Choice of retirement age in a median voter model *

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Abstract

This paper develops a positive theory of the eligibility age for retirement benefits as a parameter of a pay-as-you-go social security system in an economy where workers are of two earning profile types: low and high. Low types have a lower wage and would retire sooner than the high types in a laissez-faire equilibrium. Voters choose the eligibility age, the tax rate, the benefit amount, and whether the benefit eligibility is conditional on not working or unconditional, subject to the overall budget constraint. We show that, depending on the shares of each type in the population, their "natural" retirement ages, and their pre-tax wages, there can be an equilibrium with unconditional benefit eligibility, or an equilibrium with conditional but labor-supply non-distorting benefit eligibility. The latter equilibrium allows the low types to extract higher transfers from the high types. Changes in the parameters of the model can drive a switch from the latter to the former equilibrium and back. The model is therefore capable of sheding ligt on the pattern of eligibility ages and conditionality of bene fits observed in developed countries in the last 50 years.

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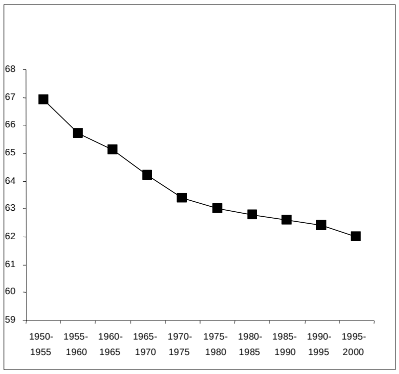
1 Introduction

A design of a pay-as-you-go (PAYG) social security system involves a number of choices: the level of the social security tax rate, the size of the benefit, the eligibility age for the benefit, and whether collection of the benefit is or is not conditional on labor income (earnings test). Previous literature on positive theories of PAYG social security has predominantly concentrated on the first two parameters, whereas the choice of the latter two has implicitly been assumed away.[1,2] These models are capable of explaining the stylized facts that PAYG systems are introduced in the first place and that they grow over time. However, they have little to say about the variation in the effective eligibility age for benefits over the last several decades. Figure 1 plots the average retirement age of men in the United States and Germany, and very similar figures could be drawn for virtually all OECD countries. Although the figures show actual retirement age as opposed to official eligibility for benefits age, empirical literature explaining the individuals' timing of retirement[3] documents that the eligibility and actual retirement ages are very closely related. Gruber and Wise (1999) summarize that in a typical developed country, there are two spikes in the distribution of the age of retirement: a large fraction of population retires when pension benefits first become available (the so-called early retirement age) and another large fraction retires at the "standard" retirement age at which they are entitled to full pension benefits. By inducing people to retire at particular ages, the governments effectively select the retirement age for a large fraction of population. Since the 1960's and 1970's the governments in most developed countries began introducing generous early retirement provisions and other pathways towards

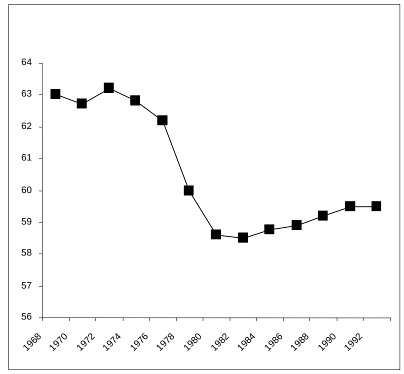
[1]Most often, this has been done by assuming an overlapping generations structure with young, (middle aged) and old agents.

[2]This literature is summarized in Mulligan and Sala-i-Martin (1997a, 1997b) and Gallasso and Profeta (2002).

[3]See, for example, Burtless and Moffit (1985), Samwick (1998), and Gustman and Steinmeier (2002).



Panel A: USA



Panel B: Germany

Figure 1: Mean retirement ages of men

Source: U.S. Bureau of Labor Statistics, Gruber and Wise (1999).

early retirement, such as disability and unemployment insurance.[4] These provisions bear a brunt of responsibility for the declines in the average retirement age shown in Figure 1. The "standard" retirement age has rarely changed since the 1960's (France or Canada are among few exceptions) but has lost much of its relevance, since most workers retire before that age, taking advantage of the early retirement provision. Having attributed the shift towards earlier retirement to a change in pension provisions, the question "what explains the earlier retirement?" moves only to a higher level: Why did the governments choose to reduce the retirement age? This is particularly puzzling since the reduction in the eligibility age came at the same time when life expectency and health were improving so that workers could have both work longer and enjoyed longer retirements. How do the governments actually determine the retirement age, i.e., how do they draw the line between those old enough to be eligible for pension benefits and those young enough to pay taxes?

Likewise, the models that focus on the tax rate and the size of the benefit have little to say about the puzzling feature that in many PAYG systems the eligibility for benefits is conditional on having a minimal labor income, often combined with a reduction in the benefit for every additional dollar of labor income. This is puzzling

[4] Conde-Ruiz and Galasso (2003) present a summary of early retirement provisions and the timing of their adoption in the OECD countries.

since such conditionality discourages labor supply and therefore creates inefficiency, even though the benefit could be paid without any conditions and hence distortion-free. Some countries have recently abolished conditionality, citing distortions in labor supply as the main reason. For example, the U.S. abolished conditionality, also called means-testing, in 2000. In signing the new law, president Bill Clinton remarked that

> "Today, one in four Americans between 65 and 69 has at least a part-time job. Eighty percent of the baby boomers say they intend to keep working past age 65... Yet, because of the Social Security retirement earnings test, the system withholds benefits from over 800,000 older working Americans, and discourages countless more - no one knows how many - from actually seeking work...This bill...will mean more baby boomers working longer, contributing more to the tax base and to the Social Security trust fund at precisely the time when the percentage of younger workers paying into the system will be dropping."

In the "popular" literature, the most frequently asserted explanation of early retirement provisions claims that they were adopted to facilitate early exit of older employees from the labor force (Ebbinghaus (2000)). A skill-biased technological change made many elderly workers redundant; they would have voluntarily retired earlier anyway (or, more accurately, remained unemployed until reaching the retirement age). The governments then simply matched the private decisions to retire earlier with pension benefits being available earlier. Borsch-Supan and Schnabel (1999) disagree with this view, however, arguing that, at least in the case of Germany, generous early retirement provisions were introduced before workers started retiring earlier.

There are only a few recent exceptions that try to address the issue of how the retirement age is set within the context of a formal model. Conde-Ruiz and Galasso (2003) construct a politico-economic model of early retirement within a two-period overlapping-generations framework in which voters decide whether to grant full or no pension benefits to workers who retire at an exogenous early age in the first period of their life. That is, voters effectively decide on the eligibility age as well as on the implicit tax on individuals who decide to work for the entire first period of their lives. Existence of an early retirement provision is driven by a past economic shock creating a political demand for early retirement combined with an intragenerational wage inequality. While the former triggers early retirement, the latter sustains it. This model, however, does not address the question of how the early retirement age is determined, although it suggests that it may be negatively correlated with the presence of intragenerational wage inequality. In fact, their model is structured to exactly replicate the "redundant elderly worker" explanation.

Butler (2000) studies political feasibility of alternative pension reform options (including an increase in retirement age) by calibrating a median voter model for the case of Switzerland. Lacomba and Lagos (2001) is so far the only general theory of retirement age. In their two-stage static model with two generations, the government first chooses a degree in which one's pension benefits depend on his wages (i.e., the level of intragenerational redistribution of the social security system). Taking this as given, voters then select the retirement age by majority voting. The main focus of the paper is on the relationship between wage inequality and retirement age; the model predicts that more redistributive social security systems and less egalitarian economies should have a higher retirement age.

Existing literature also provides only few explanations for the existence of the conditionality. Mulligan and Sala-i-Martin (1999a) argue that political competition is a time-intensive process. Hence any group that wants to be successful in the tax-transfer game needs to make sure that its members have disposable time that they can use in the political lobbing process. Hence conditionality is a feature of the system supported by the lobby of the old, since it induces retirement and enlarges the ranks of the old voter lobby.

In this paper, we aim at improving the existing literature by providing a unified model for the choice of the benefit eligibility age and the conditionality of benefits as well as for the size of the tax rate and the benefit. We assume that there are two types of agents in the economy, low and high, who are capable of working and earning a constant wage rate up until their "natural" retirement age, after which their earnings ability deteriorates. The high types have a higher wage rate and higher natural retirement age than the low types. In order to alleviate problems with majority voting in the multidimensional policy space, we assume that there are only two options for the official retirement age, equal to the natural retirement age of the low and the high type, respectively.

We show that, depending on the shares of each type in the population, their natural retirement ages, and their pre-tax wages, there can be two types of equilibria. First, there can be an equilibrium with unconditional benefit eligibility and a high eligibility age. Second, there can be an equilibrium with conditional benefit eligibility and a low eligibility age in which, however, the labor-supply of the high types is not distorted. Conditionality in the latter equilibrium allows the low types to extract more transfers from the high types. Changes in the parameters of the model can drive a switch from one equilibrium to the other, and the model is therefore capable of shedding light on the pattern of eligibility ages and conditionality of benefits observed in developed countries in the last 50 years.

The remainder of the paper is organized as follows. Section 2 introduces the formal model of the economy and the political process, and discusses individual economic decisions on labor supply and timing of retirement and individual political preferences. Section 3 characterizes equilibrium political choices of PAYG systems.

Section 4 analyzes the impact of changes in the parameters of the model on the equilibrium retirement age and uses the model to rationalize the introduction of early retirement. Section 5 offers concluding remarks. All the proofs are relegated to the Appendix.

2 Model

Time is continuous, agents are born at the age of 0, and they live until the age of 1. Therefore all live agents are of some age $a \in [0, 1)$.[5] At each time, fraction $s_L \in (0.5, 1)$ of the newborns are of the "low" type (denoted by subscript L), and fraction $s_H \equiv 1 - s_L$ are of the "high" type (H). Normalizing the overall population measure to 1, at any point in time the measure of L-agents is s_L, while the measure of H-agents is s_H. Conditional on each type, age is distributed uniformly between 0 and 1. An agent of type $i \in \{L, H\}$ is able to earn a flow wage rate of w_i up until the age of θ_i if working (and zero afterwards), while she enjoys a flow utility v in monetary terms of leisure when not working, before or after θ_i. We assume that $0 < v < w_L < w_H$ and $0 < \theta_L < \theta_H \leq 1$. That is, work pays off better than leisure, H-agents have a higher wage than L-agents, and they also have a higher "natural" retirement age.

The government imposes a social security tax at the rate τ on all labor income, determines the benefit eligibility age R, and pays a flow benefit of B to all eligible retirees. It also determines whether the eligibility for the benefit is or is not conditional on not having any labor earnings. If unconditional, every agent aged above R receives the benefit. If conditional, only agents aged above R that do not work receive the benefit. Every social security policy can therefore be summarized by a four-tuple (R, τ, B, j), $j \in \{U, C\}$, where the last entry stands for "unconditional"

[5] We exclude 1 from this set without a loss of generality to clarify the exposition because at the age of 1, an agent is indifferent among all possible policies.

or "conditional", respectively. If $R \geq \theta_H$, then it is irrelevant whether a policy is conditional or unconditional, because nobody works past the age of θ_H. Without loss of generality, we will therefore call a policy conditoinal only if $R < \theta_H$. If $R \geq \theta_H$, we will call a policy unconditional.

Utility of each agent is given by an undiscounted stream of cash flow and the stream of leisure utility.[6] The cash flow comes either from (after-tax) labor earnings or from social security benefits. First, consider social security policies with unconditional benefits. The utility of an agent of type $i \in \{L, H\}$ aged a, when the social security policy is given by $\pi \equiv (R, \tau, B, U)$, is

$$\begin{aligned} V_i(a; R, \tau, B, U) = & \max[r_i(\pi) - a, 0](1-\tau)w_i \\ & + \{1 - \max[r_i(\pi), a]\}v + [1 - \max(R, a)]\, B, \end{aligned} \tag{1}$$

where $r_i(\pi)$ is the actual retirement age of type i agents given the social security policy π. If the policy is unconditional, the eligibility age has no impact on the actual retirement behavior. In this case, agents of both types work up until their natural retirement age, provided that their after-tax wage does not fall short of the utility value of leisure. That is,

$$r_i(R, \tau, B, U) = \begin{cases} \theta_i & \text{if } (1-\tau)w_i \geq v \\ 0 & \text{if } (1-\tau)w_i < v \end{cases}. \tag{2}$$

Second, consider social security policies with conditional benefits. The utility of an agent of type $i \in \{L, H\}$ aged a, when the social security policy is given by

[6] The assumption of no discounting is mostly for expositional clarity. All of the crucial results, with the exception of the eligibility age in an unconditional system, would still go through in case of a moderate discounting.

$\pi \equiv (R, \tau, B, C)$, is

$$V_i(a; R, \tau, B, C) = \max[r_i(\pi) - a, 0](1-\tau)w_i$$
$$+ \{1 - \max[r_i(\pi), a]\}v + \{1 - \max[R, r_i(\pi), a]\}B. \tag{3}$$

In this case, agents of both types work until their natural retirement age or R, whichever one comes sooner, provided that their after-tax wage does not fall short of the utility value of leisure, and, if R comes before their natural retirement age, work past R provided that their after-tax wage does not fall short of the sum of the utility value of leisure and the benefit. That is,

$$r_i(R, \tau, B, C) = \begin{cases} \theta_i & \text{if } R \geq \theta_i \text{ or } R < \theta_i \text{ and } (1-\tau)w_i \geq v + B \\ R & \text{if } R < \theta_i \text{ and } v \leq (1-\tau)w_i < v + B \\ 0 & \text{if } (1-\tau)w_i < v \end{cases}, \tag{4}$$

We focus on steady state equilibria. In these equilibria, long-term budget balance of the social security system requires that the system is balanced in every point in time. At any point in time, the measure of working i-agents is $s_i r_i$, their labor income is $s_i r_i w_i$, and their tax payements are $\tau s_i r_i w_i$. On the other hand, at any point in time, the measure of i-agents collecting benefits is $s_i(1-R)$ if the system is conditional, and $s_i[1-\max(R, r_i)]$ if the system is unconditional. As a result, the amount of benefits collected by i-agents is $s_i(1-R)B$ if the system is conditional, and $s_i[1-\max(R, r_i)]B$ if the policy is unconditional. Therefore, in case of an unconditional policy, budget balance requires

$$\tau(s_L r_L w_L + s_H r_H w_H) = (1-R)B, \tag{5}$$

whereas in case of a conditional policy, it requires

$$\tau(s_L r_L w_L + s_H r_H w_H) = \{s_L[1 - \max(R, r_L)] + s_H[1 - \max(R, r_H)]\} B. \qquad (6)$$

A social security policy (R, τ, B, j), $j \in \{U, C\}$, is *feasible* if it satisfies the relevant budget balance condition.

Given the economic decisions on labor supply and the timing of retirement characterized by (2) and (4), (1) and (3) then determine voter preferences over alternative social security policies. The next section will characterize resulting political equilibria.

3 Political Equilibria

Voters choose among different feasible social security policies by majority vote. Because the issue space is multidimensional, non-existence of equilibria cannot be ruled out in general. To avoid this difficulty, we restrict the choice set for R to be $\{\theta_L, \theta_H\}$, the two natural retirement ages. Given this setup, an equilibrium is defined in the following way:

Definition 1 *A social security policy $\pi \equiv (R, \tau, B, j)$, $R \in \{\theta_L, \theta_H\}$, $j \in \{U, C\}$ is an equilibrium if it is feasible and if there is no alternative feasible policy that would be prefered by more than half of all voters who have a strict preference between the two policies.*

Before characterizing various equilibria, it is useful to begin by deriving some basic auxilliary results.

Lemma 2 *For any feasible policy $\pi \equiv (R, \tau, B, j)$, $j \in \{U, C\}$, $r_H(\pi) \geq r_L(\pi)$.*

Lemma 2 shows that the H types never retire before the L types, because if the former ones have an incentive to do so, then so do the latter ones since they have a smaller pre-tax wage and an earlier natural retirement age.

Lemma 3 *Consider a feasible policy* $\pi \equiv (R, \tau, B, j)$, $j \in \{U, C\}$ *that results in* $r_H(\pi) > 0$. *Suppose that there exists an alternative feasible policy* $\pi' \equiv (R, \tau', B', j)$ *with* $\tau' > \tau$ *such that* $r_i(\pi') \geq r_i(\pi)$ *for* $i \in \{1, 2\}$. *Then all the L-agents strictly prefer* π' *to* π.

Lemma 3 shows that as long as it does not introduce any new labor-market distortion and potentially removes an existing distortion, the L types prefer, keeping the eligibility age fixed, to increase the tax rate and use the resulting revenue to increase the benefit. This is because, since the benefit does not depend on type, the social security system redistributes from the H types to the L types, and, conditional on any pre-existing labor market distortions, the L types prefer to make this transfer as large as possible.

Lemma 4 *Consider any policy* $\pi \equiv (R, \tau, B, j)$, $j \in \{U, C\}$ *which results in* $r_H(\pi) < \theta_H$. *Then there exists another feasible policy that is strictly prefered to* π *by all agents.*

Lemma 4 shows that *none* of the voters prefer to distort the labor supply of the H types in equilibrium. Intuitively, the H types do not want to distort their own labor supply because such policy reduces the total pie of labor income as well as, in general, the share of H types in that income. On the other hand, the L types do not want to distort the labor supply of the H types because labor income of the latter group is a source of redistribution for the L types.

Corollary 5 *In any equilibrium* $\pi \equiv (R, \tau, B, j)$, $j \in \{U, C\}$, $r_H(\pi) = \theta_H$. *That is, labor supply of H types is never distorted in equilibrium. As a result,* $(1-\tau)w_H \geq v$, *and if* $j = C$ *and* $R < \theta_H$, *then* $(1-\tau)w_H \geq v + B$.

Because the L-agents are in the majority ($s_L > 0.5$), combining this result with Lemma 6 gives the following result:

Corollary 6 *If $\pi \equiv (R, \tau, B, j)$, $j \in \{U, C\}$, is an equilibrium, then there is no alternative feasible policy $\pi' \equiv (R, \tau', B', j)$ with $\tau' > \tau$ such that $r_i(\pi') \geq r_i(\pi)$ for $i \in \{1, 2\}$.*

Having derived these two auxilliary results, first consider equilibria that involve an unconditional policy. The following Lemma specifies necessary conditions for such equilibria:

Lemma 7 **(a)** *If*

$$s_L\theta_L/s_H\theta_H > (w_H/w_L - 1)/(w_L/v - 1), \tag{7}$$

then the policy $\pi_1 = (\theta_H, \tau_1, B_1, U)$ with $\tau_1 = 1 - v/w_L$ and $B_1 = (1 - v/w_L)(s_L\theta_L w_L + s_H\theta_H w_H)/(1-\theta_H)]$ strictly dominates any other feasible policy $\pi' = (R, \tau', B', U)$ with $\tau' \neq \tau_1$ for every L-agent, and it weakly (strictly) dominates the feasible policy $\pi'' = (\theta_L, \tau_1, B'', U)$ for every L-agent (aged $a > \theta_L$).

(b) *If*

$$s_L\theta_L/s_H\theta_H < (w_H/w_L - 1)/(w_L/v - 1), \tag{8}$$

then the policy $\pi_2 = (\theta_H, \tau_2, B_2, U)$ with $\tau_2 = 1 - v/w_H$, and $B_2 = (1 - v/w_H)s_H\theta_H w_H/(1 - \theta_H)]$ strictly dominates any other feasible policy $\pi' = (R, \tau', B', U)$ with $\tau' \neq \tau_2$ for every L-agent, and it weakly (strictly) dominates the feasible policy $\pi'' = (\theta_L, \tau_2, B'', U)$ for every L-agent (aged $a > \theta_L$).

(c) *If*

$$s_L\theta_L/s_H\theta_H = (w_H/w_L - 1)/(w_L/v - 1), \tag{9}$$

then all L-agents are indifferent between π_1 and π_2, and these two policies strictly dominate any other feasible policy $\pi' = (R, \tau', B', U)$ with $\tau' \notin \{\tau_1, \tau_2\}$ for every L-agent, and they weakly (strictly) dominate the feasible policies $\pi_1'' = (\theta_L, \tau_1, B_1'', U)$ and $\pi_1'' = (\theta_L, \tau_2, B_2'', U)$ for every L-agent (aged $a > \theta_L$).

Lemma 8 *Consider a feasible policy $\pi \equiv (\theta_L, \tau, B, j)$, $j \in \{U, C\}$ with $(1-\tau)w_H \geq v$. Then all H-agents weakly prefer the feasible policy $\pi' \equiv (\theta_H, \tau, B', j)$ to π, and there is a positive measure of H-agents for whom this preference is strict.*

Lemma 9 **(a)** *If*

$$s_L\theta_L/s_H\theta_H \geq (w_H/w_L - 1)/(w_L/v - 1), \tag{10}$$

and an equilibrium policy with unconditional benefits exists, then this policy is $\pi_1 = (\theta_H, \tau_1, B_1, U)$ with $\tau_1 = 1 - v/w_L$ and $B_1 = (1 - v/w_L)(s_L\theta_L w_L + s_H\theta_H w_H)/(1 - \theta_H)]$.

(b) *If*

$$s_L\theta_L/s_H\theta_H < (w_H/w_L - 1)/(w_L/v - 1) \tag{11}$$

and an equilibrium policy with unconditional benefits exists, then this policy is $\pi_2 = (\theta_H, \tau_2, B_2, U)$ with $\tau_2 = 1 - v/w_H$, and $B_2 = (1 - v/w_H)s_H\theta_H w_H/(1 - \theta_H)]$.

Lemma 10 *Consider any feasible policy $\pi = (\theta_L, \tau, B, C)$ with $\tau > 1 - v/w_L$. Then this policy is strictly dominated by the policy $\pi_2 = (\theta_H, \tau_2, B_2, U)$ with $\tau_2 = 1 - v/w_H$, and $B_2 = (1 - v/w_H)s_H\theta_H w_H/(1 - \theta_H)$ for all the L-agents.*

Lemma 11 *Suppose that $\pi \equiv (\theta_L, \tau, B, C)$ is an equilibrium. Then*

(a) *If*

$$v\frac{w_H - w_L}{w_L - v} \geq \frac{s_L\theta_L w_L + s_H\theta_H w_H}{1 - s_L\theta_L - s_H\theta_H}, \tag{12}$$

then the policy $\pi_3 \equiv (\theta_L, \tau_3, B_3, C)$ *with* $\tau_3 = 1 - v/w_L$ *and*

$$B_3 = (1 - v/w_L)\frac{s_L\theta_L w_L + s_H\theta_H w_H}{1 - s_L\theta_L - s_H\theta_H},$$

strictly dominates any other feasible policy $\pi' = (\theta_L, \tau', B', C)$ *with* $\tau' < \tau_3$ *for every* L*-agent.*

(b) *If*

$$v\frac{w_H - w_L}{w_L - v} < \frac{s_L\theta_L w_L + s_H\theta_H w_H}{1 - s_L\theta_L - s_H\theta_H}, \tag{13}$$

then the policy $\pi_4 \equiv (\theta_L, \tau_4, B_4, C)$ *given by*

$$\tau_4 = (w_H - v)\frac{1 - s_L\theta_L - s_H\theta_H}{w_H(1 - s_L\theta_L) + s_L\theta_L w_L} < \tau_3$$

and

$$B_4 = (w_H - v)\frac{s_L\theta_L w_L + s_H\theta_H w_H}{w_H(1 - s_L\theta_L) + s_L\theta_L w_L}$$

strictly dominates any other feasible policy $\pi' = (\theta_L, \tau', B', C)$ *with* $\tau' \neq \tau_4$, $\tau' \leq \tau_3$ *for every* L*-agent.*

Proof of Lemma 11. If (12) holds, then it follows that $(1 - \tau_3)w_H \geq v + B_3$. As a result, any alternative policy $\pi' = (\theta_L, \tau', B', C)$ with $\tau' < \tau_3$ would result in $r_H(\pi_3) = r_H(\pi') = \theta_H$ and $r_L(\pi_3) = r_L(\pi') = \theta_L$, and hence, by Lemma 3, would be strictly dominated by π_3 for all the L-agents. If (13) holds, then it follows that $(1 - \tau_4)w_H = v + B_4$. By the same argument, any alternative policy $\pi' = (\theta_L, \tau', B', C)$ with $\tau' < \tau_4$ would result be strictly dominated by π_4 for all the L-agents. Hence consider an alternative policy $\pi' = (\theta_L, \tau', B', C)$ with $\tau_4 < \tau' \leq \tau_3$. Note that it cannot be the case that $(1 - \tau')w_H \geq v + B'$, because that would imply that $r_H(\pi') = \theta_H$, and hence

$$B' = \tau' \frac{s_L\theta_L w_L + s_H\theta_H w_H}{1 - s_L\theta_L - s_H\theta_H} > \tau_4 \frac{s_L\theta_L w_L + s_H\theta_H w_H}{1 - s_L\theta_L - s_H\theta_H} = B_4,$$

and this, ogether with $\tau' > \tau_4$ and $(1-\tau_4)w_H = v+B_4$ imply that $(1-\tau')w_H < v+B'$, a contradiction. Therefore it must be the case that $(1-\tau')w_H < v + B'$. But then $r_H(\pi') = r_L(\pi') = \theta_L$, and hence

$$B' = \tau' \frac{\theta_L(s_L w_L + s_H w_H)}{1 - \theta_L}$$

Third, Corollary 6 implies that, if π is an equilibrium policy, then it cannot be the case that $\tau < 1 - v/w_L$ and $(1-\tau)w_H > v + B$, because in that case τ and B could simultaneously be increased preserving feasibility without affecting labor supply. Therefore either $\tau = 1 - v/w_L$, or $(1-\tau)w_H = v + B$, or both.

If $\tau = 1 - v/w_L$, then B is as given by (), and the inequality $(1-\tau)w_H \geq v + B$ is equivalent to (). If, on the other hand, $\tau < 1 - v/w_L$, then

$$B = \tau \frac{s_L\theta_L w_L + s_H\theta_H w_H}{1 - s_L\theta_L - s_H\theta_H},$$

and the solution for τ in () is then implied by $(1-\tau)w_H = v + B$. The inequality $\tau < 1 - v/w_L$ is then equivalent to (). It then follows that if () is satisfied, the results in (a) follow, and if () is satisfied, then the results in (b) follow. ■

Lemma 12 *Suppose that $\pi \equiv (\theta_L, \tau, B, C)$ is an equilibrium. Then*

(a) *if*

$$v\frac{w_H - w_L}{w_L - v} \geq \frac{s_L\theta_L w_L + s_H\theta_H w_H}{1 - s_L\theta_L - s_H\theta_H},$$

then $\tau = 1 - v/w_L$ *and*

$$B = (1 - v/w_L)\frac{s_L\theta_L w_L + s_H\theta_H w_H}{1 - s_L\theta_L - s_H\theta_H},$$

and it holds that $(1-\tau)w_H \geq v + B$.

(b) *if*

$$v\frac{w_H - w_L}{w_L - v} < \frac{s_L\theta_L w_L + s_H\theta_H w_H}{1 - s_L\theta_L - s_H\theta_H},$$

then

$$\tau = (w_H - v)\frac{1 - s_L\theta_L - s_H\theta_H}{w_H(1 - s_L\theta_L) + s_L\theta_L w_L} < 1 - v/w_L,$$

$$B = (w_H - v)\frac{s_L\theta_L w_L + s_H\theta_H w_H}{w_H(1 - s_L\theta_L) + s_L\theta_L w_L},$$

and it holds that $(1-\tau)w_H = v + B$.

Proof. First, because the L-agents are in majority, Lemma 10 implies that π can be an equilibrium only if $\tau \leq 1 - v/w_H$. Second, we also know from Corollary 5 that in order for π to be an equilibrium policy, it must hold that $(1-\tau)w_H \geq v + B$. Third, Corollary 6 implies that, if π is an equilibrium policy, then it cannot be the case that $\tau < 1 - v/w_L$ and $(1-\tau)w_H > v + B$, because in that case τ and B could simultaneously be increased preserving feasibility without affecting labor supply. Therefore either $\tau = 1 - v/w_L$, or $(1-\tau)w_H = v + B$, or both.

If $\tau = 1 - v/w_L$, then B is as given by (), and the inequality $(1-\tau)w_H \geq v + B$ is equivalent to (). If, on the other hand, $\tau < 1 - v/w_L$, then

$$B = \tau\frac{s_L\theta_L w_L + s_H\theta_H w_H}{1 - s_L\theta_L - s_H\theta_H},$$

and the solution for τ in () is then implied by $(1-\tau)w_H = v + B$. The inequality $\tau < 1 - v/w_L$ is then equivalent to (). It then follows that if () is satisfied, the results in (a) follow, and if () is satisfied, then the results in (b) follow.

and hence

$$B = (w_H - v)\frac{s_H\theta_H}{1 - s_L\theta_L}$$

■

We now proceed with a more intutitive characterizations of the possible equilibria that we just formally defined. The equilibrium will always reflect the preferences of the low types who have majority. They would ideally like to have the tax as high as possible, making themselves indifferent between working and not working, low retirement age θ_L, and a conditional system so that the high types do not receive the benefits when they are aged between θ_L and θ_H. This policy is be feasible only if the "participation constraint" defined by equation 12 is satisfied. In that case, there can be two types of equilibria:

1) One is policy $\pi_3 \equiv (\theta_L, \tau_3, B_3, C)$, characterized by eligibility age θ_L, the highest possible tax, and conditional benefits.

2) The other is policy or $\pi_1 = (\theta_H, \tau_1, B_1, U)$, characterized by eligibility age θ_H, the highest possible tax, and unconditional benefits.

Which of the two equilibria wins depends on which of them is preferred by the majority. $R = \theta_L$ is preferred by all low types aged $a \leq \theta_L$ since in both the tax rate is the same, the revenue and the total expenditure on benefits are the same, but the low types capture a larger fraction of the payout.

The remaining lifetime utility of the low types aged $a \in (\theta_L, \theta_H)$ if π_3 is implemented is

$$U_L(\theta_L) = (1-a)v + (1-a)\frac{\left(1 - \frac{v}{w_L}\right)E}{(1-\theta_L)\,s_L + (1-\theta_H)\,s_H}$$

while their remaining lifetime utility if π_1 is implemented is

$$U_L(\theta_H) = (1-a)v + \left(1 - \frac{v}{w_L}\right)E$$

π_3 is preferred by these agents if

$$a \leq \overline{\theta} = s_L \theta_L + s_H \theta_H$$

while low type agents with $a \in (\overline{\theta}, \theta_H)$ prefer π_1. The trade-off for the low types aged between θ_L and θ_H is simple: They are not going to work so they only care about the benefits they would receive. Under π_3, the low types as a group capture a higher fraction of the expenditure on benefits, but a given agent aged above θ_L does not collect the benefits for the maximum possible length of time.

All the high types and the low types aged above θ_H prefer π_1 over π_3. Hence the measure of agents who prefer π_3 is $s_L (s_L \theta_L + s_H \theta_H)$ and π_3 is the equilibrium policy if

$$\begin{aligned} s_L (s_L \theta_L + s_H \theta_H) &\geq \frac{1}{2} \\ (s_L \theta_L + s_H \theta_H) &\geq \frac{1}{2 s_L} \end{aligned}$$

and π_1 is the equilibrium otherwise.

If the "participation constraint" on the high types (equation 12) is violated, there are also two possible equilibria:

3) One is policy $\pi_1 = (\theta_H, \tau_1, B_1, U)$, characterized by the eligibility age $R = \theta_H$ and $\tau = (1 - v/w_L)$ (the maximum possible), and unconditional, so the participation constraint of the high types no longer applies. The expenditure on benefits is maximized, but the low types share it proportionately with the high types.

4) The other is policy $\pi_4 \equiv (\theta_L, \tau_4, B_4, C)$, characterized by the eligibility age is $R = \theta_L$, conditional benefits, and tax rate and the benefit are reduced such that the participation constraint of the high types holds as equality. While the total

expenditure on benefits is reduced, the low types capture a higher fraction of it as they collect for a longer time, and all agents pay a lower tax.

Which of the two possible equilibria will be chosen? Note that high types with $a \geq \theta_H$ clearly prefer π_4, and low types with $a \geq \theta_H$ clearly prefer π_4.

Consider now the low types aged $a \in [0, \theta_L]$. Their remaining lifetime utlity under π_4 and π_1, respectively, are

$$U_L(\pi_4) = (\theta_L - a)(1 - \tau_4)w_L + (1 - \theta_L)v + (1 - \theta_H)B_4$$
$$U_L(\pi_1) = (\theta_L - a)(1 - \tau_1)w_L + (1 - \theta_L)v + (1 - \theta_L)B_1$$

After substituting for the respective taxes and benefits, the difference between the utilities under π_4 and π_1 is

$$U_L(\pi_4) - U_L(\pi_1) = (\theta_L - a)\left[\left(1 - \frac{(w_H - v)\left((1-\theta_L)s_L + (1-\theta_H)\, s_H\right)}{w_H - s_L\theta_L(w_H - w_L)}\right) w_L - v\right] + \\ + (1-\theta_L)\frac{(w_H - v)\, E}{w_H - s_L\theta_L(w_H - w_L)} - \left(1 - \frac{v}{w_L}\right) E$$

The first term is the difference between taxes paid, while the last two terms are the difference between the benefits collected. For an agent with $a = \theta_L$, the first term is zero, since she is not going to pay any taxes and cares only about the collected. The difference between the benefits collected is

$$(1-\theta_L)B_4 - (1-\theta_H)B_1 = -\frac{w_L - v}{w_L}\frac{\theta_L w_L + (1 - s_L\theta_L)\,(w_H - w_L)}{w_H - s_L\theta_L(w_H - w_L)}E < 0$$

Therefore the difference in the benefit collection is negative, so the gain from collecting the benefits for a longer time at the expense of the high types does not exceede the loss in the benefit level. As for low types with $a \in (\theta_L, \theta_H)$ the collection under π_1 is the same while the collection under π_4 is falling in age, they also prefer π_1.

For low types aged $a < [0, \theta_L)$, the difference in benefit collection is the same but they would also pay lower lifetime taxes as they get younger, so at a sufficiently lower age $\overline{a}_L$ they would start preferring π_4. $\overline{a}_L$ is defined implicitly as

$$
\begin{gathered}
U_L(\pi_4, \overline{a}_L) - U_L(\pi_1, \overline{a}_L) = 0 \\
(\theta_L - \overline{a}_L)\left[\left(1 - \frac{(w_H - v)\left((1-\theta_L)s_L + (1-\theta_H)\, s_H\right)}{w_H - s_L\theta_L(w_H - w_L)}\right) w_L - v\right] + \\
+ (1-\theta_L)\,\frac{(w_H - v)\, E}{w_H - s_L\theta_L(w_H - w_L)} - \left(1 - \frac{v}{w_L}\right) E = 0
\end{gathered}
$$

The low types aged $a \leq \overline{a}_L$ prefer while low types aged $a > \overline{a}_L$ prefer π_1. It is not guaratneed that $\overline{a}_L \geq 0$ exists, in which case all the low types would prefer π_1.

In a similar manner, $\overline{a}_H$ can be defined for the high types younger than θ_H:

$$
\begin{gathered}
U_H(\theta_L, \overline{a}_H) - U_H(\theta_H, \overline{a}_H) = 0 \\
(\theta_H - \overline{a}_H)\left[\left(1 - \frac{(w_H - v)\left((1-\theta_L)s_L + (1-\theta_H)\, s_H\right)}{w_H - s_L\theta_L(w_H - w_L)}\right) w_H - v\right] + \\
+ (1-\theta_H)\,\frac{(w_H - v)\, E}{w_H - s_L\theta_L(w_H - w_L)} - \left(1 - \frac{v}{w_L}\right) E = 0
\end{gathered}
$$

Therefore the measure of agents who prefer π_4 over π_1 is

$$
s_L\overline{a}_L + s_H\overline{a}_H
$$

$\pi_4 \equiv (\theta_L, \tau_4, B_4, C)$ is thus an equilibrium if

$$s_L \overline{a}_L + s_H \overline{a}_H \geq \frac{1}{2}$$

while $\pi_1 = (\theta_H, \tau_1, B_1, U)$ is an equilibrium if

$$s_L \left((\theta_L - \overline{a}_L) + (1 - \theta_L) \right) + s_H \left((\theta_H - \overline{a}_H) + (1 - \theta_H) \right) \geq \frac{1}{2}$$

While π_4 is a theoretically possible equilibrium, the model errs for π_1 to be the equilibrium under realistic values of the parameters. First, it is possible that $\overline{a}_L > 0$ does not exist, in which case all low types would prefer $R = \theta_H$ and so it would be selected. Second, the low types prefer high taxes in general, so the "sharing effect", driven by a difference between θ_H and θ_L would have to be sufficiently large to compensate for the reduced redistribution from the high types. However, $\theta_H - \theta_L$ is typically a small fraction of life, while a reduction in tax rate would apply to earnings being generated throughout the agents' working lives.

4 Comparative Statics and Discussion

Despite its simplicity, the model provides possible explanations for some features of the real-world PAYG systems and for the reduction in eligibility ages in the developed countries, implemented mostly through the introduction of early retirement. The model is also able to rationalize why the eligibility for benefits is frequently conditional on not having labor earnings, particularly in the early retirement schemes. In the framework of the model, an introduction of early retirement can be thought of as a transition from equilibrium with $R = \theta_H$ to an equilibrium with $R = \theta_L$. The outcomes in the new equilibrium capture several stylized facts of the PAYG systems in developed countries, namely a low eligibility age, conditional benefits under the early retirement schemes, high tax rates, a large fraction of workers retiring at the early eligibility age, and a smaller fraction retiring at the standard eligibility age.

The transition from θ_H to θ_L can occur in two different ways, depending on whether the initial equilibrium with θ_H was induced by the binding participation constraint on the high types or not. First, if the participation constraint on the high types were not binding at θ_L, θ_H was an equlibrium because only a majority of the population preferred it over θ_L. This would occur if $s_L\theta_L + \theta_H s_H \leq 1/2s_L$. A transition to the equilibrium with θ_L would then occur when the inequality switches sign such that θ_L gains majority support.

Second, if the participation constraint on the high types were binding at θ_L, θ_H was an equilibrium since it allowed to alleviate the constraint and impose a higher tax, which the low types support. A transition to the equlibrium with θ_L would then occur if the constraint ceases to be binding.

In a sense, the first explanation is based on purely political factors, as the composition of the population shifts towards groups that prefer a different policy. The second explanation is based on economic factors, as the changes in the underlying economy allow the majority composed of youner low types to implement a more preferred policy which was not feasible before.

Consider first the latter case. The condition for the participation constraint to be binding at $R = \theta_L$ is

$$\psi = (w_H - w_L)(1 - s_L\theta_L - s_H\theta_H) - \frac{w_L - v}{v}E \leq 0.$$

If the parameters of the model change such that ψ increases, the constraint may no longer be binding. It is straightforward to verify that

$$
\begin{aligned}
&\frac{\partial \psi}{\partial w_L} < 0 \\
&\frac{\partial \psi}{\partial w_H} > 0 \text{ if } \left(1 - s_L\theta_L - s_H\theta_H \frac{w_L}{v}\right) > 0 \\
&\frac{\partial \psi}{\partial \theta_L} < 0, \frac{\partial \psi}{\partial \theta_H} < 0 \\
&\frac{\partial \psi}{\partial s_L} > 0
\end{aligned}
$$

The "redundant elderly worker" explanation for the early retirement could be represented here as a reduction in the wage rate of the low types and/or a reduction in the natural retirement age of the low types. Both changes alleviate the participation constraint and thus could lead to a transition to θ_L. This is quite intuitive, since a reduction in the wage rate of the low types requires a reduction in the tax rate and a reduction in benefit, both making it easier for the after-tax wage of the low type to exceed the sum of the benefit and the value of leisure. Similarly, a reduction in θ_L alleviates the constraint by reducing the benefits.

Somewhat surprisingly, an increase in the wage rate of the high types need not necessarily alleviate the constrait. The reason is that while an increase in the wage reduces the gap between $(1 - \tau)\, w_H$ and $B + v$, it also increases the benefit through an increase in tax revenue, thus creating an opposite effect. However, the first effect should dominate as long as the difference between w_L and v is sufficiently small. The general conclusion is that rising income inequality may also trigger the reduction in the eligibility age.

Last, an increase in the share of the low types may also trigger the transition as it reduces the benefits that can be paid out by reducing the tax base.

In the former case when the participation constraint of the high types is not binding, θ_H is implemented if

$$s_L\theta_L + \theta_H s_H \leq 1/2s_L.$$

The conditions under which a transition to an equilibrium with θ_L could occur are dramatically different from the previous case. An increase in the share of the low types is not guaranteed to increase the support for θ_L. While the sheer number of the low types (who are the only ones potentially prefering θ_L) increases, the age at which they are indifferent between θ_L and θ_H falls, so the overall effect is uncertain. Shifts in wages cannot lead to a transition to θ_L if the participation constriant was not initially binding.

More interestingly, since θ_L is preferred by low types aged below and somewhat above θ_L, the measure of agents who prefer $R = \theta_L$ is *increasing* in θ_L. The same holds for θ_H. This feature of the model provides an intriguing reconciliation of the fact that the eligibility ages decreased at a time when the capacity to work was increasing. Initially, with lower natural retirement ages of both low and high types, the equilibrium eligibility age is "too high" from the point of view of the low types but lowering it does not have sufficient support. As the natural retirement ages rise, there are more people who prefer the lower of the eligibility ages, and at some point the equilibrium R may switch from θ_H to θ_L. Although θ_L has increased, the new may still be higher than the old equilibrium θ_H.

The model also gives a simple yet powerful explanation for the conditionality of benefits. When the eligibility age is set to the natural retirement age of the low types, the low types who provided the majority support for the low eligibility age definitely prefer to make the eligibility for benefits conditional on retirement. Conditionality is used as a screening device that cuts the high types off the benefits while making sure that the explicit and implicit taxes are sufficiently low so that they continue to work until their natural retirement age. Satisfying the participation constraint guarantees that maximum revenue is extracted from the high types, while the low eligibility age and conditionality allow the low types to capture a larger fraction of revenue than they would under an unconditional system. However, the conditional system is attractive to its beneficiaries only as long as there is a sufficient number of high-wage workers willing to work beyond the eligibility age. When the revenue that can be extracted from the high types working beyond the eligibility age shrinks, such as when θ_H declines and moves closer to θ_L, the social security system may switch back to an equilibrium with (now reduced) θ_H and unconditional benefit. To an exent the recent elimination of the implicit tax in some countries could be rationalized in that way.

5 Conclusion

The literature on political economy of social security either did not consider the eligibility age for benefits as one of the choice parameters or it modelled its choice by treating several other parameters of the social security system as exogenous. In this paper we provided a unified model for the choice of the benefit eligibility age and the conditionality of benefits as well as for the size of the tax rate and the benefit. The advantage of our approach is that all parameters are indeed endogenously determined within the model, the disadvantage is that in order to alleviate problems with majority voting in the multidimensional policy space, we restricted the options for the official retirement age down to two options, equal to the natural retirement ages of the low and the high types.

We showed that changes in the parameters of the model can drive a switch from an equilibrium with a high retirement age and unconditional benefits to an equilibrium with a low retirement age and conditional benefits. The model is therefore capable of explaining the pattern of eligibility ages and conditionality of benefits observed in developed countries in the last 50 years, that were mostly characterized by introduction of early retirement provisions where benefits are available at younger ages and conditional on the agent's exit from the labor force. Calibrating the model to the data on the developed countries in order to see how well the model quantitatively explains the shifts in eligibility ages and early retirement provisions is a natural avenue for future research.

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Appendix

Proof of Lemma 7. To begin, consider an alternative unconditional feasible policy $\pi' = (\theta_H, \tau', B', U)$ that preserves the eligibility age of π_1 and π_2, but deviates from the proclaimed dominant policy in the tax rate. First, suppose that (7) holds, which is equivalent to

$$(1 - v/w_L)(s_L\theta_L w_L + s_H\theta_H w_H) \geq (1 - v/w_H)s_H\theta_H w_H,$$

and suppose that $\tau' \neq \tau_1$. If $\tau' > \tau_1$, then $(1-\tau')w_L < v$, and therefore $r_L(\pi') = 0$. If, in addition, $\tau' > \tau_2$, or, equivalently, $(1-\tau')w_H < v$, then nobody ever works, no taxes are collected, $B' = 0$, and hence

$$\begin{aligned}
V_L(a, \pi_1) - V_L(a; \pi') &= \{\max(\theta_L - a, 0)(1-\tau_1)w_L + [1 - \max(\theta_L, a)]v + [1 - \max(\theta_H, a)]\, B\} \\
&\quad - (1-a)v \\
&= \{(1-a)v + [1 - \max(\theta_H, a)]\, B\} - (1-a)v \\
&= [1 - \max(\theta_H, a)]\, B \\
&> 0.
\end{aligned}$$

Intuitively, under π', the flow payoff of every agent at every age is v. On the other hand, under π, every L-agent can guarantee himself a flow payoff of at least v at any age before θ_H, and the flow payoff is $v + B > v$ after the age of $\theta_1 000H$. Therefore consider the case $\tau_1 < \tau' \leq \tau_2$, or, equivalently, $(1-\tau')w_L < v \leq (1-\tau')w_H$. In that case $r_H(\pi') = \theta_H$, and therefore

$$\begin{aligned}
B' &= \tau' s_H\theta_H w_H/(1-\theta_H) \\
&\leq (1 - v/w_H)s_H\theta_H w_H/(1-\theta_H) \\
&< (1 - v/w_L)(s_L\theta_L w_L + s_H\theta_H w_H)/(1-\theta_H) \\
&= B_1.
\end{aligned}$$

As a result,

$$
\begin{aligned}
V_L(a,\pi_1) - V_L(a;\pi') &= \{\max(\theta_L - a, 0)(1-\tau_1)w_L + [1-\max(\theta_L, a)]v + [1-\max(\theta_H, a)]\,B\} \\
&\quad - \{(1-a)v + [1-\max(\theta_H, a)]\,B'\} \\
&= \{(1-a)v + [1-\max(\theta_H, a)]\,B\} - \{(1-a)v + [1-\max(\theta_H, a)]\,B'\} \\
&= [1-\max(\theta_H, a)]\,(B - B') \\
&> 0.
\end{aligned}
$$

Intuitively, under the policy π', the flow payoff to an L-agent is v up to the age of θ_H, and $v + B'$ afterwards. On the other hand, under the policy π, the flow payoff to an L-agent is v up to the age of θ_H, and $v+B > v+B'$ afterwards. Therefore π strictly dominates π' for every L-agent. Finally, if $\tau' < \tau_1$, then $(1-\tau')w_L \geq v$, and hence $r_H(\pi') = \theta_H$ and $r_L(\pi') = \theta_L$. But then, because $r_H(\pi') = r_H(\pi)$, $r_L(\pi') = r_L(\pi)$, and $\tau_1 > \tau'$, Lemma 3 implies that all L-agents strictly prefer π_1 to π'.

Second, suppose that (8) holds, which is equivalent to

$$
(1 - v/w_L)(s_L\theta_L w_L + s_H\theta_H w_H) < (1 - v/w_H)s_H\theta_H w_H,
$$

and suppose that $\tau' \neq \tau_2$. If $\tau' > \tau_2$, then nobody works, and using the argument from the first part of the proof (which works analogously since $B > 0$ under π_2), π strictly dominates π' for all L-agents. If $\tau_1 < \tau' < \tau_2$, then $r_H(\pi') = \theta_H = r_H(\pi)$, $r_L(\pi') = 0 = r_L(\pi)$, and $\tau_2 > \tau'$, and hence Lemma 3 implies that all L-agents strictly prefer π_2 to π'. If $\tau' = \tau_1$, then $r_H(\pi') = \theta_H$ and $r_L(\pi') = \theta_L$, and hence

$$\begin{aligned} B' &= \tau'(s_L\theta_L w_L + s_H\theta_H w_H)/(1-\theta_H) \\ &= \tau_1(s_L\theta_L w_L + s_H\theta_H w_H)/(1-\theta_H) \\ &\equiv \widehat{B} \\ &< \tau_2 s_H\theta_H w_H/(1-\theta_H) \\ &= B_2. \end{aligned}$$

As a result,

$$\begin{aligned} V_L(a,\pi_2) - V_L(a;\pi') &= \{(1-a)v + [1-\max(\theta_H,a)]\,B\} \\ &\quad - \{\max(\theta_L - a, 0)(1-\tau')w_L + [1-\max(\theta_L,a)]v + [1-\max(\theta_H,a)]\,B'\} \\ &= \{(1-a)v + [1-\max(\theta_H,a)]\,B\} - \{(1-a)v + [1-\max(\theta_H,a)]\,B'\} \\ &= [1-\max(\theta_H,a)]\,(B-B') \\ &> 0. \end{aligned}$$

Intuitively, under the policy π', the flow payoff to an L-agent is v up to the age of θ_H, and $v+B'$ afterwards. On the other hand, under the policy π, the flow payoff to an L-agent is v up to the age of θ_H, and $v+B > v+B'$ afterwards. Therefore π strictly dominates π' for every L-agent. Finally, if $\tau' < \tau_1$, then $(1-\tau')w_L > v$, and hence $r_H(\pi') = \theta_H$ and $r_L(\pi') = \theta_L$. But then, because $r_H(\pi') = r_H(\widehat{\pi})$, $r_L(\pi') = r_L(\widehat{\pi})$, and $\tau_1 > \tau'$, where $\widehat{\pi} = (\theta_H, \tau_1, \widehat{B}, U)$, Lemma 3 implies that all L-agents strictly prefer $\widehat{\pi}$ to π'. But we have shown above that all L-agents strictly prefer π_2 to $\widehat{\pi}$, and hence, strictly prefer π_2 to π'.

Third, suppose that (12) holds, which is equivalent to

$$(1-v/w_L)(s_L\theta_L w_L + s_H\theta_H w_H) = (1-v/w_H)s_H\theta_H w_H.$$

In this case, because $r_H(\pi_1) = r_H(\pi_2) = \theta_H$, $r_L(\pi_1) = \theta_L$, and $r_L(\pi_2) = 0$, it follows that

$$
\begin{aligned}
B_1 &= \tau_1(s_L\theta_L w_L + s_H\theta_H w_H)/(1-\theta_H) \\
&= \tau_2 s_H \theta_H w_H/(1-\theta_H) \\
&= B_2,
\end{aligned}
$$

and hence

$$
\begin{aligned}
V_L(a;\pi_1) &= \max[r_i(\pi) - a, 0](1-\tau)w_L + [1-\max(\theta_L, a)]v + [1-\max(R,a)]\,B_1 \\
&= (1-a)v + [1-\max(R,a)]\,B_1 \\
&= (1-a)v + [1-\max(R,a)]\,B_2 \\
&= V_L(a;\pi_2).
\end{aligned}
$$

Intuitively, even though the L-agents work until θ_L under π_1 but don't work under π_2, they are held to their reservation utility of leisure under π_1, and, as a result, receive a flow payoff of v up until the age of θ_H folowed by $v + B_1 = v + B_2$ after the age of θ_H under both policies, and they are hence indifferent. In this case, if $\tau' > \tau_2$, nobody ever works, and the same argument as before (because $B_2 > 0$) can be used to show that π_2 strictly dominates π' for all L-agents. If $\tau_1 < \tau' < \tau_2$, then $r_H(\pi') = r_H(\pi_2)$, $r_L(\pi') = r_L(\pi_2)$, and $\tau_2 > \tau'$, and hence Lemma 3 implies that all L-agents strictly prefer π_2 to π'. Finally, if $\tau' < \tau_1$, then $r_H(\pi') = r_H(\pi_1)$, $r_L(\pi') = r_L(\pi_1)$, and $\tau_1 > \tau'$, and hence Lemma 3 implies that all L-agents strictly prefer π_1 to π'.

We have shown so far that every feasible policy $\pi' = (\theta_H, \tau', B', U)$ that preserves the eligibility age of π_1 and π_2, but deviates from the proclaimed dominant policy in the tax rate, is strictly dominated by the proclaimed dominant policy. Now consider any feasible policy $\pi' = (\theta_L, \tau', B', U)$ in which the eligibility age is set at θ_L. Construct another feasible policy $\widetilde{\pi} = (\theta_H, \tau', \widetilde{B}, U)$ that deviates from π' in that the benefit eligibility age is set at θ_H rather than θ_L, and the benefit amount is adjusted to preserve the budget balance, but uses the same tax rate as π'. Because the tax rate is the same, it follows that $r_H(\pi') = r_H(\widetilde{\pi})$ and $r_L(\pi') = r_L(\widetilde{\pi})$. As a result, the tax rate and the tax base is the same under the two policies, and hence the same is true of the tax revenue. But because the eligibility age is lower under π', the measure of the beneficiary base is higher, and hence

$$\begin{aligned}
B' &= \tau'(s_L r_L w_L + s_H r_H w_H)/(1-\theta_L) \\
&< \tau'(s_L r_L w_L + s_H r_H w_H)/(1-\theta_H) \\
&= \widetilde{B}.
\end{aligned}$$

It then follows that

$$\begin{aligned}
V_L(a;\widetilde{\pi}) - V_L(a;\pi') &= \{\max(r_L - a, 0)(1-\tau')w_L + [1-\max(r_L, a)]v + [1-\max(\theta_H, a)]\,\widetilde{B}\} \\
&\quad - \{\max(r_L - a, 0)(1-\tau')w_L + [1-\max(r_L, a)]v + [1-\max(\theta_L, a)]\,B'\} \\
&= [1-\max(\theta_H, a)]\,\widetilde{B} - [1-\max(\theta_L, a)]\,B'.
\end{aligned}$$

If $a \leq \theta_L$, then $V_L(a;\widetilde{\pi}) - V_L(a;\pi') = (1-\theta_H)\widetilde{B} - (1-\theta_L)B' = 0$, and hence the L-agents aged $a \leq \theta_L$ are indifferent between π' and $\widetilde{\pi}$. If $\theta_L < a \leq \theta_H$, then

$$\begin{aligned}
V_L(a;\widetilde{\pi}) - V_L(a;\pi') &= (1-\theta_H)\widetilde{B} - (1-a)B' \\
&> (1-\theta_H)\widetilde{B} - (1-\theta_L)B' \\
&= 0,
\end{aligned}$$

and if $a > \theta_H$, then

$$
\begin{aligned}
V_L(a;\widetilde{\pi}) - V_L(a;\pi') &= (1-a)\widetilde{B} - (1-a)B' \\
&> (1-a)(\widetilde{B} - B') \\
&> 0.
\end{aligned}
$$

As a result, the L-agents aged $a > \theta_L$ strictly prefer $\widetilde{\pi}$ to π'. But, using the argument from the previous part of the proof, if τ' deviates from the tax rate of the proclaimed dominant policy, then $\widetilde{\pi}$ is strictly dominated by this policy, and, by transitivity, so is π'. On the other hand, if τ' coincides with the tax rate of the proclaimed dominant policy, then $\widetilde{\pi}$ is this dominant policy, and hence the last part of the results of the Lemma follow. ■

Proof of Lemma 8. To begin, note that because the tax rate is the same under π and π', $r_L(\pi) = r_L(\pi')$. First, suppose that $j = U$. Then

$$B = \tau(s_L r_L w_L + s_H \theta_H w_H)/(1-\theta_L)$$

and

$$B' = \tau(s_L r_L w_L + s_H \theta_H w_H)/(1-\theta_H).$$

Therefore

$$
\begin{aligned}
V_H(a,\pi') - V_H(a;\pi_1) &= \{\max(\theta_H - a, 0)(1-\tau)w_H + [1-\max(\theta_H, a)]v + [1-\max(\theta_H, a)]\, B'\} \\
&\quad - \{\max(\theta_H - a, 0)(1-\tau)w_H + [1-\max(\theta_H, a)]v + [1-\max(\theta_L, a)]\, B\} \\
&= [1-\max(\theta_H, a)]\, B' - [1-\max(\theta_L, a)]\, B.
\end{aligned}
$$

If $a \leq \theta_L$, then $V_H(a, \pi') = V_H(a; \pi_1)$. If $\theta_L < a < \theta_H$, then

$$\begin{aligned} V_H(a, \pi') - V_H(a; \pi_1) &= \tau(s_L r_L w_L + s_H \theta_H w_H) \left[1 - \frac{1-a}{1-\theta_L}\right] \\ &> 0. \end{aligned}$$

If $a \geq \theta_H$, then $V_H(a, \pi') - V_H(a; \pi_1) = (1-a)(B' - B) > 0$. Therefore all the H-agents aged $a \leq \theta_L$ are indifferent between π and π', whereas all the H-agents aged $a > \theta_L$ strictly prefer π'.

Second, suppose that $j = C$. If $(1-\tau)w_H \geq v + B$, then $r_H(\pi) = r_H(\pi') = \theta_H$, and hence all the economic outcomes are the same as if $j = U$. Therefore the argument presented in the previous paragraph applies. If $(1-\tau)w_H < v + B$, then

$$B = \tau(s_L r_L w_L + s_H \theta_L w_H)/(1-\theta_L)$$

and

$$B' = \tau(s_L r_L w_L + s_H \theta_H w_H)/(1-\theta_H).$$

Therefore

$$\begin{aligned} V_H(a, \pi') - V_H(a; \pi_1) &= \{\max(\theta_H - a, 0)(1-\tau)w_H + [1 - \max(\theta_H, a)]v + [1 - \max(\theta_H, a)]\, B'\} \\ &\quad - \{\max(\theta_L - a, 0)(1-\tau)w_H + [1 - \max(\theta_L, a)]v + [1 - \max(\theta_L, a)]\, B\} \\ &\geq [1 - \max(\theta_H, a)]\, B' - [1 - \max(\theta_L, a)]\, B. \end{aligned}$$

If $a \leq \theta_L$, then

$$\begin{aligned} V_H(a, \pi') - V_H(a; \pi_1) &= (1-\theta_H)B' - (1-\theta_L)B \\ &= \tau s_H(\theta_H - \theta_L)w_H \\ &> 0. \end{aligned}$$

If $\theta_L < a < \theta_H$, then

$$
\begin{aligned}
V_H(a, \pi') - V_H(a; \pi_1) &= (1-\theta_H)B' - (1-a)B \\
&> (1-\theta_H)B' - (1-\theta_L)B \\
&> 0.
\end{aligned}
$$

If $a \geq \theta_H$, then $V_H(a, \pi') - V_H(a; \pi_1) = (1-a)(B' - B) > 0$. Therefore all the H-agents strictly prefer π' to π. ■

Proof of Lemma 9. In case (a) with strict inequality, any feasible policy $\pi' = (R, \tau', B', U)$ with $\tau' \neq \tau_1$ is strictly dominated by π_1 for every L-agent by Lemma 7. Because L-agents are in a strict majority, π' cannot be an equilibrium. In addition, the feasible policy $\pi'' = (\theta_L, \tau_1, B'', U)$ is weakly dominated by π_1 for every L-agent, and it is strictly dominated by $\pi_1$for the L-agents aged $a > \theta_L$ (Lemma 7) and for all the H-agents (Lemma 8). Therefore, π'' cannot be an equilibrium either. As a result, if an equilibrium policy with unconditional benefits exists, then it must be π_1.

In case (a) with equality, any feasible policy $\pi' = (R, \tau', B', U)$ with $\tau' \notin \{\tau_1, \tau_2\}$ is strictly dominated by π_1 for every L-agent by Lemma 7. Because L-agents are in a strict majority, π' cannot be an equilibrium. In addition, the feasible policies $\pi_1'' = (\theta_L, \tau_1, B_1'', U)$ and $\pi_1'' = (\theta_L, \tau_2, B_2'', U)$ are weakly dominated by π_1 for every L-agent, and they are strictly dominated by $\pi_1$for the L-agents aged $a > \theta_L$ (Lemma 7) and for all the H-agents (Lemma 8). Therefore, neither of these two policies can be an equilibrium either. Finally, consider the policy π_2. All the L-agents are indifferent between π_1 and π_2, and so are all the H-agents aged $a \geq \theta_H$ because $B_1 = B_2$. However, all the H-agents aged $a < \theta_H$ strictly prefer π_1 because of the lower tax rate. As a result, if an equilibrium policy with unconditional benefits exists, then it must be π_1.

In case (b), any feasible policy $\pi' = (R, \tau', B', U)$ with $\tau' \neq \tau_2$ is strictly dominated by π_2 for every L-agent by Lemma 7. Because L-agents are in a strict majority, π' cannot be an equilibrium. In addition, the feasible policy $\pi'' = (\theta_L, \tau_2, B'', U)$ is weakly dominated by π_2 for every L-agent, and it is strictly dominated by $\pi_2$for the L-agents aged $a > \theta_L$ (Lemma 7) and for all the H-agents (Lemma 8). Therefore, π'' cannot be an equilibrium either. As a result, if an equilibrium policy with unconditional benefits exists, then it must be π_2. ■

Proof of Lemma 10. First, if $(1-\tau)w_H < v$, then nobody ever works, and hence $B = 0$. But under the policy π_2, the L-agents never work either but they receive a positive benefit when old. Therefore π_2 strictly dominates π for all the L-agents.

Second, if $v \leq (1-\tau)w_H < v + B$, then $r_H(\pi) = \theta_L$, and, therefore $B = \tau s_H \theta_L w_H/(1-\theta_L)$. It then follows that

$$
\begin{aligned}
V(a, \pi_2) - V(a, \pi) &= \{(1-a)v + [1 - \max(\theta_H, a)]B_2\} - \{(1-a)v + [1 - \max(\theta_L, a)]B\} \\
&= [1 - \max(\theta_H, a)]B_2 - [1 - \max(\theta_L, a)]B \\
&= [1 - \max(\theta_H, a)](1 - v/w_H)s_H\theta_H w_H/(1-\theta_H) \\
&- [1 - \max(\theta_L, a)]\tau s_H \theta_L w_H/(1-\theta_L) \\
&\geq (1 - v/w_H)s_H w_H \left[\frac{\theta_H}{1-\theta_H}[1 - \max(\theta_H, a)] - \frac{\theta_L}{1-\theta_L}[1 - \max(\theta_L, a)]\right].
\end{aligned}
$$

If $a \leq \theta_L$, then $V(a, \pi_2) - V(a, \pi) \geq (1 - v/w_H)s_H w_H(\theta_H - \theta_L) > 0$. If $\theta_L < a \leq \theta_H$, then $V(a, \pi_2) - V(a, \pi) \geq (1 - v/w_H)s_H w_H [\theta_H - \theta_L(1-a)/(1-\theta_L)] > 0$. If $a > \theta_H$, then $V(a, \pi_2) - V(a, \pi) \geq (1 - v/w_H)s_H w_H(1-a)[\theta_H/(1-\theta_H) - \theta_L/(1-\theta_L)] > 0$. Therefore π_2 strictly dominates π for all the L-agents.

Third, consider the case when $(1-\tau)w_H \geq v + B$. Consider the policy $\pi' = (\theta_L, \tau', B', C)$ defined by $\pi' = \pi$ if $(1-\tau)w_H = v + B$ and by $(1-\tau')w_H = v + B'$ and $B' = \tau' s_H \theta_H w_H/(1 - s_L\theta_L - s_H\theta_H)$ if $(1-\tau)w_H > v + B$. Note that, in the latter case, because $B = \tau s_H \theta_H w_H/(1 - s_L\theta_L - s_H\theta_H)$, it follows that $\tau' > \tau$, and, because $r_H(\pi) = r_H(\pi') = \theta_H$ and $r_L(\pi) = r_L(\pi') = 0$, Lemma 3 implies that all the L-agents strictly prefer π' to π. In either case, π' weakly dominates π for all the L-agents. Note that the two defining equations of π' imply that

$$\tau' = (1 - v/w_H)\frac{1 - s_L\theta_L - s_H\theta_H}{1 - s_L\theta_L}$$

and

$$B' = (1 - v/w_H)\frac{s_H\theta_H w_H}{1 - s_L\theta_L} < B_2.$$

As a result,

$$\begin{aligned}
V(a, \pi_2) - V(a, \pi') &= \{(1-a)v + [1 - \max(\theta_H, a)]B_2\} - \{(1-a)v + [1 - \max(\theta_L, a)]B'\} \\
&= [1 - \max(\theta_H, a)]B_2 - [1 - \max(\theta_L, a)]B' \\
&= (1 - v/w_H)s_H\theta_H w_H \left[\frac{1 - \max(\theta_H, a)}{1 - \theta_H} - \frac{1 - \max(\theta_L, a)}{1 - s_L\theta_L}\right].
\end{aligned}$$

If $a \leq \theta_L$, then $V(a, \pi_2) - V(a, \pi') = (1 - v/w_H)s_H\theta_H w_H [1 - (1 - \theta_L)/(1 - s_L\theta_L)] > 0$. If $\theta_L < a \leq \theta_H$, then $V(a, \pi_2) - V(a, \pi') = (1 - v/w_H)s_H\theta_H w_H [1 - (1 - a)/(1 - s_L\theta_L)] > 0$. If $a > \theta_H$, then $V(a, \pi_2) - V(a, \pi') = (1 - v/w_H)s_H\theta_H w_H(1 - a) [1/(1 - \theta_H) - 1/(1 - s_L\theta_L)] > 0$. Therefore π_2 strictly dominates π' for all the L-agents. Then, by transitivity, π_2 strictly dominates π for all the L-agents. ■