Modeling the Participation of Households and Firms in Formal and Informal Economic Activities

Bianca Pauna[1]

Introduction

The purpose of this research is to develop a dynamic labor market model that describes the mechanisms on which individuals and firms base their decision regarding the supply of and the demand for formal and informal labor, as a function of the prevailing market wage rates and other parameters. In this model, the actors, workers and firms, are faced with two choices: to supply work, respectively hire workers, formally or informally. These two decisions are made simultaneously, and can not be separated. Making use of the utility maximization framework, in the case of the individuals, and profit maximization in the case of the firms, we solve for the mix of formal and informal work that is optimal for individuals/firms to supply/demand.

The model is a dynamic one since the objective function incorporates future, as well as current period's utilities, in the case of individuals, and future, as well as current period's profits, in the case of firms. The future utility/profits are discounted by a factor less than one, which decreases towards zero the further into the future we go. The interaction of the supply and the demand in the formal and informal markets determines the equilibrium levels of employment and wage rates for the two markets. While some of the assumptions made in the construction of the model have their foundations in empirical evidence from transition economies, the results of the work are valid for the mature market economies as well.

Modeling worker's participation

In this section we derive a model of labor market behavior where the "representative" worker has to decide to allocate his/her labor between the formal and informal sectors. The worker is at the same time an utility-maximizing consumer, who faces the problem of developing a contingency plan for his consumption c_t.

There are two sectors in the economy, the formal sector, to which the worker dedicates a share n_t of his working time, and an informal sector, to which the worker allocates a time budget n^i_t. The worker maximizes the expected net present value of his utility, derived from his future consumption and supply of labor, discounted by a factor b, over an infinite horizon. In designing the contingency plan for the labor supply we extend a model introduced by Lucas and Rapping (1969) and refined by Sargent (1987).

[1] National Institute for Economic Research, email: b_pauna@yahoo.co.uk.
Acknowledgement: The research was financed through a grant by the CERGE – EI foundation under a program of the Global Development Network. Additional funds for grantees have been provided by the Austrian Government through WIIW, Vienna.

The constrained maximization problem is presented in equation [1.1] below:

$$\max \mathrm{E_t} \sum_{j}^{\infty} b^j [u_0 \cdot c_{t+j} - \left(\delta_0 + \varepsilon_{t+j}\right) \cdot n_{t+j} - \left(\delta_0' + \mu_{t+j}\right) \cdot n_{t+j}^i - \frac{\delta_1}{2} \cdot \left(n_{t+j} + n_{t+j}^i\right)^2 -$$

$$- \frac{\delta_2}{2} (n_{t+j} + \gamma \cdot n_{t+j-1})^2]; \qquad [1.1]$$

Subject to.

$c_{t+j} = w_{t+j}\ n_{t+j} + w^i_{t+j}\ n^i_{t+j} + y_{t+j} - T_{t+j}\ (w_{t+j}\ n_{t+j}\ ) - R(n^i_{t+j})$ (consumption constraint)

and:

$T_{t+j} = n_{t+j} + n^i_{t+j} + l_{t+j}$ (number of hours constraint); l_{t+j} is leisure

$u_0,\ \delta_0,\ \delta'_0,\ \delta_1,,\ \delta_2 > 0;$
$0 < b < 1;\ 0 < \gamma < 1;$
$n_{t-1}\ ,\ n^i_{t-1}$ given at t.

The expression for the utility chosen for the model is a traditional quadratic function, which increases in consumption and decreases in labor, irrespective of whether it is carried out in the formal or the informal sector. The utility function of the consumer at time t decreases with labor supplied at t. The second, third and forth terms describe the marginal disutility from formal and informal work, with the latter capturing the concave feature of the utility function, which exhibits diminishing marginal returns in employment. The utility function also decreases in the labor effort dedicated to the formal sector in the previous period $t-1$, captured by the term $\frac{\delta_2}{2}(n_{t+j} + \gamma \cdot n_{t+j-1})^2$. In other words, we assume here that working over long periods of time tires the worker and reduces one's utility by more than one-period employment alone, in the case of formal employment. We do not introduce a similar effect for informal work, as we assume that the worker views informal employment only as a casual, transitory activity, and has no intention to carry it out over his working life horizon. He works in the informal sector only temporarily, pushed by a transient adverse labor market shock, such as a shift in the formal labor market demand, triggered by transition. This way, he/she also supplements his/her formal employment earnings, which suffered a significant fall in real terms, following the sharp output contraction due to the transition. Finally, the worker can participate in the informal labor market as a means to escape the excessive payroll taxes, a characteristic of most transitional economies[2]. In other words, the expression above can be interpreted as a labor supply adjustment cost.

For simplicity all goods are aggregated into one, which enters the utility function through consumption c_t. We assume that the utility function is separable in consumption and labor supply, and that, at a later stage, the individual decides how to divide his/her income between different consumption goods. The worker/consumer is

[2] In recent past, to increase labor market flexibility, some transition countries, such as Russia, have significantly reduced labor taxation. This appears to have had a positive reaction in terms of labor supply and revenue collection. Other transitional economies contemplate a similar move.

on his/her budget constraint, meaning that he/she consumes all he/she earns in a given period. One can relax this assumption to allow for savings and loans. In this case the budget constraint would be an inter-temporal function.

The maximization of the utility is subject to two constraints. First, there is a consumption constraint that limits expenditure for a given period to the total income the worker obtains from his work in the formal and informal sectors, plus the non-labor income y_{t+j}. The latter can be transfers that he/she receives from family members or from the social security system, for example. w_t is the real gross hourly wage obtained in the formal sector, while w^i_t is the real hourly wage rate received in the informal sector. The term $T_{t+j} = t_{t+j}(w_{t+j})\cdot n_{t+j}$ captures the amount of taxes that the worker has to pay on his/her formal sector wage. It depends on the number of hours of work in the formal sector times the gross wage per hour, since it is only the formal earnings that the worker has to pay taxes on. The first derivative of taxes with respect to the formal labor income is positive. For simplicity, we assume that there is a linear relationship between taxes and income. In other words, the tax rate is constant, the same for everybody, irrespective of their income level, $\partial T_{t+j}/\partial (w_{t+j}\ n_{t+j})=t_{t+j}$. The next term $R(n^i_{t+j}) = r_{t+j}\ n^i_{t+j}$ captures the resources that the worker has to spend on concealing its informal economic activity. It is assumed that R depends on the number of hours that he/she works in the informal labor market (n^i_{t+j}), as well as on other variables that capture the official policy for dealing with informal market activities, captured by the term r_{t+j}. For the purpose of the model, r_{t+j} is considered exogenous.

The second constraint allocates the total available time between hours of work in the formal economic sector, the informal sector, and leisure. Finally, the worker takes as given the stochastic processes $\{\varepsilon_{t+j}\}_{j=0}^{\infty}$, $\{\mu_{t+j}\}_{j=0}^{\infty}$, $\{w_{t+j}\}_{j=0}^{\infty}$, $\{w^i_{t+j}\}_{j=0}^{\infty}$.

We are interested in finding the solution to the maximization process, namely the number of hours that the person works in the formal and the informal sector and its determinants. Substituting the first constraint in equation [1.1], we get the following maximization problem:

$$\max \mathrm{E}_t \sum_{j}^{T\infty} b^j [u_0 \cdot (w_{t+j}n_{t+j} + w^i_{t+j}n^i_{t+j} + y_{t+j} - t_{t+j}\cdot n_{t+j} - r_{t+j}n^i_{t+j}) - (\delta_0 + \varepsilon_{t+j})\cdot n_{t+j}$$

$$-\frac{\delta_1}{2}\left(n_{t+j} + n^i_{t+j}\right)^2 - \frac{\delta_2}{2}\left(n_{t+j} + \gamma \cdot n_{t+j-1}\right)^2; \qquad [1.2]$$

The first order conditions for the maximum, which are that the derivatives with respect to the two unknowns be equal to zero, are:

FOC: $\dfrac{\partial(.)}{\partial n_{t+j}} = 0$

$$u_0\,(w_{t+j} - t_{t+j}) - (\delta_0 + \varepsilon_{t+j}) - \delta_1\, E_{t+j}\,(n_{t+j} + n^i_{t+j}) - \delta_2\, E_{t+j}\,(n_{t+j} + \gamma\, n_{t+j-1}) -$$
$$- \delta_2\, b\, \gamma\, E_{t+j}\,(n_{t+j+1} + \gamma\, n_{t+j}) = 0 \qquad [1.3]$$

$$\frac{\partial(.)}{\partial n^i{}_{t+j}} = 0$$

$$u_0\,(w^i_{t+j} - r_{t+j}) - (\delta'_0 + \mu_{t+j}) - \delta_1\, E_{t+j}\,(n_{t+j} + n^i_{t+j}) = 0 \qquad [1.4]$$

Deducting equation [1.4] from equation [1.3], we arrive at the following expression:

$$u_0 [(w_{t+j} - w^i_{t+j}) - (t_{t+j} - r_{t+j})] - (\delta_0 + \varepsilon_{t+j}) + (\delta'_0 + \mu_{t+j}) - \delta_2 E_{t+j} (n_{t+j} + \gamma n_{t+j-1}) -$$
$$- \delta_2 b \gamma E_{t+j} (n_{t+j+1} + \gamma n_{t+j}) = 0$$

Grouping terms, the equation takes the form:

$$u_0 [(w_{t+j} - w^i_{t+j}) - (t_{t+j} - r_{t+j})] - (\delta_0 + \varepsilon_{t+j}) + (\delta'_0 + \mu_{t+j})] - \delta_2 b \gamma E_{t+j} n_{t+j+1} -$$
$$- \delta_2 (1 + b \gamma^2) E_{t+j} n_{t+j} - \delta_2 \gamma E_{t+j} n_{t+j-1} = 0 \qquad [1.5]$$

The above expression is in fact a second order difference equation. To see this we rewrite the expressions making use of the lag operator L.
Note that:
$E_{t+j} n_{t+j-1} = n_{t+j-1}$ and $E_{t+j} n_{t+j} = n_{t+j}$
$L E_{t+j} n_{t+j+1} = E_{t+j} n_{t+j} = n_{t+j}$
$L^2 E_{t+j} n_{t+j+1} = n_{t+j-1}$
Using these properties of the lag and expectation operators, the equation becomes:

$$\delta_2 b \gamma E_{t+j} n_{t+j+1} + \delta_2 (1 + b \gamma^2) L E_{t+j} n_{t+j+1} + \delta_2 \gamma L^2 E_{t+j} n_{t+j+1} = u_0 [(w_{t+j} - w^i_{t+j}) -$$
$$- (t_{t+j} - r_{t+j})] - (\delta_0 + \varepsilon_{t+j}) + (\delta'_0 + \mu_{t+j})]$$

which we can rewrite:

$$[\delta_2 b \gamma + \delta_2 (1 + b \gamma^2) L + \delta_2 \gamma L^2] E_{t+j} n_{t+j+1} = u_0 \Delta w_{t+j} - (\delta_0 + \varepsilon_{t+j}) + (\delta'_0 + \mu_{t+j})]$$

where $\Delta w_{t+j} = (w_{t+j} - w^i_{t+j}) - (t_{t+j} - r_{t+j})$. The term Δw_{t+j} captures the difference in net wages between the two sectors, since is wages minus taxes. r_{t+j} could be viewed as a tax that the informal worker pays on its informal earnings.

or:

$$\left(1 + \frac{1 + b\gamma^2}{b\gamma} L + \frac{1}{b} L^2\right) E_{t+j} n_{t+j+1} = \frac{u_0 \Delta w_{t+j} - (\delta_0 + \varepsilon_{t+j}) + (\delta'_0 + \mu_{t+j})}{\delta_2 b\gamma}$$

Consider λ_1 and λ_2 the solutions to the equation $Z^2 + \frac{1 + b\gamma}{b\gamma} Z + \frac{1}{b} = 0$. Then, the expression becomes:

$$(1 - \lambda_1 L)(1 - \lambda_2 L) E_{t+j} n_{t+j+1} = \frac{u_0 \Delta w_{t+j} - \left(\delta_0 + \varepsilon_{t+j}\right) + (\delta'_0 + \mu_{t+j})}{\delta_2 b\gamma} \qquad [1.6]$$

The necessary and sufficient condition for λ_1 and λ_2 to be real numbers is that the discriminant of the second order equation be positive. The mathematical condition is the following: $\left(\frac{1 + b\gamma^2}{b\gamma}\right)^2 - \frac{4}{b} \geq 0$.

$$\left(\frac{1}{b\gamma}+\gamma\right)^2-\frac{4}{b}=\frac{1}{b^2\gamma^2}+2\cdot\frac{1}{b\gamma}\cdot\gamma+\gamma^2-\frac{4}{b}=\frac{1}{b^2\gamma^2}-\frac{21}{b}+\gamma^2=\left(\frac{1}{b\gamma}-\gamma\right)^2\geq 0$$

We showed that the necessary and sufficient condition is fulfilled, the solutions of the equation are the following:

$$\lambda_{1,2}=\frac{-\frac{1}{b\gamma}-\gamma\pm\left(\frac{1}{b\gamma}-\gamma\right)}{2}=\frac{-\frac{1}{b\gamma}-\gamma\pm\frac{1}{b\gamma}\mp\gamma}{2}$$

which, after simplification, become: $\lambda_1=-\gamma$ and $\lambda_2=-\frac{1}{b\gamma}$.

From these expressions one can notice that λ_1 and λ_2 are both negative.

We know that applying the operator $1/(1-\lambda_2 L)$ to an expression gives a weighted sum of future values of the same expression:

$$\frac{1}{1-\lambda\cdot L}=\frac{-(\lambda\cdot L)^{-1}}{-(\lambda\cdot L)^{-1}+1}=-\frac{1}{\lambda\cdot L}\left(1+\frac{1}{\lambda}L^{-1}+\left(\frac{1}{\lambda}\right)^2L^{-2}+\ldots\right)=-\sum_{i=0}^{\infty}\left(\frac{1}{\lambda}\right)^iL^{-i}$$

Multiplying both sides of equation [1.6] by $1/(1-\lambda_2 L)$, gives:

$$(1-\lambda_1 L)\,E_{t+j}\,n_{t+j+1}=\frac{u_0\Delta w_{t+j}-(\delta_0+\varepsilon_{t+j})+(\delta'_0+\mu_{t+j})}{\delta_2 b\gamma\cdot(1-\lambda_2 L)}$$

$$(1-\lambda_1 L)\,E_{t+j}\,n_{t+j+1}=\frac{-(\delta_2 b\gamma\lambda_2)^{-1}L^{-1}}{1-\lambda_2^{-1}L^{-1}}\,[u_0\,\Delta w_{t+j}-(\delta_0+\varepsilon_{t+j})+(\delta'_0+\mu_{t+j})]$$

$$E_{t+j}\,n_{t+j+1}-\lambda_1 L\;n_{t+j+1}=$$

$$=\frac{-\lambda_1}{\gamma\delta_2}L^{-1}\left(1+\frac{1}{\lambda_2 L^{-1}}+\left(\frac{1}{\lambda_2}\right)^2L^{-2}+\ldots\right)\left(u_0\Delta w_{t+j}-(\delta_0+\varepsilon_{t+j})+(\delta'_0+\mu_{t+j})\right)$$

$$E_{t+j}\,n_{t+j+1}=\lambda_1\,n_{t+j}-\frac{\lambda_1}{\gamma\delta_2}\sum_{i=0}^{\infty}\left(\frac{1}{\lambda_2}\right)^i\left[u_0E_{t+j}\Delta w_{t+j+i}-\left(\delta_0+E_{t+j}\varepsilon_{t+j+i}\right)+\left(\delta'_0+E_{t+j}\mu_{t+j+i}\right)\right]$$

Plugging in the values for λ_1 and λ_2 we obtain the following expression for the contingency plan of the worker:

$$E_{t+j}\,n_{t+j+1}=-\gamma n_{t+j}+\frac{1}{\delta_2}\sum_{i=0}^{\infty}(-b\gamma)^i\left[u_0E_{t+j}\Delta w_{t+j+i}-\left(\delta_0+E_{t+j}\varepsilon_{t+j+i}\right)+\left(\delta'_0+E_{t+j}\mu_{t+j+i}\right)\right]$$

The contingency plan expresses current employment as a function of previous employment, current and future expected future values of the difference in net wage and disutility from work between the formal and informal sectors. Previous year's formal employment enters negatively into the determination of the current employment because of the fatigue effect that it has on workers. The current net wage influences positively formal employment, while next year's wage negatively affects the employment in the current period. The explanation is that the expectation of higher future wages induce an inter-temporal substitution effect, with workers choosing to work less at present and more when the net wage is expected to be higher. This effect appears in the case of the disutility from work variable, as well. Higher future formal disutility from work induces workers to inter-temporarily substitute work in the future for more work currently.

From the above equation one can notice that in our model a person would chose to work formally only if the term in Δw_{t+j} is large enough to compensate for the fatigue effect that formal work has on workers, considering that the disutility from work is similar for the two types of workers. Higher tax rates discourage workers to work in the formal sector, while higher costs to hide informal activity increase the participation in the formal sector. Of course, in the real world there are additional benefits from formal employment, which are not captured in our model, such as medical care, accession to social security benefits, etc.

The supply of informal work can be derived by replacing in equation [1.4] the expression of the supply of formal work:

$$E_{t+j}\, n^i_{t+j+1} = - E_{t+j}\, n_{t+j+1} + 1/\delta_1\, u_0\, (E_{t+j}\, w^i_{t+j+1} - E_{t+j}\, r_{t+j+1}) - 1/\delta_1\, (\delta'_0 + \mu_{t+j+1})$$

The contingency plan for the informal work reveals the dependency of informal employment on formal employment. This suggests that the decision to supply work formally and informally are simultaneous, and can not be separated from one another.

In order for the worker to participate in the informal sector at all, the informal wage term has to be large enough to compensate for the disutility from informal work and for employment in the formal market. The necessary condition for the worker to work informally is:

$$u_0\, (E_{t+j}\, w^i_{t+j+1} - E_{t+j}\, r_{t+j+1}) - (\delta'_0 + \mu_{t+j+1}) > \delta_1\, E_{t+j}\, n_{t+j+1}$$

The above condition can be interpreted in terms of the wage rate being above a certain reservation wage, which depends on the formal employment, ceteris paribus. The higher the formal labor supply, the higher the reservation wage of the worker. This indicates that workers manifest a preference for formal work, all else equal.

Modeling the labor demand

In this section we model the labor demand side. Similarly to workers, firms have to make decisions about their demand for production factors when they decide how much output to produce. There are two factors of production that enter a typical production function, capital and labor. To simplify the derivation of the expressions, we will leave capital out at this stage and assume that labor is the only factor of production used. Differently from the standard models, the firm has also to decide on the type of workers it hires: formal workers, informal, or a combination of the two.

Employing the work force informally could bring significant economies in term of reduced labor costs. Firms do not have to pay any wage related contributions, which represent a significant percentage of the payroll. On the other hand, if firing costs are important as well, firms may want to use informal work at times when there are temporary increases in output demand. As long as the firm is not sure that the increase in demand is permanent, it might not use formal work to accommodate it, due to the costs of firing them. Empirical evidence suggests that in general, savings on the labor costs are significant, and if the associated costs are not important, a firm could find itself in the situation of hiring its entire workforce informally.

The costs from hiring informally are expressed in term of productivity loss. We assume that formal workers are better qualified and more productive than informal workers, cetteris paribus. On the other hand, firms have to conceal their informal economic activity and, therefore, hiring and sometimes operating costs are larger, since it is not possible to look for informal workers in the usual way; informal work has to be camouflaged. Finally, firms are penalized if they are caught participating in the informal sector and, if the penalty is large enough it can act as a deterrent as well.

The maximization problem the firm faces, under the hypothesis that markets are competitive, is the following:

$$v_t = E_t \sum_{j=0}^{\infty} b^j \Pi_{t+j}$$

Where, v_t is the present value of the expected future stream of profits, Π_{t+j}.

The expression of the profit function is:

$$\Pi_{t+j} = (f_0 + a_{t+j})n_{t+j} + (f'_0 + b_{t+j})n^i_{t+j} - e_{t+j}\, n^i_{t+j} - \frac{f_1}{2}(n_{t+j} + n^i_{t+j})^2 - \frac{d}{2}(n_{t+j} - n_{t+j-1})^2$$
$$- w_{t+j}\, n_{t+j} - t' n_{t+j} - w^i_{t+j}\, n^i_{t+j} \qquad [2.1]$$

The profit function is modelled by a quadratic expression, increasing in labor and decreasing in expenditure. We normalized output prices to one, so they do not enter explicitly in the profit function. The first two terms that enter equation [2.1] captures the productivity of formal and informal labor. The marginal productivity of formal labor is different from that of informal labor, assuming that the two types of labor are not perfectly substitutable. We expect that the formal workers have higher productivity than the informal ones: $f_0 + a_{t+j} > f'_0 + b_{t+j}$. In general, unskilled work is more likely to be hired informally, especially since the wage cost of the unskilled workers are high.

The third term describes the expected costs of hiring the workforce informally. As already mentioned, firms incur costs associated with their participation in the informal market and the hiring of informal workers. We expect that the marginal product of informal labor be higher than the informal costs $f'_0 + b_{t+j} > e_{t+j}$, a necessary condition if the firm is to employ informal workers at all. The forth term captures the diminishing return to labor due to congestion. Since capital is fixed, employing more workers increases production only up to a point, after which the additional workers hinder rather than increase output. The next term captures the costs of adjusting the labor force, modeled by subtracting from the profit function the expression $\frac{d}{2}(n_{t+j}-n_{t+j-1})^2$, which is a measure of the change in employment from one year to another. We have not included a similar term for informal employment, since the employer has absolute flexibility in adjusting the size of the informal workforce. He can terminate his working relation with any informal worker at any time without cost.

The last two terms make up the total payroll costs to the firm. In the case of the formal labor force, there is an additional term t', which summarizes the firm's wage related costs. In the case of the informal labor, the firm pays only the wage to the workers.

The firm is a perfect competitor and treats, similarly to the worker, the wage as given. We compute the competitive wage at later stage, when we derive the equilibrium employment and wage in the two markets. Last, the worker takes as given the stochastic processes $\{a_{t+j}\}_{j=0}^{\infty}$, $\{b_{t+j}\}_{j=0}^{\infty}$, $\{e_{t+j}\}_{j=0}^{\infty}$, $\{w_{t+j}\}_{j=0}^{\infty}$, $\{w^i_{t+j}\}_{j=0}^{\infty}$, $\{t'_{t+j}\}_{j=0}^{\infty}$.

The maximization problem of the firm is the following:

$$\max E_{t+j} \sum_{j=0}^{\infty} b^j \ (f_0 + a_{t+j})n_{t+j} + (f'_0 + b_{t+j})n^i_{t+j} - e_{t+j}\, n^i_{t+j} - \frac{f_1}{2}(n_{t+j} + n^i_{t+j})^2 - \frac{d}{2}(n_{t+j} - n_{t+j-1})^2 - w_{t+j}\, n_{t+j} - t'_{t+j}\, n_{t+j} - w^i_{t+j}\, n^i_{t+j}$$

$f_0, f'_0, f_1, d > 0$

$0 < b < 1\backslash$

n_{t-1}, n^i_{t-1} given at t.

The first order conditions are:

$$FOC: \ \frac{\partial(.)}{\partial n_{t+j}} = 0$$

$$(f_0 + a_{t+j}) - f_1(E_{t+j}\, n_{t+j} + E_{t+j}\, n^i_{t+j}) - d(E_{t+j}\, n_{t+j} - E_{t+j}\, n_{t+j-1}) + bd(E_{t+j}\, n_{t+j+1} - E_{t+j}\, n_{t+j}) - - w_{t+j} - t'_{t+j} = 0 \qquad [2.2]$$

$$\frac{\partial(.)}{\partial n^i_{t+j}} = 0$$

$$(f'_0 + b_{t+j}) - e_{t+j} - f_1\,(E_{t+j}\, n_{t+j} + E_{t+j}\, n^i_{t+j}) - w^i_{t+j} = 0 \qquad [2.3]$$

Subtracting equation [2.3] from [2.2] we obtain:

$$(f_0 + a_{t+j}) - (f'_0 + b_{t+j}) + e_{t+j} - d\,(E_{t+j}\, n_{t+j} - E_{t+j}\, n_{t+j-1}) + bd\,(E_{t+j}\, n_{t+j+1} - E_{t+j}\, n_{t+j}) - - w_{t+j} - t'_{t+j} + w^i_{t+j} = 0$$

The above expression is a second order difference equation. Grouping terms together we derive the following expression:

$$bd\, E_{t+j}\, n_{t+j+1} - (d + bd)\, E_{t+j}\, n_{t+j} + d\, E_{t+j}\, n_{t+j-1} + (f_0 + a_{t+j}) - (f'_0 + b_{t+j}) - (w_{t+j} - w^i_{t+j}) - - (t'_{t+j} - e_{t+j}) = 0 \qquad [2.4]$$

Using the properties of the lag operator, the equation becomes:

$$bd\, E_{t+j}\, n_{t+j+1} - (d+bd)\, L\, E_{t+j}\, n_{t+j+1} + d\, L^2\, E_{t+j}\, n_{t+j+1} = \Delta\, w^f - (f_0 + a_{t+j}) + (f'_0 + b_{t+j})$$

where: $\Delta\, w^f = (w_{t+j} - w^i_{t+j}) + (t'_{t+j} - e_{t+j})$ is the difference in the firm's payroll costs for the formal and informal workers[3].

Finally, we obtain the following equation for the demand for formal employment:

$$[b\, d - d(1 + b)L + d\, L^2]\, E_{t+j}\, n_{t+j+1} = [\Delta\, w^f - (f_0 + a_{t+j}) + (f'_0 + b_{t+j})]$$

Similarly to the labor supply, the demand can be written:

$$\left[1 - \frac{d(1+b)}{bd} L + \frac{1}{b} L^2\right] E_{t+j} n_{t+j+1} = \left(\frac{1}{bd}\right) \Delta\, w^f - (f_0 + a_{t+j}) + (f'_0 + b_{t+j})$$

Consider α_1 and α_2 the solutions of the equation: $\left[Z^2 - \frac{(1+b)}{b} Z + \frac{1}{b}\right] = 0$. Then the above expression becomes:

$$(1 - \alpha_1 L)(1 - \alpha_2 L)\, E_{t+j}\, n_{t+j+1} = \left(\frac{1}{bd}\right) [\Delta\, w^f - (f_0 + a_{t+j}) + (f'_0 + b_{t+j})] \qquad [2.5]$$

The necessary and sufficient condition for α_1 and α_2 to be real numbers is for the discriminant of the second order equation to be positive. The mathematical expression of the discriminant is: $\left(\frac{1+b}{b}\right)^2 - \frac{4}{b} \geq 0$.

$$\left(1 + \frac{1}{b}\right)^2 - \frac{4}{b} = 1 + \frac{2}{b} + \frac{1}{b^2} - \frac{4}{b} = 1 - \frac{2}{b} + \frac{1}{b^2} = \left(1 - \frac{1}{b}\right)^2 > 0$$

Since the discriminant is positive, the solutions are:

$$\alpha_{1,2} = \frac{\left(1 + \frac{1}{b}\right) \pm \left(1 - \frac{1}{b}\right)}{2} = \frac{1 + \frac{1}{b} \pm 1 \mp \frac{1}{b}}{2}$$

which, after simplification, become: $\alpha_1 = 1$ and $\alpha_2 = \frac{1}{b}$.

We know that applying the operator $1/(1 - \alpha\, L)$ to an expression gives a weighted sum of future values of the same expression.

[3] Note that the expression is different from the difference in wages between the two categories of workers, since the firm has to pay work - related contributions, which have to be added to the wage bill, as well as spend resources to conceal its participation in the informal sector.

$$\frac{1}{1-\alpha\cdot L} = -\sum_{i=0}^{\infty}\left(\frac{1}{\alpha}\right)^{i} L^{-i}$$

Multiplying both sides of the equation [2.5] with $1/(1 - \alpha_2)$, we obtain:

$$(1 - \alpha_1 L)\, E_{t+j}\, n_{t+j+1} = \frac{\Delta w^{f}{}_{t+j} - \left(f_0 + a_{t+j}\right) + \left(f'_0 + b_{t+j}\right)}{bd(1-\alpha_2 L)}$$

$$(1 - \alpha_1 L)\, E_{t+j}\, n_{t+j+1} = \frac{-\left(bd\alpha_2\right)^{-1} L^{-1}}{1-\alpha_2{}^{-1} L^{-1}}\left[\Delta w^{f}{}_{t+j} - \left(f_0 + a_{t+j}\right) + \left(f'_0 + b_{t+j}\right)\right]$$

$$E_{t+j} n_{t+j+1} - \alpha_1 L E_{t+j} n_{t+j+1} = \frac{-\alpha_1}{d} L^{-1}\left(1 + \frac{1}{\alpha_2 L} + \frac{1}{\left(\alpha_2 L\right)^{-1}} + ..\right)[\Delta w^{f}{}_{t+j} - (f_0 + a_{t+j}) + (f'_0 + b_{t+j})]$$

$$E_{t+j}\, n_{t+j+1} = \alpha_1\, n_{t+j} - \frac{\alpha_1}{d}\sum_{i=0}^{\infty}\left(\frac{1}{\alpha_2}\right)^{i} [\, E_{t+j}\, \Delta w^{f}{}_{t+j+i} - (f_0 + E_{t+j}\, a_{t+j+i}) + (f'_0 + E_{t+j}\, b_{t+j+i})]$$

Replacing the values for α_1 and α_2, we obtain the following expression:

$$E_{t+j}\, n_{t+j+1} = n_{t+j} - \frac{1}{d}\sum_{i=0}^{\infty} b^{i}\, [\, E_{t+j}\, \Delta w^{f}{}_{t+j+i} - (f_0 + E_{t+j}\, a_{t+j+i}) + (f'_0 + E_{t+j}\, b_{t+j+i})]$$

The expression derived is the contingency plan of the firm in the case it decides to hire formal labor. The demand for formal employment depends positively on the previous period's employment, negatively on a weighted average of the expected future values of the difference in costs to the firm from hiring formally and informally, and negatively on the weighted average of the expected future values of the difference in productivity between the two types of workforce. The importance of future values declines as we depart from the current period. The firm chooses a current level of formal employment equal to the previous period's if the payroll costs to the firm do not differ and if the productivity of the two types of workers is similar. One can notice that there is a degree of rigidity in the formal labor demand of the firm, since there is still some demand for formal workers even when the gains in productivity from hiring formal workers is not enough to cover the losses incurred due to higher formal wages. This happens when the expression under summation is negative. The firm then adjusts its demand in time; therefore the demand curve exhibits a certain degree of inertia. The same inertia is manifested when employment needs to be adjusted upwards. In other words, policies aimed at increasing formal employment would take some time to become effective.

The demand for informal labor is obtained by replacing in equation [2.4] the expression we have derived for the formal employment:

$$E_{t+j}\, n^{i}{}_{t+j+1} = -\, E_{t+j}\, n_{t+j+1} + \frac{f'_0 + b_{t+j+1}}{f_1} - \frac{w^{i}_{t+j+1} + e_{t+j+1}}{f_1}$$

The demand for informal workers is a function of the current period's demand for formal workers, as well as productivity and informal wage. The presence of the current period optimal demand for formal workers in the demand for informal labor is an indication of the simultaneity of the decision regarding the demand for formal and informal workers. The decision of hiring informal labor can not be separated from the decision of recruiting formal workers.

The labor demand for informal workers is smaller, the larger the demand for formal work, and the larger the costs with informal hiring. On the other hand, it increases with informal labor productivity. From the expression of the demand for informal work one can notice that, unless labor productivity is high enough, the firm will not employ any informal workers. The necessary condition that has to be satisfied if the firm is to hire informal workers is the following:

$$(f'_0 + b_{t+j+1}) - (w^i_{t+j+1} + e_{t+j+1}) > f_1 E_{t+j}\, n_{t+j+1}$$

The demand for informal workers does not exhibit the inertia that the one for informal labor shows. Any increase in productivity and/or decrease in wages is immediately matched by an increase in demand, as long as the necessary condition is satisfied.

Determining the equilibrium wage and the level of informal employment

In the previous sections we constructed a supply and demand schedule for formal and informal labor. The two schedules give the desired employment as a function of initial employment and the stochastic process of the real wage. In this section we solve for the equilibrium employment level and wage assuming that both labor markets clear at all points in time.

In other words, we have to find two pairs of stochastic processes $\{w_{t+j}\}$, $\{n_{t+j}\}$ for j= 0, …, ∞, and $\{w^i_{t+j}\}$, $\{n^i_{t+j}\}$ for j= 0, …, ∞, that satisfy the conditions that when the representative worker takes the formal and informal wage as given, $\{n_{t+j}\}$ and $\{n^i_{t+j}\}$ maximize the utility function [1.1], and when the representative firm takes the formal and informal wage as given, $\{n_{t+j}\}$ and $\{n^i_{t+j}\}$ maximize its profit function.

The equilibrium in the formal market can be obtained from the following relations:

Supply: $\delta_2\, b\, \gamma E_{t+j}\, n_{t+j+1} + \delta_2 (1 + b\, \gamma^2) n_{t+j} + \delta_2\, \gamma\, n_{t+j-1} = u_0\, [(w_{t+j} - w^i_{t+j}) - (t_{t+j} - r_{t+j})] - (\delta_0 + \varepsilon_{t+j}) + (\delta'_0 + \mu_{t+j})$ [1.5]

Demand: $bd\, E_{t+j}\, n_{t+j+1} - (d + bd)\, n_{t+j} + d\, n_{t+j-1} = (w_{t+j} - w^i_{t+j}) + (t'_{t+j} - e_{t+j}) - (f_0 + a_{t+j}) + (f'_0 + b_{t+j})$ [2.4]

Multiplying the demand by u_0 and subtracting it from the supply equation, we obtain the expression:

$$(\delta_2 b\gamma - u_0 bd)\, E_{t+j}\, n_{t+j+1} + [\delta_2(1+b\gamma^2)+u_0(d+bd)]n_{t+j} + (\delta_2\gamma - u_0\, d)n_{t+j-1} =$$
$$u_0\, (f_0 + a_{t+j}) - u_0(f'_0 + b_{t+j}) - u_0\, (t_{t+j} - r_{t+j}) - u_0\, (t'_{t+j} - e_{t+j}) - (\delta_0 + \varepsilon_{t+j}) + (\delta'_0 + \mu_{t+j})$$

The above equation is also a second order difference equation. Grouping the terms together, and using the lag operator, this becomes:

$\{(\delta_2 b\gamma - u_0 bd) + [\delta_2(1+b\gamma^2)+u_0(d+bd)]L + (\delta_2\gamma - u_0 d)L^2\}\, E_{t+j}\, n_{t+j+1} =$
$u_0\,(f_0 + a_{t+j}) - u_0(f'_0 + b_{t+j}) - u_0\,(t_{t+j} - r_{t+j}) - u_0\,(t'_{t+j} - e_{t+j}) - (\delta_0 + \varepsilon_{t+j}) + (\delta'_0 + \mu_{t+j})$

Consider μ_1 and μ_2 the solutions of the equation

$Z^2 + [\delta_2(1+b\gamma^2)+u_0(d+bd)]\,(\delta_2 b\gamma - u_0 bd)^{-1}\, Z + (\delta_2\gamma - u_0\, d)\,(\delta_2 b\gamma - u_0 bd)^{-1} = 0.$ [3.1]

Then the above equation can be written as:

$(1 - \mu_1 L)(1 - \mu_2 L)E_{t+j}\, n_{t+j+1} = \{u_0\,(f_0 + a_{t+j}) - u_0(f'_0 + b_{t+j}) - u_0\,(t_{t+j} - r_{t+j}) - u_0\,(t'_{t+j} - e_{t+j}) - [(\delta_0 + \varepsilon_{t+j}) + (\delta'_0 + \mu_{t+j})]\}\,(\delta_2 b\gamma - u_0 bd)^{-1}$ [3.2]

The solutions μ_1 and μ_2 are real numbers if the discriminant of the second order equation [3.1] is positive.

The discriminant has the following expression:

$$\frac{[\delta_2(1+b\gamma^2)+u_0(d+bd)]^2}{(\delta_2 b\gamma - u_0 bd)^2} - 4\frac{(\delta_2\gamma - u_0 d)}{(\delta_2 b\gamma - u_0 bd)} \geq 0 \text{ or:}$$

$$\frac{[\delta_2(1+b\gamma^2)+u_0(d+bd)]^2 - 4(\delta_2\gamma - u_0 d)(\delta_2 b\gamma - u_0 bd)}{(\delta_2 b\gamma - u_0 bd)^2} \geq 0$$

As both the denominator and numerator are positive[4], the equation has real solutions.

The solutions of the second order equation [3.1] have to satisfy the following properties:

$\mu_1\, \mu_2 = (\delta_2\gamma - u_0\, d)\,(\delta_2 b\gamma - u_0 bd)^{-1} = 1/b$
$\mu_1 + \mu_2 = -\ \delta_2\,(1+b\gamma^2)+u_0(d+bd)]\,(\delta_2 b\gamma - u_0 bd)^{-1}$

The product of the two solutions is a positive number which implies they have the same sign, they are both either positive or negative. The value of their sum depends on the sign of the expression: $\delta_2 b\gamma - u_0 bd$. If $\delta_2 b\gamma - u_0 bd > 0$, then the solutions are negative, whereas if $\delta_2 b\gamma - u_0 bd < 0$, then the solutions are positive. Looking at the labor supply and demand, one can notice that the above expression measures the difference in slopes between the two schedules.

Refining equation [3.2] above , we obtain:

$(1 - \mu_1 L)\, E_{t+j}\, n_{t+j+1} = (1 - \mu_2\, L)^{-1}\, \{u_0\,(f_0 + a_{t+j}) - u_0(f'_0 + b_{t+j}) - u_0\,(t_{t+j} - r_{t+j}) - u_0\,(t'_{t+j} - e_{t+j}) - [(\delta_0 + \varepsilon_{t+j}) + (\delta'_0 + \mu_{t+j})]\}\,(\delta_2 b\gamma - u_0 bd)^{-1}$

[4] The denominator is positive since is a squared number. The numerator can be expanded into a sum of positive numbers, as follows :
$[\delta_2^2(1+b\gamma^2)^2 + u_0^2 d^2(1+b)^2 + 2\delta_2\, u_0\, d(1+b\gamma^2)(1+bd) - 4\,\delta_2^2 b\gamma^2 + 4\delta_2\gamma\, u_0\, bd + 4u_0 d\delta_2 b\gamma$
$- 4\, u_0^2\, bd^2) = \delta_2^2(1 - b\gamma^2)^2 + u_0^2 d^2(1-b)^2 + 2\delta_2\, u_0\, d(1+b\gamma^2)(1+bd) + 4\delta_2\gamma\, u_0\, bd + 4u_0 d\delta_2 b\gamma > 0$

Denoting $K = u_0\,(f_0 + a_{t+j}) - u_0(f'_0 + b_{t+j}) - u_0\,(t_{t+j} - r_{t+j}) - u_0\,(t'_{t+j} - e_{t+j}) - (\delta_0 + \varepsilon_{t+j}) + (\delta'_0 + \mu_{t+j})\,(\delta_2 b\gamma - u_0 bd)^{-1}$ the expression becomes:

$$(1 - \mu_1 L)\, E_{t+j}\, n_{t+j+1} = \frac{-(\mu_2^{-1} L^{-1})}{1 - \mu_2^{-1} L^{-1}} K \quad \text{or:}$$

$$E_{t+j}\, n_{t+j+1} - \mu_1 L\, E_{t+j}\, n_{t+j+1} = \frac{-\mu_1 L^{-1}}{b} \frac{1}{1 - \mu_2 L} K$$

$$E_{t+j}\, n_{t+j+1} = \mu_1\, n_{t+j} - \frac{\mu_1}{b} \sum_{i=0}^{\infty} \left(\frac{1}{\mu_2}\right)^i [\, u_0\,(f_0 + a_{t+j+i}) - u_0(f'_0 + b_{t+j+i}) - u_0\,(t_{t+j+i} - r_{t+j+i}) - u_0\,(t'_{t+j+i} - e_{t+j+i}) - (\delta_0 + \varepsilon_{t+j+i}) + (\delta'_0 + \mu_{t+j+i})]\, (\delta_2 b\gamma - u_0 bd)^{-1}$$

The expression above describes the equilibrium path of the formal employment. The term $t_{t+j+i} - r_{t+j+i}$ represents the benefit that the worker derives from participation in the informal sector, since t_{t+j+i} is the tax the worker pays on the formal wage, and r_{t+j+i} are resources spent in concealing the informal activity. Similarly, the term $t'_{t+j} - e_{t+j}$ represents the benefit the firm derives from operating in the informal sector, since t'_{t+j} is the tax the firm pays for its labor force, and e_{t+j} are the resources spent in order to cover their informal activities. Hence, the optimal formal employment dynamics is determined by the cumulative effect of the benefits derived from informal participation by both actors, the individual and the firm, as well as by the differences in productivity and disutility between formal and informal work.

If the expression $\delta_2 b\gamma - u_0 bd > 0$, the solutions are negative, and formal employment depends positively on the current productivity. Future values of the differences in productivity enter in the equation of the formal employment with signs that alternate. The observation is true in the case of the other variables as well. This indicates an inter-temporal substitution effect, with formal employment reacting to expected future values of differences in productivity, in benefits and in disutilities. We notice that the equilibrium formal employment reacts to the future expected values similarly to the labor supply schedule.

If the expression $\delta_2 b\gamma - u_0 bd < 0$, the solutions to equation [3.1] are positive, and formal employment depends positively on the difference in productivity between the two types of workers, negatively on the benefits to both firms and workers derived from operating informally, and negatively on the difference in the disutility from work. The dependence of the equilibrium formal employment on the expected future values is in this case similar to that of the labor demand schedule.

Let's move our attention to the supply and demand in the informal market.

Supply: $u_0\,(w^i_{t+j} - r_{t+j}) - (\delta'_0 + \mu_{t+j}) - \delta_1\, E_{t+j}\,(n_{t+j} + n^i_{t+j}) = 0$ [1.4]

Demand: $(f'_0 + b_{t+j}) - w^i_{t+j} - e_{t+j} - f_1\,(E_{t+j}\, n_{t+j} + E_{t+j}\, n^i_{t+j}) = 0$ [2.3]

Multiplying the supply equation by f_1 and the demand equation by δ_1 and subtracting one from the other we obtain an equation whose only variable is the informal wage:

$$(f_1\, u_0 + \delta_1)w^i_{t+j} - f_1\, u_0\, r_{t+j} - f_1\, (\delta'_0 + \mu_{t+j}) + \delta_1\, e_{t+j} - \delta_1\, (f'_0 + b_{t+j}) = 0$$

$$(f_1\, u_0 + \delta_1)w^i_{t+j} = f_1\, u_0\, r_{t+j} + f_1\, (\delta'_0 + \mu_{t+j}) - \delta_1\, e_{t+j} + \delta_1\, (f'_0 + b_{t+j})$$

$$w^i_{t+j} = \frac{f_1 u_0 r_{t+j} + f_1(\delta'_0 + \mu_{t+j}) - \delta_1 e_{t+j} + \delta_1 (f'_0 + b_{t+j})}{f_1 u_0 + \delta_1}$$

The expression above gives the equilibrium wage in the informal sector, as a function of the resources spent in concealing participation, the informal labor productivity, the cost to the firm from operating informally, and the worker's disutility from informal work.

An expression for the formal equilibrium wage can be obtained from equation [1.5] after plugging in the equilibrium level of formal and informal wage.

From equation [1.4] or [2.5] we can obtain the expression for the equilibrium path for informal work:

$$u_0\, (w^i_{t+j} - r_{t+j}) - (\delta'_0 + \mu_{t+j}) - \delta_1\, E_{t+j}\, (n_{t+j} + n^i_{t+j}) = 0 \qquad [1.4]$$

which can be rewritten as:

$$\delta_1\, (n_{t+j} + n^i_{t+j}) = u_0\, (w^i_{t+j} - r_{t+j}) - (\delta'_0 + \mu_{t+j})$$

From the expressions of the equilibrium formal employment and informal wage, we derive the equilibrium dynamics of the informal employment. After some computations, we arrive at the following formula for the informal employment:

$$n^i_{t+j} = \frac{u_0(f'_0 + b) - (\delta'_0 + \mu_{t+j}) - u_0(r_{t+j} + e_{t+j})}{f_1 u_0 + \delta_1} - n_{t+j}$$

The above expression states that the informal equilibrium employment is a function of the formal employment, informal workers' productivity, cumulative informal costs to workers and firms from informal participation, and the disutility from informal work. In order to carry out informal activity, the ratio has to be significantly larger than zero. As before, we can obtain a necessary condition in order to obtain an equilibrium informal employment different from zero, which is a combination of the previous two necessary conditions in the case of the labor demand and labor supply. This condition states that the productivity of the informal workers has to be larger than the informal costs and the informal disutility from work.

Conclusions

This paper presents a dynamic labor market model that describes the mechanisms through which individuals and firms take decisions regarding the supply of, respectively the demand for formal and informal labor as a function of the prevailing market wage rates and other parameters.

The determinants of the formal labor supply schedule are the current and the expected future differences in net wage rates and disutility from work between the formal and informal sector, as well as previous formal employment levels, which enter negatively due to the fatigue effect it has on workers. The formal labor supply schedule displays an inter-temporal substitution characteristic, through which current employment adjusts by responding to expected changes in variables. The informal labor supply schedule highlights the simultaneous feature of the decision to work formally and informally. The informal labor supply is positively influenced by the informal net wage and negatively by the disutility from informal work.. There is a reservation level of the informal wage, below which workers would not participate in the informal sector. This level is influenced to a large extent by the formal employment. Higher formal employment demands higher informal wages, ceteris paribus.

The determinants of the labor demand schedule are the current and the expected future values of difference in productivity between the two sectors, and current and expected future differences in labor costs to the firm. The formal labor demand schedule exhibits a certain degree of rigidity, with firms adjusting slowly to changes in the current and the expected future levels of the variables. Therefore, any policy aimed at increasing formal employment would take some time to become effective. The informal labor demand is determined by the current formal employment, informal productivity and informal wage. Similarly to the supply schedule, there is a reservation informal productivity level below which firms do not hire informally. The reservation level of productivity is determined to a large extent by the level of the formal employment. Higher formal employment levels result in higher reservation productivity, and vice versa, ceteris paribus.

The path of the equilibrium formal employment is determined to a large extent by the difference in the slopes of the demand and supply schedules. When demand is steeper than supply, the formal equilibrium employment path exhibits similar characteristics to the supply schedule, namely an inter-temporal substitution effect triggered by the current and the expected future changes in variables. When supply is steeper than the demand, the formal employment equilibrium path exhibits characteristics similar to the demand schedule, namely a rigidity of the equilibrium employment level vis-a-vis the current and the expected future changes in variables. The informal equilibrium employment is determined by informal labor productivity, disutility from informal work and costs to both workers and firms from informal participation. There is a necessary condition that insures the existence of an informal market equilibrium. This condition requires that the productivity of the informal workers be above a reservation level, which depends to a large extent to the formal employment level. Higher equilibrium level of formal employment demands higher informal productivity, ceteris paribus.

Bibliography:

Johnson, S., Kaufmann D. and Shleifer A. (1997). "The unofficial economy in transition", *Brookings Papers on Economic Activity* 2, pp. 159-239.

Kim, B.Y. (2002). "Poverty and informal economy participation in transition countries: evidence from Romania" (mimeo), University of Essex.

Kolev, A. (1998). "Labor supply in the informal economy in Russia during transition", Discussion Paper no. 2024, CEPR, London.

Lemieux, T., Fortin B. and Frechette P. (1994). "The effect of taxes on labor supply in the underground economy", *American Economic Review*, vol. 84, no. 1, pp. 231-255.

Lucas, R. E. Jr. and Rapping L. (1969). "Real wages, employment and inflation." *Journal of Political Economy*, Vol. 77, No. 2, pp. 1-19.

Pauna , B., Pauna C. and Scutaru C. (2002). "The process of EU integration – implications for the formal and informal economy in transition" (mimeo), Institute for Economic Forecasting, Bucharest, Romania.

Sandmo, A. (1981). "Income tax evasion, labor supply, and the equity-efficiency tradeoff", *Journal of Public Economics*, no. 265-288, pp. 265-288.

Sargent, T.J. (1987). *Macroeconomic Theory*, Academic Press, London, UK.

Thomas, J.J. (1992). *Informal Economic Activity,* Harvester Wheatsheaf, UK.