Information, Liquidity and Contagion

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September 2001

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** Working paper. Comments are welcome. We are grateful to Laurent Calvet, János Vincze and other participants at the Workshop on Liquidity of Financial Markets at Central European University, September 3, 2001 for helpful comments and suggestions. Any remaining errors are ours. This research is supported by a grant from CERGE–EI Foundation under a program of the Global Development Network. The views expressed herein are ours and have not been endorsed by CERGE–EI or the GDN.

Abstract

The purpose of this paper is to develop a simple model of liquidity based financial crisis. In our model liquidity based (i.e., non-fundamental) financial crises occur as part of the equilibrium strategy profile of imperfectly informed investors. We show that in crisis a bailout by non-participating investors need not occur. Additionally, by means of a numerical example we demonstrate that even very good quality assets may be hit by a liquidity based crisis and not bailed out. The reason is that good quality refers to a high prior probability of no default; but conditional on a crisis takiing place the posterior probability of no default can be much lower.

Further research intends to extend this model to incorporate liquidity based contagion.
Keywords: *Liquidity, Financial Contagion, Asymmetric Information*

1. Introduction

The purpose of this paper is to develop a simple model of financial markets where both fundamental and liquidity risk are present. The model enables us to explicitely study the interaction of these two risks and their effects on financial crises and vulnerability. A key underlying idea is informational asymmetry. Simply put, asset price aggregates not only fundamental infromation but also the aggregate liquidity shock, and therefore uninformed investors face a non-trivial signal extraction problem. We prove the existence of a rational expectations equilibrium under which liquidity based (non-fundamental) financial crises can occur in equilibrium. A liquidity based crisis, however, is not a sunspot; it constitutes part of the equilibrium strategy profile of market participants.

The motivation for this paper is extensive anecdotal evidence that financial contagion is at least partially caused by liquidity based financial mechanisms. It is a widespread view in policy circles in Central and Eastern Europe that the contagious effects of the Russian crisis in 1998 in the region can be partially attributed to such a mechanism. The story is roughly the following. During the crisis international investors realized significant losses on their Russian assets. Additionally these investors were subject to some domestic liquidity needs. These liquidity needs may have been endogenouos (value at risk, collateral etc.); in any case they had to withdraw money in some form. Since withdrawing money from Russia was not a viable option any more, this might have led to extensive sales in neighboring emerging markets. Consequently, the sales pressure was responsible for bearish markets and interest rate hikes in the rest of the region.

In our view there are two major problems with this story. The first is that it is not clear why international investors, when they experienced the combination of a liquidity shock at home and a price fall in Russia, chose to withdraw from these particular neighboring markets. The second problem is that if contagion in neighboring countries was not fundamentally justified, why did not some other investors enter these markets and invest at the favorable (depressed) prices.

Ultimately our purpose in this paper is to build a rigorous formal model that incorporates elements of the above story. This model will enable us to see more clearly the relevance of the aforementioned two problems. Currently, the model is capable of giving an answer to the second problem, that is, why in case of a liquidity crisis a bailout by other investors need not occur. On the other hand, in order to understand better the choice of which market to withdraw from, one needs to develop a multi-market model. Additionally, a model with several markets would be desirable in that it allows for the study of the exact dynamics of contagion. This is the subject of future research.

The fundamental ucertainty in the model is formalized as default risk: the risky asset, a fixed income instrument, has some prior probability of not delivering the promised payment. Liquidity risk is introduced in the way first proposed by Diamond and Dybvig (1983); ex ante investors do not know whether they need to consume in the first period or in the second period. The model is set up such that liquidity is uncertain in the aggregate. Similar framworks have been applied throughout the literature. Allen and Gale (1994) set up a model with aggregate uncertainty in liquidity, and study endogenous market participation decisions. Depending on parameters they show the exisence of a limited participation equilibrium, where relatively risk averse investors do not enter the market in the initial period, and therefore the price volatility is substantial in the interim period. However this interim period volatility is solely due to the aggregate liquidity shock, and lack of sufficient funds on the part of participating investors. Thus it is not clear why in the interim period non-participating investors do not enter and bail out the market. In a related paper Allen and Gale (2000) study liquidity based financial contagion. Again, there is no fundamental uncertainty and therefore contagion is due to lack of sufficient funds to finance liquidity needs. In that paper the non-participation decision of possible deep pocket investors is not formalized.

There are a number of papers that do study the interaction of liquidity based and fundamental uncertainty. Notably, there is an extensive literature on financial markets and market microstructure (for instance Kyle, 1985) where asset price imperfectly aggregates fundamental information due to liquidity (noise) traders. In these models the liquidity demand is not formalized, in contrast to the present approach. Additionally, these papers do not focus on financial crises and contagion. More related is Genotte and Leland (1990), who try to explain the 1987 stock market crash with unobserved supply shocks. Their focus is not on contagion, however. Chari and Jagannathan (1988) develop a similar model with liquidity shocks and aggregate uncertainty to study endogenous bank runs. Finally, Allen and Gale (1998) also present a model with aggregate uncertainty, but their focus is on optimal risk sharing and optimal financial crises.

It is important to stress that the present paper is preliminary. Our current research is in the direction of extending the framework of this paper to a multi-market setting.

2. The Model

2.1. SETUP

There are three time periods, 0, 1 and 2, and a continuum of identical investors of mass 1 indexed by the unit interval. Agents have preferences of the Diamond-Dybvig (1983) type. That is, they may be either early or late consumers. Early consumers only care about period 1 consumption, and late consumers only care about period 2 consumption. Each agent learns her type at the beginning of period 1. The per period utility function is linear, so that agents are risk neutral. Formally, an individual's utility function is given by

$$U(c_1, c_2) = \begin{cases} c_1 & \text{if early consumer} \\ c_2 & \text{if late consumer} \end{cases}$$

where c_i denotes consumption in period i. Each agent has some probability of becoming an early consumer. In the present model we assume that λ, the proportion of early consumers in the population, is uncertain in the aggregate. Formally, λ is drawn from a distribution $F(\lambda)$, and then independently each agent has probability λ of becoming an early consumer. The realization of λ is not observed. Thus ex ante each agent has a probability $\pi = E\lambda$ of becoming an early consumer.

Agents can trade two assets. They have access to a riskless asset (cash) with a completely inelastic supply, which has price equal to 1 unit of the consumption good each period. However, short sale (borrowing) is not allowed. The other asset in this economy is a risky one (bond). The payoff of the risky asset is as follows. There is a prior probability θ of no default, in which case the asset pays 1 unit of the consumption good in the second period. However, with probability $1 - \theta$ default occurs, and then the asset is worthless. The default distribution is independent of the distribution of λ. Whether the asset defaults or not is publicly revealed only in the second period.

The risky asset is issued in period 0, when each agent makes her investment (portfolio) decision. The quantity of issued bond (gross supply), denoted by b, is exogenously given. In period 1 the markets for both assets open, agents observe the price of the bond and trade. The price of the bond in period 1 is thus endogenously determined. In the final period it becomes public whether the bond defaulted or not, and payoffs are received.

The information structure of the model is the following. In the initial period agents only know the common prior distribution of the liquidity shock and the default event. In period 1, agents learn their type (early or late). Additionally, a mass of μ late consumers receive a signal about the quality of the bond. Here μ is an exogenous non-random parameter of the model. We assume that the signal is exactly correct, so that the μ informed agents will know whether the bond defaults or not. Who becomes an informed agent is uniformly distributed among late consumers and not known in advance. Finally, each agent observes the prevailing price of the bond in period 1. The market is anonymous, thus it cannot be observed whether informed agents are trading or not; only the price is publicly known.

We also assume that there is an unmodeled market actor, who is willing to buy any amount of the bond at some exogenous (low) price s in period 1. This actor can be thought of as the issuer of the bond; if the price is low enough, it may make sense for him to buy back the debt. This can be true regardless of whether default is going to occur or not, if one thinks that debt default is more costly (for instance because of reputational considerations) than buyback at a low price. Alternatively, we can interpret default risk as a parable of exchange rate (depreciation) risk, in which case the unmodeled market actor can be thought of as domestic investors who are exempt from that risk. For the sake of concreteness in the sequel we shall refer to this market actor as the issuer of the bond.

To sum it up, the timing of the model is the following. In the initial period agents observe the issuer's price (initial period price) of the bond and make their portfolio decision. We assume that the issuer's price is such that buying bond weakly dominates holding cash. Since agents are risk neutral, this implies that either each agent is indifferent between buying bond and not buying bond, or agents would hold all of their wealth in bonds. In the latter case the market for the bond does not clear, and instead we rely on a rationing mechanism that gives an equal amount of the bond to each agent. If agents are indifferent between holding bond and holding cash we assume that their portfolio decisions are identical. Thus in any case each agent will be holding b bonds (as the gross supply is b and there is a unit mass of investors). The amount of cash each agent holds at the and of period 0 is denoted by m. Thus if w is the initial wealth of an agent and the price of the bond in period 0 is p_0 then we have

$$w = m + p_0 b.$$

In period 1 each agent learns her type, and additionally μ late consumers learn the quality of the bond. We assume that $\lambda < 1 - \mu$ almost surely, so that there are always at least μ late consumers. Then the market for the bond opens, agents observe the price and trade. Naturally, all early consumers sell all of their bond holdings and consume the proceeds. If default is going to occur, informed late consumers sell all of their bond holdings. On the other hand, if the asset is good, informed late consumers invest all of their cash balances into bonds as long as p_1, the price of the bond in period 1, is strictly less than one. If the bond is good and $p_1 = 1$, informed late consumers are indifferent between holding cash and holding bond and their portfolio decisions will be determined by market clearing.

In this situation uninformed late consumers face a non-trivial signal extraction problem. They observe the price p_1, and from this they have to infer the quality of the bond. The price is a noisy signal because it aggregates not only the information inherent in the trade of informed late investors, but also the liquidity shock on the market. Simply put, uninformed late consumers, when they observe a low price, have to decide whether the price is low because the asset is likely to default or else because there was a huge liquidity shock (high λ). Note that the signal extraction of problem of these investors is only meaningful in a rational expectations equilibrium, where the price of the bond reflects an aggregation of both types of noises because of the actions (trades) agents make, and agents' actions are at least partially guided by the information contained in the price.

It is important to understand how a liquidity shock may depress the price of the bond. In effect, the price falls because there is not enough cash in the market, as in Allen and Gale (1994). Even if everybody knew that the asset was good, arbitrage cannot take place as going short in cash is not allowed. As long as there is shortage in cash, the price is determined by market clearing. But the price of the bond cannot be higher than 1, as in that case cash would be dominating it and nobody would wish to hold bonds.

The market for the bond in period 1 can clear in two fundamentally different ways. The first is when $p_1 > s$, so that the issuer does not buy back bonds. The second is when $p_1 = s$. In that case the issuer is willing to buy any amount of the bond, thus the price cannot fall below s. In the sequel we shall refer to the situation when $p_1 = s$ as a crisis.

Finally, in period 2 it becomes publicly known whether the bond is in default or not, payoffs are received and late consumers consume all their wealth.

2.2. EQUILIBRIUM

Finding an equilibrium in this model constitutes of specifying agents' strategy profiles and a rational expectations price function that determines p_1 as a function of the primitives of the model, notably λ and a binary variable D that indicates whether the bond is in default. We have seen that the strategy of early consumers and late informed consumers is unambiguously pinned down as long as either $p_1 < 1$ or the bond is in default. If $p_1 = 1$ and the bond is good, we assume that informed late consumers adjust their portfolio holdings so that the market clears. Thus we only need to specify the strategies of uninformed late consumers. Before doing so, we make a number of assumptions that simplify the problem.

First of all we assume that the distribution of λ, $F(\lambda)$ is absolutely continuous with respect to the Lebesgue measure, and has a compact support which is a subset of the unit interval. The density function is denoted by $f(\lambda)$, and is assumed to be continuously differentiable. Let $\lambda_{\min}$ and $\lambda_{\max}$ denote the lower and upper bounds of the support of the distribution. We assume that

$$\lambda_{\max} \leq 1 - \mu \tag{1}$$

so that there are always at least μ late consumers. Additionally, let

$$\lambda_{\min} > \frac{m}{m+b} - \mu. \tag{2}$$

This is equivalent to saying that $(1 - \lambda - \mu)m < (\lambda + \mu)b$ almost surely. This means that if all informed and all early consumers are selling their bond holdings (which equals $(\lambda + \mu)b$) then the price has to be lower than 1 (as there is only $(1 - \lambda - \mu)m$ additional cash in the market).

We also make the assumption that

$$\frac{\mu m}{(1-\mu)b} \leq s \tag{3}$$

so that if only the informed investors are buying bonds and everybody else is selling then the price has to fall down to s. This implies that informed investors alone cannot save the market from a crisis.

We need two additional assumptions, the first of which is a 'single crossing' property. As this assumption is somewhat complicated, we render it to Appendix A. The second additional assumption (the 'crisis equilibrium' property) ensures that if a crisis occurs in period 1 (so that $p_1 = s$) then uninformed agents find it in their interest to sell all their bonds, so that crisis indeed is an equilibrium outcome. This assumption is also technical, additionally it is related to the single crossing property, therefore we state it explicitly in Appendix A.

Having stated our assumptions we can formulate the following proposition.

Proposition 1. *Under assumptions (1-3), the single crossing and the crisis equilibrium properties, there exist numbers $\bar{\lambda}$ and $\bar{p}$ ($> s$) and a monotonically decreasing function $p_1(.)$ such that model has the following rational expectations equilibrium. If the asset is good, then as long as $\lambda \leq \bar{\lambda}$ the market price is $p_1 = p_1(\lambda) \geq \bar{p}$ so that there is no crisis, and all uninformed late consumers wish to buy bonds. However, if $\lambda > \bar{\lambda}$ then the market price is $p_1 = s$ and all uninformed investors sell their bonds.*

If the asset is in default, then as long as $\lambda \leq \bar{\lambda} - \mu$ the market price is $p_1 = p_1(\lambda + \mu) \geq \bar{p}$, there is no crisis in period 1 and all uninformed late investors wish to buy bonds. However, if $\lambda > \bar{\lambda} - \mu$ then the market price is $p_1 = s$ and all uninformed investors sell their bonds.

The complete proof can be found in Appendix A. However the intuition of the result is clear. Suppose that the bond is good. Then as long as the liquidity shock is not too high ($\lambda \leq \bar{\lambda}$) the market accommodates the shock, uninformed late consumers are buying bonds, and there is no crisis. This is because for $\lambda \leq \bar{\lambda}$ the expectation of the bond's terminal payoff conditional on the price $p_1(\lambda)$ is higher than the price itself, thus uninformed late consumers find it in their interest to buy. Nevertheless, the higher the liquidity shock, the lower the price ($p_1(.)$ is monotonically decreasing) because of higher selling pressure. On the other hand, if the liquidity shock is too high ($\lambda > \bar{\lambda}$), then uninformed investors believe that the bond is more likely to be in default, and therefore sell. This pushes down the price to s and a crisis takes place. Thus in equilibrium we can have a non-fundamental (liquidity) crisis.

The situation is similar if the bond is in default. However in that case a lower liquidity shock suffices to push the market into crisis ($\bar{\lambda}-\mu$ is the treshold). Thus if the bond is bad a relatively low liquidity shock will trigger the crisis, which in that case is fundamentally justified. But it is also possible that with a very low liquidity shock the market does not hit a crisis in period 1.

It follows that the model is capable of explaining non-fundamental, liquidity based crises. In particular, it has the potential of generating contagious effects, where a crisis in one market brings liquidity shortage, and this shortage spreads to a neighboring market and pushes that into crisis too, as investors in that market cannot perfectly identify liquidity shocks from fundamental weakness. Of course, to see this effect more clearly one needs to develop a multi-market extension of the present framework.

The model is also strongly related to the issue of limited market participation. There are two sides to that problem. Firstly, the model explains why an international bailout (that would be delivered by unmodeled investors) need not take place if the market is in a crisis, even if it is a non-fundamental one. This is because if the price is s, equilibrium implies that all uninformed market participants believe that it is worth to sell the bond. Thus non-participating international investors, who are unlikely to be more informed than participating investors, would not wish to invest. Although this remark may seem trivial, it is important to stress that it is a consequence of the combination of liquidity-based and fundamental uncertainty. Allen and Gale (1994) develop a related model with limited market participation. In their model there is no fundamental uncertainty, and market participation decisions have to be made in the initial period. In the middle period, when the liquidity shock occurs, non-participating late investors are not allowed to enter and buy bonds at the depressed price. It is hard to see any rationale why they cannot do so. Indeed, as there is no fundamental uncertainty, non-participating investors know the low price is a consequence of liquidity shortage. Therefore entering the market would be a free lunch, at least as long as the price is low enough so that the gains in realized return cover the entry cost.

As outlined above, the present model provides a rationale for why non-participating investors might wish not to enter the market when it is in crisis. On the other hand, it does not explain why they do not enter when the price is relatively low, but the market is not in crisis. In that case uninformed investors believe that it is worth to buy bonds; so it would seem reasonable to assume that non-participating investors might want to enter too.

To analyze this issue of providing additional funds to a market which suffers from a liquidity shock it seems natural to model the other markets that these funds are to be withdrawn from. Thus the issue of limited market participation also points in the direction of extending the present setting into a multi-market framework.

One drawback of the present model and the equilibrium of the Proposition is the possibility of multiple equilibria. In fact, whatever the liquidity shock may be, if all uninformed late consumers decide to sell their bond holdings, formula

(3) implies that the price falls to s, and thus their equilibrium strategy justifies the sale. Thus a crisis equilibrium is always possible. What the Proposition does tell us is that as long as λ is not too high there exists an other equilibrium, and if λ is high enough there cannot be any other equilibrium. So far we implicitly assumed that whenever both a no-crisis and a crisis equilibrium is possible, the no-crisis equilibrium is selected.

In our view the issue of multiple equilibria arises because of the simplicity of the model. One particularly straightforward way of curing this problem is to assume the existence of a Walrasian auctioneer, who calls prices in a monotonically decreasing fashion starting from $p_1 = 1$. For each price agents make their offers and the process stops exactly when the market clears. This procedure would select the same equilibrium we implicitly assumed. Although the procedure might seem artificial, we think it captures one important characteristics of securities market prices, namely that they are dynamic.

A different and better-founded approach to the issue of multiple equilibria in macroeconomics is that pursued by Morris and Shin (2000). They argue that by introducing some small private information to a model with sunspot equilibria, the Bayesian equilibrium of the resulting game of incomplete information is unique. We believe that their approach could be pursued in the present context as well, however it would not add to the intuition of the model therefore we refrain from doing it.

2.3. A NUMERICAL EXAMPLE

Here we develop a simple numerical example that illustrates the intuition of the model. The baseline parameters are as follows: $b = 1$, $m = 0.5$, $\mu = 0.2$ and $s = 0.125$. The density function of λ is

$$f(\lambda) = \left\{ \begin{array}{c} \gamma \exp\left\{-\frac{1}{1/9-(\lambda-7/15)^2}\right\} \\ 0 \end{array} \right.$$

where γ is chosen such that the integral of f is equal to 1. This density is plotted in Figure 1.

[Figure 1 about here]

With these parameter and distributional choices the assumptions necessary for Proposition 1 are satisfied for any θ between 0 and 1. For $\theta = 0.8$ the conditional expectation of the bond's terminal payoff given the price p_1 is plotted in Figure 2.

[Figure 2 about here]

The relevant part of the figure is where the conditional expectation curve is above the 45 degree line. In this region the conditional expectation of the terminal payoff relevant for uninformed late consumers is higher than the price of the bond. Thus uninformed late consumers are buying. The intersection of the two curves determines $\bar{p}$. This intersection is unique as the single crossing property holds. For $p < \bar{p}$ the conditional expectation would be lower than the actual price, so all uninformed late investors would be selling. Due to the setup of the model under this scenario the price would fall to s. Thus prices in the range of the interval $(s, \bar{p})$ never occur in equilibrium.

Next we examine how the equilibrium changes with θ, the quality of the bond. As Figure 3 demonstrates, $\bar{p}$ decreases in θ (the curve in the top of the figure). The intuition is clear: the better the asset, the higher price shock agents are willing to tolerate without selling the asset. This is because they attach a higher probability to the price decrease being due to liquidity reasons. Of related interest is how the quality of the bond influences the crisis outcome. In fact, no matter how good the asset, in this example the conditional expectation of the terminal payoff given that the price is s is less than s. That is, the crisis outcome does not disappear. The rationale for this is that the better the asset, the higher liquidity shock agents are willing to tolerate. Thus if the price does fall to s agents have to believe that either the liquidity shock is very high or the asset is indeed bad. The present example is such that the latter effect dominates; the posterior probability of default given that the price is s is always high enough to force agents to sell. This is also illustrated in Figure 3: the conditional expectation of the terminal payoff given that the price of the bond is s is plotted (the curve in the bottom of the figure). This curve is always below s, hence in crisis all uninformed late investors are selling. Interestingly, this conditional expectation is lowest for very good and very bad quality assets. Intuitively, if asset quality is poor, then agents tolerate only very small liquidity shocks, and hence the price practically always falls to s, therefore the posterior probability of having a good asset is also low. On the other hand, for a very good quality asset agents tolerate basically any liquidity shock, and thus if there is a crisis, they attach a very high posterior probability of its being caused by fundamental reasons.

[Figure 3 about here]

3. Conclusion

This paper developed a simple model of liquidity based financial crisis. Three main conclusions emerge from this model. First, it demonstrates that liquidity based (i.e., non-fundamental) financial crises may occur as part of the equilibrium strategy profile of imperfectly informed investors. This equilibrium is sustained by the sales pressure of uninformed investors. Presumably, non-participating outside investors are similarly uninformed, so would accordingly continue to stay out of the market should a crisis struck. This brings us to our second conclusion, namely that in case of a crisis a bailout need not occur. Thirdly, a numerical example showed that even very good quality assets may be hit by a liquidity based crisis and not bailed out. The reason was that good quality referred to a high prior probability of no default, whereas the posterior probability of no default conditional on a crisis taking place was much lower.

An unpleasant feature of the model is the presence of multiple equilibria.

Further research intends to extend the present framework to more than one market, so that it be capable of studying contagion.

A Appendix

In this appendix we formally state the single crossing and crisis equilibrium conditions mentioned in the text, and additionally we prove Proposition 1.

To do so, define the function g in the following way

$$g(p) = \frac{f(\frac{m}{m+pb})\theta}{f(\frac{m}{m+pb} - \mu)(1-\theta) + f(\frac{m}{m+pb})\theta}. \tag{4}$$

We are going to show that for a range of its domain, $g(p)$ is the conditional expectation of the bond's terminal payoff given that the price of the bond is p. Suppose that the equilibrium outlined in Proposition 1 exists, and further that $1 > p > s$. In that case we claim that the price p can occur under two possible scenarios. The first is when the bond is good, so informed agents receive a good signal, and thus are willing to spend all their cash on buying more bonds. Uninformed agents are also buying (as we are in a non-crisis situation) and thus only early consumers are selling. Therefore the market price has to be determined by the following market clearing condition

$$p = \frac{(1-\lambda)m}{\lambda b} \tag{5}$$

or equivalently

$$\lambda = \frac{m}{m + pb}. \tag{6}$$

The alternative scenario is when the bond is in default, and informed agents are selling it. Early consumers are also selling for obvious reasons, and therefore only uninformed late consumers are buying (as we are not in a crisis equilibrium). In that case the price of the bond is given by

$$p = \frac{(1 - \lambda - \mu)m}{(\lambda + \mu)b}. \tag{7}$$

Reorganizing implies that

$$\lambda = \frac{m}{m + pb} - \mu. \tag{8}$$

There cannot be any other scenario where the price is p. Indeed, assuming that uninformed late investors always act identically, the two other possibilities would be that either only informed investors are buying, or everybody is selling. However condition (3) guarantees that in either case the market price is s, and we assumed $p > s$ above. Thus uninformed agents face a signal extraction problem here: they have to determine whether the price is p because of the first scenario or because of the second scenario. Now conditions (6) and (8) coupled with Bayes' rule imply that the conditional expectation of the bond's terminal payoff given p is indeed determined by formula (4) above, that is $E\left[p_2 \,|p_1 = p\right] = g(p)$ for a range of values p. We are going to specify this range below.

Note that by assumptions (1)-(3) we have $g(0) = 0$ and $g(1) = 1$. We are in the position to state the single crossing property that we need in order for the equilibrium of Proposition 1 to exist. This property essentially requires that the two curves in Figure 2 are located in the way they are actually drawn. Formally, what we need is that the equation $g(p) - p = 0$ has a unique solution $\bar{p}$ in the open interval $(0, 1)$, at $\bar{p}$ the derivative $g'(\bar{p}) > 1$ and additionally that $\bar{p} > s$.

If this condition holds, then for $p > \bar{p}$ it has to be that $g(p) > p$, and therefore the conditional expectation of the bond's terminal payoff given p is higher than p, thus it is worth it for uninformed late consumers to buy bonds. Similarly, for $\bar{p} > p > s$ the conditional expectation of the bonds terminal payoff is lower than its actual price. Thus in that case all uninformed late consumers should be selling their bond holdings, and hence such a price can not occur in equilibrium.

Define

$$p_1(\lambda) = \frac{(1 - \lambda)m}{\lambda b}$$

a montonically decreasing function, and

$$\bar{\lambda} = \frac{m}{m + pb}$$

so that $p_1(\bar{\lambda}) = \bar{p}$. If the asset is good, formula (5) implies that the market price p_1 is given by $p_1(\lambda)$ as long as the price is not lower then $\bar{p}$ or equivalently as long as $\lambda < \bar{\lambda}$. Similalry, if the asset is in default then by formula (7) the market price is given by $p_1(\lambda + \mu)$ as long as $\lambda + \mu < \bar{\lambda}$. This is part of the statement of the Proposition.

The above discussion implies that for $p < \bar{p}$ the price should actually fall to s. We need to check if this is indeed an equilibrium. In other words, we need that for $p_1 = s$ it is worth for uninformed late consumers to sell. Note that because of the above discussion in equilibrium the price of the bond can be s under two possible scenarios. The first is when the asset is good, so informed investors are buying, but the liquidity shock λ is higher than $\bar{\lambda}$ and thus everybody else is selling. The second scenario is when the bond is in default and the liquidity shock is higher than $\bar{\lambda} - \mu$. In that case everbody is selling. The probability of the first scenario is given by $\theta(1 - F(\bar{\lambda}))$, that of the second scenario is $(1-\theta)(1 - F(\bar{\lambda} - \mu))$. By Bayes' rule the conditional expectation of the bond's terminal payoff given that the price is s is

$$\frac{\theta(1 - F(\bar{\lambda}))}{\theta(1 - F(\bar{\lambda})) + (1-\theta)(1 - F(\bar{\lambda} - \mu))}.$$

In order for $p_1 = s$ to be an equilibrium we need that this latter term is less than s. Thus our crisis equilibrium property necessary for Proposition 1 to hold can be formulated as

$$s > \frac{\theta(1 - F(\bar{\lambda}))}{\theta(1 - F(\bar{\lambda})) + (1-\theta)(1 - F(\bar{\lambda} - \mu))}. \tag{9}$$

If this condition does hold then upon observing $p_1 = s$ uninformed late consumers find it optimal to sell the bond as the conditional expectation of the terminal payoff is less than the actual price. This completes the proof of the Proposition.

Finally note that as the numerical example in the text illustrates all assumed conditions can hold at the same time so the model is meaningful.

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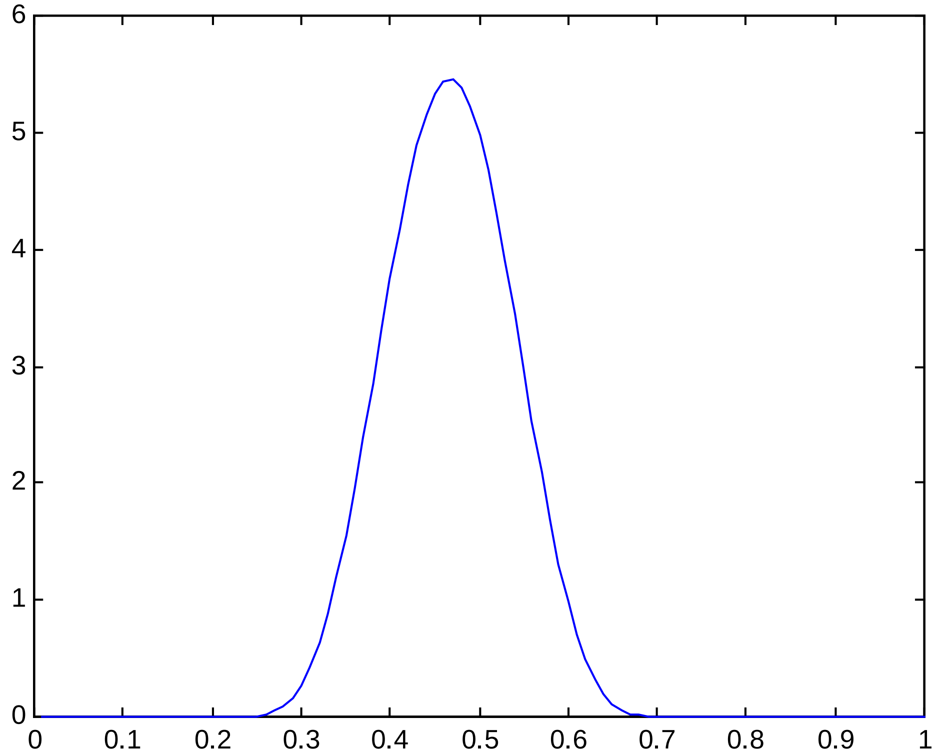

Figure 1: Density $f(\lambda)$

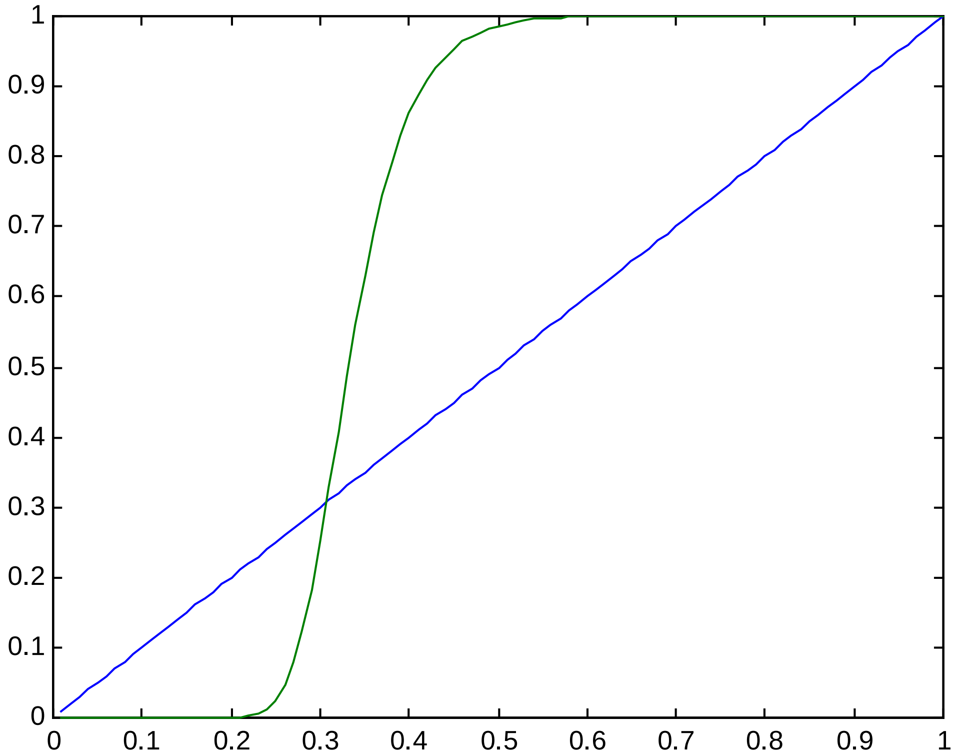

Figure 2: Conditional expectation of terminal payoff given price p_2

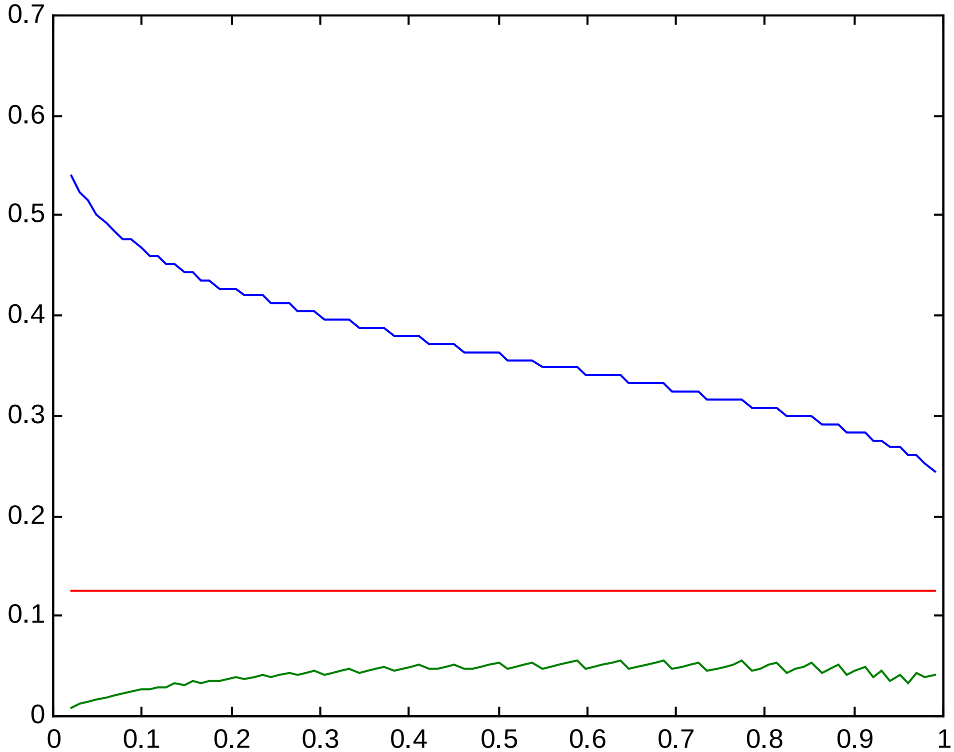

Figure 3: Plot of *p'*, *s* and the conditional expectation of terminal payoff given that the price is *s*