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Linking Education and Pensions in Transition: Educate the Young or Compensate the Old?

S. Slobodyan, V. Vinogradov*

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Abstract

In their recent paper, Boldrin and Montes (2005) analyze the "return on human capital investment" model and show that if borrowing for education is not possible, then combined public education and pension system that uses lump sum taxes and transfers, can replicate the first–best decentralized allocation achieved in an economy without taxes where borrowing for human capital accumulation (education) is allowed. Taking into account that such borrowing is either absent or inefficient in many countries, combined public education/public pensions scheme might prove welfare enhancing.

Guided by this theoretical framework, we use it to calibrate the parameters of interconnected pension and education systems for the Czech Republic under different scenarios of demographic and economic development. Both systems are undergoing deep changes, and the study proposed here might prove to be useful in informing policymakers about desirable directions of reforms of the educational and pension systems and estimating magnitude of such reforms. Our first results, estimated from the Microcensus 1996, indicate that in the Czech Republic paying for education of the next generation provides higher return than the interest "paid" on educational loans: education is inexpensive, and pensions are generous. This conclusion is consistent with currently observed 45% wage replacement rate of pensions and relatively short supply of educational services in the CR. We further discuss changes to the two systems under different paths of demographic and economic variables.

Finally, we extend the existing 1–period OLG framework of Benabou (2002) to 2 and 3 periods and ask whether a co–existing public education and pensions could be optimal from the social planner's perspective.

Keywords: Public education, OLG model, human capital, pay–as–you–go pensions.

JEL classification: H52, H55

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*Sergey.Slobodyan@cerge-ei.cz, Viatcheslav.Vinogradov@cerge-ei.cz. CERGE–EI, Prague, Czech Republic. We gratefully acknowledge funding from the Global Development Network RRC IV grant. We also thank discussants and participants in the "Economic development in modern world: Factors, constraints, and regional discrepancies" Conference in Ekaterinburg, GDN RRC 2004 Conference in Prague, and DET Workshop on Economic Growth and Institutions in Copenhagen for helpful comments and discussion. All remaining errors are ours.

1 Introduction

There exists some theoretical and empirical literature that researches into the relationship between human capital investment (e.g. public education) and the pension systems, see Boldrin and Montes (2005). This theory is referred to as "return on human capital investment" by Mulligan and Sala-I-Martin (2004) and considers public pension as a return on the investment in human capital of the next generation. This investment was made by the generation of current retirees when they were middle–aged and paid taxes partially used to educate their "offspring". On the other hand, the debt which the young incur by being educated is repaid through social security contributions when middle–aged; in a pay–as–you–go pension system, these contributions are transferred to the old as pensions. Interconnected pension and public education systems can replicate the allocation achieved by the complete markets, where young can borrow against future income. The two systems are connected through implicit rates of return on the public schooling expenditures and educational taxes. This scheme is equivalent to intergenerational transfers among three generations: the young, the old, and the middle aged.[1]

Boldrin and Montes (2005) further calibrate their model to the Spanish data. The normative prediction of the model is that the implicit rate of return i that equalizes the discounted values of educational services received and social security contributions paid equals the implicit rate of return π that equalizes the discounted values of educational taxes paid and social security contributions received, and both π and i equal the market interest rate. The authors show that this normative prediction approximately holds in the Spanish case if one assumes that institutional structure of the public education and pension systems in the last 20 years were in effect for all living cohorts. They further extrapolate their model into the future, using demographic projections and various assumptions on the behavior of taxes and expenditures over time. The projections reach two conclusions: first, demographic evolution moves the two implicit rates of return apart – individuals receive a higher rate of interest through pensions than they pay through social security contributions, and second, rates of interest paid or received by different cohorts do not monotonically depend on the year of birth. Joint consideration of education and pension systems proposed by the authors does not lead to a systematic transfer of resources from currently young and not–yet–born generations to the currently old, as is a usual conclusion in the Generational Accounting methodology (see, e.g., Auerbach, Kotlikoff, and Leibfritz (1999)).

Guided by this theoretical framework, we apply it to calibrate interconnected pension and education systems in the Czech Republic under different scenarios of demographic and economic development. Both systems are undergoing deep changes, and the study proposed here might prove to be useful in informing policymakers about desirable directions of reforms of the two and estimating magnitude of such reforms.

[1] Alternative financing schemes include a special proportional tax on capital or a special debt instrument. Without lump–sum taxation, replication of the complete markets allocation becomes impossible, though it is still possible to approximate it even when the markets are incomplete and private borrowing to finance education is not available. The analysis is very promising, because even in the developed countries the markets for financing accumulation of the human capital (education) through borrowing against future labor income are not very advanced; it sheds light on ways to complete important markets. See Patrinos (2002) for a survey of observed institutional arrangements around the world and Saint-Paul and Verdier (1993) for theoretical analysis.

In particular, it is important to understand the impact of possible changes in structure of funding of higher education and/or pension benefits. For instance, while higher education in the Czech Republic at present is mostly free, parts of the political spectrum propose to fund it privately to a larger extent. The latter scheme suggests less costly transfer to a system where education is not "free" but represents an explicit individual asset with corresponding liabilities (provided such system is socially optimal and politically feasible). We scrutinize these issues empirically by looking at the Czech Microcensus data 1996.

Though Boldrin and Montes conclude that lump sum taxes and transfers could replicate competitive allocation with no borrowing constraint, competitive allocations in OLG economies are not necessarily Pareto optimal. Moreover, in OLG framework such a conclusion might be sensitive to the specification of education subsidy. For example, Docquier and Michel (1999) have considered a similar three–period OLG model with education subsidies and old–age pensions. Education in this model costs time and money (in contrast with the preceding model where only monetary costs are present), and both costs are subsidized. Taxes are proportional, unlike lump sum ones used by Boldrin and Montes (2005). Calibrated model suggests that in such a case it might be desirable to finance significant proportion of education subsidies with lump sum *taxes* on retirees instead of proportional taxes on the middle aged combined with *pensions* to the retirees.

Yet another approach to educational subsidies was presented by Benabou (2002). In a setting with a continuum of infinitely–living heterogeneous agents, government can impose distorting (progressive) taxes, consumption taxes, and pay proportional subsidy to human capital accumulation. The model incorporates borrowing constraints: Investment in education is possible only by not consuming a part of income. In addition to a usual egalitarian motive for redistribution, government has an incentive to transfer income to the poor, because this augments resources available for human capital investment, and the latter leads to an increase in average income in presence of diminishing returns to human capital investment. However, the Benabou's approach has not been yet scrutinized in three–period OLG context.

We extend Benabou's model to overlapping generations setting and study the optimal amount of redistribution in this case. In the full–fledged model, agents live for three periods. When young, they produce using labor and inherited human capital. Part of the income is not consumed and directed to the human capital accumulation instead. In the middle age, agents consume and invest into human capital, while consumption alone takes place in the last period. In all three periods, agents are subject to age–dependent progressive tax system; in addition, they pay flat consumption tax in all three periods with the rate independent of age. They also receive education subsidy in the first period. Apart from 3–period model, we also study a simplified 2–period model, where middle–age period is not present.

In the above setup, the only channel of "savings" is through human capital investment. The agents "save" in periods one and two, but only in the first period they receive education subsidy. Functional forms chosen for the agents' utility and production functions (identical to Benabou's) guarantee that a closed form solution exists, which can be exploited to develop economic intuition. Optimal net transfers to (or from) a particular generation allow to consider intergenerational flows of resources. Heterogeneity of the agents, coupled with borrowing constraints, generates incentives to redistribute income. Our study is focused on complex interplay between net resource transfers (intergenerational transfers) and amount of redistribution (intragenerational transfers) in different generations, and the effect of initial inequality among the agents on these flows. In both 2–period and 3–period models, we show that the parameter that drives the results is the efficiency of human capital transfers of the middle aged to the young (which is treated as exogenously given).

The paper is organized as follows: Section 2 provides description of the data and discussion of different scenarios of demographic and economic development. Section 3 presents the results of our simulations and discussion on policy tools that could be used to achieve efficiency and fairness of intergenerational transfers. Section 4 describes a simplified 2–period version of our OLG model used to generate intuition for its more realistic 3–period counterpart, that is developed in Section 5. The Section 6 concludes.

2 Empirical Framework: Data, Demographic Scenarios, and Fiscal Rules.

The empirical part of the study provides an estimate of the net present value of transfers that were paid and received by every currently living cohort. Such an estimate involves stationarity assumption that might have been be justified in the Spanish case studied by Boldrin and Montes (2005), where both public school system and the social security were relatively stable within the last 20 years, but could be somehow unrealistic for the transition countries, where the real value of both education and pension expenditures have been varying greatly since 1989.[2] Therefore, this calculation will serve only as a crude benchmark for forward–looking projections.

Given the share in taxes/pensions/educations transfers for each cohort,[3] one could determine the NPV of publicly provided education, social security taxes and the taxes used to pay for education of the next cohorts, and social security pensions. Calculation of these net present values requires knowledge of period–to–period survival probabilities (available from standard mortality tables) and of the interest rates. The forward–looking projections also require assumptions regarding the future fiscal policy (in particular, social security contributions and payments, and public expenditures on education).

Forward–looking prognosis provides evaluation of NPV of contributions to and services from the public education and the pension system. Implicit rate of return along the life cycle that is paid on the debt incurred by going to school is the interest rate that equalizes NPV of education services and of social security contributions, while the implicit rate received as pensions is the one at which NPV of education taxes equals the NPV of pensions received. For computational purposes, both implicit rates could be taken as a constant spread over or below the projected market interest rate. The results of the forward projection would then be cast in terms of spread between the

[2] For instance, Czech Republic implemented first pension reform in 1990; another transformation of the pension system has been currently discussed by the Czech government; Czech educational system experienced a shift of resources in the state budget towards secondary vocational schools in 1995/96, etc.

[3] See Appendix in Boldrin and Montes (2005) for a detailed description of the methodology applied to calculate these shares.

market interest rate and the two implicit rates of returns.

To compute these implicit rates, we use Czech Microcensus as primary sources. We start our quantitative exercise with Microcensus 1996. For each individual in the sample — conditional on her age and educational status — we attempt to estimate the amount and value of public education received, the amount of taxes and the amount of pension contributions paid, and the amount of public pensions received. The list of primary and constructed variables in Microcensus used in computation of implicit interest rates is put in the Appendix.

Along with the income tax payments contributed by individuals to the state budget, we also account for the value added tax payed by physical persons. Based on the data provided by Czech Statistical Office, paid VAT is computed as a percentage of gross total money income by income decile (see Appendix for details).

We also use the available information to calculate the population shares of studying, working, unemployed and retired individuals. Further, to adjust for the demographic changes (e.g., mortality rates, or immigration flows) we use demographic projection and demographic evolution scenarios based on the data available form the national statistical office.

We consider four different scenarios of demographic development, that differ in the fertility rate, that is the number of children born to a female over her life cycle. Currently, the fertility rate in the Czech Republic is 1.19, and in 'current' scenario the fertility rate stays constant over time at the current level. We also consider moderately and highly optimistic scenarios, with the fertility parameter increasing from current level of 1.19 to 1.4 and 1.9, respectively, and the highly pessimistic scenario when the fertility drops to 1.05 from current level.[4] Given that demographic projections spanning more than 100 years are highly uncertain, in all 4 scenarios we assume that the population and its structure remain unchanged after 2100.

We start our quantitative exercise with the Czech microcensus of 1996. In sequel, our methodological approach is as follows: We include only males (to avoid fertility–related variability both in income figures and retirement age), who are either employed, self–employed, or received pensions for 12 months in the year when the microcensus was done.[5] Retirees could have additional income, either from wage or self-employment. They are considered retirees for the sample description purposes. We also stick to the assumption that according to the Czech law, currently the retirement age for males is 61. Further, pension is defined as the sum of age pension and (if any) widow pension received by an individual (orphan pensions, disability pensions, etc., are not included). We assume that all pensions are coming from the state pension fund (i.e. private pensions funds can be neglected; this assumption was fully legitimate under socialism and still applicable in large in nowadays Czech Republic).

As far as education is concerned, five types of educational level are distinguished:

[4]It is also feasible to introduce a one-time legalization of illegal migrants, similar to what was done in Spain in Spring 2005. By expert opinion, the number of illegal workers, mostly young Ukrainian males, can be as large as 200 000 that comprises a significant percentage of Czech labor force. However, this one-time measure will affect only short-run lump sum tax payments to the budget because – again by expert estimate – the vast majority of these illegal workers will return to the home country and will be unlikely to claim retirement benefits in the Czech Republic.

[5]We are aware that going along this line we are probably overestimating the tax and social security payments, and underestimating the pension benefits, while projecting our results to the total population. However, the error should not be significant.

(0) no education or incomplete elementary, (1) elementary, (2) incomplete secondary or secondary without leaving diploma (maturita), (3) complete secondary with leaving diploma (maturita), (4) high school (including Bachelor, Magisterial or Doctoral degree). Based on the yearly information of the structure and costs of education in the Czech Republic provided by the Ministry of Education, Youth and Sport, we compute that 4.4% of children aged 0–2 years attend nursery schools, while among children aged 3–6 years this proportion reaches 90%. From the same source we compute that out of pool of students enrolled in secondary education, 42.5% study in secondary vocational schools, 19.5% in gymnasia, and 38% in secondary professional schools (SOŠ in Czech).

The same source reveals the amount of educational transfers per student in the Czech Republic in 1995/96 academic year by type of school. With the above data in hand, for each cohort we are able to compute per capita educational transfers (see Appendix for the details of construction).

To evaluate transfers made by an individual to the budget in the year of Microcensus, we use the difference between gross income and net income reported by the individual in Microcensus, combined with the the VAT payment projected as a fraction of an individual's gross total money income (see the table in Appendix) While computing per-capita social security and tax payments, and education and pension transfers, we follow the methodology of Boldrin and Montes (2005). As the best available source of information, we employ Microcensus data that may be biased in a sense that statistically certain age cohorts are under–represented (e.g. students) and some are over–represented (e.g., retirees) as compared to the demographic pyramid in the 1996 population. However, by normalizing the above per-capita values for each cohort to the Microcensus shares of this cohort, we correctly measure the "participation rate" of each cohort in tax and social security contribution and education and pension transfers.

The analysis of 1996 Czech Microcensus data reveals that the tax system is progressive: less educated respondents receive smaller income and pay lower share of it in taxes (from 0.19-0.20 for those with elementary and incomplete secondary education to 0.22 for secondary school graduates with diploma to 0.25 for those with tertiary education). At the same time, pensions are increasing in the highest education level attained, but are essentially independent of age.[6] Taking into account the secular rise in real wages and essential constancy of the wage replacement rate for pensions (43.4÷45.9 in 1997-2002), the latter means that younger cohorts of retirees obtain a worse deal on their pensions. One possible reason for this phenomenon might be perfect indexation of pensions to the real wage growth.

3 Simulation Results and Policy Recommendations

In line with Boldrin and Montes (2005), our quantitative exercise provides policymakers with a good framework, as far as education and pension reforms are concerned. One way of implementation of intergenerational transfer scheme, proposed by Boldrin and Montes (2005), is to issue debt ("education bonds"), proceeds from which are used to finance education of the young cohorts. The debt is repaid through a special income tax that is proportional to the past usage of the public education system (average

[6] For very old retirees the pensions are lower, but their share in the population is tiny.

number of years of schooling) and can be understood as a tax on aggregate human capital. Transformation of human capital into a liquid asset (education bond) could therefore improve economic performance of developing countries. Moreover, addition of assets linked to the human capital (education bonds) to individual portfolios can help to diversify risks and thus to improve the welfare. A comparison of the implicit rate of returns to education transfers and pensions suggests possible direction of reforms (e.g., make pension benefits more/less generous, or increase/decrease educational transfers, or attempt to affect demographic structure of the population, etc.) By all means, the political costs of implementation of interconnected pension and public education systems may and will vary under different fiscal and demographic scenarios.

To scrutinize this idea empirically for the Czech Republic, we rely on the data from Microcensus 1996. First, we impose an assumption of 'no-change', that is to combine current budget rules (i.e. fixed education and pension transfers, and tax and social security payments) with artificially frozen current age structure. We estimate that under such assumption paying for education of the next generation provides higher return than the interest "paid" on educational loans: education is cheaper than pensions, with the gap between implicit interest rates about 1.8%. This conclusion is consistent with currently observed 45% wage replacement rate of pensions and relatively short supply of educational services in the CR.

Demographic change is bound to affect this gap. For example, consider "demand driven" budgeting: fix per capita educational transfers and pensions and preserve balanced budget assumption (considering rapid ageing of the population, this implies dramatic increase in social security contributions in the near future, as the number of workers per retiree declines). The gap disappears for cohorts born in 1980s. For still younger people, current situation is reversed: interest rate on "educational loans" exceeds that paid as pensions by as much as 1.2% under moderately and highly optimistic scenarios of the demographic development; if we assume that the fertility rate stays constant at the current rate then the difference between education and pension implicit interest rates increases to 1.5%. In the highly pessimistic case, this difference increases further and reaches 1.9% level (note, however, that highly pessimistic case is unlikely to realize in the Czech Republic).[7] Figure 1 provides an illustration.

In the distant future, as the population structure freezes at different levels determined by assumed demographic projections, it becomes feasible to equalize these two interest rates by means of various fiscal tools. Again, the efficiency of fiscal tools will be heavily conditional on the demographic developments. For instance, at current fertility rates with "demand driven" budgeting and 16.6% increment in pensions from 1996 level (without corresponding increase in social security contributions), the two interest rates will be equalized just above 3%. Note, however, that preserving balanced budget by *not* increasing pensions leads to implicit interest rate on pension benefits being 0.5% lower than that on educational loans. Also, 16.6% increase in pensions seems to undershoot if the demographic development is believed to be optimistic, and

[7] Similar patterns are observed with "supply driven" budgeting, when per capita taxes are fixed, and the total educational transfers and pensions are determined by the tax revenue, no matter what the number of students and pensioners is.

Following Boldrin and Montes (2005), we also consider "partially driven" budgeting rules which fix either per capita educational transfers — "educate the young", or per capita pension payments — "support the old".

overshoot if the budgeting is "supply driven".[8] Figures 2 and 3 visualize the point. Alternatively, the same objective can be achieved by increasing per capita educational transfers by 12.7% without corresponding raise in taxes, but the common interest rate now will become as low as 2.55% – see Figure 4 for illustration. Nevertheless, efficiency requires not only pair–wise equality of the two interest rates, but also their simultaneous equality to the market interest rate, which we estimate to be close to 3 percent.[9] Thus, pensions adjustment and education transfers adjustment lead to different outcomes in this framework. For instance, it may happen that an increase in educational transfers is deemed socially superior than an increase in pensions, but is inferior from the general macroeconomic point of view.

4 Theoretical Model: 2–Period

Though the full–fledged model allows analytical solution, we first embark on a simplified version to develop economic intuition. The agents live for two periods, invest in education in the first period and consume only in the second one. Since there is no third period, there is no pure "pension" payment as well. We consider the difference between net transfers to the young and old as an indicator of what the "pension" might look like in the full–fledged model.

4.1 The Setup

There is a continuum of agents indexed by $i \in [0,1]$. Individual agent born at time t is maximizing 2–period utility,

$$\ln c^i_{1,t} - l^{\eta}_{1,t} + \rho(\ln c^i_{2,t+1} - l^{\eta}_{2,t+1}). \tag{1}$$

The first subscript refers to the period of life, the second to the physical time period, c is consumption and l labor time. Because of the assumed functional form, labor effort depends only on aggregate parameters, thus l is not indexed. Agents generate income by combining labor time and human capital,

$$y^i_{j,t} = \left(h^i_{j,t}\right)^{\lambda} (l_{j,t})^{\mu}\,,\ j = 1,2. \tag{2}$$

Income is subject to the progressive tax, so that agent's disposable income can be expressed as

$$y^{Di}_{j,t} = \left(y^i_{j,t}\right)^{1-\tau_{j,t}} \left(y^T_{j,t}\right)^{\tau_{j,t}}\,,\ j = 1,2. \tag{3}$$

Here $y^T_{j,t}$ is introduced to discriminate between the income of the agents receiving transfers and one of those being taxed in this scheme. Such formulation of the tax scheme was used by Benabou (2002) and is called "constant marginal progression tax

[8] Ironically, with fixed current demographic structure and "supply driven" budgeting, no correction is needed at all!

[9] Bond markets in the Czech Republic are very weak and illiquid. As a measure of real interest rate, we take the average midpoint between long–term deposit and loan rates minus 12–months CPI inflation. We average years 1999 through 2003 in order to exclude extreme volatility in 1998, possibly related to Czech banking crisis of 1997.

schedule". If $\tau_{j,t} > 0$ (progressive tax schedule), poor agents with $y^i_{j,t} < y^T_{j,t}$ receive transfer while the rest is taxed. In principle, it is possible to have $\tau_{j,t} < 0$ (regressive tax).

In the first period, agent consumes and saves for educational spending, while the consumption alone occurs in the period 2:

$$c^i_{1,t}(1+\theta_t) + e^i_{1,t} = y^{Di}_{1,t}, \tag{4a}$$

$$c^i_{2,t+1}(1+\theta_{t+1}) = y^{Di}_{2,t+1} \tag{4b}$$

Here θ_t is the consumption tax rate. Finally, second period human capital is given by

$$h^i_{2,t+1} = \kappa\xi^i_t \left(h^i_{1,t}\right)^\alpha \left[(1+a_t)\, e^i_{1,t}\right]^\beta. \tag{5}$$

Here ξ^i_t is an idiosyncratic ability shock, and a_t subsidy rate to education. In the simplified setup of the model, the government can not condition education subsidy on individual characteristics other than individual educational expenditures. For a close-form expression for $l_{1,t},\ l_{2,t+1}$ and $e^i_{1,t}$ see Appendix.

Since the government does not issue debt, budget must be balanced in every period. The government budget constraint is thus given by

$$\int_0^1 \left[y^{Di}_{1,t} - y^i_{1,t} + y^{Di}_{2,t} - y^i_{2,t} + a_t e^i_{1,t} - \theta_t\left(c^i_{1,t} + c^i_{2,t}\right)\right] di = 0.$$

In new notation, the government budget constraint is reduced to

$$\left[A_{1,t}\left(1 + a_t s_t - \theta_t \frac{1-s_t}{1+\theta_t}\right) - 1\right]\overline{y}_{1,t} + \left[\frac{A_{2,t}}{1+\theta_t} - 1\right]\overline{y}_{2,t} = 0, \tag{6}$$

see Appendix for technical details. Next, let us address the government problem. The government is seeking to maximize

$$E_0 \int_0^1 \left\{ \frac{\ln c^i_{2,0} - l^\eta_{2,0}}{R} + \sum_{t=0}^{\infty} R^t \left[\ln c^i_{1,t} - l^\eta_{1,t} + \rho(\ln c^i_{2,t+1} - l^\eta_{2,t+1})\right] \right\} di, \tag{7}$$

subject to the budget constraint (6), and agents' optimal behavioral rules (18). In addition, we have to make assumptions on how human capital is transmitted across generations. The most straightforward assumption could be to postulate that

$$h^i_{1,t+1} = \varphi h^i_{2,t+1}.$$

In other words, agent with human capital $h^i_{2,t+1}$ at the second period of life transfers share φ of that human capital to child born at time $t+1$. (c.f., for instance, Fougere and Merette (1999), for a similar functional form and its justification).

Finally, the government's problem (dropping for simplicity the initial old term) could be re-written as

$$\max \sum_{t=0}^{\infty} R^t \left\{ \begin{array}{c} \lambda m_t + \mu \ln l_{1,t} + \ln A_{1,t} + \ln \frac{1-s_t}{1+\theta_t} + \frac{\lambda^2 \Delta_t^2}{2}\tau_{1,t}(2-\tau_{1,t}) \\ +\rho\left[\lambda\left(m_{t+1} - \ln\varphi\right) + \mu \ln l_{2,t+1} + \ln \frac{A_{2,t+1}}{1+\theta_{t+1}} + \frac{\lambda^2\Delta_{t+1}^2}{2}\tau_{2,t+1}(2-\tau_{2,t+1})\right] \\ -l^\eta_{1,t} - \rho l^\eta_{2,t+1} \end{array} \right\},$$

subject to

$$\begin{aligned} m_{t+1} &= (\alpha+\beta\lambda)\, m_t + \beta\mu \ln l_{1,t} + \beta \frac{\lambda^2 \Delta_t^2}{2} \tau_{1,t}(2-\tau_{1,t}) + \beta \ln\left[(1+a_t)\, s_t\right] + \\ & \quad \ln\kappa - \frac{\omega^2}{2} + \beta \ln A_{1,t} + \ln\varphi, \qquad (8a) \\ \Delta_{t+1}^2 &= \left[\alpha + \beta\lambda(1-\tau_{1,t})\right]^2 \Delta_t^2 + \omega^2, \qquad (8b) \end{aligned}$$

$$\left[A_{1,t}\left(1 + a_t s_t - \theta_t \frac{1-s_t}{1+\theta_t}\right) - 1\right] \exp\left(\mu \ln \frac{l_{1,t}}{l_{2,t}} + \lambda \ln\varphi\right) = 1 - \frac{A_{2,t}}{1+\theta_t}. \qquad (9)$$

and m_0, Δ_0^2 given.

For derivation of (8), (9), see Appendix.

4.2 Solving the Government's Problem

For the routine technical details of the derivation of the optimal stationary solution, see Appendix. In this section, let us rather focus on the economic intuition behind the formulas.

To derive intuition regarding the flow of resources across generations, two key characteristics are of interest, namely, $\frac{A_1}{A_2}$ and z. Ratio of A_1 to A_2 tells us about resource flows through tax/transfer scheme (3), while z gives total consumption of the old as a share of their income y_2 (again, not of disposable income $\widehat{y}_2$).

$$\begin{aligned} \frac{A_1}{A_2} &= \frac{R}{\rho}\frac{C_2}{1-s} = \frac{R}{\rho}\frac{1}{1-s}\exp\left(-\mu \ln\frac{l_{1,t}}{l_{2,t}} - \lambda\ln\varphi\right) = \\ &= \frac{R}{\rho}\frac{1}{1-s}\exp\left(-\frac{\mu}{\eta}\ln\frac{1-\tau_1}{1-\tau_2} - \lambda\ln\varphi\right)\exp\left(\frac{\mu}{\eta}\ln(1-s)\right) \\ &= \frac{R}{\rho}\varphi^{-\lambda}\left(\frac{1-\tau_2}{1-\tau_1}\right)^{\frac{\mu}{\eta}}(1-s)^{\frac{\mu}{\eta}-1}. \end{aligned}$$

Here

$$\begin{aligned} x &= A_1(1+a)\, s = A_1 \widetilde{s}, \\ z &= \frac{A_2}{1+\theta}, \\ C_1 &= \frac{1-R(\alpha+\beta\lambda)}{(\rho+R)\,\beta\lambda}, \\ C_2 &= \exp\left(-\mu\ln\frac{l_{1,t}}{l_{2,t}} - \lambda\ln\varphi\right), \end{aligned}$$

Recall that $\frac{1}{1-s} = 1 + \rho\lambda\beta\,(1-\tau_2)$. The value of ρ is likely to be small: this is the discount factor in an OLG model with 2–period life, and reasonable calibrations of ρ approximately equal $0.36 \approx 0.96^{25}$. Parameters λ and β are exponents in the production function and can be assumed to vary between 0.3 and 0.7. The latter yields the interval estimate [0.984;0.999] for $1-s$. The parameter μ is the production function exponent, and $\eta > 1$ with usual calibrations $2 \div 3$. Therefore, $(1-s)^{\frac{\mu}{\eta}-1}$ is likely to be a number slightly above one. If the government has no incentive to deviate from the individual discount factor, $\frac{R}{\rho}$ equals one. Therefore, the ratio $\frac{A_1}{A_2}$ depends mostly on $\frac{1-\tau_2}{1-\tau_1}$ and $\varphi^{-\lambda}$. It is reasonable to assume that much less than the whole human capital is transferred to the next generation, and therefore φ is significantly less than one. $\varphi^{-\lambda}$ is then the number definitely greater than one, as is $\frac{A_1}{A_2}$. This result could be modified if optimal marginal progressivity parameter for the old, τ_2, is significantly higher than that for the young, τ_1. However, it is very hard to get any analytical expression regarding τ_1 and τ_2.

After simplification, the expression for z yields

$$z = \frac{\rho}{\rho + R}\left(1 - \frac{R\beta\lambda}{1 - R\alpha}\right)\left[1 + \exp\left(\mu \ln \frac{l_1}{l_2} + \lambda \ln \varphi\right)\right]. \tag{10}$$

Arguments similar to those presented in the preceding paragraph confirm that if $\rho \approx R \approx 0.3$, $\tau_1 \approx \tau_2$, then the value of z will mostly depend on φ^{λ}. If φ^{λ} is significantly less than one, z is likely to be less than one.

Arguments in the preceding paragraphs also seem to indicate that the flow of funds should be directed from old to young: old should consume less than their income, and their ratio of disposable to the total income should be less than for the young. This conclusion relies on approximate equality of distortions caused by the progressive tax system to both generations (this question is uncertain and will be studied numerically). It tends to vindicate the major conclusion of Docquier and Michel (1999), where old generation pays for young's education.

4.3 Numerical Results

For numerical exercise, we selected the values of parameters mostly following Benabou (2002): $\lambda = 5/8$ and $\mu = 3/8$, $\alpha = 0.35$, $\beta = 0.4$, η either 1 or 6 (respectively, this gives elasticity of labor supply either ∞ — elastic labor, or 0.2 — inelastic labor), and the same discount factor for individual agent and government equal to 0.4. We explore the values of initial variance of the human capital distribution, Δ_0^2, corresponding to initial Gini index of 0.3, 0.5, and 0.7. In accordance with the analytical considerations stated above, we treat human capital transferability parameter φ as the most important free parameter and plot all the results against it. The issue of calibrating the parameter φ is a hard one. Benabou (2002) cites values of intergenerational persistence of human capital, $\alpha + \beta\lambda(1-\tau)$, between 0.3 and 0.6. As the parameter φ enters expression for $\ln h$ additively rather than multiplicatively, it is hard to use these numbers for calibration. Tentatively, one might say that a range of -0.3 to 0.3 for $\ln \varphi$ might be reasonable, but this question requires further consideration.

Our results for the case of elastic labor are presented on Figures 5–9. First, it is immediately obvious that for values of φ below a threshold of approximately 2.0, it is optimal to transfer resources to the young (z is less than 1). These resources flow into educational subsidy for the young, which is very significant even for extremely high values of φ such as 3.0. (Above the threshold of 2.0, young pay for the subsidy plus "pensions" to the old through consumption taxes). Subsidized savings rate $a_t s_t$ falls as φ grows. For very small values of φ, i.e., when a significant part of the human capital is lost in transition between generations, subsidized savings rate exceeds one because of the enormous educational subsidy. Curiously enough, optimal value of τ_2, the progressivity parameter for the old people, is negative for small φ: tax system is regressive. This makes perfect sense, as at these φ values the overriding objective of the central planner is to increase effective savings rate; this could be done either through higher education subsidy a or a higher savings rate $s = 1 - \frac{1}{1+\rho\lambda\beta(1-\tau_2)}$, the latter being achieved by making τ_2 negative. Negative τ_2 also increases both labor supplies, with a much stronger effect on the l_2, partially mitigating the effect of positive τ_1 which tends to repress l_1. τ_1 remains positive and large throughout the whole range of φ, which is due to its strong inequality–reducing influence on the first period disposable income. Reduced inequality increases welfare both because of egalitarian nature of the social welfare function and through partial relaxation of liquidity constraints on human capital investment. Finally, the steady-state Gini coefficient varies in a narrow range of 0.55 to 0.60. This is, essentially, an outcome predetermined by the calibration.

The only outcomes noticeably influenced by the initial Gini index are first period tax rates (predictably, higher initial inequality requires larger τ_1 to bring the system down to the required Gini of slightly above 0.55), and steady state Gini, which, surprisingly, exhibits overshooting: larger initial inequality leads to lower steady state values. Other results are, essentially, insensitive to the initial Gini.

Results for the inelastic labor are similar. However, the threshold value of φ shifts to around 1.5 (see Figure 10), subsidized investment rates does not reach above one (this is because with the inelastic labor, large and positive τ_1 does not influence first period income as much as in the elastic labor case; as a result, education subsidies are not as heavy as in the elastic labor case), second period progressivity parameter τ_2 is never negative, as there is no need to encourage savings by such drastic measures (see Figure 11), and the range of steady state Gini index is narrower.

Consider now the threshold value of φ. Assume for a second that $\tau_1 = \tau_2 = 0$. In this case, if the central planner considers shifting additional dollar from the old to the young and cares only for utilities of the currently alive, it will do it until marginal utilities of the two cohorts are equal. When $\rho = R$, this is achieved when $\frac{1}{c_1} = \frac{1}{c_2}$, or

$$\begin{aligned}\lambda \ln \varphi + \frac{\mu}{\eta} \ln(1 + \rho\beta\lambda) &= \ln(1 + \rho\beta\lambda), \\ \varphi_1^c &= (1 + \rho\beta\lambda)^{\frac{1-\frac{\mu}{\eta}}{\lambda}}.\end{aligned}$$

Given our parametrization, this expression gives $\varphi^c \approx 1.26$.

When the central planner takes into account all future generations, its decision is guided by the first order conditions derived above, in particular (10). Intergenerational transfer stops in this case when

$$\begin{aligned}z &= \frac{1}{2}\left(1 - \frac{R\beta\lambda}{1 - R\alpha}\right)\left[1 + \left(\frac{1-\tau_1}{1-\tau_2}\right)^{\frac{\mu}{\eta}} \varphi^\lambda\right] = 1, \\ \varphi_2^c &= (1 + \rho\beta\lambda)^{-\frac{\mu}{\eta\lambda}} \left(\frac{1 - R(\alpha + \beta\lambda)}{1 - R(\alpha - \beta\lambda)}\right)^{-\frac{1}{\lambda}},\end{aligned}$$

which gives the value φ^c of approximately 1.45. Therefore, no aggregate intergenerational transfer satisfies the following central planner's objectives at increasing cut–off values of φ_c : at φ_1^c, marginal utilities of currently alive are equalized, given no distortionary taxes; at φ_2^c, utility of all future generations is maximized, again without tax distortions; and finally, only $\varphi_3^c \approx 2.0$ allows maximization of all generations' utility taking into account necessity to reduce inequality through distortionary taxation. It is not a big surprise, then, that the threshold value of φ is reduced for the inelastic labor case: with less effect on labor supplies, the tax system does not distort relative incomes of the two generations so much, and thus the resulting φ_3^c becomes closer to φ_2^c. The effect is seen clearly from the expression for φ_3^c,

$$\varphi_3^c = (1+\rho\beta\lambda)^{-\frac{\mu}{\eta\lambda}} \left(\frac{1-R(\alpha+\beta\lambda)}{1-R(\alpha-\beta\lambda)}\right)^{-\frac{1}{\lambda}} \left(\frac{1-\tau_2}{1-\tau_1}\right)^{\frac{\mu}{\lambda\eta}}.$$

φ_3^c converges to φ_2^c when the optimal solution requires $\tau_2 \to \tau_1$. Presumably, this happens in the case of extremely inelastic labor supply (very high values of η).

Finally, we note that the results are highly sensitive to the treatment of initial old generation. In case its utility is included into the government's objective function and used to calculate optimal τ_1 and τ_2, the range of parameters for which it is optimal to set $\tau_2 < 0$ is sharply reduced, see Figure 12. The result is intuitive: initial olds' savings cannot be increased by regressive tax system in the second period of their life. On the other hand, given that $R = 0.4$, utility of initial old enters social planner's objective function with a weight of the same order as that of all other generations; therefore, interests of initial generation heavily affect the optimal policy.

5 Three Period Model

The simplified model elaborated above seems to indicate that income should flow from old to the young. It makes intuitive sense: old generation is "richer" than the young because is has accumulated more human capital. Furthermore, doing so is beneficial since the transfer to the young is intended mostly for the educational subsidy. Only the three period model can capture separately "human capital accumulation" motive for redistribution and pension contribution.

5.1 The Model

In the three period version of our model, we take into account limited ability of real–world old and young to work and capture it by endowing these generations with lower productivity. As savings is possible only by accumulating human capital, middle aged generation spends some amount on education, similarly to the young; however, the government pays educational subsidy only in the first period of the agent's life.

In order to simplify notation, we will follow the usual convention: variables are assumed to have time index t; variables with tilde above belong to $t+1$, with two tildes — to $t+2$, and finally, underlined variables are indexed by $t-1$.

Individual agent born at time t is maximizing 3–period utility,

$$\ln \widetilde{c}_1^i - l_1^\eta + \rho(\ln \widetilde{c}_2^i - \widetilde{l}_2^\eta) + \rho^2(\ln \widetilde{\widetilde{c}}_3^j - \widetilde{\widetilde{l}}_3^\eta). \tag{11}$$

Income is given by

$$y_j^i = \epsilon_j \left(h_j^i\right)^\lambda (l_j)^\mu, \ j = 1,2,3, \tag{12}$$

where ϵ_j captures different time endowments of generations. Without loss of generality, ϵ_2 can be set equal to one.

Tax system is modeled as in 2–period model. Budget constraints of the agent born at t are given by:

$$c_1^i (1+\theta) + e_1^i = y_1^{Di}, \tag{13a}$$
$$\widetilde{c}_2^i \left(1+\widetilde{\theta}\right) + \widetilde{e}_2^i = \widetilde{y}_2^{Di}, \tag{13b}$$
$$\widetilde{\widetilde{c}}_3^i \left(1+\widetilde{\widetilde{\theta}}\right) = \widetilde{\widetilde{y}}_3^{Di}. \tag{13c}$$

Finally, second and third period human capital are given by

$$\widetilde{h}_2^i = \kappa \xi_t^i \left(h_1^i\right)^\alpha \left[(1+a)\, e_1^i\right]^\beta, \tag{14a}$$
$$\widetilde{\widetilde{h}}_3^i = \kappa \xi_t^i \left(h_2^i\right)^\alpha \left(\widetilde{e}_2^i\right)^\beta. \tag{14b}$$

Subsidy is given only in the first period of the agent's life.

Individual is facing the following problem:

$$\max_{e_1, \widetilde{e}_2, l_1, \widetilde{l}_2, \widetilde{\widetilde{l}}_3} \ln \frac{y_1^{Di} - e_1^i}{1+\theta} - l_1^\eta + \rho(\ln \frac{\widetilde{y}_2^{Di} - \widetilde{e}_2^i}{1+\widetilde{\theta}} - \widetilde{l}_2^\eta) + \rho^2(\ln \frac{\widetilde{\widetilde{y}}_3^{Di}}{1+\widetilde{\widetilde{\theta}}} - \widetilde{\widetilde{l}}_3^\eta).$$

Similar to the 2-period model, the solution is obtained as

$$l_1 = \left[\frac{\mu}{\eta} \frac{1-\tau_1}{1-s_1}\right]^{1/\eta}, \tag{15a}$$
$$\widetilde{l}_2 = \left[\frac{\mu}{\eta} \frac{1-\widetilde{\tau}_2}{1-\widetilde{s}_2}\right]^{1/\eta}, \tag{15b}$$
$$\widetilde{\widetilde{l}}_3 = \left[\frac{\mu}{\eta}\left(1-\widetilde{\widetilde{\tau}}_3\right)\right]^{1/\eta}, \tag{15c}$$
$$e_1^i = s_1 \cdot y_1^{Di},\ s_1 = \frac{\rho\lambda\beta\left(1-\widetilde{\tau}_2\right) + \rho^2\lambda\beta\left(1-\widetilde{\widetilde{\tau}}_3\right)\left[\alpha + \lambda\beta\left(1-\widetilde{\tau}_2\right)\right]}{1+\rho\lambda\beta\left(1-\widetilde{\tau}_2\right) + \rho^2\lambda\beta\left(1-\widetilde{\widetilde{\tau}}_3\right)\left[\alpha + \lambda\beta\left(1-\widetilde{\tau}_2\right)\right]} \tag{15d}$$
$$\widetilde{e}_2^i = \widetilde{s}_2 \cdot \widetilde{y}_2^{Di},\ \widetilde{s}_2 = \frac{\rho\lambda\beta\left(1-\widetilde{\widetilde{\tau}}_3\right)}{1+\rho\lambda\beta\left(1-\widetilde{\widetilde{\tau}}_3\right)}. \tag{15e}$$

The government budget constraint is now given by

$$\int_0^1 \left[y_1^{Di} - y_1^i + y_2^{Di} - y_2^i + y_3^{Di} - y_3^i + ae_1^i - \theta\left(c_1^i + c_2^i + c_3^i\right)\right] di = 0,$$
$$\left[as_1 - \theta\frac{1-s_1}{1+\theta}\right]\overline{y}_1 + \left[A_2\left(1-\theta\frac{1-s_2}{1+\theta}\right) - 1\right]\overline{y}_2 + \left[\frac{A_3}{1+\theta} - 1\right]\overline{y}_3 = 0,$$

where $\overline{y}_j$ denotes average and aggregate income of the generation j before taxes and transfers. Note that in the expression above we have assumed $A_1 = 1$. Arguments similar to those given in the 2–period model show that only two out of three constants A_1, A_2, and A_3, are independent, and we are free to select one of them arbitrarily.

Similarly to the previous model, we assume that human capital of the young generation is transferred to the young according to the law

$$h_1^i = \varphi h_2^i.$$

We assume that at the time t, human capital of the middle generation is distributed as

$$\ln h_2 \sim N(m, \Delta^2).$$

This implies that young's human capital at time t is given as $\ln h_1 \sim N(m + \ln\varphi, \Delta^2)$. Distribution of human capital for the old generation alive at time t will be derived later.

5.2 Solving the Government Problem

Combining the expressions for the government objective function and budget constrained derived in Appendix, one can now write down the government's problem as:

$$\max \sum_{t=0}^{\infty} R^t \times$$

$$\left\{ \begin{array}{c} \ln \frac{1-s_1}{1+\theta} + \lambda m + \lambda \ln\varphi + \ln\epsilon_1 + \mu \ln l_1 + \frac{\lambda^2\Delta^2}{2}\tau_1(2-\tau_1) - l_1^\eta \\ +\rho\left[\ln \widetilde{A}_2 \frac{1-\widetilde{s}_2}{1+\widetilde{\theta}} + \lambda\widetilde{m} + \mu\ln\widetilde{l}_2 + \frac{\lambda^2\widetilde{\Delta}^2}{2}\widetilde{\tau}_2(2-\widetilde{\tau}_2) - \widetilde{l}_2^\eta\right] \\ +\rho^2 \left[\begin{array}{c} \ln \frac{\widetilde{\widetilde{A}}_3}{1+\widetilde{\widetilde{\theta}}} + \ln\epsilon_3 + \mu\ln\widetilde{\widetilde{l}}_3 + \frac{\lambda^2}{2}\left(\left[\alpha+\beta\lambda(1-\widetilde{\tau}_2)\right]^2\widetilde{\Delta}^2+\omega^2\right)\widetilde{\widetilde{\tau}}_3(2-\widetilde{\widetilde{\tau}}_3) - \widetilde{\widetilde{l}}_3^\eta \\ +\lambda\left[(\alpha+\beta\lambda)\widetilde{m} + \ln\kappa - \frac{\omega^2}{2} + \beta\left[\ln\widetilde{A}_2\widetilde{s}_2 + \mu\ln\widetilde{l}_2 + \frac{\lambda^2\widetilde{\Delta}^2}{2}\widetilde{\tau}_2(2-\widetilde{\tau}_2)\right]\right] \end{array} \right] \end{array} \right\},$$

subject to

$$0 = \left[as - \theta\frac{1-s_1}{1+\theta}\right]\overline{y}_1 + \left[A_2\left(1-\theta\frac{1-s_2}{1+\theta}\right)-1\right]\overline{y}_2 + \left[\frac{A_3}{1+\theta}-1\right]\overline{y}_3, \quad (16a)$$

$$\widetilde{m} = (\alpha+\beta\lambda)(m+\ln\varphi) + \ln\kappa - \frac{\omega^2}{2} + +\beta\left[\ln\epsilon_1(1+a)s_1 + \mu\ln l_1 + \frac{\lambda^2\Delta^2}{2}\tau_1(2-\tau_1)\right], \quad (16b)$$

$$\widetilde{\Delta}^2 = \left[\alpha+\beta\lambda(1-\tau_2)\right]^2\Delta^2 + \omega^2. \quad (16c)$$

Appendix contains technical derivations of the first order conditions.

As in the case of a simplified model, we will concentrate on time invariant policies, where $z = \widetilde{z}$. In this case we get

$$z = \frac{\frac{\rho}{R}A_2\left(1-\theta\frac{1-s_2}{1+\theta}\right) - \rho\beta\lambda\left\{\left[as-\theta\frac{1-s_1}{1+\theta}\right]\frac{\overline{y}_1}{\overline{y}_2} + \left[A_2\left(1-\theta\frac{1-s_2}{1+\theta}\right)-1\right]\right\}}{\frac{\rho}{R}A_2\left(1-\theta\frac{1-s_2}{1+\theta}\right) + \left\{\left[as-\theta\frac{1-s_1}{1+\theta}\right]\frac{\overline{y}_1}{\overline{y}_2} + \left[A_2\left(1-\theta\frac{1-s_2}{1+\theta}\right)-1\right]\right\}}. \quad (17)$$

The term in figure brackets equals total redistribution to (if it is positive) or from (if negative) young and middle generations, taking middle generation income as numerarie.

$A_2\left(1-\theta\frac{1-s_2}{1+\theta}\right)$ is the amount that is left to the middle generation after paying income and consumption taxes, its "disposable income". The case $z > 1$ (net transfer of resources to the old generation) is obtained only by taking away income of young and middle aged, which is prescribed by the government budget constraint. However, (17) shows that given total redistribution from the first two cohorts, net transfer to the old is decreasing in "disposable income" of the middle generation. In other words, if it is optimal to put relatively more burden on the young to pay transfer to the old, then transfers to the old suffer. In the opposite case ($z < 1$) when net transfer goes from the old generation, leaving more income in pockets of the middle–aged rather than young decreases the optimal transfer from the old.

To get more insight into the behavior of z, write (31a) as

$$1+\rho\beta\lambda\frac{1}{\tilde{z}} - \frac{\rho}{R}\frac{1}{z}A_2\left(1-\theta\frac{1-s_2}{1+\theta}\right)\frac{\overline{y}_2}{\overline{y}_3}$$
$$\Rightarrow \quad z = \frac{\rho}{R}\left[A_2\left(1-\theta\frac{1-s_2}{1+\theta}\right)\frac{\overline{y}_2}{\overline{y}_3} - R\beta\lambda\right].$$

Comparing expressions for $\overline{y}_2$ and $\overline{y}_3$ with (16b) and (16c), we see that $\frac{\overline{y}_2}{\overline{y}_3}$ can be written as

$$\frac{\overline{y}_2}{\overline{y}_3} = \varphi^{\lambda(\alpha+\beta\lambda)}\frac{\epsilon_1^{\beta\lambda}}{\epsilon_3}\left(\frac{(1+a)s_1}{A_2 s_2}\right)^{\beta\lambda}\exp\left\{\mu\ln\frac{l_2}{l_3} + \beta\lambda\left(\mu\ln\frac{l_1}{l_2} + \frac{\lambda^2\Delta^2}{2}\left[\tau_1(2-\tau_1) - \tau_2(2-\tau_2)\right]\right)\right\}.$$

Keeping fixed progressivity parameters τ_1, τ_2, and τ_3, we now see that net transfer to the old is likely to increase when φ increases (the same result as in the 2–period model), when ϵ_1 goes up or ϵ_3 goes down, and when the ratio of effective investment rate for the young and middle cohorts increases. All of the mentioned above changes make young generation relatively richer than the old either directly, by increasing its human capital or productivity, or indirectly, by providing more human capital in the second period.

5.3 Numerical Results

In 3–period simulations, whenever applicable, we are using the values of calibration parameters congruent to those in a 2-period case. Nevertheless, 2–period and 3–period scenarios are not fully identical. The major difference is that the time endowment parameters in the 3–period case are assumed to vary with age. More specifically, in what follows, we use ε_1 and ε_3, the time endowment of young and old relative to the middle aged, taken from Bouzahzah, la Croix, and Docquier (2002) and set equal $\varepsilon_1 = 0.65$, $\varepsilon_3 = 0.5$. To be more precise, we average that paper's time endowments for generations 1 and 2, 3 and 4, and 5 and 6, given in Table 1 of the paper, and normalize the result so that $\varepsilon_2 = 1$.

Earlier, we have learned from 2–period model, that treatment of initial old significantly influences the optimal stationary taxation policy. For the 3–period model, we have decided to include the middle-aged generation at time t_0 with the weight $1/R$, and the old generation at the same time t_0 with the weight $1/R^2$. However, initial old are assumed to stick to the behavioral pattern identical to that of the future generations, when the old generation was young and middle-aged. In order to make the theoretical model numerically tractable, we also assume that the progressivity parameter of the tax system applied to the young is the same as that applied to the middle–aged.[10]

2–period and 3–period models are similar in many respects. Notably, in both 2–period and 3–period models, the parameter that drives the results is the efficiency of human capital transfers of the middle aged to the young (which is treated as exogenously given). For small values of this parameter the social planner finds optimal to supplement educational expenditures of the young generation by means of large transfers. (A straightforward comparison of Figure 5 and Figure 13 provides an evidence). Furthermore, in the 3–period model, these transfers are now combined with generous pension benefit payments to the third generation (i.e. the old). These transfers are financed by the middle aged. As in 2–period model, transfers to the young eventually turn negative as the efficiency parameter φ increases; yet, the pensions transfers continue to grow. However, 2– and 3–period models are not identical: for the whole range of the efficiency of human capital transfer, 3–period model is characterized by lower educational transfers to the young and the heavier burden on the "tax–paying" middle–aged generation.

Furthermore, 2– and 3–period models also exhibit qualitative difference in behavior.[11] Figure 14 displays that the social planner attempts to set τ_3 as high as possible (we restricted it to be not higher than 0.5) in order to achieve an egalitarian objective while keeping $\tau_1 = \tau_2$ relatively low. Because of high τ_3, old supply a little labor and are poor, which compels the government to transfer resources to them to the tune of hundreds percent of their income. Welfare losses related to low labor supply by the old are compensated by more equal distibution of their after–transfer income. On the other hand, $\tau_1 = \tau_2$ cannot be made high, as the middle–aged generation transfers the resources to both young (for low φ) and old and has to be encouraged to supply labor.

6 Conclusion

Using the OLG framework by Boldrin and Montes, our empirical results, estimated from the Czech Republic Microcensus 1996, indicate that if current budget rules are combined with artificially frozen current age structure, paying for education of the next generation provides higher return than the interest "paid" on educational loans: education is cheaper than pensions, with the gap between implicit interest rates about 2%. This conclusion is consistent with currently observed 45% wage replacement rate of pensions and relatively short supply of educational services in the CR. Demographic change is bound to affect this gap. For example, consider "demand driven" budgeting: fix per capita educational transfers and pensions and preserve balanced budget

[10] Loosely speaking, this assumption might bind the ability of the government to achieve the optimality across the board. Note, however, that age-dependent taxes are not common, while pension systems are usually regressive and education systems – notably higher education systems – are moderately progressive. Thus by our assumption we merely say that the education system is in large as progressive as the tax system.

[11] This difference might be partially due to the our choice of age–dependent progressivity parameters. To ensure numerical tractability of 3–period model, we assume that $\tau_1 = \tau_2$. A closer look at equation (15d) reveals that first period savings, s_1, are mostly sensitive to τ_2. By equating τ_1 and τ_2 we deprive the social planner of an option to select lower (or even negative) values of τ_2 in order to stimulate savings.

assumption (considering rapid ageing of the population, this implies dramatic increase in social security contributions in the near future). The gap disappears for cohorts born in 1980s. For still younger people, current situation is reversed: interest rate on "educational loans" exceeds that paid as pensions by as much as 1.2% in moderately and highly optimistic cases (fertility increases from current level of 1.19 to 1.4 and 1.9, respectively) to 1.5% (if current fertility remains constant in the future) to 1.9% in the highly pessimistic scenario (fertility drops to 1.05).

In the distant future, as the population structure stabilizes at different levels determined by believed demographic projections, it becomes feasible to equalize these two interest rates by means of various fiscal tools. For instance, at current fertility rates with "demand driven" budgeting and 16.6% increment in pensions from 1996 level (without corresponding increase in social security contributions), the two interest rates will be equalized just above 3%. (Note, however, that preserving balanced budget by *not* increasing pensions leads to implicit interest rate on pension benefits being 0.5% lower than that on educational loans). Alternatively, the same objective can be achieved by increasing per capita educational transfers by 12.7% without the corresponding raise in taxes, but the common interest rate now will become as low as 2.55%. Nevertheless, efficiency requires not only pair-wise equality of the two interest rates, but also their simultaneous equality to the market interest rate. Thus, pensions adjustment and education transfers adjustment lead to different outcomes in this framework.

Next, using Benabou's model extended to overlapping generations setting, we find that in both 2-period and 3-period models, the parameter that drives the results is the efficiency of human capital transfers of the middle aged to the young (which is treated as exogenously given). For small values of this parameter the social planner finds optimal to supplement educational expenditures of the young generation by means of large transfers. In order to stimulate educational expenditures by the young, in 2-period model it might be optimal even to set regressive taxes on the old generation.

In the 3-period model, low and moderate values of the efficiency of human capital transfers still yield large educational transfers to the young, now combined with generous pension benefit payments to the third generation (i.e. the olds). These transfers are financed by the middle aged. As in 2-period model, transfers to the young eventually turn negative as the efficiency parameter increases; yet, the pensions transfers continue to grow.

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APPENDIX I: Description of the Data

Relevant variables in Microcensus-1996:

Variable Name	Description	Coding/Recoding
O8	Year of Birth	Two last digits of the year of birht
O9	Citizenship	Used codes: 1-CR, 2-SR
O11	Gender	Used codes: 1-male
O14	Education Level	Constructed codes: 0-no or incomplete elementary, 1-elementary, 2-secondary without maturita, 3-secondary with maturita, 4-higher
O15	Economic Activity	Used codes: 1-economically active, 3-retired, 6-students, 8-children aged 0-15 years
P806	Type of retirement	Used codes: 1-age, 4-widow
MES_ZAM	Number of months worked in this year	
P918	Retirement payment	
MES_DUCH	Number of months retirement benefits were received in this year	
HPRIJMY	Total gross yearly income	
CPRIJMY	Total net yearly income	
POJIS	Mandatory social and health insurance paid this year	
DAN	Income tax paid this year	

Expenditures per student by type of school in 1995/96 academic year

Type of school	Expenditures per student in CZK
Nursery	21618
Basic	19604
Gymnasia	25416
Secondary vocational	33586
Secondary professional	27560
University	69221

Constructed variable 'educational transfers per student' in the Czech Republic in 1996:

- `edu_transfer=21618*0.044 if student attends nursery school and aged 0--2 years;`
- `edu_transfer=21618*0.9 if student attends nursery school and aged 3--5 years;`

- edu_transfer=19604 if student attends basic school (aged 6--15 years;
- edu_transfer=25461*0.195+27560*0.38+33586*0425 if student attends secondary school (with or without maturita, aged 15+);
- edu_transfer=69221 if student attends university.

VAT as percentage of total gross money income in the Czech Republic:

Decile	The first	Second	Third	Fourth	Fifth	Sixth	Seventh	Eighth	Ninth	The last
Gross money income total	61051	79499	88890	94648	102387	111422	126941	143977	175422	244117
Consumer Expenditures	51371.1	64338.5	68747.3	75487.1	81154.5	87724.8	93276.3	101905	117552	150549
VAT paid	7469.05	9428.89	9975.06	11281.3	12025.1	12915.9	13682.2	15036.1	17097.1	22659.9
Percentage of income paid as VAT	12.23%	11.87%	11.23%	11.92%	11.74%	11.59%	10.78%	10.44%	9.75%	9.28%

APPENDIX II: Derivations

Derivation of $l_{1,t}$, $l_{2,t+1}$ and $e^i_{1,t}$ in 2-period model

Replacing $c^i_{1,t}, c^i_{2,t+1}$ by individual budget constraints (4), an individual is facing the following problem:

$$\max_{e,l_1,l_2} \ln \frac{y^{Di}_{1,t} - e^i_{1,t}}{1+\theta_t} - l^{\eta}_{1,t} + \rho(\ln \frac{y^{Di}_{2,t+1}}{1+\theta_{t+1}} - l^{\eta}_{2,t+1}).$$

Standard first order conditions for maximization yield

$$\begin{aligned} l_{1,t} &: \frac{1}{c^i_{1,t}} \frac{1}{1+\theta_t} \frac{\partial y^{Di}_{1,t}}{\partial y^i_{1,t}} \frac{\partial y^i_{1,t}}{\partial l_{1,t}} = \eta l^{\eta-1}_{1,t}, \\ l_{2,t+1} &: \frac{1}{c^i_{2,t+1}} \frac{1}{1+\theta_{t+1}} \frac{\partial y^{Di}_{2,t+1}}{\partial y^i_{2,t+1}} \frac{\partial y^i_{2,t+1}}{\partial l_{2,t+1}} = \eta l^{\eta-1}_{2,t+1}, \\ e^i_{1,t} &: -\frac{1}{c^i_{1,t}} \frac{1}{1+\theta_t} + \rho \frac{1}{c^i_{2,t+1}} \frac{1}{1+\theta_{t+1}} \frac{\partial y^{Di}_{2,t+1}}{\partial y^i_{2,t+1}} \frac{\partial y^i_{2,t+1}}{\partial h^i_{2,t+1}} \frac{\partial h^i_{2,t+1}}{\partial e^i_{1,t}}. \end{aligned}$$

After simple algebra, we obtain

$$\begin{aligned} \frac{\mu}{\eta} \frac{1-\tau_{1,t}}{c^i_{1,t}(1+\theta_t)} y^{Di}_{1,t} &= l^{\eta}_{1,t}, \\ \frac{\mu}{\eta} \frac{1-\tau_{2,t+1}}{c^i_{2,t}(1+\theta_{t+1})} y^{Di}_{2,t+1} &= \frac{\mu}{\eta}(1-\tau_{2,t+1}) = l^{\eta}_{2,t+1}, \\ \frac{1}{c^i_{1,t}} \frac{1}{1+\theta_t} &= \frac{\rho\lambda\beta(1-\tau_{2,t+1})}{e^i_{1,t}}. \end{aligned}$$

Utilizing again (4a), $l_{1,t}$, $l_{2,t+1}$, $e^i_{1,t}$ finally can be expressed as

$$l_{1,t} = \left[\frac{\mu}{\eta}(1-\tau_{1,t})(1+\rho\lambda\beta(1-\tau_{2,t+1}))\right]^{1/\eta}, \quad (18a)$$

$$l_{2,t+1} = \left[\frac{\mu}{\eta}(1-\tau_{2,t+1})\right]^{1/\eta}, \quad (18b)$$

$$e^i_{1,t} = \frac{\rho\lambda\beta(1-\tau_{2,t+1})}{1+\rho\lambda\beta(1-\tau_{2,t+1})} y^{Di}_{1,t} = s_t \cdot y^{Di}_{1,t}. \quad (18c)$$

Derivation of the government budget constraint in 2-period model

Integral of the first two terms adds up to the net transfer to the young generation through the tax/transfer scheme (3) at time t, the next two terms account for the total transfer to the old alive at t, the fifth term results in total education subsidy, and the last one totals consumption taxes. Using (4) and (18c), the government budget constraint can be rewritten as

$$\left(1 + a_t s_t - \theta_t \frac{1-s_t}{1+\theta_t}\right)\int_0^1 y^{Di}_{1,t} di + \frac{1}{1+\theta_t}\int_0^1 y^{Di}_{2,t} di = \int_0^1 \left(y^i_{1,t} + y^i_{2,t}\right) di.$$

Let us denote

$$\int_0^1 y^{Di}_{1,t} di = A_{1,t}\int_0^1 y^i_{1,t} di = A_{1,t}\overline{y}_{1,t}, \quad (19a)$$

$$\int_0^1 y^{Di}_{2,t} di = A_{2,t}\int_0^1 y^i_{2,t} di = A_{2,t}\overline{y}_{2,t}. \quad (19b)$$

In new notation, the government budget constraint is reduced to

$$\left[A_{1,t}\left(1 + a_t s_t - \theta_t \frac{1-s_t}{1+\theta_t}\right) - 1\right]\overline{y}_{1,t} + \left[\frac{A_{2,t}}{1+\theta_t} - 1\right]\overline{y}_{2,t} = 0.$$

Derivation of m_{t+1}, Δ^2_{t+1} and the government budget constraint in 2-period model

As in Benabou, all individual variables are assumed to be lognormally distributed: $\ln h^i_{1,t} \sim N(m_t, \Delta^2_t)$, $\ln \xi^i \sim N(-\omega^2/2, \omega^2)$. Given loglinearity of production technologies and assumed form of utility, log of human capital (and thus log income and log disposable income) will continue to be lognormally distributed.[12] To study the evolution of $\ln h^i_{2,t+1}$, we need to assess $\tau_{1,t}\ln y^T_{1,t}$. The latter can be derived from (19a) as

$$\tau_{1,t}\ln y^T_{1,t} = \ln A_{1,t} + \tau_{1,t}(\lambda m_t + \mu \ln l_{1,t}) + \frac{\lambda^2\Delta^2_t}{2}\tau_{1,t}(2-\tau_{1,t}). \quad (20)$$

[12]For further reference, recall that

$$\text{if } \ln x \sim N(\mu,\sigma^2), \text{ then } E[x^\lambda] = \exp\left(\mu\lambda + \frac{\lambda^2\sigma^2}{2}\right).$$

We also need state equations for m_t and Δ_t^2 (derived from (5) and (20)):

$$\begin{aligned} m_{t+1} &= (\alpha+\beta\lambda)\, m_t + \beta\mu \ln l_{1,t} + \beta\frac{\lambda^2\Delta_t^2}{2}\tau_{1,t}(2-\tau_{1,t}) + \beta\ln\left[(1+a_t)\, s_t\right] + \\ &\quad \ln\kappa - \frac{\omega^2}{2} + \beta\ln A_{1,t} + \ln\varphi, \\ \Delta_{t+1}^2 &= \left[\alpha+\beta\lambda(1-\tau_{1,t})\right]^2 \Delta_t^2 + \omega^2. \end{aligned}$$

Here m_{t+1} is the mean of the distribution of $\ln h_{1,t+1}^i$. The mean of the distribution of $\ln h_{2,t+1}^i$ is computed as m_{t+1} less $\ln\varphi$.

Now we are in a position to derive the objective function in (7).

$$\begin{aligned} \ln c_{1,t}^i &= \ln y_{1,t}^{Di} + \ln\frac{1-s_t}{1+\theta_t} \\ &= (1-\tau_{1,t})\ln y_{1,t}^i + \tau_{1,t}\ln y_{1,t}^T + \ln\frac{1-s_t}{1+\theta_t} \\ &= (1-\tau_{1,t})\,\lambda\ln h_{1,t}^i + (1-\tau_{1,t})\,\mu\ln l_{1,t} + \tau_{1,t}\ln y_{1,t}^T + \ln\frac{1-s_t}{1+\theta_t} = \\ &= (1-\tau_{1,t})\,\lambda\ln h_{1,t}^i + (1-\tau_{1,t})\,\mu\ln l_{1,t} + \ln A_{1,t} + \ln\frac{1-s_t}{1+\theta_t} + \\ &\quad +\tau_{1,t}\left(\lambda m_t + \mu\ln l_{1,t}\right) + \frac{\lambda^2\Delta_t^2}{2}\tau_{1,t}(2-\tau_{1,t}). \end{aligned}$$

Integrating $\ln c_{1,t}^i$ over all agents yields

$$\int_0^1 \ln c_{1,t}^i di = \lambda m_t + \mu\ln l_{1,t} + \ln A_{1,t} + \ln\frac{1-s_t}{1+\theta_t} + \frac{\lambda^2\Delta_t^2}{2}\tau_{1,t}(2-\tau_{1,t}).$$

Expressing $\ln c_{2,t+1}^i$ as $\ln y_{2,t+1}^{Di} - \ln(1+\theta_{t+1})$, we get

$$\begin{aligned} \int_0^1 \ln c_{2,t+1}^i di &= \int_0^1 (1-\tau_{2,t+1})\,\lambda\ln h_{2,t+1}^i di + (1-\tau_{2,t+1})\,\mu\ln l_{2,t+1} + \\ &\quad +\tau_{2,t+1}\ln y_{2,t+1}^T - \ln(1+\theta_{t+1}) \\ &= \lambda\left(m_{t+1} - \ln\varphi\right) + \mu\ln l_{2,t+1} + \ln A_{2,t+1} + \frac{\lambda^2\Delta_{t+1}^2}{2}\tau_{2,t+1}(2-\tau_{2,t+1}) \\ &\quad -\ln(1+\theta_{t+1}). \end{aligned}$$

To derive the government budget constraint, notice first that the two currently alive generations' distributions of human capital are almost identical: $\ln h_{1,t}^i \sim N(m_t,\Delta_t^2)$, while $\ln h_{2,t}^i \sim N(m_t - \ln\varphi, \Delta_t^2)$. Consequently, incomes are distributed as $\ln y_{1,t}^i \sim N(\lambda m_t + \mu\ln l_{1,t}, \lambda^2\Delta_t^2)$ and $\ln y_{2,t}^i \sim N(\lambda m_t - \lambda\ln\varphi + \mu\ln l_{2,t}, \lambda^2\Delta_t^2)$. The government budget constraint (6) thus becomes

$$\left[A_{1,t}\left(1 + a_t s_t - \theta_t\frac{1-s_t}{1+\theta_t}\right) - 1\right]\exp\left(\mu\ln\frac{l_{1,t}}{l_{2,t}} + \lambda\ln\varphi\right) = 1 - \frac{A_{2,t}}{1+\theta_t}.$$

Solving the government's problem in 2-period model

We will solve the problem by forming the following Lagrangian[13]:

$$\sum_{t=0}^{\infty} R^t \times \times \left\{ \begin{array}{c} \lambda m_t + \mu \ln l_{1,t} + \ln A_{1,t} + \ln \frac{1-s_t}{1+\theta_t} + \frac{\lambda^2 \Delta_t^2}{2} \tau_{1,t}(2-\tau_{1,t}) - l_{1,t}^{\eta} \\ +\rho \left[\lambda \left(m_{t+1} - \ln \varphi \right) + \mu \ln l_{2,t+1} + \frac{\lambda^2 \Delta_{t+1}^2}{2} \tau_{2,t+1}(2-\tau_{2,t+1}) - l_{2,t+1}^{\eta} \right] \\ +\rho \ln \left\{ 1 - \left[A_{1,t+1} \left(1 + a_{t+1} s_{t+1} - \theta_{t+1} \frac{1-s_{t+1}}{1+\theta_{t+1}} \right) - 1 \right] \exp \left(\mu \ln \frac{l_{1,t+1}}{l_{2,t+1}} + \lambda \ln \varphi \right) \right\} \\ +\Psi_t^1 \left[\begin{array}{c} m_{t+1} - (\alpha + \beta\lambda) m_t - \beta\mu \ln l_{1,t} - \beta \frac{\lambda^2 \Delta_t^2}{2} \tau_{1,t}(2-\tau_{1,t}) \\ -\beta \ln \left[(1+a_t) s_t \right] - \ln \kappa + \frac{\omega^2}{2} - \beta \ln A_{1,t} - \ln \varphi \end{array} \right] \\ +\Psi_t^2 \left[\Delta_{t+1}^2 - \left[\alpha + \beta\lambda (1-\tau_{1,t}) \right]^2 \Delta_t^2 - \omega^2 \right]. \end{array} \right\}.$$

The first set of the F.O.C.'s is given below (all time indices are shifted backward from $t+1$ to t):

$$\begin{aligned} a_t &: \\ \frac{R\beta\Psi_t^1}{1+a_t} &= \rho \frac{-\exp\left(\mu \ln \frac{l_{1,t}}{l_{2,t}} + \lambda \ln \varphi\right) A_{1,t} s_t}{1 - \left[A_{1,t} \left(1 + a_t s_t - \theta_t \frac{1-s_t}{1+\theta_t} \right) - 1 \right] \exp\left(\mu \ln \frac{l_{1,t}}{l_{2,t}} + \lambda \ln \varphi\right)}, \qquad (22a) \\ \theta_t &: \\ R &= \rho \frac{\exp\left(\mu \ln \frac{l_{1,t}}{l_{2,t}} + \lambda \ln \varphi\right) \frac{1-s_t}{1+\theta_t} A_{1,t}}{1 - \left[A_{1,t} \left(1 + a_t s_t - \theta_t \frac{1-s_t}{1+\theta_t} \right) - 1 \right] \exp\left(\mu \ln \frac{l_{1,t}}{l_{2,t}} + \lambda \ln \varphi\right)}, \qquad (22b) \\ A_{1,t} &: \\ R\beta\Psi_t^1 &= R - \rho \frac{\exp\left(\mu \ln \frac{l_{1,t}}{l_{2,t}} + \lambda \ln \varphi\right) A_{1,t} \left(1 + a_t s_t - \theta_t \frac{1-s_t}{1+\theta_t} \right)}{1 - \left[A_{1,t} \left(1 + a_t s_t - \theta_t \frac{1-s_t}{1+\theta_t} \right) - 1 \right] \exp\left(\mu \ln \frac{l_{1,t}}{l_{2,t}} + \lambda \ln \varphi\right)}. \qquad (22c) \end{aligned}$$

Excluding Ψ_t^1 from the 1^{st} F.O.C. and plugging it into the 3^{rd} one, the R/ρ ratio takes the form

$$\frac{R}{\rho} = \frac{\exp\left(\mu \ln \frac{l_{1,t}}{l_{2,t}} + \lambda \ln \varphi\right) \frac{1-s_t}{1+\theta_t} A_{1,t}}{1 - \left[A_{1,t} \left(1 + a_t s_t - \theta_t \frac{1-s_t}{1+\theta_t} \right) - 1 \right] \exp\left(\mu \ln \frac{l_{1,t}}{l_{2,t}} + \lambda \ln \varphi\right)}, \qquad (23)$$

that is an exact equivalent to the 2^{nd} F.O.C. Therefore, we conclude that that one of the three controls, a_t, θ_t, and $A_{1,t}$, is redundant (not independent).

Plugging (9) into the expression derived above, we obtain the following convenient condition:

$$\frac{\rho}{R} \frac{\frac{1-s_t}{1+\theta_t} A_{1,t}}{A_{1,t} \left(1 + a_t s_t - \theta_t \frac{1-s_t}{1+\theta_t} \right) - 1} = \frac{\frac{A_{2,t}}{1+\theta_t}}{1 - \frac{A_{2,t}}{1+\theta_t}}. \qquad (24)$$

[13]Notice that the budget constraint is plugged into currently alive generations' utility function.

This relationship has a relatively simple intuitive explanation: government continues intergenerational redistribution up to a point where marginal utility gained by one generation is equal to the marginal utility lost by another.

The next step is to consider the first order condition with respect to m_{t+1}:

$$m_{t+1} : \rho\lambda + R\lambda + \Psi_t^1 - R\Psi_{t+1}^1(\alpha + \beta\lambda) = 0. \tag{25}$$

Let consider this condition along the BGP, where all taxes and redistribution parameters are constant. In such case, (22a) guarantees that $\Psi_t^1 \equiv \Psi^1$.

Plugging (22a) into (25), we obtain

$$\begin{aligned}
& (\rho + R)\,\lambda - \frac{1}{\beta}\,(1 - R(\alpha + \beta\lambda)) \times \\
& \times \frac{\rho}{R}\frac{-\exp\left(\mu \ln \frac{l_1}{l_2} + \lambda \ln \varphi\right) A_1\,(1+a)\,s}{1 - \left[A_1\left(1 + as - \theta\frac{1-s}{1+\theta}\right) - 1\right]\exp\left(\mu \ln \frac{l_1}{l_2} + \lambda \ln \varphi\right)} \\
= \; & (\rho + R)\,\lambda - \frac{1 - R(\alpha + \beta\lambda)}{\beta}\frac{A_1\,(1+a)\,s}{\frac{1-s}{1+\theta}A_1} = 0, \\
\Rightarrow \; & \frac{\frac{1-s}{1+\theta}}{(1+a)\,s} = \frac{1 - R(\alpha + \beta\lambda)}{(\rho + R)\,\beta\lambda}.
\end{aligned} \tag{26}$$

Collecting (24), (26), and (9), we arrive at

$$\begin{aligned}
\frac{\frac{A_2}{1+\theta}}{1 - \frac{A_2}{1+\theta}} &= \frac{\rho}{R}\frac{\frac{1-s}{1+\theta}A_1}{A_1\left(1 + as - \theta\frac{1-s}{1+\theta}\right) - 1}, \\
\frac{1-s}{1+\theta} &= \frac{1 - R(\alpha + \beta\lambda)}{(\rho + R)\,\beta\lambda}\,(1+a)\,s, \\
1 - \frac{A_2}{1+\theta} &= \left[A_1\left(1 + as - \theta\frac{1-s}{1+\theta}\right) - 1\right]\exp\left(\mu \ln \frac{l_1}{l_2} + \lambda \ln \varphi\right).
\end{aligned}$$

Noticing that

$$1 + as - \theta\frac{1-s}{1+\theta} - (1+a)\,s = \frac{1-s}{1+\theta},$$

and introducing the new variables and constants

$$\begin{aligned}
x &= A_1\,(1+a)\,s = A_1\widetilde{s}, \\
z &= \frac{A_2}{1+\theta}, \\
C_1 &= \frac{1 - R(\alpha + \beta\lambda)}{(\rho + R)\,\beta\lambda}, \\
C_2 &= \exp\left(-\mu \ln \frac{l_{1,t}}{l_{2,t}} - \lambda \ln \varphi\right),
\end{aligned}$$

we can rewrite the preceding system of equations as

$$\frac{1-s}{1+\theta} = C_1\,(1+a)\,s = C_1\widetilde{s}, \tag{27a}$$

$$\frac{z}{1-z} = \frac{\rho}{R}\frac{C_1 x}{(1 + C_1)\,x - 1}, \tag{27b}$$

$$(1 + C_1)\,x - 1 = C_2(1 - z). \tag{27c}$$

Now it is straightforward that out of 4 independent tax system parameters A_1, A_2, θ, and a, only three could be determined, conditional on values of τ_1 and τ_2. The values of x and y are given by

$$x = \frac{1 + C_2}{1 + \left(1 + \frac{\rho}{R}\right) C_1}, \quad z = \frac{\rho}{R} \frac{C_1}{C_2} x,$$

and the ratio of A_1 and A_2 is given by

$$\frac{A_1}{A_2} = \frac{C_1}{1 - s} \frac{x}{z} = \frac{R}{\rho} \frac{C_2}{1 - s}.$$

It is now obvious that only the ratio of A_1 to A_2 can be determined. It is convenient to assume that $A_1 = 1$. Notice that the government optimization problem allows to calculate what share of income y_1, (not the disposable income y_1^D,) the young generation should spend on education, taking into account the educational subsidy and possible intergenerational redistribution through the tax/transfer scheme (3). However,the optimal policy mix of first period intergenerational transfer and education subsidy remains indeterminate.

Derivation of the government objective function and budget constraint in 3-period model

The values needed to derive government objective function and budget constraint are then given as follows:

$$\begin{aligned}
\int_0^1 \ln c_j^i di &= E[\ln c_j^i] = \ln \frac{1 - s_j}{1 + \theta} + E[\ln y_j^{Di}] = \ln \frac{(1 - s_j)}{1 + \theta} + (1 - \tau_j) E[\ln y_j^i] + \tau_j \ln y_j^T, \\
\text{but } \left(y_j^T\right)^{\tau_j} &= A_j \frac{E[y_j^i]}{E[\left(y_j^i\right)^{1-\tau_j}]}, \text{ or } \tau_j \ln y_j^T = \ln A_j + \ln E[y_j^i] - \ln E[\left(y_j^i\right)^{1-\tau_j}].
\end{aligned}$$

If $\ln y_j^i \sim N(\mu, \sigma^2)$, then

$$E[\ln c_j^i] = \ln A_j \frac{1 - s_j}{1 + \theta} + \mu + \frac{\sigma^2}{2} \tau_j (2 - \tau_j).$$

Distributions of $\ln y_1^i$ and $\ln y_2^i$ are easy to derive and are given by $\ln y_1^i \sim N(\lambda m + \lambda \ln \varphi + \ln \epsilon_1 + \mu \ln l_1, \lambda^2 \Delta^2)$, $\ln y_2^i \sim N(\lambda m + \mu \ln l_2, \lambda^2 \Delta^2)$. Therefore,

$$\begin{aligned}
E[\ln c_1^i] &= \ln \frac{1 - s_1}{1 + \theta} + \lambda m + \lambda \ln \varphi + \ln \epsilon_1 + \mu \ln l_1 + \frac{\lambda^2 \Delta^2}{2} \tau_1 (2 - \tau_1), \\
E[\ln \widetilde{c}_2^i] &= \ln \widetilde{A}_2 \frac{1 - \widetilde{s}_2}{1 + \widetilde{\theta}} + \lambda \widetilde{m} + \mu \ln \widetilde{l}_2 + \frac{\lambda^2 \widetilde{\Delta}^2}{2} \widetilde{\tau}_2 (2 - \widetilde{\tau}_2).
\end{aligned}$$

Distribution of $\ln y_3^i$ is more involved. First, write down

$$\begin{aligned}
\ln \widetilde{y}_3^i &= \ln \epsilon_3 + \lambda \ln \widetilde{h}_3^i + \mu \ln \widetilde{l}_3, \\
\ln \widetilde{h}_3^i &= \ln \kappa \underline{\xi} + \alpha \ln h_2^i + \beta \ln s_2 + \beta \ln y_2^{Di} = \ln \kappa \underline{\xi} + \alpha \ln h_2^i + \beta \ln s_2 + \\
&\quad + \beta \left((1-\tau_2) \ln y_2^i + \tau_2 \ln y_2^T \right), \\
\tau_2 \ln y_2^T &= \ln A_2 + \tau_2 \left(\lambda m + \mu \ln l_2 \right) + \frac{\lambda^2 \Delta^2}{2} \tau_2 (2-\tau_2), \\
\ln \widetilde{h}_3^i &= \alpha \ln h_2^i + \beta (1-\tau_2) \ln y_2^i + \ln \underline{\xi} + \ln \kappa + \\
&\quad + \beta \left[\ln A_2 s_2 + \tau_2 \left(\lambda m + \mu \ln l_2 \right) + \frac{\lambda^2 \Delta^2}{2} \tau_2 (2-\tau_2) \right].
\end{aligned}$$

We immediately see that $\ln \widetilde{h}_3^i$ is distributed as a normal variable with mean equal to

$$\begin{aligned}
& \alpha m + \beta (1-\tau_2)(\lambda m + \mu \ln l_2) - \frac{\omega^2}{2} + \ln \kappa + \beta \left[\ln A_2 s_2 + \tau_2 \left(\lambda m + \mu \ln l_2 \right) + \frac{\lambda^2 \Delta^2}{2} \tau_2 (2-\tau_2) \right] \\
= \; & (\alpha + \beta \lambda) m + \ln \kappa - \frac{\omega^2}{2} + \beta \left[\ln A_2 s_2 + \mu \ln l_2 + \frac{\lambda^2 \Delta^2}{2} \tau_2 (2-\tau_2) \right], \qquad (28)
\end{aligned}$$

and the variance given by

$$[\alpha + \beta \lambda (1-\tau_2)]^2 \Delta^2 + \omega^2. \qquad (29)$$

And so finally

$$\ln \widetilde{y}_3^i \sim N \left(\begin{array}{c} \ln \epsilon_3 + \mu \ln \widetilde{l}_3 + \lambda \left[(\alpha + \beta \lambda) m + \ln \kappa - \frac{\omega^2}{2} + \beta \left[\ln A_2 s_2 + \mu \ln l_2 + \frac{\lambda^2 \Delta^2}{2} \tau_2 (2-\tau_2) \right] \right], \\ [\alpha + \beta \lambda (1-\tau_2)]^2 \lambda^2 \Delta^2 + \lambda^2 \omega^2 \end{array} \right).$$

Using this expression, we now can derive $E[\ln \widetilde{\widetilde{c}}_3^i]$ as

$$\begin{aligned}
E[\ln \widetilde{\widetilde{c}}_3^i] &= \ln \frac{\widetilde{\widetilde{A}}_3}{1+\widetilde{\widetilde{\theta}}} + \ln \epsilon_3 + \mu \ln \widetilde{\widetilde{l}}_3 + \\
&\quad + \lambda \left[(\alpha + \beta \lambda) \widetilde{m} + \ln \kappa - \frac{\omega^2}{2} + \beta \left[\ln \widetilde{A}_2 \widetilde{s}_2 + \mu \ln \widetilde{l}_2 + \frac{\lambda^2 \widetilde{\Delta}^2}{2} \widetilde{\tau}_2 (2-\widetilde{\tau}_2) \right] \right] \\
&\quad + \frac{\lambda^2}{2} \left([\alpha + \beta \lambda (1-\widetilde{\tau}_2)]^2 \Delta^2 + \omega^2 \right) \widetilde{\widetilde{\tau}}_3 (2 - \widetilde{\widetilde{\tau}}_3).
\end{aligned}$$

We also need accumulation equations for the states m and Δ^2. Calculations are similar to those performed for $\ln \widetilde{h}_3^i$, with the only difference: as $\widetilde{m}$ is the mean of human capital distribution for the middle–aged agent at time $t+1$, this agent started life at t with human capital distributed as $N(m + \ln \varphi, \Delta^2)$. Direct comparison with (28) and (29) produces the distribution of $\ln \widetilde{h}_2$ as normal with mean

$$(\alpha + \beta \lambda)(m + \ln \varphi) + \ln \kappa - \frac{\omega^2}{2} + \beta \left[\ln (1+a) s_1 + \ln \epsilon_1 + \mu \ln l_1 + \frac{\lambda^2 \Delta^2}{2} \tau_1 (2-\tau_1) \right]$$

and variance

$$[\alpha + \beta \lambda (1-\tau_2)]^2 \Delta^2 + \omega^2.$$

Finally, average raw incomes of the three generations are easily derived given the distributions of $\ln y_j^i$ given above,

$$
\begin{aligned}
\overline{y}_1 &= \exp\left(\lambda m + \lambda \ln \varphi + \ln \epsilon_1 + \mu \ln l_1 + \frac{\lambda^2 \Delta^2}{2}\right), \\
\overline{y}_2 &= \exp\left(\lambda m + \mu \ln l_1 + \frac{\lambda^2 \Delta^2}{2}\right), \\
\overline{y}_3 &= \exp\left(\begin{array}{c} \lambda\left[(\alpha+\beta\lambda)\,\underline{m} + \ln\kappa - \frac{\omega^2}{2} + \beta\left[\ln \underline{A}_2 \underline{s}_2 + \mu \ln \underline{l}_2 + \frac{\lambda^2 \underline{\Delta}^2}{2}\underline{\tau}_2(2-\underline{\tau}_2)\right]\right] \\ + \ln\epsilon_3 + \mu \ln l_3 + \lambda^2 \frac{[\alpha+\beta\lambda(1-\underline{\tau}_2)]^2 \underline{\Delta}^2 + \omega^2}{2} \end{array}\right).
\end{aligned}
$$

Derivation of the first order conditions in 3-period model

Denoting Lagrange multipliers on (16a)-(16c) Ψ^1, Ψ^2, and Ψ^3, the first order conditions with respect to a, θ, A_2, A_3, and $\widetilde{m}$ are given by

$$a \;:\; 0 = \Psi^1 s_1 \overline{y}_1 - \Psi^2 \frac{\beta}{1+a}, \tag{30a}$$

$$\theta \;:\; 0 = -\frac{1+\frac{\rho}{R}+\frac{\rho^2}{R^2}}{1+\theta} - \frac{\Psi^1}{(1+\theta)^2}\left[(1-s_1)\,\overline{y}_1 + A_2\,(1-s_2)\,\overline{y}_2 + A_3 \overline{y}_3\right], \tag{30b}$$

$$A_2 \;:\; 0 = \frac{\rho}{R}\frac{1}{A_2}(1+\rho\beta\lambda) + \Psi^1\left(1-\theta\frac{1-s_2}{1+\theta}\right)\overline{y}_2 + R\widetilde{\Psi}^1\left[\frac{\widetilde{A}_3}{1+\widetilde{\theta}} - 1\right]\widetilde{\overline{y}}_3 \frac{\beta\lambda}{A_2}, \tag{30c}$$

$$A_3 \;:\; 0 = \frac{\rho^2}{R^2}\frac{1}{A_3} + \frac{\Psi^1}{1+\theta}\overline{y}_3, \tag{30d}$$

$$
\begin{aligned}
\widetilde{m} \;:\; 0 = {} & R\lambda\widetilde{\Psi}^1\left(\left[\widetilde{a}\widetilde{s} - \widetilde{\theta}\frac{1-\widetilde{s}_1}{1+\widetilde{\theta}}\right]\widetilde{\overline{y}}_1 + \left[\widetilde{A}_2\left(1-\widetilde{\theta}\frac{1-\widetilde{s}_2}{1+\widetilde{\theta}}\right) - 1\right]\widetilde{\overline{y}}_2\right) + \\
& + (\alpha+\beta\lambda)\,R^2 \widetilde{\widetilde{\Psi}}^1 \left(\frac{\widetilde{\widetilde{A}}_3}{1+\widetilde{\widetilde{\theta}}} - 1\right)\widetilde{\widetilde{\overline{y}}}_3 + \Psi^2 - (\alpha+\beta\lambda)\,R\widetilde{\Psi}^2 + \rho\lambda + \rho^2(\alpha+\beta\lambda) + R\lambda.
\end{aligned} \tag{30e}
$$

The system of equations above could be simplified by denoting

$$z = \frac{A_3}{1+\theta},$$

and plugging (30d) and (30a) into (30b), (30 c), and (30e) to get

$$0 = \rho + \rho^2\beta\lambda\frac{1}{\widetilde{z}} + R\Psi^1 A_2\left(1-\theta\frac{1-s_2}{1+\theta}\right)\overline{y}_2, \tag{31a}$$

$$0 = \Psi^1\left[(1-s_1)\,\overline{y}_1 + A_2\,(1-s_2)\,\overline{y}_2\right] + 1 + \frac{\rho}{R}, \tag{31b}$$

$$
\begin{aligned}
0 = {} & (\rho+R)\,\lambda + \frac{\rho^2}{R}\lambda\left(1-\frac{1}{\widetilde{z}}\right) + \rho^2(\alpha+\beta\lambda)\frac{1}{\widetilde{\widetilde{z}}} + \\
& \Psi^1\frac{1+a}{\beta}s_1\overline{y}_1 - R\,(\alpha+\beta\lambda)\,\widetilde{\Psi}^1\frac{1+\widetilde{a}}{\beta}\widetilde{s}_1\widetilde{\overline{y}}_1.
\end{aligned} \tag{31c}
$$

Next, use (16a) and (30d) to express Ψ^1 :

$$\Psi^1 = -\frac{\rho^2}{R^2}\frac{1-z}{z}\frac{1}{\left[as-\theta\frac{1-s_1}{1+\theta}\right]\overline{y}_1+\left[A_2\left(1-\theta\frac{1-s_2}{1+\theta}\right)-1\right]\overline{y}_2}$$

and plug it into (31a) to get

$$1+\rho\beta\lambda\frac{1}{\tilde{\tilde{z}}}-\frac{\rho}{R}\frac{1-z}{z}\frac{A_2\left(1-\theta\frac{1-s_2}{1+\theta}\right)\overline{y}_2}{\left[as-\theta\frac{1-s_1}{1+\theta}\right]\overline{y}_1+\left[A_2\left(1-\theta\frac{1-s_2}{1+\theta}\right)-1\right]\overline{y}_2}=0.$$

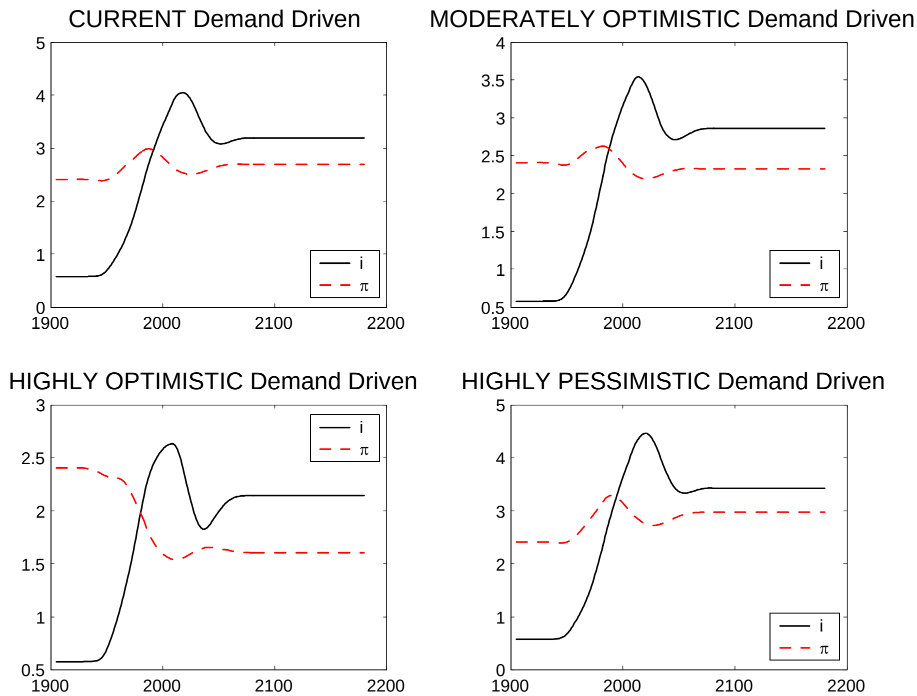


Fig. 1. Behavior of implicit interest rates i and π for the case of demand driven budgeting under different demographic scenarios.

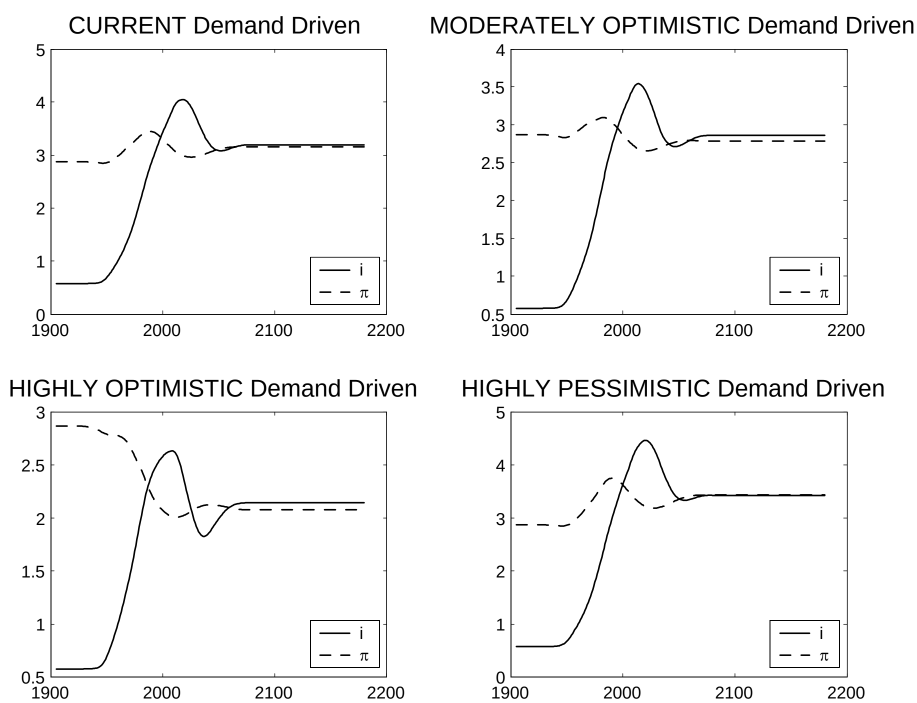


Fig. 2. Behavior of implicit interest rates i and π: demand driven budgeting and unbalanced pensions increase by 16.6%.

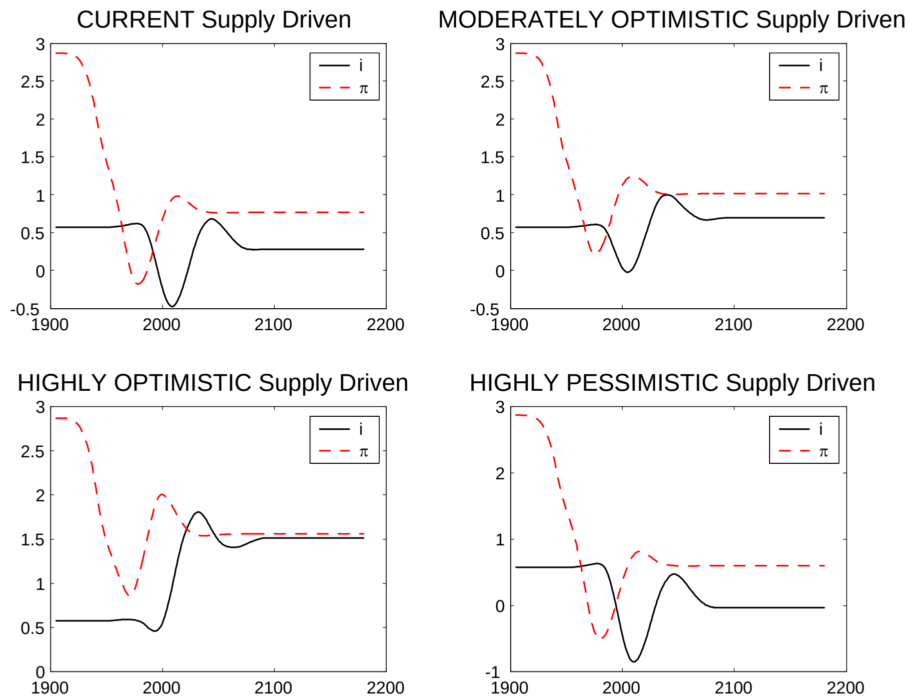


Fig. 3. Behavior of implicit interest rates i and π: supply driven budgeting and unbalanced pensions increase by 16.6%.

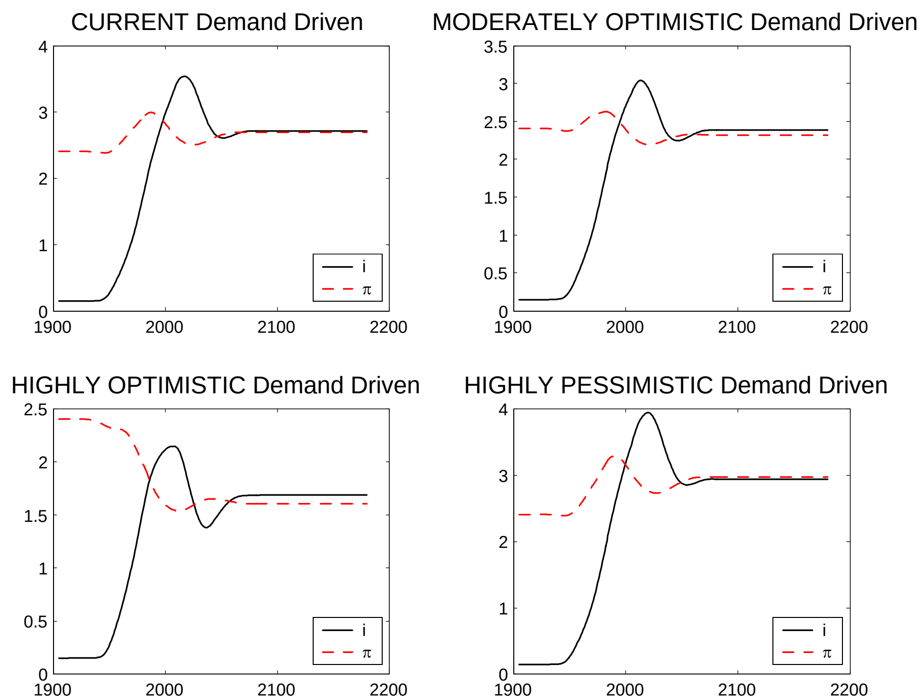
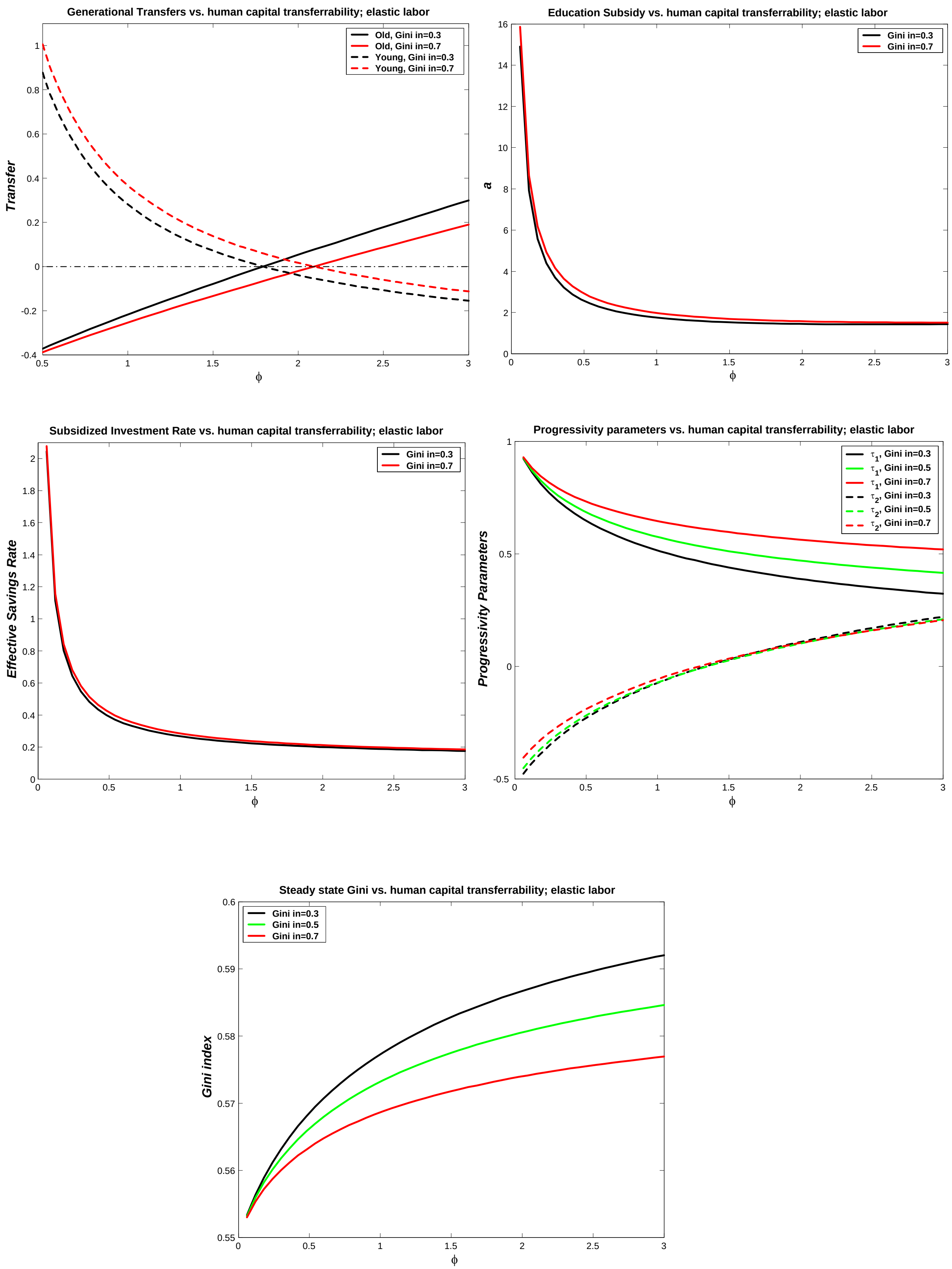


Fig. 4. Behavior of implicit interest rates i and π: demand driven budgeting and unbalanced educational expenditures increase by 12.7%.



Figs. 5-9. 2-period model, elastic labor supply.

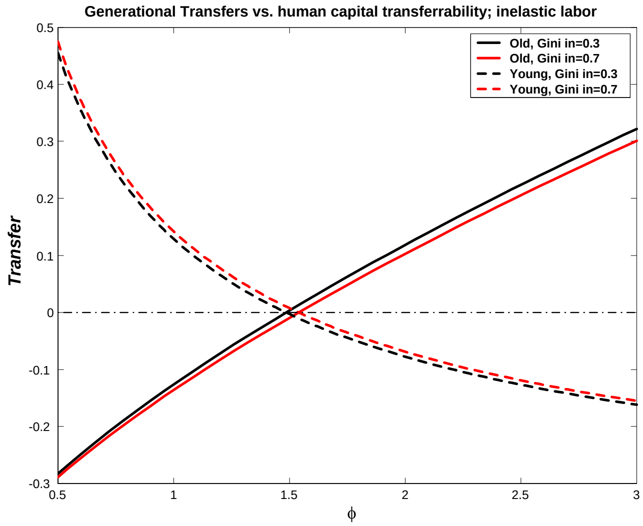


Fig. 10

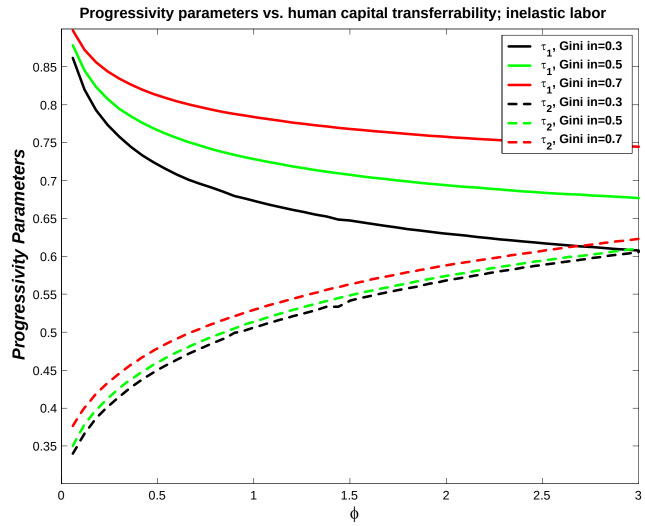


Fig. 11

2-period model, inelastic labor supply.

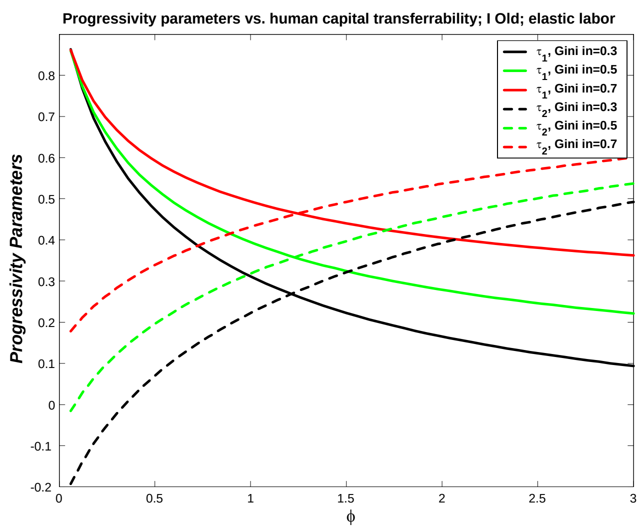
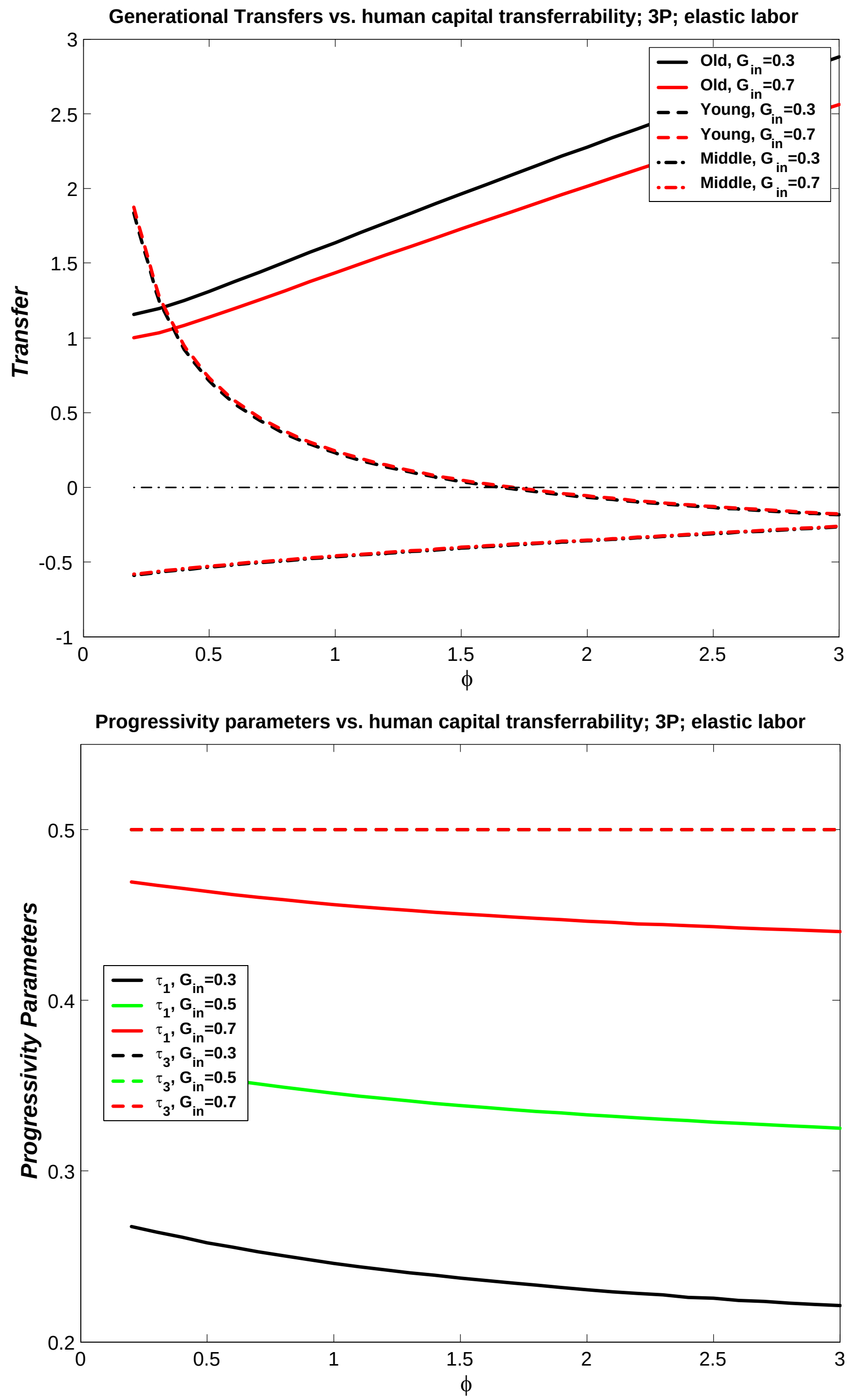


Fig. 12. 2-period model, initial old, elastic labor supply



Figs. 13-14. 3 period model, elastic labor supply.