WERE STOCKS DURING THE FINANCIAL CRISIS MORE JUMPY: A COMPARATIVE STUDY

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Abstract

This paper empirically analysis the price jump behavior of heavily traded US stocks during the recent financial crisis. Namely, I test the hypothesis that the recent financial turmoil caused no change in the price jump behavior. To accomplish this, I employ data on realized trades for 16 stocks and one ETF from the NYSE database. These data are at a 1-minute frequency and span the period from January 2008 to the end of July 2009, where the recent financial crisis is generally understood to start with the plunge of Lehman Brothers shares on September 9, 2008. I employ five model-independent and three model-dependent price jump indicators to robustly assess the price jump behavior. The results confirm an increase in overall volatility during the recent financial crisis; however, the results cannot reject the hypothesis that there was no change in price jump behavior in the data during the financial crisis. This implies that the uncertainty during the crisis was scaled up but the structure of the uncertainty seems to be the same.

Abstrakt

Tato práce empiricky analyzuje vlastnosti cenových skoků pro nejvíce obchodované americké akcie během současné finanční krize. Konkrétně testuji hypotézu, že současná finanční krize nezpůsobila žádnou změnu ve vlastnostech cenových skoků. Pro tuto analýzu používám data realizovaných obchodů pro 16 akcií a jedno ETF z databáze NYSE. Tato data jsou na jednominutové frekvenci a pokrývají období od ledna 2008 do července 2009, přičemž se obecně předpokládá, že finanční krize začala pádem akcií Lehman Brothers 9. Září 2008. V práci používám pět na modelech závislých a tři na modelech nezávislých indikátorů cenových skoků. Výsledky potvrzují vzestup celkové volatility během finanční krize, ale výsledky na druhou stranu nemohou odmítnout hypotézu, že nedošlo ke změně struktury cenových skoků. Tento výsledek říká, že nejistota během krize byla celkově zvětšená, ale samotná struktura nejistoty se nezměnila.

Keywords: financial markets, price jumps, extreme price movements, financial crisis.
JEL Classification: G01, P59.

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1 Introduction

Financial markets are uncertain even where there is no crisis. Uncertainty means that when we observe the price process for any financial instrument, we see that the price process follows a stochastic-like path. This path can be with or without a deterministic drift; however, the price process is in any case smeared by noise movements. The noise movements, known as market volatility, make the price unpredictable. However, the unpredictability of the price movements is not a priori a negative feature, it is rather the nature of financial markets since many different interests meet there. Unpredictability, though, can carry important information when the markets are working properly and noone has an inappropriate informative advantage. Thus, it is of great interest to describe the noise movements as accurately as possible (Gatheral, 2006). Such a description can then be used both in the financial industry to minimize risk and in theoretical economics, where various models of financial behavior are proposed. In addition, a deeper empirical understanding of market volatility during the recent financial crisis can shed some light on the crisis itself and thus helps to deal with future crises. In this work, I contribute to this field by studying the behavior of the extreme noise movements of high-frequency stock returns.

The literature suggests that financial markets reveal a striking characteristic of noise price movements. These noise movements can be decomposed into two components, see e.g., Giot, Laurent and Petitjean (2010), which are very different in nature. The first component, termed as regular noise, represents noise that is frequent but does not bring any abrupt changes. Regular noise stems from the statistical nature of the markets, where markets are simply a result of the interplay among many different market players with different incentives and different financial constraints. This interaction of many different agents can be mathematically described a the standard Gaussian distribution. It is the Gaussian nature of the first component that makes it easy to deal with in mathematical models of the price processes of financial instruments. Hence, various characteristics of financial instruments can be established and expectations can be calculated.

The second component, known as price jumps, is rare but very abrupt price movements. Price jumps do not fit into the description of the first noise component and thus have to be treated on their own, see e.g., Merton (1976). However, the mathematical description of price jumps cannot be easily handled. Therefore, the calculations of various market characteristics in the presence of price jumps are very difficult (Pan, 2002; Broadie and Jain, 2008). The serious problems in the mathematical description of price jumps are very often the reason why price jumps are wrongly neglected. In addition, it is still not clear what the main source of price jumps is.

A possible explanation of the source of these jumps says that they originate in the herd behavior of market participants (Cont and Bouchaud, 2000; Hirshleifer and Teoh, 2003). An illustration of such behavior is a situation when a news announcement is released, and every market participant has to accommodate the impact of that announcement. However, this herding behavior can provide an arbitrage opportunity and can be thus easily questioned. Another explanation says that the source of price jumps can lie in hidden liquidity problems (Bouchaud, Kockelkoren and Potters, 2004; Joulin, Lefevre, Grunberg and Bouchaud, 2008). A hidden liquidity problem is when either the supply or the demand side faces a lack of credit and thus is not able to prevent massive price changes. Both of the presented explanations are very different in nature. Thus, it is impossible to predict a priori what the change would be in price jump behavior in the recent financial crisis.

The two components of the noise movements together contribute to the volatility of the market. In this paper, I focus on both components of market volatility separately and study the change of each of them over time, with an explicit focus on the period of the recent financial crisis. It is widely accepted that periods of financial turbulence cause higher volatility on the market as investors become more nervous and tend to over-react to bad signals (Andersen, Bollerslev, Diebold and Vega, 2007). However, it is still not well described empirically how the two components of market volatility change during the crisis. Thus, this study focuses on this issue. Let us assume that a ratio between the two components during the not-so-bad times varies in some specific range. The question would be how would the same ratios vary during bad times, namely, how would the ratio of price jump volatility to regular noise volatility change during the recent financial crisis.

The goal of my paper is to explicitly answer two questions. First, I ask whether an overall increase in market volatility during the recent financial crisis occurred. Second, I focus on the part corresponding to price jump volatility and ask whether the behavior of price jumps changed during the recent financial crisis. To answer these questions, I employ 16 highly traded stocks and one Exchange Traded Fund (ETF) from the North American exchanges found in the TAQ database. These highly traded stocks represent a significant portion of the traded financial assets. Data from the TAQ database are originally at the tick level; thus, I have to integrate them to a 1-minute frequency. The data set spans from January 2008 to July 2009. It is found that the overall volatility significantly increased in September 2008 when Lehman Brothers filed for Chapter 11 bankruptcy protection. In addition, the periods immediately after this announcement reveal significantly higher levels of volatility. However, the ratio between the regular noise and price jump components of volatility does not change significantly during the crisis. The results suggest individual cases where the ratio increases as well as decreases. It is not possible to draw any industry-dependent conclusions.

This paper contributes to the understanding of market volatility in several ways. In addition to confirming the increase in volatility during the recent financial crisis, I extend the discussion of the decomposition of volatility into two components, which has not been well developed in the literature. I employ various technical indicators to estimate the rate of price jumps, i.e., the second component of volatility. This shows that my approach has several advantages. First, such an approach makes results more robust. Second, many papers focus on one of the indicators employed in my work and thus a direct comparative analysis is not possible. A comparative analysis, however, is one of the outcomes of my paper because I use several indicators on the same data. Third, I employ both model-dependent and model-independent indicators of price jumps on the same data. The same data set containing real prices used for both kinds of indicators is the reason why a comparison of the results can shed light on the validity of the underlying models, which are tacitly assumed to be valid when the model-dependent indicators are derived.

2 Literature Review

2.1 Motivation for Price Jumps

The literature contains a broad range of ways to classify volatility. Each classification is suitable for an explanation of a different aspect of volatility or an explanation of volatility from a different point of view, see e.g., Harris (2003) where the volatility is discussed from the financial practitioners' points of view. In the context of my work, the most important aspect is to separate the Gaussian-like component from price jumps. This separation can be seen in the first pioneering papers dealing with price jumps (see e.g., Merton, 1976 or a summary in Gatheral, 2006). Recently, the division in the Gaussian-like component and price jumps was used by Giot, Laurent and Petitjean (2010). Despite the fact that the motivation for this separation can be purely mathematical, it can be advocated by financial intuitions.

The first reason lies in the primary cause of price jumps. The literature supports two main explanations of the source of price jumps. Bouchaud, Kockelkoren and Potters (2004) and Joulin et al. (2008) advocate jumps are mainly caused by a local lack of liquidity on the market or what they call relative liquidity. In addition, the two papers also claim that an effect of news announcements on the emergence of price jumps can be neglected. On the contrary, Lee and Mykland (2008) and Lahaye, Laurent and Neely (2009) conclude that news announcements are a significant source of price jumps. They also show a connection between macroeconomic announcements and price jumps on developed markets.

Price jumps, understood as an abrupt price change over a very short time, are also related to a broad range of market phenomena that cannot be connected to the noisy Gaussian distribution. For example the inefficient provision of liquidity caused by an imbalanced market micro-structure can cause extreme price movements (see the survey in Madhavan, 2000). Price jumps can also reflect moments when some signal hits the market or a part of the market. Therefore, they can serve as a proxy for these moments and be utilized as tools to study market efficiency (Fama, 1970) or phenomena like information-driven trading, see e.g., Cornell and Sirri (1992) or Kennedy, Sivakamur and Vetzal (2006). An accurate knowledge of price jumps is necessary for financial regulators to implement the most optimal policies, see Becketti and Roberts (1990) or Tinic (1995). Finally, the non-Gaussian price movements influence the models employed in finance to estimate the performance of various financial vehicles (Heston, 1993; Bates, 1996; Scott, 1997; Gatheral, 2006).

2.2 Review of the Price Jumps Empirics

Generally, a price jump is understood as an abrupt price movement that is much larger when compared to the current market situation. The advantage of this definition is that it is model-independent: it does not require any specific form of an underlying price-generating process. On the other hand, this definition is too general and hard to explicitly define and test. The best way to treat this definition is to define the indicators for price jumps that fulfills the intuitive definition. The indicators are by definition parametrized. These parameters govern, for example, the length of the history to which returns are referred or a certain threshold.

Alternatively, price jumps can be defined in such a way where some specific form of the underlying price process is assumed. The most frequent approach in the literature is based on the assumption that the price of asset S_t follows stochastic differential equation, where the two components contributing to volatility, i.e., regular noise and price jumps, are modeled as

$$dS_t = \mu_t dt + \sigma_t dW_t + Y_t dJ_t \, , \tag{1}$$

where μ_t is a deterministic trend, σ_t is time-dependent volatility, dW_t is standard Brownian motion and $Y_t dJ_t$ corresponds to the Poisson-like jump process (see e.g., Merton, 1976). The term $\sigma_t dW_t$ corresponds to the regular noise component, while the term $Y_t dJ_t$ corresponds to price jumps. Both terms together form the volatility of the market. Based on this assumption for the underlying process, one can construct price jump indicators and theoretically assess their efficiency. Their efficiency, however, deeply depends on the assumption that the underlying model holds. Any deviation of the true underlying model from the assumed model can have serious consequences on the efficiency of the indicators.

The remaining part of this section discusses the price jump indicators based on both approaches: the model-independent price jump indicators and the model-dependent price jump indicators.

Model-independent Indicators

The model-independent price jump indicators do not require any specific form of underlying price process. This paper introduces the following indicators to measure the rate of price jumps in financial markets: extreme returns, temperature, p-dependent realized volatility, the price jump index, and the wavelet filter.

Extreme Returns

Price jumps are intuitively understood as very high or very low returns. This intuitive understanding of price jumps gives rise to the definition of an extreme returns indicator testing for the presence of a price jump at a given particular time t. Hence, a price jump is present at time t if the return at time t is above some threshold. The threshold value can be selected in either of two ways. It can be selected globally, where there is one threshold value for the entire sample, e.g., the threshold is a given centile of the distribution of returns over the entire data set. Or, it can be selected locally, i.e., some sub-samples have different threshold values. A global definition of the threshold allows us to compare the behavior of returns over the entire sample. However, the distribution of returns can vary, e.g., the width of the distribution can change due to changes in market conditions, and thus the global definition of the threshold is not suitable to directly compare price jumps over periods with different market conditions.

There are three versions of the extreme returns indicator. The first definition gives rise to absolute returns $|r_\tau|$. In this case, a price jump occurs at time τ, if the absolute return exceeds the $(100 - X)$-th centile of the entire distribution of absolute returns. This definition assumes a symmetric distribution centered around zero.

Second, the assumption about centering the distribution around zero is omitted. Then, centered absolute returns can be defined as $|r_\tau - \langle r_\tau \rangle_S|$. Hence, a price jump occurs at time τ if the centered absolute return exceeds the $(100 - X)$-th centile of the entire distribution of centered absolute returns. In this definition, $< X >_S$ stands for the mean taken over the entire sample.

Third, the extreme price jump indicator can be defined generally without any assumption about the specific symmetry of the underlying distribution. In this case, a price jump occurs at time τ if the return is either below the $X/2$-th centile or above the $(100 - X/2)$-th centile, where centiles are calculated from the entire sample.

Temperature

Kleinert (2009) shows that high-frequency returns at a 1-minute frequency for the S&P 500 and the NASDAQ 100 indices have the property that they have purely an exponential behavior for both the positive as well as negative sides.[1] The distribution can fit the Boltzmann distribution

$$B(r) = \frac{1}{2T} \exp\left(\frac{-|r|}{T}\right), \tag{2}$$

where T is the parameter of the distribution conventionally known as the temperature, and r stands for returns. The distribution is assumed to be symmetrically centered around zero. The parameter T governs the width of the distribution; the higher the temperature of the market, the higher the volatility. This follows from the fact that the second centered moment for this distribution is $\sigma_T^2 = 2T^2$. Silva, Prange and Yakovenko (2004); Kleinert and Chen (2007); Kleinert (2009) and Kleinert (2009) document that this parameter varies slowly, and its variation is connected to the situation on the market.

***p*-dependent Realized Volatility**

Realized volatility can be calculated in a standard way as the second centered moment in a given sample. This definition is a special case in the general definition of the p-dependent realized volatility

$$pRV_T^p(t) = \left(\sum_{\tau=t-T+1}^{t} |r_\tau|^p\right)^{1/p}, \tag{3}$$

where the sample over which the volatility is calculated is represented by a moving window of length T (see e.g., Dacorogna, 2001). The interesting property of this definitionis that the higher the p is, the more weight the outliers have. Since price jumps are simply extreme price movements, the property of realized volatility can be

[1]There is a small exception at the tails of the distribution.

translated into the following statement: The higher the p is, the more price jumps are stressed. Naturally, the ratio of two realized volatilities with different p can be thus used as an estimator of price jumps.

Price Jump Index

The price jump index $j_T(t)$ at time t (as employed by Joulin et al., 2008) is defined as

$$j_{T,t} = \frac{|r_t|}{< |r_t| >_T} \,, \tag{4}$$

where the history is simply calculated as $< |r_t| >_T = \frac{1}{T}\sum_{i=0}^{T-1} r_{t-i}$ and T is the market history employed.

The distribution of the price jump index $j_{T,t}$ for extreme price movements shows fat tails, i.e.,

$$P(j_T > s) \propto s^{-\alpha_T^{(f)}} \,, \tag{5}$$

where α_T is usually called the characteristic coefficient and explicitely depends on the length of the time window T, and s is a threshold value for the price jump index. Generally holds that the lower the α, the more jumpy the time series on average. The characteristic coefficient serves as a measure of the jumpiness of the data.

Wavelet Filter

The Maximum Overlap Discrete Wavelet Transform (MODWT) filter represents a technique that is used to filter out effects at different scales. In the time series case, the scale is equivalent to the frequency, thus, the MODWT can be used to filter out high frequency components of time series. This can be also described as the decomposition of the entire time series into high- and low-frequency component effects (see e.g., Gencay, Selcuk and Whitcher, 2002).[2] The MODWT technique projects the original time series into a set of other time series, where each of the time series captures effects at a certain frequency scale.

Applying the MODWT technique, the original time series $\{X_t\}$ is deconstructed as $\{X_t\} = \sum_{i=1}^{N}\{\tilde{W}_{i,t}\} + \{\tilde{V}_{2,t}\}$. The time series $\left\{\tilde{W}_{1,t}\right\}$ consists of the fastest effects. The time series $\left\{\tilde{W}_{i,t}\right\}$ with a higher index i capture effects at lower frequencies. Finally, the $\left\{\tilde{V}_{N,t}\right\}$ is a time series after filtering out the effects captured by the N previous time series $\left\{\tilde{W}_{i,t}\right\}$. The construction of the MODWT filter for $N = 2$ is defined as:

$$\tilde{W}_{1,t} = \sum_{l=0}^{L-1} \tilde{h}_{1,l} X_{t-l \bmod N} \text{ and, } \tilde{W}_{2,t} = \sum_{l=0}^{L-1} \tilde{h}_{1,l} \tilde{V}_{1,t-2l \bmod N}$$

and

$$\tilde{V}_{1,t} = \sum_{l=0}^{L-1} \tilde{g}_{1,l} X_{t-l \bmod N} \text{ and, } \tilde{V}_{2,t} = \sum_{l=0}^{L-1} \tilde{g}_{1,l} \tilde{V}_{1,t-2l \bmod N},$$

where $\tilde{h}_l$ and $\tilde{g}_l$ are coefficients defining a given wavelet filter.

The most straightforward way to study the contribution of processes at certain scales is to calculate the energy decomposition of the price time series. The energy decomposition of the time series for $N = 2$ is defined as $\|X\|^2 = \left\|\tilde{W}_1\right\|^2 + \left\|\tilde{W}_2\right\|^2 + \left\|\tilde{V}_2\right\|^2$, where $\|X\|$ is the standard L^2 norm.

Model-dependent Indicators

Model-dependent indicators assume a specific form of the underlying price process. The remaining part of this section follows the main stream in the literature and assumes that the price process is governed by eq. (1). Three indicators are introduced in this paper: the integral and differential indicators based on the difference between the bi-power variance and standard deviation, and the bi-power statistics

[2] The high-frequency component is intuitively connected to price jumps.

for the identification of price jumps.

The Difference between Bi-power Variance and Standard Deviation

Barndorff-Nielsen and Shephard (2004) discuss the role of the standard variance, or the second centered moment, in the models where the underlying process follows eq. (1). In such a case, the standard variance captures the contribution from both the noise and the price jump process. In addition, the authors show that a definition exists for the realized variance, which does not take into account the term with price jumps. Such a definition is called the realized bi-power variance. The difference between the standard and the bi-power variance can be used to define indicators that assess the jumpiness of the market. Generally, there are two ways to employ bi-power variance: the differential approach and the integral approach.

The Differential Approach The standard variance is defined as

$$\hat{\sigma}_t^2 = \frac{1}{T-1} \sum_{\tau=t-T}^{t-1} \left(r_\tau - < r_{\tau'} >_T\right)^2 \tag{6}$$

with $< r_{\tau'} >_T = \frac{1}{T} \sum_{i=0}^{T-1} r_{t-i}$.

The bi-power variance is defined according to Barndorff-Nielsen and Shephard (2004) as

$$\hat{\hat{\sigma}}_t^2 = \frac{1}{T-2} \sum_{\tau=t-T+2}^{t-1} |r_\tau| \, |r_{\tau-1}| \ . \tag{7}$$

The ratio between the two variances, defined as $R_t^{S/BP} = \hat{\sigma}_t^2 / \hat{\hat{\sigma}}_t^2$, satisfies by definition $R_t^{S/BP} \geq 1$. The higher the ratio, the more jumps are contained in the past T time steps back. This method is called a differential since it treats the jumpiness of the markets at every time step.

The Integral approach The integral approach is motivated by the work of Pirino (2009). The integral approach employs the two cumulative estimators for the total volatility over a given period. The first one is the cumulative realized volatility estimator defined as

$$RV_{Day} = \sum_{Day} (r_\tau)^2 \ ,$$

where the sum runs over all prices inside a given period.

The second estimator is the bi-power cumulative volatility estimator defined in an analogous way to equation (7):

$$BPV_{Day} = \frac{\pi}{2} \sum_{Day} |r_\tau| \, |r_{\tau-1}| \ ,$$

where the sum runs over all entries inside a given period and $\pi/2$ is a normalization constant. This estimator does not take into account the contribution of price jumps. Analogously to the previous case, the ratio of the two cumulative estimators defined as $R_{Day}^{RV/BPV} = RV_{Day}/BPV_{Day}$ serves as a measure of the relative contribution of price jumps to the overall volatility over the particular period.

Bi-power Test Statistics

The bi-power variance can be used to define the proper statistics for the identification of price jumps one by one. This means testing every time step for the presence of a price jump as defined in equation (1). These statistics were developed by Andersen, Bollerslev and Dobrev (2007) and Lee and Mykland (2008) and are defined as $\mathcal{L}_t = r_t/\hat{\hat{\sigma}}_t$, where all the symbols are in agreement with the previous definitions. Following Lee and Mykland, the variable ξ is defined as

$$\frac{\max_{\tau \in A_n} |\mathcal{L}_\tau| - C_n}{S_n} \rightarrow \xi \,, \tag{8}$$

where A_n is the tested region with n observations and the employed parameters are $C_n = \frac{(2\ln n)^{1/2}}{c} - \frac{\ln \pi + \ln(\ln n)}{2c(2\ln n)^{1/2}}$, $S_n = \frac{1}{c(2\ln n)^{1/2}}$ and $c = \sqrt{2}/\sqrt{\pi}$.

The variable ξ has in the presence of no price jumps the cumulative distribution function $P(\xi \leq x) = \exp\left(e^{-x}\right)$. The knowledge of the underlying distribution can be used to determine the critical value ξ_{CV} at a given significance level. Whenever ξ is higher than the critical value ξ_{CV}, the hypothesis of no price jump is rejected, and such a price movement is identified as a price jump. In contrast, when ξ is below the critical value, we cannot reject the null hypothesis of no price jump. Such a price movement is then treated as a noisy price movement. These statistics can be used to construct a counting operator for the number of price jumps in a given sample.

3 Data and Descriptive Statistics

3.1 Data Selection

I employ a set of 16 stocks and one ETF from the Trade and Quote database (TAQ) established by NYSE. Data ranges from the beginning of January 2008 to the end of July 2009. The selected time span covers the critical period of the financial crisis, whose peak was in September 2008. Table 1 summarizes the selected stocks, where stocks are ordered alphabetically according to their tickers.

The stocks used for this analysis accord to several criteria. First, all the stocks are heavily traded with a large intraday stock flow. This fact is important for the derivation of the homogeneous time series, which is extensively described in the following section. Second, the stocks and the ETF selected for this study are of the high market importance. Market importance can be judged in several ways. The most obvious is the market capitalization of the company and its inclusion into the main stock market indices. Therefore I include stocks that are a substantial part of the S&P 500 index. Since the S&P 500 index is a capitalization-weighted stock market index, the larger its weight, the more capitalized a company is. I also include stocks with a large weight in the Dow Jones Industrial Average index.[3]

[3] The exact composition of the Dow Jones Industrial Average index is discussed in the Appendix.

Table 1: List of stocks and ETF used in this analysis.

ID	Ticker	Name	Sector	Reason/Index	Avg. Daily Vol. Traded 11/09
1	APPL	Apple Inc.	Information Technology	S&P500	15,856,500
2	BAC	Bank of America Corp.	Financials	S&P500; DJI	163,920,700
3	C	Citigroup, Inc.	Financials	see Note A	300,960,700
4	CVX	Chevron Corp.	Energy	S&P500; DJI	9,415,900
5	GE	General Electric Co.	Industrials	S&P500; DJI	80,429,200
6	GOOG	Google Inc.	Information Technology	S&P500	2,131,500
7	HPQ	Hewlett-Packard	Information Technology	S&P500; DJI	15,752,200
8	IBM	Intl. Business Machines Corp.	Information Technology	S&P500; DJI	5,852,500
9	JNJ	Johnson & Johnson	Health Care	S&P500; DJI	12,211,600
10	KO	Coca Cola Company	Consumer Staples	S&P 500; DJI	8,671,900
11	MSFT	Microsoft Corp.	Information Technology	S&P500; DJI	53,121,400
12	PFE	Pfizer	Health Care	S&P 500; DJI	50,452,700
13	PG	Procter & Gamble	Consumer Staples	S&P500; DJI	12,742,300
14	SPY	S&P 500 ETF	ETF	Note B	179,587,000
15	T	AT&T Inc.	Telecommunications Services	DJI	26,854,300
16	WFC	Wells Fargo & Co.	Financials	S&P500	38,866,300
17	XOM	Exxon Mobil Corp.	Energy	S&P500; DJI	18,893,700

Note: Stocks and ETF are sorted in alphabetical order by the TICKER name. The column Reason/Index captures the main reason for the stock to be included. S&P500 or DJI means that the stock has a substantial weight in the S&P500 index or the Dow Jones Industrial Average index, respectively. The reason to include Citigroup is its very high daily volume traded on exchanges. In addition, this stock is quite specific. Despite its very low nominal value, price movements of this stock influence other stocks. The abrupt changes in this stock are very often taken as a signal for changes in the financials sector. The SPY Exchange Traded Fund by was chosen because it tracks the performance of the S&P 500 index, very often considered as a benchmark for the performance of US stocks. The last column shows the average daily volume traded at US exchanges in November 2009. Data were taken from Yahoo! Finance (http://finance.yahoo.com)

This index is price-weighted, and therefore a large share in the index is taken by companies whose shares have the highest price. In addition, this index is considered to be a representative benchmark of the industrial performance of the US economy. Therefore selecting companies with a large weight in this index enables me to track changes in US industrial performance.

In addition to the companies selected due to their weights in the two main indexes, I have also included Citigroup, Inc. This stock was badly hit during the recent financial crisis and its value dramatically declined. However, this company traditionally has a large impact on financial markets, which advocates its inclusion.

Finally, I have also included an Exchange Traded Fund (ETF), which tracks the performance of the S&P 500 index.[4] This ETF serves as a vehicle—in reality it is quite popular and highly traded—for those who want to be exposed to the S&P 500 index performance as a whole. The ETF represents the benchmark for the performance of the US economy, which is why it is important to this study. Therefore, this ETF reflects the overall trends on the market since the excess movements by any single stock is smoothed out by other stocks.

In conclusion, the stocks selected for this analysis are important representatives of the US stock markets. They cover the markets from a market capitalization point of view as well as from an industrial point of view. However, the selection is still small and thus enables me to keep track of each stock during the analysis.

3.2 Data Frequency

The TAQ database contains two separate databases: realized trades and quotes. Data with quotes are useful to calculate the depth of the market, to study the market micro-structure or to estimate a fair price at a given tick, but a database with realized trades cannot be used on its own for any estimation of the price process on the tick-level or to study the market micro-structure. In this work, I derive the

[4]The ETF does not track the S&P 500 index precisely since the value of the ETF is less maintenance fees.

data at a 1-minute frequency from the database with realized trades. The data at a 1-minute frequency are defined as an equally weighted average over all trades inside a given minute. Such an average captures fully the trading activity over the entire period. In addition, this method smooths out the possible discrepancies in the data as well as the known problem with the bid-ask bounce (Huang and Stoll, 1997; Hasbrouck, 2002).

The equally weighted average of realized trades requires a sufficient amount of tick data in every minute. The selection of stocks, as described above, assures this requirement. To illustrate this, Table 1 contains the average daily traded volumes for March 2010. The very large volumes demonstrate that the selected stocks and the ETF have very large intraday activity.[5]

3.3 Data Filtering

Following the official data description provided by the NYSE (see the NYSE official website), I discard observations with the CORR flag—an indicator denoting an ex post correction of the given tick—different from zero as well as all entries with the COND flag equal to Z—COND with a value of Z denotes delayed entries. Following the paper of Brownlees and Gallo (2006), I then test the data for the presence of significant outliers. These outliers also have to be carefully discarded from the data. However, when the activity is high, the net effect of the outlier is averaged out when taking the equally weighted average over a given minute. I employ the condition used in Brownlees and Gallo (2006)

$$|p(t) - p_k(t)| < 3\sigma_k(t) + \gamma\,,$$

[5]The small number of realized trades per minute does not smooth out the bid-ask bounce. This consequently leads to a wrong estimate of the price in this particular minute. Therefore, I have included a formal check, which counts the number of realized trades per minute. Whenever the number of realized trades is less than 15—an empirically chosen threshold—the price for this minute is obtained by interpolation. This check assures that no spurious price jumps will be created.

where $p_k(t)$ is an average calculated for the moving window running $\pm k$ periods around time t, and $\sigma_k(t)$ is a standard deviation calculated on the same time window. Based on the Brownlees and Gallo (2006), I have chosen $\gamma = 0.005$ and $k = 5$.

3.4 Trading Hours

The data from the database come in tape time from 4:00:00 to 19:59:59. The trading hours for the exchanges included in the database are from 9:30:00 to 15:59:59. The trades realized before the official trading hours are in what is known as pre-opening market hours, while the trades realized after the official trading hours are in after-market hours. During the trading hours, I calculate the price of the stock at a given minute as an average of prices for all valid realized trades in this minute.

In this work, I study the main trading period and therefore completely discard after-market hours. The pre-opening hours, however, cannot be easily discarded. This comes from the fact that in a few cases, I need to estimate the current situation on the market, which is simply some statistics over a moving window going a given number of time steps back. Naturally, this can cause some problems for the initial moments of the trading day, where no data in the trading hours are available. Therefore, I employ the data from the pre-opening period to estimate the situation on the market before the official opening occurs.

Since the pre-opening period is not heavily traded, I have introduced the following empirical rule to estimate the situation on the market in the pre-opening period: I separate the two hours preceding the opening hours into 10-minute blocks, where each block will have a separate price. The price in the block is calculated as an average over all trades in the block. If the activity in the block is not high enough, if the number of trades is less than 50, the price for a given block is taken as the same as the price in the first minute following the block. The prices are estimated in a backward direction starting at the immediate moments preceding the opening of the markets.[6] This procedure utilizes more information for a given trading day compared to the case where the pre-opening hours would be completely cut off.

3.5 Descriptive Statistics

The descriptive statistics of returns provide the first insight into the behavior of price jumps. Returns are defined in a standard way as $r_t = \log(R_t/R_{t-1})$, where R_t is the average price of the stock (or ETF) for time t. Time is measured in minutes. Figure 1 depicts the first four moments of the distribution of returns—the measures of mean, standard deviation, skewness and kurtosis—calculated daily, i.e., every trading day.

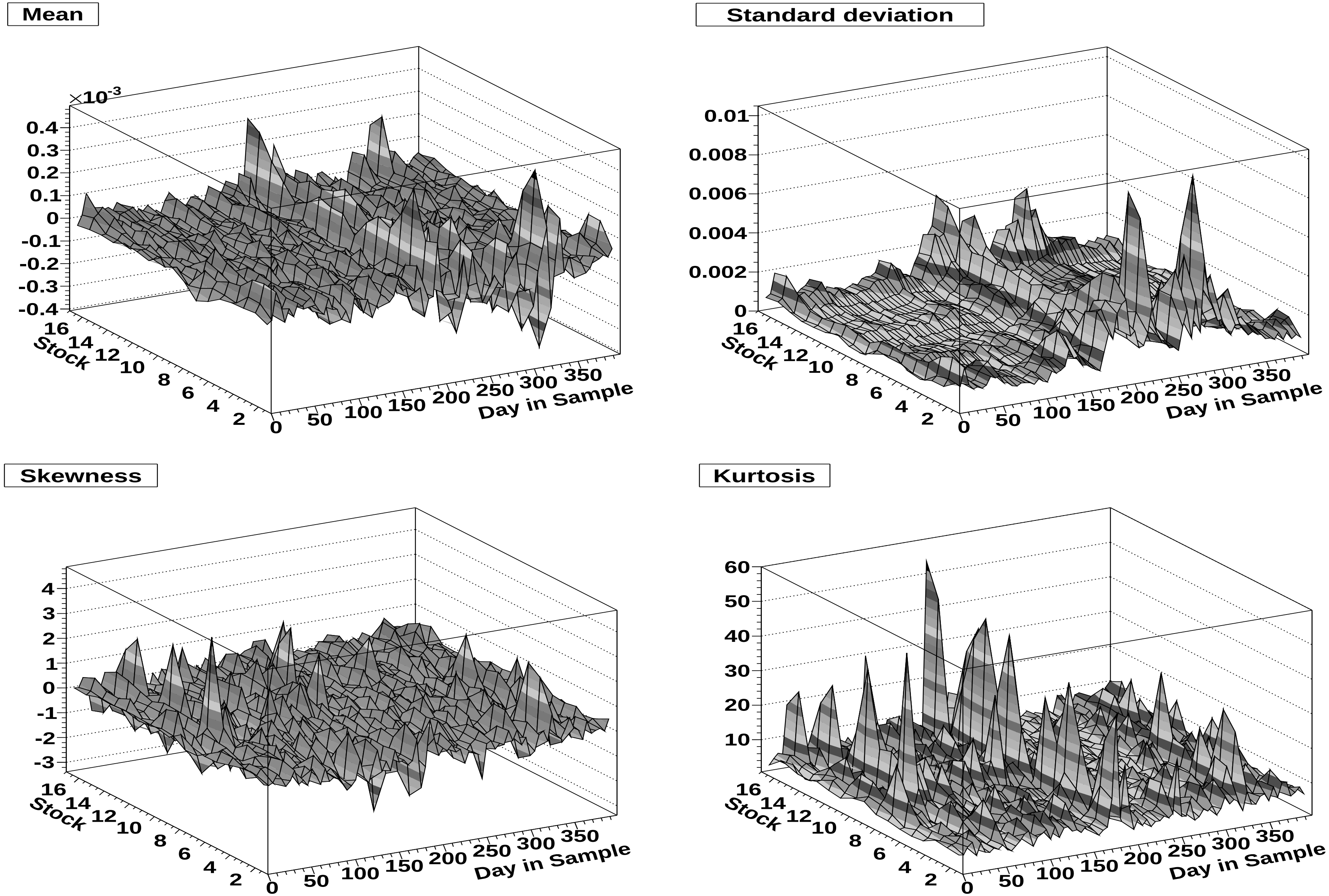
The results show that the mean fluctuates around zero. The rate of fluctuations has increased during the crisis. The first swing does not come directly after Lehman Brothers filing for bankruptcy protection.[7] However, it took some time for the markets to realize the oncoming problems. One month after the plunge of Lehman Brothers' shares, the markets were in crisis. At this time, we observed a big swing in the fluctuations of shares. In addition, mainly the stocks from the banking sector (Bank of America, Citigroup and Wells Fargo) experienced other significant turbulent periods starting in January 2009 and continuing in the first three months of 2009. The excessive movements in the means of returns can be explained by the market mood changing every day and stocks soaring one day and falling the next day.

Similar patterns can be concluded from the figure with standard deviation. In this case, however, the period with increased volatility started directly after the Lehman Brothers problems. The problems escalated and the volatility was reaching towards new heights. In addition, the banking sector and oil industry show strong

[6]First, I estimate the price for the period 9:20 to 9:29. If there is a low number of trades, the price is taken as the price at 9:30. Second, I estimate the price for the period 9:10 to 9:19. If there is a low number of trades, I take the price at 9:20, which is the price of the entire first block.

[7]The big plunge of Lehman Brothers shares occurred on September 9, 2008. This date corresponds to the Day174 in the Sample.

Figure 1: The first four moments of returns calculated day by day for every stock and ETF.



increases in volatility in the first months of 2009.

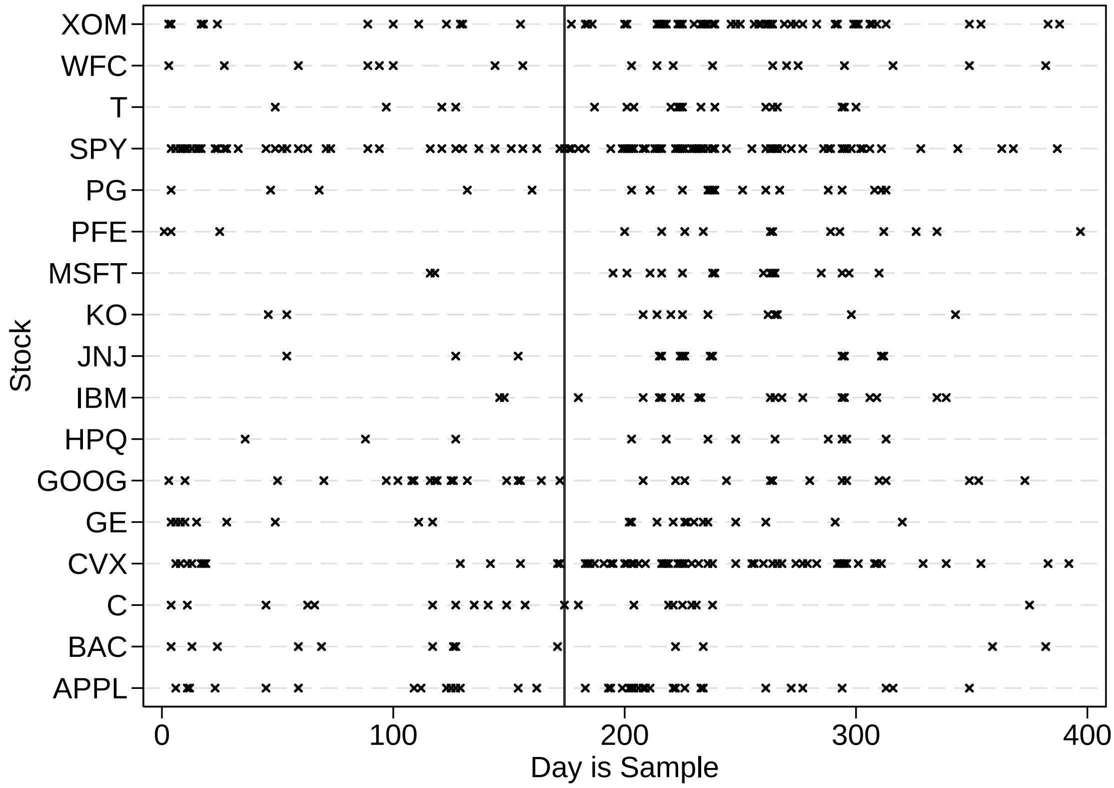
Skewness, on the other hand, does not reveal any striking systematic difference during the crisis. The measure of skewness oscillates but without any systematic pattern or without any change of rate in oscillations during the crisis. The measure of kurtosis, on the other hand, reaches very high values at the beginning of the crisis. After these heights, the kurtosis seems to be significantly lower. This means that the underlying distribution of returns was at the beginning of the crisis highly leptokurtic, i.e., with fatter tails and thus with a higher rate of price jumps, while after the first weeks of crisis, at the end of October 2008, the kurtosis reaches pre-crisis levels and the values show lower variance. This suggests “slimmer” tails on average with a low rate of price jumps.

3.5.1 Jarque-Bera statistics

In addition to the first four moments, a more subtle test is needed to test for the non-normality of returns and, thus, for the presence of price jumps. A standard test to address this question is to employ the Jarque-Bera statistics (Jarque and Bera, 1980) defined as $JB = \frac{N}{6}\left(S^2 + \frac{(K-3)^2}{4}\right)$, with S being the measure of skewness, K the measure of kurtosis and N the number of observations. The test is asymptotically equal to χ^2_2 and specifies the null hypothesis that data are iid and come from a Gaussian distribution. The alternative hypothesis means either a deviation from a Gaussian distribution or a non-iid feature of the underlying generating process.

Figure 2 depicts the result of the Jarque-Bera test. Namely, the Jarque-Bera statistics is calculated for every stock on a daily basis. Every day, the Jarque-Bera statistics is compared to the critical value of the χ^2_2 distribution at the 95% confidence level. For every stock and every trading day, there are two possible outcomes: the test statistics is either equal to or below the critical value or it exceeds the critical value. In the former case, we fail to reject the null hypothesis and tend to accept the fact that the underlying process is iid and follows a Gaussian distribution.

Figure 2: Jarque-Bera statistics for returns.



Note: The Jarque-Bera statistics was calculated for every stock and every trading day separately. Then, the statisitics was compared with the critical value at the 95% confidence level. The Figure contains 17 lines–one for eeach stock. Whenever there is a cross, the Jarque-Bera statistics did not exceed the critical value and therefore the null hypothesis of returns to come from iid Gaussian distribution cannot be rejected. The vertical line corresponds to September 9th, 2009–the day when the Lehman Brothers problems started.

In the latter case, we reject the null hypothesis and accept the alternative hypothesis. The situation where the Jarque-Bera statistics did not exceed the critical value is marked by a cross. In addition, the figure contains a vertical line denoting the day when the Lehman Brothers problems occured.

An eye check of the results confirms the observations inferred from the previous figure measuring a kurtosis. After the emergence of the crisis in October, there is a significant period of time where the Jarque-Bera statistics is rather low and even below the critical value. This, unsurprisingly, corresponds to the period with low levels of kurtosis. In addition, a visual inspection suggests that the ETF behaves according to the iid Gaussian distribution more often than other stocks. This comes from the fact that the ETF mimics the composition of the S&P 500 index and is thus composed of many underlying stocks, where extremes coming from a single stock are averaged out and only those extremes which occurred at the same time remain.

4 Methodology

I first summarize the price jump indicators used for this study and then outline the procedure for how the indicators were employed.

4.1 Indicators

The Literature Review section contains an extensive overview of the price jump indicators. To summarize, I shall employ the following set of price jump indicators:

1. Model-independent indicators
   (a) Extreme returns
   (b) Temperature
   (c) p-dependent realized volatility
   (d) The price jump index
   (e) The wavelet filter
2. Model-dependent indicators
   (a) The difference between bi-power variance and standard deviation
      (i) The differential approach
      (ii) The integral approach
   (b) Bi-power test statistics

The indicators, as they are explained in the previous sections, are by construction very different. Besides the obvious division of model-independent and model-dependent, they can also be divided from another point of view: whether they aim to exactly identify price jumps or rather to assess the jumpiness of the markets. The jumpiness of the markets is understood as a measure of the rate of price jumps occurring during a specified period without counting price jumps explicitly. In both cases, I shall refer it as the price jump measure.

The price jump indicators that identify price jumps explicitly are extreme returns and bi-power test statistics. The rest of the indicators can be utilized to construct exact price jump indicators, however, they are employed as a measure of jumpiness throughout this work. Therefore, whenever the two periods are compared with respect to price jumps, they are either compared by counting the number of price jumps or by comparing the measure of price jumps, i.e., the jumpiness.

4.2 Definition of the Financial Crisis

The main goal of this work is to answer the question whether the current financial turmoil caused any change in the price jump behavior in the financial markets using high-frequency data. I approach the problem by dividing the entire sample into sub-samples corresponding to individual trading days. For every day, I assess the number of price jumps or the measure for jumpiness. Then I compare the statistics of these measures for days before the crisis with the statistics for days during the crisis.

Deciding when the crisis started is not clear and cannot be done explicitly, I rather choose the start of the crisis based on the main events that happened in the financial markets. I consider as the first main event triggering the financial crisis the plunge of the shares of Lehman Brothers on September 9, 2008. Based on this event, I define the financial crisis as a structural break in the behavior of financial markets. I employ two different versions of the breaking scheme:

- **The Permanent Break (PB):** The crisis started on September 9, 2008 and lasted until the end of the sample.

- **The Temporary Break (TB):** The crisis started on September 9, 2008 and lasted 30 trading days or for 1.5 months.

The first scheme is intuitive and is based on the fact that the crisis started with the problems of Lehman Brothers. The effect of the crisis was permanently present on financial markets at least until the end of July 2009. The second scheme, however, focuses solely on the most problematic days following the plunge of shares. The period of 30 working days was chosen based on the news and the behavior of financial markets. The two schemes thus provide different pictures. The first scheme answers the question about a permanent change in the behavior of financial markets, while the second scheme rather focuses on the immediate panic that spread through the financial markets and affected the trading habits of market participants.

The estimation of price jumps, or jumpiness, itself is done by employing the battery of tests described above. The tests were developed in the literature and very often require a fine-tuning process to obtain unbiased results. The fine-tuned indicators then allow us to measure the number of price jumps, or the jumpiness, at absolute levels. This defines the cardinal measure of price jumps. Having in hand the cardinal measure, the absolute numbers of price jumps can be interpreted on their own as well as the price jumps being identified with particular events at given moments. Such a formulation is, however, too strong to answer the main question: how can we compare the days relatively.

The weak formulation of employing the indicators is to use them as an ordinal measure. The indicators used in this way are not required to be absolutely unbiased. The bias in the number of price jumps can be present as soon as it is proportional to the number of price jumps. This still allows me to compare days with respect to the number of price jumps truthfully, i.e., to assess which of the days, or generally periods, were more jumpy.

In the remaining part of this section, I employ the battery of tests described in the preceding sections. I explicitly test the following hypothesis: The recent

Table 2: Conversion table for “Day in Sample” units and calendar days.

DiS	Calendar	DiS	Calendar	DiS	Calendar
1	Jan 2nd, 2008	148	Aug 1st, 2008	254	Jan 2nd, 2009
22	Feb 1st, 2008	169	Sep 2nd, 2008	274	Feb 2nd, 2009
42	Mar 3rd,2008	174	Sep 9th, 2008	293	Mar 2nd, 2009
62	Apr 1st, 2008	190	Oct 1st, 2008	315	April 1st, 2009
84	May 1st, 2008	203	Oct 20th, 2008	336	May 1st, 2009
105	Jun 2nd, 2008	213	Nov 3rd, 2008	356	Jun 1st, 2009
126	Jul 1st, 2008	232	Dec 1st, 2008	378	Jul 1st, 2009

Note: The table includes September 9th, 2008, when the Lehman Brothers' shares plunged, the beginning of the financial crisis. The table also includes October 20th, 2008. This day is used to define the end of the temporary break.

financial crisis caused no statistically significant change in the price jump properties of the price time series.

4.3 The Trading Days

The sample of price times series employed in this work covers the period spanning from January 2, 2008 to July 31, 2009. I divide the entire sample into sub-samples, each corresponding to one trading day. On every sub-sample, I apply the price jump indicators. Then I test for differences across days. The length of the sub-sample was chosen intuitively. This enabled me to obtain reasonable statistics within a day as well as between days.

Days in the sample are denoted in “Day in Sample” (DiS) units. The advantage of these units compared to calendar days is it makes the figures smooth, without gaps corresponding to weekends and holidays. The seeming disadvantage is the calendar days cannot be easily identified. Therefore, Table 2 provides a frame of reference for a conversion between DiS and calendar days. In addition, some important dates are mentioned in the table explicitly.

4.4 Hypotheses to Test

The indicators employed in this work measure the jumpiness of the financial markets on a daily basis. The indicators can be divided into two groups according to the way the daily measure is achieved. First, there are indicators that by construction estimate one number for every day. The second group of indicators gives an estimate of jumpiness for every tick. Then, the measure of jumpiness per day is obtained based on these tick estimates. These two different groups of indicators also imply different hypotheses to test with different meanings.

Thus, I form four different hypotheses in this work, two for each of the two groups of price jump indicators.

Group I: One Number per Trading Day The first group of indicators gives exactly one number per trading day. I divide the sample of trading days into two sub-samples. These two sub-samples are formed by trading days occuring during the crisis or not during the crisis. The period of the financial crisis is defined above.

Hypothesis I-A: The null hypothesis of this test says that the two sub-samples come from the same distribution. The main scope of this test is to compare whether the estimated price jump measures changed during the crisis.

Test: I employ the two-sample Wilcoxon test (see the Appendix) and test whether the estimated price jump measures for the two sub-samples come from the same distribution.

Hypothesis I-B The null hypothesis states that the variance of the two sub-samples, i.e., during and not during the crisis, are the same. This test questions whether the trading days in either of the two sub-samples were on average more heterogeneous. In other words, this procedure tests the heterogeneity of the trading days between the sub-samples.

Test: I employ the standard F-test and compare whether the variance of the estimated price jump measures changed during the crisis. The F-test is defined as

$$\frac{S_C^2}{S_{No-C}^2} \sim F_{(N_C-1, N_{No-C}-1)},$$

where S^2 is the standard deviation of the characteristic coefficient calculated during the crisis "C" and outside the crisis "$No - C$". The N_C is the number of days the crisis lasts and N_{No-C} is the complement to the total number of days in the sample.

Group II: One Number per Tick The second group of price jump indicators gives one number pertick, in my case one number per minute. Having in hand these numbers, I calculate mean and variance of these numbers per trading day. Analogously to the previous case, I divide the sample into two sub-samples.

Hypothesis II-A The null hypothesis of this test says that the means of the two sub-samples come from the same distribution. The main scope of this test is to compare whether the estimated price jump measures changed during the crisis.

Test: I employ the two-sample Wilcoxon test and test whether the daily means of the estimated price jump measures for the two sub-samples come from the same distribution.

Hypothesis II-B The null hypothesis of this test says that the means of the two sub-samples come from the same distribution. The main scope of this test is to question whether the heterogeneity inside the trading days changed during the financial crisis.

Test: I employ the two-sample Wilcoxon test and test whether the daily variances of the estimated price jump measures for the two sub-samples come from the same distribution.

5 Results

This section summarizes the results when all the price jump indicators were employed. The price jump indicators are described in the same order as above.

5.1 Model-independent Indicators

5.1.1 Extreme Returns

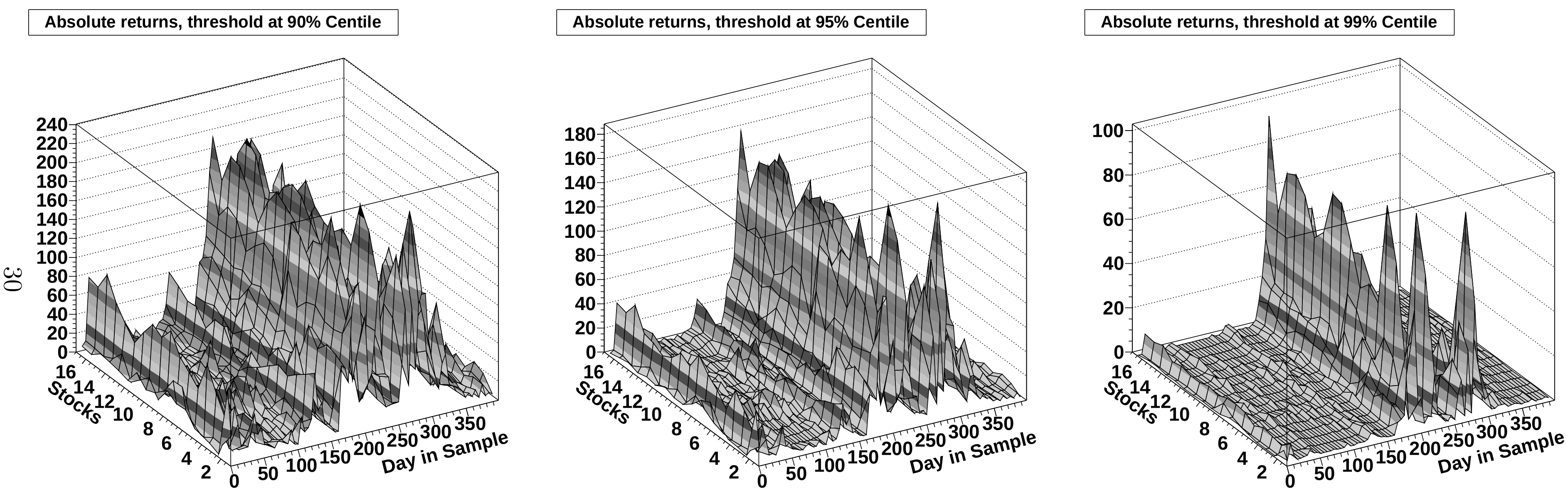
Extreme returns define price jumps globally. Figure 3 shows the number of absolute returns per day above the 90-th, 95-th and 99-th centile calculated from the distribution of the same quantity over the entire period. Figure 4 shows the number of absolute centered returns per day above the 90-th, 95-th and 99-th centile calculated from the distribution of the same quantity over the entire period. Figure 5 shows the number of returns per day below/above the 5/95-th, 2.5/97.5-th and 0.5/99.5-th centiles calculated from the distribution of the same quantity over the entire period, respectively.

The figures suggest that the period following the plunge of Lehman Brothers' shares is characterized by an increase in extreme returns. However, this does not directly respond to the question about the behavior of price jumps when price jumps are understood as extreme movements much bigger compared to the current market situation. Rather, the increased levels of the extreme returns indicator suggests a rise in market volatility. In addition to the period following the problems of Lehman Brothers, the turmoil period also appeared in the beginning of 2009, when extreme returns also rocketed up.

5.1.2 Temperature

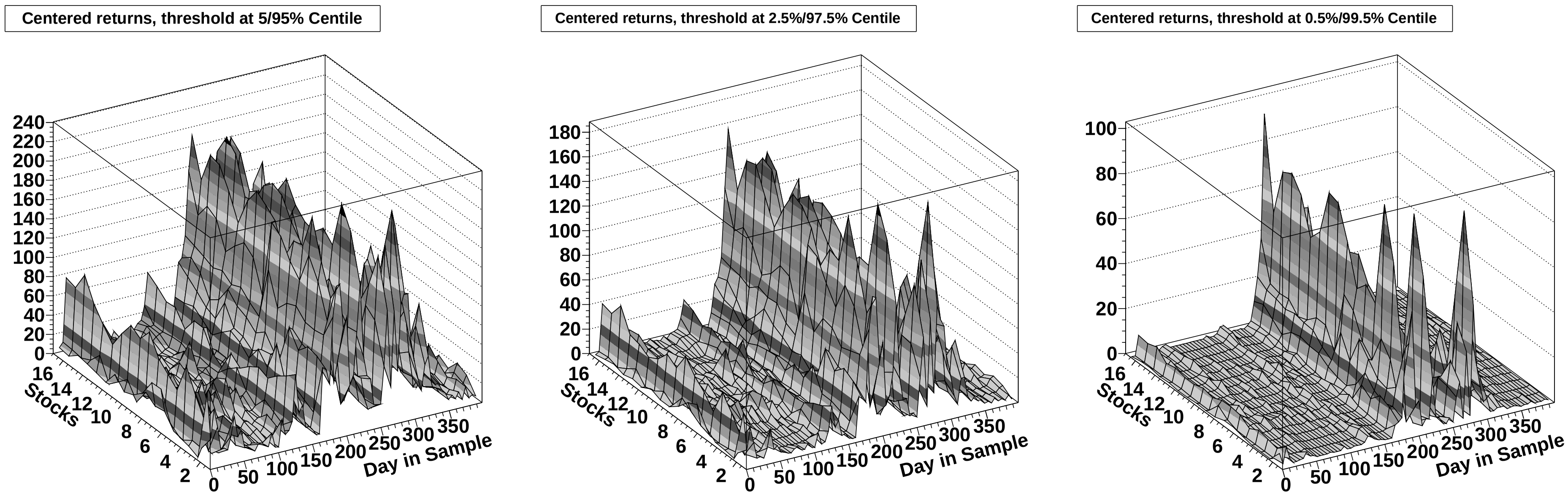
The temperature T is estimated according to equation (2). The equation is nonlinear, however, a practical way to approach it is to log-linearize it and then apply the standard least squares method. The linearized equation (2) reads

Figure 3: Price jump indicator based on absolute returns.



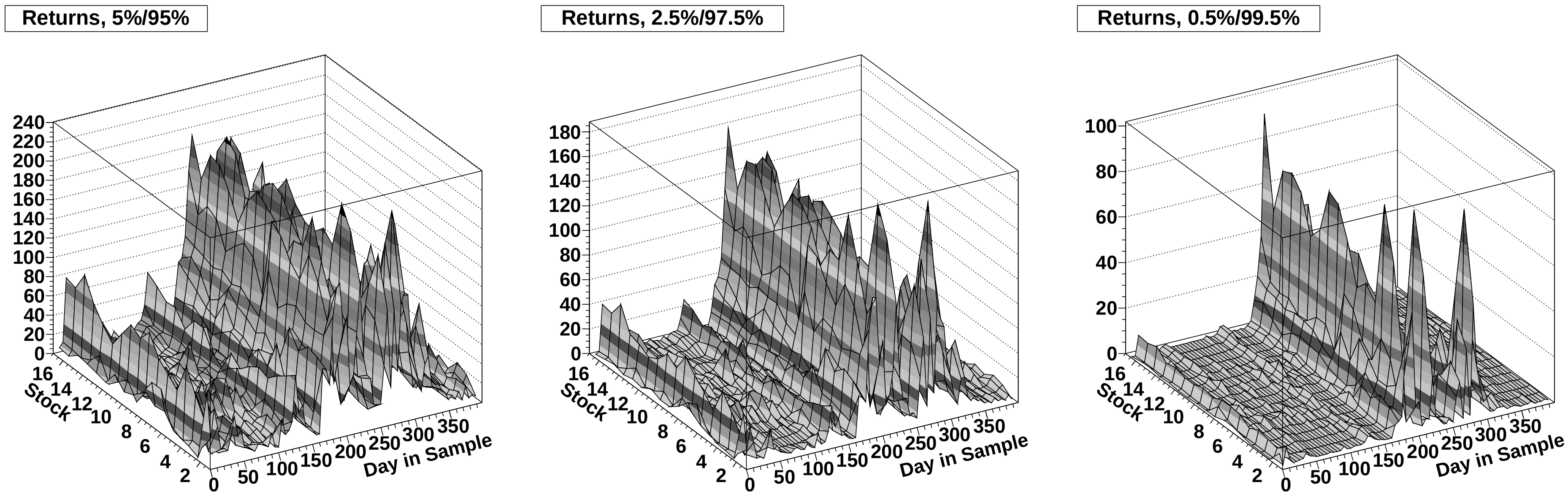
Note:Centiles are taken over the entire sample. The number of absolute returns above the threshold is calculated for every trading and every stock. The stocks are numbered according to Table 1.

Figure 4: Price jump indicator based on absolute centered returns.



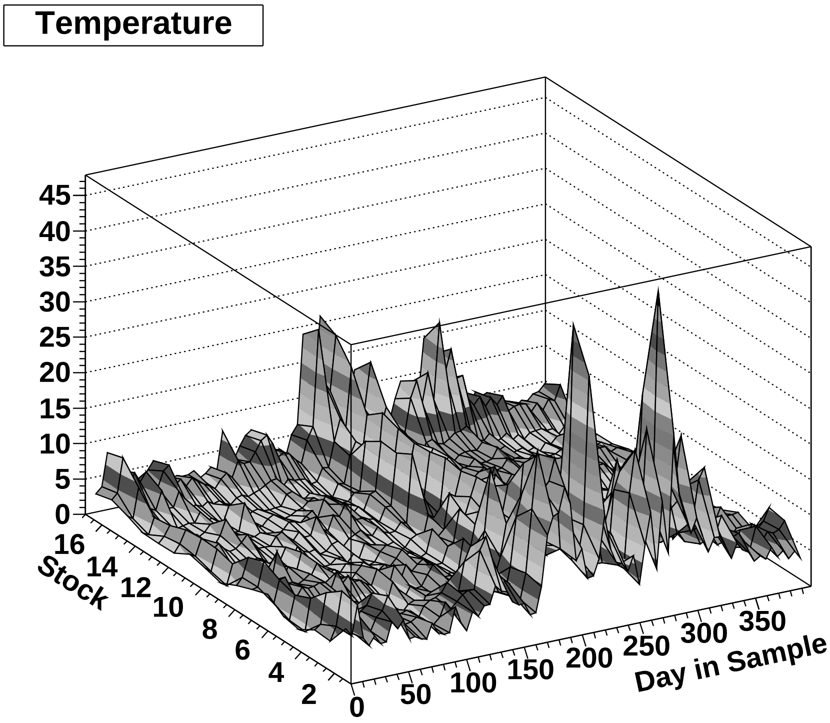
Note: Centiles are taken over the entire sample. The number of centered absolute returns above the threshold is calculated for every trading day and every stock. The stocks are numbered according to Table 1.

Figure 5: Price jump indicator based on extreme returns.



Note: Centiles are taken over the entire sample. The number of returns below/above the threshold is calculated for every trading day and every stock. The stocks are numbered according to Table 1.

Figure 6: Temperature.



Note: Temperature is estimated for returns for every stock and every trading day based on eq. (2). The stocks are numbered according to Table 1.

$$\ln B(r) = \ln \frac{1}{2T} - \frac{1}{T} r \,,$$

from which the inverse of the temperature can be directly estimated. When the estimation is carried out, returns are by definition assumed to be symmetric with respect to the origin.

Figure 6 shows the estimated temperature for every stock calculated day by day. The temperature does not distinguish price jumps. Therefore, these results support the same conclusion as those obtained from the extreme returns indicator. The period after Lehman Brothers' problems emerged is characterized by increased market volatility. In addition, the banking sector (Bank of America, Citigroup, and Wells Fargo) shows significantly higher market volatility at the beginning of 2009.

5.1.3 p-dependent Realized Volatility

I employ equation (3) for two particular values of $p = 1$ and $p = 4$. The realized volatility with $p = 4$ is relatively more sensitive to price jumps compared to the realized volatility with $p = 1$. I employ this difference between them to construct the following price jump indicator defined as

$$pRV_T^{p/p'}(t) \equiv pRV_T^p(t)/pRV_T^{p'}(t)\,, \tag{9}$$

where $pRV_T^p(t)$ is defined by eq. (3), and the two parameters governing the sensitivity to price jumps are $p = 4$ and $p' = 1$. In addition, this price jump indicator is defined for each time and takes into account the history of the T preceding time steps, including the current time. I employ two time windows for history: $T = 60$ and $T = 120$. The history at the beginning of the trading day is calculated from the pre-opening period, as is extensively discussed in the preceding sections.

This indicator captures the change in price jumps in the following way: If there is an extreme movement in either of the T time steps of the preceding window, the indicator will be higher compared to the situation without any price jump. The indicator keeps its higher value until the price jump is present in the moving window. The first occurrence of the high value of the indicator suggests the occurrence of a price jump.

I will explicitely test Hypothesis II-A and Hypothesis II-B, i.e., whether the means and the variances of the estimated ratio defined by equation (9) come from the same distribution during and not during the crisis. I do not report figures in this case since they do not provide any strong visible hints about the behavior of this indicator.

Hypothesis II-A: Table 3 shows the result of the Wilcoxon statistics for this hypothesis. The table contains the z-value of the test. The stars denote at what level of significance we can reject the null hypothesis, stating that means are the same over

Table 3: Result of the two-sample Wilcoxon test for the mean of $pRV_T^{p/p'}(t)$.

ID/Ticker	Permanent break		Temporary break	
	60	120	60	120
1 APPL	-3.332***	-4.362***	1.019	0.828
2 BAC	4.538***	3.283***	-0.118	-1.506
3 C	-3.110***	-0.828	0.195	0.775
4 CVX	6.021***	5.666***	1.754*	1.030
5 GE	2.543**	0.071	-1.310	-2.187**
6 GOOG	-1.956*	-1.427	0.511	0.205
7 HPQ	5.184***	4.102***	0.954	0.724
8 IBM	5.006***	3.633***	0.459	1.078
9 JNJ	5.442***	5.243***	0.457	0.110
10 KO	5.752***	5.329***	-0.248	-0.931
11 MSFT	2.133**	1.209	2.128**	1.578
12 PFE	1.821*	1.482	0.214	-0.294
13 PG	5.101***	4.967***	-0.055	-0.092
14 SPY	1.259	3.590***	3.307***	3.005***
15 T	4.265***	3.408***	1.129	0.503
16 WFC	4.133***	0.787	-3.455***	-3.160***
17 XOM	4.904***	2.913***	-0.034	-1.298

Note: The mean was calculated for every stock and every trading day. I have used the two definitions of the financial crisis. Permanent Break and Temporary Break, and two time window, $T = 60$ and $T = 120$ time steps. The table captures the z-statistics for the test. The additional stars denote whether we can reject H_0 that the two samples come from the same distribution and the corresponding confidence levels: 90% (*), 95% (**) and 99% (***). The overall positive/negative value of the z-statistics suggests that the median of the means is lower/higher during the financial crisis.

the entire period. In addition, the excessively high z-value of the Wilcoxon statistics corresponds to a situation when the median of means during the crisis is smaller than the median of means outside the crisis. The excessively low z-values mean the opposite. For illustration, the case of Apple using $T = 60$, and the financial crisis defined as a Permanent Break gives a z-value equal to -3.332, which means that we can reject the null hypothesis at the 99% confidence level. In addition, the negative z-value suggests that the median of means after the emergence of Lehman Brothers' problems is higher compared to the previous period. This means that Apple stocks were more jumpy during the crisis.

To summarize, the table shows that when the financial crisis is defined through

Table 4: Result of the two-sample Wilcoxon test for the variance of $pRV_T^{p/p'}(t)$.

ID/Ticker	Permanent Break		Temporary Break	
	60	120	60	120
1 APPL	-4.844***	-5.467***	0.888	1.899*
2 BAC	5.742***	4.933***	-0.218	-0.839
3 C	6.705***	7.286***	1.078	1.628
4 CVX	4.100***	3.991***	0.908	0.821
5 GE	1.579	-0.510	-0.148	-0.503
6 GOOG	-1.533	-0.807	0.307	-0.113
7 HPQ	2.653***	2.239**	1.303	0.930
8 IBM	2.098**	1.526	2.168**	1.879*
9 JNJ	3.466***	3.443***	1.140	0.485
10 KO	3.099***	3.382***	-0.059	-0.428
11 MSFT	0.685	0.494	2.115**	1.637
12 PFE	-0.840	-0.683	-0.526	-0.875
13 PG	4.761***	4.808***	1.060	0.701
14 SPY	-1.219	2.933***	-0.661	0.916
15 T	1.999**	1.979**	0.714	0.436
16 WFC	1.723*	-0.674	-0.614	-1.825*
17 XOM	0.483	-0.241	-1.423	-1.654*

Note: The variance was calculated for every stock and every trading day. I have used two definitions of the financial crisis. Permanent Break and Temporary Break, and two time window, $T = 60$ and $T = 120$ time steps. The table captures the z-statistics for the test. The additional stars denote whether we can reject H_0 of the two samples come from the same distribution and the corresponding confidence levels: 90% (*), 95% (**) and 99% (***). The overall positive/negative value of the z-statistics suggests that the median of variances is lower/higher during the financial crisis.

the Permanent Break, the distributions of the mean are more likely to be different. This suggests that no matter how turbulent the days were following the plunge of Lehman Brothers' shares, the crisis emerged in the subsequent months. In addition, the z-values tend to be positive, which suggests that the median of means for the p-ratio is lower during the crisis. This means that the rate of price jumps decreased during the crisis, or alternatively, price jumps were overwhelmed by the overall increase in the magnitude of returns.

Hypothesis II-B: Table 4 shows the results of the Wilcoxon test. The results are in agreement with the previous test in several aspects. First, the case of a Temporary Break does not lead to a situation where we can reject the null hypothesis about the agreement of the distributions of variance during and not during the crisis. Second, the stocks that had significantly different distributions of the ratio $pRV_T^{p/p'}(t)$ also tend to have significantly different distributions of the estimated variances.

5.1.4 The Price Jump Index

I estimate the characteristic coefficient α introduced in eq. (5). I estimate the coefficient for every trading day in the sample. I use two time windows, namely $T = 60$ and $T = 120$ time steps back. The estimation of the characteristic coefficient was done for the log-linearized version of equation (5). Then I employed OLS for the tail parts of the distribution to estimate α.[8] The price jump index captures the behavior of extreme price movements normalized by the current market situation and thus assesses the jumpiness of the financial markets. This indicator gives one number per trading day, thus I shall test Hypothesis I-A and Hypothesis I-B.

Hypothesis I-A: The results of the Wilcoxon test are in Table 5. The results show that the difference in the price jump behavior is more likely to occur when the financial crisis is defined using the Permanent Break. In addition, the results suggest that the banking sector was hit hard by the problems of Lehman Brothers. This observation follows from the fact that the three banks show a difference in the characteristic coefficient for both definitions of the financial crisis. In every case, the z-ratio is positive for the banking industry, which means that the median of the distribution of the characteristic coefficient is lower during the crisis, and therefore, returns for the banking indsutry's stocks were more jumpy. In addition, the results show that the price jump index captures different aspects of extreme price movements when compared to the previous indicator.

[8]The other methods to estimate α are discussed, for example, by Vaglica, Lillo, Moro and Mantegna (2008), who use MLE for estimation.

Table 5: Result of the two-sample Wilcoxon test for the means of the price jump index.

ID/Ticker	Permanent Break		Temporary Break	
	60	120	60	120
1 APPL	-1.543	-2.534**	1.433	2.704***
2 BAC	3.736***	6.006***	2.324**	2.018**
3 C	5.889***	7.057***	3.211***	2.601***
4 CVX	3.211***	3.428***	0.080	0.209
5 GE	2.645***	2.073**	0.999	1.206
6 GOOG	-1.880*	-1.740*	1.203	1.061
7 HPQ	0.967	0.987	1.563	0.391
8 IBM	0.494	0.773	3.442***	2.655***
9 JNJ	2.174**	2.912***	1.293	0.151
10 KO	1.976**	2.718***	1.335	0.704
11 MSFT	1.610	0.257	2.228**	1.470
12 PFE	1.106	0.852	1.086	-0.100
13 PG	1.719*	2.058**	1.764*	1.163
14 SPY	0.263	0.423	0.533	1.562
15 T	0.710	1.489	-0.502	-0.383
16 WFC	5.561***	4.610***	4.173***	2.805***
17 XOM	2.975**	2.538**	1.758*	1.488

Note: The characteristic coefficient was calculated for every stock and every trading day. I have used two definitions of the financial crisis. Permanent Break and Temporary Break, and two time window, $T = 60$ and $T = 120$ time steps. The table captures the z-statistics for the test. The additional stars denote whether we can reject H_0 of the two samples come from the same distribution and the corresponding confidence levels: 90% (*), 95% (**) and 99% (***). The overall positive/negative value of the z-statistics suggests that the median of characteristic coefficients is lower/higher during the financial crisis.

Hypothesis I-B: Table 6 shows the F-statistics defined above. The results clearly show that the characteristic coefficients for the price jump index tend to have different variances during both definitions of the financial crisis when the length of the moving window is rather short. Generally, the value of an F-statistic higher/lower than one suggests that the variance during the crisis was higher/lower relative to the variance outside the crisis, respectively.

The implication of this claim brings another interesting insight. There are stocks for which the F-statistic is significantly lower using one definition of the financial crisis and significantly higher for the other definition. This is the case, for example,

Table 6: Result of the two-sided F-test for the variance of the characteristic coefficients of the price jump index.

ID/Ticker	Permanent Break		Temporary Break	
	60	120	60	120
1 APPL	1.887***	1.112	1.160**	1.077
2 BAC	0.581***	0.859	2.021***	0.653
3 C	1.039	2.084***	1.345	1.071
4 CVX	2.874***	1.839***	0.973	0.437***
5 GE	1.541***	1.526***	1.077	0.941
6 GOOG	0.661***	0.858	0.928	0.685
7 HPQ	1.165	1.197	0.845	0.844
8 IBM	1.624***	0.985	2.113***	1.055
9 JNJ	1.296*	1.242	1.029	0.801
10 KO	1.169	1.657***	2.586***	1.949***
11 MSFT	1.057	0.720**	0.259***	0.422***
12 PFE	0.466***	0.501***	0.595*	0.602*
13 PG	1.108	1.158	0.780	0.988
14 SPY	1.204	0.933	2.594***	1.805**
15 T	1.499***	1.116	0.479**	0.714
16 WFC	2.173***	1.811***	1.361	1.299
17 XOM	1.216	1.586***	1.573*	1.488

Note: The null hypothesis says that the variances during and not during the crisis match. Stars denote, at what confidence level we can reject the null hypothesis: 90% (*), 95% (**) or 99% (***). In addition, the value of F-statistics higher/lower than one means that variance of the characteristic coefficient during the crisis was higher/lower when compared to the period not during the crisis. The two F-distributions are $F_{225,172}$ for the Permanent Break and $F_{29,368}$ for the Temporary Break.

for Bank of America stocks. In this case, the variance for the Permanent Break is lower during the crisis, while it is significantly higher for the Temporary Break. The period immediately following the plunge of Lehman Brothers shares was then dominated by huge movements in the characteristic coefficient. This means that a short, volatile period was followed by a long period with a rather stable characteristic coefficient, which causes a decrease of volatility.

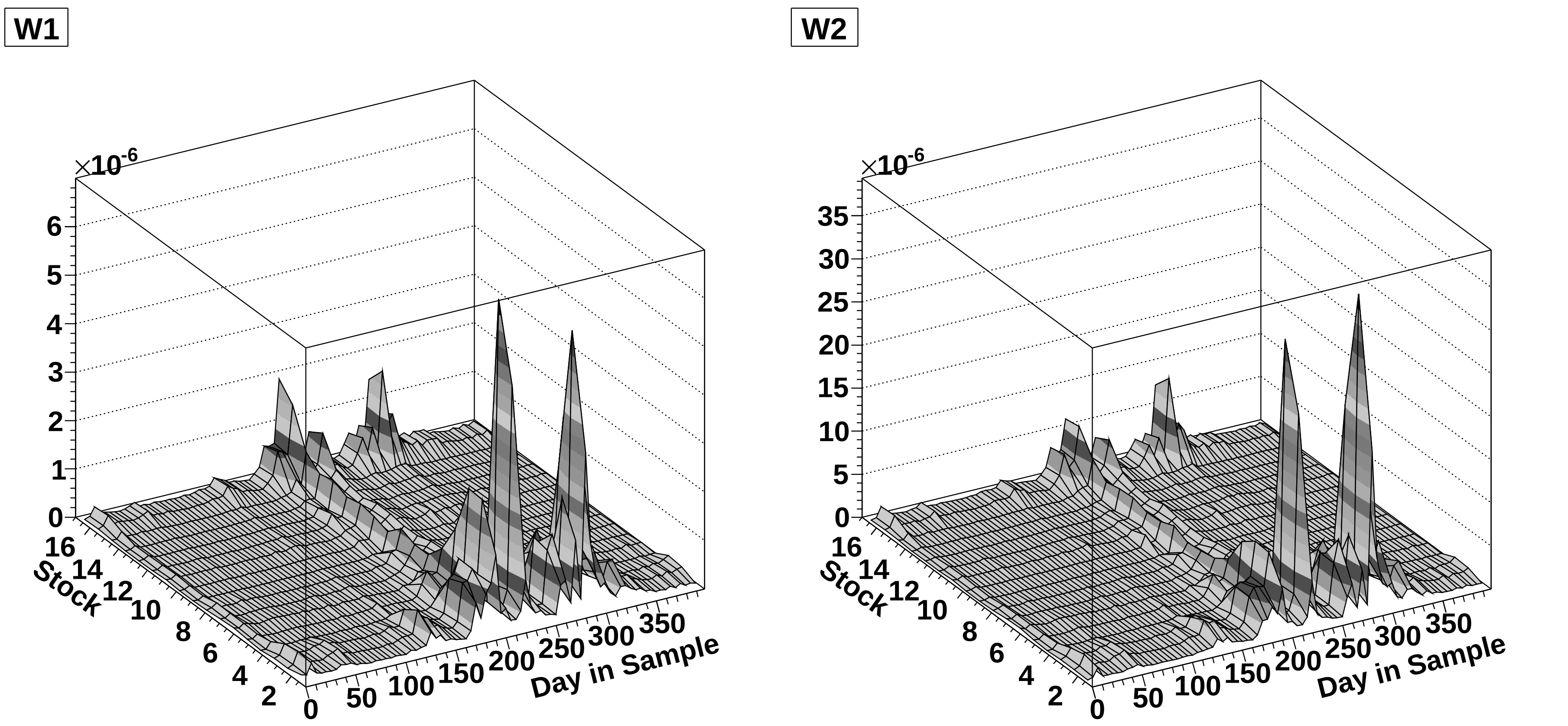
The opposite is true, for example, for Chevron Mobil stocks, where the short period following the plunge of Lehman Brothers shares was dominated by a rather stable characteristic coefficient, which turns out tobe more volatile in the long term.

5.1.5 Wavelets

I employ the Daubechies LA wavelet filter with length $L = 8$. The length of the filter is sufficient to compensate for the possible non-stationarity in the price time series (see Gencay, Selcuk and Whitcher, 2002). The non-stationarity in the data is usually treated by taking first differences, while for the MODWT analysis, price levels are employed directly. I perform the MODWT decomposition using the first two levels as described above. As a measure of jumpiness, I perform an energy decomposition for trading days. Then, I calculate the ratio of the total energy for a given day corresponding to each of the two levels of MODWT decomposition: $\left\|\tilde{W}_1\right\|^2 / \|X\|^2$ and $\left\|\tilde{W}_2\right\|^2 / \|X\|^2$. The higher the first ratio, the more high-frequency processes the time series contains. Therefore, a high ratio suggests an increased period of price jumps; however, this indicator can also reach high values even for non-jumpy periods. The increased ratio thus suggests an increased level of volatility caused by a high-frequency process, which does not necessarily coincide with the intuitive definition of price jumps.

Figure 7 contains two sub-figures: on the LHS they are depicted as $\left\|\tilde{W}_1\right\|^2 / \|X\|^2$, while on the RHS they are depicted as $\left\|\tilde{W}_2\right\|^2 / \|X\|^2$. The figures show an increase in the energy corresponding to the high-frequency processes after the emergence of the financial crisis. Since the high-frequency processes do not correspond solely to price jumps, the increased ratio of the energy corresponding to high-frequency processes can also be caused by the increased rate of noise. This ratio thus presents a necessary condition for the presence of price jumps. In addition, the ratio serves as an indicator for an increase in high-frequency volatility rather than solely for the identification of price jumps.

Figure 7: The relative energy of the first MODWT level (LHS) and the second MODWT level (RHS).



The relative energy is defined as $\left\|\tilde{W}_i\right\|^2 / \left\|X\right\|^2$, where $\{\tilde{W}_i\}$, with $i = 1, 2$, stands for the first-level and second-level wavelet coefficients and $\|X\|^2$ is the total energy over a trading day. The ratio reveals the first significant increase at the beginning of September 2008, where the Lehman Brothers' problems started. We can also clearly recognize a big increase in the ratio for all the big banks (Bank of America, Citi and Wells Fargo) and for both oil companies (Chevron and Exxon Mobile).

Table 7: The two-sided Wilcoxon statistics for the mean of $R_t^{S/BP}$.

ID/Ticker	Permanent Break		Temporary Break	
	60	120	60	120
1 APPL	-1.818*	-0.166	1.435	0.999
2 BAC	-3.087***	-5.482***	-1.167	-1.155
3 C	-1.698*	-3.147***	-0.179	-0.347
4 CVX	1.495	1.322	0.370	0.691
5 GE	-4.460***	-6.036***	-0.694	-0.865
6 GOOG	1.814*	2.127**	-0.004	-0.237
7 HPQ	1.026	0.984	0.324	0.355
8 IBM	0.093	0.162	1.353	1.501
9 JNJ	1.349	0.989	-0.757	-0.854
10 KO	1.900*	1.604	0.069	0.120
11 MSFT	2.010**	1.206	0.752	0.408
12 PFE	-2.337**	-2.167**	-0.837	-1.028
13 PG	2.743***	2.234**	0.404	0.237
14 SPY	2.195**	3.175***	-1.397	-1.233
15 T	1.070	1.165	1.323	1.422
16 WFC	-6.037***	-7.405***	-1.977**	-1.850*
17 XOM	-1.820*	-2.085**	-1.939*	-1.998**

Note: The mean was calculated for every stock and every trading day. I have used two definitions of the financial crisis. Permanent Break and Temporary Break, and two time window, $T = 60$ and $T = 120$ time steps. The table captures the z-statistics for the test. The additional stars denote whether we can reject H_0 of the two samples come from the same distribution and the corresponding confidence levels: 90% (*), 95% (**) and 99% (***). The overall positive/negative value of the z-statistics suggests that the median of means is lower/higher during the financial crisis.

5.2 Model-dependent Indicators

In this part, I present the results for model-dependent indicators as they were introduced in the previous sections.

5.2.1 The Difference between Bi-power Variance and the Standard Deviation: A Differential Approach

First, I calculate the ratio $R_t^{S/BP} = \hat{\sigma}_t^2/\hat{\hat{\sigma}}_t^2$. The two variances in the ratio require certain time windows. I choose $T = 60$ and $T = 120$. The ratio is defined for every

Table 8: The two-sided Wilcoxon statistics for the variance of $R_t^{S/BP}$.

ID/Ticker	Permanent Break		Temporary Break	
	60	120	60	120
1 APPL	-2.677***	-0.141	1.560	1.032
2 BAC	1.254	-3.471***	-0.632	-0.963
3 C	0.424	-1.840*	0.411	-0.228
4 CVX	2.323**	2.083**	0.826	0.841
5 GE	0.100	-3.635***	1.191	0.467
6 GOOG	0.345	0.960	-0.465	-0.602
7 HPQ	1.822*	1.604	0.571	0.658
8 IBM	1.851	1.347	1.330	1.183
9 JNJ	2.732***	2.230**	-0.107	-0.503
10 KO	2.266**	2.216**	-0.123	-0.363
11 MSFT	2.845	1.788*	2.131**	1.570
12 PFE	-1.205	-1.558	0.788	-0.019
13 PG	3.874***	3.718***	1.070	0.767
14 SPY	1.482	2.050**	-1.781*	-1.870*
15 T	1.103	1.318	1.473	1.707*
16 WFC	-0.040	-3.940	0.044	-0.505
17 XOM	0.783	-0.266	-1.680*	-1.988**

Note: The variance was calculated for every stock and every trading day. I have used two definitions of the financial crisis. Permanent Break and Temporary Break, and two time window, $T = 60$ and $T = 120$ time steps. The table captures the z-statistics for the test. The additional stars denote whether we can reject H_0 of the two samples come from the same distribution and the corresponding confidence levels: 90% (*), 95% (**) and 99% (***). The overall positive/negative value of the z-statistics suggests that the median of variances is lower/higher during the financial crisis.

time step t, where the history at the beginning of the trading day is calculated from the pre-opening period. Since $\hat{\sigma}_t$ is the variance, which also takes into account the price jumps, a high level of the ratio means the presence of price jumps. In addition, the ratio remains at increased levels as long as the price jump is contained in the history of up to T time steps back.

Since the ratio is by its nature very similar to the ratio constructed using p-dependent realized volatility I shall test Hypotheis II-A and Hypotheis II-B.

Hypothesis II-A: The results of the test are summarized in Table 7. The test shows that in the case of the Temporary Break, we cannot reject the null hypothesis stating that the means of $R_t^{S/BP}$ come from the same distribution. In the case of the

Permanent Break, three titles show more significant differences: Bank of America, General Electric and Wells Fargo. In addition, in all three cases the ratio is negative, which means that median of means during the crisis is higher. This also means that these three assets had the most significant increase in price jumps during the crisis. Since there is no immediate increase in the ratio during the initial days of the crisis, the increase in price jumps appear on the long term horizon. In addition, the ETF shows the opposite behavior, i.e., a decrease in the jumpiness after the emergence of the financial crisis.

Hypothesis II-B: Table 8 shows the result of the test. In the case of the Temporary Break, the variance is rather stable. On the other hand, for the Permanent Break, there are a few cases where the variance varies during the financial crisis. The most striking difference is in Procter and Gamble shares, where the variance decreased during the financial crisis, and thus, the trading activity was on average more uniform over the trading day.

5.2.2 The Difference between Bi-power Variance and the Standard Deviation: An Integral Approach

For the next step, I employ the ratio $R_{Day}^{RV/BPV} = RV_{Day}/BPV_{Day}$. I calculate the ratio for every trading day and every stock. Then, I test Hypothesis I-A and Hypothesis I-B.

Hypothesis I-A: The result of the two-sample Wilcoxon test is summarized in Table 9. The results show that the only asset that has significantly non-zero z-values for both definitions of the financial crisis is the one of the ETF. On the other hand, the assets for the banking industry show no significant deviation and therefore suggest no change in the price jump behavior.

Hypothesis I-B: The results of the F-test are summarized in Table 10. The table suggests that the most significant difference in the variance is for the stocks of Bank of America. In this, the Permanent Break definition of the financial crisis

Table 9: The two-sided Wilcoxon statistics for the ratio $R_{Day}^{RV/BPV}$.

ID/Ticker	Permanent Break	Temporary Break
1 APPL	0.355	0.395
2 BAC	0.037	1.177
3 C	1.472	0.215
4 CVX	0.729	1.116
5 GE	-0.948	-1.845*
6 GOOG	-0.812	0.994
7 HPQ	0.858	-0.075
8 IBM	1.348	1.873*
9 JNJ	1.134	0.029
10 KO	2.048**	-0.266
11 MSFT	1.031	-1.292
12 PFE	2.087**	0.345
13 PG	1.249	0.525
14 SPY	2.101**	2.461**
15 T	2.900***	1.558
16 WFC	-0.142	-0.418
17 XOM	-1.301	1.392

Note: The ratio $R_{Day}^{RV/BPV}$ was calculated for every stock and every trading day. I have used two definitions of the financial crisis. Permanent Break and Temporary Break, and two time window, $T = 60$ and $T = 120$ time steps. The table captures the z-statistics for the test. The additional stars denote whether we can reject H_0 of the two samples come from the same distribution and the corresponding confidence levels: 90% (*), 95% (**) and 99% (***). The overall positive/negative value of the z-statistics suggests that the median of ratios is lower/higher during the financial crisis.

gives a significantly higher variance during the crisis. On the other hand, in the Temporary Break case, the variance during the financial crisis is significantly lower. This favors the claim that after the emergence of Lehman Brothers' problems, the trading days were rather uniform with the same rate of market panic. On the other hand, in the long term, the trading days became more heterogeneous.

However, the opposite observation is true for the returns of Johnson and Johnson stocks. In this case, the days following the Lehman Brothers' problems were, on average, very different from each other. The difference subsequently smoothed out in the long term. In addition, the sectors do not share the same characteristics. For example, companies from the sensitive banking sector show very different behavior,

Table 10: Results of the two-sided F-test for the variance of the ratio $R_{Day}^{RV/BPV}$.

ID/Ticker	Permanent Break	Temporary Break
1 APPL	0.834	0.974
2 BAC	5.518***	0.195***
3 C	1.090	0.829
4 CVX	0.605***	0.697
5 GE	1.467***	1.177
6 GOOG	1.247	2.368***
7 HPQ	0.952	4.330**
8 IBM	1.018	0.636
9 JNJ	0.450***	1.662**
10 KO	0.374***	1.035
11 MSFT	1.184	0.738
12 PFE	1.041	1.767**
13 PG	1.009	4.952***
14 SPY	1.000	1.089
15 T	0.912	0.597*
16 WFC	1.116	1.116
17 XOM	0.915	1.094

Note: The null hypothesis says that the variances of the ratio $R_{Day}^{RV/BPV}$ during and not during the crisis match. Stars denote, at what confidence level we can reject the null hypothesis: 90% (*), 95% (**) or 99% (***). In addition, the value of F-statistics higher/lower than one means that variance of the ratio $R_{Day}^{RV/BPV}$ during the crisis was higher/lower when compared to the period not during the crisis. The two F-distributions are $F_{225,172}$ for the Permanent Break and $F_{29,368}$ for the Temporary Break.

as can be illustrated by Bank of America and Citigroup.

5.2.3 Bi-power Test Statistics

Finally, I employ the test statistics developed by Lee and Mykland (2008) and introduced above. The test statistics require choosing a moving window, as can be seen in equation (8). Lee and Mykland (2008) suggest using $T = 270$ time steps back for a 5-minute frequency. However, such a moving window cannot be satisfied in my framework since I do not allow for overlap between trading days. I choose instead two moving windows $T = 60$ and $T = 120$, the lengths used in the previous cases. The possible bias stemming from this choice of moving windows is again compensated for by considering the relative differences of the number of jumps. For

Table 11: The two-sided Wilcoxon statistics for the counting indicator.

ID/Ticker	Permanent Break		Temporary Break	
	60	120	60	120
1 APPL	-0.666	-0.141	0.099	-0.156
2 BAC	6.286***	5.416***	0.526	0.298
3 C	1.732*	3.312***	-1.334	-1.446
4 CVX	2.251**	0.004	0.433	-0.045
5 GE	7.561***	6.082***	1.722*	0.567
6 GOOG	-2.411**	-2.644***	1.623	2.016**
7 HPQ	1.607	2.143**	-1.706*	-1.991**
8 IBM	0.659	1.656*	-1.333	-1.418
9 JNJ	-0.824	-1.337	0.675	-0.566
10 KO	-1.142	-1.869*	1.578	1.948*
11 MSFT	0.163	-0.405	0.624	-0.702
12 PFE	1.427	1.668*	-0.028	0.280
13 PG	-2.989***	-2.674***	-3.673***	-2.289**
14 SPY	-0.198	-0.128	-1.598	-0.781
15 T	0.754	0.2400	-1.861*	-2.186**
16 WFC	5.144***	4.786***	0.120	0.032
17 XOM	2.293**	2.308**	-1.022	-0.848

Note: The mean of the counting indicator was calculated for every stock and every trading day. I have used two definitions of the financial crisis. Permanent Break and Temporary Break, and two time window, $T = 60$ and $T = 120$ time steps. The table captures the z-statistics for the test. The additional stars denote whether we can reject H_0 of the two samples come from the same distribution and the corresponding confidence levels: 90% (*), 95% (**) and 99% (***). The overall positive/negative value of the z-statistics suggests that the median of means of the counting indicator is lower/higher during the financial crisis.

the purpose of test statistics, I consider the 95% confidence level. The test statistics enable me to identify price jumps exactly and thus construct a counting indicator for the number of price jumps. I shall explicitly test Hypothesis I-A and Hypothesis I-B.

Hypothesis I-A: The results of the test are summarized in Table 11. In the case of the Permanent Break, there are several cases where the number of price jumps differs during the crisis. All the banks, General Electric and Exxon Mobil are characterized by positive z-values and thus by a lower number of price jumps during the crisis. On the other hand, Google and Procter and Gamble show a higher number of price jumps. In addition, Procter and Gamble is the only one that

Table 12: Results of the two-sided F-test for the variance of the counting indicator.

ID/Ticker	Permanent Break		Temporary Break	
	60	120	60	120
1 APPL	1.217	1.281*	1.315	1.855**
2 BAC	0.956	0.997	1.582*	2.245***
3 C	0.823	0.890	1.229	1.619**
4 CVX	0.704**	0.765*	0.948	0.698
5 GE	1.277*	1.054	1.253	1.213
6 GOOG	0.902	0.933	1.637**	0.901
7 HPQ	0.873	1.083	0.971	1.557*
8 IBM	0.936	1.048	1.189	1.510*
9 JNJ	0.982	1.028	1.291	1.049
10 KO	0.951	0.975	1.217	1.506*
11 MSFT	0.998	0.832	1.562*	1.091
12 PFE	1.004	0.905	1.019	1.231
13 PG	1.033	1.036	1.477	1.103
14 SPY	0.976	0.946	1.235	1.514*
15 T	0.900	0.759*	1.188	1.107
16 WFC	1.224	1.034	0.875	1.169
17 XOM	1.248	1.112	1.043	0.735

Note: The null hypothesis says that the variances of the mean of the counting indicator during and not during the crisis match. Stars denote, at what confidence level we can reject the null hypothesis: 90% (*), 95% (**) or 99% (***). In addition, the value of F-statistics higher/lower than one means that variance of the mean of the counting indicator during the crisis was higher/lower when compared to the period not during the crisis. The two F-distributions are $F_{225,172}$ for the Permanent Break and $F_{29,368}$ for the Temporary Break.

shows the same change of price jumps also for the Temporary Break. This suggests that the short period immediately after Lehman Brothers' problems was dominated by a huge increase in price jumps. In the case of the remaining stocks, there are no agreements between the different number of price jumps using the Permanent Break and the different number of price jumps using the Temporary Break. This means that the main change in the number of price jumps occurred in the long-time horizon.

Hypothesis I-B: The results of the F-test are in Table 12. In the case of the Permanent Break, the variance in the number of price jumps is not present. On the other hand in the case of the Temporary Break, the difference in the variance is present, namely for Bank of America where the F-statistics are higher than one. This suggests that the variance was higher during the crisis, i.e., the days were very different during the crisis than they were not during the crisis.

6 Conclusion

I performed an extensive technical analysis of price jumps using high-frequency market data (1-minute frequency) covering 16 major traded stocks and one ETF traded on the main North American stock exchanges. The data spans the period from January 2009 until the end of July 2009. The main question of this paper was to decide whether the behavior of price jumps, understood as extreme and irregular price movements different by their nature from regular Gaussian noise, changed during the recent financial crisis.

The paper provides a broad range of model-dependent as well as model-independent price jump indicators. Using these indicators, I measure the number of price jumps, or the jumpiness, of every trading day for every stock. Then, I compare the days of the financial crisis with those not during the crisis. I define a financial crisis as a structural break. First, I define it as a permanent break starting the day when Lehman Brothers' shares plunged. Second, I define the financial crisis as a temporary break starting the same day but lasting only 30 trading days. Having in hand such tools, I test the hypothesis that days during the financial crisis are the same with respect to price jump properties as those not during the crisis.

First of all, the results support the claim that volatility increased during the financial crisis. The volatility soars after the Lehman Brothers problems were announced and the peak lasts until mid-October. Then the volatility decreases but keeps above its pre-crisis level. In the first two months of 2009, the volatility increases again, mainly for the banking industry. The increased levels of volatility are in agreement with general knowledge since they reflect the increase in overall market impatience. The results, however, do not show an increase in price jumps. An overall increase in price jumps would mean a higher rate of market panic and more irrational behavior. A rather stable rate of price jumps, on the other hand, suggests that the proportion of market panic with respect to general impatience remained the same. However, there are some individual cases where the rate of price jumps increased and decreased during the crisis. In addition, it is not possible to draw any industry-dependent conclusions, which is surprising for the banking industry.

Finally, this paper also proves that different price jump indicators measure price jumps very differently. The difference in sensitivity between the indicators, however, is not so easy to describe; this would require a detailed numerical analysis. Such an analysis would be worth, while paying since the exact quantitative connection between the various price jump indicators would enable us to perform a meta-analysis of the results from various papers that use different indicators. The synergy obtained from such a study would draw more complex picture about the market mechanisms governing the spread of information. Such mechanisms play key role when market panic is formed. In addition, this would enable us to better quantitatively describe the irrational behavior of financial markets and thus, hopefully, understand them more deeply.

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Appendix

Wilcoxon test

Wilcoxon test is a non-parametric test comparing whether the two observed samples come from the same distribution (Mann and Whitney, 1947; Wilcoxon, 1945). The observations in each of the two samples are ranked and then compared. Finally, z-statistics is defined based on the results of comparison between the two samples. This z-statistics follows for large samples[9] a standard normal distribution. The null hypothesis of the test states that both observed samples come from the same distribution. When the calculated z-statistics exceeds the critical value, we reject the null hypothesis. In addition, the sign of the z-statistics can suggest something about the position of medians of the two compared samples.

Composition of the Dow Jones Industrial Average index

Table 13 shows the composition of the Dow Jones Industrial Average index including weights. The stocks included in this work are in bold.

[9] Usually above 20; Mann and Whitney (1947) and Wilcoxon (1945).

Table 13: Composition of the Dow Jones Industrial Average index.

Company	Ticker	Weight	Company	Ticker	Weight
International Business Machines Corp.	IBM	9,27%	American Express Co.	AXP	3,01%
Chevron Corp.	CVX	5,69%	Merck & Co. Inc.	MRK	2,63%
3M Co.	MMM	5,62%	E.I. DuPont de Nemours & Co.	DD	2,51%
Exxon Mobil Corp.	XOM	5,47%	Verizon Communications Inc.	VZ	2,27%
United Technologies Corp.	UTX	4,97%	Walt Disney Co.	DIS	2,20%
McDonald's Corp.	MCD	4,63%	**Microsoft Corp.**	MSFT*	2,16%
Procter & Gamble Co.	PG	4,54%	Home Depot Inc.	HD	1,99%
Johnson & Johnson	JNJ	4,53%	Kraft Foods Inc. Cl A	KFT	1,98%
Coca-Cola Co.	KO	4,21%	**AT&T Inc.**	T	1,94%
Caterpillar Inc.	CAT	4,20%	Cisco Systems, Inc.	CSCO*?	1,73%
Wal-Mart Stores Inc.	WMT	3,95%	Intel Corp.	INTC*	1,40%
The Travelers Companies, Inc.	TRV	3,83%	**Pfizer Inc.**	PFE	1,34%
Boeing Co.	BA	3,81%	**Bank of America Corp.**	BAC	1,18%
Hewlett-Packard Co.	HPQ	3,69%	**General Electric Co.**	GE	1,16%
JPMorgan Chase & Co.	JPM	3,13%	Alcoa Inc.	AA	0,94%

Note: The able contains the weights of stocks included in the Dow Jones Industrial Average index. The stocks whose ticker is marked by (*) are traded on NASDAQ. The rest of the stocks are traded on NYSE. Figures are taken from November 2009. Bold names are those included in this study.

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