

# The Social Welfare Effect of Tying in a Vertically Differentiated Market for System Goods: The Case of Microsoft\*

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## Abstract

The present paper is concerned with welfare analysis of the equilibrium outcome from tying an application to a system platform in the market for PC software systems. The issue has gained popularity in the last decade around the anti-trust cases against Microsoft. What is new in the present paper is that the market for PC software systems is assumed to be vertically differentiated. That is, there are well-defined leaders in quality whose brands dominate the market. First, the paper aims to show that the tying arrangement of Microsoft does not need to be driven by market-power-leverage incentives. In fact, the solution of the model shows that at the subgame-perfect equilibrium the tying decision of Microsoft must rather be driven by market-share-saving incentives. Second, the social welfare effect of tying in this case is measured and compared to the social welfare in the benchmark non-tying case. The results imply that tying leads to increase in the average quality in the market which is sufficient to compensate for the negative welfare effect of having the market non-covered. As a net effect both consumer surplus and social welfare would increase while production surplus would decrease.

*Keywords:* product tying, vertical differentiation, market foreclosure, system goods, social welfare, anti-trust policy

*JEL classification:* L11, L13, L15

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## Introduction

There is a continuous debate among economists whether really merging two firms producing different components of a final good is always motivated by innocent incentives.

The debate gained popularity in the last decade around the anti-trust cases against Microsoft on allegations that Microsoft abused monopoly power by tying its operating system with other software applications. With few exceptions all the models of system-good markets used both by the opponents and the proponents of Microsoft in the debate assume a standard Chamberlinian market structure where goods are horizontally differentiated. However, there are authors (Etro (2006b)) who recognize a clear pattern of quality leadership in the market for PC software systems which implies that the system goods sold on these markets are rather vertically differentiated<sup>1</sup>.

The present paper steps on the assumption that PC software systems are vertically differentiated in which case, tying behavior of Microsoft could be motivated by own-sales-saving rather than predatory (entry-deterrence) incentives. However, this possibility itself does not imply that product tying would be a socially desirable practice per se. Even when the existing level of market competition is not threatened, product tying could still hurt the society. The main objective of the paper is to clarify that issue by measuring the welfare effect of product tying in the case when it could be considered as a defensive strategy by the multi-product firm.

To the extent to which the setup of the model used in the paper is chosen to correspond closely to the structure of the market for PC software systems in the late 1990s, the implications of the paper are meant also to contribute to the economic debate on the anti-trust cases against Microsoft. Therefore, in the remainder of the introductory section, a description will be provided of the antitrust cases against Microsoft, existing literature on the problem, research objectives and suggested methodology for their fulfillment.

In 1998 the U.S. Department of Justice starts a trial against Microsoft on allegations that Microsoft abused monopoly power on Intel-based personal computers in its handling of operating system sales and web browser sales (DOJ Complaint 98-12320). In 2000, the findings of fact are issued in favor of the plaintiffs on almost all points including use of monopoly power to exclude rivals and harm competitors. The adopted legal remedies include splitting Microsoft into two companies, and imposing severe business conduct restrictions. Microsoft appealed and in 2011 the case is settled with agreement which does not prevent Microsoft from tying its operating system with other software applications.

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<sup>1</sup> For the difference between horizontal and vertical differentiation see (Shaked and Sutton, 1983, p. 1469).

However, independently from U.S. authorities, in 1998 the European Commission starts an investigation on how Microsoft streaming media application is integrated into its operating system. In 2003 Microsoft is ordered to disclose part of the code of its operating system as well as to offer a version of it without media player. In 2004 after a year of nonfeasance of the order, Microsoft is penalized to pay the largest fine ever handed out by the EU at the time (Commission Decision of 24.03.2004 relating to a proceeding under Article 82 of the EC Treaty (Case COMP/C-3/37.792 Microsoft)). As a result, in 2009 Microsoft executes the order of the European Commission and provides the latest version of its operating system with an option to choose with which media player to be installed.

The anti-trust cases against Microsoft have raised serious debate among economists. There are different arguments for and against Microsoft's tying arrangement but consensus is missing between the two sides in the dispute.

The proponents of Microsoft use three arguments why tying its operating system to an application would have a positive impact on the social welfare. First, Davis, MacCrisken and Murphy (1999) claim that the expansion of the system software functionality to include additional features of standalone applications is inevitable result of the evolution of the PC operating systems. So, the tying of Microsoft's products must be considered as a technological tying which increases the value of the operating system for the end user and thus increases the consumer surplus. Second, Economides (2000) shows that operating system is a network good i.e. its value for the end consumers increases in their number. Therefore, it is beneficial for the end users if they are served by a product with larger market share. Third, Etro (2006b) applies the theory of aggressive market leaders suggesting that Microsoft stays on top of the market because as a market leader it has the greatest incentives to innovate and keep its leadership position. Hence, it acts more competitively and thus serves society better as a monopolist than if there were not a dominant player in the market.

The opponents of Microsoft rather rely on the classical definition of tying as a market foreclosure practice. Carlton and Waldman (1998) state that by tying its operating system to different software applications Microsoft could leverage its market power at the operation system market to the respective market for standalone software applications. This would have a negative effect on social welfare if in this adjacent market Microsoft is a follower rather than a leader. Fudenberg and Tirole (2000) apply an overlapping generation model to show that even with network externalities there could be a negative social welfare effect of market foreclosure when the new users are relatively more than old ones. That is, the welfare loss by the presence of monopoly might not be fully compensated by the network externalities of expanding the pool of consumers served by the monopolist.

Neither the proponents, nor the opponents of Microsoft, however, have so far studied the possibility that Microsoft might have decided to tie its operating system with its software applications as a response to an entry in the monopoly market for personal-computer operating systems.

Although not in the context of the antitrust cases against Microsoft, the potential existence of such an incentive for tying is suggested by Kovac (2007). He shows that when multi-product goods are assumed to be vertically differentiated, market foreclosure incentive to bundle changes. The multi-product firm might be interested to apply tying not because in this way it would become a monopolist on both integrated markets. Rather it would tie its product as a response to a foreclosure threat from a potential lower quality entrant.

What makes the conjecture of Kovac (2007) interesting for reconsideration in the context of the anti-trust cases against Microsoft is that it seems consistent with the market facts at the time when Microsoft has taken the decision to tie its products. With the development of Internet in the beginning of 1990s, a commercial version of UNIX, the dominant brand in the server operating system market, enters the personal-computer system software market under the brand Linux. This allows consumers to use both Microsoft's browsing application Internet Explorer and its single competitor Netscape Navigator not only on Microsoft's operation system Windows but also on Linux. The resulting market situation of 3 potential entrants (Microsoft, Linux, Netscape), 2 types of components (operation system and browser), and 4 possible goods (WindowsNetscape, WindowsExplorer, LinuxNetscape, LinuxExplorer) matches exactly the market setting suggested by Kovac (2007).

Particularly, the conjecture made by Kovac (2007) when applied to the system-software market of 1990s implies that if Microsoft's browsing application Internet Explorer were not tied to its operating system Windows, consumers would not be willing to buy Internet Explorer based on Windows. They would either use it on Linux or purchase Netscape Navigator based on Windows instead. So, Microsoft would have no option to save the sales of WindowsExplorer other than tying the browser to the operating system. Tying itself would result in foreclosure of the sales of Linux which would benefit the both quality leaders, Microsoft and Netscape.

In order to improve the general validity of the main implications of, Burlakov (2013b) applies an alternative model of a vertically differentiated market. The model is originally developed by Burlakov (2011a) to fit better the standard definition of a system good. In Burlakov's (2013a) setup the different types of components of the

The present paper aims to apply a modified version of the model of Burlakov (2013b) which allows for a multi-product firm to enter the market similarly to Microsoft in the late 1990s. Distinct from Kovac (2007) system goods are assumed to be sold together no matter whether tying

arrangement is imposed or not. Respectively, by solving the new model it is shown that the non-bundling subgame equilibrium outcome described by Kovac (2007) could still hold in a pure system-good market but at a narrower spread of consumer tastes. Moreover, in order the multi-product firm's good to be effectively excluded in the non-tying equilibrium, an additional condition needs to be introduced. Namely, the quality of the multi-product firm's good must be exogenously restricted to be sufficiently low relative to the quality of the best good in the competitive market.

Based on the equilibrium solution of the model, the social welfare is measured in case of tying<sup>2</sup> and non-tying. The comparison of the two measures allows checking whether society would have gained if Microsoft had abstained from tying and let instead its browser be sold separately from its system platform. It also allows for policy implications to be derived how adequate to the considered market situation would be the measures taken by the U.S. and the EU.

The paper is organized as follows. In Section 2 the modified version of the Burlakov's (2013b) model is presented. Section 3 presents the solution of the model and specifies the equilibrium outcomes with and without tying arrangement, respectively. The welfare analysis is given in Section 4.

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<sup>2</sup> In the case of Microsoft, the system platform is tied to the internet browser but not the other way around as suggested by Kováč (2007). However, as it is shown in the present paper the implications of Kováč (2007) still hold with the actual tying strategy of Microsoft if consumers are allowed to install the Netscape's browser together with Microsoft's one on the Windows system platform. In fact, in reality they could never be prevented to do so. Therefore, taking that into account in the current analysis not only does not restrict the validity of its results but makes them even more consistent with the real-world case being analyzed.

## II. The Model

Assume three firms: multi-product firm M which offers good A1 in market A and B2 in market B, single-product firm A2 which offers good A2 in market A and single-product firm B1 which offers good B1 in market B.

The strategic interaction between firms is modeled as a four-stage game.

Stage 1: Firms make entry decision.

If a firm decides to enter a market it pays an infinitesimally small entry fee  $\varepsilon$  which incentivizes it to stay out of the market if its profit is zero.

Stage 2: After observing the rest of the entrants, each firm chooses the quality of its products.

If both firms M and A2 enter market A the qualities of their products are ranked in decreasing order as follows:

$$0 < \underline{v} \leq v_2 \leq \bar{v}_2 < v_1 \leq \bar{v}_1 \quad (1)$$

where:

$v_1$  - quality of good A1

$v_2$  - quality of good A2

$\underline{v}$  - lower bound on the range in which qualities could vary in market A

$\bar{v}_1$  - upper bound on the quality of good A1

$\bar{v}_2$  - upper bound on the quality of good A2

If both firms M and B1 enter market B the qualities of their products are also ranked in decreasing order as follows:

$$0 < \underline{u} \leq u_2 \leq \bar{u}_2 < u_1 \leq \bar{u}_1 \quad (2)$$

where:

$u_1$  - quality of good B1

$u_2$  - quality of good B2

$\underline{u}$  - lower bound on the range in which qualities could vary in market B

$\bar{u}_1$  - upper bound on the quality of good B1

$\bar{u}_2$  - upper bound on the quality of good B2

Assuming different upper bounds on the first and second quality in each market implies that firms face different technological possibility frontiers. As it is shown in the next section, this assumption ensure strict quality differentiation in equilibrium even though firms choose their qualities simultaneously.

Stage 3: Given the qualities chosen at the previous stage, firm M decides to tie A1 to B2 or not<sup>3</sup>

Stage 4: Firms compete in prices<sup>4</sup>

For simplicity, production cost (of quality) is assumed to be zero<sup>5</sup>. Since the market entry fee  $\varepsilon$  is close to zero, it could be ignored in the solution for optimal prices. So, the profits of firms

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<sup>3</sup> It could be shown that the other tying strategies are strictly dominated. For detailed analysis see Kovac (2007).

<sup>4</sup> (Bertrand) Competition in prices at the last stage is a standard assumption in the vertical-differentiation models which fits well the market for software systems where products are easily multipliable, so quantity supplied tends to infinity.

coincide with their total revenues and the pricing subgame equilibrium is given by the solution of the following system of maximization problems:

$$\begin{cases} \underset{p_1^A, p_2^B}{Max} \Pi^M = TR^M = p_1^A D_1^A + p_2^B D_2^B, \text{ if A1 and B2 are sold separately} \\ \underset{p_1^A, p^M}{Max} \Pi^M = TR^M = p_1^A D_1^A + p^M D^M, \text{ if B2 is tied to A1} \end{cases} \quad (3)$$

$$\underset{p_1^B}{Max} \Pi_1^B = TR_1^B = p_1^B D_1^B \quad (4)$$

$$\underset{p_2^A}{Max} \Pi_2^A = TR_2^A = p_2^A D_2^A \quad (5)$$

where:

$\Pi^M, \Pi_1^B, \Pi_2^A$  - the profits of firms M, B1 and A2, respectively

$TR^M, TR_1^B, TR_2^A$  - the total revenues of firms M, B1 and A2, respectively

$p_1^A, p_2^A, p_1^B, p_2^B, p^M$  - the prices of goods A1, A2, B1, B2, and bundle A1B2, respectively

$D_1^A, D_2^A, D_1^B, D_2^B, D^M$  - the demands for goods A1, A2, B1, B2, and bundle A1B2,

respectively

The demands for goods are given as a sum of the demands for the product pairs in which they take part. For example, the demand for good A1 is given by the sum of the market shares of the product pairs A1B1 and A1B2:

$$D_1 = D_{11} + D_{12} \quad (6)$$

where:

$D_{11}, D_{12}$  - market shares of the product pairs A1B1 and A1B2, respectively

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<sup>5</sup> Shaked and Sutton (1983) show that zero-cost assumption would yield the same optimal solution as in case positive unit cost is assumed which increases slowly (i.e. with less than unit) in quality.



The market shares of the product pairs are determined by consumer preferences.

As it is standard for the models of product differentiation, here it is assumed that consumers make indivisible and mutually exclusive purchases<sup>6</sup> in a sense that a consumer either buys a pair of goods  $A_i B_j$  or do not buy in the market at all. If the consumer does not buy in the market, she is assumed to consume instead an outside option which is available for free.

Another standard assumption for the models of vertical product differentiation is that consumers are characterized by their taste (variable) for quality  $\theta$  which is assumed to be uniformly distributed on the interval  $[\underline{\theta}, \bar{\theta}]$ .

The utility function measures the individual surplus of consumer  $k$  from purchasing given product pair  $A_i B_j$  and takes the following form:

$$U_{ij}^k = \chi_{ij} \cdot \theta_k - p_{ij}, \{i = 1, 2, j = 1, 2\} \quad (7)$$

where:

$U_{ij}^k$  - reduced-form utility function of consumer  $k$  from purchasing product pair  $A_i B_j$

$\theta_k$  - taste (variable) for quality of consumer  $k$

$\chi_{ij}$  - quality of product pair<sup>7</sup>  $A_i B_j$ ,  $\chi_{ij} = (v_i)^\alpha + (u_j)^\alpha$ ,  $\{\alpha \geq 0\}$

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<sup>6</sup> In the context of the software-system market, by purchasing a good, here it is meant paying the license fees for the operating system and the application software which the consumer runs on it. Since software license fees are paid per workplace and a PC user is not expected to use the same application on two computers simultaneously, her utility is defined on a unit purchase. Respectively, since the ordinary software applications (excl. client-server systems) are not meant to be used simultaneously by two users on the same computer, the purchase is considered exclusive.

<sup>7</sup> Note that the quality of a product pair is derived by the qualities of the goods in it according to a modified CES accumulation function. The so chosen functional form allows expressing different degrees of substitutability between the two qualities dependent on the value of the CES parameter  $s = \frac{1}{1 - \alpha}$ .

$p_{ij}$  – price of product pair  $A_i B_j$ , when  $A_i$  and  $B_j$  are sold separately the pair price is given as a pure sum of the goods' prices,  $p_{ij} = p_i^A + p_j^B$

Because consumers have unit consumption and their tastes are uniformly distributed within the market range  $(\underline{\theta} \leq \theta \leq \bar{\theta})$ , the bounds of the market shares of the product pairs are marked by the taste parameters of the so-called “marginal consumers”. Each marginal consumer  $\theta_{ij/i'j'}$ ,  $\{i \neq i' \text{ and/or } j \neq j'\}$  is indifferent between given distinct pair of available qualities  $A_i B_j$  and  $A_{i'} B_{j'}$ .

The expression for the marginal consumer's taste variable could be directly derived from (7) and looks as follows:

$$\theta_{ij/i'j'} = \frac{p_{ij} - p_{i'j'}}{\chi_{ij} - \chi_{i'j'}} = \frac{p_{ij} - p_{i'j'}}{d_{ij/i'j'}} \quad (8)$$

where:

$d_{ij/i'j'}$  – the difference in qualities between product pairs  $A_i B_j$  and  $A_{i'} B_{j'}$ ,

$$d_{ij/i'j'} = \chi_{ij} - \chi_{i'j'}$$

For example, the demand for product pair A1B1 is given by the difference between the upper bound of the market range  $\bar{\theta}$  and the marginal taste at which consumers are indifferent between pair A1B1 and the next most preferred pair in the market. If the next most preferred product pair is A1B2, the market share of A1B1 can be expressed as follows:

$$D_{11} = \bar{\theta} - \theta_{11/12} \quad (9)$$

where:

$\theta_{11/12}$  - marginal taste at which consumers are indifferent between pairs A1B1 and A1B2

Similarly, the demand for product pairs A1B2, A2B1 and A2B2 could be derived:

$$D_{12} = \theta_{11/12} - \theta_{12/21} \quad (10)$$

$$D_{21} = \theta_{12/21} - \theta_{21/22} \quad (11)$$

$$D_{22} = \begin{cases} \theta_{21/22} - \theta_{22/O}, & \text{for } \underline{\theta} < \theta_{22/O} \\ \theta_{21/22} - \underline{\theta}, & \text{for } \underline{\theta} \geq \theta_{22/O} \end{cases} \quad (12)$$

where:

$\theta_{12/21}$  - marginal taste at which consumers are indifferent between pairs A1B2 and A2B1

$\theta_{21/22}$  - marginal taste at which consumers are indifferent between pairs A2B1 and A2B2

$\theta_{22/O}$  - marginal taste at which consumers are indifferent between pair A2B2 and the outside

option.

Note, however, that expressions (10), (11) and (12) could take only non-negative values. Thus, for example if  $\theta_{21/22}$  exceeds  $\theta_{12/21}$ , product pair A2B1 will be driven out from the market. So, the next most preferred product pair after A1B2 would be A2B2 instead. Furthermore, if  $\theta_{12/22}$  exceeds  $\theta_{11/12}$ , also product pair A1B2 will be driven out from the market. As it is shown in the next section, for this outcome to occur in the subgame equilibrium without tying it is sufficient the upper bound on the quality of good B2 ( $\bar{u}_2$ ) to be sufficiently low<sup>8</sup>. In addition, if the lower bound  $\underline{\theta}$  on the market range is large enough, all consumers will be served by product pairs A1B1 and A2B2 only<sup>9</sup>:

$$D_{11}^* = \bar{\theta} - \theta_{11/22} \quad (13)$$

$$D_{22}^* = \theta_{11/22} - \underline{\theta} \quad (14)$$

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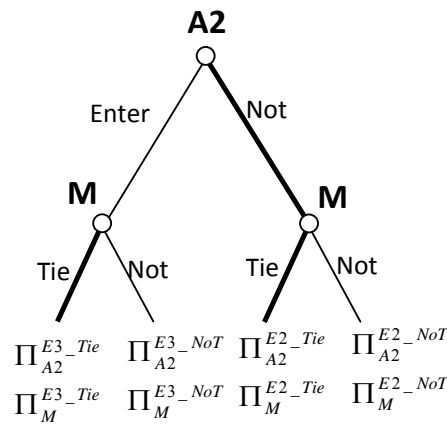
<sup>8</sup> Similar condition could also be derived for the case when only single-product firms enter the market, see Burlakov (2013b).

<sup>9</sup> To distinguish between hypothetical expressions (9), (10), (11), (12) and the equilibrium ones (13), (14), in the latter demands are marked by ‘NoT’ for “not tying” and asterisk.

### III. Equilibrium Solution

Since the model introduced in the previous section steps on a dynamic game of complete information, its solution is given by the corresponding subgame-perfect Nash equilibrium. It could be derived by backward induction. Figure 1 below depicts the solution by means of an extensive-form game tree:

Figure 1: Extensive-form game representation of the equilibrium solution<sup>10</sup>



The so-presented subgame-perfect equilibrium outcome implies that if allowed firm M will find it optimal to tie B2 to A1 in the third stage no matter whether firm A2 is in the market or not. As a result, firm A2 would prefer not to enter and the market will remain non-covered. That is, part of consumers would purchase the outside option while the rest would go for either A1B1 or (A1B2). This equilibrium solution as well as the rigorous definition of the conditions at which it would hold is stated in lemma 1 below.

<sup>10</sup> For simplicity here only a reduced-form game tree is shown. Though not explicitly analyzed in the paper, the subgame equilibrium outcomes in case of firm M entering as a single-product firm or using different tying strategies were also considered when determining the subgame perfect equilibrium outcome on figure 1.

**Lemma 1.** Let the range in which the quality of B2 varies is defined by the following conditions:

$$\bar{u}_2 \leq \text{Min} \left\{ \sqrt[\alpha]{\frac{3((\bar{u}_1)^\alpha + (\bar{v}_1)^\alpha) - 11(\underline{v})^\alpha + 8\chi_o}{11}}, \sqrt[\alpha]{\frac{4((\bar{u}_1)^\alpha + (\underline{v})^\alpha) - 5(\bar{v}_1)^\alpha + \chi_o}{5}} \right\} \quad (15)$$

$$\underline{u} > \sqrt[\alpha]{\frac{((\bar{u}_1)^\alpha + (\bar{v}_1)^\alpha) - 5(\underline{v})^\alpha + 4\chi_o}{5}} \quad (16)$$

and the lower bound on market range satisfies the following inequality:

$$\frac{2\bar{\theta}d_{11/22}}{4d_{22/O} + 3d_{11/22}} \leq \underline{\theta} < \frac{\bar{\theta}}{2} \quad (17)$$

Then, if tying is allowed only firms M and B1 will enter the market, they will choose the best feasible qualities for their goods:

$$v_1 = \bar{v}_1; u_1 = \bar{u}_1; u_2 = \bar{u}_2 \quad (18)$$

and firm M will tie good B2 to A1 so that at the end only product pairs A1B1 and (A1B2) will have positive market shares in equilibrium.

The corresponding optimal prices chosen in the last stage would be as follows<sup>11</sup>:

$$p_{A1}^{E2\_Tie*} = \frac{\bar{\theta}}{6} (2d_{11/12}^{E2} + 3d_{12/O}^{E2}), p_{B1}^{E2\_Tie*} = \frac{\bar{\theta}}{2} d_{12/O}^{E2}, p_{B1}^{E2\_Tie*} = \frac{\bar{\theta}}{3} d_{11/12}^{E2} \quad (19)$$

Proof: see the Appendix

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<sup>11</sup> To distinguish the equilibrium with tying from the equilibrium without tying, the solution for the former is denoted by “E2\_Tie\*” which stands for “2 entrants, tying” while the solution for the latter is denoted by “E3\_NoT\*” which stands for “3 entrants, not-tying”.

If tying is not allowed, firm M would not have an incentive to sell B2 as part of the pair A1B2. In order B2 to be sold as a separate part of A1B2 the price of A1 needs to be decreased. In this case, however, the share of the high-taste consumer surplus that M can obtain through selling A1B1 will shrink. Therefore, firm M would rather prefer to keep the high price of A1 and sell B2 as part of the pair A2B2. Thus, it will accommodate firm A2. So, the latter will enter and cover the market in equilibrium. The not-tying equilibrium outcome is defined in lemma 2 below.

**Lemma 2.** Given that conditions (15), (16) and (17) hold, all the three firms M, B1 and A2 will enter the market, they will choose the best feasible qualities for the top-ranked goods and the worst feasible qualities for the bottom-ranked goods:

$$v_1 = \bar{v}_1; u_1 = \bar{u}_1; v_2 = \underline{v}; u_2 = \underline{u} \quad (20)$$

so that at the end only product pairs A1B1 and A2B2 will have positive market shares in equilibrium.

The corresponding optimal prices chosen in the last stage would be as follows:

$$\begin{aligned} p_{A1}^{E3-NoT*} &= \frac{\theta}{2} (2d_{22/O}^{E3} + d_{11/22}^{E3}), p_{A2}^{E3-NoT*} = \frac{2\bar{\theta} - 3\underline{\theta}}{4} d_{11/22}^{E3}, \\ p_{B1}^{E3-NoT*} &= \frac{(2\bar{\theta} - \underline{\theta})}{4} d_{11/22}^{E3}, p_{B2}^{E3-NoT*} = \underline{\theta} d_{22/O} - \frac{(2\bar{\theta} - 3\underline{\theta})}{4} d_{11/22}^{E3} \end{aligned} \quad (21)$$

Proof: see the Appendix

Finally, it remains to assess the social welfare effect of tying by comparing the total surplus (resp. consumer and producer surplus) in the case of the tying equilibrium in lemma 1 to the total surplus in the case of the not-tying equilibrium in lemma 2. The results imply that even though the market would not be covered when tying is allowed, still the increase in the average quality in the market will compensate for the negative effect on social welfare due to the corresponding dead-weight loss. As a result, consumer surplus will be higher in case of tying. The producer surplus will decrease but the total social welfare effect of tying will remain positive (see the Appendix):

$$\begin{aligned}
 CS^{E2\_Tie*} &> CS^{E3\_NoT*} \\
 PS^{E2\_Tie*} &< PS^{E3\_NoT*} \\
 SW^{E2\_Tie*} &> SW^{E3\_NoT*}
 \end{aligned}
 \tag{22}$$



## V. Conclusion

The present paper is concerned with welfare analysis of a specific equilibrium outcome that could occur as a result of tying arrangement in a vertically differentiated market for system goods. Correspondingly, the market for system software during the 1990s is chosen as illustrative example of such a market. The evaluation of the social welfare effect of tying is therefore presented in the context of the anti-trust cases against Microsoft.

Under initially stated market conditions the paper establishes two distinct equilibrium outcomes – non-tying and tying-to-platform, respectively. What makes them special is that in the particular market situation being considered, it could be argued that the multi-product firm chooses to tie its products not driven by predatory pricing incentives but just to save the sales of its system combination which otherwise would be foreclosed.

The key question is whether the tying behavior of the multi-product firm should be socially admissible provided that it is not strictly anti-competitive at the particular market conditions.

The main findings have the following implications:

- at the established special market conditions there exists a subgame-perfect equilibrium in which multi-product firm would choose to tie its low-quality product to the high-quality one and sell them at a single wholesale price. This tying arrangement maximizes the multi-product firm's profit but leaves the market non-covered which has a negative partial effect on the social welfare.
- tying has a potential to improve the social welfare by increasing the average quality offered in the market. However, this outcome is related to foreclosure of the sales of the low-quality good in the market where the multi-product firm is a leader.
- the non-tying subgame equilibrium (corresponding to the unbundling measure imposed by the European Commission on Microsoft) is strictly Pareto dominated by the tying outcome. So, making Microsoft supply a Linux version of its application software emerges as a stronger competition-enhancing policy measure than unbundling.

Finally, it needs to summarize the main restrictions on the general validity of the paper's implications. The model in the paper relies on several assumptions without which its results would not hold.

First, it takes as an example the system-software market from the 1990s when all suppliers were profit-maximizing firms. This is not the case nowadays when many freeware products are offered by non-profit suppliers which use voluntary labor and finance their production activities by charity.

Second, it is assumed that the spread of consumer tastes is restricted and the market is sufficiently narrow to accommodate exactly two goods in equilibrium. In the last decade, the demographics of PC users expanded significantly and as a result there is a greater variety of software products available in the market.

Third, it is assumed that the system-software market is vertically differentiated and goods in it consist of only two products. In fact, though Microsoft's operation platform Windows dominates the market, there is a small segment of IT professionals who use Linux not because it is cheaper but because they find it more valuable. The model used in the current paper rather ignores them. Moreover, the decision what operation system to buy depends not only on the quality of the browsing application that goes with it but also on the quality of a number of other applications that could be run on it.

In compliance with the above, the following suggestions could be made for future research on the topic. A non-profit version of the model of a vertically differentiated market for system goods could be developed and used to check how competitive outcome and social welfare change when entrants are not profit-maximizers. Respectively, a market setting with wider spread of consumer tastes needs to be considered. Finally, it would be worth if the implications for two-product system goods could be shown to hold generally for combinations of larger number of products.

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## Appendix

### A. Proof of lemma 1 and lemma 2

Both equilibrium outcomes defined by lemma 1 and lemma 2, respectively, follow from the profit-maximization solutions for the two subgames following the entry of firm A2.

Initially, let's skip the quality-choice stage by simply assuming the following notations for the differences between the chosen qualities:

- given 3 entrants M, B1, A2 (E3):

$$d_{ij/i'j'}^{E3}, \{i = 1,2; j = 1,2, i' = 1,2; j' = 1,2\}$$

- given 2 entrants M, B1 (E2):

$$d_{ij/i'j'}^{E2}, \{i = 1; j = 1,2, i' = 1; j' = 1,2\}$$

These notations allows to derive conditions for having exactly two product pairs with positive market shares at equilibrium in each of the subgames.

In case of 3 entrants and tying, the profit-maximization expressions of (3), (4) and (5) would look as follows:

$$\text{Max}_{p_1^A, p_M} \Pi_M^{E3-Tie} = p_1^A \left( \bar{\theta} - \frac{(p_1^A + p_1^B) - p_M}{d_{11/12}^{E3}} \right) + p_M \left( \frac{(p_1^A + p_1^B) - p_M}{d_{11/12}^{E3}} - \frac{p_M - (p_2^A + p_1^B)}{d_{12/21}^{E3}} \right)$$

$$\text{Max}_{p_1^B} \Pi_{B1}^{E3-Tie} = p_1^B \left[ \left( \bar{\theta} - \frac{(p_1^A + p_1^B) - p_M}{d_{11/12}^{E3}} \right) + \left( \frac{p_M - (p_2^A + p_1^B)}{d_{21/21}^{E3}} - \frac{p_2^A + p_1^B}{d_{21/O}^{E3}} \right) \right]$$

$$\text{Max}_{p_2^A} \Pi_{A2}^{E3-Tie} = p_2^A \left( \frac{p_M - (p_2^A + p_1^B)}{d_{12/21}^{E3}} - \frac{p_2^A + p_1^B}{d_{21/O}^{E3}} \right)$$

For the sales of A2 to be foreclosed, it must not have demand even if priced down at zero. Solving for the optimal prices when  $p_2^A = 0$  and substituting the results in the expression for the demand of good A2 yields the following inequality:

$$\bar{\theta} \geq \frac{4d_{11/12}d_{12/21} + d_{21/o}(d_{11/12} + 3d_{12/21})}{8d_{11/12}d_{12/21} + 6d_{21/o}(d_{11/12} + 3d_{12/21})} \bar{\theta}$$

However, note that  $\bar{\theta} \geq \frac{\bar{\theta}}{4} > \frac{4d_{11/12}d_{12/21} + d_{21/o}(d_{11/12} + 3d_{12/21})}{8d_{11/12}d_{12/21} + 6d_{21/o}(d_{11/12} + 3d_{12/21})} \bar{\theta}$  for

$d_{21/o} \geq 4d_{12/21} > \frac{4d_{11/12}d_{12/21}}{d_{11/12} - 3d_{12/21}}$ . Hence, the sales of good A2 would always be efficiently

foreclosed as long as  $d_{21/o} \geq 4d_{12/21}$  which after being re-written in terms of good qualities (using the definitions for  $d_{ij/i'j'}$  and  $\chi_{ij}$  in (7) and (8)) would look as follows:

$$u_2 \leq \sqrt[4]{\frac{4((u_1)^\alpha + (v_2)^\alpha) - 5(v_1)^\alpha + \chi_o}{5}} \quad (i)$$

Provided that (i) holds the market would be covered by two product pairs only, A1B1 and (A1B2). The corresponding equilibrium solution is derived from the following profit-maximization problems:

$$\text{Max}_{p_1^A, p_M} \Pi_M^{E3-Tie} = p_1^A \left( \bar{\theta} - \frac{(p_1^A + p_1^B) - p_M}{d_{11/12}^{E3}} \right) + p_M \left( \frac{(p_1^A + p_1^B) - p_M}{d_{11/12}^{E3}} - \bar{\theta} \right)$$

$$\text{Max}_{p_1^B} \Pi_{B1}^{E3-Tie} = p_1^B \left( \bar{\theta} - \frac{(p_1^A + p_1^B) - p_M}{d_{11/12}^{E3}} \right)$$

In case of 3 entrants and not-tying, the profit-maximization expressions of (3), (4) and (5) would look as follows:

$$\text{Max}_{p_1^A, p_1^B} \Pi_M^{E3-NoT} = p_1^A \left( \bar{\theta} - \frac{(p_1^A + p_1^B) - (p_2^A + p_2^B)}{d_{11/12}^{E3}} \right) + p_2^B \left( \frac{(p_1^A + p_1^B) - (p_2^A + p_2^B)}{d_{11/22}^{E3}} - \frac{(p_2^A + p_2^B)}{d_{22/O}^{E3}} \right)$$

$$\text{Max}_{p_1^B} \Pi_{B1}^{E3-NoT} = p_1^B \left( \bar{\theta} - \frac{(p_1^A + p_1^B) - (p_2^A + p_2^B)}{d_{11/22}^{E3}} \right)$$

$$\text{Max}_{p_2^A} \Pi_{A2}^{E3-NoT} = p_2^A \left( \frac{(p_1^A + p_1^B) - (p_2^A + p_2^B)}{d_{11/22}^{E3}} - \frac{p_2^A + p_2^B}{d_{22/O}^{E3}} \right)$$

The optimal solution implies that market will be exactly covered in equilibrium. The corresponding optimal prices in lemma 2 are derived from the following profit-maximization problems:

$$\text{Max}_{p_1^A, p_1^B} \Pi_M^{E3-NoT} = p_1^A \left( \bar{\theta} - \frac{(p_1^A + p_1^B) - (p_2^A + p_2^B)}{d_{11/12}^{E3}} \right) + p_2^B \left( \frac{(p_1^A + p_1^B) - (p_2^A + p_2^B)}{d_{11/22}^{E3}} - \underline{\theta} \right)$$

$$\text{Max}_{p_1^B} \Pi_{B1}^{E3-NoT} = p_1^B \left( \bar{\theta} - \frac{(p_1^A + p_1^B) - (p_2^A + p_2^B)}{d_{11/22}^{E3}} \right)$$

$$\text{Max}_{p_2^A} \Pi_{A2}^{E3-NoT} = p_2^A \left( \frac{(p_1^A + p_1^B) - (p_2^A + p_2^B)}{d_{11/22}^{E3}} - \underline{\theta} \right)$$

For having a positive price of B2 at which the market is covered, the lower bound on the market range must be sufficiently large as follows:

$$\underline{\theta} > \frac{2\bar{\theta}d_{11/22}^{E3}}{4d_{22/O}^{E3} + 3d_{11/22}^{E3}} \quad (\text{ii})$$

However, in order for A2B2 to have positive market share, the market range should also be wide enough:

$$\underline{\theta} < \frac{\bar{\theta}}{2} \quad (\text{iii})$$



The quality of B2 should be sufficiently large to ensure that the right-hand side of (ii) does not exceed the right-hand side of (iii):

$$u_2 > \sqrt[\alpha]{\frac{((u_1)^\alpha + (v_1)^\alpha) - 5(v_2)^\alpha + 4\chi_o}{5}} \quad (\text{iv})$$

Finally, for A1B2 to have zero demand the following relation between quality differences must hold:

$$d_{22/o} \leq \frac{3}{8} d_{11/22} \quad (\text{v})$$

which after being re-written in terms of good qualities (using the definitions for  $d_{ij/v_j}$  and  $\chi_{ij}$  in (7) and (8)) takes the form of the following restriction on the quality of B2 from above:

$$u_2 \leq \sqrt[\alpha]{\frac{3((u_1)^\alpha + (v_1)^\alpha) - 11(v_2)^\alpha + 8\chi_o}{11}} \quad (\text{vi})$$

Assume that conditions (i)-(vi) hold. Then, the optimal payoffs of the three firms could be defined in terms of their qualities for each of the four subgames on figure 1. Their expressions are given in Table 1 below:

Table 1: Subgame equilibrium payoffs of the three firms M, B1 and A2

Subgame	Firm M	Firm B1	Firm A2
E3_Tie	$\Pi_M^{E3\_Tie} = \frac{\bar{\theta}^2}{9} d_{11/12}^{E3} + (\bar{\theta} - \underline{\theta}) \underline{\theta} d_{12/O}^{E3}$	$\Pi_{B1}^{E3\_Tie} = \frac{\bar{\theta}^2}{9} d_{11/12}^{E3}$	$\Pi_{A2}^{E3\_Tie} = -\varepsilon$
E3_NoT	$\Pi_M^{E3\_NoT} = \frac{(16\bar{\theta}\underline{\theta} - 4\bar{\theta}^2 - 11\underline{\theta}^2)}{16} d_{11/12}^{E3} + (\bar{\theta} - \underline{\theta}) \underline{\theta} d_{22/O}^{E3}$	$\Pi_{B1}^{E3\_NoT} = \frac{(2\bar{\theta} - \underline{\theta})^2}{16} d_{11/22}^{E3}$	$\Pi_{A2}^{E3\_NoT} = \frac{(2\bar{\theta} - 3\underline{\theta})^2}{16} d_{11/22}^{E3}$
E2_Tie	$\Pi_M^{E2\_Tie} = \frac{\bar{\theta}^2}{36} (4d_{11/12}^{E2} + 9d_{12/O}^{E2})$	$\Pi_{B1}^{E2\_Tie} = \frac{\bar{\theta}^2}{9} d_{11/12}^{E2}$	$\Pi_{A2}^{E2\_Tie} = 0$
E2_NoT	$\Pi_M^{E2\_NoT} = \frac{\bar{\theta}^2}{4} d_{12/O}^{E2}$	$\Pi_{B1}^{E2\_NoT} = \frac{\bar{\theta}^2}{4} d_{11/12}^{E2}$	$\Pi_{B1}^{E2\_NoT} = 0$

It is straightforward to show that the payoff of firm M in each of the four subgames is strictly increasing in the qualities of both of its goods, A1 and B2. Therefore, it would always be optimal for firm M to set the qualities of its goods to their upper bounds:

$$v_1^{E3\_Tie*} = v_1^{E3\_NoT*} = v_1^{E2\_Tie*} = v_1^{E2\_NoT*} = \bar{v}_1 \quad (\text{vii})$$

$$u_2^{E3\_Tie*} = u_2^{E3\_NoT*} = u_2^{E2\_Tie*} = u_2^{E2\_NoT*} = \bar{u}_2 \quad (\text{viii})$$

Similarly, the payoff of firm B1 is also strictly increasing in the quality of its good in each of the four subgames. Therefore, it would always be optimal for firm B1 to set the quality of its good to its upper bounds:

$$u_1^{E3\_Tie*} = u_1^{E3\_Not*} = u_1^{E2\_Tie*} = u_1^{E2\_Not*} = \bar{u}_1 \quad (\text{ix})$$

$$v_2^{E3\_Not*} = \underline{v} \quad (\text{x})$$

The so-derived results (vii-x) imply that the optimal choices of qualities do not vary among the subgames:

$$d_{11/12}^{E3} = d_{11/12}^{E2} = d_{11/12} = \chi_{11} - \chi_{12} = (\bar{u}_1)^\alpha - (\bar{u}_2)^\alpha \quad (\text{xi})$$

$$d_{12/O}^{E3} = d_{12/O}^{E2} = d_{12/O} = \chi_{12} - \chi_o = (\bar{u}_1)^\alpha + (\bar{u}_2)^\alpha - \chi_o \quad (\text{xii})$$

This makes the payoffs of firm M easily comparable. As a result, the last-stage solution depicted on figure 1 is confirmed. Namely, if allowed firm M would always find it optimal to tie B2 to A1 independent on whether A2 has decided to enter or not in the first stage. Accordingly, for firm A2 it would be optimal not to enter. So, the subgame-perfect equilibrium solution coincides with the subgame equilibrium in case of 2 entrants and tying (E2\_Tie).

Alternatively, if firm M is not allowed to use tying arrangements, firm A2 would find it optimal to enter in the first stage as shown on figure 1.

It remains only to substitute for the optimal quality choices of firm M and B1 in the assumed conditions (i)-(vi) for exactly two product pairs to have positive demand in equilibrium. Thus, it could be shown that the presented equilibrium solution would hold provided that the common initial conditions (15), (16) and (17) of lemma 1 and 2 hold. With this result the proof of lemma 1 and lemma 2 is complete.

Table 2 below presents the expressions for total social welfare surplus (SW), consumer surplus (CS) and producer surplus (PS) in the two alternative subgame equilibria, with (E2\_Tie) and without tying (E3\_Not) as well as the comparison between them.

Table 2: Subgame equilibrium payoffs of the three firms M, B1 and A2

Subgame	E2_Tie	E3_Not
CS	$\frac{(4\chi_{11} + 5\chi_{12} + 27\chi_o)\bar{\theta}^2 - 36\chi_o\theta^2}{72}$	$\frac{4(\chi_{11} + 3\chi_{22})\bar{\theta}^2 - 4(\chi_{11} + 7\chi_{22} - 8\chi_o)\bar{\theta}\theta + (\chi_{11} + 15\chi_{22} - 32\chi_o)\theta^2}{32}$
PS	$\frac{(8\chi_{11} + \chi_{12} - 9\chi_o)\bar{\theta}^2}{36}$	$\frac{4(\chi_{11} - \chi_{22})\bar{\theta}^2 + (\chi_{22} - \chi_o)\bar{\theta}\theta - (\chi_{11} - \chi_o)\theta^2}{16}$
SW	$\frac{(20\chi_{11} + 7\chi_{12} + 9\chi_o)\bar{\theta}^2 - 4\chi_o\theta^2}{72}$	$\frac{4(3\chi_{11} + \chi_{22})\bar{\theta}^2 - 4(\chi_{11} - \chi_{22})\bar{\theta}\theta - (\chi_{11} + 15\chi_{22})\theta^2}{32}$
$CS^{E2\_Tie}$ – $CS^{E2\_NoT}$	$\frac{-9(\chi_{11} + 15\chi_{22} - 32\chi_o)\theta^2 + 36(\chi_{11} + 7\chi_{22} - 8\chi_o)\bar{\theta}\theta - 4[5(\chi_{11} - \chi_{12}) + 27(\chi_{22} - \chi_o)]\bar{\theta}^2}{288} > 0$	
$PS^{E2\_Tie}$ – $PS^{E2\_NoT}$	$\frac{(9\chi_{11} + 135\chi_{22} - 144\chi_o)\theta^2 - 144(\chi_{22} - \chi_o)\bar{\theta}\theta - 4[(\chi_{11} - \chi_{12}) - 9(\chi_{22} - \chi_o)]\bar{\theta}^2}{144} < 0$	
$SW^{E2\_Tie}$ – $SW^{E2\_NoT}$	$\frac{(9\chi_{11} + 135\chi_{22} - 16\chi_o)\theta^2 + 36(\chi_{11} - \chi_{22})\bar{\theta}\theta - 4[7(\chi_{11} - \chi_{12}) + 9(\chi_{22} - \chi_o)]\bar{\theta}^2}{288} > 0$	

The inequalities in (22) follow straight from the last three rows of Table 2.