

# Secondary schools efficiency in the Czech Republic<sup>1</sup>

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## Abstract

The present paper estimates efficiency of secondary schools in the Czech Republic using Data Envelopment Analysis. The estimated efficiency is then related to school and teacher characteristics using Tobit model. It was found that schools differ significantly in their efficiency, which ranges from 0.6 to 1. The total efficiency of secondary schools was estimated to be 0.83 (CCR model) and 0.87 (BCC model). The second stage analysis found that the teacher-student ratio, percentage of internal teachers, availability of students advice center, cooperation with foreign schools and sorting of students have significant effect on schools performance. The DEA results were also proved to be robust using a jackknife procedure.

**JEL Classification:** I20, I21, P20

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## 1. Introduction

A strong educational system is a driving force of economic prosperity and, hence, a question about what determines educational efficiency is of special importance. This question was explored in many previous studies mainly for US and Western European countries. However, there was not much research done for transitional countries, even though this question is especially important at the time of economic changes when educational system goes through a number of reforms. The aim of this paper is, therefore, to estimate efficiency of secondary gymnasium schools in the Czech Republic and to analyze the reasons why some gymnasiums perform better than the other ones.

Analysis of educational system can identify efficient and less efficient schools and find determinants of educational efficiency, which gives educational policy makers valuable information on how educational system can be improved. An increase in educational efficiency would result in better student achievements without an increase in resources.

The first aim of this paper is achieved by means of Data Envelopment Analysis (DEA), a method specially designed for estimating performance of public institutions that use multiple inputs and produce multiple outputs. The DEA method does not require specification of relative weights of the inputs and outputs. And since such weights are difficult to specify, the DEA provides a valuable alternative technique for estimation and comparison of school productivity.

The second aim of the paper, identification of the determinants of school efficiency, is achieved using Tobit model. The second part analyzes the effect of different teacher and school characteristics on the efficiency estimated in the first part of the paper.

The notion of efficiency differs from the one of effectiveness. The later means that a school achieves high results no matter what its resources are. On the other hand, efficiency means that a school achieves maximum results possible with the limited resources it has. This corresponds to the definition of a production function which shows maximum possible output at a given level of input. In this sense schools may be viewed as production units that use inputs and produce outputs. And, even though, one cannot say what the maximum level of output possible is, one can estimate it by observing the schools that produce most outputs at the given level of input. Inefficiency is then measured using the distance between a given school and the most efficient schools. This is the basic principle of the DEA analysis.

To estimate efficiency one should consider both outputs and inputs of a production unit. Moreover in case of schools there are several outputs, such as scores in different subjects. Some schools may perform better in one output while the other in another output. To compare these schools one should use both outputs. The conventional regression analysis can only use one output and so it does not reflect different aspects of the school performance. It is also not possible to combine different outputs in a single index since it is difficult to specify weights for achievements in different subjects. The DEA is an

appropriate alternative since it can use multiple-input and output nature of the school production.

In brief, the DEA is a useful method since it truly estimates efficiency by comparing a unit to the best performing units not to average as regression does; and it can model multi-input and output nature of educational production.

This paper uses unique data set on secondary schools in the Czech Republic, which covers all gymnasiums and their students in the country. This provides the most complete information on school characteristics and student achievements, and allows precise estimation of school efficiency. The paper also analyzes what would be the total gain in school efficiency if all schools were operating on the production frontier.

The paper is organized as follows: Section 2 presents testable hypothesis, Section 3 reviews recent literature on the DEA; the mathematical formulation of the DEA methodology is presented in Section 4, after description of the data in Section 5, Section 6 presents results; Section 7 concludes.

## **2. Hypothesis about determinants of school efficiency**

Several school and teacher characteristics were selected in order to explain efficiency. These are teacher quality, school quality, school policy on distributing students between classes and school size. Teachers input can have significant effect on achievement. It is measured by teacher-student ratio, percent of internal teachers, age of teachers, gender composition, years director is in function and fluctuation of teachers.

There are two effects that each characteristic may have. For example, higher teacher-student ratio results in smaller class size and so positively affects achievements. On the other hand, it leads to less instructional hours per teacher and consequently lower wages which negatively affect motivation and achievements. Similarly there are two effects of teachers age. Younger teachers are usually more enthusiastic, but older teachers have more experience of working with students. Which of these effects is stronger is to be tested.

The quality of school input such as existence of school council, students career advice center, psychological advice center, public relations and meeting with parents is hypothesized to have positive effect on efficiency. For example, students career advice center may positively affect students motivation and as a result improve school performance.

Distribution of students between classes is a choice made by school. Some schools form a class for the best students, other have classes with equally distributed skills. Intuitively sorting of students may lead to better efficiency since advanced and less advanced students may study at their level of skills and consequently may better learn the subject. School size may affect efficiency since bigger schools may benefit from economies of scale, in other

words, different classes may use the same facilities and, therefore, the school expenditures on facilities per student is lower.

In brief, the following hypotheses will be tested.

H1: Quality of teachers input such as teacher-student ratio, percent of internal teachers, teachers age, gender composition, years director is in function and fluctuation of teachers affect efficiency.

H2: Quality of school input such as existence of students career advice center, public relation and meeting with parents, school council, and psychological advice center positively affect efficiency.

H3: Sorting of students by ability have a positive effect on efficiency.

H4: Bigger schools benefit from economies of scale and so are more efficient.

### 3. Literature review

The idea of efficiency was developed by Farrel (1957) who proposed to measure efficiency using the distance from a given unit to the production frontier. Later, a mathematical model was formulated in order to compute this efficiency by Charnes et al. (1978) and it received the name Data Envelopment Analysis (DEA). The model was further extended to allow for a production frontier with variable return to scale by Banker et al. (1984). Since then the DEA become a standard model to estimate efficiency and it was employed with data on schools, hospitals and other public organizations. A review of few recent studies that use data on schools are presented in Table 1.

Most of the studies use the DEA to estimate individual school performance and total system performance. However, some studies went further and tried to relate efficiency to some school and local characteristics using a second stage analysis (Bradley 2001, Kirjavainen and Loikkanen 1998).

Table1. Summary of previous studies

|                           | Sample                       | Inputs  | Outputs  | Notes  |
|---------------------------|------------------------------|---|--|--|
| Chakraborty et al. (2001) | Public schools in Utah       | Student-teacher ratio, percentage of teachers with advanced degree and percentage of teachers with more than 15 years of experience | Test scores in 11 <sup>th</sup> grade in reading writing and mathematics | Compare results of DEA and stochastic frontier models              |
| Bradley et al. (2001)     | Secondary schools in England | Socio-economic background and staff qualification   | Attendance rate and examination results                                  | Use Tobit model to explain efficiency, perform jackknife procedure |

Table 1 (continued)

|                                  | Sample                                | Inputs   | Outputs   | Notes   |
|----------------------------------|---------------------------------------|--|---|---|
| Ruggiero and Vitaliano (1999)    | New York school districts             | Operating expenditures per student   | Scores on standardized tests, dropout rate and graduation rate                  | Use Tobit model to relate efficiency to family and local characteristics                                    |
| Colbert et al. (2000)            | Top US MBA programs                   | Faculty to student ratio, average GMAT score and average years of work experience  | Measures of student and recruiter satisfaction and average starting salary      |   |
| Kirjavainen and Loikkanen (1998) | Finnish senior secondary schools      | Teaching and non-teaching hours per week, experience and education of teachers, admission level and education of parents | Number of graduates, score in compulsory and additional subjects                | Use Tobit model to explain efficiency by school, teacher and local characteristics. Use jackknife procedure |
| Noulas and Ketkar (1998)         | Public schools in state of New Jersey | Student to teacher ratio, student to administrator ration and student to non-certificate staff ratio                     | Percentage of students that pass ninth-grade level High School Proficiency Test | Use OLS to relate efficiency to local characteristics   |

All studies agree that choice of inputs and outputs is important. Moreover there is a question which inputs to include in the DEA model and which to reserve for the second stage. Generally, inputs that are under school control are employed in the first stage and ones outside school control are included in the second stage.

The inputs most frequently used are teacher-student ratio, qualification of teachers, school expenditures and equipment. Some studies also include initial skills of pupils on entering school. Initial skills are an important input since they in large extent determine the final achievements. This type of the DEA model can be given a value-added interpretation, since it uses difference between final and initial skills.

The outputs usually employed are scores on such tests as SAT or graduate exams. Some studies also use attendance and graduation rates. These outputs capture a broad range of school production. The factors used in the second stage to explain efficiency are various teachers characteristics, school organization, local characteristics such as regional unemployment rate or population density. For example, Bradley (2001) uses degree of competition and number of selective schools in the region.

#### 4. Methodology

The DEA is a mathematical model which computes efficiency of a unit relatively to the other units. The efficiency measure can be represented as a ratio of the total weighted

outputs to the total weighted inputs. In a model with  $n$  units that uses  $m$  inputs denoted by  $x_{ik}$ ,  $k=1..m$  and produces  $s$  outputs denoted by  $y_{rk}$ ,  $r=1..s$ , the efficiency measure can be presented as follows:

$$h_k = \max_{u_r, v_i} \left( \frac{\sum_{r=1}^s u_r y_{rk}}{\sum_{i=1}^m v_i x_{ik}} \right) \quad (1)$$

The weights of the inputs and outputs are chosen such that the efficiency measure is maximized. However there is an additional constraint that using these weights no other unit can achieve efficiency score greater than one. This is represented by the following inequality:

$$\frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}} \leq 1, \quad j=1, \dots, n \quad (2)$$

The efficiency measure  $h_k$  is then a number between zero and one, with the units that achieve one being efficient and the units below one being inefficient. The efficient units then form a hyperplane which is called efficiency frontier.

The maximization problem specified above can be presented in the linear form, which is more convenient for solving as follows:

$$h_k = \max \sum_{r=1}^s u_r y_{rk}, \quad (3)$$

$$s.t. \sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^s u_r y_{rj} \geq 0, \quad j=1, \dots, n \quad (4)$$

$$\sum_{i=1}^m v_i x_{ik} = 1 \quad (5)$$

$$u_r \geq 0, v_i \geq 0, \quad r=1, \dots, s, \quad i=1, \dots, m. \quad (6)$$

Condition (4) requires that the sum of weighted outputs is not greater than the sum of weighted inputs. Condition (5) requires that the sum of weighted inputs is equal to one. And condition (6) requires that the weights be positive. The model is solved for each unit in order to find the specific weights that maximize unit's efficiency.

The last formulation was developed by Charnes et al. (1978) and is called a CCR model. The basic assumption of the model is that production function has constant return to scale, in other words a proportional increase in inputs leads to a proportional increase in outputs which means that the production function is a straight line.

Another model was developed by Banker et al. (1984). The model allows variable return to scale production function by adding a unit-specific constant  $c_k$ . This model is called a BCC model and is represented by:

$$h_k = \max \sum_{r=1}^s u_r y_{rk} + c_k \quad (7)$$

$$s.t. \sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^s u_r y_{rj} - c_k \geq 0, \quad j=1, \dots, n \quad (8)$$

$$\sum_{i=1}^m v_i x_{ik} = 1 \quad (9)$$

$$u_r \geq 0, v_i \geq 0, \quad r=1, \dots, s, \quad i=1, \dots, m. \quad (10)$$

The CCR and BCC models can be input or output oriented. In the input oriented models we seek a proportional decrease of inputs that brings a unit to the production frontier. In the output oriented model we seek a proportional increase of outputs that brings a unit to the production frontier. The input and output oriented models give the same results under constant return to scale assumption, but they give different results under variable return to scale assumption. An excellent review of different DEA models is presented by Seiford and Thrall (1990), Johnes (1993) and Charnes et al. (1994).

An inefficient point can be projected on the production frontier and a corresponding frontier point can be found. This frontier point is a combination of the other efficient points and it can be presented as their weighted sum. In order to find the corresponding weights it is convenient to rewrite the model in the following form which automatically computes the weights:

$$\max f_k$$

$$\sum_{j=1}^n L_{kj} y_{rj} \geq f_k y_{rk} \quad (11)$$

$$\sum_{j=1}^n L_{kj} x_{ij} \leq x_{ik} \quad (12)$$

$$L_{kj} \geq 0, \quad j=1, \dots, n. \quad (13)$$

$$\sum_{j=1}^n L_{kj} = 1 \quad (14)$$

The  $L_k$  is a vector of weights for construction of an efficient point which is called an efficient reference of the point  $k$ . If a point is efficient then the weights of the other points are zero, its own weight is one, and the corresponding efficiency  $f_k$  is also equal to one. In case of an inefficient unit some weights of the other units are positive and the efficiency score  $f_k$  is less than one. Condition (14) requires that the sum of all weights is one, which is a necessary condition for the variable return to scale property. One should also mention that the DEA does not require explicit specification of the functional form of production technology and, therefore, does not put additional restrictions on the model.

The present paper estimate both CCR and BCC model. Both models are estimated with output orientation, since schools usually have fixed inputs and they maximize their output.

The Tobit model, used in the second stage, can be represented as follows:

$$\begin{aligned} y_i &= X\beta + e_i \\ y_i &= y_i^* \quad \text{if } y_i^* < 1 \\ y_i &= 1 \quad \text{if } y_i^* \geq 1 \end{aligned} \tag{15}$$

Since the efficiency is limited by an upper limit of 1, a standard regression analysis, used with this data, would give biased estimates. Therefore, the Tobit model is used, which allows computing correct estimates of the coefficients.

## 5. Data

The data for the study was received from the following three sources – data on graduate exams (Maturant 98), data on university applicants (Applicant 98) and secondary schools data (SET). These data were collected by the Institute for Information in Education (UIV). The SET data was collected in 1997. The Maturant 98 and Applicant 98 data was collected in 1998. The Maturant 98 data contains information about the scores on graduate exams in Mathematics, Czech, English and German languages. Each student must take these exams in order to be able to apply to a university.

The Applicant 98 data includes information about all applicants that applied to university in year 1998. For each applicant one can identify which school he comes from and if he was admitted or not. Therefore, we can compute the total number of admitted students by school.

The SET data contains information on school resources such as number of classrooms and other facilities. The number of classrooms per student is used as one of the school inputs. Another input is the school facility index which is composed of three indicators – number of books per student, number of computers per student and number of other facilities per student. Other facilities include dining hall, cafeteria, lodging facility, sport hall, gym, swimming pool, artistic classroom and study room. Each indicator is equal to one if the given facility is greater than the corresponding average level. The physical facility index is then computed as a sum of the three indicators.

Maturant 98 data also contains information about the students scores in 8<sup>th</sup> grade, the one immediately preceding admission to gymnasium. Students score are available in four subjects – Mathematics, Czech, English and German. An average grade of students is computed as an indicator of students initial skills which is used as an input. Initial score in 8<sup>th</sup> grade allow also to identify sorting of students between classes.



The SET data provides information about a number of teachers characteristics – teacher-student ratio, percentage of internal teachers, teachers age, gender composition, years director is in function and fluctuation of teachers.

There are also the following characteristics of school organization – existence of school council, student career advice center, psychological advice center, public relation and meetings with parents, cooperation with foreign schools, school age and school size. The school size allows to test if there is an effect of economies of scale.

Two control variables are included to account for heterogeneity of schools – private ownership and 6 or 8-year school. Overall the sample consists of 270 gymnasiums. A description of input and output variables is presented in the Table 1 and their descriptive statistics are presented in the Appendix.

Table 2. Description of the variables

| <b>Variable</b>                           | <b>Explanation</b>   |
|---|--|
| <b>DEA model inputs</b>                   |  |
| Students skills at admission to gymnasium | Five minus school average grade of students at completion of primary school <sup>2</sup>                             |
| Classrooms per student ratio              | Number of available classrooms divided by the total number of students   |
| Physical facility index                   | Composed of number of books per student, number of computers per student and other facilities per student            |
| <b>DEA model outputs</b>                  |  |
| Score in mathematics                      | School average score in standardized graduation test in mathematics  |
| Score in Czech language                   | School average score in standardized graduation test in Czech language   |
| Admission rate to university              | Number of students admitted to university divided by number of students in final year                                |
| <b>Second stage – dependent variables</b> |  |
| <b>Teachers</b>                           |  |
| Teacher-student ratio                     | Total number of teachers in school divided by total number of students in school                                     |
| Teachers age                              | School average age of teachers   |
| Percent of internal teachers              | Percent of teachers that work fulltime   |
| Percent of female teachers                | .  |
| Fluctuation of teachers                   | Total number of teachers that were teaching graduation students in years 1994-1998 divided by the number of students |
| Years director is in function             | How long current director is working in his position   |
| <b>School</b>                             |  |
| Percent of male students                  | .  |
| Students career advice center             | Existence of a center that consult students about their future career  |
| School council                            | .  |

<sup>2</sup> 5-grade transformation is used since grading is from 1 to 5 with 1 being the best grade

Table 2 (continued)

| <b>Variable</b>                           | <b>Explanation</b>   |
|---|--|
| Public relations and meeting with parents | Equal to one if level of activity higher than average  |
| School age                                | .  |
| Cooperation with foreign schools          | Number of foreign schools a given school is cooperating with   |
| <b>Sorting</b>                            |  |
| Sorting of students by their skills       | Difference between minimum and maximum class average initial grades divided by minimum class average grade |
| <b>School size</b>                        |  |
| Number of students in school              |  |
| <b>Controls</b>                           |  |
| Private ownership                         | Equals to one if school is private   |
| 6 and 8 years school                      | Equals to one if school has more than 4 years study program  |

## 6. Results

The results of the DEA model are presented in Table 3. The table includes mean and standard deviation of efficiency as well as inputs and outputs for 10 most and least efficient schools. The total efficiency is estimated at 0.83 (assuming constant return to scale) and 0.87 (assuming variable return to scale); which means that the total inefficiency is 17% and 13% respectively. The efficiency ranges from 0.6 to 1.

It is clear from Table 3 that schools achieve efficiency either if their output is high or if their input is low. This corresponds to the definition of efficiency which says that a unit is efficient if it achieves maximum possible output with his level of inputs. Schools 3, 4, 8 achieve efficiency of 1 because their outputs are high. Schools 6, 9 also achieve efficiency of 1, even though they do not have highest output, but their inputs are very low. The least efficient schools achieved lower than average scores even though their inputs are high.

Selected characteristics are presented for top and bottom schools in Table 4. There is significant variation in characteristics for both groups but efficient schools have on average lower teacher-student ratio and higher sorting of students between classes.

Similarly to Kirjavainen and Loikkanen (1998) and Bradley (2001), the present study uses jackknife procedure to test robustness of results. The jackknife procedure consists of dropping one efficient point at a time and estimating efficiency with the rest of the sample. If the new results are similar to the old one then the point which was dropped does not have a significant effect on the estimated efficiency and thus the results are robust against outliers at the frontier. The correlation computed by the jackknife method ranges from 0.93 to 0.99, which shows that the results are robust.

Table 3. Summary of the efficiency results<sup>3</sup>

| School                      | Efficiency |      | Outputs              |                         |                                | Inputs                 |                              |                         |
|-----------------------------|------------|------|----------------------|-------------------------|--------------------------------|------------------------|------------------------------|-------------------------|
|                             | CRS        | VRS  | Score in mathematics | Score in Czech language | Percent admitted to university | Classrooms per student | 5-average grade at admission | Physical facility index |
| Ten most efficient schools  |            |      |                      |                         |                                |                        |                              |                         |
| 1                           | 1          | 1    | 21                   | 32                      | 0.71                           | 0.05                   | 3.0                          | 2                       |
| 2                           | 1          | 1    | 23                   | 39                      | 0.63                           | 0.08                   | 2.9                          | 2                       |
| 3                           | 1          | 1    | 22                   | 41                      | 0.79                           | 0.05                   | 3.4                          | 0                       |
| 4                           | 1          | 1    | 24                   | 40                      | 0.41                           | 0.08                   | 3.0                          | 3                       |
| 5                           | 1          | 1    | 15                   | 41                      | 0.36                           | 0.04                   | 3.3                          | 1                       |
| 6                           | 1          | 1    | 22                   | 39                      | 0.29                           | 0.05                   | 3.0                          | 1                       |
| 7                           | 1          | 1    | 23                   | 33                      | 0.41                           | 0.06                   | 3.0                          | 1                       |
| 8                           | 1          | 1    | 23                   | 40                      | 0.59                           | 0.05                   | 3.2                          | 1                       |
| 9                           | 1          | 1    | 20                   | 42                      | 0.64                           | 0.06                   | 2.9                          | 0                       |
| 10                          | 1          | 1    | 20                   | 44                      | 0.72                           | 0.05                   | 3.5                          | 0                       |
| Mean                        | 0.83       | 0.87 | 18.2                 | 36.0                    | 0.51                           | 0.07                   | 3.29                         | 1.43                    |
| S.d.                        | 0.10       | 0.10 | 2.9                  | 3.9                     | 0.16                           | 0.02                   | 0.23                         | 0.92                    |
| Ten least efficient schools |            |      |                      |                         |                                |                        |                              |                         |
| 261                         | 0.60       | 0.60 | 11                   | 26                      | 0.32                           | 0.08                   | 3.0                          | 1                       |
| 262                         | 0.62       | 0.65 | 8                    | 29                      | 0.04                           | 0.09                   | 3.2                          | 1                       |
| 263                         | 0.63       | 0.65 | 15                   | 28                      | 0.29                           | 0.08                   | 3.2                          | 3                       |
| 264                         | 0.63       | 0.68 | 15                   | 30                      | 0.42                           | 0.07                   | 3.3                          | 3                       |
| 265                         | 0.64       | 0.67 | 13                   | 31                      | 0.50                           | 0.10                   | 3.4                          | 3                       |
| 266                         | 0.64       | 0.64 | 6                    | 28                      | 0.29                           | 0.13                   | 3.0                          | 2                       |
| 267                         | 0.64       | 0.71 | 16                   | 31                      | 0.39                           | 0.07                   | 3.5                          | 1                       |
| 268                         | 0.64       | 0.67 | 14                   | 30                      | 0.30                           | 0.07                   | 3.2                          | 2                       |
| 269                         | 0.64       | 0.71 | 15                   | 32                      | 0.47                           | 0.06                   | 3.5                          | 2                       |
| 270                         | 0.65       | 0.67 | 16                   | 24                      | 0.29                           | 0.19                   | 3.1                          | 3                       |

<sup>3</sup> Efficiency was computed using DEAP 2.1 computer program, Coeli (1996).

Table 4. Selected characteristics for most and least efficient schools

| School                      | Index of sorting | Percent of male students | Percent of internal teachers | Number of cooperating foreign schools | Teacher-student ratio | Students career advice center |
|-----------------------------|------------------|--------------------------|------------------------------|---------------------------------------|-----------------------|-------------------------------|
| Ten most efficient schools  |                  |                          |                              |                                       |                       |                               |
| 1                           | 0.01             | 0.60                     | 90                           | 2                                     | 0.09                  | 0                             |
| 2                           | 0.00             | 0.86                     | 100                          | 1                                     | 0.15                  | 0                             |
| 3                           | 0.25             | 0.42                     | 100                          | 3                                     | 0.07                  | 1                             |
| 4                           | 0.00             | 0.41                     | 94                           | 2                                     | 0.08                  | 0                             |
| 5                           | 0.00             | 0.24                     | 97                           | 2                                     | 0.14                  | 0                             |
| 6                           | 0.00             | 0.64                     | 100                          | 0                                     | 0.08                  | 1                             |
| 7                           | 0.00             | 0.30                     | 100                          | 0                                     | 0.09                  | 0                             |
| 8                           | 0.25             | 0.46                     | 93                           | 2                                     | 0.10                  | 0                             |
| 9                           | 0.01             | 0.51                     | 97                           | 3                                     | 0.07                  | 1                             |
| 10                          | 0.26             | 0.39                     | 100                          | 0                                     | 0.07                  | 0                             |
| Mean                        | 0.15             | 0.42                     | 90.46                        | 2.63                                  | 0.10                  | 0.49                          |
| S.d.                        | 0.12             | 0.11                     | 13.16                        | 2.13                                  | 0.05                  | 0.50                          |
| Ten least efficient schools |                  |                          |                              |                                       |                       |                               |
| 261                         | 0.13             | 0.55                     | 95                           | 2                                     | 0.13                  | 1                             |
| 262                         | 0.00             | 0.00                     | 86                           | 2                                     | 0.13                  | 0                             |
| 263                         | 0.00             | 0.57                     | 78                           | 3                                     | 0.19                  | 0                             |
| 264                         | 0.09             | 0.24                     | 96                           | 1                                     | 0.07                  | 1                             |
| 265                         | 0.00             | 0.35                     | 86                           | 2                                     | 0.12                  | 0                             |
| 266                         | 0.08             | 0.30                     | 94                           | 1                                     | 0.11                  | 0                             |
| 267                         | 0.00             | 0.30                     | 47                           | 0                                     | 0.13                  | 0                             |
| 268                         | 0.21             | 0.44                     | 43                           | 0                                     | 0.13                  | 1                             |
| 269                         | 0.05             | 0.42                     | 88                           | 3                                     | 0.10                  | 0                             |
| 270                         | 0.00             | 0.50                     | 20                           | 0                                     | 0.29                  | 0                             |

The results of the second stage analysis are presented in Table 5. Coefficients on teacher-student ratio and percent of internal teachers are marginally significant at 10-percent level. However, these effects disappear when variable return to scale is assumed. The negative effect of higher teacher-student ratio can be explained by the following aspect of school funding system. The teachers wage depends on the number of students they teach and their teaching hours. Since an increase in teacher-student ratio leads either to forming smaller classes or decreasing teaching hours, it consequently leads to lower wages and less motivation for a teacher. Therefore the total effect of the teacher-student ratio is negative. However, the effect of square of teacher-student ratio is positive, which means that efficiency declines less than proportionately with teacher-student ratio. This is probably because at higher level of teacher-student ratio the effect of class-size becomes stronger.

Among school characteristics significant are existence of students career advice center, percentage of male students in class, cooperation with foreign schools and sorting of students. Existence of a students advice center improves motivation of students and leads to a better performance. Sorting of students between classes appear to have a positive

effect which corresponds to the initial assumption that allowing students to learn at their level of skills leads to more productive learning.

The influence of percent of male students can be explained by the fact that boys received significantly better results in mathematics but almost the same results in Czech language compared to girls. Boys received 10% higher scores in mathematics and only 1% lower score in Czech compared to girls. So, one of the outputs for school with mostly boys is unusually high which leads to higher estimates of efficiency. Cooperation with foreign schools is found to increase productivity which is an effect of learning from other schools experience.

The effect of school size is found to be insignificant. Similarly, Kirjavainen and Loikkanen (1998) did not find an effect of school size; however, Bradley (2001) did find a small significant effect. The author argues that besides economies of scale there is another effect that bigger schools are more difficult to manage, which may compensate the first effect.

Table 5. Tobit analysis results.

| Variables                                 | Efficiency (constant return to scale) |           |         | Efficiency (variable return to scale) |           |         |
|---|---------------------------------------|-----------|---------|---------------------------------------|-----------|---------|
|   | Coef.                                 | Std. Err. | P> t    | Coef.                                 | Std. Err. | P> t    |
| Teacher-student ratio                     | -1.4836                               | (0.9208)  | 0.108*  | -1.1232                               | (0.9233)  | 0.225   |
| Teacher-student ratio squared             | 5.8708                                | (3.0030)  | 0.052*  | 4.0786                                | (3.0079)  | 0.176   |
| Teachers age                              | -0.0014                               | (0.0020)  | 0.490   | -0.0016                               | (0.0021)  | 0.449   |
| Percent of internal teachers              | 0.0010                                | (0.0006)  | 0.082*  | 0.0008                                | (0.0006)  | 0.177   |
| Percent of female teachers                | 0.0001                                | (0.0006)  | 0.885   | 0.0002                                | (0.0006)  | 0.749   |
| Fluctuation of teachers                   | 0.0209                                | (0.0544)  | 0.701   | 0.0615                                | (0.0547)  | 0.262   |
| Years director is in function             | -0.0006                               | (0.0018)  | 0.712   | 0.0009                                | (0.0018)  | 0.599   |
| Percent of male students in class         | 0.1027                                | (0.0533)  | 0.055*  | 0.1278                                | (0.0539)  | 0.019** |
| Students career advice center             | 0.0252                                | (0.0121)  | 0.039** | 0.0201                                | (0.0122)  | 0.100*  |
| School council                            | 0.0133                                | (0.0124)  | 0.287   | 0.0086                                | (0.0125)  | 0.493   |
| Public relations and meeting with parents | -0.0102                               | (0.0138)  | 0.462   | -0.0070                               | (0.0138)  | 0.612   |
| Age of school                             | -0.0001                               | (0.0001)  | 0.427   | 0.0000                                | (0.0001)  | 0.599   |
| Cooperation with foreign schools          | 0.0054                                | (0.0028)  | 0.055*  | 0.0072                                | (0.0028)  | 0.011** |
| Sorting of students by their skills       | 0.2197                                | (0.0598)  | 0.000** | 0.1851                                | (0.0598)  | 0.002** |
| School size                               | 0.0000                                | (0.0002)  | 0.916   | 0.0001                                | (0.0002)  | 0.554   |
| Private ownership                         | -0.0685                               | (0.0245)  | 0.005** | -0.0712                               | (0.0244)  | 0.004** |
| 6 or 8 years gymnasium                    | 0.0806                                | (0.0158)  | 0.000** | 0.0904                                | (0.0161)  | 0.000** |
| Constant                                  | 0.7729                                | (0.1130)  | 0.000** | 0.7788                                | (0.1136)  | 0.000** |
| Pseudo $R^2$                              |                                       | 0.26      |         |                                       | 0.31      |         |
| Number of observations                    |                                       | 270       |         |                                       | 270       |         |

\* Indicates significance at 10% level

\*\* Indicates significance at 5% level

Such teacher characteristics as teachers age, percent of female teachers, fluctuation of teachers and years since current director is in function showed no significant effect. Also

such school characteristics as existence of school council, meeting with parents and age of school do not show a significant effect. Both control variables – private ownership and 6 or 8-years gymnasium are found significant. Private schools perform little worse than public ones. Six and eight-years gymnasiums performed better than four-years ones. Even though it is known that the six and eight-years gymnasiums accept the best students, the fact that we control for the initial skills allows stating that these schools are more efficient than the four-years schools.

## **7. Conclusion**

The Data Envelopment Analysis proved to be an appropriate method to analyze efficiency of educational institutions. First of all, it can model multi-input and multi-output nature of school production. Second, it produces a true measure of efficiency, which is robust against the effect of outliers at the frontier. The last statement was confirmed by the jackknife procedure.

The efficiency analysis of 270 gymnasiums in the Czech Republic found that the total efficiency of these schools is 0.83 (under CRS) and 0.87 (under VRS) which is quite high. However, efficiency scores of individual schools range from 0.6 to 1, which shows that some schools are significantly less productive.

The second stage analysis found that such factors as teacher-student ratio, percentage of internal teachers, existence of a career guidance center, cooperation with foreign schools, percentage of male students and sorting of students significantly affect schools efficiency. Other qualities such as age of teachers, percentage of female teachers, fluctuation of teachers, school age and school size do not significantly affect efficiency.

Interestingly, the teacher-student ratio is found to have negative effect which can be explained by an additional effect it has on the teachers wage. Since the teachers wage depends on number of students they teach and hours of instructions, an increase in the teacher-student ratio leads to a decrease in wage and consequently to lower motivation. As a result the total effect is negative. This finding shows that one should take into account all the links within the educational system in order to estimate a true effect.

Two factors – cooperation with foreign schools and sorting of students – show an especially strong effect on schools efficiency. The first effect can be explained by schools benefiting from foreign schools experience, by allowing their students and teacher to study or practice abroad for some time. The second finding shows that grouping of students by their ability leads to a better performance which corresponds to the initial hypothesis on the positive effect of students sorting.

In conclusion, the results of the analysis show that some of the school and teacher characteristics significantly affect school productivity. These findings contribute to the educational debate and make an additional step on the way to achieve higher efficiency of the educational system.

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## Appendix A.

Table 6. Descriptive statistics

| Variable                                  | Mean  | Std. Dev. | Min  | Max  |
|---|-------|-----------|------|------|
| Score in mathematics                      | 18.17 | 2.93      | 6    | 24   |
| Score in Czech language                   | 35.97 | 3.92      | 24   | 46   |
| Admission rate to university              | 0.51  | 0.16      | 0.04 | 0.98 |
| Students skills at admission to gymnasium | 3.29  | 0.23      | 2.60 | 3.80 |
| Classrooms per student ratio              | 0.07  | 0.02      | 0.04 | 0.23 |
| Physical facility index                   | 1.43  | 0.92      | 0    | 3    |
| Teacher-student ratio                     | 0.09  | 0.03      | 0.06 | 0.29 |
| Teachers age                              | 40.83 | 3.57      | 27   | 48   |
| Percent of internal teachers              | 90.46 | 13.16     | 19   | 100  |
| Percent of female teachers                | 63.90 | 10.38     | 3    | 94   |
| Fluctuation of teachers                   | 0.18  | 0.16      | 0.03 | 1    |
| Years director is in function             | 5.78  | 3.38      | 1    | 25   |
| Percent of male students in class         | 0.42  | 0.11      | 0    | 1    |
| Students career advice center             | 0.49  | 0.50      | 0    | 1    |
| School council                            | 0.56  | 0.50      | 0    | 1    |
| Public relations and meeting with parents | 0.31  | 0.46      | 0    | 1    |
| Age of school                             | 60.41 | 69.15     | 3    | 411  |
| Cooperation with foreign schools          | 2.63  | 2.13      | 0    | 12   |
| Sorting of students by their skills       | 0.15  | 0.12      | 0    | 0.51 |
| School size                               | 73.21 | 38.22     | 11   | 198  |
| Private ownership                         | 0.16  | 0.37      | 0    | 1    |
| 6 or 8 years gymnasium                    | 0.19  | 0.39      | 0    | 1    |

## Appendix B. Overview of secondary education system and educational finance

After completion of a basic school a student chooses between a secondary general school (gymnasium), a secondary technical or a secondary vocational school. Admission to a secondary school is based on competition and all secondary schools require students to pass an entry examination. At the end of secondary school students may take Graduate Examination (Maturita). In case of gymnasiums all students take this exam since it is required for applying to university.



Since 1996 all schools are under direct jurisdiction of the Ministry of Education. Many school responsibilities have been decentralized. Schools have more power to choose their curriculum, manage their own budget – still provided by the state – and many other matters related to school facilities, staff and administration (UIV, 1999).

During the 1990s private schooling rose. Private schools receive 60 to 90 percent of the standard support state schools receive, but in addition they receive financial contribution from students. This contribution is necessary for school investment, since state funding only covers teachers salaries and operational costs.

Education is funded from the state central budget and municipal budgets. Schools receive funds for three different purposes. First, a school receives normative funds, which depend on the number of students enrolled. The funds are calculated by multiplying number of students by a fixed normative sum. In 1997, 88% of the school funds were allocated using the normative system. Normative funds are used to pay teacher wages, insurance and other school equipment like books or computers. The share of normative funds allocated for wages is fixed.

Second, a school receives funding for the maintenance of the school building, heating and other services. The Ministry of Education covers all the costs of school maintenance for public schools.

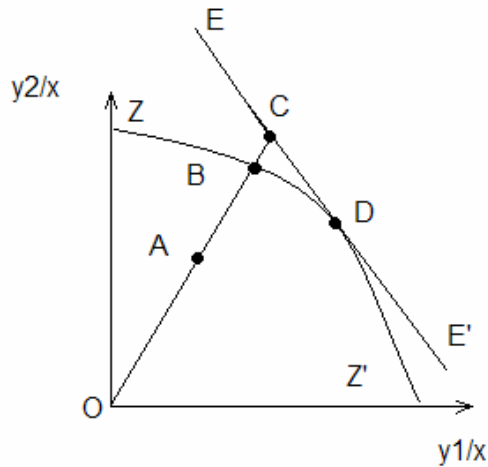
Third, a school receives investment funds, which are used for the construction of new school buildings and classrooms. These funds may also include equipment for new classrooms. The investment funding differs from year to year and is only directed for a major improvement in school facilities. In a given year some schools may receive huge investments, while other schools may not receive investment funds at all. Approximately 98 percent of educational funding comes from the state budget. About 80 percent of these funds come from the national budget and the rest from the municipal budgets.

## **Appendix C. Examples describing the DEA model**

### *Example 1:*

This example illustrates the principle of efficiency calculation in the DEA model. For illustrative purposes we may assume that school has one input and two outputs. The outputs are, for example, student achievements in mathematics and English. This case is represented on the Graph 1.

Graph 1: Efficiency estimation using data envelopment analysis.



The two schools B and D are on the efficiency frontier  $ZZ'$ . The third school A is below the line and, therefore, represents an inefficient school. We may see that under any output weighting scheme this school is below the efficiency frontier.

The school A has corresponding efficient point B. Efficiency is then estimated as the ratio  $OA/OB$ , i.e. how much is current output lower than the one of the efficient school. This is so called technical efficiency.

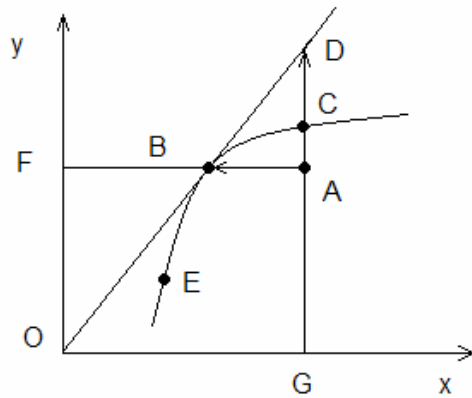
In case if prices of outputs are available we can estimate allocative efficiency as well. Suppose we know how much one output is valued relative to the other. Let  $EE'$  present the isoprofit line. The school D uses same inputs as school B and is also efficient, however, producing different combination of outputs it achieves higher total output value. By changing proportion of outputs school B can move to point D and achieve higher total value as in point C. The allocative efficiency is then measured as  $OB/OC$  for point B or  $OA/OC$  for point A.

However, in case of schools such prices of outputs are not available; therefore, we can only estimate technical efficiency. Timmer (1971) points out that the difference between DEA and the production frontier estimated using OLS is that the former truly estimate technical efficiency, whereas, the later estimate the total of technical and allocative efficiency.

*Example 2:*

This example illustrates the property of constant return to scale (CRS) and variable return to scale (VRS) as well as input and output orientation. We use a simple model with one input and one output presented on Graph 2. Let points A,B,C, and E be schools. The line OD then presents the CRS frontier, and the curve CBE presents the VRS frontier.

Graph 2: Production frontier under constant and variable return to scale.



The difference between input and output oriented models can also be illustrated using Graph 2. In case of input-oriented model we want to know by how much inputs can be decreased given fixed output. This value is presented on the graph by ratio  $BF/AF$ . In case of output-oriented model we want to know by how much the current output is lower than the maximum feasible output. This value is presented by the ratio  $AG/DG$  in case of CRS and  $GA/GC$  in case of VRS models. The input and output-oriented models give the same efficiency results under CRS assumption, however it is not so under VRS assumption.