Nonparametrically estimated diffusion with Poisson jumps: Theory and application on exchange rate.

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Juraj Valachy

Financial Policy Institute, Ministry of Finance, Stefanovicova 5, 81782, Bratislava, Slovakia. E-mail: jvalachy@mfsr.sk.

I propose a model of exchange rate with the presence of Poisson jumps and a procedure for nonparametric estimation of the diffusion function that originates in Johannes (2000). In my model, I assume a constant and then linear jump intensity, which enables me to use nonparametric framework for smaller data samples. The model is calibrated on two Central European (CE) and two EMS currencies. The CE currency benefits from the presence of the Poisson jump component because emerging markets exhibit significantly lower liquidity than the advanced ones. Results suggest that the currencies of the Cenral European countries exhibit a higher volatility generated from the jump component than the currencies of EMS.

Key Words: nonparametrics, diffusion, exchange rate, Central Europe, volatility, interest rate parity, EMS, asset pricing

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1. INTRODUCTION

The econometric models of asset return which use only Brownian motion as an error term are not able to explain certain stylized facts of financial time series, like fat tails or high skewness. These models, therefore, do not mimic real data and, consequently, misprice derivatives is based on these models. The most obvious way to allow asset return distribution to have fat tails and excess kurtosis is to include an additional source of variance —Poisson jumps— into the model.

One of the first asset price models with Poisson jumps is in Merton (1976). After this seminal paper, the jump process has been applied in the literature of asset price modeling in various contexts, e.g. interest rate or exchange rates.[1]

Merton's (1976) model assumes a simple specification of the jump component. In his model the log of jump size is assumed to have a normal distribution with a constant mean value and constant jump intensity. In later papers, the jump size distribution is allowed to be sensitive to trends in the market. Ball and Roma (1994) enriched the Vasicek (1977) model by time varying jump diffusion. For the purposes of their study, the jump size was modeled as a function of displacement from the central parity of EMS currencies. Bates and Craine (1999) specify a model where the volatility factor drives the intensity of jumps. Also Bates (2000) uses a time-varying arrival rate of jumps. All the above-mentioned papers assume the parametric specification of all model functions: drift, continuous (Brownian) diffusion and jump intensity.

Ait-Sahalia (1996) relaxes the assumption of the parametric diffusion function and offers a procedure for estimating the nonparametric diffusion function of the interest rate process. He estimates the diffusion function nonparametrically while drift is still parametric. Stanton (1997) extends this paper and presents a methodology for the estimation of both drift and diffusion nonparametrically. However, neither of these two papers assume a jump component. Bandi and Nguyen (2000) extend the methodology even further and provide a complete asymptotic theory for nonparametric estimates of drift, diffusion and jump intensity functions. Their paper is based on Johannes (2000) who justifies the nonparametric extraction of the parameters and functions controlling the arrival of a jump from the estimated infinitesimal conditional moments.

However, there is one important problem connected with otherwise general nonparametric methodology of Ait-Sahalia and Stanton or Johannes, namely the data requirement. Ait-Sahalia (1996) uses 5500 daily observations (around 20 years), and Johannes (2000) and Bandi and Nguyen (2000) use more than 8000 observations.

My motivation is to devise a technique that can be used for samples of moderate size to alleviate the shortcomings of previous studies. This can be used to study emerging markets where data are not available for longer time spans. Naturally, this can be applied to transition countries as well. For example, in the case of the exchange rate of Central European currencies I have about 2500 daily observations that cover approximately 10 years.

[1]Examples are Ball and Torous (1983), Jorion (1988) and Bates (1996).

I aim to propose a model of the exchange rate process as well as a procedure for the estimation of this process. For this purpose I modify the procedure described in Johannes (2000) for exchange rate. The modification for exchange rate requires imposing certain restrictions on and modifications to the model. The resulting methodology allows the nonparametric estimation of diffusion on samples of moderate size.

This paper tries to achieve two goals: (1) to propose a model for exchange rate with the presence of Poisson jumps, and (2) to offer an appropriate estimation technique which would not be data-demanding. It has been recognized in the finance literature that one of the most important features for derivative pricing is the specification of the diffusion function. Therefore, the separation of continuous from discontinuous volatility should increase the precision of derivative pricing and the two types of noise have different should hedging requirements and possibilities. In fact, the ability to disentangle jumps from volatility is the essence of risk management, which should focus on controlling large risks leaving aside the day-to-day Brownian fluctuations

In my paper I want to modify the Johannes (2000) model. To assess the effect of this modification I perform a simulation study. The last part of this paper is devoted to the calibration of my model on two Central European currencies and two EMS currencies. Results reveal that the currencies of the Central European region exhibit higher volatility generated by the Poisson jump component. In addition, assumptions about the functional form of the jump intensity function are reasonable.

The rest of this paper is organized as follows. Section 2 is devoted to an overview of the literature. The specification of my model is in section 3 and the proposed estimation procedure is in section 4. In section 5 I describe the simulation studies and empirical results for the four different currencies. A brief conclusion is at the end.

2. OVERVIEW OF LITERATURE

2.1. Models with parametric drift, diffusion functions and Poisson jumps

Continuous time models in finance typically rest on one or more stationary diffusion processes with dynamics represented by Itô stochastic differential equation. The evolution of interest rate is governed by the process:

$$dr_t = \mu\left(r_t\right)dt + \sigma\left(r_t\right)dW_t$$

where functions $\mu(\cdot)$ and $\sigma(\cdot)$ are drift and diffusion functions respectively, and $\{W_t, t \geq 0\}$ is a standard Brownian motion. Usually drift and diffusion functions are parameterized, $\mu(r, \theta)$ and $\sigma(r, \theta)$, $\theta \in \Theta \subset R^K$.

Using parametric specification of the process definitely has its advantages. We can express the process analytically and employ the maximum likelihood estimation procedure to obtain consistent estimates of parameters. However, a disadvantage lies in the possible misspecification of the model. There is no reason to prefer one functional form of parametrization of drift or diffusion to another.

An example of a parametric model of interest rate with Poisson jumps is Das's (1999) model, which includes mean-reverting drift, Brownian diffusion and the Poisson jump process. This model may be written as:

$$dr_t = \kappa (\theta - r_t) dt + \sigma dW_t + \xi dJ (\lambda)$$

where ξ is a random jump whose size is lognormally distributed with constant mean and volatility. The parameter θ represents the long-run mean of the process and κ the speed of adjustment to this mean. The arrival of jumps is governed by a Poisson process with a frequency parameter λ that indicates the average number of jumps per year. The diffusion and Poisson process are independent of each other and independent of ξ as well. Thus, the interest rate evolves with a mean-reverting drift and two sources of volatility: continuous Brownian diffusion and discontinuous Poisson jumps.

2.2. Models with nonparametric drift and diffusion (no jump)

Ait-Sahalia (1996), in his seminal paper, relaxes the assumption of parametric specification of diffusion and allows diffusion to be a nonparametric function. More precisely, he uses parametric drift and a nonparametric diffusion function to the model interest rate behavior (r_t). He constructs the diffusion function from the marginal distribution $\pi(\cdot)$ and the drift parameter vector θ. Ait-Sahalia (1996) uses the following model:

$$dr_t = \mu (r_t) dt + \sigma (r_t) dW_t$$

where $\{W_t, t \geq 0\}$ is a standard Brownian motion. $\mu(\cdot)$ and $\sigma(\cdot)$ are the drift and the diffusion functions of the process. Let $\pi(\cdot)$ be the marginal density of the spot rate, and $\mu(\cdot)$ estimated parametric drift. The diffusion function is calculated by the formula:

$$\sigma^2 (r) = \frac{2}{\pi (r)} \int_o^r \mu (u, \theta) \pi (u) du.$$

Stanton (1997) develops a procedure for estimating *both functions* $-\mu(\cdot)$ and $\sigma^2(\cdot)$ – nonparametrically from data observed only at discrete time intervals. He uses Taylor expansions to construct a family of *approximations* to functions of drift, diffusion, and market price of risk. Stanton's estimate of drift confirms the indication formulated in Lo and Wang (1995) about the misspecification of linear drift and shows evidence of substantial nonlinearity of drift. The estimated diffusion, $\sigma(\cdot)$, is similar to that estimated (parametrically) by Chan, Karolyi, Longstaff and Sanders (1992).

2.3. Models with nonparametric drift, diffusion and jump intensity

Johannes (2000), as one of the first, presents a methodology for nonparametric estimation of drift, diffusion and jump intensity functions of the interest rate process. Bandi and Nguyen (2000) generalize his procedure, provide the asymptotic theory and confirm that his proposed estimators are consistent and efficient. Since the Johannes (2000) methodology is more illustrative, I will describe his procedure.

Consider a transformation of the process into logarithms

$$d\log(r_t) = \mu(r_-)\,dt + \sigma(r_-)\,dW_t + \xi dJ_t$$

where $\{W_t, t \geq 0\}$ is a scalar Brownian motion. $\mu(\cdot)$ and $\sigma(\cdot)$ are the drift and the diffusion functions of the process modeled as the functions of interest rate level; J_t is a time-homogeneous Poisson jump process. The jumps arrive with intensity $\lambda(r)$ and the jump sizes are assumed to be normally distributed $\xi \sim N\left(0, \sigma_\xi^2\right)$.

The key to estimation is to identify the characteristics of the jump-diffusion dynamics through instantaneous moment conditions. Intuitively, the drift function should describe the size of expected change in interest rate, the diffusion function should capture the magnitude of noise of the expected change, the lambda function should tell how often the interest rate makes a sudden jump and the parameter of jump volatility should measure the size of these jumps. The four moments encompass all four characteristics of the process, which allows me to express them analytically.

It is shown in Bandi and Nguyen (2000) that under regularity conditions the first two moments are specified as

1M $\quad \lim\limits_{\triangle\downarrow 0} \frac{1}{\triangle} E\left[\log\left(\frac{r_{t+\triangle}}{r_t}\right) \mid r_t = r\right] = \mu(r)$

2M $\quad \lim\limits_{\triangle\downarrow 0} \frac{1}{\triangle} E\left[\log\left(\frac{r_{t+\triangle}}{r_t}\right)^2 \mid r_t = r\right] = \sigma^2(r) + \lambda(r) E\left[\xi^2\right].$

The higher moments can be expressed by this formula; $\lim\limits_{\triangle\downarrow 0} \frac{1}{\triangle} E\left[\log\left(\frac{r_{t+\triangle}}{r_t}\right)^j \mid r_t = r\right] = \lambda(r) E[\xi^j]$, where $E[\xi^j] = \sigma_\xi^j \prod_{n=1}^{j/2}(2n-1)$ if j is even number, $E[\xi^j] = 0$ otherwise.

Therefore, the fourth and sixth moments are

4M $\quad \lim\limits_{\triangle\downarrow 0} \frac{1}{\triangle} E\left[\log\left(\frac{r_{t+\triangle}}{r_t}\right)^4 \mid r_t = r\right] = 3\lambda(r)\left(\sigma_\xi^2\right)^2$

6M $\quad \lim\limits_{\triangle\downarrow 0} \frac{1}{\triangle} E\left[\log\left(\frac{r_{t+\triangle}}{r_t}\right)^6 \mid r_t = r\right] = 15\lambda(r)\left(\sigma_\xi^2\right)^3.$

The identification scheme uses the fact that the 1st, 2nd, 4th and 6th moments identify $\mu(r), \sigma^2(r), \lambda(r)$, and σ_ξ^2.

The 4th and 6th moments completely identify the jump components. Given the jump components, the second moment identifies the diffusion function, $\sigma^2(r)$, and the first moment identifies the drift. For estimation of the conditional j-th moments (while assuming $\triangle = 1$), Johannes (2000) proposes nonparametric kernel estimators in the following form:

$$M^j(a) = \frac{\frac{1}{h}\sum_{i=1}^{n-1} K\left(\frac{r_i - a}{h}\right)\left[r_{i+1} - r_i\right]^k}{\frac{1}{h}\sum_{i=1}^{n} K\left(\frac{r_i - a}{h}\right)}.$$

Bandi and Nguyen (2000) prove that the estimation scheme outlined above is consistent.

3. MODEL

The prime interest of this paper is the modeling of the exchange rates of the Central European countries. For this purpose I modify the procedure described in Johannes (2000) for exchange rate. The modification for exchange rate requires certain restrictions and modifications to the model. The resulting methodology allows the nonparametric estimation of diffusion on samples of moderate size.

Using the concept of interest rate parity as a background motivation (Keynes 1923) I model drift as a function of instantaneous expected rate of appreciation of the foreign currency which is equal to the interest rate differential (IRD). Interest rate parity is a concept challenged by the empirical literature. In early papers, we can find a rejection of this hypothesis (Fama 1984, Frankel and Froot 1987, among others). Baille and Bollerslev (2000) claim that failure to find evidence for the presence of the interest rate parity condition can be due to incorrect statistical modeling.

The same factor, IRD, influences the diffusion and jump intensity. Bilson (1999) argues that the volatility is related to the difference between the interest rates of the two currencies. Large interest rate differentials can only exist in the presence of high currency volatility, otherwise arbitrage opportunities would arise. Moreover, the IRD variable can attain a negative value, whereas the diffusion and jump intensity can attain only positive values. Therefore, I decided to model the diffusion and jump intensity as a function of the absolute value of IRD.

The last modification lies in the different parametrization of model functions. Johannes (2000) allows all model functions to be nonparametric. In order to decrease the requirement for sample size I decided to use a parametric specification of jump intensity and jump volatility while diffusion is still a nonparametric function. In other words, I replace the nonparametric estimate of jump diffusion moments by linear parametric estimates.

My model for exchange rate has the following specification:

$$d\log S = \mu(b_t)\,dt + \sigma(b_t)\,dW_t + \xi_t dJ$$
$$prob\,(dJ = 1) = \lambda(b_t)\,dt \text{ and } \xi_t \sim N\left(0, \sigma_\xi^2\right)$$

where

S	is the nominal price of foreign currency in terms of domestic currency
$\mu(b_t)$	is the parametric mean-reverting drift function: $R \to R$
$\sigma(b_t)$	is the diffusion function: $R \to R^+$
$\lambda(b_t)$	is the jump intensity function: $R \to R^+$
$\{W_t, t \geq 0\}$	is a standard Brownian motion
$\xi_t \sim N\left(0, \sigma_\xi^2\right)$	is the jump size (normally distributed random variable), and
$b_t = r_t^{domestic} - r_t^{foreign}$	is the short-term interest rate differential.

The jumps arrive with intensity $\lambda(b)$ and the jump sizes are assumed to be normally distributed. It is important to assume that mean jump size is 0. For some assets this could be problematic, since jumps usually move price in a certain direction. However, for the sake of simplicity I assume that jumps just increase volatility rather than move the exchange rate systematically in a certain direction. Of course, this assumption can be relaxed. Moreover, specifying the process in logarithms with mean zero jumps ensures that $\mu(b)$ retains its interpretation as the local mean of the process.

The proposed model is derived from the Johannes (2000) model and retains its spirit. Using this analogy, I am able to express four instantaneous moment conditions and then find the estimates of the model functions. The four instantaneous moments are as follows:

1M . $\lim\limits_{\triangle \downarrow 0} \frac{1}{\triangle} E\left[\log\left(\frac{S_{t+\triangle}}{S_t}\right) \mid S_t = S\right] = \mu(b_t)$

2M $\lim\limits_{\triangle \downarrow 0} \frac{1}{\triangle} E\left[\log\left(\frac{S_{t+\triangle}}{S_t}\right)^2 \mid S_t = S\right] = \sigma^2(b_t) + \lambda(b_t) E\left[\xi^2\right]$

4M $\lim\limits_{\triangle \downarrow 0} \frac{1}{\triangle} E\left[\log\left(\frac{S_{t+\triangle}}{S_t}\right)^4 \mid S_t = S\right] = 3\lambda(b_t)\left(\sigma_\xi^2\right)^2$

6M $\lim\limits_{\triangle \downarrow 0} \frac{1}{\triangle} E\left[\log\left(\frac{S_{t+\triangle}}{S_t}\right)^6 \mid S_t = S\right] = 15\lambda(b_t)\left(\sigma_\xi^2\right)^3.$

For the rest of the paper I use the notation b_t for the interest rate differential, e.g. $b_t = r_t - r_t^*$.

4. ESTIMATION PROCEDURE

4.1. Jump intensity is constant

In the specification of my model I do not mention the exact parametrization of the jump intensity function. As the first candidate, I take the constant jump intensity. This very simple specification should ilustrate the influence of Poisson jumps on the diffusion function estimate. In the next section, I assume a more realistic functional form of jump intensity, namely a linear function of IRD. These restrictions on shape of jump intesity function, e.g.linear versus nonparametric specification, are key for an efficient estimation procedure in moderate sample sizes.

Let us assume constant jump intensity with respect to IRD, e.g. $\lambda(b) = \lambda = const$. In the presence of Poisson jumps and constant jump intensity, the shape of the diffusion function will be the same as without jumps, but will be shifted downward. The exact formula for diffusion function is

$$\sigma^2(b_t) = \sigma_T^2(b_t) - \lambda\sigma_\xi^2,$$

where $\sigma^2(b_t)$ is the diffusion of the continuous part, $\sigma_T^2(b_t)$ is the total diffusion (or 2nd moment), and σ_ξ^2 is the variance of jumps.

The intuition behind this result is the following. The sizes of jump and jump intensity are not conditioned on the actual level of interest rate differential b and therefore, they have no influence on the shape of the diffusion function. They just have an influence on the scale of this function.[2]

Under the assumption of constant jump intensity, the proposed estimation procedure is as follows:

1. Estimate parametrically the drift $\mu(\cdot)$. Since I assume drift to be a linear mean-reverting function of IRD, ordinary least squares identifies the parameters α and β, $E[\log S_{t+1} - \log S_t \mid b_t] = \alpha + \beta b_t$.

2. Estimate λ and σ_ξ^2. Based on the calculation of moments, the ratio of the 6th and 4th moment will give me the desired estimate of σ_ξ^2. Consequently, the estimate of λ will be $\widehat{\lambda} = \frac{4th\ Moment}{3\left(\sigma_\xi^2\right)^2}$. Particular moments are calculated as follows: 4th Moment $= \frac{1}{n-1}\sum_{i=1}^{n-1}\log\left(\frac{S_{t+\triangle}}{S_t}\right)^4 = 3\lambda\left(\sigma_\xi^2\right)^2$ and 6th Moment $= \frac{1}{n-1}\sum_{i=1}^{n-1}\log\left(\frac{S_{t+\triangle}}{S_t}\right)^6 = 15\lambda\left(\sigma_\xi^2\right)^3$.

3. The diffusion function can be completely identified by subtracting the 2nd moment estimated nonparametrically from constant volatility generated by Poisson jumps, e.g. $\sigma^2(b) = 2nd\ Moment(b) - \widehat{\lambda}\sigma_\xi^2$, whereas $2nd Moment = \lim_{\triangle\downarrow 0}\frac{1}{\triangle}E\left[\log\left(\frac{S_{t+\triangle}}{S_t}\right)^2 \mid b_t = b\right]$.

4. The diffusion estimator can be used to correct for heteroscedasticity in the residuals from the OLS regression in step 1.

[2] This model for exchange rate evolution can be applied for option pricing. The continuous and discontinuous diffusions have different impacts on option prices, and, therefore, by separating them from each other, I should achieve a higher precision of option pricing.

4.2. Jump intensity is not constant

In the previous section I use a very simple and unrealistic assumption of constant jump intensity. Bilson (1999) argues that the higher the IRD, the higher the volatility, and consequently the higher the jump intensity. Therefore, I allow the jump intensity to be parametrically dependent on IRD. Using parametric specification will allow me to use a smaller data sample and, at the same time, benefit from the good properties of econometric models that allow for a nonparametric diffusion function and contain a Poisson jump component.

The first candidate for parametric specification of $\lambda(\cdot)$ is a simple linear function, namely $\lambda(b_t) = \gamma + \delta |b_t|$. Using the absolute value of IRD means that the jump intensity is symmetric in magnitude of IRD. In other words, a negative value of the differential has the same effect on jump intensity as a positive differential of the same size. By letting $\delta = 0$, the linear specification collapses to the previous case of constant jump intensity. In the case of linear jump intensity, the continuous diffusion, $\sigma^2(b)$, will have a different shape than in case without Poisson jumps.

The estimation of all functions of the exchange rate process is the following

1. Estimate parametrically the drift $\mu(\cdot)$. Since I assume drift to be a linear mean-reverting function of IRD, ordinary least squares identifies the parameters α and β, $E[\log S_{t+1} - \log S_t \mid b_t] = \alpha + \beta b_t$.

2. Estimate $\lambda(b)$ and σ_ξ^2. Based on the calculation of moments, the ratio of the 6th and 4th moment will give me the desired estimate of σ_ξ^2. Consequently, the parameters of $\lambda(b) = \gamma + \delta|b|$ are identified via OLS regression $\frac{E\left[\log^4\left(\frac{S_{t+\triangle}}{S_t}\right)|S_t=S\right]}{3\left(\sigma_\xi^2\right)^2} = \gamma + \delta |b_t|$ with restriction $\gamma > 0$.

3. The diffusion function can be completely identified by subtracting the 2nd moment estimated nonparametrically from the volatility generated by Poisson jumps, e.g. $\sigma^2(b) = 2nd\ Moment(b) - \lambda(b)\sigma_\xi^2$, whereas $2ndMoment = \lim\limits_{\triangle\downarrow 0} \frac{1}{\triangle} E\left[\log\left(\frac{S_{t+\triangle}}{S_t}\right)^2 \mid b_t = b\right]$.

4. The diffusion estimator can be used to correct for heteroscedasticity in the residuals from the OLS regression in step 1.

5. EMPIRICAL ANALYSIS

5.1. Monte Carlo Simulation

As the first step in the evaluation of my methodology, I compare the estimated diffusion with those obtained in Johannes (2000). Using his dataset I was able to completely replicate his results. In addition I used my methodology on this dataset to see differences originating in particular assumption of jump intensity. The estimated diffusion functions are shown in Figure 1.

Johannes (2000) estimates the jump intensity as a nonparametric function of the interest rate. My assumption of the constant jump intensity clearly yields a biased estimate of the diffusion function. The linear jump intensity, on the other hand, produces a similar diffusion function as the nonparametric jump intensity. The reason for this interesting result could be that the actual (nonparametric) shape of jump intensity is not far from a linear shape, and in addition, the arising differences affect the overall diffusion only marginally. This finding underlines the idea of the simple specification of the jump intensity. However, the finding of near-linear jump intensity cannot be generalized for any time series.

In order to show the sampling properties of the new method I run a Monte Carlo simulation. Using the Bernoulli approximation first introduced by Ball and Torous (1983), I downgrade the continuous model to a discrete version. The assumption is that in each time interval either only one jump occurs or no jump occurs. The drawback of discretization of the continuous model lies in bias. This bias, however, should not be pronounced when using daily frequency.

I generate the sample path that has parameter values similar to the Johannes (2000) data. More precisely, I take the parameter estimates of the jump volatility, σ_ξ^2, constant (average) jump intensity, $\overline{\lambda}$, and the nonparametric diffusion function σ^2. The simulated process has the following specification:

$$\log\left(r_{t+1}\right) - \log\left(r_t\right) = \mu\left(r_t\right) dt + \sigma\left(r_t\right)\left(W_{t+1} - W_t\right) + \xi_{t+1} J_{t+1}$$

where $\mu\left(r_t\right) = -0.01(r_t - 0.09)$, $P\left[J_{t+1} = 1 | r_t\right] = \lambda\left(r\right) = 0.09$, $\sigma_\xi^2 = 0.0013$, and $\sigma\left(r_t\right)$ is a nonparametric function equal to the diffusion function estimated by Johannes (2000). The properties of this time series are similar to the Johannes (2000) interest rate data set. The difference lies in the functional form of jump intensity and drift. The Johannes (2000) data contains a nonlinear jump intensity, whereas my simulated series assumes a constant intensity with respect to interest rate level.

The simulation study is done in the following way. I generated a sample path with 10,000 observations. Then using the Monte Carlo method I chose 2000 observations, because the typical sample size for the transition country is around this value. Then, I estimate the parameters of the model by my procedure. I repeat this process 5000 times. The results from the simulation are in Table 1 below.

Both methods, Johannes's (2000) and mine, calculate the diffusion in same way, e.g. substracting the (nonparametric) second moment from the (nonparametric or parametric) volatility generated by Poisson jumps,

$\sigma^2(r) = 2nd\,Moment\,(r) - \lambda(r)\,\sigma_\xi^2$. The second moment is estimated in the same way (with the same error) by both methods. Therefore, for assessing the effect of my modifications on the diffusion function, it is sufficient to evaluate the volatility generated from Poisson jumps.

The data generating process uses a constant jump intensity. In order to check whether I do not detect spurious linear jump intensity, I also run a regression of jump intensity on the interest rate level. In 8.3% of cases I was able to reject the hypothesis of constant relationship; even the true relationship is constant. This can be viewed as the error of model selection.

Table 1
Monte Carlo median, 10% and 90% confidence bands for the model parameters

	Sigma of Jumps	Lambda of Jumps	Linearity detected
True value	0.0013	0.09	0%
Median	0.001133	0.1225	8.3% cases
Std.Error	0.000157	0.0233	
10%	0.000927	0.0960	
90%	0.001337	0.1556	

It is interesting that the method produces slightly biased coefficients. The jump size volatility is biased downward, whereas jump intensity parameter lambda upward. I do not know the source of this bias nor the way to modify the estimation to correct for it. Nevertheless, the effect of this bias on the diffusion function is marginal.

5.2. Exchange rate

For the purpose of calibration of the model on exchange rates, I use the nominal exchange rates expressed in terms of the Deutsche mark [3] to calculate changes in exchange rate over two consecutive trading days. I use interest rates of one-month maturity to calculate the required interest rate differentials. In the literature we may also find shorter maturities used to calculate IRD. However, one-month maturity is the maturity that is published in each country for the longest period. It is also a standard reference interest rate for most central banks.

I choose two currencies from the Central European (CE) region, the Hungarian forint and Czech crown. For comparative purposes I choose two currencies from the EMS region, the Belgian franc and French franc. The sample for both EMS currencies starts on January 1, 1991 and lasts untill December 31, 1998. The sample for the Czech crown starts on January 1,

[3] After the introduction of Euro in Janury 1, 1999, the exchange rate of the Deutsch mark is calculated using the Euro exchange rate.

1993 ends on July 31, 2002, and for the Hungarian forint starts on August 1, 1996 and ends on July 31, 2002.

The estimated diffusion functions are shown in Figure 2. There is a strong nonlinear relationship between the diffusion and interest rate differential that cannot be *a priori* analytically described. The volatility of EMS currencies starts to rise when the IRD reaches 5 percentage points. On the other hand, the volatility of CE currencies starts to rise when the IRD reaches 10 percentage points. These findings support the use of the nonparametric method even more. The jump intensity is constant in the case of the Czech crown, Hungarian forint and Belgian frank. The French frank is the only case where I find the jump intensity to be a linear function of interest rate differential. Naturally, I could model the relationship with a richer analytical form, but this is a topic for further research. Nevertheless, there seems to be a tendency of the constant jump intensity that is in line with my proposed methodology. The estimations of the drift parameters show that none of the currencies comply with interest rate parity. This result is line with the literature on exchange rate, where exchange rate is considered as unpredictable (Andersen, Bollerslev, Diebold, and Labys 2001).

The values of estimated volatilities of jump size and jump intensity are in Table 2 below. The jump volatilities of Central European currencies are significantly higher than EMS currencies. Higher jump volatilities are, on the other hand, compensated for lower jump intensity. Yet, the total volatility generated by Poisson jumps is higher for Central European currencies than EMS currencies.

Table 2

Estimates of the model

$$\log\left(S_{t+1}\right)-\log\left(S_t\right)=\alpha+\beta\left(i_t-i_t^*\right)+\sigma\left(b\right)\left(W_{t+1}-W_t\right)+\xi dJ$$

$$prob\left(dJ=1\right)=\alpha_2+\beta_2\lambda\left(b\right)dt \text{ or } prob\left(dJ=1\right)=\overline{\lambda}dt \text{ and } \xi\sim N\left(0,\sigma_\xi^2\right)$$

	β	σ_ξ^2	$\overline{\lambda}$	β_2	Total Jump Volatility
CZK/DEM	-0.00152	0.000915	0.0070	0.1888	0.00000643
HUF/DEM	0.00234	0.000272	0.0237	-0.319	0.00000649
FRF/DEM	-0.00841	0.000025	0.0512	3.976***	0.00000132
BEF/DEM	0.00024	0.000141	0.0178	4.508	0.00000253

Note: *** denotes significance at the 1% level.

The estimated diffusions presented in Figure 2 require mention of one important drawback to using nonparametrics. This is the most pronounced in the diffusion of the Hungarian forint. The diffusion exhibits quite a volatile shape. This volatility in shape can have two reasons. The first reason is that the true relationship between diffusion and IRD is trully volatile. The more probable, second reason, is the lack of values of exchange rate for certain IRD levels. The OLS would simply bridge the gaps in the IRD level, but the nonparametric regression is not able to make such a bridge. The result is the sudden decreases in diffusion to low levels or even levels near zero. These sudden decreases would not be so problematic unless I would not need to subtract from it the volatility of the jump component. At the end, I would reach negative values of volatility. This result is, of course, not correct. Therefore, I report the 2nd moment instead of the continuous diffusion function. The effect of jump volatility is in most cases constant with respect to IRD and would not have the impact on shape.

6. CONCLUSION

In this paper I analyzed the volatility of the exchange rates of Central European countries (Hungary and the Czech Republic) and the European Union countries participating in the former European Monetary System (Belgium and France). In the proposed model I modify the Johannes (2000) methodology of nonparametric estimation of diffusion function of interest rate with Poisson jumps. In my methodology I assume a constant or linear jump intensity that enables me to use a nonparametric framework for smaller data samples.

Using the simulation studies I show that my proposed methodology should be better in terms of error band than the general Johannes (2000) methodology for the case of constant jump intensity, though I introduce a slight bias. Further, I calibrate the model on the four above-mentioned currencies. I find that CE currencies exhibit constant jump intensity whereas one EMS currency exhibits a linear jump intensity with respect to interest rate differential (IRD) and one a constant jump intensity.

In general, the currencies of the CE countries exhibit a higher volatility generated from the jump component than the currencies of EMS. Moreover, the nonparametric estimates of conditional volatility reveal a higher sensitivity of volatility to the size of the interest rate differential. In particular, the volatility of EMS currencies starts to rise when the IRD reaches 5 percentage points. On the other hand, the volatility of CE currencies starts to rise when the IRD reaches 10 percentage points.

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FIGURE 1

Estimated diffusion functions on the Johannes (2000) dataset using three different assumptions about the jump intensity function: (1) nonparametric, (2) linear, and (3) constant.

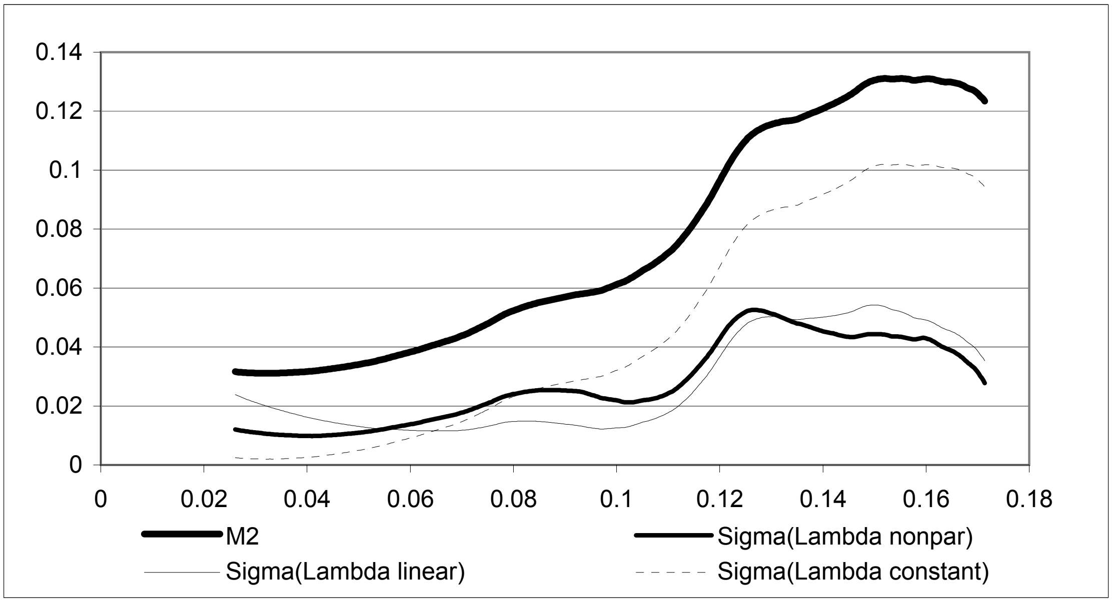

Figure 2
Estimated diffusion function for Czech koruna, Hungarian forint, French and Belgian frank