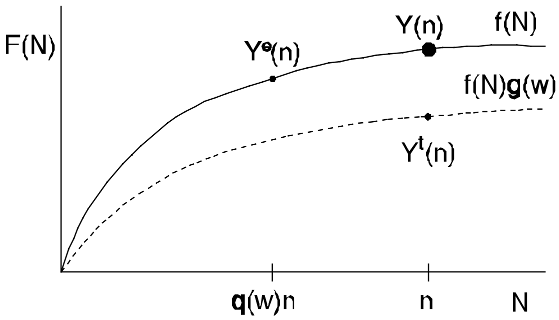

Figure 1

Here we can see the main difference between the approaches. It is assumed in the efficiency wage theory that it is possible to transform n units of unskilled labor to q(w)n units of skilled labor (and to use the same production function for both levels of skills). In contrast, the wage trap model assumes that this **transformation is not possible**. If the level of skills is changed, the different production function must be used to reflect the work of n units of unskilled labor.

Certain reasons exist to support the assumption that unskilled labor is impossible to transform into skilled labor. There are two sources of differences in labor skills: either skilled workers are more able to produce than unskilled workers, or skilled workers better utilize the work period.

If we suppose that skill differences are due to unequal workers' abilities, then the efficiency wage theory must intrinsically assume a perfect manager, who is able to "extract skills" from workers (e.g. to "create" one skilled worker from two unskilled workers). Moreover, this transformation is possible only if the skills of unskilled workers are "additive", i.e. if, for example, one group of workers manages the first half of the production process, while another group of workers manages the second half. This requirement is very restrictive because it is reasonable to assume that additiveness is unlikely to exist in practice.

If we assume that skill differences are due to the unequal utilization of work time, then n units of unskilled work can really be transformed into q(w)n units of skilled work. Yet even in this case there must be a perfect manager. The manager must be able to organize production to assure that at any time the constant number of work units will be executed (e.g. to let the first half of workers work in the morning, while the second half works in the afternoon). This is necessary to assure that the marginal product of labor will be maximal (corresponding to q(w)n units of skilled work). However, if workers have control over the utilization of their work time and if they decide "randomly" when to have their rest time (given by 1-g(w) of the period of work), then again production can be described only by the production function with the lower marginal product of labor on each level of N, which corresponds to the work of unskilled, non-transformable labor (i.e. by the wage trap model).

Generally, the wage trap production function better describes reality when skill differences are due to unequal abilities. If skill differences are due to unequal utilization of work time, the efficiency wage production function can be used only for production with the work control manager[2]. In all other cases the wage trap production function is more appropriate.

Since this paper will address the situation of firms in planned and transition economies, it is more suitable to use the wage trap production function for the description of firm behavior, as it is reasonable to assume that perfect work managers are not likely to exist in those economies (and then this type of production function better describes reality without respect to the source of skill differences).

Mathematical formulation:

[2] It should be noted that e.g. assembly line may serve as the work control.

We will treat a simple profit maximization problem, i.e.

$$\max_{x,y,w}\ py - wx$$
$$s.t.\ \ x \geq 0 \qquad \textbf{(3)}$$
$$y \leq f(x)g(w)$$

where parameters p, w denote the prices of output and input, respectively. An interior solution is ensured because Inada conditions for the technological production function are assumed.

The solution to this problem must fulfil the following conditions:

$$w = pf'(x)g(w)$$
$$f'(x)\frac{x}{f(x)} = g'(w)\frac{w}{g(w)} \qquad \textbf{(4)}$$

The main difference between this solution and the solution of the efficiency wage model is the uniqueness of an optimum. The production function of the latter model is strictly concave. This ensures (together with Inada conditions) the existence and uniqueness of an optimum (because element wx is linear, it does not have any impact on the concavity of a profit function). There is a completely different situation in the wage trap problem. The element wx is no longer linear (both components are variables). More importantly, although both technological production and effort functions are concave (and even $g(w_1) \leq 1$), **the resulting product** (and therefore generalized production function) **does not have to be concave.** Thus the profit function for problem (3) need not be concave as well.

Moreover, the following theorem can be stated:

Theorem 1: Let

$$\pi(x,w) = pf(x)g(w) - wx \qquad \textbf{(5)}$$

where f is a technological production function and g is continuous, piece-wise and linear. Then there is no point such that the second derivative of π exists and π is concave in it.

Proof: see Appendix

It was found that not all the generalized production functions with a continuous, piece-wise linear effort function are concave. From now on we will deal only with this particular form of an effort function in problem (3) [3]. The **economic interpretation of the piece-wise linear effort function** may be the following: This function may **describe different skill levels of workers. The common sign of workers in each particular level is that their maximal share of input utilization is equal to the minimal share of input utilization of workers from a higher successive skill level. Workers within each particular group increase their shares of input utilization linearly in response to a wage increase**.

We will assume that the wage levels that distinguish workers with different skills are fixed, given in advance and known to everybody. It should be noted, however, that in a dynamic framework, these values would be endogenous, influenced by relative sizes of workers in different skill levels.

From now on, uniqueness of optimum of a profit maximizing problem is not assured and, actually, we will show that there are cases when this problem may have multiple optima. We will call this phenomenon **multimodality**.

[3] Note that even this function (although not differentiable) fulfils the mathematical definition of concavity (needed in the definition of the problem).

Conditions for multimodality in the profit maximization problem

Though non-concavity of a generalized production function is an indicator of the existence of multimodality, it is not sufficient evidence in itself. Moreover, there are points where this function does not have the second derivative. Therefore it is not possible to use the standard tools of mathematical analysis (values of the first and second derivative at the maximum) to determine the maximum. The following theorem may be useful for this purpose.

Theorem 2: Let f be a n-dimensional function and let there be a neighborhood of x^* from $\mathbf{R^n}$ of a form $I_1*I_2*..*I_n$, where I_j is a closed interval in **R**. Let all the partial derivatives exist for all the points of this neighborhood except x^*. Let these partial derivatives be continuous and fulfill the condition:

$$\frac{\partial f}{\partial x_i}(\underline{x_1}\ldots\underline{x_n})=0 \quad \Rightarrow \quad \frac{\partial f}{\partial x_i}(\underline{x_1}\ldots x_i\ldots\underline{x_n}) \begin{cases} >0 & x_i<\underline{x_i} \\ <0 & x_i>\underline{x_i} \end{cases} \quad i=1\ldots n-1 \tag{6}$$

Let just one vector $(\underline{x}_1,...,\underline{x}_{n-1})$ exist for all x_n in I_n s.t.

$$\frac{\partial f}{\partial x_i}(\underline{x_1}\ldots\underline{x_{n-1}},x_n)=0 \quad i=1\ldots n-1 \tag{7}$$

Let the derivative of $z(x_n)=f(x_1(x_n),..,x_{n-1}(x_n),x_n)$ (z is created from (7)) be integrable. Then the following holds: If all the partial derivatives w.r.t. $x_1,...,x_{n-1}$ are equal to 0 and:

$$\frac{\partial f}{\partial x_n}(\underline{x_1}\ldots\underline{x_n}) \begin{cases} >0 & \underline{x_n}<x^*_n \\ <0 & \underline{x_n}>x^*_n \end{cases} \quad \text{for} \quad \frac{\partial f}{\partial x_i}(\underline{x_1}\ldots\underline{x_n})=0 \quad i=1\ldots n-1 \tag{8}$$

then $f(x^*_1,...x^*_n) > f(x_1,...x_n)$ for all $(x_1,...,x_n)$, i.e. $(x^*_1,...x^*_n)$ is the local maximum of the function f.

Proof: see Appendix

Results of theorem 2 will now be used to formulate of sufficient conditions for the existence of multimodality in problem (3). We will deal with the following piece-wise linear form of a function g:

$$g(\sum_{j=0}^{m-1} d_j + e_m) = \sum_{j=0}^{m-1} k_j d_j + k_m e_m \quad , \quad \sum_{j=0}^{m-1} d_j + e_m < \sum_{j=0}^{m} d_j$$
$$k_0 > k_1 > \ldots > k_z \quad , \quad \sum_{j=0}^{z} k_j d_j = 1 \qquad \textbf{(9)}$$

where d_j determines the length of an interval, where a derivative of the function g is constant; z denotes the number of skill levels.

Formulation of Conditions for Multimodality

Our task is to find sufficient conditions for a production function to have (at least) two optima (and we must find at least one such function, of course).

We start from the problem:

$$\max \ \pi(x,w) = pf(x)g(w) - wx \qquad \textbf{(10)}$$

which is equivalent to (3). Function g is of the form in equation (9). First of all, it is clear that the maximum of function U can be only at a point where function g changes its slope. Otherwise it would be at a point where function U has continuous partial derivatives of the second order. The necessary and sufficient condition for the maximum is then a negative definiteness of the Hessian of the function π, but this condition is not fulfilled at any point, at which the derivative exists (see theorem 1). Therefore the task is to find two such vectors $(x,w)^*_1$, $(x,w)^*_2$ and corresponding neighborhoods satisfying the system:

$$p\frac{\partial f(x)}{\partial x}g(w) = w$$
$$pf(x)\frac{dg(\overline{w})}{dw} - x > 0 \quad \overline{w} < w \qquad \textbf{(11)}$$
$$pf(x)\frac{dg(\overline{w})}{dw} - x < 0 \quad \overline{w} > w$$

where derivatives are computed in given vectors. We could use the results of theorem 2 for a formulation of these conditions, since all the assumptions required by this theorem are satisfied[4] (the existence of a function z is guaranteed by the properties of a function f - by Inada conditions and strict concavity that is necessary for the uniqueness of z)[5].

The condition (11) can be revised with respect to the special forms of the assumed functions:

$$\frac{1}{pk_i} < \frac{f(x_1^*)}{x_1^*} < \frac{1}{pk_{i+1}} \qquad \frac{\partial f(x_1^*)}{\partial x} = \frac{w_1^*}{pg(w_1^*)}$$
$$\frac{1}{pk_j} < \frac{f(x_2^*)}{x_2^*} < \frac{1}{pk_{j+1}} \qquad \frac{\partial f(x_2^*)}{\partial x} = \frac{w_2^*}{pg(w_2^*)} \qquad \textbf{(12)}$$

where i<j and

$$w_1^* = \sum_{m=0}^{i} d_m \ , \ g(w_1^*) = \sum_{m=0}^{i} k_m d_m \ , \ w_2^* = \sum_{m=0}^{j} d_m \ , \ g(w_2^*) = \sum_{m=0}^{j} k_m d_m \qquad \textbf{(13)}$$

symbols k_m and d_m have the same meaning as in definition (9).

The Existence of Multimodality in the Wage Trap Model

This section will be devoted to proving the existence of multimodality in the wage trap problem of profit maximization.

We will consider the following form of a technological production function:

$$f(x) = Cx^a \qquad a < 1 \qquad \textbf{(14)}$$

where x denotes labor input (it should be noted that the considered form of the

[4] We need not prove the existence of a neighborhood required in theorem 2. Those intervals exist for x due to the continuity of the first derivative of a function f and for w due to the continuity of f and x.

[5] The following reason explains the strict inequalities in the conditions (11): If there were a maximum and one of the non-strict inequalities was 0, then we would have a whole interval, $<w_1,w_2>$, where all the functional values at points with the remaining derivations equal to 0 would be the same. But there is at least one point among them which has continuous partial derivations in some neighborhood of that point. Such a point cannot be the maximum according to theorem 1 and this is a contradiction to the assertion that this point had the same value as the maximum.

technological production function does not substantially restrict the solution of the problem). The condition for parameter a ensures strict concavity, and this function also fulfills the other assumptions, and thus meets all the requirements used in theorems which will be referred to in later proofs.

The existence of a multimodality of solutions in the profit-maximization problem (3) for a producer with the technological production function (14) will be proved in the following theorem. It will be proved that (at least) two maxima may exist independently of the value of output price p (i.e. this pair will be ensured for all values of p).

Theorem 3: There always exists an effort function $g(w_1)$ for each technological production function f(x) that satisfies:

$$\frac{\partial f(x)}{\partial x} = a\frac{f(x)}{x} \qquad a<1 \tag{15}$$

such that problem (3) has (at least) two solutions for all values of p.[6]
Proof: See Appendix

Multimodality in the decision problem of a profit maximizing producer - an interpretation

Example 1: Let us have the following production function: $F(x,w)=Cx^a g(w)$, where x denotes labor, w is wage and:

g(w) =1/2 w for w in [0,1]
=1/4 (w-1) + 1/2 for w in [1,2]
=1/8 (w-2) + 3/4 for w in [2,4]
=1 for w > 4

i.e., there are 4 effort levels: unskilled, semi-skilled, skilled and professional (the forms of both production and effort functions do not restrict results in any way). We will look for the solution to the problem:

$$\max_{x,w} V(x,w) = pCx^a g(w) - wx \qquad x,w \geq 0 \tag{16}$$

[6] Note that technological production function (14) satisfies condition (15)).

[redacted]ormulae:

[redacted] $ax^{a-1}g(w)-w=0$ **(17)**

[redacted] have different forms for different intervals:

[redacted]): $\frac{1}{2}Cx^{a}p-x$

[redacted]): $\frac{1}{4}Cx^{a}p-x$ **(18)**

[redacted]: $\frac{1}{8}Cx^{a}p-x$

[redacted]: $0Cx^{a}p-x$

[redacted]xima with different values of w:

[redacted] $x^{a}p-x<0 \Rightarrow Cx^{a-1}<\frac{2}{p}$

[redacted] $^{a}p-x<0 \Rightarrow \frac{2}{p}<Cx^{a-1}<\frac{4}{p}$ **(19)**

[redacted] $p-x<0 \Rightarrow \frac{4}{p}<Cx^{a-1}<\frac{8}{p}$

[redacted] $-x<0 \Rightarrow Cx^{a-1}>\frac{8}{p}$

[redacted] simultaneously to obtain the optima

[redacted] p $\Rightarrow$ a

[redacted] p $\Rightarrow$ a

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[redacted]ameter a, which ensure two optima **(20)**

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[redacted]orms of the

effort function are used).[7]

It was proven that the problem (16) has two solutions $(x,1)_1^*$ and $(x,2)_2^*$ for all prices p. This means there are always two (continuous) supply and two corresponding factor demand functions for all prices. The explicit form of demand as well as supply functions can be expressed by equations (17):

$$x(p)=\left(\frac{g(w)Cpa}{w}\right)^{\frac{1}{1-a}} \qquad y(p)=C\left(\frac{g(w)Cpa}{w}\right)^{\frac{a}{1-a}} g(w)$$

$$\Rightarrow w=1: \quad x(p)=\left(\frac{1}{2}Cpa\right)^{\frac{1}{1-a}} \quad y(p)=\left(\frac{1}{2}C\right)^{\frac{1}{1-a}}(pa)^{\frac{a}{1-a}} \tag{21}$$

$$w=2: \quad x(p)=\left(\frac{3}{8}Cpa\right)^{\frac{1}{1-a}} \quad y(p)=\left(\frac{3}{4}C\right)^{\frac{1}{1-a}}\left(\frac{pa}{2}\right)^{\frac{a}{1-a}}$$

Now the profit function π=py-wx can also be obtained:

$$\pi(p)=Cpg(w)\left(\frac{g(w)Cpa}{w}\right)^{\frac{a}{1-a}}(1-a)$$

$$\Rightarrow \quad w=1: \quad \pi(p)=\left(\frac{1}{2}Cp\right)^{\frac{1}{1-a}} a^{\frac{a}{1-a}}(1-a) \tag{22}$$

$$w=2: \quad \pi(p)=\left(\frac{3}{4}Cp\right)^{\frac{1}{1-a}}\left(\frac{a}{2}\right)^{\frac{a}{1-a}}(1-a)$$

The resulting functions of the same type (demand, supply, profit) for the different values of w are mutually distinguished only according to the multiplicative constant. They never intersect each other unless the curves are coincident. This occurs if:

$$\frac{1}{2}=\frac{3}{4}\frac{1}{2^{a^*}} \qquad \Rightarrow \qquad a^*=\frac{\ln\frac{3}{2}}{\ln 2} \tag{23}$$

for supply and profit functions. In other words, it is possible that the final effect for the level of profit and the supplied quantity of a product is the same for both

[7] We proceeded by the solving in the way that we found some group of production functions, in which there is multimodality, for the given effort function. The process may be operated just conversely, i.e. to find the effort function for the exactly defined production function.

types of producers, although they use different production strategies (the amounts of workers and their qualities are different). It should be noted that the coincidence may never occur in input demand curves.

Interpretation: If a profit-maximizing producer takes a wage as an endogenous variable, then under the conditions posed by the model (4) he will never utilize input factors nor produce on the highest border of technological possibilities (as he would in the neoclassical standard model of profit maximization). Rather, he will produce on a certain **production level** (below a production function surface) with workers that are paid a wage w, and therefore their production function is f(x)g(w). An increase of productivity (due to higher value of an effort function) must be paid for by a higher wage.

We have shown that it is possible for a producer to set several different production levels (by choosing different wages) and to be still in a (locally) optimal production situation. Inevitably, the question rises about possibilities of motion along and between the particular levels. This problem is very important especially in profits, since there are two maxima of the task (3) for all prices, to which generally different values of profit correspond. An obvious endeavour of a producer will be to reach the maximal level of profit.

There are two possible ways to look at a production function. The first is to assume that a producer knows his production function and uses this knowledge in decision making. The second case is to look at a production function as the description of producer behavior (a producer does not know his production function, but he behaves "unwittingly" in the way described by this function). The second meaning will be considered throughout this paper.

The problem of how to reach the maximal profit level is easily solved in the standard efficiency wage theory. The assumption of profit maximization implies that a producer beginning to produce with certain (maybe even randomly chosen) amounts of input factors always tries to improve his/her profit. But it also means that a producer moves according to the trial-error method in the direction of the highest increase of the profit function. An assumption of strict concavity of the production (and then profit as well) function assures that each production plan will converge in the case via "continuous gradual approximation", at the unique optimum, stable in the sense that an arbitrary attempt to move out of this optimum induces a loss in profit.

The situation is completely different in the case of the wage trap decision problem (3). Existence of more optima causes a production plan, chosen by a producer who adheres to the above described method to converge at the one local optimum, and there is no way to continuously arrive at a possible higher optimum (this case can be called **a wage trap solution**). This means that there is no way to ensure by market principles that the maximal value of profit will be attained in the decision problem of a profit-maximizing producer, who schedules the magnitude of labor costs himself (herself). The shift to a better local optimum is only possible by a jump (not continuously), when a producer may be faced with potential barrier in the form of a temporary decline of profitability or production, if the initial jump is not "big enough".[8]

The practical application of such a process is rather difficult, because the producer does not know his/her production function and therefore also does not know (in contrast to the classical efficiency wage model) whether there is ever a higher optimum in the "direction" in which he/she is modifying the production plan. It may therefore be supposed that a successful decision depends to a high degree on the quality of a manager and/or on experience, which could be provided by a consulting firm, for example.

Remark: It should be noted that there is no general rule as to which type of labor produces a higher profit. It can be read from (22) and (23) that if parameter $\underline{a}$ is greater than $\underline{a}^*$ (which indicates more labor intensive production), it is more profitable to use less skilled workers (by setting w=1) whereas if $a<a^*$, then the production of more skilled workers (with w=2) leads to a higher profit (in other words, a higher production level does not necessarily bring greater profit).

Example 2 (multiple optima - restricted case): It is possible to create models where multiple optima are connected only with some values of price p.

Let us have the following problem:

$$\max \; V(z,m)=pz^{a}g(m)-rz-m \qquad m\geq 0 \;,\; z\geq 0 \tag{24}$$

It describes the maximization problem of a producer who uses both non-labor (z; r is its price) and fixed labor ($x=\underline{x}$=const) inputs in production with a technological production function $y=z^a$. A producer can influence workers by labor costs $m=w\underline{x}$ (w is a wage) according to the mechanism discussed above.

[8] We can imagine that this transition runs in more steps. Then the initial small jumps lead to the lower profit levels between considered two local profit maxima.

If we consider the effort function to be in the same form as in example 1, then after derivations analogous to those of example 1 we get input demand curves conditioned by the value of ratio p/r:

$$
\begin{aligned}
m=1: &\quad 2^{1-a}\left(\frac{2}{a}\right)^a < \frac{p}{r} < 4^{1-a}\left(\frac{2}{a}\right)^a &\Rightarrow\quad z=\left(a\frac{1}{2}\frac{p}{r}\right)^{\frac{1}{1-a}}\\
m=2: &\quad 4^{1-a}\left(\frac{4}{3a}\right)^a < \frac{p}{r} < 8^{1-a}\left(\frac{4}{3a}\right)^a &\Rightarrow\quad z=\left(a\frac{3}{4}\frac{p}{r}\right)^{\frac{1}{1-a}}\\
m=4: &\quad 8^{1-a}\left(\frac{1}{a}\right)^a < \frac{p}{r} &\Rightarrow\quad z=\left(a\frac{p}{r}\right)^{\frac{1}{1-a}}
\end{aligned}
\qquad \textbf{(25)}
$$

That is, (if we set r=1) price intervals exist, where problem (24) has two solutions: $(4^{1-a}[4/3a]^a;4^{1-a}[2/a]^a)$ and $(8^{1-a}[1/a]^a;8^{1-a}[4/3a]^a)$. In a>1/2, then there are even three solutions in the interval $(8^{1-a}[1/a]^a;4^{1-a}[2/a]^a)$.

Interpretation: Most of the results of example 1 remain valid. The only difference is that there is now a way in which a market can help a producer to change a production level "continuously" - to change the price p (when r remains fixed). When price p rises above a certain value p_1, a potential barrier disappears and a producer can continuously attain some higher value of a profit without its decline. An analogous situation, but one connected with a transition to the lower production level, occurs in the case of a decline of a price below p_2.[9] If a producer stays only in the local optima, then, with respect to the previously described ways of motion in the set of admissible production possibilities, he/she cannot choose the level with momentary higher value of profit, but he/she is "locked in" on one particular level as long as the potential barrier does not disappear. Therefore, it is better to speak about a "supply net" rather than a supply curve, since it is possible for the producer to move in a "circle" rather than along a line, where a price comes up and then down. A production net is then formed from severale such "circles" tied to each other.

[9] $p_2 < p_1$ defines an interval, for which (it was proven) there exist optima for two subsequent values of m and therefore as well as for two subsequent production levels (m influences productivity).

CONCLUSION

The purpose of this paper has been to create a model that could describe the difference between producer behavior in market and planned economies, in order to explain changes in economies that move from planned towards market structures (changes in firm production and profitability, wage differential increase, etc).

It has been shown that if the standard efficiency wage model is modified to reflect the impossibility of perfectly managing labor utilization, there may exist multiple optima in the profit maximization problem. The producer can then move to the higher profit optimum only if he substantially changes the wage paid to his workers.

These facts have quite substantial implications for firms in transition economies. For example, the model can explain why foreign firms acting in transition economies may be more profitable and productive than domestic ones even if they use the same capital inputs - simply because they pay workers more, and are therefore able to attract higher quality workers.[10] There is another substantial advantage of foreign firms. If domestic firms want to pick the correct skill level of workers (through setting the optimal wage), they often have to incur high fixed costs connected with either retraining or laying off the present labor force in their firms. On the other hand, foreign firms entering markets in transition economies can often immediately choose the appropriate quality of workers, thus avoiding these fixed costs. The existence of these "transaction" costs may cause, even though the highest profit maximum is definitely optimal from the long-run perspective, some domestic firms (with financial difficulties) to choose, in the short-run, to stay in the (initial) lower local maximum to avoid retraining (lay off) costs.

It has also been shown that if the change of wages is executed in several small steps this process may lead to a temporary decline in profits.

Generally, the model suggests the inappropriateness of all wage restriction policies, because they could prevent the producer from getting out of the wage trap. This fact is especially important for transition economies, where production

[10] Production levels can be also viewed as the levels of X-efficiency. In that case the wage increase (or possibly any production change based on foreign experience) leading to the movement towards the higher optimum is caused by the increase in X-efficiency (which may support the original Leibenstein's theory -see [3]).

plans were originally distorted and are now to be restored. The wage restrictions should not be implemented even in the case of a fall in firm profitability, because a producer may be just overcoming the potential barrier connected with movement between profit maxima; the wage increase may be a step in the right direction.

Appendix

This appendix will be devoted to mathematical proofs.

Theorem 1: (page 11)
Proof:

$$H(\pi)=\begin{vmatrix} pf_{xx}g & pf_x g'-1 \\ pf_x g'-1 & pfg'' \end{vmatrix}=p^2(fg''gH(f)+H(\pi_0)) \qquad \textbf{(26)}$$

$H(\pi_0)$ is f_{xx} bordered by ($f_x g'$-1) with zero at (2,2), where f_{xx} (f_x) mean the second (first) derivative of f w.r.t. x, g' is the derivative w.r.t. w. For the function π to be concave, the principal minor determinants must have sign$(-1)^k$=1,2. The first minor fulfils this condition due to the strict concavity of the function f. The minor determinant of the order 2 is $H(\pi)$. Because g''=0 according to the assumptions, the first element of (26) is equal to 0. We use the following theorem (H. Varian [9], p. 309) to determine the sign of the second element:

A matrix of an order (n,n) is negative definite s.t. bx=0 (b,x are n-dimensional vectors) if and only if all the principal minor determinants of a matrix A bordered by the vector (b_1,...,b_k) have sign $(-1)^k$=2,...,n.

Because f_{xx} is negative, the matrix created from this element bordered by an arbitrary number has the sign (-1). But that means, $H(\pi)$ has the same sign. Since a necessary condition for a concavity of U is sign$(H(\pi))=(-1)^2$, function π is not therefore concave at any point, where the second derivative exists.

Q.E.D.

Theorem 2: (page 12)
Proof:Let us take an arbitrary point from the given neighborhood (x_1,...,x_n). When there is a vector with the same last element ($\underline{x}_1$,...,$\underline{x}_n$), partial derivatives of f w.r.t. the first n-1 vector elements are equal to 0 (see assumptions). Now we will use the proposition about an increment of a function in the following form to compare functional values of both points:

Let g be a function and B($\underline{x}$,r) be a cube, where the first derivative exists in all the points. Then

$$g(\underline{x})-g(x)=\sum_{j=1}^{n}\frac{\partial g(u^j)}{\partial x_j}(\underline{x}_j-x_j) \qquad \textbf{(27)}$$

holds for an arbitrary point of this cube, where the following holds for vectors

u^j=1,...,n: they belong to B($\underline{x}$,r) and their j-th elements are in the interval defined by the elements $\underline{x}_j$ and x_j.

The inequality $f(\underline{x}_1,...,\underline{x}_n) > f(x_1,...,x_n)$ follows then from the proposition, because all the elements of the sum expressing the difference of functional values are positive (see assumptions) except the last one, which is equal to 0 due to the equality of vector components.

We will define the function $z(x_n) = f[x_1(x_n),...,x_{n-1}(x_n),x_n]$, which assigns the value of the function f to x_n at the point where partial derivatives w.r.t. the first n-1 variables are 0 (the possibility of such a definition is assured in assumptions). Then:

$$f(x_1^* ... x_n^*) - f(\underline{x}_1 ... \underline{x}_n) = z(x_n^*) - z(\underline{x}_n) = \int_{\underline{x}_n}^{x_n^*} z(u)' du = \int_{\underline{x}_n}^{x_n^*} \sum_{j=1}^{n} \frac{\partial f(x_j)}{\partial x_j} \frac{\partial x_j(x_n)}{\partial x_n} dx_n > 0 \quad \textbf{(28)}$$

because the function z is constructed in such a way that partial derivatives of the function f w.r.t. $x_1,...,x_{n-1}$ are 0 and the sign of the derivative of f w.r.t. x_n depends on the difference of limits of an integral, the assumptions assure positivity of the resulting integral at the same time. But then $f(x_1,...,x_n) < f(\underline{x}_1,...,\underline{x}_n) < f(x_1^*,...,x_n^*)$ holds for an arbitrary vector of the given neighborhood and the vector $(x_1^*,...,x_n^*)$ is the maximum of the function f.
Q.E.D.

Theorem 3: (page 15)
Proof: We will even prove that maxima exist for $w^*=d_1$ and $w^*=d_1+d_2$, where $d_1=d_2=1$.
i) We choose arbitrary $k_1<(1/2)$ and d_1 s.t. $k_1d_1<1$. We will calculate maximizing value of a function f from the equation:

$$f'(x) = \frac{w}{pk_1d_1} \quad \textbf{(29)}$$

and denote it $(x)_1^*$. A solution exists and is unique due to properties of f - monotonicity, concavity. Because f'(x).x < f(x) holds, then obviously:

$$\frac{1}{pk_1} < \frac{f(x_1^*)}{x_1^*} \quad \textbf{(30)}$$

for every value of p.

ii) We choose k_2: We want:

$$\frac{1}{a}\frac{1}{pk_1}=\frac{f(x_1^*)}{x_1^*}<\frac{1}{pk_2} \tag{31}$$

(the first equality is obvious from (29)). On the other hand, we also want:

$$\frac{1}{pk_2}<\frac{f(x_2^*)}{x_2^*}=\frac{1}{a}\frac{d_1+d_2}{k_1d_1+k_2d_2} \tag{32}$$

(analogy of (30)), where x_2^* is the corresponding maximizing value of f calculated from equation (29) analogously as in (i). Both these conditions give:

$$\frac{ak_1}{2-a}<k_2<ak_1 \tag{33}$$

(because we suppose that $d_1=d_2=1$)
iii) Finally, constants k_3 and d_3 will be determined as to satisfy $k_3<k_2$, $k_1d_1+k_2d_2+k_3d_3<1$ (it is possible, because we determined $k_2<k_1<(1/2)$ and

$$f'(x_2^*)<\frac{1}{pk_3} \tag{34}$$

We found constants k_1,k_2,k_3,d_1,d_2,d_3 in the sense of definition (9), price vector (p,w) and two values x_1^* and x_2^* satisfying relations:

$$\begin{aligned}\frac{1}{pk_1}<\frac{f(x_1^*)}{x_1^*}<\frac{1}{pk_2} \qquad f'(x_1^*)=\frac{w}{pk_1d_1}\\ \frac{1}{pk_2}<\frac{f(x_2^*)}{x_2^*}<\frac{1}{pk_3} \qquad f'(x_2^*)=\frac{w}{p(k_1d_1+k_2d_2)}\end{aligned} \tag{35}$$

According to the previously proved theorems, this means that vectors $(x,w)_1^*$ and $(x,w)_2^*$, where $w_1^*=d_1$ and $w_2^*=d_1+d_2$, are maxima of the decision problem (3). Moreover, the proof was made independent of the choice of p. Q.E.D.

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