

Mispricing and Lasting Arbitrage between Parallel Markets in the Czech Republic

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July, 1998

Abstract:

If co-existing parallel markets are efficient, then arbitrage will maintain a correct pricing relationship. A related question is whether two parallel emerging markets offering more or less the same securities but using different institutional designs, can behave as a single, fully integrated market. In this paper we introduce an explicit model of price convergence (with transaction costs), in which price differences are studied using levels of arbitrage activity. For the empirical analysis we use two parallel markets in the Czech Republic—the Prague Stock Exchange (PSE) and the RMS (over-the-counter system). In particular, we study the degree of arbitrage activity for different segments of the PSE and the evolution of arbitrage in the early history of these emerging markets. The empirical results provide evidence of market linkage for actively traded stocks. We find a significant relationship between the segment of the market to which a given firm belongs and the estimated level of arbitrage trading. Moreover, the level of arbitrage activity increases over time for all market segments, and, as the markets mature, the differences among the segments gradually disappear.

Keywords: arbitrage, co-movements of financial markets, emerging markets,
integration of emerging markets, mispricing

JEL Classification: D40, E44, G12, G14

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1. Motivation

One of the basic principles underlying modern financial theory is the law of one price or the no-arbitrage condition: If the payoff scheme of a given security may be replicated by a portfolio of other assets in a frictionless market, the price of that portfolio should equal the price of the security. The failure to fulfill this condition indicates inefficiency of the market and non-optimal behavior of the agents.

This condition has been extensively tested in the empirical literature. The setup of these studies, however, is usually far from the theoretical assumption of no market frictions under which the no-arbitrage condition is derived. There are usually transaction costs involved in any trading, including arbitrage trading. Also, it is often difficult to find a *riskless* arbitrage portfolio. The majority of existing studies have therefore concentrated on investigating the pricing relationship between spot and futures markets. Given the stochastic nature of security prices, any changes in the prices of a given future contract and the underlying security must be closely linked if the markets efficiently process all the available information. The majority of the empirical studies¹ apply Engle and Granger (1987) cointegration analysis to examine the causal links between the spot and futures security or financial markets. Such studies usually use intra-day transaction data, do not account for transaction costs explicitly, and focus primarily on the lead-lag relationships between the two markets.

The present study circumvents some of the limitations of the previous research and provides a more direct test for inter-market arbitrage behavior, explicitly taking into account the transaction costs of arbitrage trading. Since the payoffs to the underlying security and future contract take place at different points in time, studies examining

¹ See, e.g., Arshanapalli and Doukas (1994), Hung and Zhang (1995), Shyy, Vijayraghavan and Scott-Quinn (1996) or Wang and Yau (1994).

arbitrage between spot and futures markets require a model of or assumptions about the interest rate in order to construct the arbitrage portfolio. In this paper, a specific model of arbitrage between two spot markets trading the same security is developed. The model is based on Garbade and Silber's (1983) model of price convergence and accounts for the transaction costs of the arbitrage trades. Because the paper studies the trading of the same security on two markets at the same time, it should provide evidence much closer to the theoretical notion of (riskless) arbitrage trading as defined in financial theory.

One reason such models have not been empirically tested very often is the lack of appropriate data. For reasons of transparency and liquidity, each security is usually traded on only one market. This is not the case, however, in the Czech Republic. Due to the somewhat nonstandard evolution of the capital market in the Czech Republic after voucher privatization, two equity markets, the Prague Stock Exchange (PSE) and the RM-System (RMS, secondary market) exist. A large set of identical stocks are traded on both of these markets. This situation provides a unique opportunity to investigate directly *on-the-spot* arbitrage trading for a large group of individual securities, differing in size, volume of trade or other factors. Because the Czech capital market is rather young, it also allows to study the dynamics of arbitrage trading over time.

Despite the general perception that the Czech capital market is one of the worst in the region in terms of transparency and investor protection,² Němeček (1997) found little evidence of insider trading of liquid stocks on the first two tiers of the PSE.

On the other hand, empirical evidence exists confirming the inefficiency of the Czech capital markets. Němeček (1998) found that a simple one-factor market model relating a stock's return to the return of the market index does not explain the variability in expected

² The Economist (April 1996) and the Wall Street Journal (May 1996), among others, reported on "dealing in Prague as a losers' guide to investment," and characterized the Czech capital market as "a muddy market" and as "anarchy to the outsider, sweet profit to those in the know."

returns of individual stocks. Thus, the market seems unable to exploit profitable opportunities offered by the co-variability of the individual firms' returns.

Hanousek and Němeček (1997) applied Granger-causality tests to market indices to study interactions between different segments of the PSE and the RMS. Because of missing links between some market segments, they concluded that the PSE and RMS did not behave as one integrated market during the period studied (1995-1996). In addition, Hanousek and Filer (1998) showed that even the most liquid segment of the PSE does not exhibit semi-strong efficiency.

Recognizing the arbitrage opportunities when prices of the same security differ at a given point in time on two markets, however, does not require any sophisticated econometric analysis. It might be expected, therefore, that agents operating on the Czech capital market were able to identify and exploit arbitrage opportunities due to different prices on the PSE and RMS, or learned to do so over time, even if the market failed to be efficient in more subtle ways.

In this paper we study inter-market links for individual stocks by estimating the level of arbitrage activity between the PSE and RMS for a set of actively traded stocks.³ Our methodology is based on the market-linkage test developed by Wang and Yau (1994) in the context of Garbade and Silber's (1983) model of price convergence. Wang and Yau (1994) show that a Dickey-Fuller (1979) test for a unit root in the mispricing series (the difference in prices on the two markets) can be used to test for market linkage as well as to estimate the level of arbitrage activity between the two markets. They show that if the two price series are cointegrated, failing to reject the null hypothesis that a unit root is present in the mispricing series is equivalent to failing to reject the null hypothesis that

³ Empirical evidence of existing intra-market linkages [Hanousek and Němeček (1997)] also confirms the

this two markets are not linked. If statistically different from one, the estimated first-order autoregressive coefficient of the mispricing series measures the degree of market linkage.

We extend Wang and Yau's (1994) approach by taking transaction costs into account.⁴ The resulting model has the form of a modified AR(1) process, with transaction costs as an additional explanatory variable. This model specification is then estimated for the ninety-five most liquid firms traded on both the PSE and RMS. The empirical evidence confirms our expectations. Although arbitrage opportunities were not exploited by the agents in very early trading on the two markets, the extent of arbitrage increased over time and eliminated those differences in prices between the two markets beyond the level of transaction costs. While there are many possible explanations for a first-order autocorrelation in the mispricing series (including autocorrelation in news coming to the market, convergence of the characteristics of the sets of agents operating on the two markets, features of market microstructure, etc.), additional evidence suggests that arbitrage activities are an important explanation of the empirical findings.

The paper is organized as follows. In the following section the price convergence model is formulated and the procedure used for testing market linkages and the level of arbitrage activity is described. Section 3 discusses the institutional features of the PSE and RMS. Section 4 then presents empirical results for a set of liquid stocks traded on both markets. Discussion of these results closes the paper.

leading role played by liquid stocks in incorporating common information signals into stock prices.

⁴ Various institutional barriers like transaction costs and settlement procedures could affect the relationship between parallel markets. Here, only the transaction fees are considered.

2. The Model and Methodology

Suppose there are no costs trading besides the explicit transaction fees charged by one of the two markets (denoted here by A and B) where the stock is traded. Assume that these transaction costs are stated as a percentage of the volume of trade and do not depend on the direction of trade (buying or selling). Denote by tc_A and by tc_B the costs (percentage) of trades on markets A and B, respectively. Similarly, let P_t^A and P_t^B denote the price of a stock at time t on markets A and B. The total costs of a one-directional arbitrage trade, i.e., the simultaneous selling of one share on market A and the buying of one share on market B or vice versa, is then

$$TC(P_t^A, P_t^B) = tc_A P_t^A + tc_B P_t^B. \quad (1)$$

One can then write the upper limit on the theoretical price on market A implied by the price on market B, P_t^{*A} , as

$$P_t^{*A} = P_t^B + TC(P_t^{*A}, P_t^B) \quad (2)$$

or, after substitution for transaction costs TC,

$$\begin{aligned} P_t^{*A} (1 - tc_A) &= P_t^B (1 + tc_B) \\ P_t^{*A} &= P_t^B \frac{(1 + tc_B)}{(1 - tc_A)}. \end{aligned} \quad (3)$$

Similarly, the upper limit on the theoretical price on market B implied by the price on market A, P_t^{*B} is

$$P_t^{*B} = P_t^A \frac{(1 + tc_A)}{(1 - tc_B)}. \quad (4)$$

When the observed price P_t^A exceeds the theoretical price P_t^{*A} , the price discrepancy can be exploited by simultaneously buying one share on market B and selling one share on market A, thereby increasing demand and prices on market B while increasing supply and

depressing prices on market A. Thus, arbitrage activity ensures that prices on both markets are closely linked.

Garbade and Silber (1983) presented a model of the dynamic price relationship between cash and futures, assuming arbitrage and zero transaction costs. They assert that the rate of price convergence depends on the supply elasticity of arbitrage activities, since the greater the supply elasticity of these activities, the more quickly the price differential will be arbitrated away. In our context, with transaction fees as the only cost of arbitrage trading, this model becomes

$$P_t^A = P_{t-1}^A - \beta_1 (P_{t-1}^A - P_{t-1}^{*A})^+ + \beta_2 (P_{t-1}^B - P_{t-1}^{*B})^+ + \varepsilon_t^A \quad (5)$$

$$P_t^B = P_{t-1}^B - \beta_1 (P_{t-1}^B - P_{t-1}^{*B})^+ + \beta_2 (P_{t-1}^A - P_{t-1}^{*A})^+ + \varepsilon_t^B, \quad (6)$$

where $x^+ = \max(x, 0)$, β_1 and β_2 are elasticities of the arbitrage supply, and ε_t^A and ε_t^B are error terms. When $\beta_1 = \beta_2 = 0$, the prices on the two markets have no relationship.

Instead of estimating β_1 and β_2 separately, equation (6) can be subtracted from equation (5) to obtain

$$P_t^A - P_t^B = (P_{t-1}^A - P_{t-1}^B) - (\beta_1 + \beta_2) \left((P_{t-1}^A - P_{t-1}^{*A})^+ - (P_{t-1}^B - P_{t-1}^{*B})^+ \right) + (\varepsilon_t^A - \varepsilon_t^B) \quad (7)$$

Further

$$\begin{aligned} & (P_{t-1}^A - P_{t-1}^{*A})^+ - (P_{t-1}^B - P_{t-1}^{*B})^+ = \\ & = \left(P_{t-1}^A - P_{t-1}^B - \frac{tc_A + tc_B}{1 - tc_A} P_{t-1}^B \right)^+ - \left(P_{t-1}^B - P_{t-1}^A - \frac{tc_A + tc_B}{1 - tc_B} P_{t-1}^A \right)^+ = \\ & = \left(P_{t-1}^A - P_{t-1}^B - \frac{tc_A + tc_B}{1 - tc_A} P_{t-1}^B \right)^+ + \left(P_{t-1}^A - P_{t-1}^B + \frac{tc_A + tc_B}{1 - tc_B} P_{t-1}^A \right)^- = \\ & = (P_{t-1}^A - P_{t-1}^B) + F_t, \end{aligned} \quad (8)$$

where $x^- = \min(x, 0)$ and

$$F_t = \begin{cases} \frac{tc_A + tc_B}{1 - tc_B} P_{t-1}^A & \text{if } (P_{t-1}^A - P_{t-1}^B) < -\frac{tc_A + tc_B}{1 - tc_B} P_{t-1}^A \\ - (P_{t-1}^A - P_{t-1}^B) & \text{if } -\frac{tc_A + tc_B}{1 - tc_B} P_{t-1}^A \leq (P_{t-1}^A - P_{t-1}^B) \leq \frac{tc_A + tc_B}{1 - tc_A} P_{t-1}^B \\ -\frac{tc_A + tc_B}{1 - tc_A} P_{t-1}^B & \text{if } \frac{tc_A + tc_B}{1 - tc_A} P_{t-1}^B < (P_{t-1}^A - P_{t-1}^B) \end{cases} \quad (9)$$

Thus, we get

$$P_t^A - P_t^B = (1 - \beta_1 - \beta_2)(P_{t-1}^A - P_{t-1}^B) - (\beta_1 + \beta_2)F_t + (\varepsilon_t^A - \varepsilon_t^B) \quad (10)$$

or

$$P_t^A - P_t^B = (1 - \beta)(P_{t-1}^A - P_{t-1}^B) - \beta F_t + \varepsilon_t, \quad (11)$$

where $\beta = \beta_1 + \beta_2$ and $\varepsilon_t = \varepsilon_t^A - \varepsilon_t^B$.

The situation in which there is no relationship between the two markets corresponds to the coefficient of $(P_{t-1}^A - P_{t-1}^B)$ equal to one in equation (11). Thus, testing for a unit root in equation (11) is equivalent to testing for the absence of market linkage between the two markets. An augmented Dickey-Fuller test is used for this purpose. Moreover, the estimate of coefficient β measures the level of arbitrage activity for a given security.

3. Comparison of the PSE and the RMS

The structure of the Czech capital markets was to a large extent determined by voucher privatization in the Czech Republic. About 1,700 firms were privatized during the two waves of voucher privatization. As a byproduct of voucher privatization the majority of citizens of the Czech Republic became shareholders of the previously state-owned firms. To allow people to trade shares acquired in the voucher privatization, two capital markets opened soon after the end of the first wave of voucher privatization. While, by its institutional design, the RM-System was (from its opening in May 1993) more suited to the trading needs of small individual shareholders, the Prague Stock Exchange (which opened in April 1993) hoped to attract trading of institutional investors. Originally all the securities were traded on both markets, although after the period studied here, less liquid firms were dropped from the PSE. For more historical and institutional details on the two

markets, see Hanousek and Němeček (1997).

Main differences between the PSE and the RMS:

- While the transfer of securities and money (“delivery against payment”) on the PSE is processed three days after the trade (T+3), on the RMS settlement is done on the day of the trade (T+0).
- The vast majority of stocks offered on the PSE are traded at a fixed daily price, while all stocks on the RMS are traded at a continuously variable price.
- While the PSE has several tiers (and trading groups) with different listing requirements, there are no listing requirements on the RMS (See Table I for details on trading group definitions and Table A.I for quantitative characteristics of the groups).
- The RMS is not based on a membership principle. Anyone can access it directly through one of over one hundred locations throughout the Czech Republic. Trades on the PSE, however, require use of a member broker.
- Total trading volume (including block and direct trades) of the PSE exceeded the RMS trading volume on average by about four to one during the period studied (see Table II). If block and direct trades are excluded, however, the trading volumes are much closer. The PSE continued to dominate the RMS in terms of trading volume, though the PSE-to-RMS ratio fell from 4:1 (in April 1995) to 5:3 (in February 1997). Interestingly, trading volume on the least transparent section of the PSE was almost as low as on the RMS.

Table I. Listing Requirements for the PSE by Tiers (1996)

Trading Group	Requirements			Segment notation
	Disclosure**	Liquidity*	Capital	
Tier one	Quarterly	> 300,000	Public offer > 200 mill. Registered capital > 500 mill.	A1, A2
Tier two	Semi-annually	N/A	Public offer > 100 mill. Registered capital > 250 mill.	B2
Tier three	Annually	N/A	N/A	C2, C3

All figures are in CZK.

* Average volume per session (last five months)

** Level of disclosure varies across tiers.

Trading group (segment notation) A1 represents securities traded on the first tier at variable prices; segment A2 represents securities traded on the first tier at fixed (afternoon-auction) prices. Segments B2 and C2 represent securities from the second and third tiers, respectively, all traded at fixed (afternoon-auction) prices. And lastly, third-tier securities, which have very low liquidity and therefore are traded only twice a week in the fixed-price afternoon auction, are represented in C3. More information on the quantitative characteristics of the groups is given in Table A.I.

Table II. The trading volume on registered capital markets (CZK billion)

Trading volume (CZK billion)	1993*	1994	1995	1996	1997	1998**
PSE Central market	2.0	16.0	22.0	28.8	22.1	5.4
PSE – direct and block trades	7.0	46.0	173.4	364.4	657.5	161.1
RMS Central market	2.9	4.4	5.8	9.5	7.6	2.9
RMS – direct and block trades	–	–	19.4	90.9	151.1	64.0
PSE total over RMS total	3.1	14.1	7.8	3.9	4.3	2.5
PSE central over RMS central	0.7	3.6	3.8	3.0	2.9	1.9

* April-December (PSE), July-December (RMS)

** January-April

Price determination on each market⁵

PSE: During each day's morning auction buy and sell orders are matched and cleared. This clearing procedure establishes a price that maximizes the number of shares traded given the buy and sell orders submitted at various prices (subject to the maximum admissible daily price change of $\pm 5\%$). At 11 a.m. the outcomes (prices, volumes and index of the excess supply or demand) are made available to the general public. A given security may then, in accordance with its trading group, be traded in the afternoon session at a fixed or variable price. If fixed-price trading applies (groups A2 through C3), any afternoon transactions must be conducted at the price set during the morning auction. Also, these afternoon transactions must first clear any possible excess demand (supply) remaining from the morning auction. Only a small set of the most liquid stocks (group A1) are traded in the variable-price continuous auction, which opens with the morning-auction price.

RMS: In a continuous auction, computer algorithms are used to execute trades and to match buy and sell orders. The auction price is set to maximize trade volume, given the set of buy and sell orders admissible in a given auction round (in which the maximum admissible daily price change is $\pm 10\%$). In the event that several prices yield the same volume of trade, the arithmetic mean is applied. A trading day begins with the auction of all orders not satisfied in the previous day of trading, including those received after the closing of the market. Continuous trading follows during normal trading hours. Any event changing demand and supply patterns (such as the arrival of a new trading order or cancelation/expiry of an existing order) initiates possible matching of trading orders.

⁵ The price-setting mechanisms are described as they operated during the period studied. They have subsequently been changed extensively, such that currently all stocks traded on the PSE trade continuously.

Possibilities for arbitrage trading

Table A.II of the Appendix shows the timetable of daily trading on both the PSE and RMS. The trading hours of the continuous auctions on each market overlap between 11:30 and 14:00, thus providing an opportunity for immediate arbitrage trading. Table A.III gives an overview of the transaction costs on both markets.

The theoretical concept of arbitrage requires arbitrage trading to be riskless. Nevertheless, in reality there are always some risks involved in such trading, most frequently due to the time lag between the buying of stock on one market and selling of that stock on the other market. In this paper, all trading activities which exploit the difference in prices on the two markets are called arbitrage trading (despite the risks of one part of the buy-and-sell transaction not being executed or the risks of a price change in the stock between submitting and executing trading orders) as long as the expected payoff of such a transaction is positive.

Given the trading schedules and procedures of the PSE and those of RMS, there are two possible types of arbitrage. Within-day arbitrage can be done by agents who have online access to both the PSE and RMS and can, during the afternoon sessions, submit orders for the (fixed-price) afternoon continuous auction of the PSE and the (variable-price) continuous auction of the RMS. This kind of arbitrage requires, however, an excess demand (or supply) after the morning auction on the PSE when the price on the RMS is lower (or higher). Moreover, online access to the continuous trading auctions is relatively costly, given the low trading volumes of the PSE and RMS. In order to model these intra-day arbitrage activities, the transaction data and data on the demand/supply imbalances of the PSE after the morning auction would be required. Unfortunately, these data are not publicly available. However, some evidence regarding the extent of intra-day arbitrage activities can be obtained from changes in the opening versus closing RMS prices

compared to the fixed PSE prices. More precisely, if arbitrage trading takes place during the continuous auctions of the PSE and RMS, then the price difference between the (fixed) PSE price and the opening RMS price will be higher than the difference between the (fixed) PSE price and the RMS closing price as part or all of the difference is eliminated through the intra-day arbitrage trading.

Despite this type of activity, end-day prices on the PSE and RMS still exhibit significant differences. This provides possible space for arbitrage trading, even for agents without instantaneous access to the market, since the end-day prices determine admissible price intervals for the following day. By examining the price difference at the end of the trading day and submitting relevant orders for the next day's auctions, an agent can avoid the costs of online access to the market and exploit arbitrage opportunities for costs basically equal to the transaction fees of the buy-and-sell trade on the PSE and RMS. On the other hand, such a procedure contains larger risks of price changes than intra-day arbitrage (where the PSE price is fixed for most stocks). Given the differences in the population of agents operating on the two markets, prices on the RMS are (on average) lower than prices on the PSE. The price risks of arbitrage trading may thus be reduced by executing buy transactions on the RMS during the continuous auction on trading day t and submitting the sell order for the morning auction on the PSE at day $t+1$. This procedure may be conveniently executed even without inventories of the arbitrated stock, given that the RMS settlement is done in time $t+0$.

While diverging slightly from the theoretical concept of (riskless) arbitrage, an investigation of this type of near-arbitrage trading may provide important insights into the efficiency of the markets and the behavior of the agents. The model of arbitrage trading with transaction costs derived in section 2 will be used for empirical analysis of inter-day arbitrage trading in the following section.

4. Empirical Evidence

Data description

We omitted data from the very early stages of both markets, focusing instead on the period between April 3, 1995 and December 20, 1996, i.e., after most second-wave shares had entered the market (allowing for a one-month “settling period”) and before the first major delisting from the PSE.

The estimation of equation (11) in section 2 requires that a given stock be traded on both markets at times t and $t-1$. To minimize the effects of non trading on estimating the model, we use only the most actively traded stocks. Namely, we estimate the model for each of the 95 firms⁶ that fall into segment A1, A2 or B2 of the PSE.

It is likely that there will be a larger difference in prices for less liquid stocks. Moreover, it is also expected that the effect of arbitrage opportunities will depend on liquidity. Therefore, we ordered the stocks according to the average volume of trading on the PSE.

Figure 1 plots the average mispricing (defined as the absolute value of percentage difference in daily average prices with the PSE price taken as a base) versus the average depth of the market. The depth of the market (i.e., the average minimum of daily trading volume on the PSE or RMS) ranges broadly from 5,000 to about 1.3 mill. CZK, while the average absolute value of percentage differences in prices, which is clearly inversely related to the trading volume, ranges from 1.17% to 7.45%. Figure A.1 in the Appendix graphs, by subperiod and market segment, mispricing in two subsequent trading sessions.

⁶ Two firms in segment B2 are not studied because of an insufficient number of observations.

Estimation of intra-day arbitrage

In section 3 we described the possibility of immediate arbitrage trading during the afternoon continuous auctions of the PSE and RMS. Because the transaction data are unavailable, this type of arbitrage trading could not be investigated by means of the proposed model of arbitrage in the presence of transaction costs. As mentioned in the above discussion, however, some inference regarding intra-day arbitrage trading can be made by comparing the opening and closing RMS prices to the (fixed) price on the PSE. If arbitrage trading occurs during the afternoon auctions of the PSE and RMS, then this trading should (at least partially) eliminate the price differential of the PSE and RMS. The price difference between the (fixed) PSE price and the opening RMS price will then be higher than the difference between the PSE price and the RMS closing price. To test this hypothesis one could use a classical pair t-test. Since the underlying distributions of these price differences can be quite different from the normal distribution, however, we opted to use a distribution free test – the sign test [see for instance Pratt and Gibbons (1981) or Lehmann (1975)]. Though the sign test is not very powerful for normally distributed samples, it provides a robust check over a very wide class of distributions.⁷ Therefore, rejecting the null hypothesis of no elimination of the price differential will give us strong support for intra-day co-movements of prices.

The test is conducted as follows:

- 1) For each security we mark by “+” a day in which the difference in opening prices was bigger than the difference in closing prices, and by “-” otherwise.
- 2) The null hypothesis is that “+” and “-” are equally likely in the population (i.e., no change in price differences at opening and closing), $H_0: \text{Prob}(“+”) = 1/2$. We are of

⁷ For any continuous, symmetric distribution, the asymptotic efficiency of the sign test relative to the t-test is never lower than 1/3 (but it can also be infinity). The relative efficiency is $2/\pi = .64$ in the case of normal distribution, 1/3 for uniform distribution and 2 for double exponential distribution [see Pratt and Gibbons

course interested only in one side alternative, that the closing price difference is smaller than the opening price difference, i.e., $H_A: \text{Prob}(\text{"+"}) > 1/2$.

- 3) Denote by S_+ the number of plus signs observed. For testing the null hypothesis we can use the critical values of the sign test [see for instance Lehman (1975)], or an asymptotic normal approximation that works very well for sample sizes greater than 20:

$$z = \frac{2S_+ - (n+1)}{\sqrt{n}}.$$

Therefore, if the z statistics exceed the critical values of the standard normal distribution we reject the null hypothesis. In this case we find empirical evidence that trading (at least partially) eliminates the price differential of the PSE and RMS.

We conducted the sign test for each of the 95 securities mentioned earlier, testing the null hypothesis for the whole period as well as for each quarter. It is interesting that for a significant portion of securities we found empirical evidence that the price differences between the PSE and RMS prices were partly eliminated during a trading day; see Table III (i.e., rejecting the null hypothesis of no elimination of the price differentials).⁸

Table III. Summary of sign test results.

	Total	95Q2	95Q3	95Q4	96Q1	96Q2	96Q3	96Q4	97Q1*
10% level: rejected	45	54	53	43	43	42	28	44	31
- not rejected	51	36	38	50	52	53	67	51	65
5% level: rejected	40	47	46	34	33	34	25	34	26
- not rejected	56	43	45	59	62	61	70	61	70

*The first quarter of 1997 is incomplete (January-February, 17).

(1981)].

⁸ Detailed results are available from the authors upon request.

When analyzing the elimination of price differentials between the PSE and RMS during intra-day trading, one would not expect the decline pattern presented in Table III. As the market matures and agents become more experienced, one would expect the number of firms for which price difference gets eliminated during a single day to increase. But Table III summarizes only the number of firms for which we observed elimination of price differentials during a trading day. It does not take into account the liquidity constraints and/or the market capitalization of the firms studied. Let us note that we used opening and closing prices as a proxy for the intra-day change in price. This simplification will work very well for segments of the PSE traded with a fixed price (A2, B2), but it is inappropriate for the segment traded at a variable price (A1); therefore we excluded this segment from our analysis. We suggest analyzing the trading groups A2 and B2 (basically using market indices) to adjust for market capitalization of the firms. The results are presented in Table IV.

Table IV. Results of the sign test: Segment average prices (z-statistics are in parentheses)

	Total	95Q2	95Q3	95Q4	96Q1	96Q2	96Q3	96Q4	97Q1*
A2	392 (15.63) ^a	57 (7.03) ^a	58 (7.29) ^a	51 (6.40) ^a	56 (6.77) ^a	53 (5.46) ^a	48 (3.72) ^a	44 (3.18) ^a	25 (3.23) ^a
B2	394 (15.82) ^a	55 (6.51) ^a	55 (6.51) ^a	50 (6.12) ^a	56 (6.77) ^a	54 (5.72) ^a	52 (4.71) ^a	49 (4.45) ^a	23 (2.51) ^a

a denotes significance at the 1% level. The first quarter of 1997 is incomplete (January-February, 17).

It is clear that despite the decline of significance levels, we we reject the hypothesis of no price elimination on 1% significance level for both segments and all periods.

Estimation of inter-day arbitrage

The actual transaction costs of trading on the PSE and RMS, given in the Appendix, Table A.III, decrease with the volume of a given trading order. For the firms traded on the first and second tiers of the PSE, transaction costs range from about 2.12%⁹ for orders less than 1,000 CZK to 0.32% for orders over 7 mill. CZK.

To measure the transaction costs of one-way arbitrage trade from the PSE to the RMS or vice versa [the variable F_t specified in equation (11)], we make the following assumptions. First, in assessing arbitrage activities, we ignore the effect of marginal arbitrage activity on prices on both markets. Thus, we assume that the volume of these activities is relatively small compared to the depth of the market. The level of transaction costs used for determining F_t is set (*ad hoc*) by assuming that the volume of the arbitrage trade is one tenth the depth of the market and using corresponding level of transaction costs. Three levels of total transaction costs are used: 2.12%, 1.62%, and 1.12% for arbitrage trades of less than 1,000 CZK, 1,000 – 10,000 CZK, and 10,000 – 100,000 CZK, respectively.¹⁰ In terms of our model, we have $t_{C_{PSE}} = 0.12\%$ and $t_{C_{RMS}} = 2\%, 1.5\%$, and 1.0% for the three volume categories.

Each of the price series was first tested if they are difference stationary using an augmented Dickey-Fuller test. All of the price series studied were difference stationary.¹¹ The cointegration of the prices of a given stock on the PSE and RMS was not rejected using the Engle and Granger (1987) procedure for any firm tested.

⁹ Under a simplifying assumption of 2% RMS transaction fees for orders with volume lower than 1,000 CZK.

¹⁰ There are only two firms in our sample with a depth higher than 1 million CZK (and assumed volume of arbitrage activities higher than 100,000 CZK); for these two firms, transaction costs of 1.12% are used.

¹¹ The results of these tests are not reported here, but are available from the authors on request.

Since it has been established that the price series are difference-stationary and cointegrated, we can turn to estimating the proposed model of arbitrage with transaction costs as derived in section 2. Using data from April 3, 1995, to February 1997, the following model

$$P_t^{PSE} - P_t^{RMS} = (1 - \beta)(P_{t-1}^{PSE} - P_{t-1}^{RMS}) - \delta F_t + \varepsilon_t \quad (12)$$

was estimated for each of the 95 firms studied. In addition, we tested the validity of the restriction $\delta = \beta$ imposed in equation (11). The results of these estimations are summarized in Table V (on the following page). Figure 2 shows the estimated coefficients of β and δ for each of the 95 firms (recall that the firms are sorted in descending order by volume of trade on the PSE).

In the interest of precision, we have to address the problems of testing unit roots in equation (12). In addition to a classical AR(1) process, equation (12) also contains the variable F_t . Instead of using resampling methods, like bootstrap, for computing the critical values of the unit root test, we prefer to use the classical ADF test. Because F_t is stationary (or trend stationary with a very small trend), the ADF test with a constant and a trend should give us a quite reasonable approximation of the critical values of the unit root test for equation (12).

The null hypothesis of unit root in the model ($\beta = 0$) is rejected at any reasonable significance level. The ADF test statistics for individual firms range from -9.09 to -27.39 (see Table V for the average ADF statistics by market segment¹²), while the critical value for the Augmented Dickey-Fuller test with a constant and time trend in large samples at a 1% significance level is -3.96 . Thus, for all firms the hypothesis of no market linkage between the PSE and RMS is clearly rejected. Table V shows that for many firms we reject the null hypothesis of validity of restriction $\beta = \delta$ and/or the null

hypothesis of stability of coefficients over time. This is not very surprising given the duration of the sample used for the estimation, the dynamic nature of the emerging markets, and the attributes of the model — arbitrage activities *should* clearly depend on particular market conditions.

Table V. Summary of whole-sample estimates of the model

	Number of firms	Average of				Number of firms (% of firms) where we reject* the	
		ADF statistics	$1-\beta$ (stdev($1-\beta$))	δ (stdev(δ))	Adj. R^2	restriction $\beta = \delta$	stability of coefficients*
All firms	95	-20.201	0.403 (0.078)	-0.599 (0.245)	0.149	25 (26.3%)	48 (50.5%)
Segment A1	8	-20.157	0.285 (0.079)	-0.633 (0.165)	0.131	2 (25.0%)	5 (62.5%)
Segment A2	37	-21.647	0.386 (0.072)	-0.597 (0.209)	0.119	13 (31.1%)	22 (56.5%)
Segment B2	50	-19.139	0.435 (0.082)	-0.595 (0.284)	0.173	10 (20.0%)	21 (42.0%)

* A 5% significance level is used for both tests. The Chow test with the sample split in half is used as a test for the stability of coefficients.

We expect that the supply of arbitrage activities (the number and experience of agents pursuing arbitrage opportunities) has increased over time, making the two markets more closely integrated and increasing their efficiency. To assess the extent of arbitrage over different stages in the markets' development and for firms of various sizes and liquidity, the following procedure is used.

We take a “rolling window” of 90 trading days and, for each firm, run a sequence of regressions to estimate coefficients β and δ of the model on the rolling samples of trading days 1 to 90, 2 to 91, and so on. For each of the regressions, we also evaluate the model specification by testing for the stability of the coefficients (Chow test) and by testing for the validity of the restriction $\beta = \delta$. Results of these estimations are summarized in Figures 3 through 6.

Almost all of the estimates of $(1-\beta)$ are lower than 0.5 (see Figure 2). The pattern in the estimates of the coefficient $(-\delta)$ follows the pattern of $(1-\beta)$ estimates, although estimates

of $(-\delta)$ are significantly more volatile across firms. This is not surprising given the upper and lower limits on the F_t variable bounded by the level of transaction costs, which result in lower sample variability, making a precise estimation of the $(-\delta)$ coefficient more difficult. The time averages of the 90-day “rolling sample estimates” of the regression coefficients $(1-\beta)$ and $(-\delta)$ for individual firms are reported in Figure 3. Slight differences in Figures 2 and 3 suggest that the coefficients will not be stable across the whole period studied.

The cross-firm averages of the 90-day “rolling sample” estimates of $(1-\beta)$ and $(-\delta)$ are reported in Figure 4. The starting date of the 90-day interval is reported on the horizontal axis. This graph documents the increasing level of arbitrage activity captured by the decrease in the average estimate of $(1-\beta)$. Further, as time proceeds, the difference between the average estimates of $(1-\beta)$ and $(-\delta)$ approaches 1, i.e., the restriction $\delta = \beta$ becomes valid over time. This restriction was a feature of the theoretical model as given in the original specification (11). Testing for this restriction in the estimated specification (12) thus allows for an assessment of the theoretical model’s validity. Goodness-of-fit statistics from the “rolling sample” estimations are summarized in Figures 5 and 6. Figure 5 shows the time averages of the adjusted R^2 and p-values of the tests for stability of coefficients and for the validity of the restriction $\delta = \beta$. Figure 6 plots the cross-firm averages of the same statistics for different dates. Both graphs indicate the stability of the coefficients in the 90-day periods used for estimation and the validity of the restriction on these coefficients implied by the model.

However, these results *per se* do not provide any evidence that the size of the firm may influence the level of arbitrage activity or the performance of the model. To further clarify the possible dependence of the level of arbitrage activity on the characteristics of a given stock, we present averages by market segment (i.e., across trading groups) of the PSE.

Figures 7 and 8 show the averages of the estimated coefficients and of the adjusted R^2 , respectively. They indicate a significant relationship between the segment of the market to which a given firm belongs and the level of arbitrage trading in shares of this firm. The supply of arbitrage activity is highest for the most transparent and most liquid segment of the “blue chips.” The level of arbitrage activity increases in all studied segments of the market over time, and the differences among the segments diminish.

4. Conclusions

In this paper we studied arbitrage links between two parallel markets trading the same security. A model of arbitrage trading was developed that explicitly models the trading costs of arbitrage trades. A methodology for testing the inter-market links and measuring the level of arbitrage activities was described. The model extends the market-linkage test developed by Wang and Yau (1994) in the context of Garbade and Silber’s (1983) model of price convergence. We tested a modified AR(1) process of mispricing series which takes into account transaction costs for the existence of unit roots. The estimated first-order autoregressive coefficient of the mispricing series suggests the degree of arbitrage activities (with a coefficient close to zero implying greater linkage between the markets while a coefficient equal to one means no linkage).

In the empirical part of the paper we carried out an investigation of arbitrage trading on the parallel equity markets in the Czech Republic. As data limitations do not allow a detailed analysis of intra-day arbitrage trading during the afternoon continuous auctions of the PSE and RMS, the main focus of the analysis was on inter-day arbitrage trading studied via a model of arbitrage trading with transaction costs. Because of the very low liquidity of the majority of firms traded on the market, only ninety-five of the most liquid firms on the PSE were analyzed. The results indicate significant links between the two

markets and document autocorrelation patterns in the mispricing series as predicted by the model.

There are, of course, other possible explanations besides arbitrage trading for the observed autocorrelation in the mispricing series. A major problem is probably the decline of trading volume on the central markets (for both the PSE and RMS) from starting twenty-five to less than five percent of the trading volume. This means that the price discovery process in this case would lead to inefficiencies *per se*. In addition, the autocorrelated structure mentioned above may stem from different characteristics of the traders on each of the two markets (the PSE versus the RMS), who could react differently to information signals. New information may have a different impact on traders' expectations or traders may react to the signals with different time lags. Finally, the information signals themselves may be serially correlated. The observed decrease in the autocorrelation of the mispricing series over time may then be a result of traders processing information signals more quickly and efficiently, traders' characteristics becoming more uniform over the two markets, and/or the information signals becoming less serially correlated.

A different structure of traders probably played some role in the first months of trading. There is, however, significant evidence for the increasing importance of arbitrage trading in explaining the behavior of and links between PSE and RMS prices over time. In the Wang and Yau (1994) model of arbitrage trading, a decrease in the first order autocorrelation of the mispricing series is taken as an indication of an increase in the elasticity of supply of arbitrage activities. This type of decrease occurred in the serial correlation of the mispricing series between the PSE and RMS over time. Moreover, the importance of arbitrage trading is documented by the validity of the restriction on coefficients that is implied by the model of arbitrage trading.

Although a detailed analysis of intra-day arbitrage trading is not possible, the results of simple nonparametric tests provide evidence of the increasing role of arbitrage trading for maintaining close links between the prices of stocks on the PSE and RMS. As intra-day changes in the pricing gap between the PSE and RMS can hardly be attributed to changes in the characteristics of the traders operating on the markets, the results of these tests provide further support for an arbitrage-based explanation of the behavior of the mispricing series.

We can therefore summarize the empirical findings as follows: As a new market emerged, the use of arbitrage opportunities ensured co-movements of stock prices on the two parallel Czech capital markets, the Prague Stock Exchange and the RM-System. There was a significant relationship between the segment of the market to which a given firm belonged and the level of arbitrage trading in shares of this firm. The level of arbitrage activities was highest for the segment of continuously traded stocks (i.e., for the segment with the most transparency and the most liquidity). Moreover, the level of arbitrage activities increased over time at least in the top three segments of the market (all the segments studied), while differences among these segments gradually disappeared.

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Figure 1. Trading volumes and mispricing

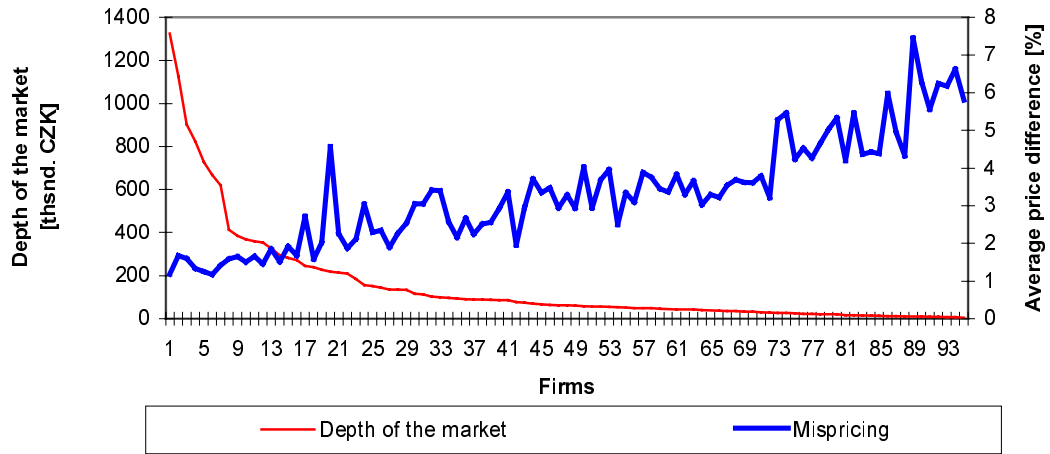


Figure 2. Whole-sample estimates of $(1-\beta)$ and $(-\delta)$ for individual firms

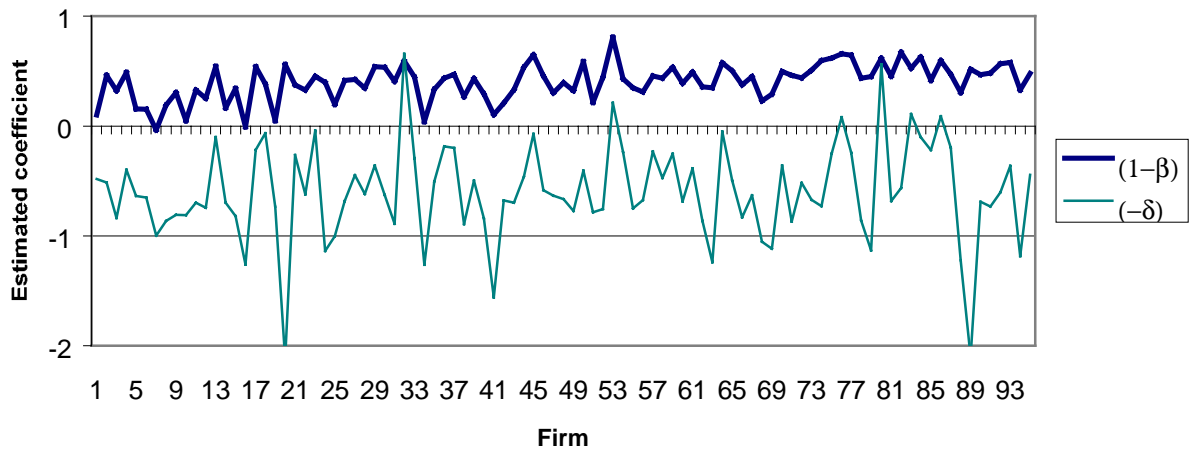


Figure 3. Time averages of the 90-day “rolling sample” estimates of $(1-\beta)$ and $(-\delta)$ for individual firms

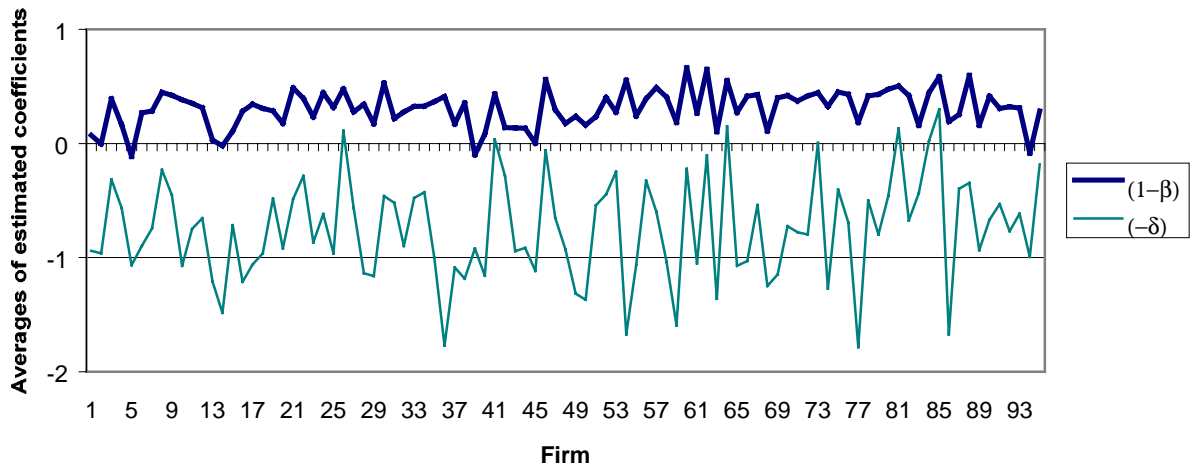


Figure 4. Cross-firm averages of the 90-day “rolling sample” estimates of $(1-\beta)$ and $(-\delta)$ for different dates

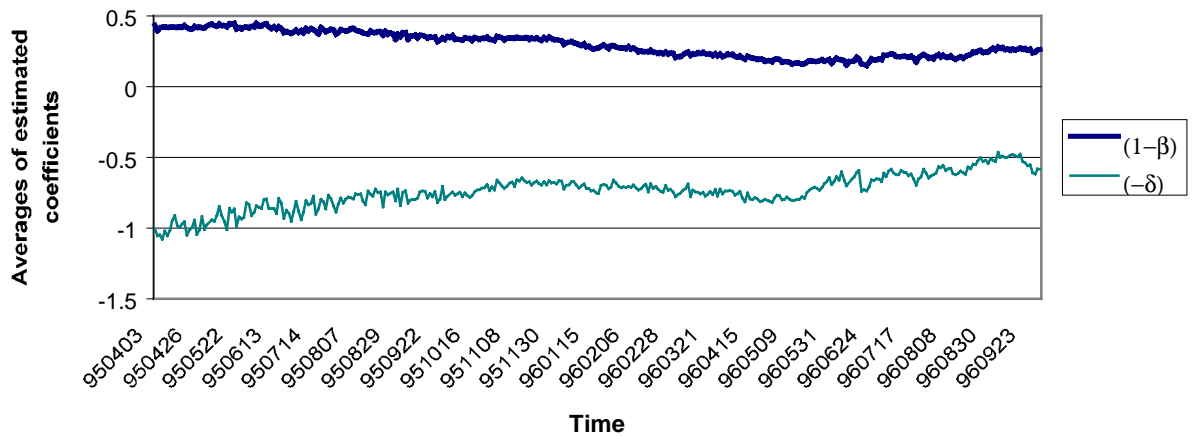


Figure 5. Time averages of the 90-day “rolling sample” specification statistics for individual firms

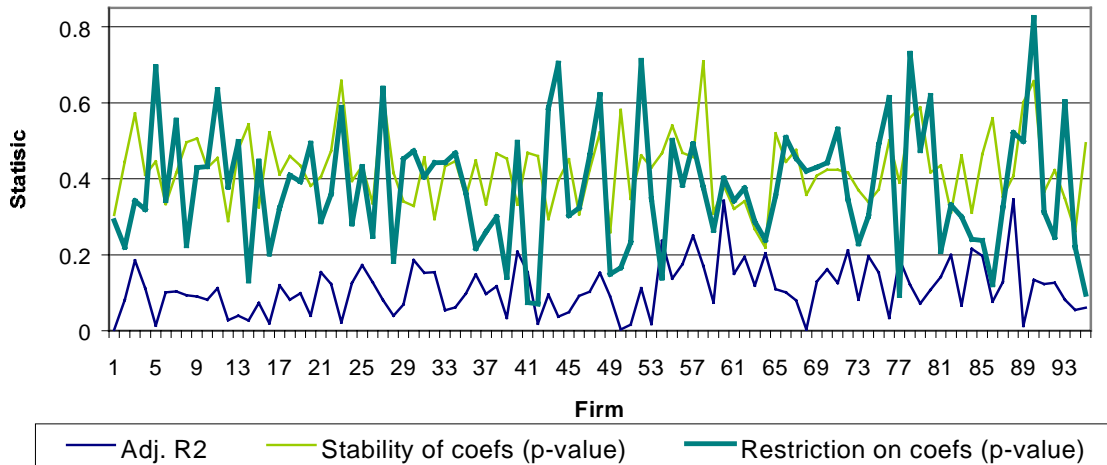


Figure 6. Cross-firm averages of the 90-day “rolling sample” specification statistics for different dates

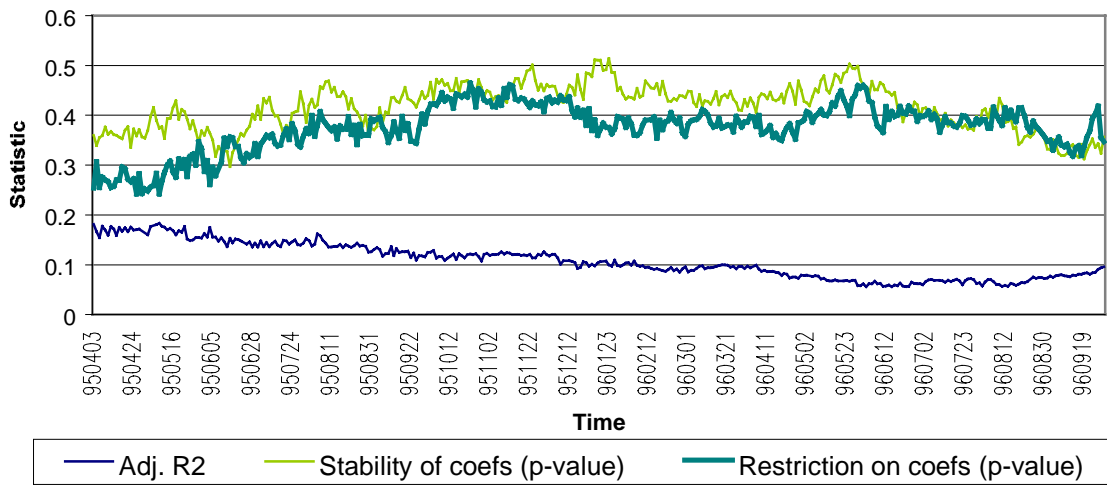


Figure 7. Cross-firm averages of the 90-day “rolling sample” estimates of $(1-\beta)$ by trading group (segment of the market) for different dates

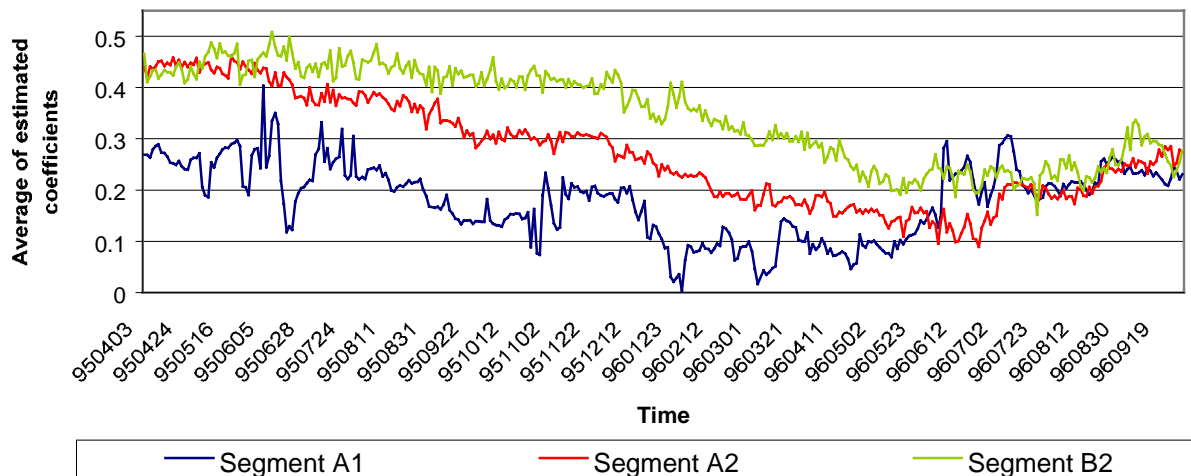
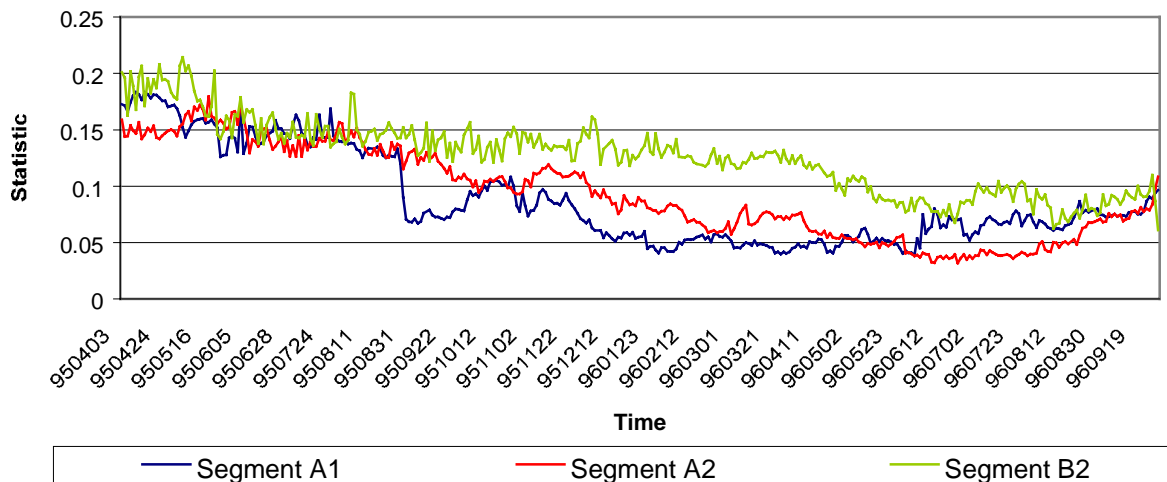


Figure 8. Cross-firm averages of the Adjusted R^2 from the 90-day “rolling sample” estimations, by trading group (segment of the market) and for different dates



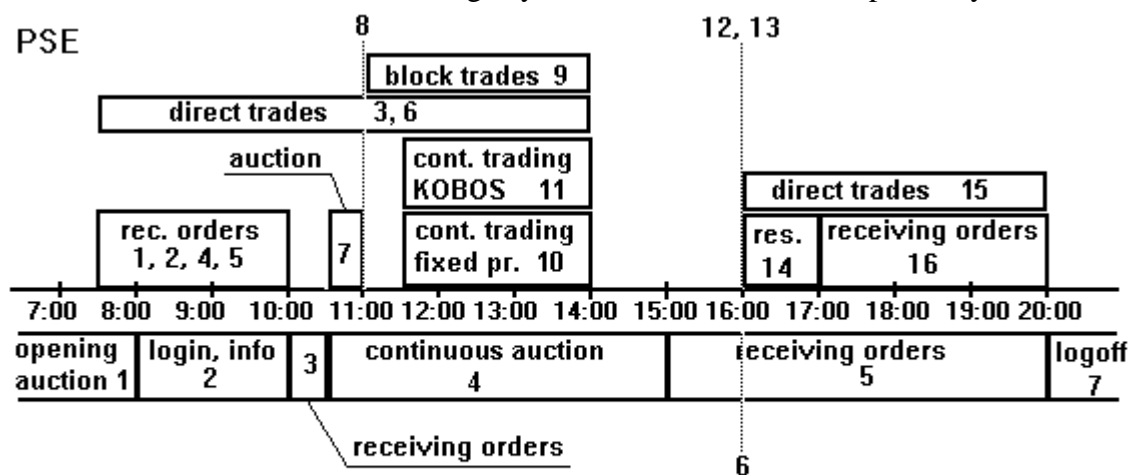
Appendix

Table A.I: Characteristics of the particular trading groups

Date	A1	A2	B2	C2	C3	On RMS only*
April 1995						
# of stocks	7	36	48	568	1050	256
Ratio of Volume of Trade PSE/RMS	7.95	4.84	5.07	3.48	3.26	-
% of market cap. on PSE	39.52%	23.53%	9.08%	22.98%	4.89%	-
% of market cap. on RMS	38.20%	22.43%	8.57%	22.04%	5.47%	3.29%
February 1997						
# of stocks	8	37	52	564	1030	399
Ratio of volume of trade PSE/RMS	6.49	3.33	3.92	2.76	1.39	-
% of market cap. on PSE	44.57%	29.25%	10.46%	13.68%	2.04%	-
% of market cap. on RMS	43.30%	28.31%	10.11%	13.27%	2.09%	2.92%

* a group of stocks not traded on the PSE

Table A.II: The timetable of a trading day on the PSE and RMS, respectively



RMS

PSE	From	Till	
1	7:30	9:30	Receiving of online external orders (orders filled in by PSE members through terminals in their remote offices)
2	7:30	10:00	Receiving of online internal orders
3	7:30	14:00	Receiving of on-line orders for direct trade
4	8:00	9:30	Receiving of external orders from the floor
5	8:00	10:00	Receiving of internal orders from the floor
6	8:00	14:00	Receiving of orders for direct trade from the floor
7	10:30	11:00	<u>Auction</u> of received orders
8	11:00		Announcement of mid-day prices
9	11:00	14:00	Trading with blocks of securities
10	11:30	14:00	<u>Continuous trading at a fixed price</u>
11	11:30	14:00	<u>Continuous trading at a variable price</u> (KOBOS)
12	16:00		Announcement of the final prices of traded securities
13	16:00		Announcement of the list of securities for next-day trading
14	16:00	17:00	Delivery of the final daily results of trading
15	16:00	20:00	Receiving of online orders for direct trade
16	17:00	20:00	Receiving of online orders for auction

RMS	From	Till	
1	0:00	8:00	<u>Opening auction</u> of orders from previous trading day
2	8:00	10:00	Logging-in, inspection of demand and supply patterns after the opening auction
3	10:00	10:30	Receiving of orders for trade
4	10:30	15:00	<u>Continuous auction</u> : receiving of orders for trade, validation and matching of orders, implementation of trade
5	15:00	20:00	Receiving of orders for the next trading day
6	16:00		Announcement of the admissible price spreads for the next trading day
7	20:00	22:00	Logging-off, transfer of the data on trades made by an investor during a given day

Table A.III. Transaction costs of trading on the PSE and RMS

Prague Stock Exchange

Fee (as a percentage of volume) for trades concluded, bonds excluded	(valid since Jan 1, 1996)
a) main market	0.12%
b) secondary market	0.12%
c) free market	0.40%
Fee (as a percentage of volume) for trades concluded in bonds	
a) main market	0.01%
b) secondary market	0.01%
c) free market	0.01%
Fee for the direct trade (per trade)	300.00 CZK
Fee for the block trade (per trade)	1,000.00 CZK

RM-System

Volume of trade (CZK)	since Jan 1, 1995	since Jan 1, 1997
till 1,000	20 CZK	20 CZK
1,000 - 10,000	20 CZK + 1.5% over 1,000	20 CZK + 1.7% over 1,000
10,000 – 100,000	155 CZK + 1.0% over 10,000	173 CZK + 1.2% over 10,000
100,000 - 1 mil.	1055 CZK + 0.8% over 100,000	1253 CZK + 0.8% over 100,000
1 mill. - 3 mill.	8255 CZK + 0.5% over 1 mill.	8453 CZK + 0.5% over 1 mill.
3 mill. - 7 mill.	18255 CZK + 0.3% over 3 mill.	18453 CZK + 0.3% over 3 mill.
over 7 mill.	30255 CZK + 0.2% over 7 mill.	30453 CZK + 0.2% over 7 mill.

- since Jan 1, 1997, the fee for trading bonds has been 0.2% of the trading volume, at least 20 CZK
- the fee for direct trade is 10% of the usual transaction fee, at least 20 CZK, at most 200 CZK
- there is a fee for submitting an order for trade ranging from 20 to 130 CZK
- a trading order can include special features — limit price, stop price, 15- or 90-day validity, all-or-nothing condition, or a combination of these — which increase the usual transaction cost by a rate ranging from 0% to 10%.

Figure A.1

A1	ε

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