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RNDr Miloslav Kopecký

ÚVOD DO THEORIE ROZLOŽENÍ A VÝSKYTU SLUNEČNÍCH SKVRN NA SLUNEČNÍM DISKU

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THE OUTLINE OF THE THEORY OF DISTRIBUTION AND OCCURENCE OF SUNSPOTS ON THE SOLAR DISC

By

MILOSLAV KOPECKÝ

INTRODUCTION

The aim of this paper is to gather all information concerning the theory of distribution and occurence of sunspots up to this date. It comes from a series of papers of ther authors, as well as from papers of the author of this publication, published in various scientific journals. These papers are supplemented with some new knowledge and in this way, it is an attemp to give a complete surwey of the fundamentals this theory.

This paper however, does not deal with the theory of distribution of spots from the usual point of view, for the more common theory has not yet been thoroughly workedout up to the present time. This theory of distribution of spots is developed under certain simplifying assumption, of which the most important are the following:

1.) ***All sunspots occur only on the equator, which always pass through the center of the solar disc. I. e. „the royal strip" of the occurence of sunspots around the equator, may be substituted by a cylinder, at which we are looking in the direction of the normal to its mantle.***

2.) ***Practically we assume, that the groups of sunspots are formed by only one circular shaped spot.***

Thus these assumptions enable us to establish, as completely as possible, the fundamentals of this theory of spots, which will be a starting point for the more common theories.

The task of this theory is, first of all, to define some basic functions and relations affecting the distribution and presence of sunspots on the solar disc, and to find a method of their determination (Chapter I and II). Based on the knowledge of these functions and relations some observed systematic occurrences in the distribution and presence of sunspots on the solar disc are then derived theoretically. It is, firstly, the decrease of sunspots, in approaching the Sun-limb (Chapter III), which is shown not to *be proportional to the cosine of the angular distance from the central meridian, an assumption mistakenly made by several authors.*

Further, a theoretical derivation is given of the number of formed and vanished sunspot groups at different distances from the central meridian (Chapter IV). Here it is shown that these numbers are affected essentially by the fact that the sunspots are being observed only once in 24 hours.

Finally, methods are developed of computing the number of sunspot groups formed on the total surface of the Sun, and of their average life-time (Chapter V). These two characteristics are of great importance for the investigation of the sunspot periodicity.

Finally I should like to thank Dr M. Blaha *and Mr.* P. Mayer *for suggestive discussions.*

The Astronomical Institute

Ondřejov, July, 1954.

THE BASIC NOTATION

S ... area of the sunspot group
S_M . . maximum area during the development of the sunspot
k_1, k_2 ... tangents of increase and decrease of the area of sunspots

$$k = \frac{k_2 - k_1}{k_1 k_2}$$

t ... time counted from the real origin of the group
T ... life-time of the sunspot group
T_0 ... average life-time of the sunspot group
λ ... angular distance from the central meridian
$\Phi(\lambda)$ ... function of visibility
$\Theta(\lambda)$ ... function of foreshortening
f ... number of groups with life-time from T to $T + dT$ originated on the whole sun during a time unit
f_0 ... **number of all groups originated on the whole sun during a time unit and its life-time is greater than L.**

$$\Delta f = \frac{f}{2\pi}\,;\ \Delta f_0 = \frac{f_0}{2\pi}$$

S_0 ... **minimum visible area of sunspots in the central meridian**
N ... **number of groups existing on the whole solar surface**
$\bar{N}$ ... number of observed sunspot groups
$\bar{f}$ or $\bar{f}_1$ ... number of apparently originated sunspot groups
$\bar{f}_2$ ... number of apparently decayed groups

$$F(T) = \frac{f}{f_0}$$

ω ... angular velocity of the solar rotation
$N(S)$ ... the number of groups on their instantaneous area
L ... latent life-time of sunspot groups.

I. BASIC FUNCTIONS AND RELATIONS

§ 1. *The Life Curve of the Area of Spot Groups*

Let us first make a simplified assumption, that the area of a spot group S increases linearly with time to its maximum, and then again decreases linearly with time, in such a way, as illustrated in Figure 1.

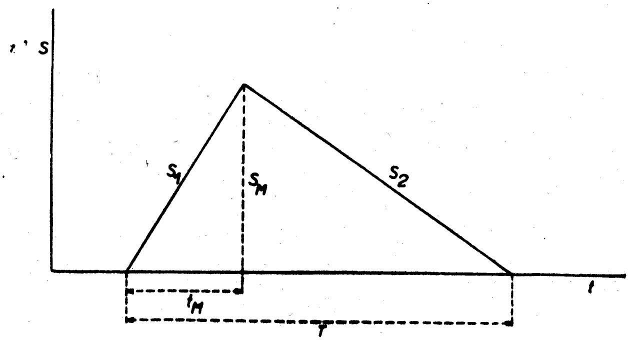

FIG. 1.

How far this simplifying supposition proves satisfactory, is demostrated in paper [1].

The area of the spot group on the ascending part of the development of the group, S_1, and that one on the descending part, S_2, and the maximum area of the group, S_M, are given by the relations [1],

$$S_1 = k_1 t; \quad S_2 = k_2(t - T) \tag{1,1}$$

$$S_M = k_1 t_M = k_2(t_M - T) \tag{1,2}$$

where t is the time counted from the origin of the group and where

$$k_1 > 0; \quad k_2 < 0 \tag{1,3}$$

are the tangents of the increase and decrease of the group area, t_M is the time when the area of the group reached its maximum and T is the life-time of the spot group. By eliminating t_M from the relation (1,2) we obtain

$$T = \frac{k_2 - k_1}{k_1 k_2} S_M = k S_M \,. \tag{1,4}$$

M. N. GNEVYSHEV [2] found a linear relation between T and S_M, whereby if S_M is expressed in millionths of the area of the solar hemisphere and T in days, the coefficient

$$k = 0{,}1 \,. \tag{1,5}$$

From the equation (1,4) it follows that between k_1 and k_2 the relation

$$k_2 = \frac{k_1}{1 - 0{,}1 k_1} \tag{1,6}$$

is valid.

On the basis of the mean curves of the development of area constructed in Greenwich [3] and by M. WALDMEIER [4], it is possible to obtain the dependence of k_1 on the maximum area of group S_M [1], for which we may write the following relation:

$$k_1 = 0{,}094 \,.\, S_M + 9{,}3 = 0{,}94 \,.\, \mathrm{T} + 9{,}3 \,. \tag{1,7}$$

The dependence of the tangents k_1 and k_2 on the maximum area of group S_M or on the life-time T, is determined by the equations (1,6) and (1,7).

However, at the first approximation, we can always assume that

$$|k_1| > |k_2| \tag{1,8}$$

i. e., the area of the group increases faster than it decreases after reaching its maximum. This approximation is in agreement with observations.

§ 2. *The Dependence of the Number of Groups on their Life-time*

On the Sun there exist groups of the most different life-times. However, we are able to observe only such groups of spots, which have a life-time longer or equal to a minimum life-time L. This minimum life-time L have such groups, which with S_M as their maximum area, have reached the minimum area of visibility in the central meridian, S_0, so that with respect to the relation (1,4), L is given by the following expression:

$$L = kS_0 \,. \tag{1,9}$$

On the whole Sun there originate in a unit of time $f\,\mathrm{d}T$ groups with a life-time T. From all the groups with various life-times originating in the course of a time unit, we can observe f_0 groups, where

$$f_0 = \int_L^\infty f\,\mathrm{d}T \,.$$

Then we can write [5]:

$$\frac{f}{f_0}\,\mathrm{d}T = F(T)\,\mathrm{d}T \,, \tag{1,10}$$

where the function $F(T)$ determines the relative number of spot groups with various life-times and therefore, with regard to the relation (1,4) also with various maximum areas S_M, in the total material of observation.

The function $F\ (T)$ is limited by the conditions

$$\int_L^\infty F(T)\,\mathrm{d}T = 1 \,, \tag{1,11}$$

$$\lim_{T\to\infty} \mathrm{F}(T) = 0 \,. \tag{1,12}$$

The sum of life-times ΣT of all groups of spots, with a life-time from T to $T + \mathrm{d}T$, is given by the expression

$$\Sigma T = f_0 T F(T)\,\mathrm{d}T \,. \tag{1,13}$$

The sum of all life-times of all spots which have life-times from L to ∞ is therefore given by the integral on the right side of the equation (1,13). As the number of all groups of spots entering our statistics is f_0, the mean life-time T_0 of all groups observed by us is given by the following expression:

$$T_0 = \int_L^\infty T F(T)\,\mathrm{d}T \,. \tag{1,14}$$

The course of the function $F(T)$ varies during the eleven year cycle of sunspots, because the mean life-time T_0 changes as was shown in papers [1,9].

§ 3. *The Basic Relation in the Statistics of Sunspots*

First let us take into consideration only groups with life-times from T to $T + \mathrm{d}T$. The number of groups originating per unit of time on the entire Sun is $f\,\mathrm{d}T$. Then it holds, that the number of all existing groups $\mathrm{d}N$ on the visible and invisible solar hemisphere, having life-times from T to $T + \mathrm{d}T$, is given by the expression [6]

$$\mathrm{d}N = fT\,\mathrm{d}T\,. \tag{1,15}$$

Hence we get the number of all existing groups N on the whole Sun at a given instant by the integral of the equation (1,15), where for f we substitute from equation (1,10)

$$N = f_0 \int_L^\infty TF(T)\,\mathrm{d}T\,. \tag{1,16}$$

With respect to equation (1,14), the equation (1,16) then changes to the required form

$$N = f_0 T_0\,, \tag{1,17}$$

which indicates, that the number of spot groups N, occuring at a given instant on the whole Sun, is determined by the number of all originated groups on the entire Sun and by their mean life-time. This relation is very significant for the investigation of the periodicity of sunspots, as will be shown in Chapter V.

§ 4. *The Function of Visibility and the Function of Foreshortening of Areas of Spot Groups*

We define the function of foreshortening in the following manner: A group of spots, which is at a distance λ from the central meridian, has an area S. Due to the projection and other, up to this date unknown reasons, we see this group of spots with an area S'. Then the function of foreshortening of area of spot groups, $\Theta(\lambda)$, is given by the relation

$$S = S'\Theta(\lambda) . \tag{1,18}$$

We define the function of visibility in the following manner:

A group of spots, which is in the central meridian, must have a minimum area S_0, depending on the resolving power of the telescope. In order to be able to see a spot group at a distance λ from the central meridian, it must have the actual area S larger or equal to the area $\bar{S}$, which is determined by the relation

$$\bar{S} = S_0\Phi(\lambda) . \tag{1,19}$$

The course of the functions $\Theta(\lambda)$ and $\Phi(\lambda)$ depends, most probably, both on the size of the group and its structure, and eventually even on other factors, and therefore will differ for various groups. Thus far however, we are obliged to assume that $\Theta(\lambda)$ and $\Phi(\lambda)$ are the same for all groups, as we practically know nothing, up to this date, of the above mentioned dependences. Furthermore, we assume that both the functions, are purely monotone, increasing functions, whereby for $\lambda = 0$ is $\Phi(\lambda) = \Theta(\lambda) = 1$, for $\lambda > 0$ is $\Phi(+\lambda) = \Phi(-\lambda)$ and $\Theta(+\lambda) = \Theta(-\lambda)$, and for $+\lambda_1 < +\lambda_2$ is

$$\left(\frac{d\Phi(\lambda)}{d\lambda}\right)_{\lambda=+\lambda_1} < \left(\frac{d\Phi(\lambda)}{d\lambda}\right)_{\lambda=+\lambda_2}$$

and

$$\left(\frac{d\Theta(\lambda)}{d\lambda}\right)_{\lambda=+\lambda_1} < \left(\frac{d\Theta(\lambda)}{d\lambda}\right)_{\lambda=+\lambda_2} .$$

The functions $\Phi(\lambda)$ and $\Theta(\lambda)$ need not be identical, as was incorrectly assumed by the majority up to now [1, 17, 18, 5, 11, 12, 8]. We may demonstrate it in this manner:

Let us assume that all groups are formed by one circular shaped spot and that the foreshortening of area of spots towards the solar limb is caused by nothing else than by mere projection. Then

$$\Theta(\lambda) = \sec\lambda . \tag{1,20}$$

In order to be able to see the spot in the central meridian, it must have a minimum radius r_0, which depends on the resolving power of the telescope. A spot with a minimum radius r_0 has a minimum visible area S_0,

$$S_0 = \pi r_0^2 . \qquad (1,21)$$

A spot at a distance λ has a radius r and therefore an actual area

$$S = \pi r^2$$

and its apparent area S' is

$$S' = \pi r^2 \cos \lambda .$$

It has been assumed up to now, that in order to see a spot at a distance λ from the central meridian, its apparent area S' must equal the minimum observable area S_0 so that

$$S_0 = \pi r^2 \cos \lambda = S \cos \lambda . \qquad (1,21a)$$

Hence it follows that

$$\bar{S} = S_0 \sec \lambda$$

and with regard to equations (1,18) and (1,19),

$$\Phi(\lambda) = \Theta(\lambda) = \sec \lambda .$$

The resolving power of a telescope however, reveal to us which smallest linear dimensions we are still capable of determining. Hence follows the second possible condition for the visibility of a spot at a distance λ from the central meridian. In order to see a spot at a distance λ from the central meridian, its apparent radius must be in the direction of the heliographical longitude, (i. e. its small semi-axis, as the spot in this case is apparently elipse shaped), again equal to r_0. Its real radius then is [7,23]

$$r = r_0 \sec \lambda \qquad (1,22)$$

and its actual area S is given with regard to the equation (1,21) by the expression

$$S = \pi r_0^2 \sec^2 \lambda = S_0 \sec^2 \lambda .$$

Hence according to the definition it follows that

$$\Phi(\lambda) = \sec^2 \lambda . \qquad (1,23)$$

From equations (1,20) and (1,23) it follows that in case of the existence of projection itself and the validity of the second condition (1,22), which is more probable than the first condition (1,21a), the following relation between the function of visibility $\Phi(\lambda)$ and the function of foreshortening $\Theta(\lambda)$ holds true for circular spots:

$$\Phi(\lambda) = [\Theta(\lambda)]^2 .$$

It follows, therefore, that not even in a common case need the function $\Phi(\lambda)$ and $\Theta(\lambda)$ be identical. Up to the present time it is not evident, when the visibility of the sunspot is determined by its linear diameter or area. It is necessary to solve definitely this problem on the basis of material gathered by observation.

§ 5. *Minaert's Diagram*

Very important for the theory of statistics of sunspots, is so-called Minaert's diagram, constructed by M. MINAERT to explain intuitively the reasons for asymetry of the origin and decay of spot groups on the eastern and western half of the solar disc [8]. However, for its grafic representation, this diagram is significant also for solving other problems concerning the statistics of spots.

Minaert's original diagram is shown in Figure 2 and its schematic explanation in Figure 4. The distance from the central meridian is plotted on to the axis x and the actual area of spots to the axis y. The curve $\bar{S}$ is the curve of the visibility of spots, the course of which is given by equation (1,19). It determines the area which the group of spots must have, so that it can be observed at a distance λ from the central meridian.

Due to the influence of the rotation of the Sun, the spot in Minaert's diagram moves from the east to the west, so that during a time t it moves through an angle λ,

$$\lambda = \omega t , \qquad (1,24)$$

where ω is the angular velocity of the solar rotation. Due to the development of spot groups, the axis x in Minaert's diagram is, as follows from equation (1,24) the axis of time. We can therefore directly draw into this diagram the developing curves of spot groups in such a way as it is in Figure 2, or as it is in the case of linear developments of spot groups in Figure 4. (see § 1.). The group is observable as long as the cur-

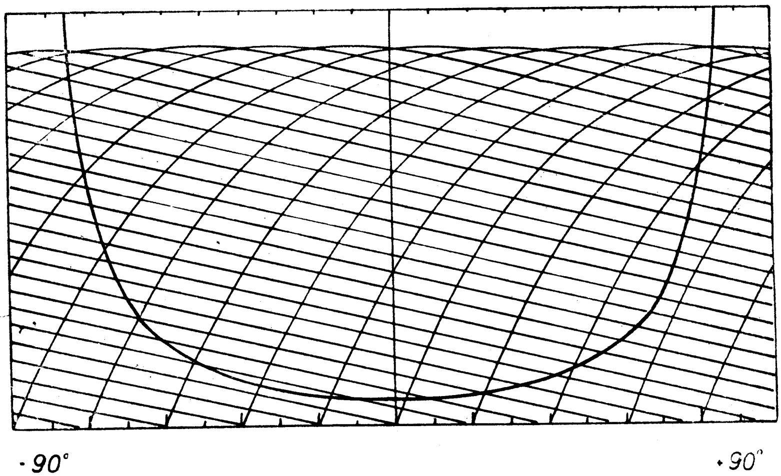

FIG. 2.

ve of the development of the area of the spot lies above the curve of visibility $\bar{S}$. The points of intersection of both these curves then indicate the distance from the central meridian, in which both groups can be observed either for the first or for the last time.

The linear development of the area of spot groups in Minaert's diagram, is with regard to equations (1,1), (1,4) and (1,24) given by the relations

$$\dot{S}_1 = \frac{k_1}{\omega}\lambda\,; \qquad S_2 = \frac{k_2}{\omega}(\lambda - \lambda_T)\,, \tag{1,25}$$

$$\lambda_T = \omega k S_M\,, \tag{1,26}$$

where λ_T is the angle, through which the Sun turns during a life-time T.

Minaert's illustrative diagram will be used in the next chapters to derive certain relations.

II. THE COURSE OF THE FUNCTIONS $F(T)$, $\Phi(\lambda)$ AND $\Theta(\lambda)$

§ 1. *The Determination of the Course of the Function $F(T)$*

The cours of the function $F(T)$ was determined for the first time by M. N. Gnevyshev [2], by means of the calcules of probability without assumption of linear development of the sunspots area. To determine the course of the function $F(T)$, he used only those groups of spots, whose origin as well as decay was observed on the visible hemisphere of the Sun, whereby he took for the ,,visible side of the Sun" the region from $-73°$ to $+73°$ from the central meridian. Then the probability p, that the group of spots with a life-time T will originate and decay on the visible side of the Sun, is given by the expression

$$p = \frac{146° - \omega T}{360°},$$

where ω is the angular velocity of the solar rotation. If the group decays in the following rotation, then the probability p is given by the expression

$$p = \frac{146° + 360° - \omega T}{360°}$$

etc. The number of groups with a life-time T, will then be determined in such a way, that from observation determined number of groups with a life-time T — whose origin as well as decay was observed — we multiply by the reciprocal value of probability $1/p$. The development of the function $F(T)$, determined by Gnevyshev in this way, is shown in Figure 3, by the dashed curve.

However, we do not actually see all groups up to the distance of

73° from the central meridian and the determination of their real origin is rather problematical, as follows from Minaert's diagram.

M. Kopecký and P. Mayer therefore, worked out a new method to determine the function of $F(T)$ [9]. On the visible as well as the invi-

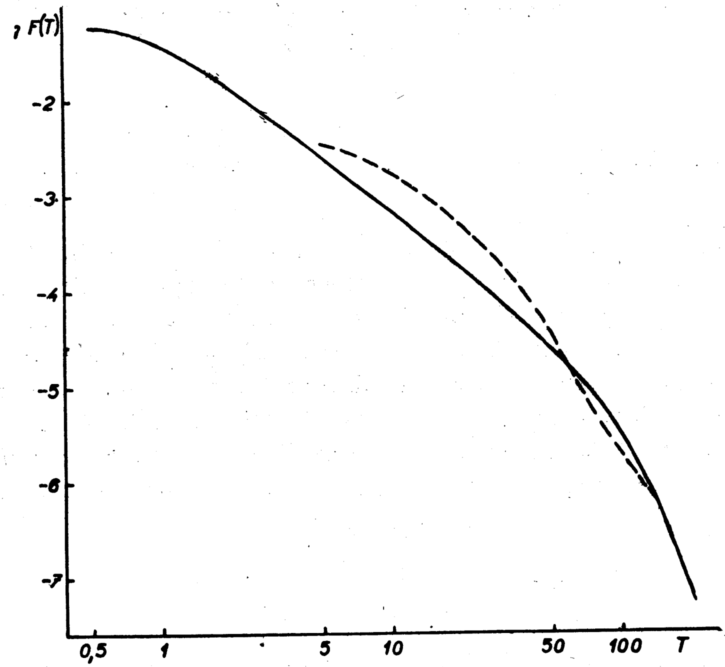

Fig. 3.

sible hemisphere of the Sun there exist on the whole N groups, of which N_1 groups have at the given instant an area S_1, N_2 groups an area S_2, N_z groups an area S_z, etc.

This relative distribution $N(S)$ of existing groups of spots with different areas in the total number of existing groups of spots N, at the given moment depends on how many spots with a certain life-time T, (or having a maximum area S_M) originate per unit of time, in other words, it depends on the function $F(T)$. Hence we can write

$$N(S) = \varphi[F(T)] . \tag{2,1}$$

We can easily statistically determine the course of the function $N(S)$, because it is given by relative distribution of groups of spots with different areas in the close neighbourhood of the central meridian, where we see all spots to their minimum determinable area S_0, as the function of visibility in the close vicinity of the central meridian $\Phi(\lambda) \approx 1$. With regard to the given form of the function φ in equation (2,1) and to the statistically determined course of the function $N(S)$, we are also able to determine the course of the function $F(T)$. We may determine the form of the function φ in the following manner:

Let us derive how many groups of spots with areas between S and $S + \Delta S$ exist at the given instant. Spots on the increasing part of their development remain in the interval of areas ΔS for a period of time Δt_1,

$$\Delta t_1 = \frac{\Delta S}{k_1} .$$

Spots on the decreasing part of their development remain in the interval of areas ΔS during the time Δt_2,

$$\Delta t_2 = - \frac{\Delta S}{k_2} .$$

With regard to equation (1,15) we get that in the interval of areas ΔS there exist at every moment $\mathrm{d}N(S)$ groups with a life-time from T to $T + \mathrm{d}T$, given by equations

$$\mathrm{d}N(S) = f(\Delta t_1 + \Delta t_2) = kf\Delta S \,\mathrm{d}T$$

In reality, there originate spots with various life-times on the Sun, the distribution of which is given by the function $F(T)$. By using the equation (1,10) we then get

$$\mathrm{d}N(S) = kf_0 F(T) \Delta S \,\mathrm{d}T .$$

In the interval of the area from S to $S + \Delta S$, there may exist only such spots, which have a maximum area $S_M \geqq S$, i. e., a life-time $T \geqq kS$. The number of all groups having at the given moment an area from S to $S + \Delta S$ is then given by the relation

$$N(S) = kf_0 \Delta S \int_{T=kS}^{\infty} F(T) \,\mathrm{d}T . \qquad (2,2)$$

The relation (2,2) is the required relation (2,1), where the number of existing spot groups with an area S to $S + \Delta S$, is the function of the lower limit of the integral of the function $F(T)$. The course of the function $F(T)$ we then determine from the known function $N(S)$ by means of the derivative of the equation (2,2) according to the area, i. e., on the basis of the equation

$$F(T) = -\frac{1}{k^2 f_0} \cdot \frac{\mathrm{d}}{\mathrm{d}S}\left(\frac{N(S)}{\Delta S}\right). \tag{2,3}$$

If we do not know the value of the fraction $\frac{1}{k^2 f_0}$ when computing the function of $F(T)$, we may determin from equation (2,3) only the development of the function $F(T)$, and we then determine its value at individual points using the condition (1,11).

The course of the function $F(T)$ determined by this method is given in figure 3 and is marked by the full line [9].

§ 2. *Archenhold's Method to Determine the Function $\Phi(\lambda)$*

The function $N(S)$ determines the number of groups having at a given instant an area S, and we may easily determine it statistically near the central meridian. The number of all groups $\sum_{S''} N(S)$, having an area larger than a certain area S'', then equals

$$\sum_{S''} N(S) = \int_{S''}^{\infty} N(S)\,\mathrm{d}S\,. \tag{2,4}$$

The number of groups $\Delta N_0 \delta\lambda$ observed at a distance λ to $\lambda + \delta\lambda$ is given by the equation

$$\Delta N_0 \delta\lambda = \int_{S_0 \Phi(\lambda)}^{\infty} N(S)\,\mathrm{d}S\,,$$

as will be shown in Chapter III, § 1. We can easily statistically determine even $\Delta N_0 \delta\lambda$. From the known form $N(S)$ and known $\Delta N_0 \delta\lambda$ for a certain λ we determine on the basis of the relation

$$\Delta N_0 \delta\lambda = \int_{S''}^{\infty} N(S)\,\mathrm{d}S \tag{2,5}$$

the unknown lower limit S'' of the integral. From the known minimum visible area S_0 and the limit area S'' determined in relation (2,5), we determine the value $\Phi(\lambda)$ for given λ from the relation

$$\Phi(\lambda) = \frac{S''}{S_0} .$$

This method was used by G. H. A. ARCHENHOLD to determine $\Phi(\lambda)$ for individual spots [7]. This method has a general validity, because the assumption linearity of sunspots development is used.

§ 5. *The Deviation of the Function of Foreshortening $\Theta(\lambda)$ from* sec λ

If a spot group S' is observed, its actual area S is given by equation (1,18). Its area S_r, reduced by means of a secant, which we consider equal to the actual area in the first approximation, is then given by expression

$$S_r = S \frac{\sec \lambda}{\Theta(\lambda)} = S \frac{1}{\varphi(\lambda)} . \qquad (2,6)$$

The number of groups $\sum\limits_{S''} N(S)$, having an actual area larger than S'', is given by equation (2,4).

Now let us examine the decay of the number of groups towards the solar limb, while considering only groups having a larger corrected area than S_r'. If $\Theta(\lambda) = \sec \lambda$, then the number of these groups do not decay from the center of the disc up to the distance of λ' from the central meridian, given by the condition

$$\Phi(\lambda') = \frac{S_r'}{S_0} .$$

However, if $\Theta(\lambda) = \sec \lambda$, we substitute for S'' in the equation (2,4) from the equation (2,6), and consequently we get

$$\sum_{S'_r} N(S) = \int_{S'_r \varphi(\lambda)}^{\infty} N(S) \, \mathrm{d}S . \qquad (2,7)$$

From statistics worked out by F. Link [10], it follows, that the number of nonrecurrent spot groups larger than 50 millionths of the reduced area decays towards the solar limb. Hence it follows that the value of the function $\varphi(\lambda)$ in equation (2,7) must increase with increasing λ, and therefore

$$\frac{d\varphi(\lambda)}{d\lambda} > 0 . \qquad (2,8)$$

Because for $\lambda = 0$ is $\varphi(\lambda) = 1$, is for $\lambda > 0$ $\varphi(\lambda) > 1$, and hence it follows that for $\lambda > 0$ is

$$\Theta(\lambda) > \sec\lambda . \qquad (2,9)$$

From relation (2,9) follow that the areas corected by $\sec\lambda$ are smaller than the real areas. This conclusion has the general validity, because the assumption linearity of sunspots development is used.

III. THE DECAY OF SUN SPOTS TOWARDS THE SOLAR LIMB

In accordance with the rules of projection the observed sunspot areas, when approaching the Sun-limb, appear to diminish, in proportion with the $\cos \lambda$. From this geometrical relation many authors, similarly, derived that the number of sunspot groups, also, and the total of their observed areas ought to decrease with the decreasing $\cos \lambda$. It is the main concern of this Chapter to show that this reasoning is erroneous, and to find, further, by which factors the decrease of sunspots, in approaching the Sun-limb, is affected. This derivation will be carried out on the assumption that we know function $N(S)$ or $F(T)$, respectively, function $\Phi(\lambda)$, and the smallest visible size of areas S_0.

§ 1. *The Decay of Number of Groups*

Let us denote by $\Delta N_0(\lambda)$ the number of all groups which we observe in a unit of heliographic longitude and by Δf_0 the number of originated groups per unit of time in a unit of heliographic longitude, i. e.,

$$\Delta f_0 = \frac{f_0}{2\pi} . \qquad (3,1)$$

The number of groups with an area larger than S'', is given by equation (2,4). At a distance λ to $\lambda + \delta\lambda$ from the central meridian, we are able to observe — due to the function of visibility — only such groups having a larger area than $S_0\Phi(\lambda)$ and their number then, according to M. Hotinli [11], is given by equation

$$\Delta N_0(\lambda)\delta\lambda = \int_{S_0\Phi(\lambda)}^{\infty} N(S)\, \mathrm{d}S\delta\lambda , \qquad (3,2)$$

which determines the decay of the number of groups towards the solar limb.

We can also express the decay of the number of groups towards the solar limb by means of the function $F(T)$, first in such a way that for

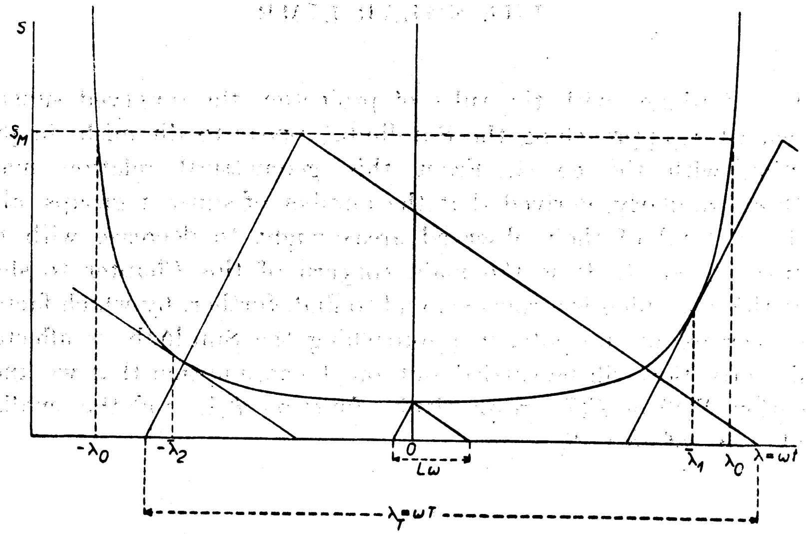

Fig. 4.

the function $N(S)$ we substitute from equation (2,2) where, however, instead of f_0 there will be Δf_0. Hence we get

$$\Delta N_0(\lambda)\delta\lambda = k\Delta f_0 \int\limits_{S_0\Phi(\lambda)}^{\infty} \int\limits_{T=kS}^{\infty} F(T)\,\mathrm{d}T\,\mathrm{d}S\delta\lambda\,. \tag{3,3}$$

We may express the course of $\Delta N_0(\lambda)\delta\lambda$ by means of the function $F(T)$ also in another way [1,5]. First let us consider groups having a life-time between T to $T + \mathrm{d}T$. In the central meridian we can see $\Delta N\delta\lambda$ of these groups, which are given by the expression (see Figure 4.)

$$\Delta N\delta\lambda\,\mathrm{d}T = \Delta f(T - L)\delta\lambda\,\mathrm{d}T = k\Delta f(S_M - S_0)\delta\lambda\,\mathrm{d}T\,.$$

At a distance λ to $\lambda + \delta\lambda$, the minimum visible area $S_0\Phi(\lambda)$ and the number of groups, which we observe at this distance from the central meridian, is given by the expression

$$\Delta N(\lambda)\delta\lambda \, \mathrm{d}T = k\Delta f[S_M - S_0\Phi(\lambda)]\delta\lambda \, \mathrm{d}T = \Delta f[T - L\Phi(\lambda)]\delta\lambda \, \mathrm{d}T \, . \quad (3,4)$$

At a distance λ to $\lambda + \delta\lambda$ from the central meridian however, we observe all groups which have an area larger than $S_0\Phi(\lambda)$ i. e. groups, which have a life-time longer than $L\Phi(\lambda)$. Their number $\Delta N_0(\lambda)\delta\lambda$ is then, with regard to the equation (1,10) given by the expression

$$\Delta N_0(\lambda)\delta\lambda = \Delta f_0 \int_{L\Phi(\lambda)}^{\infty} F(T)[T - L\Phi(\lambda)] \, \mathrm{d}T\delta\lambda \, . \quad (3,5)$$

If we solve the integral on the right side of equation (3,5) by the method per partes, we get the right side of the equation (3,3).

In the previous chapter (II, § 3) it has already been shown, that the number of groups, which have an area larger than S'', ought not to decay from the center of the disc up to the distance λ, determined by the condition, that $\Phi(\lambda') = \frac{S''}{S_0}$, unless the so-called corrected area by means of sec λ, would really equal the actual group area S. However, as in reality for $\lambda > 0$ is $\Theta(\lambda) > \sec \lambda$, the number of groups larger than a sertain corrected area S'_r, must decay from the central meridian towards the solar limb. This is caused by the fact that if we take into consideration groups, with an area larger than a certain corrected area S'_r, in reality we take groups with an actual area larger than $S'_r \frac{\Theta(\lambda)}{\sec \lambda}$ as follows from equation (2,6), which area steadily increases, with regard to the relation (2,8) and (2,9) towards the solar limb.

Let us now consider only such groups having a reduced area in the interval of reduced areas $S_{r2} - S_{r1} = \Delta S_r$. Unless the reduced area would equal the real one, their number again ought not to decay from the central meridian up to the distance λ and their number $N(S)$ would be given by equation (2,2). However as the reduced area S_r and the actual area S are not equal for $\lambda > 0$, the number of groups $N(S)$ with a reduced area within the limits of reduced areas $S_{r2} - S_{r1} = \Delta S_r$ is given with respect to equations (2,2), (3,1) and (2,6) by the expression

$$N(S)\delta\lambda = k\Delta f_2 \Delta S_r \varphi(\lambda) \int_{T = kS_{r_1}\varphi(\lambda)}^{\infty} F(T) \, \mathrm{d}T\delta\lambda \, . \quad (3,6)$$

We get the change of the number of these groups with the distance from the central meridian by forming the derivative of equation (3,6) with respect to λ

$$\frac{\mathrm{d}N(S)}{\mathrm{d}\lambda} = k\Delta f_0 \Delta S_r \frac{\mathrm{d}\varphi(\lambda)}{\mathrm{d}\lambda} \left\{ \int\limits_{T=kS_{r_1}\varphi(\lambda)}^{\infty} F(T)\,\mathrm{d}T - k\,S_{r_1}\varphi(\lambda)F(kS_{r_1}\varphi(\lambda)) \right\}. \tag{3,7}$$

As is evident from equation (3,7), we cannot determine in general, whether the number of groups with reduced areas between S_r and $S_r + \Delta S_r$, will decrease toward the limb, or whether it will increase, or whether it will remain constant.

This is caused by the fact that with the approach toward the solar limb we take not only steadily larger groups but also steadily larger ranges between the considered actual areas ΔS.

§ 2. *The Decay of Sum of Areas of Spot Groups*

We are going to derive the decay of sum of areas of spot groups toward the solar limb in the following manner [5]:

The number of groups with a life-time from T to $T + \mathrm{d}T$ observed from a distance λ to $\lambda + \mathrm{d}\lambda$ from the central meridian, is given by equation (3,4). The actual mean area p of these groups is given by the expression

$$p = \frac{S_0\Phi(\lambda) + S_M}{2}. \tag{3,8}$$

From the equation (3,4) and (3,8), we get the expression for the sum of all areas $P(\lambda)$ of groups at a distance λ to $\lambda + \delta\lambda$ from the central meridian, having life-times from T to $T + \mathrm{d}t$: [5]

$$\left.\begin{aligned} P(\lambda)\delta\lambda &= p\Delta N(\lambda)\delta\lambda\,\mathrm{d}T = k\Delta f\,\frac{S_M^2 - S_0^2\Phi(\lambda)^2}{2}\,\delta\lambda\,\mathrm{d}T = \\ &= \Delta f\,\frac{T^2 - L^2\Phi(\lambda)^2}{2k}\,\mathrm{d}T\delta\lambda \end{aligned}\right\} \tag{3,9}$$

From equation (3,9) and (1,10), we then get the sum of actual areas of groups of all life-times at distances λ to $\lambda + \delta\lambda$ from the central meridian $P_0(\lambda)$ [5],

$$P_0(\lambda)\delta\lambda = \frac{\Delta f_0}{2k}\int\limits_{L\Phi(\lambda)}^{\infty} F(T)[T^2 - L^2\Phi(\lambda)^2]\,\mathrm{d}T\delta\lambda\,. \qquad (3{,}10)$$

Equation (3,10) gives us the decay of the real group areas of spots toward the solar limb. From it we then get the decay of the total observed areas $\overline{P}_0(\lambda)\delta\lambda$ of spot groups toward the solar limb in the form,

$$\overline{P}_0(\lambda)\delta\lambda = \frac{\Delta f_0}{2k\Theta(\lambda)}\int\limits_{L\Phi(\lambda)}^{\infty} F(T)[T^2 - L^2\,\Phi(\lambda)^2]\,\mathrm{d}T\delta\lambda\,. \qquad (3{,}11)$$

The decay of the total actual areas $P_0(\lambda)$ of spot groups toward the solar limb is caused by the fact that at a distance λ from the central meridian we may observe only such spot groups, which have an actual area larger than $S_0\Phi(\lambda)$. We are unable to observe groups having a real area smaller than $S_0\Phi(\lambda)$, therefore they are not included in our statistics, but their number and their area increase toward the solar limb.

According to M. Hotinli [11] the decay of the total actual areas of spot groups $P_0(\lambda)\delta\lambda$ towards the solar limb can also be described by the expression

$$P_0(\lambda)\delta\lambda = \int\limits_{S_0\Phi(\lambda)}^{\infty} SN(S)\,\mathrm{d}S\delta\lambda\,. \qquad (3{,}12)$$

Now if we again substitute for $N(S)$ from equation (2,2), we get

$$P_0(\lambda)\delta\lambda = k\Delta f_0 \int\limits_{S_0\Phi(\lambda)}^{\infty} S \int\limits_{T=kS}^{\infty} F(T)\,\mathrm{d}T\,\mathrm{d}S\delta\lambda\,. \qquad (3{,}13)$$

Again by solving the integral in equation (3,10), using the method per partes we get the expression (3,13).

§ 3. *Some Conclusions*

If we proceed from the central meridian towards the solar limb, we can observe only groups with a steadily larger and larger area, so that the mean actual area of spot groups increases towards the solar limb. Its course for groups with life-times form T to $T + \mathrm{d}T$ is given by equation (3,8). The course of mean area of spot groups p_0 of all life-times is given with regard to equation (3,2) by the ratio

$$p_0(\lambda)\delta\lambda = \frac{\int\limits_{S_0\Phi(\lambda)}^{\infty} SN(S)\, dS\delta\lambda}{\int\limits_{S_0\Phi(\lambda)}^{\infty} N(S)\, dS\delta\lambda} \tag{3,14}$$

The decay of number as well as of areas of spots, essentially depends on the course of the function $F(T)$ [5], as follows from equations (3,3), (3,5), (3,10) and (3,13). The course of the function $F(T)$ however,

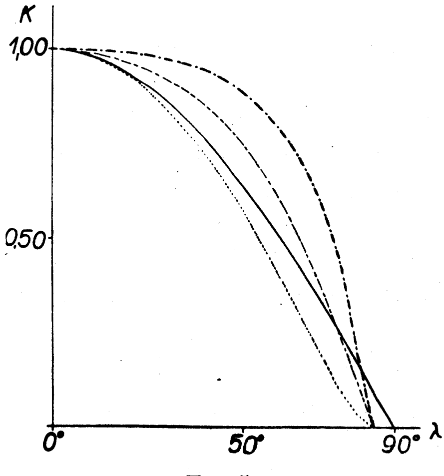

FIG. 5.

changes during the eleven year cycle (see chapter I § 2.). Therefore, it changes during the cycle, as well as the course of the decay of the numbers of groups and their areas towards the solar limb.

The decay of the number and spot areas, as follows from § 1 and § 2, also chiefly depends on the minimum visible area S_0 in the central meridian, which depends on the resolving powers of the telescope used. The course of the decay of number and areas of spot groups is, therefore, dependent on the resolving powers of the telescope, and will differ for various observatories, as is already obvious from statistics worked-out by L. PAJDUŠÁKOVÁ-MRKOSOVÁ [13,5].

Some authors up to now suppose, that in case when the function of foreshortening is $\Theta(\lambda) = \sec \lambda$, the number and the whole observed area of groups aught to decay towards the solar limb like $\cos \lambda$. Already G. H. A. ARCHENHOLD [7], however, drew attention to the fact that this was not possible, which evidently follows from equations determining the decay towards the solar limb derived in § 1 and § 2. It is especially obvious in Figure 5, where the decay is compared with the course of function $\cos \lambda$ [the full line]. The decay of the number of groups is given in Figure 5 by the computations from equation (3,5), the decay of the total area from equations (3,10) and (3,11), and that as the ration of the number or area at a distance λ to the number or area in the central meridian, while in the first approximation we used for simplicity $\Phi(\lambda) = \Theta(\lambda) = \sec \lambda$, $L = 1$, $F(T) = \frac{a}{T} + b$ and for the upper limit of the integral we put $T = 10$. In that case the decay of the number of groups K_N in Figure 5 is given by equation (the dashed curve)

$$K_N = \frac{\int\limits_{\sec \lambda}^{10} \left(\frac{a}{T} + b\right)(T - \sec \lambda)\, \mathrm{d}T}{\int\limits_{1}^{10} \left(\frac{a}{T} + b\right)(T - 1)\, \mathrm{d}T},$$

the decay of the sum of actual areas (the dot-and-dashed curve)

$$K_P = \frac{\int\limits_{\sec \lambda}^{10} \left(\frac{a}{T} + b\right)(T^2 - \sec^2 \lambda)\, \mathrm{d}T}{\int\limits_{1}^{10} \left(\frac{a}{T} + b\right)(T^2 - 1)\, \mathrm{d}T}$$

and the decay of the sum of observed areas, $K_{\bar{P}} = K_P \cos \lambda$ (the dotted curve).

The values a and b are determined from the relations (1,11) and (1,12).

IV. THE NUMBER OF APPEARING AND DISAPPEARING GROUPS AT VARIOUS DISTANCES FROM THE CENTRAL MERIDIAN

§ 1. *In the Case of uninterrupted Observation*

A. S. D. Maunder [14] found statistically, that on the eastern half of the solar disc more groups of spots originate, than on the western half. The results of her statistics were confirmed by papers of Rodes [15] and W. Brunner [10].

W. Gleissberg [17] showed, that this effect is explainable in a natural way, if we assume that a certain time passes by between the real origin of the group and the moment when we observe it on the solar disc for the first time. This supposition is correct, because the group must already have a certain area, namely, a diameter, depending on the resolving power of the telescope in use — in order to be able to see it.

A spot appearing to us for the first time in the central meridian, originated in reality d days ago. A spot appearing to us at a distance λ from the central meridian, originated — as is assumed by Gleissberg [17] — d sec λ days ago, at a distance of $13° . d . \sec \lambda$ from the central meridian, if the Sun turns approximately 13° daily. Groups of spots appearing for the first time on the eastern half of the disc between the central meridian and $-70°$ from the central meridian, in reality originated within the limits of heliographical longtitudes $70° + 13° . d . (\sec 70° - 1)$. Groups of spots appearing for the first time on the western half of the disc between the central meridian and $+70°$ from the central meridian, in reality originated within the limits of heliographical longtitudes $70° - 13° . d . (\sec 70° - 1)$. The ratio of the appearing groups on the eastern half E to the number of appearing groups on the western half W of the solar disc is then — according to Gleissberg — given by the expression

$$\frac{E}{W} = \frac{70 + 13d\,(\sec 70^\circ - 1)}{70 - 13d\,(\sec 70^\circ - 1)}\,.$$

The ratio of $\frac{E}{W}$ is larger than *1*, which corresponds qualitatively to the observation. Later on W. GLEISSBERG devoted more time to this problem in a more detailed manner also with regard to heliographical latitudes of spot groups [18].

The fact, that on the eastern half of the solar disc more groups must apparently originate than on the western half of the solar disc and that on the western half of the solar disc more groups must apparently decay than on the easter half of the disc, is very clearly illustrated in Minaert's diagram (see figure 2), which was also constructed for this purpose [8]. The appearing or disappearing of a group of spots, is given on it by the point of intersection of the developing curve of the spot groups with the curve of visibility.

The most common solution of Minaert's diagram for the origin and the decay of the spot groups was given by F. LINK [10]. If the spot group has its real origin at a distance λ_1 from the central meridian and its real decay at a distance λ_2 from the central meridian, then its area $\overline{S}$ at its first appearance or disappearance, at a distance λ from the central meridian, is given with regard to the relations (1,25) by the equations:

$$\overline{S} = k_1\,\frac{\lambda - \lambda_1}{\omega}\,; \qquad \overline{S} = k_2\,\frac{\lambda - \lambda_2}{\omega}\,. \tag{4,1}$$

The minimum area, which a group of spots must have, in order to see it at a distance λ from the central meridian is given by the relation (1,19). For the apparent origin of groups we then obtain the relation

$$S_0\Phi(\lambda) = k_1\,\frac{\lambda - \lambda_1}{\omega}\,. \tag{4,2}$$

If the spot group really originates at a distance λ_1 from the central meridian, we then see it for the first time at a distance λ from the central meridian, determined by the relation (4,2). The group of spots, which really originated at distances λ_1 to $\lambda_1 + \mathrm{d}\lambda_1$ we shall then see originating at distances λ to $\lambda + \mathrm{d}\lambda$. We obtain the relation between $\mathrm{d}\lambda$ and $\mathrm{d}\lambda_1$, by differentiating the equation (4,2), in the following form,

$$\mathrm{d}\lambda_1 = \left[1 - \frac{\omega S_0}{k_1} \cdot \frac{\mathrm{d}\Phi(\lambda)}{\mathrm{d}\lambda}\right] \mathrm{d}\lambda \,. \tag{4,3}$$

Let us suppose that the number of really originated groups $\mathrm{d}f_1$ in the interval of lengths $\mathrm{d}\lambda_1$ is independent of λ_1, so that

$$\mathrm{d}f_1 = \Delta f \, \mathrm{d}\lambda_1 \,. \tag{4,4}$$

The number of groups $\mathrm{d}f_1$ actually originated in the intervals of lengths $\mathrm{d}\lambda_1$, is the same as the number of apparently originated groups $\mathrm{d}\bar{f}_1$, in the intervals of lengths $\mathrm{d}\lambda$, corresponding to $\mathrm{d}\lambda_1$. With regard to this and to equations (4,3) and (4,4) we obtain that the number of apparently originated groups $\mathrm{d}\bar{f}_{1,1}$ at distances λ to $\lambda + \mathrm{d}\lambda$ is given by the relation

$$\mathrm{d}\bar{f}_{1,1} = \Delta f \left[1 - \frac{\omega S_0}{k_1} \cdot \frac{\mathrm{d}\Phi(\lambda)}{\mathrm{d}\lambda}\right] \mathrm{d}\lambda, \tag{4,5}$$

which determines the dependence of the number of apparently originated groups on their distances from the central meridian in the ascending part of their development.

In the same manner we can derive the dependence of the number of apparently decayed groups $\mathrm{d}\bar{f}_{2,1}$ on their distances from the central meridian, in the descending part of their development.

$$\mathrm{d}\bar{f}_{2,1} = \Delta f \left[1 - \frac{\omega S_0}{k_2} \cdot \frac{\mathrm{d}\Phi(\lambda)}{\mathrm{d}\lambda}\right] \mathrm{d}\lambda \,. \tag{4,6}$$

If we substitute $\frac{S_0}{k_1} = d_1$ into equation (4,5), where d_1 is the time between the actual origin of the group and its appearance in the central meridian and $-\frac{S_0}{k_2} = d_2$, where d_2 is the time between the apparent decay of group in the central meridian and its real decay, we obtain the relations which were derived by M. Waldmeier and A. Liepert [19].

The difference between the number of apparently originated and apparently decayed spot groups is then, with respect to equations (4,5), (4,6) and (1,4), given by the expression

$$\mathrm{d}\bar{f}_{1,1}(\lambda) - \mathrm{d}\bar{f}_{2,1}(\lambda) = - k\Delta f\omega S_0 \frac{\mathrm{d}\Phi(\lambda)}{\mathrm{d}\lambda}\,\mathrm{d}\lambda\,. \qquad (4,7)$$

The so-called total balance $B(\lambda)$ [10], introduced by F. Link, defined as the difference between the number of apparently originated and decayed spot groups at symmetrical distances from the central meridian, is then given by the relation

$$B(\lambda) = \mathrm{d}\bar{f}_{1,1}(+\lambda) + \mathrm{d}\bar{f}_{1,1}(-\lambda) - \mathrm{d}\bar{f}_{21}(+\lambda) - \mathrm{d}\bar{f}_{21}(-\lambda) =$$
$$= - k\Delta f\omega S_0 \left[\frac{\mathrm{d}\Phi(+\lambda)}{\mathrm{d}\lambda} + \frac{\mathrm{d}\Phi(-\lambda)}{\mathrm{d}\lambda}\right]\mathrm{d}\lambda = 0,$$

with regard to the fact that

$$\left(\frac{\mathrm{d}\Phi(\lambda)}{\mathrm{d}\lambda}\right)_{\lambda<0} = -\left(\frac{\mathrm{d}\Phi(\lambda)}{\mathrm{d}\lambda}\right)_{\lambda>0}. \qquad (4,8)$$

From Minaert's normal diagram it then follows that the total balance $B(\lambda)$ always ought to equal O, which however, does not coincide with observations as was shown by F. Link [10].

From relations derived by F. Link, we can also find the difference between the number of apparently originated groups on the eastern half of the disc $\mathrm{d}\bar{f}_{1,1}(-\lambda)$ and apparently decayed on the western half of the disc $\mathrm{d}\bar{f}_{2,1}(+\lambda)$ and between the number of apparently decayed on the eastern half of the disc $\mathrm{d}\bar{f}_{2}(-\lambda)$ and apparently originated on the western half of the disc $\mathrm{d}\bar{f}_{1}(+\lambda)$. Then with respect to (1,8) and (4,8) we obtain,

$$\mathrm{d}\bar{f}_{1,1}(-\lambda) - \mathrm{d}\bar{f}_{2,1}(+\lambda) = \Delta f\omega S_0 \frac{\mathrm{d}\Phi(\lambda)}{\mathrm{d}\lambda}\left(\frac{k_1 + k_2}{k_1 k_2}\right) < 0\,, \qquad (4,9)$$

$$\mathrm{d}\bar{f}_{1,1}(+\lambda) - \mathrm{d}\bar{f}_{2,1}(-\lambda) = - \Delta f\omega S_0 \frac{\mathrm{d}\Phi(\lambda)}{\mathrm{d}\lambda}\left(\frac{k_1 + k_2}{k_1 k_2}\right) < 0\,. \qquad (4,10)$$

From relations (4,9) and (4,10) it then follows, that the number of apparently decayed groups on the western half of the solar disc is larger than the number of apparently originated groups on the eastern half of the disc, and that the number of apparently originated groups on the western half of the solar disc, is larger than the number of apparently decayed groups on the eastern half of the disc. Both of these theoretically derived relations are in accordance with observation [20].

Relations (4,5) and (4,6), derived by F. Link, hold true however, only within the limits of distances from the central meridian from $-\bar{\lambda}_2$ to $+\bar{\lambda}_1$. Angle $\bar{\lambda}_1$ determines the distance from the central meridian from which — on the western half of the solar disc — we are still able to see an apparently originating group, and is given by the condition (see Figure 4)

$$S_0\left(\frac{\mathrm{d}\Phi(\lambda)}{\mathrm{d}\lambda}\right)_{\lambda=\bar{\lambda}_1} = \frac{k_1}{\omega}\,. \tag{4,11}$$

Analogously angle $-\bar{\lambda}_2$ determines the distance from the central meridian, from which — on the eastern half of the solar disc — we may still see an apparently decaying spot group and it is given by the condition (see Figure 4)

$$S_0\left(\frac{\mathrm{d}\Phi(\lambda)}{\mathrm{d}\lambda}\right)_{\lambda=-\bar{\lambda}_2} = \frac{k_2}{\omega}\,. \tag{4,12}$$

Relations (4,5) and (4,6) derived by Link and the relations derived from them, are valid only for such groups of spots, which apparently originate on the ascending part of their development or which apparently decay on the descending part of their development. There are groups even on the descending part of their development which apparently originate, namely, which appear as new groups on the eastern half of the solar disc, at distances $-\lambda < -\bar{\lambda}_2$ from the central meridian, and their number is given by the expression

$$\mathrm{d}\bar{f}_{1,2} = -\Delta f\left[1 - \frac{\omega S_0}{k_2}\cdot\frac{\mathrm{d}\Phi(\lambda)}{\mathrm{d}\lambda}\right]\mathrm{d}\lambda \tag{4,13}$$

and the total number of apparently originated groups between λ to $\lambda + \mathrm{d}\lambda$ for $-\lambda < -\bar{\lambda}_2$ is hence given, with regard to (4,5) and (4,13), by the equation

$$\mathrm{d}\bar{f}_1 = \mathrm{d}\bar{f}_{1,1} + \mathrm{d}\bar{f}_{1,2} = -\Delta f k\omega S_0\frac{\mathrm{d}\Phi(\lambda)}{\mathrm{d}\lambda}\mathrm{d}\lambda\,. \tag{4,14}$$

In a similar way, there are groups also on the ascending part of their development which apparently decay on the western half of the solar

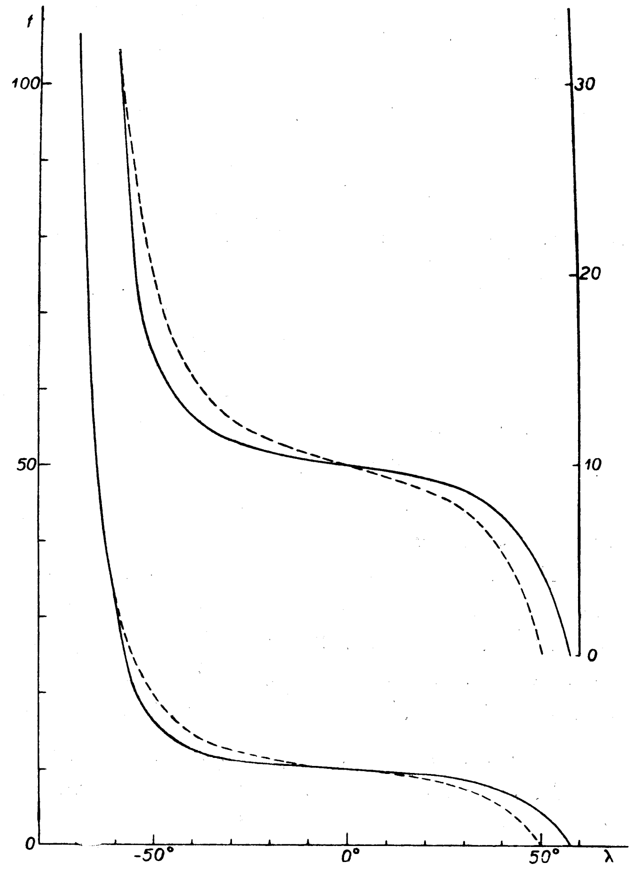

FIG. 6.

disc at distances $\lambda > \bar{\lambda}_1$ from the central meridian, and their number is given by the equation

$$\mathrm{d}\bar{f}_{2,2} = -\Delta f \left[1 - \frac{\omega S_0}{k_1} \cdot \frac{\mathrm{d}\Phi(\lambda)}{\mathrm{d}\lambda}\right] \mathrm{d}\lambda \qquad (4,15)$$

and the number of all apparently decaying groups on the western half of the disc at distances $\lambda > \bar{\lambda}_1$ from the central meridian, with regard to (4,6) and (4,15), given by the equation

$$d\bar{f}_2 = d\bar{f}_{2,1} + d\bar{f}_{2,2} = \Delta fk\omega S_0 \frac{d\Phi(\lambda)}{d\lambda} d\lambda \,. \tag{4,16}$$

From relations (1,8), (4,11) and (4,12), we obtain that

$$\bar{\lambda}_1 > \bar{\lambda}_2 \tag{4,17}$$

and then from the relations (4,14) and (4,16) with respect to relation (4,8) we get that for $|\lambda| > |\lambda_1|$ it holds true, that

$$d\bar{f}_2 = d\bar{f}_1$$

i. e., that the number of apparently originating groups at the eastern limb of the solar disc, is equal to the number of apparently decaying groups at the western limb of the disc.

The mutual course of the numbers of apparently originated and decayed spot groups according to relations (4,5), (4,6), (4,14) and (4,16) for $k_1 = 27{,}7$, $k_2 = -15{,}6$, $S_0 = 10$ and $\Phi(\lambda) = \sec^2 \lambda$ is given in Figure 6 and that in such a manner, that the number of apparently decayed groups at a distance $+\lambda$ from the central meridian is plitted on to the distance $-\lambda$ and vice versa, so that the mutual course of $d\bar{f}_1$ and $d\bar{f}_2$ is more evident. (Full line $\bar{f}_1$, dashed line $\bar{f}_2$.)

The theory of the number of the apparently originated and decayed spot groups at diferent distances from the central meridian — as it is worked out in this § — holds true only under the following two assumptions:

1. It holds true only for the group of spots with the same life-time T or maximum area S_M, as all the spot groups taken into consideration must have the same tangents k_1 and k_2 (see chapter I, § 1).

2. It holds true only in the case of uninterrupted observation of spots, for only in that case we are able to determine precisely, at what distance from the central meridian the spot groups apparently originated or apparently decayed (se the following §).

§ 2. *In Case of an Observation once in 24 Hours*

Computations of the apparently originated or decayed spot groups, worked-out up to now by means of Minaert's diagram — as it was presented in the previous § — are valid only under the assumption of uninterrupted observations of spots, as it is only in that case when we can determine precisely the distance from the central meridian, in which the group apparently originated or decayed.

However, in reality we observe sunspots, on the average, always once in 24 hours. Consequently, the group of spots — which became visible at a distance λ from the central meridian — can be observed for the first time perhaps even after 24 hours, when its distance from the central meridian is, due to the solar rotation already $\lambda + 13{,}2°$. Nevertheless, in spite of this, we state, that it apparently originated at the distance $\lambda + 13{,}2°$ from the central meridian.

The problem of the influence of an observation once in 24 hours on the number of apparently originated groups at various distances from the central meridian, was treated for the first time by G. H. A. ARCHENHOLD [21], by means of the calculus of probability. His method is based on the following consideration:

Let us take into account for example a group, which becomes visible for the first time on the eastern half of the disc at a distance — 72° from the central meridian. Then the probabilities at observation once in 24 hours, that we see this group for the first time in regions — 80° to — 70°, — 70° to — 60° and — 60° to — 50°, are successively 2/13,2; 10/13,2 and 1,2/13,2. If a group is above the range of visibility for a period less than 24 hours then it is necessary to take this into consideration. If for example, the above considered group is visible only from — 72° to — 66°, then the probabilities, that we see this group for the first time in the above considered regions, are 2/13,2; 4/13,2 and 0. The total probability, that we would see this group at all, is 6/13,2; it is therefore, more probable, that this group will escape our observation.

Theoretical curves of the number of originated groups at various distances from the central meridian, gained by this method, are in very good accordance with observation [21].

The number of apparently originated groups at various distances from the central meridian in case of an observation once in 24 hours, may also be derived in the following manner [20]:

When determining newly appearing groups, we actually determine the number of existing groups, which we did not see during the last observation 24 hours ago. Such groups could have become visible at any arbitrary instant between the last and present observation and

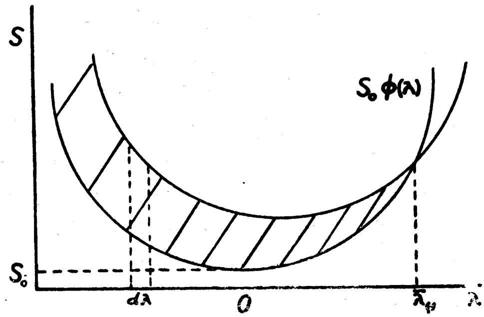

FIG. 7.

due to the solar rotation could have been carried away up to 13.2° to the west from their first real visible position, where we stated that they had only just appeared for the first time. This means that the area of such a group in Minaert's diagram will be at such a stage of development which lies between the curve of visibility $S_0\Phi(\lambda)$ (see Figure 7) and curve $\Omega(\lambda)$, while the straight lines between $S_0\Phi(\lambda)$ and $\Omega(\lambda)$ are of the same lenght and represent the development of the area of the groups in 24 hours, i. e., for a time between two consecutive observations. In this case

$$\Omega(\lambda) = k_1 + S_0\Phi(\lambda - \omega) , \tag{4,18}$$

where ω is the angular velocity of the solar rotation and k_1 is the tangent of the increase of the area of spot groups.

In this way we obtain that the number of apparently originated spot groups $\mathrm{d}f_{1,1}$ at a distance λ to $\lambda + \mathrm{d}\lambda$ from the central meridian, is proportional to the difference of both curves and to the interval $\mathrm{d}\lambda$,

$$\mathrm{d}\bar{f}_{1,1} = C[\Omega(\lambda) - S_0\Phi(\lambda)]\,\mathrm{d}\lambda , \tag{4,19}$$

where C is the constant of the proportionality denoting the number of existing groups in a unit of area in Minaert's diagram and we shall derive its size in the following manner:

Part of Minaert's diagram is given in Figure 8. If in a unit of time Δf groups originate in the unit λ, then during a time t, in a part of the heliographical longitudes λ to $\lambda + \lambda_1$ there originated f_1 spot groups,

$$f_1 = \Delta f \lambda_1 t ,$$

which are to be found within the rhomboid in Figure 8. The area of this rhomboid P is according to (1,1),

$$P = (S_2 - S_1)\lambda_1 = k_1 t \lambda_1 .$$

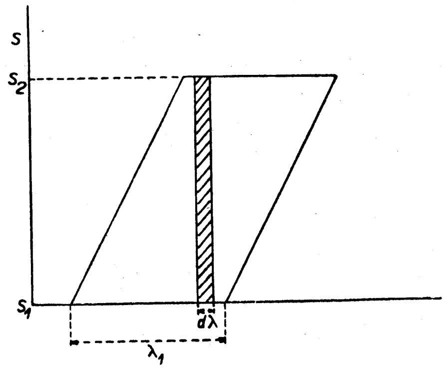

Fig. 8.

In a unit of area of the rhomboid and hence even in a unit of area in Minaert's diagram there exist then C groups, where

$$C = \frac{f_1}{P} = \frac{\Delta f}{k_1} . \tag{4,20}$$

The number of apparenly originated groups on the ascending part of their developing area $\mathrm{d}\bar{f}_{1,1}$ at a distance λ to $\lambda + \mathrm{d}\lambda$ from the central meridian, we then obtain by substituting (4,8) and (4,20) into (4,14),

$$\mathrm{d}\bar{f}_{1,1} = \frac{\Delta f}{k_1} [k_1 + S_0\Phi(\lambda - \omega) - S_0\Phi(\lambda)] \, \mathrm{d}\lambda . \tag{4,21}$$

On the western half of the solar disc we see these groups originating up to the distance of $\lambda_{1,1}$ from the central meridian (see Figure 7), which is given by the condition

$$\Phi(\lambda_{1,1} - \omega) - \Phi(\lambda_{1,1}) = -\frac{k_1}{S_0} . \tag{4,22}$$

Even in this case we only consider groups with the same life-time T, i. e. with a maximum area S_M, by which the tangents k_1 and k_2 are determined. With respect to this we observe these groups on the eastern half of the disc to originate apparently up to the distance $-\lambda_0$ from the central meridian, which is given by the condition

$$\Phi(\lambda_0) = \frac{S_M}{S_0} = \frac{T}{L} . \tag{4,23}$$

On the eastern half of the disc for $|\lambda| > |\lambda_{1,2}|$ there also appear as new ones, such groups, which already are on the descending part of the development of their area. Their number $d\bar{f}_{1,2}$ we derive similarly, as in the previous case and is given by the expression

$$\mathrm{d}\bar{f}_{12} = \frac{\Delta f}{k_2} [S_0\Phi(\lambda) - k_2 - S_0\Phi(\lambda - \omega)] \,\mathrm{d}\lambda . \tag{4,24}$$

The limit-angle $\lambda_{1,2}$ is given by the condition

$$\Phi(\lambda_{1,2}) - \Phi(\lambda_{1,2} - \omega) = \frac{k_2}{S_0} . \tag{4,25}$$

From equations (4,21) and (4,24) we obtain that the number of all apparently originated groups between $\lambda_{1,2}$ and $-\lambda_0 + \omega$ is given by the expression

$$\mathrm{d}\bar{f}_1 = \mathrm{d}\bar{f}_{1,1} + \mathrm{d}\bar{f}_{1,2} = \Delta f k S_0[\Phi(\lambda - \omega) - \Phi(\lambda)] \,\mathrm{d}\lambda . \tag{4,26}$$

Finally at distances $-\lambda_0$ to $-\lambda_0 + \omega$ from the central meridian, the number of all newly appearing groups is given by the expression

$$\mathrm{d}\bar{f}_1 = \Delta f k S_0[\Phi(\lambda_0) - \Phi(\lambda)] \,\mathrm{d}\lambda . \tag{4,27}$$

When determining the number of apparently decayed groups we again actually determine the number of existing groups, which we had yet seen during the last observation, however during the present observation we do not see them any longer. In reality, these groups could have ceased being visible, due to the solar rotation up to a point of 13.2° more to the west, than where we last observed them 24 hours ago.

In the same way like apparently originating groups, we obtain that on the eastern half of the solar disc we observe spot groups apparently decaying up to the distance of $\lambda_{2,1}$ from the central meridian, whereby

$$\lambda_{2,1} = \lambda_{1,2} - \omega \,, \tag{4,28}$$

while the number of apparently decayed groups on the descending part of the development of their area is given by the expression

$$\mathrm{d}\bar{f}_{2,1} = \frac{\Delta f}{k_2} \left[k_2 - S_0\Phi(\lambda + \omega) + S_0\Phi(\lambda)\right] \mathrm{d}\lambda \,. \tag{4,29}$$

On the western half of the solar disc at distances from the central meridian which are larger than $\lambda_{2,2}$ we also see apparently decaying groups, which are onthe ascending part of the development of their area. Their number $\mathrm{d}f_{2,2}$ is given by the expression

$$\mathrm{d}\bar{f}_{2,2} = \frac{\Delta f}{k_1} \left[S_0\Phi(\lambda + \omega) - k_1 - S_0\Phi(\lambda)\right] \mathrm{d}\lambda \,, \tag{4,30}$$

while angle $\lambda_{2,2}$ is determined by relation

$$\lambda_{2,2} = \lambda_{1,1} - \omega \,. \tag{4,31}$$

From equations (4,24) and (4,30) we obtain that the number of decayed groups $\mathrm{d}f_2$ from $\lambda_{2,2}$ to $\lambda_0 - \omega$ is given by the expression

$$\mathrm{d}\bar{f}_2 = \mathrm{d}\bar{f}_{2,1} + \mathrm{d}\bar{f}_{2,2} = \Delta f S_0 k[\Phi(\lambda + \omega) - \Phi(\lambda)] \,\mathrm{d}\lambda \tag{4,32}$$

and from $\lambda_0 - \omega$ to λ_0 by

$$\mathrm{d}\bar{f}_2 = \Delta f S_0 k[\Phi(\lambda_0) - \Phi(\lambda)] \,\mathrm{d}\lambda \,. \tag{4,33}$$

In case that $|\lambda_{1,2}| < |\lambda_0 - \omega|$ does not hold true or if $\lambda_{2,2} < \lambda_0 - \omega$ does not hold true, it is necessary to change in a certain manner the expressions for the number of apparently originated and decayed groups at distances $|\lambda| > |\lambda_0 - \omega|$ from the central meridian.

With respect to the fact that $|\lambda_{2,2}| > |\lambda_{1,2}|$ and that on the eastern half of the disc $\lambda < 0$ and that on the western half $\lambda > 0$, it follows from equations (4,26), (4,27), (4,32) and (4,33) that for $|\lambda| > |\lambda_{2,2}|$

$$d\bar{f}_1 = d\bar{f}_2$$

is valid, which means, that the number of apparently originated groups at individual distances between $-\lambda_{2,2}$ to $-\lambda_0$ from the central meridian, are the same as the number of apparently decayed groups at individual distances from the central meridian between $+\lambda_{2,2}$ to $+\lambda_0$. We obtain then the same results for the solar limb as in the previous §. The total course of the number of apparently originated and decayed spot groups is given in Figure 9, for $k_1 = 27.7$, $k_2 = -15.6$; $S_M = 250$, $S_0 = 10$, $\Delta f = 10$, $\omega = 13.2$, $\Phi(\lambda) = \sec^2 \lambda$, i. e. for the same constants as in the previous §. Even here for illustrative purposes of the mutual course, the number of apparently decayed groups at a distance $+\lambda$ from the central meridian, was plotted out to a distance $-\lambda$. The course of both curves in the close vicinity of the central meridian is simultaneously plotted on larger scale on the same figure. The difference in the course of apparently originated and decayed groups during an uninterrupted observation and during an observation once in 24 hours, is clearly evident when comparing Figure 6 to Figure 9.

The total number of originated groups from $-\lambda_{2,2}$ to $+\lambda_{1,1}$ is equal to the number of all decayed groups from $\lambda_{2,1}$ to $\lambda_{2,2}$, i. e.

$$\int_{-\lambda_{2,2}}^{\lambda_{1,1}} d\bar{f}_{1,1} + \int_{-\lambda_{2,2}}^{\lambda_{1,2}} d\bar{f}_{1,2} = \int_{\lambda_{2,1}}^{\lambda_{2,2}} d\bar{f}_{2,1} .$$

Let us now examine the difference between the number of all apparently originated and all apparently decayed spot groups in the neighbourhood of the central meridian from $-\lambda'$ to $+\lambda'$ for $|\lambda'| < |\lambda_{1,2}|$. With regard to the fact, that

$$\int_{-\lambda'}^{+\lambda'} \Phi(\lambda)\, d\lambda = 2 \int_{0}^{+\lambda'} \Phi(\lambda)\, d\lambda ,$$

$$\int_{-\lambda'}^{+\lambda'} \Phi(\lambda - \omega)\, d\lambda = \int_{-\lambda'-\omega}^{0} \Phi(\lambda)\, d\lambda + \int_{0}^{+\lambda'-\omega} \Phi(\lambda)\, d\lambda ,$$

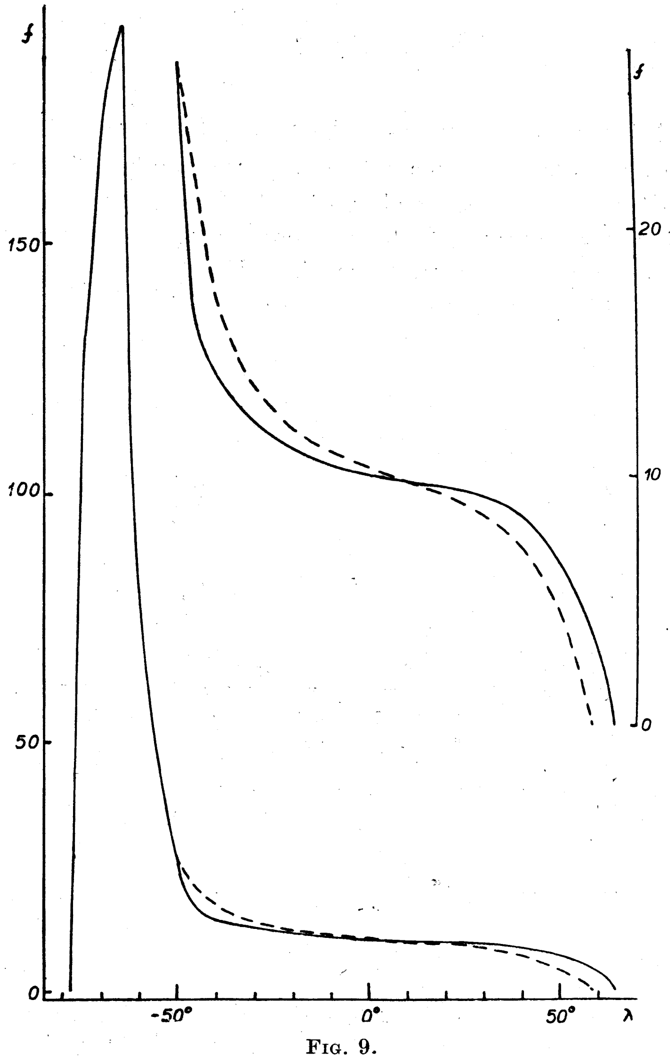

FIG. 9.

$$\int_{-\lambda'-\omega}^{0} \Phi(\lambda)\,\mathrm{d}\lambda = \int_{0}^{+\lambda'+\omega} \Phi(\lambda)\,\mathrm{d}\lambda\,,$$

we obtain from equations (4,21) and (4,29) and with regard to (1,8)

$$\int_{-\lambda'}^{+\lambda'} \mathrm{d}\bar{f}_{1,1} - \int_{-\lambda'}^{+\lambda'} \mathrm{d}\bar{f}_{2,1} = \Delta f S_0 \frac{k_1 + k_2}{k_1 k_2} \left[\int_{+\lambda'}^{+\lambda'+\omega} \Phi(\lambda)\,\mathrm{d}\lambda - \int_{\lambda'-\omega'}^{+\lambda'} \Phi(\lambda)\,\mathrm{d}\lambda \right] < 0, \tag{4,34}$$

because $\Phi(\lambda)$ is a purely monotone increasing function.

From relations (4,34), it then follows that the number of all apparently decayed groups in the neighbourhood of the central meridian is larger, than the number of all apparently originated groups in the neighbourhood of the central meridian. This result is in complete accordance with observation [20].

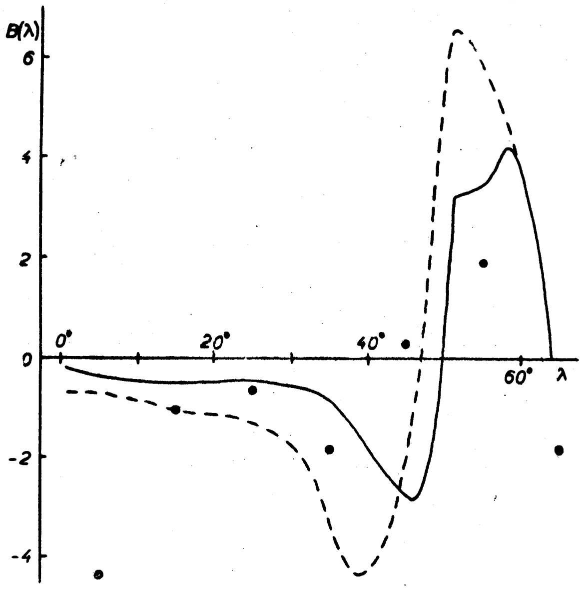

Fig. 10.

Let us now examine the course of the total balance $B(\lambda)$ introduced by F. Link [10] (see the previous §). In the neighbourhood of the central meridian from $\lambda = 0$ to $\lambda_{1,2}$, the course of $B(\lambda)$ is given by the expression

$$B(\lambda) = \Delta f S_0 \frac{k_1 + k_2}{k_1 k_2} [\Phi(+\lambda + \omega) + \Phi(+\lambda - \omega) - 2\Phi(+\lambda)] < 0,$$

which in reality already follows from the relation (4,34). With regard to relations between the limit-angles λ_{ii}, the course of $B(\lambda)$ for $\lambda_{1,2} < < \lambda < \lambda_{2,2}$ is given by further relations, while $B(\lambda) \neq 0$. For $\lambda > \lambda_{2,2}$ is $B(\lambda) = 0$. The total course of the total balance $B(\lambda)$ for constants, already previously introduced (full line) for $k_1 = 24.7$ and $k_2 = -9$ (dashed line), is given in Figure 10, where individual points are the values observed [10, 20]. We see, that besides the close vicinity of the central meridian, the observed and theoretical course of the total balance $B(\lambda)$ is practically identical. From the course of the number of apparently originated and decayed groups at various distances from the central meridian at observation once in 24 hours (see Figure 9) we also find that the number of all apparently decayed groups on the western half of the solar disc is larger than the number of all apparently originated groups on the eastern half of the disc is less than the number of all apparently originated groups on the western half of the disc. Hence we obtain a result identical with the result obtained in the previous §. This result is in complete agreement with observation [20].

On the whole we can make the following summary: The theoretic computation of apparent numbers of originated and decayed spot groups at various distances from the central meridian for an observation once in 24 hours, coincides, on the whole, much better with the observation than a theoretic computation for the uninterrupted observation.

§ 3. *The Dependence of the Size of the Asymmetry of the Origin and Decay of Spot Groups on the Size of Life-time of Spot Groups*

The asymetry of the origin, A_1 and the decay, A_2, of groups of spots is defined as a ratio between the number of all apparently originated or decayed groups of spots on the eastern half of the solar disc to the number of all apparently originated or decayed groups of spots on the western half of the solar disc.

The derivation of the dependence of the size of the asymetries on the life-times of spot groups will be worked-out again only for groups with the same life-time T, i. e. even with the same maximum area S_H and for simplicity, also for uninterrupted observations, which qualitatively cannot influence the result.

First of all let us take into consideration only such groups, which apparently originate on the ascending part and apparently decay on the descending part of the development of their area, i. e. the groups for which it is valid, that $\bar{\lambda}_1 > \lambda_0$ and $\bar{\lambda}_2 > \lambda_0$ (see Figure 4). Then from equations (4,5) and (4,6) with respect to (4,23) we obtain

$$A_1 = \frac{\lambda_0 + \frac{\omega}{k_1}(S_M - S_0)}{\lambda_0 - \frac{\omega}{k_1}(S_M - S_0)} = \frac{\lambda_0 + \frac{\omega}{kk_1}(T - L)}{\lambda_0 - \frac{\omega}{kk_1}(T - L)}, \tag{4,35}$$

$$A_2 = \frac{\lambda_0 + \frac{\omega}{k_2}(S_M - S_0)}{\lambda_0 - \frac{\omega}{k_2}(S_M - S_0)} = \frac{\lambda_0 + \frac{\omega}{kk_2}(T - L)}{\lambda_0 - \frac{\omega}{kk_2}(T - L)}. \tag{4,36}$$

With increasing T, λ_0 grows faster than $\frac{\omega}{kk_i} \cdot (T - L)$ so that with the increasing life-time T the asymmetry of originating, A_1, also grows and with regard to (1,3) the asymmetry of decaying, A_2, decreases.

With increasing S_M we reach the case — with regard to (4,17) — when $\lambda_0 < \bar{\lambda}_1$ but $\lambda_0 > \bar{\lambda}_2$ at the same time. In this case at distances $\lambda > \bar{\lambda}_2$ from the central meridian there also originate groups on the descending part of the development, by which the number of apparently originated groups on the eastern half of the disc becomes still larger. So that also with regard to equation (4,13) we obtain

$$A_1 = \frac{\bar{\lambda}_2 - \frac{\omega}{kk_2}[T - L\Phi(\bar{\lambda}_2)] + \frac{\omega}{kk_1}(T - L)}{\lambda_0 - \frac{\omega}{kk_1}[T - L]}, \tag{4,37}$$

so that the asymmetry A_1 of the originated groups grows again with increasing T.

In this case we observe the apparently decaying groups on the eastern half of the disc only up to the distance of $\bar{\lambda}_2$, so that

$$A_2 = \frac{\bar{\lambda}_2 + \frac{\omega}{kk_2} L(1 - \Phi(\bar{\lambda}_2))}{\lambda_0 - \frac{\omega}{kk_2}(T - L)} . \tag{4,38}$$

This means, that with the exception of a slight change of k_2 the number of apparently decayed groups in the east is independent of T, whilst the number of apparently decayed groups on the west grows with increasing T. With increasing T the value of the asymmetry of the decaying groups decreases.

Finally there arises the case most frequent in practice when $\lambda_0 > \bar{\lambda}_1$ and $\lambda_0 > \bar{\lambda}_2$. Then we receive

$$A_1 = \frac{\bar{\lambda}_2 - \frac{\omega}{kk_2}[T - L\Phi(\bar{\lambda}_2)] + \frac{\omega}{kk_1}(T - L)}{\bar{\lambda}_1 + \frac{\omega}{kk_1} L[1 - \Phi(\bar{\lambda}_1)]} , \tag{4,39}$$

$$A_2 = \frac{\bar{\lambda}_2 + \frac{\omega}{kk_2}[1 - \Phi(\bar{\lambda}_2)]}{\bar{\lambda}_1 - \frac{\omega}{kk_2}(T - L) + \frac{\omega}{kk_1}[T - L\Phi(\bar{\lambda}_1)]} . \tag{4,40}$$

We can see that even in this case, which is most frequent in practice, the asymmetry of the originating groups grows with the increasing life-time T and the asymmetry of the decaying groups decreases. At the same time the number of apparently decayed spot groups on the east and of the apparently originated groups on the west is practically independent of the life-time. These theoretically derived conclusions concerning the asymmetry of the originating and decaying groups of spots were for the apparently originated groups — confirmed also by the observation, while, not directly the life-time, but the mean life-time of spot groups [1] was taken into consideration.

V. THE COMPUTATION OF THE NUMBER OF ORIGINATED GROUPS ON THE ENTIRE SUN AND OF THEIR MEAN LIFE-TIMES

§ 1. *From the Entire Solar Disc*

While computing the number of all originated groups on the whole Sun f_0 and of their life-times T_0 we shall start with the relation (1,17),

$$N = f_0 T_0 \, , \tag{5,1}$$

where N is the number of all existing groups on the entire Sun at the given moment, and with the analogical relation to relation (5,1),

$$\overline{N} = \overline{f} \tau \, , \tag{5,2}$$

where $\overline{N}$ is the number of all observed groups on the visible solar disc at the given instant, $\overline{f}$ is the number of all originated and risen groups on the visible solar disc for a unit of time and τ is the mean time during which we observe the group of spots on the solar disc.

It is imposible to derive a sufficiently precise method for the computation of f_0 and T_0 from the entire solar disc. That is to say, we are obliged to assume again, that we are observing the sunspots steadily and that all groups of spots have a same life-time T, so that on the entire Sun there originate f of them (1).

First let us derive, according to this assumption, the relation between the number of the existing group N on the whole Sun and the number of observed groups $\overline{N}$ on the visible solar disc. As we can observe only the groups with an area larger than S_0, on the disc, we have to take into consideration groups larger than S_0, on the entire Sun. At every moment there exist N of them on the whole Sun:

$$N = f(T - L) = 2\pi \Delta f(T - L) \, . \tag{5,3}$$

From the equation (3,4) we get the number of all observed groups on the solar disc $\overline{N}$,

$$\overline{N} = \int_{-\lambda_0}^{+\lambda_0} \Delta N(\lambda)\,\mathrm{d}\lambda = \Delta f[2T\lambda_0 - L\int_{-\lambda_0}^{+\lambda_0} \Phi(\lambda)\,\mathrm{d}\lambda]\,. \tag{5,4}$$

Hence from (5,3) and (5,4) we obtain

$$N = \overline{N}\,\frac{2\pi[T - L]}{2T\lambda_0 - L\int_{-\lambda_0}^{+\lambda_0} \Phi(\lambda)\,\mathrm{d}\lambda}\,. \tag{5,5}$$

We receive the number of all originated and risen groups $\overline{f}$ on the visible disc of the Sun from integration of equations (4,5) and (4,13), in range of the limit-angles of visibility.

$$\overline{f} = \Delta f\left[\overline{\lambda}_1 + \overline{\lambda}_2 + \omega T - \omega L\,\frac{k_2\Phi(\overline{\lambda}_1) - k_1\Phi(\overline{\lambda}_2)}{k_2 - k_1}\right] \tag{5,6}$$

The relation between T and τ we then get from equations (5,4) and (5,6) with respect to relation (5,2):

$$\tau = \frac{\overline{N}}{\overline{f}} = \frac{2T\lambda_0 - L\int_{-\lambda_0}^{+\lambda_0} \Phi(\lambda)\,\mathrm{d}\lambda}{\overline{\lambda}_1 + \overline{\lambda}_2 + \omega T - \dfrac{\omega L}{k_2 - k_1}\,[k_2\Phi(\overline{\lambda}_1) - k_1\Phi(\overline{\lambda}_2)]}\,. \tag{5,7}$$

The computation of T from τ known from the observation can be made from relation (5,7) only by means of the graphical method. Then by means of T, determined in this way, we are able to solve the number of all originated groups on the entire Sun f from the observed number originated and risen groups on the visible disc $\overline{f}$ by means of the relation, which is obtainable from (3,1) and (5,6):

$$f = \overline{f}\,\frac{2\pi}{\overline{\lambda}_1 + \overline{\lambda}_2 + \omega T - \dfrac{\omega L}{k_2 - k_1}\,[k_2\Phi(\overline{\lambda}_1) - k_1\Phi(\overline{\lambda}_2)]}\,. \tag{5,8}$$

The equation (5,7) and (5,8) are then the required relations. These relations were already used for practical computations of annual values of f and T [1].

§ 2. *From the Part close to the Central Meridian*

In order to expel the influence of the function of visibility and in order to enable us to consider the groups of all life-times, let us restrict ourselves to the close vicinity of the central meridian from $-\lambda'$ to $+\lambda'$, where we can assume that at the first approximation $\Phi(\lambda) =$ = constant. In this case the assumption of linear development of sunspots is useless. Then we can derive for the computation of f and T the following method [22].

Regarding relations (1,10) and (1,15) the relation between the number of existing groups, dN', from $-\lambda'$ to $+\lambda'$ and the number of existing groups, dN, on the whole Sun, whose life-time is between T to $T + dT$, is given by the ratio

$$dN' = \frac{\lambda'}{\pi} dN = \frac{\lambda'}{\pi} f_0 F(T) T \, dT . \tag{5,9}$$

The number of all existing groups N' between $-\lambda'$ to $+\lambda'$ with respect to the relation (1,14) is hence given by equation

$$N' = \frac{\lambda'}{\pi} f_0 \int_L^\infty F(T) \, T \, dT = \frac{\lambda'}{\pi} f_0 T_0 . \tag{5,10}$$

By a similar consideration we obtain that the number of recently appeared groups f' between $-\lambda'$ to $+\lambda'$ — which is formed by the number of originated groups within a time unit and by the number of groups that were carried here by rotation, i. e., by the number of groups which exceeded the distance from the central meridian $-\lambda'$ within a time unit due to rotation — is given by the expression

$$f' = \frac{\lambda'}{\pi} f_0 \int_L^\infty F(T) \, dT + \frac{\omega f_0}{2\pi} \int_L^\infty F(T) T \, dT = f_0 \frac{2\lambda' + \omega T_0}{2\pi} . \tag{5,11}$$

If τ' means the average time of the existence of groups in the range of distances from the central meridian from $-\lambda'$ to $+\lambda'$, the relation between τ' and T_0 is given by the expression

$$\tau' = \frac{N'}{f'} = \frac{2\lambda' T_0}{2\lambda' + \omega T_0}$$

and consequently

$$T_0 = \frac{2\lambda'\tau'}{2\lambda' - \omega\tau'} . \qquad (5,12)$$

If we set T_0 from (5,11) into the equation (5,12) we obtain

$$\frac{f_0}{f'} = \frac{\pi}{\lambda'} - \frac{\pi\omega}{2\lambda'^2}\,\tau' . \qquad (5,13)$$

Evidently there is a linear dependence between $\frac{f_0}{f'}$ and τ'.

The equations (5,12) and (5,13) are the required relations, as by means of them we can determine the number of all originated groups within a unit of time on the entire Sun f_0 and their mean life-time T_0 on the basis of τ' and f' determined from the observation.

The number of all originated groups f_0 (not however their mean life-time) can also be determined by the method proposed by F. LINK [22]. At distances $-\lambda'$ to $+\lambda'$ from the central meridian, we shall determine the number f_1 of actually originated spot groups. Hence the number of all originated groups f_0 on the whole Sun is given by the relation

$$f_0 = f_1 \frac{\pi}{\lambda'} . \qquad (5,14)$$

§ 3. *The Importance of f_0 and T_0 for the Investigation of the Periodicity of Spots. The Comparison of the Methods of their Determination*

The investigation of periodicity of sunspots was carried out, up to this date, by means of the relative number which in a certain manner, gives us the quantity of sunspots existing at the given moment on the solar hemisphere facing the earth and hence it can be a characteristic, agreeing to a certain degree with the geoactivity of the Sun. It is certain, that as a characteristic of the solar activity the relative number does not prove to be quite suitable; from the point of view of the periodicity of the solar activity however the moving averages of the relative number represent quite well a certain mean course of the solar geoactivity, even when not absolutely satisfying in all respects.

However, the value of the relative number is dependent, first of all, on the number of the observed spot groups, which appears when determining the relative number in the multiple of ten. The fact that the relative number is determined first of all by the number of observed groups $\overline{N}$, is evident from the dependence between R and $\overline{N}$, which can be expressed in the following form [1]:

$$R = 10{,}57\,\overline{N}\,.$$

Hence it follows from this relation, that the averages of the relative numbers within a longer period of time at the first approximation are directly proportional to the averages of the daily number of observed groups on the solar disc $\overline{N}$ in the same period of time.

The number of observed groups on the solar disc $\overline{N}$ is dependent on the number of all groups N existing on the whole Sun, i. e., on its visible as well as invisible hemisphere. Then we can write that

$$R = K\overline{N} = K\psi(N)\,, \qquad (5{,}15)$$

where $\psi(N)$ is at the first approximation given by equation (5,5). Hence it follows from equation (5,15) that the relative number is determined essentially by the number of all existing spot groups on the whole Sun N.

From relation (1,17) it follows, that the number of existing spot groups on the entire Sun, and with regard to the relation (5,15) consequently the relative number as well, is determined by the number of groups originated within a unit of time on the whole Sun f_0 and by their mean life-time T_0. The course of f_0 and T_0 changes during the eleven years cycle [1], but their mutual courses can be practically independent of each other. The course of R and $\overline{N}$ is then determined by the course of f_0 and T_0. Hence it further follows [1], that the course of f_0 and T_0 is in a much closer connection with the causes of the periodicity of solar acitivity than the course of $\overline{N}$ and R and that it will therefore be necessary to continue to follow the course of f_0 and T_0 and their mutual relation, as far as the investigation of the regularity of periodicity of the sunspots and their causes will be concerned.

Now it is necessary to chose the most suitable method for determining the number of all originated spot groups f_0 and their mean life--times T_0.

The method of computing f_0 and T_0 derived in Chapter V, § 1, is based on certain assumptions, which are not in complete accordance with the facts: with the assumption of a certain growth curve of a group area, with the assumption of the course of the function of visibility, with an assumed certain minimum observable area of spot groups and with the assumption that the life-time of all spot groups is

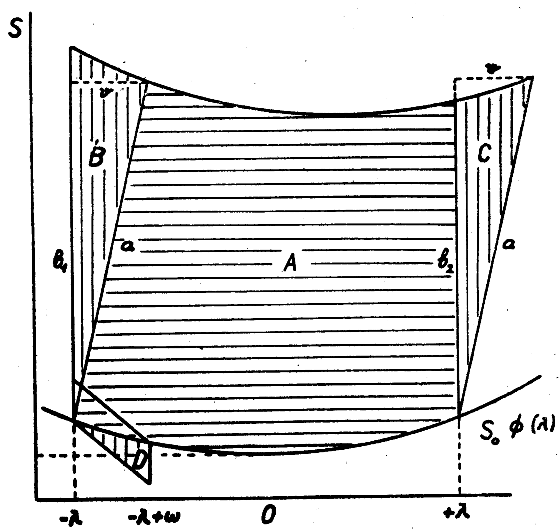

FIG. 11.

the same and that it is exactly the average one. It is not necessary to use even one of these assumptions in methods derived in Chapter V, § 2, except for the assumption that in the close vicinity of the central meridian the function of visibility is practically constant, and except for the assumption concerning the proportional origin of spot groups on the entire Sun, the assumption of which is difficult to avoid. From this it already follows that methods derived in Chapter V, § 2, will lead to more correct values of f_0 and T_0 than the methods derived in Chapter V, § 1, which agrees even with the statistical results [22].

But even methods derived in Chapter V, § 2, do not lead to the same results, as far as the number of originated groups f_0 are concerned [22]. This difference between the results of both methods is explainable by the fact that sunspots are not observed continuously, but always once in 24 hours. We shall prove this by means of Figure 11, which is a modification of Minaert's diagram [8] similar as in Chapter IV, § 2, and where $S_0\Phi(\lambda)$ is the function of visibility and represents the growth of group area during 24 hours. In case of an uninterrupted observation, the number of originated groups at distances $-\lambda$ to $+\lambda$ from the central meridian, is given by the number of the points of intersection of the curves indicating the growth of the spot group areas, having a visibility curve of $S_0\Phi(\lambda)$ and this number of points of intersection in 24 hours is equivalent to the area $A + C$. But in reality we observe once in 24 hours and by Link's method we consider the range from $-\lambda$ to $+\lambda$ only for such groups, which did not exist at all on the solar disc 24 hours ago. Their number is equivalent to the area $A + B$. We do not indicate the number of groups C having originated between $-\lambda$ and $+\lambda$ in this manner as having originated at these distances from the central meridian, and vice versa, the number of groups B which originated $-\lambda$ ago, we indicate as having originated between $-\lambda$ and $+\lambda$. Because for triangles B and C, it holds true, that

$$v_1 = v_2 \; ; \quad b_1 > b_2 \Rightarrow B > C \, ,$$

we obtain that

$$\frac{A + B}{A + C} > 1 \, .$$

Hence it follows that the number of originated spot groups according to F. Link's method will always be larger than in reality.

On the other hand, according to the author's method we always obtain a smaller number of originated groups than in reality, because the number of groups B is included in this method among the groups appearing owing to the solar rotation, but the number of originated groups C as well as the number of groups D appearing owing to the solar rotation, are not included into our statistics.

VI. THE COMMON CURVE OF THE DEVELOPMENT OF THE AREA OF THE GROUP OF SPOTS

Up to now, we mostly assumed that the area of the group of spots increases linearly with time up to the maximum and then again linearly decreases with time (see Chapter I. § 1.). The aim of this Chapter is to show which of the derived relations is valid even in case of a common development of a spot group area and how it would, in some simpler cases, be necessary to adjust formerly derived relations for a common development of spot group areas.

Let the common developing curve of a spot group area be given by functions

$$S = g(t) = G(\lambda) \tag{6,1}$$

and let it be limited by Gnevyshev's relation [2] (see Chapter I § 1.)

$$T = 0{,}1\ S_M = kS_M\ . \tag{6,2}$$

The same coefficient of proportionality is not valid anymore however, between the latent life-time L and the minimum area of visibility S_0. The relation between them may be written in the form

$$L = \bar{k}S_0\ , \tag{6,3}$$

where $\bar{k}$ is the constant only for a given form of function (6,1) and for given S_0, for it depends on both of them.

The limit-angle of visibility λ_0 from the central meridian in which we are still able to see the spot with a maximum area S_M, is then given by the condition

$$\Phi(\lambda_0) = \frac{S_M}{S_0} = \frac{\bar{k}T}{kL}\ . \tag{6,4}$$

It is now necessary to determine the limit-angles $\bar{\lambda}_1$ nad $\bar{\lambda}_2$ in which we may see the group of spots still apparently originating or decaying, which were formerly determined by conditions (4,11) and (4,12), in the following manner:

we write the equation (6,1) in the form

$$S = G(\lambda + K)\,, \tag{6,5}$$

where K is a constant. Now let us look for the points of intersection of this curve with the curve determined by the function of visibility $S_0\Phi(\lambda)$. Then there exist only two such K's that for them two real intersection points coincide into one real double intersection point. By finding these two K's we have fundamentally also found angles $\bar{\lambda}_1$ and $\bar{\lambda}_2$.

Even in the case of the common function of the development of a spot group area, the number of $\Delta N \mathrm{d}\lambda \,.\, \mathrm{d}T$ groups with a life-time T to $T + \mathrm{d}T$, which we observe in the close vicinity of the central meridian $\mathrm{d}\lambda$, is given by the equation

$$\Delta N\, \mathrm{d}\lambda\, \mathrm{d}T = \Delta f(T - L)\, \mathrm{d}\lambda\, \mathrm{d}T \tag{6,6}$$

and therefore the number of all groups existing on the visible and invisible solar hemisphere is given by equation

$$N\, \mathrm{d}T = f(T - L)\, \mathrm{d}T\,. \tag{6,7}$$

If we make the assumption that L is the same for all groups with various life-times, we obtain by means of equation (1,10) the expression for the number of all existing spot groups N, whose life-time T is from L to ∞:

$$N = f_0 \int_L^\infty F(T)\,(T - L)\, \mathrm{d}T = f_0(T_0 - L)\,. \tag{6,8}$$

However, from equation (6,6) we cannot derive the decay of the number of groups and of the sum of areas of groups towards the solar limb. The solution of this problem is made difficult by the fact that the coefficient of proportionality between the minimum observable area a $S_0\Phi(\lambda)$ and the appertaining latent life-time continually changes, namely according to the form of the function (6,1) and to the distance λ from the central meridian. The decay of the number of groups $\Delta N_0(\lambda)\,\mathrm{d}\lambda$ and of the sum of actual areas $P_0(\lambda)\,\mathrm{d}\lambda$ towards the solar limb, we may however, even in this case, express by Hotinli's relations (3,2) and (3,12) [11]

$$\Delta N_0(\lambda)\,\mathrm{d}\lambda = \int_{S_0\Phi(\lambda)}^{\infty} N(S)\,\mathrm{d}\lambda\,\mathrm{d}S\,,$$

and

$$P_0(\lambda)\,\mathrm{d}\lambda = \int_{S_0\Phi(\lambda)}^{\infty} SN(S)\,\mathrm{d}\lambda\,\mathrm{d}S\,,$$

whole validity is absoluty common.

Finally it is most certainaly understandable, that even in this case, the relation for the number of all existing groups on the whole Sun,

$$N = f_0 T_0\,,$$

holds true, which is not dependent on the form of the developingt curve of the spot areas (6,1).

Analogously, the method of computing f_0 and T_0, is in no way influenced by the form of the growth curve of the spot areas (6,1), which was derived in Chapter V, § 2, as when deriving this method, no assumptions had been made concerning the form of the function (6,1).

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РЕЗЮМЕ

Целью настоящего труда является обобщение сведений из теории распределения и появления солнечных пятен на диске Солнца. В этой своей работе автор исходит как из ряда трудов других авторов, так и из своих собственных трудов, помещенных в разных журналах, дополняя эти труды некоторыми новыми сведениями. Эта работа является попыткой создать обобщенное представление о теории распределения и появления солнечных пятен на диске Солнца.

Настоящая работа, однако, не занимается освещением вопросов этой теории с общей точки зрения, так как более общая теория не была до сих пор в совершенстве разработана. Эта теория основывается на определении упрощенных предположений, из которых самыми простыми являются следующие:

1) Все солнечные пятна появляются только на экваторе, который всегда проходит по середине диска Солнца, т. е. что ,,королевский пояс'' появления солнечных пятен можно себе представить в виде цилиндра, на кторый смотрим в направлении нормали к его поверхности.

2) В действительности предположим, что групыы солнечных пятен составляются из одного пятна округлой формы.

Эти предположения позволяют нам создать возможно полную основную теорию распределения и появления пятен, которая необходима для оценки результатов разных статистик солнечных пятен и из которой можно исходить для создания более общей теории.

Настоящая работа занимается прежде всего следующими вопросами: определением основных отношений и функций, уменьшением количества пятен в направлении к краю солнечного диска, количеством появившихся и исчезнувших пятен на разных расстояниях от центрального меридиана, подсчетом количества возникших групп на всем Солнце и средней продолжительности их жизни и значением этих величин для исследования периодичности солнечных пятен.

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