

WHY ARE THERE MOBILITY RESTRICTIONS?

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ABSTRACT

Mobility restrictions (e.g., suburban payment, life-long tenure, non-ivory) are widely observed. This paper is an attempt to understand why there are these restrictions. I present a model economy that features production technologies that focus on the search for partners in the formation and break-up of teams. Under no restrictions, there is too much searching and break-up as the individual decision problem ignores the loss of the search partner's utility. Arrangements such as search payment and break-up payment can improve welfare. Thus the paper rationalizes mobility restrictions as welfare-improving arrangements.

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Key words: mobility restriction; production team; search; break-up

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1. INTRODUCTION

Monetary restrictions such as variable payments and life-long tenure in the labor market have significant effects on aggregate economic activity. The high unemployment in Europe, in comparison to the US, has long been attributed to regulations restricting labor mobility. Lizardo (1990) finds that variable payments reduce employment in his estimation of European data. Hopmann and Rogerson (1993) report similar findings in a labor market equilibrium model. The authors further report significant negative welfare effects of firing costs.¹ These findings lead me to question why there are these monetary restrictions: if they are so costly, why do we observe them? Monetary restrictions are also widely found in the marriage market. Divorce payments are common in divorce cases and widely observed in developing countries in the history of developed countries. Another example of monetary restriction is in union membership. For example, the Japanese union system, *keiretsu*, works under the implicit understanding that the firm would be obliged to stay in the network (see Fukuyama 1996). On my argument that these restrictions lead to inefficient matching and thus reduce welfare, economists otherwise. Then again, why do we observe these restrictions? This is the question that I address in this paper.

One reason why there are monetary restrictions may be that the people who gain by having these restrictions somehow have the power to have their way. Thus the workers' unions are often blamed for imposing firing restrictions. In this vein, Saint-Paul (1997) argues that the employer is politically strong enough to resist employment-protection legislation in Europe. Although this argument has been intuitively appealing, it also has a drawback. Implicit in the argument is that these restrictions have overall negative welfare effects

¹ There is ambiguity about the employment effect of firing costs. Bentolila and Bertola (1990) and Alvarez and Veracierto (1999) report positive effect using calibrated models. See Ljungqvist (1999) for a good evaluation of the literature on this issue. To my knowledge, however, all researchers report, if any, negative welfare effect of firing costs.

that they're an unfortunate outcome given political and institutional constraints. The argument then is simply a question of why there is not a compensation scheme whereby the gig inns (firms or unemployed workers) from removing the restrictions on compensation to the losers (employed workers) so that everybody comes out better off. Such a compensation scheme is not a new brilliant idea (see Rorik 1996). In this paper, I take a different perspective, simply, propose to understand the mobility restrictions that exist without such a scheme. I explore how the restrictions may improve overall welfare, thereby justifying their presence.

The basic story is as follows. Imagine two people in partnership. Suppose one of them makes a labor-intensive partner. If he picks up with his current partner in forms a new partnership with the labor-intensive partner, this is a net partner suffers welfare loss. However, without any restrictions on a break-up, the welfare loss of the labor-intensive partner is attributed to the decision problem of the starting partner. Due to this externality, the partnership picks up too easily and further there is too much search for a labor-intensive partner since making a labor-intensive partner is over-valued. In this situation, a break-up restrictions such as a break-up payment or a break-up fee can improve the joint welfare of the partners by internalizing (some of) the externality.

The elements of the story are not new. The idea that a search payment may improve welfare is central in the implicit contract theory (see, for example, Arnott, Hosios, and Stiglitz 1988). The focus of the theory, however, is on the issues of wage rigidity and involuntary unemployment. Further, the driving force behind the implicit contract is the need to insure risk-averse workers against negative shocks, on which the above story does not depend. In terms of modeling, the story is simply placed in the framework of the search and matching theory. In the theory, various types of externality have been recognized

in the model.² However, the particular externalities present in the above story has not been explored to my knowledge.³

Before proceeding to the model, I will make some qualifications so that I don't oversell the story unwittingly. First, the model is kept to quite a strict level. For instance, the model does not feature inter-symmetry between current partners, present between the firm and workers or between men and women. I recognize that modeling such symmetry is important if I will later take the model to explore the quantitative implications in the labor or marriage markets. However, I think that the nature of search and re-employment in the labor or marriage markets is similar enough to that in symmetric partnership (e.g., unions, partnership, friendship) so that we can grasp the main argument of the story at the most in a level of abstraction without sacrificing its applicability in various contexts. Second, I do not limit that the model captures the solution for mobility restrictions. Rather, I view the reason as rather comprehensive than others, such as the one seen in political economy. Third, the model does not plan cross-country or cross-period differences in mobility restrictions. Thus the model does not answer why the European labor market has more restrictions than the US market or why the divorce rate was high in the 70's in Italy and Brazil. The model, however, does provide a reason why there *may* be mobility restrictions and thus, I think, is set up for an interesting theoretical discussion in the future.

The main part of the paper is organized as follows. In Section 2, I present the model of search and re-employment that formalizes the above story. For the narrative purpose, I consider the economic situation that shares the same environment apart with respect to the re-employment restriction. The natural market economy has no restrictions and consequently there is too much search and re-employment. The social economy restricts re-employment requiring

² See Hosios (1990) and the references therein.

³ See Burdett and Coles (1999) for a standard formalization of partnership formation. The main difference of the model in this paper is that partnerships can be voluntarily broken, which gives rise to the above-mentioned externality.

the starting point to pay the start point in the amount equal to the welfare loss of the start point. This break-up payment arrangement is higher than the optimal value of the start point. The third economy restricts break-up by the break-up payment on current points' optimality. Although this break-up arrangement is not higher than optimality, it improves welfare on the nonhomogeneous economy. In Section 3, I discuss some further implications and interpretations of the model. In Section 4, I conclude by evaluating the merits and shortcomings of the model.

2. A MODEL ECONOMY OF SEARCH AND BREAK-UP

In this section, I will formalize the value of mobility restrictions by comparing the economy with the future start point break-up and in terms of break-up restrictions. But, first, I will lay out the common environment of the economy. There are many people. Everyon's own-period utility is $c-x$, where c is consumption and x is search effort or search cost. Everyon counts future consumption and search costs by own-period discount rate β . At the beginning of the period, a person is either alone, or part of a two-person production team. The first event of the period is that people work to produce output. A person working alone produces y_1 units of output. The output of the team depends on how compatible the two partners are. Let $y \in [y_1, y_2]$, where $y_1, y_2 > 0$, not the compatibility of the partners. A team with compatibility y produces $2y$ units of output. Partners divide output equally so that each partner takes y units of output. The equal division of the output is a natural assumption since, as will be discussed below, the two partners' search and matching prospects are identical and thus they are in equal bargaining positions.

After production, people search for new partners. The search and matching technology is as follows. A person paying x units of search cost meets potential partner with probability $m(x)$, that satisfies $m(0) = 0$ and for all $x \geq 0$, $m'(x) > 0$ and $m''(x) < 0$. The concavity of the function m ensures that the equilibrium search level for the partners of

any production technology is interior and unique. The compatibility of any two technologies is determined by the distribution of the function $F(y)$. Implicitly, this is the assumption that although person A and person B differ in terms of their compatibility with person C, their pairwise compatibility is independent in terms of the distribution of compatibility.

To focus the analysis on the issues of coordination between current partners, I will make two simplifying assumptions. First, for any current technology, the other of the two partners must be a potential partner: either one of the two matches is a potential partner or neither is. This will approximate the case if the length of the period is small, even if the search outcomes of the two current partners are independent of each other: the probability of the two current partners meeting potential partners is assumed to be y .⁴ To be consistent with this assumption, assume that for all $x \geq 0$, $m(x) < 1/2$. Second, if a person whose compatibility with his current partner is y matches a potential partner, this potential partner's compatibility with *his* current partner is also y . Similarly, a person without a match is only a person without a match. Without this assumption, a person's search history and tenure on the other side are affecting the distribution of searches: the search of a person with low current output has a positive tenure on the other side, so he would be more willing to form a new match; conversely, the search of a person with high current output has a negative tenure. Thus, this assumption allows the analysis to start away from the complication resulting from this type of tenure and to focus on the tenure present between current partners. To concentrate the institution learning must be consistent with this assumption, suppose that the current output is observable to everyone and that there is a costless technology that lets people to various search methods depending on their current output.

⁴ This assumption is implicit in continuous-time models, which are obtained in the limit of shortening the length of period. See Burdett and Coles (1999).

After surviving in a market, a person who must eventually participate in a market to form a new team with the eventual participant or to stay with the current participant, taking as given any risk-up restriction. Not that, given the assumptions on a market technology, two newly-met people eventually in terms of the competitiveness with their respective current partners in their future prospects. Thus their risk-up decision will be the same: either they both want to form a new team with each other or neither wants to. This simplifies the analysis greatly. After that, the main form of risk-up, or provision, is a provision of current output, not of any risk-up payment. Finally, the nature of the prior distribution of all (new and old) teams is risk-up only. The analogous risk-up is simply a way of generating non-diminishing returns to risk-up activities in the long-run.

In this environment, a person's key decisions are that is how much to survive, if he must eventually participate, whether to form a new team. Important in the person's decision problem is what constraints he faces in coordinating the survival with his current partner. In the following, I will consider the economic situation in this respect.

2.1 Economy 1: No Restriction

As an remark, let us first consider an economy where survival and risk-up are not coordinated between current partners: for survival they cannot jointly commit to survival or risk-up rule nor can they negotiate forming an eventual partner. This environment, although meant to capture the terms for the non-cooperative purposes, has some realism. First, committing to a rule is difficult in many circumstances: a person's survival is difficult for his partner to observe in any enforcing rule on a (pre-)post unwilling partner is not easy. Second, people may not have enough means of payment either for honoring a rule or for a post-negotiation: the current income may be small relative to the payment in the market they are involved in.

To formalize a person's decision problem in this economy, let $v(y)$ be the present discounted utility of a person with the current output y . The Bellman's equation is

$$\begin{aligned} v(y) = \max_{x,g} \{ & y - x + \beta(1 - \delta)m(x) \int_{y_1}^{y_2} g(y')v(y')dF(y') \\ & + \beta(1 - \delta)[1 - m(x) \int_{y_1}^{y_2} g(y')dF(y') - \phi(y)]v(y) \\ & + \beta[\delta + (1 - \delta)\phi(y)]v(y_1)\}. \end{aligned} \quad (1)$$

In this equation, $g(y')$ is the r -k-up decision made upon meeting a laborer partner whose compatibility is y' ($g(y') = 1$ if a new match is formed in the current time is broken up and $g(y') = 0$ if not) and $\phi(y)$ is the probability of finding a suitable partner. The person takes this probability as given since there is no coordination between partners. Let $x(y)$ and $g(y, y')$ denote the search and r -k-up functions associated with the Bellman's equation. Since the current partners of a man solve the same decision problem, in equilibrium the probability for a person to leave his partner will be the same as the probability for his partner to leave him. Thus we have

$$\phi(y) = m(x(y)) \int_{y_1}^{y_2} g(y, y')dF(y'). \quad (2)$$

An equilibrium of this economy is a value function $v(y)$, a search function $x(y)$, a r -k-up function $g(y, y')$, and a r -k-up probability function $\phi(y)$ that satisfy equations 1 and 2. The following proposition characterizes the equilibrium.

Proposition 1. There is a unique equilibrium of Economy 1 with the following properties. For all y , v is continuous and increasing; x is decreasing; ϕ is decreasing; $g(y, y') = 0$ if $y' \leq y$; and $g(y, y') = 1$ if $y' > y$.

Proof: Let T denote an operator that maps w to $T(w)$ as follows:

$$\begin{aligned} T(w)(y) = \max_{x,g(\cdot)} \{ & y - x + \beta(1 - \delta)m(x) \int_{y_1}^{y_2} g(y')w(y')dF(y') \\ & + \beta(1 - \delta)[1 - m(x) \int_{y_1}^{y_2} g(y')dF(y') - \phi_w(y; w)]w(y) \\ & + \beta[\delta + (1 - \delta)\phi_w(y; w)]w(y_1)\}, \end{aligned} \quad (3)$$

where

$$\phi_w(y; w) = m(x_w(y; w)) \int_{y_1}^{y_2} g_w(y, y'; w) dF(y'). \quad (4)$$

In this question, x_w and g_w are the solution to the minimization problem in question 3. We can show that if w is continuous, $T(w)$ is continuous; for any continuous w and w' , if $w(y) \geq w'(y)$, $T(w)(y) \geq T(w')(y)$; and for any continuous w and any constant c , $T(w+c) \leq T(w) + \beta c$. Thus T satisfies the Blackwell's condition and is a contraction mapping. Now suppose w is continuous and non-increasing. We have $g_w(y, y'; w) = 0$ if $y' \leq y$ and $g_w(y, y'; w) = 1$ if $y' > y$ (imposing the shut-off convention when g_w is not known with respect to y). Given the assumptions on m , $x_w(y; w)$ is unique and non-increasing and continuous. Then $\phi_w(y; w)$ is continuous and increasing. Using these properties of g_w , x_w , and ϕ_w , we can show that $T(w)(y)$ is continuous and increasing in y . Thus by the Contraction Mapping Theorem, there is a unique continuous and non-increasing v that solves equations 1 and 2 and further, v is increasing. Repeating the steps involved in deriving the properties of g_w , x_w , and ϕ_w , we have the stated properties of g , x , and ϕ . Q.E.D.

To understand this proposition intuitively, a person is indifferent with respect to the amount of (greater) output. If a person makes a lot of rent from a farm, he forms a new farm if he is more compatible with the lot of rent from the farm than with the amount of rent, and he stays with the amount of rent from the farm otherwise. Given this shut-off rule on farm formation and a risk-up, the low rent output person has, the more likely the person will form a new farm with a lot of rent from the farm. Thus making a lot of rent from the farm is more valuable to a person with low rent output and consequently he will search more. Due to greater search and low rent shut-off compatibility for a risk-up, a person with low rent output is more likely to risk up (or search).

In this economy, the equilibrium search and risk-up behavior is socially efficient: so a social planner can increase welfare by increasing search and risk-up behavior. To see this, let us consider the social planner's problem. Note that the search and rent from the farm is such that there is no scope for the social planner to transfer the joint utility (i.e., the farm's rent plus utility) of a pair of output from the farm with that of another. Thus the social planner's problem is to maximize the joint utility of a pair of output

partners. Let $v(y)$ be the present discounted utility of a person with the current output y under the planner's allocation. The Bellman's equation associated with the planner's problem is

$$\begin{aligned} v(y) = \max_{x,g} \{ & y - x + \beta(1 - \delta)m(x) \int_{y_1}^{y_2} g(y')v(y')dF(y') \\ & + \beta(1 - \delta)[1 - 2m(x) \int_{y_1}^{y_2} g(y')dF(y')]v(y) \\ & + \beta[\delta + (1 - \delta)m(x) \int_{y_1}^{y_2} g(y')dF(y')]v(y_1) \} \end{aligned} \quad (5)$$

in the corresponding probability of a rick-up is

$$\phi(y) = m(x(y)) \int_{y_1}^{y_2} g(y, y')dF(y') \quad (6)$$

where x and g are the same as the rick-up functions associated with equation 5. The sufficient conditions for a person's allocation problem (equation 1) in the planner's allocation problem (equation 5) is that the loss of the current partner's utility is equal to the form where it is equal to the latter. The optimum of this economy is a value function $v(y)$, a search function $x(y)$, a rick-up function $g(y, y')$, and a rick-up probability function $\phi(y)$ that satisfy equations 5 and 6. The following proposition characterizes the optimum.

Proposition 2. The optimum has the following properties: v is continuous and increasing for all y ; there is $y \in (y_1, y_2)$ such that x and ϕ are decreasing if $y < y$ and $x(y) = \phi(y) = 0$ if $y \geq y$; $g(y, y') = 0$ if $y' \leq \gamma(y)$ and $g(y, y') = 1$ for $y' > \gamma(y)$ where $\gamma(y) = \min \{y' : v(y') - v(y) \leq v(y) - v(y_1)\}$.

Proof: Let T be an operator that maps w to $T(w)$ according to the right-hand side of equation 5. We can show that T preserves our assumptions on continuity and satisfies the Blackwell's conditions of monotonicity and discounting. (See the proof of Proposition 1.) Thus T is a contraction and there is a unique continuous v that solves equation 5. Further, we can show that if w is non-decreasing, $T(w)$ is increasing. Thus v is increasing. Using this, we can derive the stated properties of g , x , and ϕ . Q.E.D.

The optimal break-up behavior further suggests that the level of compatibility that is higher than the current level causes the break-up to be lower only when the current partners' joint utility is greater than their joint utility loss. The optimal search level is set so that the marginal search cost is equal to the present joint utility gain from the greater level of search, net of the present joint utility loss. In equilibrium, on the other hand, a person breaks up with his current partner as long as his utility gain is positive, ignoring the utility loss of his current partner. Due to this externality, the market too easily breaks up. Further, since a person only searches out his utility gain, matching potential partners is over-valued when the return is too much searching as well.⁵ The following proposition summarizes this discussion:

Proposition 3. The equilibrium of Economy 1 is sub-optimal: $v(y) > v(y)$ for all y .

Proof: Let v be the equilibrium value function and T the operator in the proof of Proposition 2. Comparing equations 3 and 5, we can show that $T(v)(y) \geq v(y)$ with strict inequality for $y = y_1, y_2$. Using this and the monotonicity of T , we can show that $T(T(v))(y) \geq v(y)$ with strict inequality for $y = y_2$. Using this, we can in turn show that $T(T(T(v)))(y) > v(y)$ for all y . Given that T is a contraction, this implies that v is a strict inequality. Q.E.D.

Recall that in this benchmark economy coordination between the current partners is ruled out: they cannot commit to rule nor can they negotiate for forming a long-term partnership. In the remainder of Section 2, I will consider the search and break-up behavior in its effect on welfare when these assumptions are relaxed in some coordination is allowed.

⁵ The intuition that there is too much searching is in the context of the one period decision problem (i.e., fixing v). Over the infinite horizon, the externality would in general affect the 'shape' of v as well, which also affects the current search level. The direction and the size of this effect are not intuitively clear. In any case, the aggregate levels of search and break-up are likely to be lower under the planner's dictation. See Section 3.

rank-up probability function $\tilde{\phi}(y) (\equiv \phi_z(y, z(y)))$ that satisfies equations 7 and 8. The following proposition characterizes the equilibrium.

Proposition 4. The equilibrium of Economy 2 exists and is optimal: $\tilde{v} = v$; $\tilde{x} = x$; $\tilde{g} = g$; $\tilde{\phi} = \phi$. Further, $z(y) = \beta(1 - \delta)(\tilde{v}(y) - \tilde{v}(y_1))$ for all y .

Proof: The right-hand side of equation 7 is equivalent to

$$m_z \left\{ y - x_z(y, z) + \beta(1 - \delta)m(x_z(y, z)) \int_{y_1}^{y_2} g_z(y, y', z) [\tilde{v}(y') - 2\tilde{v}(y) + \tilde{v}(y_1)] dF(y') + \beta[\delta\tilde{v}(y_1) + (1 - \delta)\tilde{v}(y)] \right\}.$$

Furthermore we show that $g_z(y, y', z) = 0$ if $\beta(1 - \delta)(\tilde{v}(y') - \tilde{v}(y)) \leq z$ and $g_z(y, y', z) = 1$ if $\beta(1 - \delta)(\tilde{v}(y') - \tilde{v}(y)) > z$. Then the maximum will occur when $z = \beta(1 - \delta)(\tilde{v}(y) - \tilde{v}(y_1))$. Substituting this in equation 7, we can show that the maximization problems in equation 5 and 7 are the same: the same x and g solve both problems and the maximum is the same for both problems. Thus the equilibrium exists and satisfies the stated equivalent to the optimum. From Proposition 2, we know that \tilde{v} is increasing. Repeating the above steps involving inverting z that solve the maximization problem, we can show that for all y , $z(y)$ is unique and satisfies the stated property. Q.E.D.

In this economy, the rank-up payments serve to internalize the loss of the short-run return's utility so that the rank-up decision is optimal. Without the externality, the short-run level is optimal as well: the short-run level is where the marginal surplus cost of meeting potential profit is equal to the value of meeting potential profit net of the negative effect on the current profit.

2.3 Economy 3: Break-up Bonus

In Economy 2, the optimal short-run rank-up behavior results from the assumption that people have the same net present value and commit to do so. Even if people do not have the same net present value, the short-run rank-up behavior would be optimal if they can commit to a rule that allows rank-up payments on the completeness of the alternative profit: the

rule would allow a rick-up only when the utility gain by the saving pattern is greater than the utility loss by the saving pattern. However, such a rule would require that the saving pattern not satisfy the optimality condition with his current pattern and his alternative pattern, an assumption that is not necessarily in the rick-up permanent economy. It seems more plausible to assume otherwise: a person's optimality with his alternative pattern is sufficient for his current pattern to be optimal.

With this motivation, in this subsection let us consider an economy that features a simple rick-up rule that does not depend on the optimality of the alternative pattern. The assumption is that, as in Economy 2, people do not commit to a rule for saving but, as in Economy 1, people do not have the means to pay a further third party who does not know the current pattern's optimality with his alternative pattern. In equilibrium the current patterns will choose to rick-up if and only if their output level (i.e., the optimality condition) is higher than the cut-off level. The key result is that under this rick-up arrangement, unsurprisingly, the saving rick-up behavior is welfare-improving on Economy 1 but, unlike in Economy 2, falls short of the optimum.

As in Economy 2, although current patterns together decide on the rick-up decision, the decision problem now considers if it is optimal to save given the patterns' relative income prospects and thus would prefer the same hold. Let $\bar{v}(y)$ be the present discounted utility of a person with the current output y in this economy. The Bellman's equation is

$$\begin{aligned} \bar{v}(y) = \max_{x,g,h} \{ & y - x + \beta(1 - \delta)m(x) \int_{y_1}^{y_2} hg(y')\bar{v}(y')dF(y') \\ & + \beta(1 - \delta)[1 - m(x) \int_{y_1}^{y_2} hg(y')dF(y') - \phi_h(y, h)]\bar{v}(y) \\ & + \beta[\delta + (1 - \delta)\phi_h(y, h)]\bar{v}(y_1). \end{aligned} \quad (9)$$

In this equation, h is the rick-up rule ($h = 0$ if rick-up is not allowed and $h = 1$ if it is) and $\phi_h(y, h)$ is the probability of saving given h . This probability satisfies

$$\phi_h(y, h) = m(x_h(y, h)) \int_{y_1}^{y_2} hg_h(y, y', h)dF(y') \quad (10)$$

where $x_h(y, h)$ and $g_h(y, y', h)$ are the solution to the right-hand side of equation 9, taking h as given. An equilibrium of this economy is a value function $\bar{v}(y)$, a k-up function $h(y)$, a s.r.h function $\bar{x}(y) (\equiv x_h(y, h(y)))$, a k-up function $\bar{g}(y, y') (\equiv g_h(y, h(y)))$, and a k-up profitability function $\bar{\phi}(y) (\equiv \phi_h(y, h(y)))$ that satisfy equations 9 and 10. The following proposition characterizes the equilibrium.

Proposition 5. There is a unique equilibrium of Economy 3 with the following properties. For all y , \bar{v} is continuous and increasing; there is $\bar{y} \in (y_1, y_2)$ such that $h(y) = 1$ if $y < \bar{y}$ and $h(y) = 0$ if $y \geq \bar{y}$; \bar{x} and $\bar{\phi}$ are decreasing if $y < \bar{y}$ and $\bar{x}(y) = \bar{\phi}(y) = 0$ if $y \geq \bar{y}$. For $y < \bar{y}$, $\bar{g}(y, y') = 0$ if $y' \leq y$ and $\bar{g}(y, y') = 1$ if $y' > y$.

Proof: Let \bar{T} denote the operator that maps w to $\bar{T}(w)$ as follows:

$$\begin{aligned} \bar{T}(w)(y) = \max_{x, g, h} \{ & y - x + \beta(1 - \delta)m(x) \int_{y_1}^{y_2} hg(y')w(y')dF(y') \\ & + \beta(1 - \delta)[1 - m(x) \int_{y_1}^{y_2} hg(y')dF(y') - \phi_{hw}(y, h; w)]w(y) \\ & + \beta[\delta + (1 - \delta)\phi_{hw}(y, h; w)]w(y_1) \}, \end{aligned} \quad (11)$$

where

$$\phi_{hw}(y, h; w) = m(x_{hw}(y, h; w)) \int_{y_1}^{y_2} g_{hw}(y, y', h; w)dF(y'). \quad (12)$$

In this equation, x_{hw} and g_{hw} are the solution to the minimization problem in equation 11. We can show that T preserves our assumptions and satisfies the Blackwell's conditions of monotonicity and discounting. (See the proof of Proposition 1.) Thus \bar{T} is a contraction. Now suppose w is continuous and non-decreasing. Equation 11 can be rewritten as

$$\bar{T}(w)(y) = \max_h \{ \bar{T}(w; h)(y) \} \quad (13)$$

where $\bar{T}(w; h)(y)$ is the minimum taking h as given. We can show that for any y , $\bar{T}(w; 0)$ is continuous and increasing. Also, when $\bar{T}(w; 1) = T(w)$ and thus $\bar{T}(w; 1)$ is continuous and increasing. (See the proof of Proposition 1.) Then $\bar{T}(w)$ is continuous and increasing. Thus there is a unique continuous and non-decreasing \bar{v} that solves equations 9 and 10 and further, \bar{v} is increasing. Now by comparing $\bar{T}(\bar{v}; 0)$ and $\bar{T}(\bar{v}; 1)$, we can show that for any y , $h = 0$ if and only if $\int_y^{y_2} [\bar{v}(y') - 2\bar{v}(y) - \bar{v}(y_1)]dF(y') \leq 0$. Since \bar{v} is

in rising, there is a cut-off \bar{y} as stated in the proposition. Given this cut-off rule and using that \bar{v} is increasing, we can show that the properties of \bar{x} , \bar{g} , and $\bar{\phi}$. Q.E.D.

In other words, there is a cut-off output level so that partitions with output greater than the cut-off level have to be a rick-up. Consequently they do not survive and thus never rick-up a point in the space of endogenous shocks. This is also true for this cut-off level, the point utility gain from surviving for a certain amount is smaller than the point utility loss of the would-be surviving partition. For partitions with output less than the cut-off level, however, the joint utility is greater with surviving than with not surviving. Thus they have to allow a rick-up and to survive without restrictions in Economy 1.

Given the simplicity of the rick-up rule (i.e., that it does not depend on the competitiveness of the alternative partition), the equilibrium surviving rick-up behavior would be sub-optimal. However, the rule should survive greater welfare than the no-rule arrangement of Economy 1: intuitively, the rule is only allowed to survive if it is a current partition has the option to survive than in Economy 1 by allowing a rick-up. Thus we have the following proposition.

Proposition 6. The equilibrium of Economy 3 is sub-optimal but welfare-improving on the equilibrium of Economy 1: $v(y) > \bar{v}(y) > v(y)$ for all y .

Proof: Let T and \bar{T} be the operators in the proofs of Propositions 2 and 5. Comparing equations 5 and 11, we can show that $T(\bar{v})(y) \geq \bar{v}(y)$ for all y with strict inequality for $y \in (y_1, \bar{y})$. Using this and the monotonicity of T , we can show that $T(T(\bar{v}))(y) \geq v(y)$ for all y with strict inequality for $y \in [y_1, \bar{y})$. Using this, we can in turn show that $T(T(T(\bar{v}))) (y) > v(y)$ for all y . Given that T is contraction, this implies that $v(y) > \bar{v}(y)$ for all y . Now let v be the equilibrium value function for Economy 1 and \bar{T} the operator in the proof of Proposition 1. Comparing equations 3 and 11, we can show that there is $\hat{y} \in (y_1, y_2)$ such that $\bar{T}(v)(y) \geq v(y)$ with strict inequality for $y \in (\hat{y}, y_2)$. Using this and the monotonicity of \bar{T} , we can show that $\bar{T}(\bar{T}(v))(y) \geq v(y)$ with strict inequality for $y = y_2$. Using this, we can in turn show that $\bar{T}(\bar{T}(\bar{T}(v))) (y) > v(y)$ for all y . Given that \bar{T} is contraction, this implies that $\bar{v}(y) > v(y)$ for all y . Q.E.D.

3. FURTHER DISCUSSION ON THE MODEL ECONOMY

The outcome of the analysis in Section 2 can be summarized as follows. If a person's risk-up decision has a negative net utility to his current portfolio, restrictions on risk-up can improve welfare. In particular, if a planner cannot rule out the means of payment, a risk-up payment rule induces the optimal risk-up behavior. If a planner cannot rule out the means of payment, a risk-up rule induces sub-optimal welfare-improving risk-up behavior. The analysis is meant to illustrate the reason for such a preference for long employment, not vice versa.

Since the focus of the analysis was on the value of mobility restrictions, I did not model the remaining issues, in which a planner cannot rule out the means of payment. Let us briefly consider this case. In this case, current portfolios are negotiated from a net utility portfolio and this would lead to an efficient risk-up decision: current portfolios will be risk-up only if the joint gain from risk-up is greater than the joint loss. However, the payment will be made only if the would-be seller portfolio to the would-be buying portfolio, the opposite from the risk-up payment arrangement. Making potential portfolio will then overvalue and there will be too much risk-up. Thus although negotiation would improve welfare, it would not achieve optimality, similar to the risk-up arrangement.

In the model, the mobility restrictions were assumed to be self-imposed by the current portfolios. However, this is not essential. The restrictions could be imposed by a third party such as government and this would lead to the same results. A third party is useful in helping to enforce rules. Thus we observe that long-run payments are not vice versa. On the other hand, a third party may have less information than the first person on portfolio, limiting its usefulness. Thus the long-run payment is not vice versa to be more uniform across different markets: in terms of the model in this paper, it is possible to model government-enforced risk-up payment and rules

to in percent of the current output, whose simplicity would rule out the value of those rules.⁶ A related issue, which stems from the model itself, is how to implement the restrictions. Some sophisticated searchers may be able to circumvent the restrictions. Others may be able to find long-term employment in Japan or implement a custom rule.

Although the focus of the analysis was on the effect of mobility restrictions on welfare, the model has implications for some other aspects of the economy. I will briefly mention these in the conclusion. The main results of numerical exercises that I carried out for steady states (i.e., the aggregate variables under the stationary distributions of population over current output) of the model are as follows. First, the aggregate levels of search and recruitment are likely to be lower with mobility restrictions, that is, in Economy 2 than in Economy 1. Intuitively, mobility restrictions lower the value of search and recruitment (Economy 2) or directly reduce mobility (Economy 3). Second, the distribution of current output is more equal with mobility restrictions. Equivalently, with the restrictions there is less fluctuation in a person's consumption over time. This is due to search and recruitment, which leads to mobility, to disperse the distribution of current output: people move to higher output levels while those who do not move to the lowest output level. As a result, the fraction of population without recruitment is lower with mobility restrictions. Third, the effect of mobility restrictions on the aggregate output is not necessarily positive on the parameter values. The aggregate output depends on the distribution of population over current output. Since with restrictions there are less people with high current output and less people with low current output, the net effect is unclear.⁷

⁶ A further complication of restrictions imposed and enforced by a third-party is the institutional arrangement by which the restrictions are formulated and implemented. This process of formulation and implementation is more complicated than if the restrictions are self-imposed, and thus leaves the room for inefficiency.

⁷ The sum of aggregate output (as defined in the model) and aggregate search cost may correspond better to the usual concept of aggregate output. This sum is likely to be lower with mobility restrictions due to the low levels of search.

4. CONCLUDING REMARKS

In this paper, I address the question of why mobility restrictions are widely observed in employment, marriage, and other partnerships. I present a model economy that features production teams and focuses on the search for partners and formation and breakup of teams. The main results are that when there are no restrictions, there is too much searching and too much re-king-up due to the negative externality of a person's re-king-up decision on the search partners, and that restrictions such as re-king-up payment and re-king-up and improvement fees. Thus the model rationalizes mobility restrictions as welfare-improving arrangements. This explanation for mobility restrictions is different from those based on political economy with respect to optimality. In the review of the issue, I consider this explanation's complement rather than an alternative to the others.

In the remainder, I will state two limitations of the current model. First, the model is stylized and useful for thinking about mobility restrictions in a reasonably encompassing manner. The drawback is that the model cannot be used as a platform for quantitative analysis in its current form. Making the model more participatory by including future, such as symmetry between partners, relevant to understanding the quantitative implications would be useful for evaluating the merits of the story. I leave this task for future research. Second, the current model does not explain observed differences in the degree of mobility restriction across countries and time periods (e.g., the greater search payment in Europe than in the US, the nonmonotonic increase in the 70's in Italy and Brazil).⁸ This limitation applies to existing models as well. For an explanation for mobility restriction to be a complement, the differences across countries and time periods that lead to the different degrees of restriction must be identified. In this paper, I only

⁸ A reason may be that there are differences in the efficiency of search and matching technology across countries and periods. In terms of the model, the matching function m may be scaled-up for the US in comparison to Europe and for the post-70's in Italy and Brazil in comparisons to the pre-70's.

took on the minor task of explaining why the *may* mobility restriction is not the largest task for future research.

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