Stable GARCH and Temporal Aggregation

(Non-technical summary)

by

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The paper presents estimates of stable power GARCH models for returns aggregated with varying temporal frequency. GARCH models are the single most important econometric tools for describing the return process of securities. These models reasonably depict some typical characteristics of the return series recorded at exchange markets (e.g. volatility clustering, heavy tails), but they still fail in many cases. The proliferation of alternative GARCH models indicates the intensity of the effort to improve the performance of these models.

A possible reason for the failures of GARCH may be the assumption about the distribution of innovations: diverse GARCH variants frequently assume normally distributed innovations, however, that is often not supported by empirical evidence.

Choosing a distribution to model GARCH innovations is still an open problem. None of the candidates tried so far (e.g. normal, Student-t, generalized exponential, Pareto stable distributions) have proven to be significantly better in general than the others. Stable distributions (normal is a special case) can be favoured against the others by their special role in probability theory: by the generalized central limit theorem. Stable distributions are the only non-degenerate distributions arising as limits of normalized sums of independent and identically distributed random variables. Thus if we think of innovations as sums of random effects too numerous and difficult to incorporate into the model, then stable distributions are a natural choice to describe them. The name stable refers to stability under addition: the distribution of appropriately normalized sums of iid stable distributions is the same as the distribution of the summands. The key parameter of stable distributions is

the index of stability (this parameter is invariant under convolution) $0 < \alpha \leq 2$. For the normal distribution $\alpha = 2$.

We have a dual goal in this paper. On the one hand, we compare the performance of GARCH models with Gaussian and Lévy distributions (stable with $\alpha < 2$). We use return series with different frequencies, to analyse the stability of the risk process. On the other hand, we compare the properties of two investments at two very different markets: one major stock from NASDAQ (CISCO), and one from the Budapest Stock Exchange (MOL). We start from transaction level data for 1998, and we derive the analysed time series from these.

The Budapest Stock Exchange is a recently (re)established, small, thin market with little tradition and experience in the proper management of an exchange market. However, it probably was the best-regulated and most transparent market in the Central and Eastern European region. By the mid-1990's it could develop into a 'normal' market, leaving behind most of the initial peculiarities. Both market participants and regulatory authorities acquired the skills necessary for operating smoothly on the market (*c.f.,* Johnson and Schleifer, 1999).

The question whether a series of returns is stable under addition can be investigated by forming non-overlapping sums of size n of successive returns and estimating the index of stability of the sequence of aggregated returns. If the original sequence of returns is stable, then the index of stability is a constant independent of the aggregation.

However, if we assume that the returns series comes from a specific data generating process, the above temporal aggregation should be performed on the random component of the stochastic process. As we assume that stock market returns were generated by a stable power GARCH process with Lévy distributed innovations, we should only aggregate the innovations themselves, not the original time series.

The consequences of temporal aggregation of a GARCH generated returns series are well-known if the GARCH process is driven by innovations with finite moments up to the fourth order (*c.f.,* Drost and Nijman 1993). However, Lévy distributions have infinite moments beyond the first one, thus, the theoretical properties of the aggregated series are unknown. Consequently, we had to develop an indirect test: we first estimated the stable power GARCH model from high frequency data, and examined the stability of innovation distribution under summation, using the residuals of the estimated GARCH models.

We had to deal with an additional difficulty: as the underlying distribution has no finite variance, (unless it is the Gaussian limit) the usual asymptotic theory does not apply. We had to simulate critical values for the test statistics, and bootstrap confidence interval for the estimated parameters. One important result in our paper is that the distribution of the diagnostic test statitistics strongly depends both on the actual value of the index of stability, and, more disturbingly, on the sample size, even for relatively large samples.

One important result of our study is that Lévy stable GARCH models clearly outperform normal GARCH ones. Thus, the probability of extreme shocks is severely underestimated by the standard risk models, based on the usual Gaussian GARCH process. This may lead to incorrect Value at Risk calculations. The deposit requested from day traders, and various other capital adequacy measures are all based on such calculations. As the true process is better described by an unrestricted Lévy GARCH model, market actors and regulators may well incurr unexpected (and unwanted) excess risk in their operations.

Another very important result is that the gain in using a Lévy GARCH model is much larger for MOL. Thus, extreme events are more likely to drive an emerging market than a mature one. Further, Gaussian GARCH estimated from MOL is much more sensitive to sample adjustments than from CISCO, which also indicates that risk analysis, based on Gaussian innovations, may be very misleading on an emerging capital market.

However, even though unrestricted stable GARCH models dominate Gaussian ones, they are not perfect descriptions of the return process either. We can clearly reject stability of the residuals. Thus, these models are just specific approximations of the true data generating processes, and they can only be applied with due caution.