Viliam Druska

Essays on Efficiency Measurements

Dissertation

Prague, July 2012

CERGE
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Charles University Prague

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To my family

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Abstract

The unifying theme of this dissertation is the measurement of production efficiency, covering both parametric and non-parametric approaches to efficiency assessment. The first chapter considers the estimation of a fixed-effect panel data model with disturbances that are spatially correlated, based on a geographic or economic proximity measure. For when the time dimension is small (the usual panel data case), the study develops a generalized moments estimation approach based on a cross-sectional model from Kelejian and Prucha (1999). This approach is then applied in a stochastic frontier framework to a panel of Indonesian rice farms. Within this framework, spatial correlations are based on geographic proximity, and represent productivity shock spillovers across the production units. Using a Moran I test statistic, the first chapter empirically demonstrates that productivity shock spillovers may exist in this (and perhaps other) data sets, and that these spillovers have profound effects on technical efficiency estimation. The second chapter represents a logical extension of the first as it theoretically develops a random effect panel data model that accounts for spatial correlation across disturbance terms. The model is then applied within the framework of production frontier to the same data set of Indonesian rice farms. The study empirically confirms the impact of spatial correlation on the estimates of technical efficiency and compares the results with the outcomes from the first chapter. The empirical results also indicate that the technique developed here provides a viable alternative to the incorporation of time-invariant regressors in the equation specification. The third chapter addresses the drawbacks of the routine use of ratio analysis in the assessment of retailing performance. Applying multiple input-multiple output data envelopment analysis (DEA), the study assesses the technical and scale efficiency of the retail chain operation of a European mobile operator and identifies input excesses and means of reducing them. It also provides a review of parametric methodologies (COLS and SFA) and their use in testing the hypothesis of the constant returns to scale of the employed technology. The study concludes with policy recommendations for improvements in the productive efficiency of retail chain operations.

Jednotícím tématem této dizertační práce je měření efektivity výroby s využitím jak parametrických, tak i neparametrických přístupů k hodnocení efektivnosti. První část dizertace se zabývá odhadem panelových dat modelem pevných efektů s náhodnou složkou, která je prostorově korelována na základě určité geografické blízkosti, nebo ekonomické závislosti. Pro případ, kdy časová dimenze dat je malá (což je u panelových dat obvyklý případ) článek rozvíjí metodu obecných momentů založenou na modelu studie Kelejiana a Průchy (1999). Tato metoda je pak aplikována v kontextu stochastické produkční hranice na panelu indonéských rýžových farem. V tomto kontextu vyplývají prostorové korelace z geografické blízkosti a představují vedlejší účinky šoků ovlivňující produktivní efektivitu výrobních jednotek. Pomocí testovací statistiky Moran I, článek empiricky dokazuje možnou

přítomnost vedlejších účinků těchto šoků v těchto (a možná i dalších) datech a to, že tyto šoky mají významný dopad na odhady technické efektivity. Druhá část dizertace představuje logické rozšíření první části tím, že teoreticky rozvíjí model náhodných efektů, který reflektuje přítomnost prostorové korelace v náhodné složce. Model je následně aplikován v kontextu stochastické produkční hranice na stejná data indonéských rýžových farem. Článek empiricky potvrzuje vliv prostorové korelace na odhad technické efektivity a srovnává výsledky této analýzy s výsledky dosaženými v prvním článku. Empirické výsledky rovněž naznačují, že postup vyvinutý zde nabízí alternativu k zahrnutí časově invariantních regresorů v specifikaci odhadované funkce. Třetí část dizertace adresuje nedostatky běžně používané analýzy poměrových výkonnostních ukazatelů pro účely hodnocení efektivity maloobchodu. Uplatněním metody více vstupů- více výstupů studie hodnotí technickou a rozsahovou efektivitu fungování řetězce maloobchodních prodejen evropského mobilního operátora a identifikuje míru excesů využívaných produkčních vstupů a způsoby jejích redukce. Studie také poskytuje přehled parametrických metod (COLS a SFA) a jejich využití pro testování hypotézy konstantní ekonomie rozsahu pro použitou technologii. Studie závěrem poskytuje doporučení pro zlepšení produkční efektivity obchodního řetězce maloobchodních prodejen.

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Czech Republic, Prague Viliam Druska
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Introduction[1]

The unifying theme of my dissertation is the measurement of production efficiency, covering both parametric and non-parametric approaches to efficiency assessment.

The first chapter considers the estimation of a fixed-effect panel data model with disturbances that are autocorrelated across cross-sectional units. It is assumed that the disturbances are *spatially correlated*, based on some geographic or economic proximity measure. If the time dimension of the data is large, feasible and efficient estimation proceeds by using the time dimension of the data to estimate the spatial dependence parameters. For the case where the time dimension is small (the usual panel data case), the study develops a generalized moments estimation approach that is a generalization of a cross-sectional model from Kelejian and Prucha (1999). This approach is then applied in a stochastic frontier framework to a panel of Indonesian rice farms where spatial correlations are based on geographic proximity, altitude, and weather. The correlations represent productivity shock spillovers across the rice farms in different villages on the island of Java. Using a Moran I test statistic, the first chapter empirically demonstrates that productivity shock spillovers may exist in this (and perhaps other) data sets, and that these spillovers have profound effects on technical efficiency estimation.

The second chapter represents a logical extension of the first chapter as it theoretically develops a random effect model that accounts for correlation across disturbance terms and empirically demonstrates how this correlation can bias the estimates of individual specific terms. The model is then applied in the framework of a stochastic production frontier where firm-level output is an additive function of inputs and a random error term composed of technical inefficiency and statistical noise. Viewing statistical noise as productivity shocks

[1] All remaining errors are the responsibility of the author.

due to the geographical or economic proximity of cross-section units study shows that productivity spillovers (correlations) may exist in the statistical noise component, and demonstrate the effect of these spillovers on the estimates of technical efficiency. Moreover, the results indicate that the technique presented here provides an alternative to the incorporating of time-invariant regressors in the equation specification.

Performance in retailing is usually evaluated by the routine use of ratio analysis, but due to the univariate nature of this simple management tool there are many drawbacks to the obtained results. The last chapter aims to demonstrate the successful employment of parametric and non-parametric methods for evaluating technical performance in retailing. The results of this study are used to optimize the retail chain of a European mobile telecommunication network operator by providing estimates of and recommendations for improvements in the productive efficiency of the chain operations. Estimates of store-level technical and scale efficiency indicate that a majority of stores are operating in the decreasing returns-to-scale region of the production possibility set. The employed methodology enables identification of input excesses and means of reducing them.

Chapter 1

Generalized Moments Estimation of a Spatially Correlated Panel Data Model[2]

Much has been written about spatial dependence in cross-sectional economic data that can be distinguished by absolute or relative location. For example, data on employment or wealth can be organized by county, state, census tract, or country, and spatial dependence can be modeled across these units. Anselin (1988) and Anselin and Rey (2010) provide an excellent textbook treatment of the analysis of spatially dependent data. Theoretical or empirical spatial issues have also been addressed in Anselin (2010); Case (1991); Conley (1999); Delong and Summers (1991); Dubin (1988); Fishback, Horrace, and Kantor (1999); Kelejian and Robinson (1993); Moulton (1990); Quah (1992); and Topa (1996). These cross-sectional specifications address the important phenomena of spatial aggregation, infrastructure effects, and economic spillovers, to name a few.

Kelejian and Prucha (1999) consider a generalized moments estimation of regression models that allows the spatial autocorrelation of *disturbances* across cross-sectional units. Estimation hinges on the *ex ante* specification of a "spatial weighting matrix" in the regression error. The form of the weighting matrix is at the discretion of the analyst, but often it can be based on meteorological theory. "Of course, if panel data are available one can consider, for example, a seemingly unrelated regression model, or an error component model to permit for cross-sectional correlation, and estimate the cross-sectional correlations via the time dimension of the panel if the time dimension is large" (Kelejian and Prucha (1999),

[2]A previous version of this work was published as Druska V. and Horrace W.C. (2004). "Generalized Moments Estimation for Spatial Panel Data: Indonesian Rice Farming." *American Journal of Agricultural Economics,* 86 (1), 185-198.

footnote 2, p. 509). Unfortunately, in the usual panel data case, the time dimension is *small* (fixed), so consistent estimation of the cross-sectional correlations is typically not justified.

This study extends the Kelejian and Prucha estimator to the usual panel-data case, based on certain restrictions on the evolution of spatial dependence over time. It is important to stress that the panel-data theory presented is for the case where T is fixed; consequently, the current discussion also hinges on the *ex ante* specification of a spatial weighting matrix. Once we allow the time dimension to grow, the specification of the weighting matrix becomes unnecessary, as the estimation techniques presented herein become empirically inferior to approaches that rely on T-asymptotics, such as seemingly unrelated regression models or error component models.

We apply these spatial techniques to a stochastic frontier model in which a common production function and farm-level technical efficiencies are estimated for a sample of farm inputs and outputs. Cross-sectional estimation of these models is due to Aigner, Lovell and Schmidt (1977) and Meeusen and van den Broeck (1977), while panel estimation is due to Schmidt and Sickles (1984). Our concern is, of course, the panel specification, and we select a panel of 171 Indonesian rice farms observed over six periods for our example. The output is rice and the inputs are things such as seed, fertilizer, and land acreage. The time dimension of the data is small, so consistent estimation of cross-sectional correlations in the error process is not justified. Consequently, we specify a spatial weighting scheme in the error process that allows for spillovers across farms based on geographic proximity and weather conditions. The results indicate that spatial correlations exist in the data and have an impact on the magnitude and variability of the production function and technical efficiency estimates obtained.

1.1 A Panel Model with Spatial Disturbances

Consider the standard fixed effect (FE) model

$$y_{it} = \alpha_i + \beta' x_{it} + u_{it}, \quad i = 1, \ldots, N, \quad t = 1, \ldots, T,$$

where β is $(k\times1)$ and x_{it} is $(1\times k)$. Here we assume that T is fixed, so we cannot rely on T-asymptotics. Forming a vector of observations in i, the model becomes

$$y_t = \alpha + x_t\beta + u_t, \qquad t = 1, \ldots, T, \tag{1}$$

where $\alpha' = [\alpha_1, \ldots, \alpha_N]$ and x_t is $(N\times k)$. Now suppose that the error term is spatially lagged such that

$$u_t = \rho_t M_t u_t + \varepsilon_t, \qquad t = 1, \ldots, T, \tag{2}$$

where ρ_t is a scalar, spatial autoregressive parameter, M_t is a ($N \times N$) spatial weighting matrix of known constants (diagonal elements equal to 0) that captures the spatial correlations across cross-sectional units, and ε_t ($N \times 1$) is a zero-mean disturbance. (Later we allow for a time-invariant spatial parameter and weighting matrix.) The elements of M_t are m_{ijt}, and are chosen based on some geographic or economic proximity measure such as contiguity, physical, economic or climatic distances, or dissimilarities. For example, in section 1.3, we select m_{ijt} to be the inverse of the physical distance (km^{-1}) between unit i and unit j in time period t.

The application of interest is the stochastic frontier model, where y_{it} and x_{it} are the productive output and exogenous inputs, respectively, of farm i in period t. Stochastic frontier models specify output as a linear function of (a) farm level technical (in)efficiency (an unobserved factor imputed to each farm, embodied in α_i), (b) a representative log-production function (deterministic, within the control of each farm, and represented by $x_t\beta$), and (c) productivity shocks (random, out of the farmer's control, and represented by u_t). Therefore, equation (1) is a stochastic frontier specification. When augmented by equation (2), the specification implies that, in each period t, productivity shocks are correlated across i, and specifically that the productive output of farm i is a function of the spatial lag of productivity shocks, $\rho_t M_t u_t$, experienced by other farms in the sample. This would seem reasonable if productivity shocks included geographic or climatic unobservables that affected farms in similar ways but were location- or climate-specific (e.g., unmeasured rainfall, temperature, and sunlight). Notice that there is no spatial lag of y_t on the right-hand side of equation (1). Therefore, the specification *implicitly assumes* that, in each period t, the productive output of farm i is *not* a function of the output of other farms in the sample. This seems reasonable if the production function is viewed as a purely deterministic (engineering) process, where the farmer controls all the inputs. We need the following additional assumptions:

Assumption 1: The elements of ε_t are independently and identically distributed with zero mean and finite variance σ_t^2 , the fourth moment of ε_t is finite, and ε_t is independent of ε_s , $\forall\ t \neq s$.

Assumption 2: All diagonal elements of M_t are zero. The matrix (I_N - $\rho_t M_t$) is non-singular. $|\rho_t| < 1$.

Notice that under Assumptions 1 and 2, $u_t = (I_N - \rho_t M_t)^{-1}\varepsilon_t$, so $E(u_t) = 0$ for all t, but $E(u_t u_t')$ has a general, non-spherical structure, which is a function of ρ_t, M_t and σ_t^2 . Since M_t

is known, E($u_t u_t'$) is known up to ρ_t and σ_t^2, the parameters that we will ultimately estimate. Estimation of ρ_t and σ_t^2 allows feasible and efficient estimation of equation (1). Also, notice that if ρ_t = ρ, M_t = M, and σ_t^2 = σ^2, then E($u_t u_t'$) is a constant, which can be consistently estimated as $T\rightarrow\infty$. Here, we assume that T is fixed, so the consistent estimation of E($u_t u_t'$) is unreasonable, and we must assume that M_t is known to identify an estimate of equation (1). For now, assume that ρ_t and σ_t^2 are known. Forming vectors in t from the vectors of observations in i,

(3) $y = \iota_T \otimes \alpha + x\beta + u,$

$u = (\rho \otimes IN)M^*u + \varepsilon,$

where ι_T is a T-dimensional column vector of ones, and

$$M^* = \begin{bmatrix} M_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & M_T \end{bmatrix}$$

$$\rho^* = \begin{bmatrix} \rho_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \rho_T \end{bmatrix}.$$

Notice that

$$E(\varepsilon\varepsilon') = \begin{bmatrix} \sigma_1^2 I_N & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \sigma_T^2 I_N \end{bmatrix},$$

so the disturbance in equation (2) is heteroskedastic. Define $\Phi_t = (I_N - \rho_t M_t)/\sigma_t$ and then we can pre-multiply the model in equations (1) and (2) to get

(4) $y_t^* = \alpha_t^* + x_t^* \beta + \varepsilon_t^*,$

where $y_t^* = \Phi_t y_t$, $x_t^* = \Phi_t x_t$, $\alpha_t^* = \Phi_t \alpha$, and $\varepsilon_t^* = \Phi_t \varepsilon_t = \varepsilon_t / \sigma_t$. Stacking observations in t,

(5) $y^* = \alpha^* + x^*\beta + \varepsilon^*,$

where $\alpha^{*\prime} = [\alpha_1^{*\prime}, ..., \alpha_T^{*\prime}]$, a TN dimensional vector. Equation (5) possesses a "well-behaved" disturbance, that is, E(ε^*) = 0 and E($\varepsilon^* \varepsilon^{*\prime}$) = I_{TN}. The identification of any estimates of the parameters in equation (5) hinges on the estimation of the unknown parameters M_t, ρ_t, and σ_t^2, which will be undertaken later. Kelejian and Prucha's cross-sectional procedure could be directly applied to equation (4) T times over N observations to recover estimates of ρ_t and σ_t^2

for known M_t. These estimates could then be used to estimate the parameters in equation (5).[3] We refer to this estimation technique as "unrestricted estimation." Our application implies some equality restrictions on the model in equation (5). In particular, our definitions of spatial dependence are based on the distinct physical characteristics of the farming villages on the island of Java (longitude, latitude, infrastructure, etc.), which are certainly constant over the short time period of the data (six years). Therefore, we impose some equality restrictions on equation (5) to identify alternative estimators of the model parameters.

1.1.1 Fully Restricted Specification

One obvious restriction is to assume that some subset of the weighting matrices, autoregressive parameters, and variance parameters are equal. As an extreme case we could assume that $M_1 = \ldots = M_T = M$, $\rho_1 = \ldots = \rho_T = \rho$, and $\sigma_1^2 = \ldots = \sigma_T^2 = \sigma^2$, implying that $\Phi_1 = \ldots = \Phi_T = \Phi$. Then $\alpha_t^* = \Phi\alpha^*$ in equation (4) and $\alpha^* = \iota_T \otimes \Phi\alpha$ in equation (5). Of course, the error term ε of equation (3) is no longer heteroskedastic, it has variance matrix $E(\varepsilon\varepsilon') = \sigma^2 I_{TN}$, so Φ need not be a function of σ for efficiency. FE estimation of equation (5) under this full restriction will then be efficient for α^* and β, if ρ and σ^2 are known, and if the restriction is true. It is also consistent for fixed T as $N \rightarrow \infty$. Additionally, an estimate of α can be recovered by transforming the estimate of α^* with Φ. Of course, ρ and σ^2 are not known, so the challenge is to consistently estimate them, so that equation (5) can be *feasibly* estimated; this is undertaken in section 1.2.

1.1.2 Partially Restricted Specification

As another example of a reasonable restriction on the parameters of the model, briefly consider the empirical example. We observe $N = 171$ Indonesian rice farms over $T = 6$ periods. Periods 1, 3, and 5 are "wet or rainy seasons" and periods 2, 4, and 6 are "dry seasons". It may be reasonable to suspect that $\rho_1 = \rho_3 = \rho_5 = \rho_W$ (wet) and $\rho_2 = \rho_4 = \rho_6 = \rho_D$ (dry), and we can suspect a similar equation for M_t, σ_t^2, and Φ_t. (This may be true on the island of Java, since during the rainy season many roads in the low country are impassable, and hence spill-overs based on infrastructure are potentially diminished.) Then,

$$\alpha^* = [(\Phi_W\alpha)' \ (\Phi_D\alpha)' \ (\Phi_W\alpha)' \ (\Phi_D\alpha)' \ (\Phi_W\alpha)' \ (\Phi_D\alpha)']'$$

[3] Conley's technique could also be applied here and could conceivably produce more flexible results since Conley's technique accommodates less restrictive assumptions on the error process. However, our intent is to specifically examine the Kelejian and Prucha results.

in equation (5) is a TN dimensional column vector that consists of $2N$ parameters. The system in (5) then consists of $2N + k$ parameters and can effectively be treated as $2 \times 171 = 342$ farms observed over $6/2 = 3$ periods, so the FE estimation of equation (5) is feasible, since it has been assumed that realizations of the error ε_t are independent across both t and i. Of course, there will be an efficiency loss in the estimate of α^*, relative to the fully restricted estimate, since the time series dimension has been effectively cut in half from 6 to 3, but the slope parameter β will still be efficient (and consistent in N) since it is still based on the same number of observations, TN. Again the challenge is the estimation of ρ_W, ρ_D, σ_W^2, and σ_D^2, which is undertaken in the following section.

1.2 Feasible Estimation

Kelejian and Prucha (1999) develop a moments estimator of the parameters ρ_t and σ_t^2 in the cross-sectional setting ($T = 1$). We now generalize their results for the case where ρ_t and σ_t^2 are different across t.[4] Using their notation, let $\tilde{u}_t$ be a predictor of u_t from the FE (or within) regression implied by equation (1), ignoring equation (2). That is, $\tilde{u}_t$ converges in distribution to the random variable u_t. Additionally, let $\bar{\tilde{u}}_t = M_t\tilde{u}_t$, $\bar{\bar{\tilde{u}}}_t = M_t\bar{\tilde{u}}_t$, $\bar{\varepsilon}_t = M_t\varepsilon_t$, and $\bar{\bar{\varepsilon}}_t = M_t\bar{\varepsilon}_t$. Consider the following $3T$ moment conditions implied by equations (1) and (2) and assumptions 1 and 2.

$$E[N^{-1}\varepsilon_t'\varepsilon_t] = \sigma_t^2,$$

$$E[N^{-1}\bar{\varepsilon}_t'\bar{\varepsilon}_t] = \sigma_t^2 N^{-1} tr(M_t'M_t),$$

$$E[N^{-1}\bar{\varepsilon}_t'\varepsilon_t] = 0,$$

$t = 1, ..., T$. Noting that $\varepsilon_t = (I_N - \rho_t M_t)u_t$, these moment conditions imply the following system of $3T$ equations

$$\Gamma_t[\rho_t, \rho_t^2, \sigma_t^2] - \gamma_t = 0$$

where

$$\Gamma_t = \begin{bmatrix} \frac{2}{N}E(u_t'\bar{u}_t) & \frac{-1}{N}E(\bar{u}_t'\bar{u}_t) & 1 \\ \frac{2}{N}E(\bar{\bar{u}}_t'\bar{u}_t) & \frac{-1}{N}E(\bar{\bar{u}}_t'\bar{\bar{u}}_t) & \frac{1}{N}tr(M_t'M_t) \\ \frac{1}{N}E(u_t'\bar{\bar{u}}_t + \bar{u}_t'\bar{u}_t) & \frac{-1}{N}E(\bar{u}_t'\bar{\bar{u}}_t) & 0 \end{bmatrix},$$

[4] We present no proofs of our results, because they are all straightforward extensions of Kelejian and Prucha's proofs.

$$\gamma_t = \begin{bmatrix} \frac{1}{N}E(u_t'u_t) \\ \frac{1}{N}E(\bar{u}_t'\bar{u}_t) \\ \frac{1}{N}E(u_t'\bar{u}_t) \end{bmatrix},$$

$t = 1, \ldots, T$. The sample analogs based on $\tilde{u}_t$ are

$$G_t[\rho_t, \rho_t^2, \sigma_t^2] - g_t = \nu_t(\rho_t, \sigma_t^2) \tag{6}$$

$$G_t = \begin{bmatrix} \frac{2}{N}\tilde{u}_t'\tilde{\bar{u}}_t & \frac{-1}{N}\tilde{\bar{u}}_t'\tilde{\bar{u}}_t & 1 \\ \frac{2}{N}\tilde{\bar{\bar{u}}}_t'\tilde{\bar{u}}_t & \frac{-1}{N}\tilde{\bar{\bar{u}}}_t'\tilde{\bar{\bar{u}}}_t & \frac{1}{N}tr(M_t'M_t) \\ \frac{1}{N}\tilde{u}_t'\tilde{\bar{\bar{u}}}_t + \tilde{\bar{u}}_t'\tilde{\bar{u}}_t & \frac{-1}{N}\tilde{\bar{u}}_t'\tilde{\bar{\bar{u}}}_t & 0 \end{bmatrix},$$

$$g_t = \begin{bmatrix} \frac{1}{N}\tilde{u}_t'\tilde{u}_t \\ \frac{1}{N}\tilde{\bar{u}}_t'\tilde{\bar{u}}_t \\ \frac{1}{N}\tilde{u}_t'\tilde{\bar{u}}_t \end{bmatrix},$$

$t = 1, \ldots, T$. Here ν_t is the usual error associated with a sample of statistical realizations (i.e. it will ultimately be squared, summed, and then minimized by selecting parameters optimally). The system consists of $3T$ equations and $3T$ unknowns, but the system is actually T separate subsystems of three equations and three unknowns. If these T subsystems satisfy Assumptions 1 and 2 above and Assumptions 3, 4, and 5 of Kelejian and Prucha (1999), then Theorem 1 of Kelejian and Prucha (1999) is applicable to the individual subsystems.[5] That is, $\hat{\rho}_t$ and $\hat{\sigma}_t^2$ that solve the non-linear optimization

$$(\hat{\rho}_t, \hat{\sigma}_t^2) = \arg\min_{r,s^2}[\nu_t(r,s^2)'\nu_t(r,s^2) : s^2 \geq 0] \tag{7}$$

are consistent for ρ_t and σ_t^2 as $N \to \infty$. For a proof see Kelejian and Prucha (1999). Let

$$\hat{\Phi}_t = (I_N - \hat{\rho}_t M_t) / \sqrt{\hat{\sigma}_t^2} \ .$$

(We can substitute $\hat{\Phi}_t$ for Φ_t and estimate equation (5), but we will ultimately choose to restrict the model.) Let us call $\hat{\rho}_t$ and $\hat{\sigma}_t^2$ *unrestricted* estimates.

[5] Assumptions 3, 4, and 5 of Kelejian and Prucha (1999) are contained in the Appendix.

1.2.1 Feasible Estimation of the Fully Restricted System

If we can assume that $M_1 = \ldots = M_T = M$, $\rho_1 = \ldots = \rho_T = \rho$, and $\sigma_1^2 = \ldots = \sigma_T^2 = \sigma^2$ as before, then we can impose the assumption in equation (6) and estimate $\hat{\rho}_t$ and $\hat{\sigma}_t^2$, $t = 1, ..., T$, as above.[6] Then average estimates of ρ and σ^2 are

(8) $\hat{\rho} = T^{-1}\sum_t \hat{\rho}_t$ and $\hat{\sigma}^2 = T^{-1}\sum_t \hat{\sigma}_t^2$.

We shall call these estimates the *fully restricted average* estimates. The estimates will be consistent as $N\to\infty$, as long as the restriction is true. These are two-stage estimates, where in the first stage unrestricted estimates are calculated ($\hat{\rho}_t$ and $\hat{\sigma}_t^2$, $t = 1, ..., T$), and the restriction is imposed in the second stage of averaging over t. Since the estimates are based on the unrestricted estimates they do not exploit all the information in the data set simultaneously. That is, each $\hat{\rho}_t$ and $\hat{\sigma}_t^2$ is calculated from one of T separate sub-samples of the data. These estimates imply

$$\hat{\Phi} = (I_N - \hat{\rho}M),$$

which can be substituted into equation (5). Then FE estimation of equation (5) with $\alpha^* = \iota_T \otimes \hat{\Phi}\,\alpha$ is unbiased for α^* and β.

If we can (a) impose the restriction, (b) estimate the parameters in a single step, and (c) do so such that the data is not divided into T subsamples, then the resulting parameter estimates should be more efficient than the average fully restricted estimates. One such estimate is based on the moment conditions:

$$E[(TN)^{-1}\varepsilon'\varepsilon] = \sigma^2,$$

$$E[(TN)^{-1}\bar{\varepsilon}'\bar{\varepsilon}] = \sigma^2 (N)^{-1} tr(M'M),$$

$$E[(TN)^{-1}\bar{\varepsilon}'\varepsilon] = 0,$$

where $\bar{\varepsilon} = M\varepsilon$ and $\bar{\bar{\varepsilon}} = M\bar{\varepsilon}$.[7] Letting $\tilde{u}$ be a predictor of u from the FE (or within) regression implied by equation (3), $\bar{\tilde{u}} = M\tilde{u}$ and $\bar{\bar{\tilde{u}}} = M\bar{\tilde{u}}$, equation (6) becomes

(9) $G[\rho, \rho^2, \sigma^2] - g = \nu(\rho, \sigma^2)$,

where

[6] The fact that we estimate ρ_t , $t = 1, ..., T$, implies a test of the hypothesis $\rho_1 = \ldots = \rho_T = \rho$. We are not aware of any such test, nor are we aware of a standard error calculation for the estimate of ρ_t. Of course, the standard error could be boot-strapped. Later we use the Moran I and Lagrange Multiplier test to test the significance of the overall weighting scheme in each period.

[7] Notice that the middle moment condition contains N^{-1} and not $(TN)^{-1}$, since it is based on M and not M^*.

$$G = \begin{bmatrix} \frac{2}{TN}\tilde{u}'\widetilde{\overline{u}} & \frac{-1}{TN}\widetilde{\overline{u}}'\widetilde{\overline{u}} & 1 \\ \frac{2}{TN}\widetilde{\overline{\overline{u}}}'\widetilde{\overline{u}} & \frac{-1}{TN}\widetilde{\overline{\overline{u}}}'\widetilde{\overline{\overline{u}}} & \frac{1}{N}tr(M'M) \\ \frac{1}{TN}\tilde{u}'\widetilde{\overline{\overline{u}}} + \widetilde{\overline{u}}'\widetilde{\overline{u}} & \frac{-1}{TN}\widetilde{\overline{u}}'\widetilde{\overline{\overline{u}}} & 0 \end{bmatrix},$$

$$g = \begin{bmatrix} \frac{1}{TN}\tilde{u}'\tilde{u} \\ \frac{1}{TN}\widetilde{\overline{u}}'\widetilde{\overline{u}} \\ \frac{1}{TN}\tilde{u}'\widetilde{\overline{u}} \end{bmatrix}.$$

The system consists of three equations and three unknowns and is exactly the Kelejian and Prucha (1999) result. Then estimates $\tilde{\rho}$ and $\tilde{\sigma}^2$ follow from

(10) $$(\tilde{\rho}, \tilde{\sigma}^2) = \arg\min_{r,s^2}[\nu(r,s^2)'\nu(r,s^2) : s^2 \geq 0].$$

We shall call these the *fully restricted moment estimates* (to differentiate them from the fully restricted average estimates). The potential efficiency gain of $\tilde{\rho}$ and $\tilde{\sigma}^2$ estimates over the estimates $\hat{\rho}$ and the $\hat{\sigma}^2$ hinges on the fact that equation (9) exploits the information contained in *TN* observations and imposes a hypothetically correct restriction, while equation (6) exploits the information contained in *N* observations over $t = 1, \ldots, T$, with no restriction. Again, $\tilde{\rho}$ and $\tilde{\sigma}^2$ imply $\tilde{\Phi}$, which can be inserted in equation (5); then FE estimation of equation (5) with $\alpha^* = \iota_T \otimes \tilde{\Phi}\, \alpha$ is unbiased for α^* and β. Consistent estimation of α follows by transforming the estimate of α^* by $\tilde{\Phi}$.

1.2.2 Feasible Estimation of the Partially Restricted System

For our 171 Indonesian rice farms observed over six periods, if we can assume that $M_1 = \ldots = M_6 = M$, $\rho_1 = \rho_3 = \rho_5 = \rho_W$, $\rho_2 = \rho_4 = \rho_6 = \rho_D$, $\sigma_1^2 = \sigma_3^2 = \sigma_5^2 = \sigma_W^2$, and $\sigma_2^2 = \sigma_4^2 = \sigma_6^2 = \sigma_D^2$, then we can impose the assumption $M_1 = \ldots = M_6 = M$ in equation (6) and estimate $\hat{\rho}_t$ and $\hat{\sigma}_t^2$, $t = 1, \ldots, 6$ as above. Then consistent estimates of ρ_W, ρ_D, σ_W^2, and σ_D^2 are the *partially restricted average estimates*.

(11) $$\hat{\rho}_W = \tfrac{1}{3}(\hat{\rho}_1 + \hat{\rho}_3 + \hat{\rho}_5),$$

$$\hat{\rho}_D = \tfrac{1}{3}(\hat{\rho}_2 + \hat{\rho}_4 + \hat{\rho}_6)$$

and

$$\hat{\sigma}_W^2 = \tfrac{1}{3}(\hat{\sigma}_1^2 + \hat{\sigma}_3^2 + \hat{\sigma}_5^2),$$

$$\hat{\sigma}_D^2 = \tfrac{1}{3}(\hat{\sigma}_2^2 + \hat{\sigma}_4^2 + \hat{\sigma}_6^2).$$

Again, these are two-stage estimates, which imply that

$$\hat{\Phi}_W = (I_N - \hat{\rho}_W M)/\sqrt{\hat{\sigma}_W^2} \quad \text{and}$$

$$\hat{\Phi}_D = (I_N - \hat{\rho}_D M)/\sqrt{\hat{\sigma}_D^2}\,,$$

which can be substituted into equation (5). Then FE estimation of equation (5) with

$$\alpha^* = [\ (\hat{\Phi}_W\alpha)',\ (\hat{\Phi}_D\alpha)', (\hat{\Phi}_W\alpha)',\ (\hat{\Phi}_D\alpha)', (\hat{\Phi}_W\alpha)',\ (\hat{\Phi}_D\alpha)']'$$

is consistent for α^* and β.

Define $\varepsilon_W' = [\varepsilon_1'\ \varepsilon_3'\ \varepsilon_5']$, $\varepsilon_D' = [\varepsilon_2'\ \varepsilon_4'\ \varepsilon_6']$, $\tilde{u}_W' = [\tilde{u}_1'\ \tilde{u}_3'\ \tilde{u}_5']$, and $\tilde{u}_D' = [\tilde{u}_2'\ \tilde{u}_4'\ \tilde{u}_6']$. Additionally, let $\bar{\tilde{u}}_j = M\tilde{u}_j$, $\bar{\bar{\tilde{u}}}_j = M\bar{\tilde{u}}_j$, $\bar{\varepsilon}_j = M\varepsilon_j$, $\bar{\bar{\varepsilon}}_j = M\bar{\varepsilon}_j$, and $j = W, D$.

It follows analogously that the single stage estimates are

(12) $\quad (\tilde{\rho}_j, \tilde{\sigma}_j^2) = \arg\min_{r,s^2}[\nu_j(r,s^2)'\nu_j(r,s^2) : s^2 \geq 0],\ j = W, D,$

where

$$\nu_j(\rho_j, \sigma_j^2) = G_j[\rho_j, \rho_j^2, \sigma_j^2] - g_j,\ j = W, D,$$

and where G_j and g_j are G_t and g_t of equation (6), but with j substituted for t and $3N$ substituted for N. Call these estimates the *partially restricted moment estimates*. $\tilde{\rho}_j$ and $\tilde{\sigma}_j^2$ imply $\tilde{\Phi}_j$ for wet and dry seasons, and FE estimation of equation (5) is again consistent for α^* and β.

To summarize, the unrestricted estimation procedure yields $\hat{\rho}_t$ and $\hat{\sigma}_t^2$ by solving equation (7); this is simply the application of the Kelejian and Prucha procedure T times. These estimates imply fully restricted average estimates ($\hat{\rho}$ and $\hat{\sigma}^2$) by averaging over T in equation (8) or partially restricted average estimates ($\hat{\rho}_j$ and $\hat{\sigma}_j^2$, $j = W, D$) by averaging over wet and dry seasons in equation (11). These are two-stage estimates. Fully restricted moment estimates ($\tilde{\rho}$ and $\tilde{\sigma}^2$) are produced by solving (10) and partially restricted moments estimates ($\tilde{\rho}_j$ and $\tilde{\sigma}_j^2$, $j = W, D$) are produced by solving equation (12). These are single stage estimates.

1.3 Application to Indonesian Rice Farms

We now estimate the models with a balanced panel of Indonesian rice farms. The data were previously analyzed by, for example, Erwidodo (1990), Lee (1991), Lee and Schmidt (1993), and Horrace and Schmidt (1996, 2000). For detailed discussion of the data see Erwidodo (1990). For the panel specification of a stochastic frontier model, y is the natural logarithm of output (ln(rice)), x is a vector of inputs (e.g. seed and fertilizer) and α_i embodies farm-level technical inefficiency. This is a standard stochastic frontier specification based on a Cobb-Douglas production function. Per Schmidt and Sickles (1984), a measure of technical efficiency for farm i is calculated plugging the estimate of α_i into the expression $\exp(\alpha_i - \max_j\alpha_j)$. In order to perform the spatial analysis we first specify the spatial weighting matrix M_t for the error process, which captures productivity spillovers across farms.

1.3.1 Geographic and Climatic Characteristics of West Java

In 1977 the Indonesian Ministry of Agriculture began to survey 171 rice farms concerning farming practices over six (three wet and three dry) growing seasons. The farms were selected from six villages located in the production area of the Cimanuk River Basin in West Java. Of the six villages included in the sample, two are located on the north coast of the island in an area with average altitudes of 10–15 meters above sea level. Another three villages are in an area (600–1100 meters above sea level) in the central part of West Java. The last village is in the center of the island with an altitude of 375 meters. The infrastructure in the Cimanuk River Basin is fairly heterogeneous. Some of the villages (in both highland and lowland areas) lack reliable transportation systems and local roads are almost impassable in the wet season. Other villages located in close proximity to province capital cities are highly accessible along paved, all-weather roads.[8]

Based on these facts, we constructed and performed our analysis using two different weighting matrixes $M1_t$ and $M2_t$. The first one, $M1_t$, is based on the *inverse of the geographical distance* between individual farms.[9] We used the geographical coordinates of the villages to determine the physical distances between producing units. The distances between individual villages are between 31 and 91 km. The individual distances between farms within the same village is unavailable and is therefore arbitrarily chosen to be 10 km.[10] The $M1$ weighting matrix then consists of the inverse values of these distances. That is, m_{ijt} equals the inverse of the distance between farms i and j. In the second weighting matrix we

[8] The survey ended in 1983, so the infrastructure description may be different from the current state.

[9] Cliff and Ord (1973,1981) first measured the potential interactions between spatial units using a combination of distance measures and the relative length of the common border (contiguity). Since there is no measure of contiguity available in our case we use physical distance only as a proxy for interdependence between spatial units.

[10] Experimentation with the weighting matrix suggested that the analysis was fairly robust to this arbitrary selection.

employ an intra-village contiguity scheme.[11] For $M2_t$ we let m_{ijt} equal 1 if farms i and j are in the same village and 0 otherwise. That is, the weighting scheme is based on *common villages*. For computational simplification and as a standard practice in forming weighting matrixes we *normalize* each weighting matrix so the elements of each row sum to 1. Additionally, all the weighting schemes are assumed time invariant, so the t subscript can be dropped.

1.3.2 Spatial Analysis of Indonesian Rice Farms

We first estimate the standard FE model of stochastic production frontier described by (1). Inputs to the production of rice included in the data set are seed (kg), urea (kg), trisodium phosphate (TSP) (kg), labor (labor-hours), and land (hectares). Output is measured in kilograms of rice. The data also include dummy variables. *DP* equals 1 if pesticides were used and 0 otherwise. *DV*1 equals 1 if high yield varieties of rice were planted and *DV*2 equals 1 if mixed varieties were planted. *DSS* equals 1 if it was a wet season and 0 otherwise. The results are in column I of Table 1.1 and are based on the restriction that $\rho_1 = ... = \rho_6 = 0$. These results are identical to those contained in Horrace and Schmidt (1996).

Before embarking on a spatial analysis, we use the residuals from the standard FE estimation to determine whether or not spatial dependence (based on each of our three weighting schemes) exists in the data. As before, let the usual FE residuals in period t be $\tilde{u}_t$. We employ two tests for spatial dependence; the first is the *Moran I statistic* (see e.g. Anselin, 1988). To preclude confusion with the symbol for the identity matrix we adopt the script ϑ. The ϑ statistic for period t is

$$\vartheta_t = [N/S]\{[\tilde{u}_t' M\tilde{u}_t]/\tilde{u}_t'\tilde{u}_t\},$$

where N is the number of farms and S is the sum of all the elements in the weighting matrix M. The null hypothesis for this test is the "absence of spatial dependence".[12] Notice that we have dropped the t subscript on the weighting matrix M, because our empirical analysis assumes time invariance for this matrix. As shown by Cliff and Ord (1972), the asymptotic distribution for the statistic is standard normal if ϑ is transformed in the usual manner:

$$z_t = \{\vartheta_t - E[\vartheta_t]\}/ V[\vartheta_t]^{1/2},$$

[11] Moran (1948) and Geary (1954) advanced initial measures of spatial dependence (spatial correlation) that were based on the notion of binary contiguity between spatial units. That is, if spatial units have a common border (are contiguous) a value of 1 is assigned to the spatial correlation and 0 otherwise.

[12] According to Anselin (1988), the interpretation of the test is not always straightforward, even though it is by far the most widely used approach. Indeed, while the null hypothesis is obviously absent spatial dependence, a precise expression for the alternative does not exist.

where $E[\vartheta_t]$ is the mean, and $V[\vartheta_t]$ is the variance of the statistic in period t, derived under the null of "no spatial dependence". In the general case of a non-normalized weighting matrix these can be expressed in the form:

$$E[\vartheta_t] = (N/S)tr(PM)/(N-k)$$

$$V[\vartheta_t] = (N/S)^2\{tr(PMPM') + tr(PM)^2 + [tr(PM)]^2/(N-k)(N-k+2) - \{E[\vartheta_t]\}^2,$$

where P is the projection matrix $I_N - x_t(x_t'x_t)^{-1}x_t'$ and x_t is a matrix of the demeaned exogenous variables from the standard model in equation (1). The test was conducted for both weighting schemes in each time period t = 1,..., 6. The z_t-scores for weighting scheme *M1* are in the third row (z_t) of Table 1.2 and range from 6.0702 in period t=2 to 26.4159 in period t=4. It is therefore safe to conclude that at the 95% confidence level we reject the hypothesis of "no spatial dependence" based on weighting scheme *M1*. Test results for weighting schemes *M*2 were similar and are contained in the third row (z_t) of Table 1.4.

The Moran *I* statistic is sensitive to heteroskedasticity and tends to over-reject the standard normal critical value. An alternative language multiplier (LM) test procedure for the null hypothesis of no spatial dependence is presented by Anselin, Bera, Florax, and Yoon (equation (9)). The test statistic

$$LM_t = \frac{[\tilde{u}_t'M\tilde{u}_t/\sigma_t^2]^2}{tr[(M'+M)M]}$$

is distributed χ_1^2 with critical values of 3.84 (95% level) and 6.63 (99% level). Results are in the last rows of Tables 1.2 and 1.4 for weighting schemes *M1* and *M2*, respectively, and confirm the Moran *I* results. We reject the null hypothesis in each case.

Based on these test results, our proposed weighting schemes appear justified in each period. Consequently, we estimated the unrestricted spatial autoregressive parameters and error variances for each period for each scheme using equation (6). Notice that the autoregressive and variance parameter estimates are identified for each period, even though the parameters α^* and β in equation (5) are not. Estimation results are contained in Tables 1.2 and 2.4 for *M1* and *M2*, respectively. Note that for both weighting schemes, the ρ-parameter tends to be larger in period 1 than in period 2, larger in period 3 than period 4, and larger in period 5 than in period 6. These differences correspond to differences in wet seasons (t = 1, 3, 5) and dry seasons (t = 2, 4, 6).

To identify parameter estimates for α^* and β in equation (5) we estimated the fully and partially-restricted systems described in sections 1.2.1 and 1.2.2, respectively. The fully restricted system, $\rho_1 = \ldots = \rho_6 = \rho$, was estimated using both the average autoregressive parameter of equation (8), $\hat{\rho}$, and the moments autoregressive parameter of equation (10), $\tilde{\rho}$, for each weighting scheme. Estimation results for $\hat{\rho}$ = 0.7248 and for $\tilde{\rho}$ = 1.0557 using weighting scheme *M1* are given in columns II and III in Table 1.1. There is little difference in the slope parameter estimates based on $\hat{\rho}$ or $\tilde{\rho}$ of the standard FE model of column I. This is not surprising, since ignoring the spatial dependence causes an efficiency loss in the slope parameter estimates (not a bias). Indeed, the most noticeable differences in the estimates of columns I, II and III are in the standard error estimates, with Columns II and III being generally smaller than column I, the standard model. The sign of the coefficient on the pesticide variable (DP) changes from positive to negative when we include spatial effects, however, it is always insignificant. The difference in the magnitudes of $\hat{\rho}$ and $\tilde{\rho}$ is troublesome. Perhaps this difference indicates that the restriction $\rho_1 = \ldots = \rho_6 = \rho$ does not hold. We did not attempt to test this, however it would be possible if the variance matrix of ρ_t were estimable. The results of the fully restricted model under weighting scheme *M2* is in columns II and III in Table 1.3. The results are similar to the *M1* case: slope coefficients do not change much, standard error estimates decrease, and there is a large difference between the two estimates of ρ.

The feasible estimation of the partially-restricted system follows the same pattern, except that instead of only one correlation coefficient fixed for all time periods now we estimate and utilize two correlation coefficients: one for wet and one for dry seasons. We calculate the average parameter estimates of equation (11) ($\hat{\rho}_W$, $\hat{\rho}_D$) and the moments estimates of equation (12) ($\tilde{\rho}_W$, $\tilde{\rho}_D$) for each weighting scheme. FE estimation results for ($\hat{\rho}_W$, $\hat{\rho}_D$) and ($\tilde{\rho}_W$, $\tilde{\rho}_D$), based on weighting scheme *M1*, are in columns IV and V of Table 1.1. The differences between the average and moments parameter estimates are much less pronounced than the fully restricted case (compare estimates $\hat{\rho}_W$ = 0.7584 to $\tilde{\rho}_W$ = 0.8218, estimates $\hat{\rho}_D$ = 0.6914 to $\tilde{\rho}_D$ = 0.7476, and estimates $\hat{\rho}$ = 0.7248 to $\tilde{\rho}$ = 1.0557). One might conclude that the partially restricted model seems to fit the data better, however this is not formally tested. (Additionally, the fact that the estimates are now all less than unity suggests that the partially restricted model may be favored over the fully restricted model.) Again, the standard errors of the slope parameter estimates are smaller for the partially restricted model than for the standard model (column I). The coefficient on the season variable (DSS) is not identified, since it is effectively time invariant now that the data set has been dichotomized into "wet" and "dry" sub-samples.[13] The coefficients on the partially restricted system are generally higher than those of the fully restricted system (columns II and III) and the standard model (column I). As in the fully restricted system, the coefficient on the pesticide variable (DP) is negative and insignificant. Even though it is insignificant, this is troubling, since economic theory usually dictates that the production function to be nondecreasing in its arguments. However, one could argue that too much pesticide might have a negative effect on output. Alternatively, one could argue that we have not adequately controlled for the interaction between pesticides (DP), output (y), and weather (DSS, ρ_W, and ρ_D). Perhaps, pesticide use is higher during the wet season (more water, more insects) and our simple dummy variable for pesticide does not adequately capture a more complex relationship. Nonetheless, the implications are compelling and the coefficient *is* insignificant. Estimation results for weighting scheme *M2* are similarly presented in columns IV and V of Table 1.3. Again, the results are similar to scheme *M1* for this particular sample.

1.3.3 Technical Efficiency Rankings

Stochastic frontier models are often concerned with estimating firm-level technical inefficiency and, in particular, determining the relative magnitudes of the resulting inefficiency measures, using a rank or order statistic. In the following analysis we demonstrate how the various weighting schemes affect the technical efficiency rankings of the farms. Specifically, for each weighting scheme we estimate and rank the estimated technical efficiencies, $\exp(\alpha_i - \max_j \alpha_j)$, for each farm. This is done for the standard FE model (column I of Table 1.1) and for *the fully restricted moments estimator* (column III of Tables 1.1 and 1.3). The idea is to see how the rankings differ between the standard model and the spatial model for both of the weighting schemes. Order statistics for each model are contained in Table 1.5. The first three columns of the table are results for the standard FE model. Since there are 171 farms we only report results for the four farms with the highest technical efficiency, the four farms with the median technical efficiency, and the four farms with the lowest technical efficiency. Column I contains the farm number, column II contains the ordered estimates of farm-level technical efficiency, and column III contains the ordinal rankings for the standard FE model (numbered 1 to 171). To see the effects of the spatial dependence on technical efficiency estimation, we also report the ordinal rankings for the

[13] Even though the time dimension has effectively been cut in half by this dichotomy, the estimates of the slope parameters are still based on the entire sample (*TN*) after the observables have been demeaned based on whether they are "dry" or "wet".

same 12 farms for the fully restricted spatial model under weighting schemes *M1* and *M2* in columns IV and V. While there are some changes across weighting schemes in the rank ordering of the most- and least-efficient farms, these are minor. For instance, in the standard model, farm 152 had a technical efficiency rank of 4, but it has a rank of 6 under the weighting schemes. Notice that the ranking of the most efficient farm (farm 164) is always 1 and that of the least efficient farm (farm 45) is always 171. The largest differences in ranking appear in the median farms. For example, farm 166 has a standard FE ranking of 85 but spatial rankings of 116. These are potentially large changes in the median technical efficiency ranking, which could only be detected with a spatial analysis.

To further summarize the changes in the efficiency ranking in Table 1.5, we calculate *Spearman's rho* (r_s) for each weighting scheme, using the standard FE model as the baseline. Spearman's rho is a standard measure of rank correlation between two rank statistics given by

$$r_s = 1 - \frac{6\sum_i \delta_i^2}{N^3 - N},$$

where δ_i is the difference in the rankings for the i^{th} farm. For example when comparing the rank statistic for the standard model and the *M1* model in Table 1.5, $\delta_{164} = 0$ and $\delta_{15} = 86 - 62 = 24$. Here we always compare the rankings of the *M1*and *M2* models to the standard model ranking. It is true that $r_s \in [-1, 1]$, $r_s = 1$ when the two rank statistics are identical and $r_s = -1$ when the rank statistics are completely reversed (i.e. as we move from one order statistic to the other, the most efficient farm becomes the least efficient, the second most efficient farm become the second-least efficient, etc.). The Spearman statistics are in the last row of Table 1.5 and are on the order of 0.8 for both of the weighting schemes. We can interpret this result as saying that only 80% of the rank statistic is preserved when we use a spatial weighting specification over the standard specification.

To better understand the changes in technical efficiency under the various weighting schemes, we present some density plots of the estimates of the parameters, α_i. Technically, there is no distribution of α_i to speak of, since it is assumed to be a fixed parameter and not a random variable. The *estimates* of the α_i are indeed random, and each estimate has its own marginal distribution from the joint distribution of the estimate of the N-dimensional vector α. However, for the purposes of exposition, we treat the estimates of α_i as if they are random draws from a univariate distribution in what follows. According to the panel data specification of Schmidt and Sickles, $\alpha_i = \alpha^{max} - \tau_i$, where τ_i is the nonnegative technical efficiency of farm *i* and α^{max} is a parameter representing maximal efficiency. The implication is that for fixed α^{max}, the "distribution" of α_i is just a relocation of the "distribution" of technical inefficiency. Therefore to infer the effects of various weighting schemes on the estimates of technical efficiency is to make inferences on the estimates of α_i.

Density plots for the various models are given in Figure 1.1. Density estimates are based on maximum likelihood, cross-validation bandwidth selection, and a standard Gaussian kernel. Fixing α^{max} across models, some generalizations about this data set can be made. First, the standard fixed effect model (FE in the figure) without spatial lags in the errors tends to underestimate α_i (overestimate technical inefficiency) in comparison to the spatial models (*M1* and *M2*). This is technical inefficiency in an absolute sense, since we are fixing α^{max} across models at some unknown value. This is reflected in Figure 1.1 as the density of the standard FE model being shifted to the left of the densities for the spatial models (with little to no rescaling). This has implications for the predictions of the conditional mean output implied by equation (1): the FE model (on average) gives lower predictions of productive output than the spatial models (all else being equal). That is, for fixed technology and input factors, the spatial models impute more of the observed output to unobserved technical ability (α_i) and less of it to luck (u_{it}) in this data set. Indonesian rice farms may be operating closer to the efficiency frontier than previous studies suggest.

1.4 Conclusions

This study presents a generalization of the cross-sectional model of Kelejian and Prucha (1999). Because economic agents and entities have finite lives, one cannot always rely on large *T* in panel data sets. Most panel data sets (with the exception of perhaps microeconomic financial data) have large *N* and small *T*. Additionally, if *T* is somewhat large, the usually time-invariant unobserved heterogeneity models (e.g., FE) may not be applicable, since it is widely held that heterogeneity may change in long-run, dynamic economic systems (particularly when it is viewed as technical inefficiency). The result is that consistency arguments usually must hinge on *N*-asymptotics. This is fine for estimating conditional means (the model's slope parameters). However, any second moment parameters that embody cross-sectional dependence cannot be consistently estimated in the sense that they will necessarily rely on *T*-asymptotics.

When faced with this dilemma, researchers have two recourses: collect more data or impose more structure on the model and hope that the structure will be testable. Given the aforementioned arguments against large *T*, it would seem that we have only the alternative of imposing more structure on our models. The question then becomes: what structure is reasonable? Spatial weighting schemes seem to be a reasonable and natural approach. The theoretical economic literature is rife with arguments for economic spillovers, and spatial analysis provides a means to make these spillovers explicit. Moreover, tests of “no spatial dependence” do exist in the literature. Therefore, if we must make assumptions about the second moments of our data, spatial weighting schemes may be a viable approach.

Dynamic spatial dependence in the second moment of our estimators has implications for dynamics in the first moment. The FE model has time-invariant heterogeneity parameters, but the transformed model has dynamic parameters. It is this loss of time-invariance that makes the general model “not identified,” and forces us to impose some restrictions on the dynamics of the spatial dependence. This could be important. Most panel data models that attempt to make the heterogeneity parameters dynamic do so by imposing structure on the first moments of the models. For instance, several articles in the stochastic frontier literature impose a special structure on the conditional mean of the heterogeneity parameters. (For examples see Cornwell, Schmidt, and Sickles 1990; Lee and Schmidt 1993; Battese and Coelli 1992; and Kumbhakar 1990.) The models presented here create dynamic heterogeneity through *second moment* conditions on the error process. The implications of this difference for models of dynamic heterogeneity are unknown, but it is interesting to point this difference out.

Additionally, spatial dependence may be a way to indirectly incorporate time-invariant regressors into a FE model. For example, Horrace and Schmidt (1996) incorporate dummy variables for the six villages into a random effects specification but are forced to exclude these dummy variables from a FE specification, because they are time-invariant at the farm level. In the application presented here, village effects are incorporated into the second moment of the residual for the FE model. While there are commonly employed techniques for incorporating time-invariant regressors into a FE model (see Hausman and Taylor 1981), the research presented here provides analysts with an alternative means of accomplishing this.

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1.A Tables and Figures

Table 1.1 Weighting Scheme *M1* – Inverse of Distance

	Standard FE Model I	Fully Restricted Average II	Fully Restricted Moment III	Partially Restricted Average IV	Partially Restricted Moment V
$\hat{\rho}$	-	0.7248	-	-	-
$\tilde{\rho}$	-	-	1.0557	-	-
$\hat{\rho}_w$	-	-	-	0.7584	-
$\hat{\rho}_D$	-	-	-	0.6914	-
$\tilde{\rho}_W$	-	-	-	-	0.8218
$\tilde{\rho}_D$	-	-	-	-	0.7476
Seed	0.1208	0.1038	0.0998	0.1292	0.1248
	(0.030)	(0.025)	(0.024)	(0.024)	(0.024)
Urea	0.0918	0.0894	0.0901	0.1405	0.1440
	(0.021)	(0.018)	(0.017)	(0.015)	(0.015)
TSP	0.0892	0.0353	0.0244	0.0340	0.0307
	(0.013)	(0.012)	(0.012)	(0.011)	(0.011)
Labor	0.2431	0.2366	0.2379	0.2254	0.2204
	(0.032)	(0.029)	(0.028)	(0.026)	(0.026)
Land	0.4521	0.4879	0.4931	0.5046	0.5141
	(0.035)	(0.031)	(0.030)	(0.027)	(0.027)
DP	0.0338*	-0.0178*	-0.0298*	-0.0224*	-0.0212*
	(0.032)	(0.028)	(0.028)	(0.025)	(0.025)
DV1	0.1788	0.1084	0.0935	0.1250	0.1320
	(0.041)	(0.038)	(0.038)	(0.034)	(0.035)
DV2	0.1754	0.1060	0.0952	0.0917	0.0947
	(0.057)	(0.049)	(0.048)	(0.048)	(0.048)
DSS	0.0533	0.0759*	0.1062*	-	-
	(0.022)	(0.063)	(0.302)	-	-
R-sq	0.910228	0.9246	0.9271	0.9190	0.9177

Note: Numbers in parentheses are standard errors. All estimates are significant at least at the 5% significance level except those marked with an asterisk.

Table 1.2 Unrestricted Estimates, Weighting Scheme *M1*

Time period	1	2	3	4	5	6
$\hat{\rho}_t$	0,62	0,52	0,87	0,84	0,77	0,71
$\hat{\sigma}_t^{\ 2}$	0,04	0,08	0,08	0,07	0,05	0,07
z_t	8,24	6,07	24,95	26,42	14,16	12,30
LM_t	65,41	30,50	1461,01	1680,70	254,85	175,99

Table 1.3 Weighting Scheme *M2* – Common Villages

	Standard FE Model I	Fully Restricted Average II	Fully Restricted Moment III	Partially Restricted Average IV	Partially Restricted Moment V
$\hat{\rho}$	-	0.6604	-	-	-
$\tilde{\rho}$	-	-	0.9882	-	-
$\hat{\rho}_w$	-	-	-	0.6811	-
$\hat{\rho}_D$	-	-	-	0.6398	-
$\tilde{\rho}_W$	-	-	-	-	0.7388
$\tilde{\rho}_D$	-	-	-	-	0.6999
Seed	0.1208 (0.030)	0.1035 (0.025)	0.0996 (0.024)	0.1255 (0.024)	0.1248 (0.024)
Urea	0.0918 (0.021)	0.0909 (0.018)	0.0901 (0.017)	0.1435 (0.015)	0.1446 (0.015)
TSP	0.0892 (0.013)	0.0356 (0.012)	0.0239 (0.012)	0.0326 (0.011)	0.0301 (0.011)
Labor	0.2431 (0.032)	0.2385 (0.029)	0.2376 (0.028)	0.2201 (0.026)	0.2198 (0.026)
Land	0.4521 (0.035)	0.4855 (0.031)	0.4934 (0.030)	0.5131 (0.028)	0.5148 (0.027)
DP	0.0338* (0.032)	-0.0189* (0.028)	-0.0306* (0.028)	-0.0208* (0.025)	-0.0219* (0.025)
DV1	0.1788 (0.041)	0.1116 (0.038)	0.0928 (0.038)	0.1335 (0.034)	0.1326 (0.035)
DV2	0.1754 (0.057)	0.1080 (0.049)	0.0947 (0.048)	0.0970 (0.049)	0.0961 (0.049)
DSS	0.0533 (0.022)	0.0789* (0.051)	0.0844* (1.424)	- -	- -
R-sq	0.910228	0.9240	0.9271	0.9171	0.9174

Note: Numbers in parentheses are standard errors. All estimates are significant at least at the 5% significance level except those marked with an asterisk.

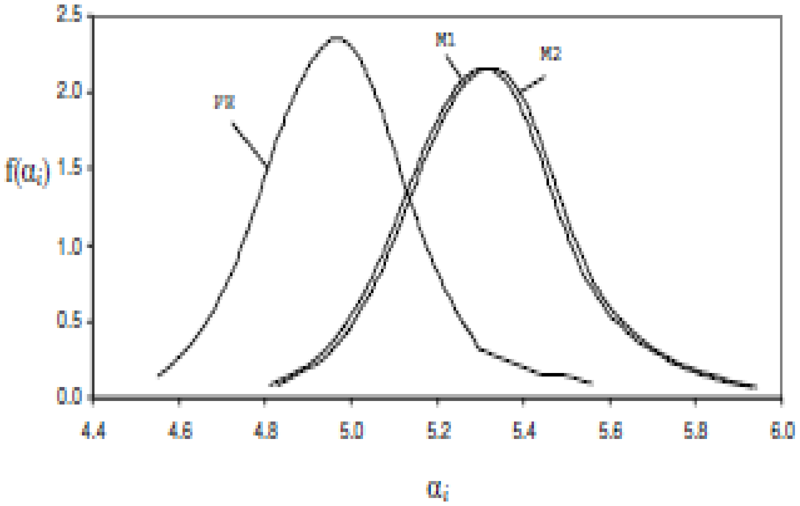
Table 1.4 Unrestricted Estimates, Weighting Scheme *M2*

Time period	1	2	3	4	5	6
$\hat{\rho}_t$	0,57	0,48	0,79	0,79	0,69	0,65
$\hat{\sigma}_t^{\,2}$	0,04	0,08	0,08	0,07	0,05	0,07
z_t	7,50	5,09	23,54	23,51	13,66	11,18
LM_t	57,03	21,62	1409,13	1386,30	256,73	153,03

Table 1.5 Orders Statistics, Various Models

Standard FE Model		Spatial Models			
Farm #	Standard FE Efficiency	Standard FE Model	Weight Scheme *M1*	Weight Scheme *M2*	Weight Scheme *M3*
164	100%	1	1	1	1
118	93.23%	2	2	2	3
163	93.03%	3	3	3	2
152	89.93%	4	6	6	4
13	55.62%	84	106	106	114
166	55.47%	85	116	116	108
15	55.40%	86	62	62	72
40	55.35%	87	54	54	64
86	39.80%	168	165	165	166
143	38.37%	169	169	169	170
117	37.90%	170	168	168	168
45	36.55%	171	171	171	171
	r_s:	1.0000	0.8027	0.8095	0.8674

Figure 1.1 Density estimates of α_i for various models



FE = fixed effect with no spatial weighting
M1 = *M1* weighting scheme
M2 = *M2* weighting scheme

Appendix

The following are the assumptions 3, 4, and 5 from Kelejian and Prucha (1999). Let $P(\rho_t) = (I_N - \rho_t M_t)^{-1}$ with typical element $p_{ij}(\rho_t)$.

Assumption 3: (*i*) The sums $\Sigma_i|m_{ijt}|$ and $\Sigma_j|m_{ijt}|$ are bounded by, say, $c_m < \infty$ for all $1 \leq i, j \leq N$, $N \geq 1$. (*ii*) The sums $\Sigma_i| p_{ij}(\rho_t)|$ and $\Sigma_j| p_{ij}(\rho_t)|$ are bounded by, say, $c_p < \infty$ for all $1 \leq i, j \leq N$, $N \geq 1$, $|\rho_t| < 1$.

Assumption 4: Let $\tilde{u}_{it}$ be the i^{th} element of $\tilde{u}_t$. There exists finite dimensional random vectors d_{it} and Δ_t such that $|\tilde{u}_{it} - u_{it}| \leq ||d_{it}||\ ||\Delta_t||$ with $N^{-1}\Sigma_i||d_{it}||^{2+\delta} = O_p(1)$ for some $\delta > 0$ and $N^{-1/2}\Sigma_i||\Delta_t|| = O_p(1)$.

Assumption 5: The smallest eigen value of $\Gamma_t'\Gamma_t$ is bounded away from zero.

Section 2

A Random Effect Model with Spatial Dependence in the Error Term

The merits and appropriateness of the two central modes of panel data treatment, fixed effect (FE) and random effect (RE) models, have been discussed by Mundlak (1978), Chamberlain (1978), and Hausman and Taylor (1981), among many others. The goal of this study is to deal with a problem that may arise in the application of these models. Although these panel data models differ by design in the treatment of the unit-specific effect, they both share the same independence assumption across a statistical noise component. The violation of this assumption yields inefficient estimates of regression coefficients and leads to a bias in the estimates of the unit-specific coefficients in FE and RE models.

Independence assumptions across cross-sectional units are often at odds with economic theory. For example, real shocks in remote areas can have an impact through price change on the rest of the economy, currency crises in one part of the world can travel via financial liaisons and cause repercussions in seemingly secluded areas, and to price the quality of air one must consider local weather conditions. Spatial dependence is pertinent to urban, development, growth, environmental and other areas of economics. This issue has been addressed in theoretical or empirical form in Anselin (2010); Anselin and Rey (2010); Case (1991); Conley (1999); Delong and Summers (1991); Dubin (1988); Fishback, Horrace, and Kantor (1999); Kelejian and Robinson (1993); Moulton (1990); Quah (1992); and Topa (1996). Interdependencies among economic agents, firms or countries demonstrated in reality and not parametrically accounted for in an econometric model specification represent from a statistical point of view dependence among individuals' unobservables.

In this paper I consider the panel data case when the disturbances are *spatially correlated* across the cross-sectional units, as these are related in the economic or geographic dimension. If the time dimension of data is large enough one can consistently estimate cross-sectional correlation with standard approaches that rely on T-asymptotics (see e.g. Kmenta 1990). When the time dimension is small (the typical panel data case) and hence when one cannot rely on T-asymptotics, this paper develops an RE panel data estimator that extends the cross-sectional GMM model developed by Kelejian and Prucha (1999). The GMM-based treatment of the same spatial phenomena in the FE model was developed by Druska and Horrace (2004); henceforth this paper will be referred to as "DH". This paper thus represents a natural extension of this estimator from the panel data case when individual unit-specific coefficients are treated as fixed to the case when they are treated as random.

Kelejian and Prucha (1999) consider a generalized moment estimation technique that permits correlation across the disturbances in the case of cross-sectional data. This cross-sectional correlation can be likened to time series correlation and its estimation requires the specification of a spatial weighting matrix that captures the interdependency among cross-sectional units. The construction of the weighting matrix is usually guided by underlying economic, geographic or meteorological theory.[14] Although this seems to be a strong parametric assumption imposed on the model, it is a testable assumption and it represents the price to be paid for the lack of the time dimension in the data. Obviously, if the time dimension in the panel data is large enough, one can proceed with a seemingly unrelated regression model or an error component model to estimate cross-sectional correlation via the time dimension. However, for the typical panel data case characterized by small time dimension the application of these techniques cannot be justified. In this paper I consider the usual panel data case when N, the number of units, is large, while T, the number of time observations per unit, is small, and adopt the Kelejian and Prucha cross-sectional estimator to the RE panel data treatment.

The empirical part of this paper presents an application of the herein-derived spatial technique to an efficiency measurement problem within a stochastic frontier framework. In this framework the firm-level output is modeled as an additive function of inputs and a random error term composed of technical inefficiency and statistical noise. While there are many different ways to estimate these types of models15 they all hinge on an independence assumption across the statistical noise component. The RE specification16 of the models considered in this paper is not an exception. Within this framework statistical noise can be viewed as productivity shocks due to the geographical or economic proximity of cross-

[14] A comprehensive treatment of spatial phenomena and estimation techniques is provided by Anselin (1988).

[15] A comprehensive summary of these techniques and theory is provided by Kumbhakar and Lovell (2000).

[16] A panel data treatment of this type of model in the form of FE and RE is in Schmidt and Sickles (1984).

sectional units. Given this viewpoint, it is not unreasonable to suspect that productivity shocks within a given industry may be correlated across realizations. That is, productivity spillovers (correlations) may exist in the statistical noise component, and the usual independence assumption across the realizations of statistical disturbances may be violated.

Using a panel of 171 Indonesian rice farms observed over six periods I empirically demonstrate that the productivity shocks (cross sectional correlation in the random part of the error term) potentially exist in this data set, and that spillovers have an impact on the estimates of technical efficiency.[17] In this data set the production function output is rice, and inputs are things like seed, labor, fertilizer, and land acreage. Since the time dimension of the data is quite small, the consistent estimation of cross-sectional correlations in the error process is of little comfort. Therefore I apply the GMM methodology and proceed with the specification of a spatial weighting scheme in the error process. The proposed specification of the weighting matrix allows for spillovers across farms, utilizing information on geographical coordinates of the villages, to determine the physical distances between producing units. The results indicate the presence of spatial correlations in this data generation process, which has ramifications for the estimation of the production function and the estimation of farm-level technical efficiency.

The chapter is organized as follows. In section 2.2 the RE model with a spatially correlated error term is formulated and identifying parameter restrictions are set. Section 2.3 presents the feasible estimation technique based on the Kelejian and Prucha (1999) generalized moment method. The following section illustrates the application of the technique on the sample of Indonesian rice farm data and compares RE estimates with the results from the FE model in DH. Section 2.5 concludes.

[17] To provide a complete set of results from the extension of Kelejian and Prucha to the panel data setting I use the same data set of 171 Indonesian rice farms observed over six time periods, and the same weighting matrix definitions as DH, and report the results of the Kelejian and Prucha extension to both the fixed effect and random effect model.

2.1 RE Model with Spatial Disturbances

Consider the following specification of a RE panel data model (Hsiao 1990):

(1) $$y_{it} = \mu + x_{it}\beta + \eta_{it}\,,\quad i = 1,\ldots,N,$$
$$t = 1,\ldots,T\,,$$

where μ represents a constant term, β is (k×1), and x_{it} is (1×k). η_{it} is assumed to consist of a random unit specific component α_i and a random time- and unit-specific disturbance term u_{it}:

(2) $$\eta_{it} = \alpha_i + u_{it}\,.$$

Standard (and fairly restrictive) assumptions of the basic RE model are:

$$E(\alpha_i) = E(u_{it}) = 0, \qquad E(\alpha_i u_{it}) = 0,$$

$$E(\alpha_i \alpha_j) = \begin{cases} \sigma_\alpha^2 & i = j, \\ 0 & i \neq j, \end{cases}$$

(3) $$E(u_{it} u_{js}) = \begin{cases} \sigma_u^2 & i = j, t = s, \\ 0 & otherwise, \end{cases}$$

$$E(\alpha_i x_{it}^{'}) = E(u_{it} x_{it}^{'}) = 0.$$

The variance-covariance matrix of η_i is $E(\eta_i \eta_i') = \sigma_u^2 I_T + \sigma_\alpha^2 e'e = V$, e being 1xT vector of ones. Its inverse is (Nerlove 1971 and Maddala1971)

(4) $$V^{-1} = \frac{1}{\sigma_u^2}\left[I_T - \frac{\sigma_\alpha^2}{\sigma_u^2 + T\sigma_\alpha^2} ee' \right] = \frac{1}{\sigma_u^2}\left[Q + \psi \frac{1}{T} ee' \right],$$

where

$$\psi = \frac{\sigma_u^2}{\sigma_u^2 + T\sigma_\alpha^2}$$

and

$$Q = I_T - \frac{1}{T} ee'.$$

Forming a vector of observation in t and rearranging the right-hand side of equation (1) the model becomes

(5) $$y_i = \tilde{x}_i \delta + \eta_i\,,$$

where $\tilde{x}_i = (e, x_i), \delta' = (\mu, \beta'), \eta_i = (\eta_{i1},\ldots,\eta_{iT})$. Under the above assumptions an efficient parameter estimator of (5) is a GLS estimator of δ with normal equations

(6) $$\left[\sum_{i=1}^{N} \tilde{x}_i' V^{-1} \tilde{x}_i\right] \hat{\delta}_{GLS} = \left[\sum_{i=1}^{N} \tilde{x}_i' V^{-1} y_i\right].$$

Note that $V^{-1} = P'P$, where $P = \left[I_T - (1-\psi^{1/2})(1/T)ee'\right]$. Were the P known, one could estimate $\hat{\delta}_{GLS}$ by pre-multiplying (5) by the transformation matrix P and applying the OLS method to the transformed model. This is equivalent to first subtracting $(1-\psi^{1/2})$ of individual means $\bar{y}_i$ and $\bar{x}_i$ from their corresponding values of y_{it} and x_{it}, and then running OLS on

(7) $$\left[y_{it} - (1-\psi^{1/2})\bar{y}_i\right] = const + \left[x_{it} - (1-\psi^{1/2})\bar{x}_i\right]\beta + \omega_{it}.$$

Since σ_u^2 and σ_α^2 are unknown, two-step feasible GLS (FGLS) estimation is used. In the first step variance components are estimated using within-group and between-group residuals:

(8) $$\hat{\sigma}_u^2 = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T}\left[(y_{it} - \bar{y}_i) - \hat{\beta}_W'(x_{it} - \bar{x}_i)\right]^2}{N(T-1) - K},$$

(9) $$\hat{\sigma}_\alpha^2 = \frac{\sum_{i=1}^{N}(\bar{y}_i - \mu - \hat{\beta}_B'\bar{x}_i)^2}{N-(K+1)} - \frac{1}{T}\hat{\sigma}_u,$$

where $\hat{\beta}_W$ are within, and $(\mu, \hat{\beta}_B)$ between, estimates of model's coefficients. In the second step, the estimated values of variance components are substituted into (6) and the coefficients of the RE model are estimated.

As stated in (3), the fundamental assumption of the standard RE model is that the random disturbance term, u_{it}, is not correlated across cross-sectional units or over time. In the following I relax the assumption of no cross-sectional correlation; instead I allow for spatial dependence across units and suppose that the disturbance term is spatially correlated so that (after forming vectors of observations in *i*)

(10) $$u_t = \rho_t M_t u_t + \varepsilon_t,$$

where u_t is a Nx1 vector of the random disturbance term in (1), scalar ρ_t represents a spatial correlation parameter, ε_t is a (Nx1) zero-mean disturbance term, and M_t is a ($N\times N$) spatial weighting matrix of known constants, which captures the spatial correlations across cross-sectional units. Elements of M_t and m_{ijt} are specified to be nonzero if cross-sectional unit *i* relates to unit *j* in a meaningful way and are chosen based on some geographic or economic proximity measure such as contiguity, physical distance, or economic or climatic dissimilarities.[18] If m_{ijt} is nonzero, units i and j are referred to as neighbors. All diagonal elements of M_t are zero, i.e. no unit is viewed as its own neighbor. The variable $M_t u_t$ is referred to as a spatial lag of u_t. I need to invoke two more assumptions:

> *Assumption 1:* The elements of ε_t are independently and identically distributed with zero-mean and finite variance σ_t^2, the fourth moment of ε_t is finite, and ε_t is independent of ε_s, $\forall\ t \neq s$.
>
> *Assumption 2:* The matrix $(I_N - \rho_t M_t)$ is non-singular.

Equation (10) and assumptions 1 and 2 imply that $u_t = (I_N - \rho_t M_t)^{-1}\varepsilon_t$, $E(u_t) = 0$ for all t, and $E(u_t u_t')$ has a general, non-spherical structure that is a function of ρ_t, M_t and σ_t^2.

Since M_t is known, $E(u_t u_t')$ is known up to the ρ_t and σ_t^2 parameters, which I will ultimately estimate. Estimates of ρ_t and σ_t^2 allow the feasible estimation of equation (1), producing efficient estimates of regression parameters and unbiased estimates of unit-specific coefficients when the disturbance term is spatially correlated across cross-sectional units.[19] For now, assume that M_t, ρ_t, and σ_t^2 are known and that the disturbance term in (1) is spatially correlated according to (10). Forming vectors of observations in i I can rewrite the model in (1) as

(11) $$y_t = \mu + x_t\beta + \alpha + u_t$$
$$u_t = \rho_t M_t u_t + \varepsilon_t$$

where $y_t' = [y_{1t},...,y_{Nt}], \alpha' = [\alpha_1,...,\alpha_N], x_t' = [x_{1t},...,x_{Nt}], u_t' = [u_{1t},...,u_{Nt}]$, and $t = 1,..., T$. Further forming vectors in t from the vectors of observations in i, the model in (11) becomes

(12) $$y = e' \otimes \mu + x\beta + e' \otimes \alpha + u$$
$$u = (\rho^* \otimes I_N)\, M^* u + \varepsilon,$$

where e is 1xT vector of ones, $y' = [y_1',..., y_T'], x' = [x_1',..., x_T'], u' = [u_1',..., u_T]$, and

[18] For example, in section 2.3, I set m_{ijt} to one if farms are located in the same village and to 0 otherwise.
[19] It has to be stressed that the presented panel data technique is for the case when one cannot rely on T-asymptotics. As $T \to \infty$, the specification of the weighting matrix becomes unnecessary and the approach presented here becomes inferior to approaches relying on T-asymtotics, such as seemingly unrelated regression or error component models.

$$M^* = \begin{bmatrix} M_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & M_T \end{bmatrix},\ \rho^* = \begin{bmatrix} \rho_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \rho_T \end{bmatrix}, \text{ and } E(\varepsilon\varepsilon') = \begin{bmatrix} \sigma_1^2 I_N & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \sigma_T^2 I_N \end{bmatrix}.$$

Note that the disturbance term ε in (12) is heteroskedastic over time. Defining $\Phi_t = (I_N - \rho_t M_t)/\sigma_t$, I can pre-multiply the model given by (11) to get

(13) $$y_t^* = \mu_t^* + x_t^* \beta + \alpha_t^* + \varepsilon^*{}_t,$$

where $y_t^* = \Phi_t y_t, \mu_t^* = \Phi_t \mu, x_t^* = \Phi_t x_t, \alpha_t^* = \Phi_t \alpha$ and $\varepsilon_t^* = \Phi_t u_t = \varepsilon_t / \sigma_t$. Pooling observations in *t* yields

(14) $$y^* = \mu^* + x^* \beta + \alpha^* + \varepsilon^* \ ,$$

where $\alpha^*{}' = [\alpha_1^*{}',...,\alpha_T^*{}']$ and $\mu^*{}' = [\mu_1^*{}',...,\mu_T^*{}']$ are *TN* dimensional vectors and the well-behaved disturbance term ε^* satisfies $E(\varepsilon^*) = 0$ and $E(\varepsilon^* \varepsilon^*{}') = I_{TN}$. The feasible estimation of (14) hinges on the knowledge of M_t, ρ_t, and σ_t^2. The GMM estimator developed for the cross-sectional case by Kelejian and Prucha could be directly applied to each cross section of N observations and parameters ρ_t, and σ_t^2 could be estimated using the predicted values of the spatially correlated disturbance term u_t from equation (11). I refer to this approach as *unrestricted estimation.* However, in this specification the unobserved heterogeneity term varies across cross-sectional units *and* the time series. As a result, this system consists of *TN* observations and *TN* + *k* parameters, so estimates are not identified.

It is not unreasonable to assume that in practical application the characteristics determining the spatial dependence across units tend to be stable over the short time span of the usual panel data—the case of our interest. Actual applications are thus likely to imply certain equality restrictions on the model in (14) that allow for the identification of the model's parameters. In the application used in this paper the definitions of spatial dependence are based on geographical characteristics (e.g. physical locations and distance between units), which are certainly constant over the short time period of the given data. This allows us to set equality restrictions on equation (14), leading to the identification of the model parameters.

2.1.1 Fully Restricted Specification

The specifications of spatial dependence reflect the distinct physical characteristics of the farming villages located in the west part of Java island (e.g. longitude, latitude, infrastructure, etc.), which can be deemed constant over the relatively short time period of six years.

Therefore the first and straightforward equality restriction is to assume that the components of weighting matrices, autoregressive and variance parameters, do not change over time, i.e. that $M_1 = \ldots = M_T = M$, $\rho_1 = \ldots = \rho_T = \rho$, and $\sigma_1^2 = \ldots = \sigma_T^2 = \sigma^2$, implying that $\Phi_1 = \ldots = \Phi_T = \Phi$. In this case $\alpha_t^* = \Phi\alpha$ and $\mu_t^* = \Phi\mu$ in equation (13) and $\alpha^* = e \otimes \Phi\alpha$ and $\mu_t^* = e \otimes \Phi\mu$ in equation (14). Moreover the disturbance term ε in equation (10) is no longer heteroskesdastic. Its variance matrix is $E(\varepsilon\varepsilon') = \sigma^2 I_{TN}$ and Φ does not have to be a function of σ for the sake of efficiency. Were ρ and σ^2 known, the RE would provide efficient parameter estimates of (14) provided the imposed restriction is true. The unbiased estimates of α could then be arrived at by transforming the estimated α^* with Φ. To compute the FGLS of equation (14), consistent estimates of ρ and σ^2 are required. An extended Kelejian and Prucha methodology will be used to estimate these parameters in section 3.

2.1.2 Partially Restricted Specification

To arrive at another reasonable parameter restriction, consider the empirical example. In our sample the 171 Indonesian rice farms, selected from six villages located in the production area of the Cinamuk River Basin in West Java, are observed over six numbered time periods from which 1, 3, and 5 represent "wet seasons", W, and periods 2, 4, and 6 "dry seasons", D.[20] Given this fact together with the specifics of the farm locations,[21] I impose the restriction[22] that $\rho_1 = \rho_3 = \rho_5 = \rho_W$ (wet) and $\rho_2 = \rho_4 = \rho_6 = \rho_D$ (dry), and similarly for M_t, σ_t^2, and Φ_t. Then $\alpha^* = [\Phi_W\ \Phi_D\ \Phi_W\ \Phi_D\ \Phi_W\ \Phi_D]' \otimes \alpha$ and $\mu^* = [\Phi_W\ \Phi_D\ \Phi_W\ \Phi_D\ \Phi_W\ \Phi_D]' \otimes \mu$ in equation (14) are TN dimensional column vectors, consisting of only $2N$ and 2 different parameters respectively. The system in (14) then consists of $2N + k + 2$ parameters and can effectively be treated as 2×171 = 342 farms observed over 6/2 = 3 periods. Although relative to the fully restricted case there will be an efficiency loss in the estimate of α^* (as the time series dimension is cut in half), the slope parameters will be efficient and consistent (in N) as all NT observations are utilized. As in the fully restricted case, consistent estimates of ρ_W, ρ_D, σ_W^2, and σ_D^2 are required to proceed with a feasible GLS estimation of model in (14).

[20] The specification reflects actual weather conditions on Java island.
[21] This partial restriction is motivated by the fact that during the rainy season many roads in the low country of Java island are impassable, and hence spillovers based on infrastructure are potentially diminished relative to the dry season.
[22] The same partially restricted specification is used in DH.

2.2 Feasible Estimation

This section develops a feasible estimator of the RE model with a spatially inter-correlated disturbance term that extends the cross-sectional model of Kelejian and Prucha to the panel data case. I propose the following four-step estimation procedure. First, estimate the standard RE model in (12) neglecting the spatial correlation in the disturbance term. Second, formulate the spatial weighting scheme *M,* test the statistical validity of the specified spatial dependence, and use a GMM technique to estimate the cross-sectional correlation in the disturbance term. Third, transform the model (12) by the estimate of Φ via a Cochrane-Orcutt-type transformation to account for the spatial correlation and re-estimate the transformed model using the RE estimator. Fourth, to arrive at consistent estimates of μ and α, transform the estimates of μ^* *and* α^* by the estimate of Φ.

2.2.1 Feasible Estimation of the Fully Restricted System

In what follows I assume that ρ_t, σ_t^2, and M_t are constant for all time periods and equal to ρ, σ^2, and *M*, respectively.[23] In this case I can obtain an asymptotically efficient estimate of β and consistent estimates of μ and α by the following four-step procedure:

Step 1. Estimate the RE model in (12) ignoring the correlation in *u* by a standard FGLS technique to obtain a sample predictor of the spatially correlated disturbance term *u*, denoted by $\tilde{u}$.

Step 2. Based on a geographic or economic proximity measure specify elements m_{ij} of *M*. Before proceeding with the spatial transformation of the RE model one should verify the validity of this spatial dependence assumption e.g. by using a *Moran I statistic* as follows. As above let $\tilde{u}_t$ denote the estimate of the standard RE residual in period *t*. The test statistics for the period *t* is defined as

$$\vartheta_t = [N/S]\{[\tilde{u}_t' M\tilde{u}_t]/\tilde{u}_t'\tilde{u}_t\},$$

[23] Note that if spatial correlation is constant over time and the time dimension of data is large enough, one may use estimation techniques relying on *T*-asymptotics (such as seemingly unrelated regression or error component model) to arrive at an empirically superior estimate of spatial correlation. The technique developed herein is for the case when time dimension of the data is small.

where N is the number of cross-sectional observations and S is the sum of all elements of M. The null hypothesis for the Moran I test is the "absence of spatial correlation". Cliff and Ord (1972) show that the standard normal distribution is the asymptotic distribution for the statistic when ϑ_t is transformed in the standard manner:

$$ZI_t = \{\vartheta_t - E[\vartheta_t]\}/ V[\vartheta_t]^{1/2},$$

where $E[\vartheta_t]$ represents the mean and $V[\vartheta_t]$ the variance of the statistic in period t derived under the above null hypothesis. These are expressed for the general case as

$E[\vartheta_t] = (N/S)tr(PM)/(N - k)$

$V[\vartheta_t] = (N/S)^2\{tr(PMPM') + tr(PM)^2 + [tr(PM)]^2/(N - k)(N - k + 2) - \{E[\vartheta_t]\}^2$.

$P = I_N - \hat{x}_t(\hat{x}_t'\hat{x}_t)^{-1}\hat{x}_t'$ is the projection matrix and $\hat{x}_t$ is the matrix of demeaned exogeneous variables from standard model in equation (1).

After the rejection of the *"no spatial dependence"* hypothesis follows the estimation of the correlation coefficient ρ (and σ^2) using a $\tilde{u}$ estimate. To do so I extend the GMM method of Kelejian and Prucha from the cross-sectional case to the case of panel data. Consider the following moment conditions implied by the spatially correlated model in (12) and Assumptions 1 and 2:

(15) $$E[(TN)^{-1}\varepsilon'\varepsilon] = \sigma^2,$$

$$E[(TN)^{-1}\bar{\varepsilon}'\bar{\varepsilon}] = \sigma^2(N)^{-1}tr(M'M),$$

$$E[(TN)^{-1}\bar{\varepsilon}'\varepsilon] = 0,$$

where $\bar{\varepsilon} = M^*\varepsilon$ and $\bar{\bar{\varepsilon}} = M^*\bar{\varepsilon}$.[24] Moreover let $\bar{u} = M^*u$ and $\bar{\bar{u}} = M^*\bar{u}$. Recalling that $\varepsilon = (I_{TN} - \rho M^*)\, u$, the above moment conditions (15) imply the following system of three equations and three unknowns[25]

(16) $$\Gamma\theta - \gamma = 0,$$

where

$$\Gamma = \begin{bmatrix} \frac{2}{TN}E(u'\bar{u}) & \frac{-1}{TN}E(\bar{u}'\bar{u}) & 1 \\ \frac{2}{TN}E(\bar{\bar{u}}'\bar{u}) & \frac{-1}{TN}E(\bar{\bar{u}}'\bar{\bar{u}}) & \frac{1}{N}tr(M'M) \\ \frac{1}{TN}E(u'\bar{\bar{u}} + \bar{u}'\bar{u}) & \frac{-1}{TN}E(\bar{u}'\bar{\bar{u}}) & 0 \end{bmatrix}, \quad \gamma = \begin{bmatrix} \frac{1}{TN}E(u'u) \\ \frac{1}{TN}E(\bar{u}'\bar{u}) \\ \frac{1}{TN}E(u'\bar{u}) \end{bmatrix}.$$

Let $\bar{\tilde{u}} = M^*\tilde{u}$, $\bar{\bar{\tilde{u}}} = M^*\bar{\tilde{u}}$, and $\theta = [\rho, \rho^2, \sigma^2]'$. The sample analogs based on $\tilde{u}$ then are

[24] Note that the second moment condition is based on M not M^* and therefore contains N^{-1} not $(TN)^{-1}$.

[25] The system of three equations follows from the individually squaring and summing of $u - \rho\bar{u} = \varepsilon$ and $\bar{u} - \rho\bar{\bar{u}} = \bar{\varepsilon}$, from multiplying the first with the second expression and summing, and finally by dividing all the elements by the sample size TN.

(17) $$G\theta - g = \nu(\rho,\sigma^2),$$

where ν can be viewed as regression residuals associated with sample observations (i.e. parameters θ will be found by minimizing the sum of squares of its elements) and

$$G = \begin{bmatrix} \frac{2}{TN}\tilde{u}'\tilde{\tilde{u}} & \frac{-1}{TN}\tilde{\tilde{u}}'\tilde{\tilde{u}} & 1 \\ \frac{2}{TN}\tilde{\tilde{\tilde{u}}}'\tilde{\tilde{u}} & \frac{-1}{TN}\tilde{\tilde{\tilde{u}}}'\tilde{\tilde{\tilde{u}}} & \frac{1}{N}tr(M'M) \\ \frac{1}{TN}\tilde{u}'\tilde{\tilde{\tilde{u}}}+\tilde{\tilde{u}}'\tilde{\tilde{u}} & \frac{-1}{TN}\tilde{\tilde{u}}'\tilde{\tilde{\tilde{u}}} & 0 \end{bmatrix},\ g = \begin{bmatrix} \frac{1}{TN}\tilde{u}'\tilde{u} \\ \frac{1}{TN}\tilde{\tilde{u}}'\tilde{\tilde{u}} \\ \frac{1}{TN}\tilde{u}'\tilde{\tilde{u}} \end{bmatrix}.$$

This system consists of three equations and three unknowns and corresponds directly to Kelejian and Prucha's results. If this system satisfies Assumptions 1 and 2 above and Assumptions 3, 4, and 5 of Kelejian and Prucha[26] then Theorem 1 of Kelejian and Prucha applies: the $\tilde{\rho}$ and $\tilde{\sigma}^2$ that solve the nonlinear optimization

(18) $$(\tilde{\rho},\tilde{\sigma}^2) = \arg\min_{r,s^2}[\nu(r,s^2)'\nu(r,s^2) : s^2 \geq 0]$$

are consistent estimates of ρ and σ^2 as N $\rightarrow \infty$. Following DH, I refer to these as the *fully restricted moment estimates*.[27]

Step 3. Use the fully restricted moment estimate $\tilde{\rho}$ from the transformation matrix $\tilde{\Phi} = (I_{TN} - \tilde{\rho}M^*)$. Apply a Cochrane-Orcutt-type transformation to (12) substituting $\tilde{\Phi}$ for Φ to form the RE model with the well-behaved disturbance term given by (14). Estimate the transformed equation (14) by the FGLS technique to arrive at consistent estimates of μ^*, α^*, and β.

Step 4. Consistent estimates of μ and α follow from the transformation of μ^* and α^* by $\tilde{\Phi}$.

2.2.2 Feasible Estimation of the Partially Restricted System

Following the discussion in 2.2, consider the following equality restrictions on the model in (12): $M_1 = \ldots = M_6 = M$, $\rho_1 = \rho_3 = \rho_5 = \rho_W$, $\rho_2 = \rho_4 = \rho_6 = \rho_D$, $\sigma_1^2 = \sigma_3^2 = \sigma_5^2 = \sigma_W^2$, $\sigma_2^2 = \sigma_4^2 = \sigma_6^2 = \sigma_D^2$. Let $\tilde{u}_W' = [\tilde{u}_1'\tilde{u}_3'\tilde{u}_5']$ and $\tilde{u}_D' = [\tilde{u}_2'\tilde{u}_4'\tilde{u}_6']$ denote the predictor of the disturbance term *u* in equation (12) for the wet and dry season, respectively. I

[26] Kelejian and Prucha's Assumptions 3, 4, and 5 are provided in the Appendix.
[27] The proofs of results are not presented as they represent straightforward extensions of Kelejian and Prucha's proofs.

can obtain a feasible estimate of the parameters in (12) analogously to the fully restricted case.

First define $\varepsilon_W' = [\varepsilon_1' \ \varepsilon_3' \ \varepsilon_5']$ and $\varepsilon_D' = [\varepsilon_2' \ \varepsilon_4' \ \varepsilon_6']$, and let $\bar{\tilde{u}}_j = M\tilde{u}_j$, $\bar{\bar{\tilde{u}}}_j = M\bar{\tilde{u}}_j$, $\bar{\varepsilon}_j = M\varepsilon_j$, and $\bar{\bar{\varepsilon}}_j = M\bar{\varepsilon}_j$, $j = W, D$. The estimates of ρ_j and σ_j^2 are then given by minimizing

(19) $$(\tilde{\rho}_j, \tilde{\sigma}_j^2) = \arg\min_{r,s^2}[\nu_j(r,s^2)'\nu_j(r,s^2) : s^2 \geq 0], \ j = W, D,$$

where

$$\nu_j(\rho_j, \sigma_j^2) = G_j[\rho_j \quad \rho_j^2 \quad \sigma_j^2]' - g_j, \ j = W, D.$$

Here G_j and g_j are special forms of the G and g of equation (16), each collecting the observations for wet or dry periods only, and $3N$ is substituted for TN. In this case the application of the Kelejian and Prucha methodology requires that each of the two "3N" subsystems satisfies Assumptions 1 through 5 so that Theorem 1 can be applied to arrive at consistent estimates of ρ_j and σ_j^2, $\tilde{\rho}_j$ and $\tilde{\sigma}_j^2$. Following DH let us refer to these estimates as the *partially restricted moment estimates*.[28]

As above, $\tilde{\rho}_j$ and $\tilde{\sigma}_j^2$ allow the formulation of $\tilde{\Phi}_j = (I_{TN} - \tilde{\rho}_j M^*)/\sqrt{\tilde{\sigma}_j^2}$ for wet and dry seasons. The $\tilde{\Phi}$ can be substituted for Φ in equation (14). FGLS applied to (14) again yields consistent estimates of μ^*, α^*, and β.

2.3 Application of Stochastic Production Frontier to Indonesian Rice Farms

In what follows I utilize the stochastic production frontier framework defined for the panel data case in Schmidt and Sickles (1984). In the stochastic production function equation

(20) $$y_{it} = \mu + x_{it}\beta + u_{it} - \alpha_i, \qquad i = 1,\dots,N, \quad t = 1,\dots,T,$$

y_{it} is firm production output, x_{it} is a vector of production inputs, and u_{it} stands for random production shocks. The firm-specific time-invariant term α_i represents technical inefficiency, and $\alpha_i \geq 0$ for all i. (For the case of a cost function the sign of the one-sided term would be

[28] In addition to this approach DH estimates the correlation coefficient for each cross-section allowing these parameters to vary in time and then use the average over time. DH refers to these estimates as *partially restricted average estimates*. The method used here may be more efficient as it utilizes all the available data to arrive at the correlation estimates.

positive.) Let $E(\alpha_i) = \bar{a} > 0$ and define

$$\breve{\mu} = \mu - \bar{a}, \qquad \breve{\alpha}_i = \alpha_i - \bar{a}.$$

Using this notation, the model in (20) becomes a panel data model with both error terms having zero mean:

(21) $$y_{it} = \breve{\mu} + x_{it}\beta + u_{it} - \breve{\alpha}_i.$$

Treated as an RE model with spatial dependence in the error term, u_{it} (21) can be estimated by the four-step FGLS method described above. To arrive at the estimates of technical efficiency, further define

$$\mu_i = \mu - \alpha_i = \breve{\mu} - \breve{\alpha}_i,$$

so that the model becomes

(22) $$y_{it} = \mu_i + x_{it}\beta + u_{it}.$$

Denote the N estimates of intercepts from (22) as $\hat{\mu}_1,\ldots,\hat{\mu}_N$, and define

$$\hat{\mu} = \max(\hat{\mu}_i) \text{ and}$$

$$\hat{\alpha}_i = \hat{\mu} - \hat{\mu}_i, \qquad i=1,2,\ \ldots,\ N.$$

A measure of technical efficiency for production unit i is calculated by plugging the estimates of α_i into the expression: $TE_i = \exp(\mu_i - \hat{\mu})$. Given the FGLS estimates $\hat{\beta}$ of the slope parameters and regression residuals $\hat{\varepsilon}_{it} = y_{ti} - x_{it}\hat{\beta}$ from the standard and spatially corrected RE model, the efficiency estimates $\hat{\alpha}_i$ can be recovered from the estimates of μ_i:

(23) $$\hat{\mu}_i = \frac{1}{T}\sum_T \varepsilon_{it}, \qquad i = 1,\ldots,N.$$

Provided that $\hat{\beta}$ is consistent (which it is, if $N\rightarrow\infty$), these estimates are consistent as $T\rightarrow\infty$. As an alternative, one can use the best linear unbiased predictor (BLUP) defined by Taub (1979) as follows:

$$\hat{\alpha}_i{}^* = \frac{-\hat{\sigma}_\alpha^2 \sum_t (y_{it} - \hat{\mu}^* - x_{it}\hat{\beta})}{(T\hat{\sigma}_\alpha^2 + \hat{\sigma}_u^2)},$$

and the resulting estimate of μ_i is

(24) $$\hat{\mu}_i = \hat{\mu}^* - \hat{\alpha}_i{}^*.$$

With $T\rightarrow\infty$ this estimator converges to (23). The differences in results were negligible. Both of the estimators of μ_i rely on a well-behaved disturbance term u_{it}. In case this component is polluted by the presence of cross-sectional correlation, both of the above predictors yield biased estimates of μ_i and hence lead to incorrect estimates of unit-specific technical efficiency.

As the illustration of the spatially corrected FGLS estimator of the RE model I use the balanced panel data of rice farms for which I estimate the stochastic production frontier function and unit-specific technical efficiency. The sample comes from the survey of the Indonesian Ministry of Agriculture consisting of 171 rice farms, located in six villages, that were observed over a period of three years, each consisting of a wet and a dry season (N=171, T=6). Villages are scattered around the Cimanuk River Basin in West Java. Weather conditions vary considerably among villages (highland areas receive much more precipitation and are characterized by lower average temperatures than lowlands) and so does the village infrastructure (while some villages are close to capital cities and have a fairly developed infrastructure other villages are virtually inaccessible during the rainy season.)[29] To capture the spatial dependence across production units, akin to DH I construct two different weighting schemes, M1 and M2, based on the geographical proximity of individual rice farms. The first weighting matrix represents the inverse of the distance between villages, while the construction of the second matrix employs the contiguity principle (i.e. m_{ij} is set to 1 if farms i and j are both from the same village and zero otherwise).

Using this balanced panel I first estimate a stochastic production frontier model, in which the dependent variable, y, is the natural logarithm of the production output (the amount of rice measured in kilograms), the production inputs in natural logarithms (seed, labor, land and fertilizer) with dummy variables (DP = 1 if pesticides were used and 0 otherwise, DV1 = 1 if a high-yield variety of rice was planted, DV2 = 1 if mixed varieties of rice were planted; DR1–5 are dummies for five out of the six villages) represent a vector of explanatory variables, x, and α_i embody farm-level technical inefficiency. This is a stochastic frontier specification based on a Cobb-Douglas production function that has been extensively applied to this data set previously (see Erwidodo 1990, Horrace and Schmidt 1996, 1999 and Lee and Schmidt 1993).[30]

Estimation results from the standard RE model (assuming no presence of spatial correlation) are reported in column IV of Table 2.1 (next to the results from the FE model of DH). Based on the Hausman specification test I can reject the correlation of individual effects with the other regressors in this data set and therefore I can proceed further with the RE

[29] The survey ended in 1983, so the infrastructure description may be different from the current state.
[30] A detailed discussion of the data is provided by Erwidodo (1990).

model.[31] In the next step I perform a test for spatial dependence in the error term for which I employ the *Moran I statistic*. The test results achieved for both weighting schemes *M1* and *M2* in each time period t = 1, ..., 6 are reported in Table 2.3. I reject the null hypothesis (of *"no spatial correlation"*) at the 95% significance level for each time period. The test results thus support the specification of the weighting schemes and hence I proceed with the estimation of the spatial correlation coefficients and the FGLS of the spatially corrected RE model.

2.3.1 Estimation Results

Tables 2.1 and 2.2 summarize the estimation results for the RE model and compare them with the outcome from DH. The Hausmann test for this data set indicates that both FE and RE provide consistent estimates, with RE being the more efficient estimator. In line with this result, as can be seen in Table 2.1, standard RE model estimates of the slope coefficients have smaller standard errors than standard FE. This difference narrows after we correct FE for spatial correlation using fully restricted moment estimates and it disappears completely in case we apply a partially restricted method (correction of the FE model for spatial correlation increases the efficiency of the slope coefficient estimates).

Note that unlike the typical FE model, in the RE framework we can also analyze the effect of time-invariant regressors (regional dummies DR1–5) on the production output. As in the case of the FE model (DH), we can see an improvement in the efficiency of the RE model slope estimates as their estimated standard errors decrease after correction for spatial correlation. The regional dummy variables and a dummy variable for the wet season represent a startling exception to this trend. The significance of their estimated coefficients markedly decreases in the spatially adjusted RE model. My interpretation of this result is that it demonstrates (and hence proves the assertion in DH) that the spatial dependence structure imposed on the second moment conditions has the potential to capture the regional effect modeled by these time-invariant dummy regressors. This result indicates an additional benefit of the proposed approach to the modeling of the spatial dependence. The loss of the significance of the seasonal dummy (DSS) coefficient (for both FE and RE) indicates that the proposed spatial dependence modeling technique likely also captures a seasonal effect. This is particularly evident for the case when we allow the cross-sectional correlation to vary between wet and dry seasons. In this case the DSS coefficient is not identified in the spatially

[31] The value of the χ - test statistic for Hausman test with eight degrees of freedom is 12.691, with a P-value = 0.1229.

corrected FE model and in the spatially corrected RE model its estimated coefficient is far from significant.

The sum of the estimated coefficients (excluding the regional dummy variables) for all FE and RE models exceeds unity and indicates that rice farms operate in the increasing-returns-to-scale region. Unlike for the FE model there is no general tendency for the estimated coefficients to increase as we move from the standard model to its spatial version. Consistently with the FE model, the size of the TSP (fertilizer) coefficient drops after correcting for spatial correlation, indicating a more complex relationship between this production input and production output. While the size of the slope coefficients estimated by the RE model tend to stay the same or decline as we move from the standard to the spatial model(s), the estimated size of the constant term increases as we move from the standard to the partially and to the fully restricted spatial model.

2.3.2 Technical Efficiency Estimates

Stochastic frontier analyses provide a means to assess firm-level technical inefficiency in terms of the relative magnitudes of individual units' inefficiency by using a rank or order statistic. Hence I estimate and rank the estimated technical efficiencies $TE_i = \exp(\mu_i - \hat{\mu})$ for each farm. I apply both estimators (23) and (24) to produce the estimates of the unit-specific terms. The comparison of the results from these two estimators revealed no differences in farm ranking and the density distributions of the estimated unit-specific regression coefficients were not significantly different.

I report efficiency levels and relative ranking results for the RE model in Table 2.4 and compare them with the FE results from DH shown in Table 2.5. Since there are 171 farms I only report (along the lines of DH) results for the four farms with the highest technical efficiency, the four farms with the median technical efficiency, and the four farms with the lowest technical efficiency. In Tables 2.4 and 2.5, column I shows the farm number, column II contains the ordered estimates of farm-level technical efficiency for the standard model, and column III provides the ordinal rankings for the standard model (numbered 1–171). To assess the impact of spatial dependence on the estimates of technical efficiency, columns IV and V report the ordinal rankings for the same 12 farms for the fully restricted spatial model under weighting schemes M1 and M2. While there are some changes across weighting schemes in the rank ordering of the most and least efficient farms, these are minor for both the RE and FE model. For instance, in the standard RE model, farm 163 had a technical efficiency rank of 3, but it has a rank of 4 under the weighting schemes. Notice that the ranking of the most and the least efficient farms (farm 164 and 143 for RE; and 164 and 45 for FE) is constant across the models. While the least efficient farm from RE has a higher rank in the FE model and vice versa, they are still ranked among the bottom four farms across the models. For both the FE and RE models the largest differences in ranking appear in the median farms. To further assess the difference in relative efficiency ranking, Tables 2.4 and 2.5 provide the *Spearman's rho* (r_S) statistic. *Spearman's rho* (r_S) is a standard measure of rank correlation between two rank statistics given by

$$r_s = 1 - \frac{6\sum_i \delta_i^2}{N^3 - N},$$

where δ_i is the difference in the rankings for the *i*th farm. In column III the rank correlation between the standard FE and standard RE model is reported. Column IV (V) gives the rank correlation between M1 (M2) and the standard model. The value of 0.96 in Table 2.4 for both M1 and M2 indicates that 96% of the rank statistics is preserved after I correct the RE model for spatial dependence. In comparison with FE (with a rank correlation for M1 or M2 of 0.8) the use of the spatial weighting specification in RE changes the relative rank of technical efficiency very little. The rank correlation between the standard FE and RE (column III) is 0.93, indicating that moving from the FE to the RE model preserves a high proportion of the ranking in this data set.

To further assess the impact of spatial dependence on the estimate of technical efficiency I constructed (along the lines of DH) the density distributions of the estimated unit-specific regression coefficients μ_i for the standard and spatially corrected RE models (M1, M2). These are shown in Figure 2.1 and can be compared with the density distributions of α_i (fixed effect coefficients) for the standard and spatially corrected FE model taken from DH. Density estimates are based on maximum likelihood, cross-validation bandwidth selection, and a standard Gaussian kernel. Fixing the maximum value of α_i (for FE) and μ_i (for RE) across models, some generalizations about the analyzed data set can be made. First, both the standard FE and RE models without correction for spatial autocorrelation in the errors tends to underestimate α_i and μ_i (overestimate technical inefficiency) in comparison with the spatially corrected models (M1 and M2). This is technical inefficiency in an absolute sense, since we are fixing a maximum value of α_i (for FE) and μ_i (for RE) across models at some unknown value. This is reflected in Figure 2.1 and 2.2 as the densities of the standard FE and RE models are shifted to the left of the densities for their spatial counterparts. This has implications for the predictions of the conditional mean output implied by equation (1): the standard FE and RE models (on average) yield lower predictions of productive output than the spatial models (all else equal). That is, for fixed technology and input factors, the spatial models impute more of the observed output to unobserved technical ability (α_i for FE and μ_i for RE) and less of it to luck (u_{it}) in this data set. Indonesian rice farms may be operating closer to the efficient frontier than previous studies suggest.

2.4 Conclusions

Interdependency across cross-sectional units is a frequent phenomenon in economic theory as well as in economic data sets and it presents a formidable estimation challenge for empirical econometricians. In the case when one can rely on a large T dimension in panel data, a seemingly unrelated model or error component model can, for example, be used to properly address the issue of cross-sectional dependence. However, panel data sets with a large T dimension are not very common when studying real world phenomena. In practice the researcher is typically faced with data sets that are observation-rich in the N dimension but poor in the T dimension. Hence the consistency of the estimates of these models often has to rely on $N\rightarrow\infty$. This works well for estimates of the model's slope coefficients. However, any second moment parameters that embody cross-sectional dependence cannot be consistently estimated in the sense that they will necessarily rely on $T\rightarrow\infty$. As a result, the unit-specific heterogeneity components of these models cannot be consistently estimated.

To achieve consistent estimates of the unit-specific constant parameters, DH propose for the typical panel data case (when the N dimension is large and the T dimension is small) a generalization of the GMM estimation approach (due to Kelejian and Prucha 1999) for the FE model that enables accounting for cross-sectional correlation in the disturbance term. This paper complements DH as it adopts the GMM framework to the RE model that accounts for correlation across the disturbance terms and, using the framework of the stochastic production frontier, it empirically demonstrates how this correlation can bias the estimates of unit-specific technical efficiency.

The cost of this approach is the necessity to impose additional structure on the model in the form of a spatial weighting scheme, which captures the spatial correlations across cross-sectional units. However, these structures can typically be supported by solid arguments for economic spill-overs stemming from the theoretical economic literature. Moreover, the research stream devoted to the study of spatial phenomena provides a means that let us explicitly account for these spill-over effects. Finally, these structures are testable and hence the spatial weighing schemes appear as a viable approach to create assumptions about the second moments of given data.

In the general version of the spatial RE model above I impose a dynamic spatial dependence structure on the error process of the RE model that allows the weighting scheme and cross-sectional correlation parameter to vary for each time period. In the transformed RE model the time-invariant, unit-specific parameters of the standard RE model thus also become fully dynamic. While in its general form the transformed model is not identified, this technique (after invoking additional restrictions on second moment dynamics) provides a valuable alternative means to model the dynamics in the heterogeneity parameters.[32] In practical applications the characteristics determining the spatial dependence across units tend to be stable over the short time span of the usual panel data (the case of our interest) and can provide a natural means to restrict model dynamics. In the application used in this study the definitions of spatial dependence are based on stable geographic characteristics, which allow us to set restrictions on the dynamics of the second moment that lead to the identification of the model parameters. The partially restricted version of the production frontier model capturing the differences between wet and dry seasons presented in this paper demonstrates the ability of this approach to model dynamics in the second moment with dynamic implications for the first moment parameters estimates.

Moreover, the estimation results of this study bear out that imposing spatial structure on the second moment conditions can indeed provide a means to indirectly incorporate time-invariant regressors into a panel data model (a hypothesis put forward by DH). This is important especially for the FE model where, for example, in this particular data set, the effect of the dummy variables for individual villages cannot be directly accounted for using an FE specification.[33]

[32] The usual approach to modeling the dynamics of the heterogeneity parameters is to impose a special structure on the conditional first moment of unit-specific coefficients. For examples of this technique see Kumbhakar (1990) and Lee and Schmidt (1993).

[33] This approach thus provides a viable alternative to the commonly employed techniques for incorporating time-invariant regressors into a fixed effect model, see Hausman and Taylor (1981).

2.5 References

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2.A Tables and Figures

Table 2.1 Rice Regressions, Weighting Scheme *M1* – Inverse of Distance

	Fixed Effect Model			Random Effect Model		
	Standard FE Model	Fully Restricted Moment	Partially Restricted Moment	Standard RE Model	Fully Restricted Moment	Partially Restricted Moment
	I	II	III	IV	V	VI
$\tilde{\rho}$	-	1,0557	-	-	0,7110	-
$\tilde{\rho}_W$	-	-	0,8218	-	-	0,7513
$\tilde{\rho}_D$	-	-	0,7476	-	-	0,6742
Seed	0,1208	0,0998	0,1248	0,1327	0,1154	0,1142
	(0,030)	(0,024)	(0,024)	(0,027)	(0,023)	(0,024)
Urea	0,0918	0,0901	0,1440	0,1131	0,1125	0,0950
	(0,021)	(0,017)	(0,015)	(0,018)	(0,016)	(0,017)
TSP	0,0892	0,0244	0,0307	0,0762	0,0367	0,0399
	(0,013)	(0,012)	(0,011)	(0,012)	(0,011)	(0,011)
Labor	0,2431	0,2379	0,2204	0,2231	0,2295	0,2421
	(0,032)	(0,028)	(0,026)	(0,029)	(0,026)	(0,026)
Land	0,4521	0,4931	0,5141	0,4770	0,5084	0,4845
	(0,035)	(0,030)	(0,027)	(0,031)	(0,028)	(0,028)
DP	0,0338	-0,0298	-0,0212	0,0141	-0,0166	-0,0057
	(0,032)	(0,028)	(0,025)	(0,029)	(0,026)	(0,026)
DV1	0,1788	0,0935	0,1320	0,1772	0,1256	0,1197
	(0,041)	(0,038)	(0,035)	(0,038)	(0,036)	(0,036)
DV2	0,1754	0,0952	0,0947	0,1446	0,0965	0,1033
	(0,057)	(0,048)	(0,048)	(0,052)	(0,046)	(0,045)
DSS	0,0533	0,1062		0,0492	0,0614	0,0444
	(0,022)	(0,302)		(0,021)	(0,060)	(0,055)
DR1				-0,0512	-0,0486	-0,2425
				(0,050)	(0,130)	(0,236)
DR2				-0,0442	-0,0646	-0,0520
				(0,059)	(0,119)	(0,210)
DR3				-0,0725	-0,0759	-0,0739
				(0,062)	(0,122)	(0,219)
DR4				0,0116	0,0336	0,0237
				(0,059)	(0,132)	(0,232)
DR5				0,0749	0,0690	0,0682
				(0,060)	(0,129)	(0,229)
Const				5,0635	5,2462	5,6190
				(0,194)	(0,203)	(0,266)
R-sq	0,9102	0,9271	0,9177	0,8840	0,8543	0,8590

Table 2.2 Rice Regressions, Weighting Scheme *M2* – Common Villages

	Fixed Effect Model			Random Effect Model		
	Standard FE Model	Fully Restricted Moment	Partially Restricted Moment	Standard RE Model	Fully Restricted Moment	Partially Restricted Moment
	I	II	III	IV	V	VI
$\widetilde{\rho}$	-	0,9882	-	-	0,6501	-
$\widetilde{\rho}_W$	-	-	0,7388	-	-	0,6750
$\widetilde{\rho}_D$	-	-	0,6999	-	-	0,6259
Seed	0,1208	0,0996	0,1248	0,1327	0,1330	0,1114
	(0,030)	(0,024)	(0,024)	(0,027)	(0,024)	(0,024)
Urea	0,0918	0,0901	0,1446	0,1131	0,1140	0,0958
	(0,021)	(0,017)	(0,015)	(0,018)	(0,016)	(0,016)
TSP	0,0892	0,0239	0,0301	0,0762	0,0347	0,0404
	(0,013)	(0,012)	(0,011)	(0,012)	(0,011)	(0,011)
Labor	0,2431	0,2376	0,2198	0,2231	0,2627	0,2379
	(0,032)	(0,028)	(0,026)	(0,029)	(0,026)	(0,027)
Land	0,4521	0,4934	0,5148	0,4770	0,4680	0,4876
	(0,035)	(0,030)	(0,027)	(0,031)	(0,028)	(0,028)
DP	0,0338	-0,0306	-0,0219	0,0141	-0,0045	-0,0104
	(0,032)	(0,028)	(0,025)	(0,029)	(0,026)	(0,026)
DV1	0,1788	0,0928	0,1326	0,1772	0,1306	0,1200
	(0,041)	(0,038)	(0,035)	(0,038)	(0,037)	(0,036)
DV2	0,1754	0,0947	0,0961	0,1446	0,0933	0,1045
	(0,057)	(0,048)	(0,049)	(0,052)	(0,047)	(0,045)
DSS	0,0533	0,0844		0,0492	0,0743	0,0156
	(0,022)	(1,424)		(0,021)	(0,074)	(0,041)
DR1				-0,0512	-0,3059	-0,0856
				(0,050)	(0,141)	(0,256)
DR2				-0,0442	-0,0496	-0,0467
				(0,059)	(0,127)	(0,238)
DR3				-0,0725	-0,0493	-0,0168
				(0,062)	(0,134)	(0,244)
DR4				0,0116	0,0955	-0,0039
				(0,059)	(0,138)	(0,262)
DR5				0,0749	-0,3257	-0,0029
				(0,060)	(0,142)	(0,240)
Const				5,0635	5,5189	5,3111
				(0,194)	(0,227)	(0,264)
R-sq	0,9102	0,9271	0,9174	0,8840	0,8500	0,8612

Table 2.3 Moran I Test of Weighting Scheme *M1* and *M2*

	Time period	1	2	3	4	5	6
M1	Z I	3,822	3,466	23,855	22,465	11,136	9,904
M2	Z I	3,413	2,814	22,488	19,957	10,714	9,013

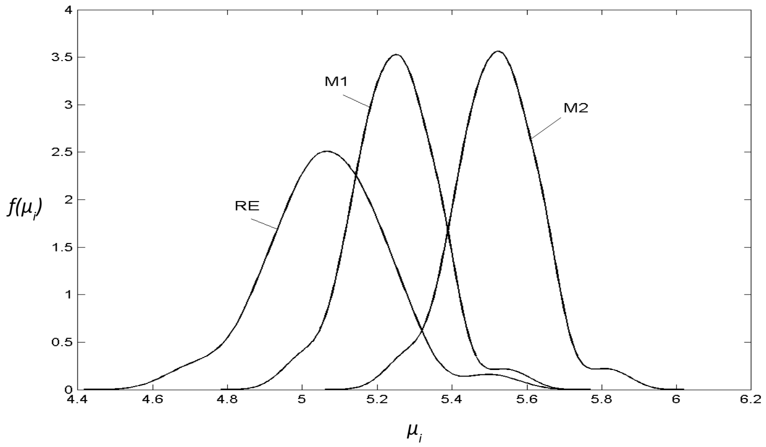
Table 2.4 Technical Efficiency Orders Statistics, RE Models

Standard RE Model			Spatial RE Models	
Farm No.	Standard RE Efficiency (%)	Standard RE model	Weight Scheme M1	Weight Scheme M2
I	II	III	IV	V
164	100,00%	1	1	1
118	98,91%	2	2	2
5	93,91%	3	4	4
163	91,66%	4	3	3
146	61,14%	84	71	71
119	61,26%	85	101	101
51	61,24%	86	127	127
38	61,23%	87	79	79
45	61,17%	168	168	168
142	41,98%	169	167	167
145	41,51%	170	169	169
143	39,56%	171	171	171
r_s		0,9287	0,9666	0,9669

Table 2.5 Technical Efficiency Orders Statistics, FE Models

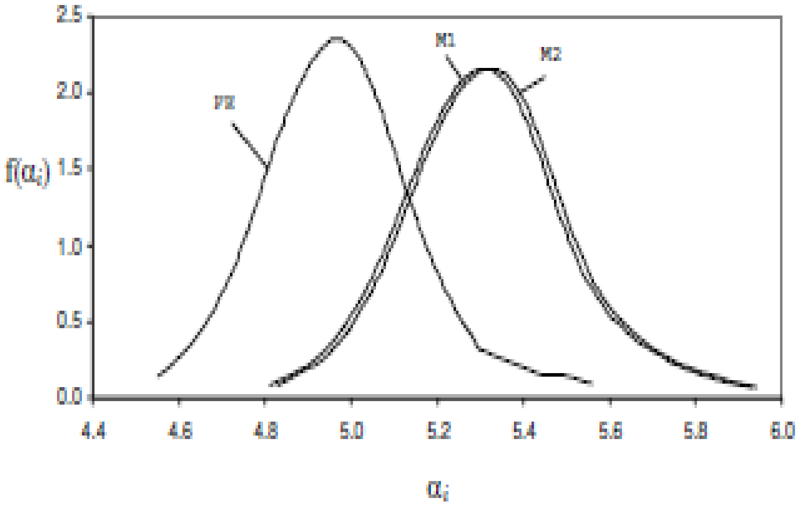
Standard FE Model			Spatial FE Models	
Farm No.	Standard FE Efficiency (%)	Standard FE model	Weight Scheme M1	Weight Scheme M2
I	II	III	IV	V
164	100%	1	1	1
118	93.23%	2	2	2
163	93.03%	3	3	3
152	89.93%	4	6	6
13	55.62%	84	106	106
166	55.47%	85	116	116
15	55.40%	86	62	62
40	55.35%	87	54	54
86	39.80%	168	165	165
143	38.37%	169	169	169
117	37.90%	170	168	168
45	36.55%	171	171	171
r_s		0,9287	0,8027	0,8095

Figure 2.1 Density estimates of μ_i for RE models



Notes:
RE = random effect with no spatial weighting
M1 = *M1* weighting scheme
M2 = *M2* weighting scheme

Figure 2.2 Density estimates of α_i for FE models



Notes:
FE = fixed effect with no spatial weighting
M1 = *M1* weighting scheme
M2 = *M2* weighting scheme

Appendix

The following are the assumptions 3, 4, and 5 from Kelejian and Prucha (1999). Let $P(\rho_t) = (I_N - \rho_t M_t)^{-1}$ with typical element $p_{ij}(\rho_t)$.

Assumption 3: (*i*) The sums $\Sigma_i|m_{ijt}|$ and $\Sigma_j|m_{ijt}|$ are bounded by, say, $c_m < \infty$ for all $1 \le i, j \le N$, $N \ge 1$. (*ii*) The sums $\Sigma_i|\, p_{ij}(\rho_t)|$ and $\Sigma_j|\, p_{ij}(\rho_t)|$ are bounded by, say, $c_p < \infty$ for all $1 \le i, j \le N$, $N \ge 1$, $|\rho_t| < 1$.

Assumption 4: Let $\tilde{u}_{it}$ be the i^{th} element of $\tilde{u}_t$. There exists finite dimensional random vectors d_{it} and Δ_t such that $|\tilde{u}_{it} - u_{it}| \le ||d_{it}||\ ||\Delta_t||$ with $N^{-1}\Sigma_i||d_{it}||^{2+\delta} = O_p(1)$ for some $\delta > 0$ and $N^{-1/2}\Sigma_i||\Delta_t|| = O_p(1)$.

Assumption 5: The smallest eigen value of $\Gamma_t'\Gamma_t$ is bounded away from zero.

Section 3

Too large or too small? Returns to scale in a retail network[34]

While increasing market penetration sets a limit on the future revenue growth of mobile network operators, high earnings expectations persist on the side of shareholders. These expectations fuel incentives for improvement of the productivity of operators resources. Therefore, to secure optimal allocation of operators resources, managers are often interested in supporting their decisions by the use of academic methodologies.

This technical efficiency and returns to scale study supports a wider scope project on the optimization of a mobile network operator retail network. We focus on identifyng returns to scale, because mobile network operators in environments with high rate of changes are constantly forced to grapple with competitors by creating economies of scale. Further, according to the managerial literature (e.g. Stabell and Fjeldstadt 1998) on chain value creation, key determinants of costs of the retail chain are capacity utilization and scale of operations.

When analyzing the productivity of their operations, retailers usually rely on aggregate measures like sale per unit of size or unit of labor. In operations research literature (e.g. Athanassopoulos and Ballantine 1995, Beamon 1999, Reynolds 2004), it is argued that the use of ratio analysis (common for managerial evaluation of performance) is not sufficient to properly assess performance of analyzed decision making units because the major limitation of ratio analysis is its univariate nature. To deal with this drawback, our study illustrates the application of a more comprehensive framework for assessing the performance of retail network units by comparing the results of non–parametric, parametric production frontier methods and ratio analysis. Parametric methods are capable of handling single output–multiple input technology, and require specification of production function form. The non–

[34] A previous version of this work was published as Brázdik F. and V. Druska (2005) "Too large or too small? Returns to scale in a retail network". CERGE EI Working Paper no. 273

parametric method used in this work, allows for multiple input–multiple output technologies and does not require specification of the production function form. Moreover, the non–parametric method provides information on returns to scale on an individual level. The use of parametric methods (Corrected Ordinary Least Squares – COLS and Stochastic Frontier Analysis – SFA) along with a non–parametric method (Data Envelopment Analysis – DEA) provides a means of assessing the robust- ness of estimated efficiency levels.

The retail units have only limited control over their outputs, which are mostly determined by the sales potential of the unit's location. Therefore, the suitable behavioral objective for retail network managers would be input minimization, rather than output maximization. To determine the efficiency of retail network units, production frontier analysis is assesses the actual levels of inputs with respect to the estimated optimal levels of inputs. This input oriented efficiency measure detects managerial failures to minimize use of inputs for a given level of output. Moreover, this approach provides an indication of the possible gains from exploiting technical and scale efficiencies.

The paper is organized as follows. Section 3.1 provides details of the retail technology and discusses the input–output specification used to model the production frontier. Section 3.2 lays out the theoretical DEA framework and specifies the linear programming problem used to evaluate the technical and scale efficiency of the retail units. It also provides a review of the parametric methodologies (COLS and SFA) used to test the hypothesis of constant returns to scale of e mployed technology. The section 3.3 reports a ratio analysis and production frontier results and section 3.4 concludes with a summary of policy implications.

3.1 Retail technology

A formulation of the DEA problem and specification of the production function form requires an understanding of the production process and the identification of production inputs and outputs, respectively. This section briefly describes the production function of the retail outlets and defines the corresponding measures of production inputs and outputs.

The key function of retail outlets of a mobile network operator is to acquire new subscriptions to services provided by the operator. The acquisition of a new customer involves the sale of a SIM card[35], a mobile phone (and accessories), and the selection of a price plan suiting the subscriber's preferences. While revenues reported by a store are derived from the sale of equipment (and prepaid credit vouchers), these are not considered to be the key output of the unit.[36] Rather, definition of key unit's output is motivated by the store's primary acquisition function. Once a customer signs up for one of the price plans (or purchases a prepaid card) he starts using the services provided by the network operator and he generates revenues that are collected either via monthly bill or via sale of prepaid vouchers. In fact this revenue is also used to cover the costs of running retail stores thus revenues collected from acquired subscribers represent the key (financial) benefit derived from the operation of retail stores. Therefore, the number of SIMs sold and monthly revenue generated by the number of subscribers acquired in a store were chosen as an output measures.

In total we use three different production output specifications. For two one–output models we use either the number of SIM cards sold to customers or the revenue generated by these customers, respectively. In the case of the two–output models (DEA only) we describe outputs together as SIM cards sold and revenue generated by customers using the sold SIM cards.

The mobile phone operator at the time of this study was running a retail network with over forty outlets across the country and was considering closing or relocating some of the existing units as well as opening additional units at new locations. Some key questions management faces when setting up a retail outlet are: What should the size of the sales area be? How many sales people are required to achieve the sales potential in that unit's sales area? These two factors appear to be the key determinants of the

[35]SIM, a Subscriber Identity Module, is a card commonly used in a GSM phone. The card holds a microchip that stores information and encrypts voice and data transmissions. The SIM card also stores data that identifies the caller to the network service provider.

[36]The biggest revenue item (from total revenue reported by a store), the handset revenue, would be a misleading output indicator as phones sold together with an activation of a tariff plan are sold at subsidized prices and the margins (difference between retail and wholesale price) are hence in these cases negative. The negative handset margin is thus treated by the operator and the industry as the component of Subscriber Acquisition Cost (SAC) rather than as revenue derived from providing services.

outlet performance in addition to the regional and location aspects.[37] At the same time, the costs of these two production factors represent 98% of the units' total operating costs.[38] Therefore the number of sales representatives (employees) and the sales area of the store were identified as the production inputs. This specif cation of inputs is used for all models.

The stores' sizes and their locations were determined by the retail chain manager at the outset of the retail network roll out on the basis of the initial sales potential estimates of individual regions. The size of each location allows for a variable number of employees, up to the point of its capacity given by the sale area. The total headcount per individual store is decided jointly by the local store manger, regional manager, and central retail network manager. Based on the observed traffic pattern, the store manager is able to adjust the capacity of the sale force by drawing on part–time staff. Store opening hours are set so as to ref ect the sales potential of the location; i.e. stores in shopping malls are open whole weekends, while stores in other locations are e.g. open for limited hours on Saturdays. As one employee represents 40 working hours per week, the measure of the number of employees captures the differences in opening hours across stores.

Following the literature on retail productivity other criteria relevant for retail productivity assessment including employees' personal characteristics such as training level and motivation (Bush, Bush, Ortinau and Hair 1990, Lusch and Serpkenci 1990); wage rate (Bucklin 1978) and such as attitudes (MecKenzie, Podsakoff and Fetter 1993); behavioral outcomes such as service quality (Parasuraman, Zeithaml and Berry 1994) and assortment differences (Grewal, Levy, Mehrotra and Sharma 1999) were considered. However, as explained below there is no evidence that these factors differ across the studied retail stores and employees; thus these characteristics are not helpful in explaining productivity variation across stores in our sample.

[37]The regional aspects and sales potential of individual sales areas were assessed in a separate study and are beyond the scope of this paper.

[38]Utilities and site maintenance represent remaining units' costs.

At the time of this study, the strong market growth and increasing penetration forced operators to allocate their resources primarily in the subscriber's acquisition activities. Acquisition is thus seen as the primary goal of the retail stores, and the work effort of the sales persons is stimulated by the incentives that ref ect this goal. Because incentives and overall reward scheme of the sales staff is centrally designed and is homogeneous across stores, the variable pay of the sales staff is driven by the number and the value of the SIMs sold. The value of the SIM is measured by the (expected) revenue the sold SIM will generate, which in turn is proxied by the price tariff the customer with the given SIM subscribes to.

The time spent with an individual customer purchasing a specific service does not differ from location to location, but does differ from customer to customer due to the differences in tariffs being sold. The key discriminator here is the payment type associated with the tariff sold. The prepaid tariffs (also known as 'pay as you go' tariffs) take a shorter time to sell than do postpaid tariffs. Consumers with prepaid tariffs, as the attribute indicates, pay for the services in advance, i.e. before the services are consumed. On the other hand the postpaid tariffs allow the subscribers to consume services before being is charged for them. Postpaid fees are collected from these subscribers via invoices that are sent to their home addresses. This system requires that the customer register personal data with the operator (and often also f nalization of the time specific service contract) and thus requires longer sale time. Postpaid tariffs bind subscribers to paying a regular monthly fees, thus guarantying operator a recurring monthly revenues which, as a consequence, increases the value of the postpaid customer above the prepaid one (it is common knowledge that mobile subscribers on prepaid tariffs generate on average a lower monthly revenue than do postpaid subscribers). This fact is captured by the incentive scheme, and the longer time spent acquiring postpaid subscriber is rewarded by the higher number of points earned by the sales staff for the acquisition of this type of customer.

However, the busiest locations may produce a perverse effect despite this feature of the incentive scheme. The extreme workload (high number of incoming shoppers) at these busy locations can cause the sales staff to go after quantity rather than quality of customers, as total points earned is under these conditions thus maximized. As a consequence the busy locations can sell a high volume of SIMs, though their average value may be lower compared to the SIMs activated by other less busy stores whose locations show a smaller sales potential. We capture this specific feature of the retail sale technology in one of the DEA models by using two output measures: number of SIMs sold and total revenue generated by the store.

Even though some time of the sales staff is spent on serving the current subscribers, this is not seen as the primary goal of the store and does not take a big share of staff time; hence it is not ref ected by the incentive scheme. Based on staff and store manager experience and on a comparison across stores the share of time spent on service activities is minor and approximately the same in all stores (and varies across stores by about +/- 5%).

All sales persons must pass the same sales skills training so that the high quality of service is homogeneous and preserved across stores.[39] Quality of service is measured regularly, and quality assurance test results from the time period of this study indicate that there is very small variation in key quality indicators across stores and sales persons. The consistently high homogeneous quality of sales staff thus limits possible explanations for the varying degree of eff ciency across stores. Homogeneity of the other service attributes (e.g. types of offered handsets) is ensured by the systems and technology supporting the seamless operation of all stores (e.g. central inventory system).

While the output potential (number of shoppers coming to the store) is assumed to be given and determined by the regional characteristics and intensity of the operator's nationwide advertising campaigns the realization of that output potential is (assumed to be) the function of the input mix (quality of inputs is assumed homogeneous) and it is

[39]Quality of both sale and product skills are ensured. Moreover, each introduction of new products or services by the operator is coupled with the appropriate product/service specif c knowledge training of the sales staff.

the optimum input mix we seek to identify for each location.

3.1.1 Data description

Table 3.1 in the Appendix, summarizes the descriptive statistics of inputs and outputs used to specify the production mix. Stores in our sample were observed over a 3–month of steady mobile market growth. The 42 stores represent the total number of the operator's stores, and the number of SIMs reported are those sold over the period of one quarter of the year. The revenues reported were accounted for in the third month of the quarter following the sale (acquisition) period. The number of employees are cumulative over the 3–month period and thus ref ect total man-hour capacity devoted to sales activity in the time period we study. All variables are measured with minimum error as the information systems in place provide automatic data collection and their accuracy has been tested over time prior to the study period.

Figure 3.1 (see Appendix), presents the matrix of scatter plots for each pair of input and output variables. The high positive correlation between number of employees and both measures of output is clearly visible. However, the relationship is less clear when sales area is considered. While output seems to increase with number of employees it does not seem to measure up with increasing sale area size. This observation clearly corresponds to the actual flexibility of input adjustments. The number of employees can be adjusted fairly flexibly to reflect the varying demand conditions of each store. However the size of the sale area is fixed over the time of the lease and once the location and the size of the store is decided (based on the initial estimates of the location sales potential) there is limited possibility to adjust this production input.

This study (efficiency and input slack estimates) thus gauges the quality of two separate skills: the ability of the central manager to determine optimum size–location and total retail chain headcount mix, and the ability of individual retail store managers to optimally schedule employees. The identified labor input slacks can thus be directly translated into a reduction of employees working hours. The size of sale areas are, however, more diff cult to adjust in practice. It is up to the manager to consider the possibilities for store size adjustment or alternative solution in addressing low incoming customer traffic if slacks are small. When slacks are substantial, store relocation may be inevitable to evade further wasteful consumption of resources.

3.2 Methodology

There is no single widely accepted approach to assessing retail store productivity (Donthu and Yoo 1998). However, in recent retailing studies (e.g. Donthu and Yoo 1998, Reynolds 2004) production frontier methodologies are most frequently used. In this paper we use both parametric and non–parametric production frontier methods to create a comprehensive framework for technical eff ciency analysis.

In their original paper on production frontier methods, Marschak and Andrews (1944) sketched out the terms "technical" and "economic eff ciency" (paragraph 1 and 11) of a firm. Later, Farrell (1957) defined the efficiency of a decision making unit (DMU), which consists of two components: technical efficiency (TE) and allocative efficiency (AE). In Farrell's efficiency concept, overall efficiency (OE) is defined as multiplicative combination of technical and allocative eff ciency, so that OE=TE*AE. Allocative effi - ciency expresses the extent to which an analyzed DMU uses its inputs in proportions which minimize the costs of production, assuming that the unit is already fully technically efficient. Technical efficiency measures the extent to which inputs are converted to outputs relative to the best practice frontier.

An important feature of the retailing network in a competitive market is that the retail stores must meet the demand for their services but are not able to choose the level of output they will offer due to competition limitations. Further in our case, we have to abstract from the role of prices of mobile network services because the retail units do not affect these prices; all retail units are offering the same services for the same prices. Regional and location aspects (demographic, social, economic and competitive supply patterns of towns/districts) that determine the sales level of mobile network services by retail units are considered to be beyond the unit's control. These facts provide a rationale for considering the levels of outputs as given by local characteristics and by the general operator's sales strategy. Given the exogeneity of the output levels, the retail network maximizes prof t simply by minimizing the levels of used inputs for producing a given level of output.

We also abstract from the role of prices in allocation of resources because rent and size of the retail unit is predominantly affected by location characteristics and availability of space for rent. Due to these restrictions, we do not assume that the unit location and size is chosen with respect to allocative eff ciency. Therefore, rather than considering the cost or profit efficiency level of retail units, we focus our analysis on the technical efficiency of units, especially input efficiency.

The following sections review input–oriented DEA methodology and two parametric methods (COLS and SFA) used to search for the production frontier that allows us to evaluate technical and scale efficiency of retail stores.

3.2.1 Non–parametric frontier approach: DEA

Using Farrell's (1957) concept, Charnes, Cooper and Rhodes (1978) introduced the first DEA model to evaluate technical efficiency in a multi input–output environment. Since then, the CCR model and its modifications have become a widely used tool for operations analysis and production frontier search in many sectors including schools, hospitals or banks. The DEA models measure the efficiency of DMUs by identifying of the best performing units. These best performing units are then used to construct the "best practice frontier" through a piecewise linear envelopment of observed data. Therefore the efficiency scores estimated by the DEA models are relative measures of efficiency within the sample of the analyzed DMUs.

The DEA approach assumes that each of the considered DMUs is described by a vector x_j, $x_j = (x_{1j}, \ldots, x_{mj})^T$ of m inputs amounts that are used to produce s outputs in amounts described by vector y_j, $y_j = (y_{1j}, \ldots, y_{sj})^T$. To aggregate these vectors into matrices of inputs and outputs the following notation will be used:

matrix of inputs vectors	$X_{m\times n} = (x_1, \ldots, x_n)$
i^{th} row of "input" matrix X	${}_ix = (x_{i1}, \ldots, x_{in})$, $i = 1, \ldots, m$
matrix of outputs vectors	$Y_{r\times n} = (y_1, \ldots, y_n)$
r^{th} row of "output" matrix Y	${}_ry = (y_{r1}, \ldots, y_{rn})$, $r = 1, \ldots, s$.

When using the DEA approach to search for the "best practice frontier", it is assumed that the set of production mixes used by DMUs is described by the production possibility set

$$T = \{(x,y)|\text{ using inputs } x \geq 0 \text{ outputs } y \geq 0 \text{ are produced}\}$$

and it is assumed that the technology set has the following properties:

1. Convexity: If $(x_j, y_j) \in T$, $j = 1, \ldots, n$ and $\lambda \in \mathbb{R}^n_+$, $\Rightarrow (X\lambda, Y\lambda) \in T$.

2. Ineff ciency property: If $(x,y) \in T$ and $\bar{x} \geq x$, then $(\bar{x}, y) \in T$.
   If $(x,y) \in T$ and $\bar{y} \leq y$ then $(x, \bar{y}) \in T$.

3. Minimum extrapolation: T is the intersection of all sets satisfying the conve- xity and ineff ciency property and such that each of the observed production mix $(x_j, y_j) \in T, j = 1, \ldots, n$.

4. No free lunch: $(0,y) \notin T$, for $y > 0$.

The production frontier ("best–practice frontier") determine the minimum level of inputs that needed to produce a given level of outputs. The input–oriented DEA models examine the levels of inputs needed for the production of DMU's output mix, and the efficiency measure indicates whether the DMU under consideration uses the minimum necessary level of inputs. The simplest input–oriented eff ciency score of the DMU$_j$ measures the maximum proportional reduction of inputs that allow production of the same output mix as the examined DMU_j. However, due to its simplicity, this measure fails to uniquely identify efficient units. Therefore, we will use a more complex eff - ciency measure, described later.

We use the input–oriented DEA models to evaluate the input–oriented pure technical efficiency and scale efficiency score of the DMU_j, for $j = 1, \ldots, n$ which are characterized by the following general linear programming problem:

$$\begin{aligned}
\min_{\lambda_j, \theta_j, e_j, s_j} \quad & \theta_j - \varepsilon(\mathbf{1}^T e_j + \mathbf{1}^T s_j) && (1) \\
s.t. \quad & \theta_j x_{ij} - {}_i x \lambda_j - e_{ij} = 0, \quad i = 1, \ldots, m; \\
& {}_r y \lambda_j - s_{rj} = y_{rj}, \quad r = 1, \ldots, s; \\
& \varphi(\mathbf{1}^T \lambda_j) = \varphi; \\
& \lambda_j, e_j, s_j \geq 0,
\end{aligned}$$

where $\lambda_j \in \mathbb{R}^n_+$ (intensity variable), $\theta_j \in \mathbb{R}_+$ (proportional input reduction), $e_j \in \mathbb{R}^m_+$ (non–proportional input excess), $s_j \in \mathbb{R}^s_+$ (output slack), φ takes the value 0 for the CCR input–oriented model introduced by Charnes et al. (1978) and 1 for the BCC input–oriented model by Banker, Charnes and Cooper (1984).[40] Formulation of these models as in Problem 1 is referred to as envelopment form because the optimal solution $(\theta^*_j, \lambda^*_j, e^*_j, s^*_j) \in \mathbb{R}^{1+n+m+s}_+$ identif es the projection of DMU_j on the envelopment surface ("best practice frontier") in the direction of proportional inputs reduction.

θ^*_j expresses the minimal, proportionally reduced, levels of inputs for the DMU_j while keeping the outputs at the same levels, in order to improve the technical efficiency of this unit.[41] Low value of θ^*_j indicates excessive use of all inputs in the production mix. This property of θ^*_j provides a rationale for using θ^*_j as an efficiency measure. Due to the fact that this measure ignores non–proportional reduction of inputs, additional conditions on input excess and output slack are needed to identify efficient units.

The constant ε in Problem 1 is a non–Archimedean infinitesimal that allows the problems of the search for maximal input reduction and the search for frontier projection to be condensed into a single optimization problem. In a chapter on computa-

[40] Here, $\mathbb{R}_+$ denotes the set of positive real numbers and $\mathbf{1}$ is a column vector of ones.

[41] $1 - \theta^*_j$ expresses the maximal proportional input reduction of input levels.

tional aspects of the DEA, Charnes, Cooper, Lewin and Seiford (1994) argue that the value of ε should be determined by an analyzed sample. Therefore, we choose $\varepsilon = 10^{-6}\min_{j=1,\dots,n} 1/(\sum_{i=1,\dots,m} x_{ij})$ for our analysis. This choice of ε means that proportional input reduction effectively preempts the optimization that involves non–proportional slacks e_j and s_j. The DEA models as stated in Problem 1 are solved by implementing of the primal–dual interior point method designed by Mehrotra (1992).[42]

The elements e_{ij} express the input excess for each of the inputs, and the vector $e_j \in \mathbb{R}^m_+$ is formed to express the non–proportional input excess for DMU_j's input–output mix. At least one element of e^*_j (part of the optimal solution) should be zero, otherwise there exists the possibility of proportional input reduction. Similarly for outputs, the outputs slacks s_{rj} form a vector of outputs slacks $s_j \in \mathbb{R}^s_+$ and s^*_j expresses the possible output augmentation. Further, individual slack analysis can help retail managers allocate resources more effectively and improve performance.

The eff ciency of DMU$_j$ is evaluated using the optimal solution $(\lambda^*_j, \theta^*_j, e^*_j, s^*_j)$ of Problem 1 under the assumption of the selected returns to scale (RTS) type. In the DEA literature (Charnes et al. 1994, Banker et al. 1984, Sueyoshi 1997) the efficiency of DMU$_j$ is evaluated according to the following theorem:

Theorem 1. *Efficient DMU$_j$: The DMU$_j$ is DEA efficient if both of the following conditions are satisfied: 1) $\theta^*_j = 1$; and 2) All values of slacks are zero: $\mathbf{1}^T e^*_j = 0$ and $\mathbf{1}^T s^*_j = 0$. Otherwise DMU$_j$ is inefficient.*

If DMU$_j$ is identified as inefficient according to Theorem 1, the optimal values of slacks e^*_j, s^*_j and the optimal value θ^*_j identify the sources and levels of present ineffi - ciency.[43] To take into account the presence of proportional and non–proportional slacks we use the efficiency measure introduced by Tone (1993) to evaluate efficiency in a comprehensive yet simplified fashion by defining the following input oriented efficiency

[42] To analyze sensitivity of solutions with respect to the choice of ε, we used $\varepsilon = 0$ to calculate new efficiency scores. No significant changes in efficiency scores were recorded.

[43] According to Theorem 1 $\theta^*_j = 1$ is just a necessary condition but not sufficient to evaluate the DMU$_j$ as efficient. Consider the case of DMU$_6$ in Figure3.2, where $\theta^*_6(VRS) = 1$ but because $e^*_j > 0$ this unit is dominated in efficiency by DMU$_4$.

measure:

$$\chi_j = \left(\theta_j^* - \frac{\mathbf{1}^T e_j^*}{\mathbf{1}^T x_j}\right) \frac{\mathbf{1}^T y_j}{\mathbf{1}^T Y \lambda_j^*.} \tag{2}$$

This efficiency measure has the following properties:

1. $\chi_j = 1 \Leftrightarrow$ The DMU_j is efficient

2. $\chi_j = \theta_j^* \Leftrightarrow$ The values of all slacks are zero

3. $0 \leq \chi_j \leq 1$

4. χ_j is a units invariant measure

5. χ_j is monotonically increasing in inputs and outputs of DMU_j

6. χ_j is decreasing in the relative values of the slacks.

The first efficiency measure property guarantees that this efficiency measure uniquely identifies the efficient DMU while the fourth, fifth and sixth property provide a rationale for use of this measure to create efficiency an ranking for analyzed DMUs.

Further, after identifying efficient DMUs and identifying of projections onto the production frontier (potential efficiency improvements), we use the DEA methodology to examine scale efficiencies of DMUs. Scale efficiency measures the extent to which DMU_j can take advantage of returns to scale by a change in its size towards the optimal scale, characterized by the constant returns to scale property.[44]

Charnes et al. (1994) and Sueyoshi (1997) provide an extensive summary of the relationships between various DEA model specif cations and estimated types of efficiency (technical, pure technical, scale, cost, and allocative). Following the outlined methodology, we estimate the pure technical efficiency of DMU_j using the BCC model (setting $\varphi = 1$ in Problem 1), and the technical and scale efficiency by utilizing the CCR model

[44]As defined in the glossary of the Steering Committee for the Review of Commonwealth/State Service Provision (1997).

(setting $\varphi = 0$ in Problem 1). Because of the multiplicative nature of technical efficiency, the scale efficiency of production frontier elements can be evaluated by breaking down the technical efficiency score into a scale of operations component and a "pure" technical efficiency score.

We also estimate the model under the assumption of non–increasing returns to scale (NIRS). Such a model is derived from Problem 1 by replacing the intensity variable constraint with inequality $\mathbf{1}^T\lambda_j \leq 1$. For these input oriented DEA models the following property for optimal solution θ_j^* holds:

$$0 < \theta_j^*(CRS) \leq \theta_j^*(NIRS) \leq \theta_j^*(VRS) \leq 1. \tag{3}$$

The amount of scale inefficiency can be imagined as the distance between the constant returns to scale (CRS) and the variable returns to scale (VRS) frontier, because this distance is determined by the scale efficiency component of technical efficiency. Figure 3.2 illustrates the comparison of the CRS frontier (CCR model) with the VRS frontier (BCC model). In Figure 3.2, the VRS frontier and production possibility set are divided according to the RTS type of frontier elements into subsets of increasing returns to scale (IRS, dashed line), scale efficient (bold solid line) and decreasing returns to scale (DRS, dot–dash line). Elements from the horizontally shaded area can be projected in the input reduction direction onto the part of the frontier with the IRS property, and from vertically shaded area onto the part of the frontier with the DRS property. Projections of elements from the cross–shaded area belong to the scale efficient part of the production frontier. Figure 3.2 thus illustrates the situation in which DMU_2 and DMU_3 are scale efficient units where as the rest of the analyzed units are scale inefficient due to the presence of either IRS or DRS.

As illustrated above, the analyzed DMU_j can be identified as operating in the region of the production possibility set with a) increasing RTS, b) decreasing RTS or c) scale efficiency property. In addition to quantifying the scale efficiency level we also determine for each unit the type of RTS region it operates in. The literature on RTS identification presents various approaches to extracting qualitative information on returns to scale of the frontier. Löthgren and Tambour (1996) summarize four different (but equivalent) approaches to estimating of returns to scale using a primal or dual solution to the DEA models stated in Problem 1. To determine the RTS type for an individual retail unit we employ the scale efficiency method. The concept of the scale efficiency method introduced by Färe and Grosskopf (1985) is in detail discussed by Zhu and Shen (1995) and can be given as the following theorem from Löthgren and Tambour (1996):

Theorem 2. *Scale efficiency method: For the specific DMU_j let define scale efficiency measure $SE_j = \frac{\theta_j^*(CRS)}{\theta_j^*(VRS)}$. Then $SE_j = 1$ iff the DMU_j exhibits CRS (the DMU_j is scale efficient); if $SE_j < 1$, then $\frac{\theta_j^*(CRS)}{\theta_j^*(NIRS)} = 1$ iff the DMU_j exhibits IRS; if $SE_j < 1$, then $\frac{\theta_j^*(CRS)}{\theta_j^*(NIRS)} < 1$ iff the DMU_j exhibits DRS.*

An important part of the DEA analysis is the test for sensitivity of results to the selection of inputs and outputs for productivity mix description and returns to scale assumption. For this purpose efficiency scores are calculated using alternative model specifications. Besides the descriptive statistics comparison, the sensitivity of efficiency rankings constructed according to the efficiency measure of Tone (1993) is examined by use of the Spearman rank correlation coefficient. The rank correlation coefficient and statistics by Spearman (1904) test the hypothesis of rank independence. Spearman's (1904) correlation coefficient is commonly used to compare rankings in statistical studies.[45]

3.2.2 Parametric frontier approach: COLS

Further, to assess the robustness of efficiency and RTS estimates, we complement the DEA methodology with results of the parametric production frontier approach using corrected ordinary least squares (COLS) and stochastic frontier (SFA) methodology.

[45] For implementation details of the Spearman rank correlation coeff cient and statistics, see the manual by Stata Corporation (2003). For properties of the correlation coeff cient see Kendall (1955).

Winsten (1957), in his discussion of Farrell's (1957) paper, suggested a parametric alternative to DEA that is based on a two–stage estimation of production frontier, known as corrected ordinary least squares.

The deterministic production frontier of Cobb–Douglas production technology with variable returns to scale is represented by the following model:

$$\ln(y_j) \quad = \quad \beta_0 + \sum_{k=1}^{m} \ln(x_{jk})\beta_j - u_j, \tag{4}$$

where the inputs $x_{jk} \in \mathbb{R}_+$ are used to produce single output $y_j \in \mathbb{R}_+$ for $j = 1,\ldots,n$ and inefficiency component $u_j \geq 0$ is assumed to be iid distributed with non-negative mean and constant variance. Equation 4 is in the first stage estimated by OLS which produces the unbiased and consistent slope parameters estimates of the frontier model. In this stage, a consistent but biased estimate of intercept parameter β_0 is obtained.

In the second stage, the unbiased intercept is estimated consistently by:

$$\hat{\beta}_0^* = \hat{\beta}_0 + \max_j\{\hat{u}_j\}, \tag{5}$$

and the OLS residuals are corrected according to:

$$-\hat{u}_j^* = \hat{u}_j - \max_j\{\hat{u}_j\}.$$

This correction makes all residuals non–negative and at least one of them is zero. The corrected residuals $-\hat{u}_j^*$ are used to provide consistent estimates of technical efficiency. The technical efficiency of producer i is calculated according to the following function:

$$TE(COLS)_j = \exp(-\hat{u}_j^*).$$

Using the technical efficiency score $TE(COLS)_j$ we construct an efficiency ranking and compare this ranking to the DEA efficiency ranking to evaluate the sensitivity of our results.

3.2.3 Parametric frontier approach: SFA

The COLS approach, summarized in the previous section, does not take into account the possible effect of random shocks that may also cause variation in output. Therefore, we also employ a method of stochastic frontier which accounts for random shocks and technical inefficiency effect on variation in output. Stochastic frontier analysis method (SFA) was ficst introduced by Meeusen and van den Broeck (1977) and Aigner, Lovell and Schmidt (1977). Since then, SFA has become a very popular tool that competes with the DEA approach in estimating production frontiers.

Assuming that the production function is linear in logarithms, the stochastic production frontier can be defined as follows:

$$\ln(y_j) = \beta_0 + \sum_{i=1}^{m} \beta_i \ln(x_{ij}) + v_j - u_j,$$

where u_j represents the non–negative technical inefficiency component and v_j is the symmetric two–sided random shock component.

Various specifications of the inefficiency term distribution lead to distinct frontier models. The most popular are half–normal (u_j *iid* $N^+(0, \sigma_u^2)$), truncated normal (u_j is *iid* with $N(\mu, \sigma_u^2)$ truncated at 0) and exponential model (u_j *iid* exponentially distributed). We estimate these models by maximum likelihood method.

Based on Kumbhakar and Lovell's (2000) remark on the low sensitivity of efficiency ranking to inefficiency distributional assumptions (conf rmed in our sample), we estimate stochastic production frontier under the assumption of a half–normal distribution of the inefficiency term. Under this assumption the likelihood–ratio test is used to test for the presence of an inefficiency component in the model. This test compares values of likelihoods functions under $H_0 : \sigma_u^2 = 0$ against alternative hypothesis $H_1 : \sigma_u^2 > 0$. For more details on one–sided likelihood–ratio test statistics see Gutierrez, Carter and

Drukker (2001).

In addition assessing the robustness of efficiency estimates, we use the deterministic (COLS) and stochastic (SFA) frontier methods to validate the conclusions of data envelopment analysis on returns to scale. To complete the parametric frontier analysis, we test the null hypothesis that retail units employ the CRS technology by use of the Wald test (Kmenta 1990) to test if the sum of production factor elasticities sums to 1 (testing restriction $\sum_{k=1}^{m} \beta_k = 1$).

3.3 Results

In this section a summary of performance analysis results obtained by the DEA and parametric production frontier methodology is presented. Ratio analysis is discussed as well. For all technology specifications, inputs are described by the size of the sale area and the number of employees. As mentioned in previous sections, we use three different specif cations of outputs to describe the retail technology of mobile network services.[46]

To assess technical efficiency we constructed four output/input ratios and ranked units according to these four productivity indicators. Results of this analysis are summarized in Table 3.2. For each ratio four top, middle and bottom performing units are shown. There are two ratios per each output. We report the Spearman rank correlation coefficients for each output ratio and a low consistency of rankings is observed across ratio measures (0.2811 and 0.4661) with respect to choice of output. These results illustrate the problem with the univariate nature of ratio analysis. As mentioned in introduction, ratio analysis is of capturing to capture the multivariate nature of the considered retailing technology. These results thus provide a rationale for use of more complex measures of productivity.

Using three different specifications of outputs we f rst compute efficiency scores by the input–oriented DEA models. Two models use a single output specification: 1) number of SIMs sold (referred to as SIMs model), and 2) revenue generated by acquired

[46] Descriptive statistics of the models' inputs and outputs are summarized in Table3. 1.

subscribers (referred to as Revenue model). The third model uses two outputs: number of SIMs sold and revenue generated by customers owning these SIMs (referred to as SIMs&Revenue model). To estimate the production frontier under the SFA and COLS approach, we use only the first and second one–output model specif cation.

Table 3.3 summarizes the descriptive statistics for the efficiency scores (χ_j), technical and scale efficiency (θ_{CCR}), pure technical efficiency (θ_{BCC}) and scale efficiency for all three specifications of the DEA models. Table 3.3 shows that on average the retail units are from 88 to 94 % scale efficient and that average pure technical efficiency (θ) ranges from 52 to 58% depending on model specification. This result suggests that pure technical inefficiency is the main source of technical inefficiency and that DMUs on average are operating close to full scale efficiency.

We used the efficiency score χ_j to create performance rankings of DMUs. We assessed the sensitivity of results with respect to model specif cation by calculating Spearman rank correlation coeff cients and by testing statistics for signif cance of rank correlation coefficients. To assess the extent of correlation we used Mortimer's (2002) review of studies on parametric and non–parametric methods comparison as a benchmark. Also, from analysis of the relation between sample size and extent of rank correlation in studies reviewed by Mortimer (2002), we were not able to identify any bias in extent of correlation with respect to sample size. In general, for values of Spearman rank correlation of 0.9 to 1, the correlation is considered very strong; for values between 0.7 and 0.9, correlation is considered strong; and for values between 0.5 and 0.7, correlation is considered moderate.

The robustness of results was also tested by recalculating scores after the units identified as efficient and outlier units were removed from the full sample (42 observations). The sample of 39 observations used in the test focused on the consistency of DEA results, where we removed 3 units from the full sample that were identified as efficient by the 2 outputs–2 inputs models. Further, we removed the four busiest sales locations according to a complementary study on store location aspects, where these four stores acquired a high number of customers (high number of SIMs acquired) when compared to the rest of the stores. The decision to remove these four units is also supported by hat matrix analysis, when these units are characterized by high values of leverage (0.1496–0.3536, the small sample cutoff $3p/n$ suggested by Vellman and Welsch (1981), is 0.1428) and Cook's distance (0.2276–0.7083, while the cutoff $4/(n-k-1)$, suggested by Belsey, Kuh and Welsch (1980), is 0.1025).

Table 3.4 shows a summary of Spearman rank correlation coefficients for all considered models under either a CRS or VRS assumption. All estimated correlation coefficients are significant, and the high values of correlation coefficients values suggest a low sensitivity of results to inputs and outputs specification among the considered models. Spearman's ranking correlation coefficient for these DEA technical efficiency rankings ranges from 0.7543 to 0.9728 in the case of SIMs models, $0.7115 - 0.9882$ in the Revenue model and $0.7816 - 0.9882$ for the SIMs&Revenue model.

The differences between θ_{CCR} and θ_{BCC} suggest that after eliminating pure technical inefficiency (projecting observations onto the VRS frontier) inputs can be reduced on average by additional 4–8 % without affecting level of outputs when CRS technology is used. Table 3. 5 presents a detailed view on computed input reduction parameters and scale eff ciency scores. The results presented in Table 3.5 are consistent with ordering condition 3 on theta. Table 3.5 also shows the levels of scale efficiency in the SE–score column.

The differences between means of efficiency scores (χ_j) and means of proportional reduction parameters (θ_j) within the model specif cation arise from the presence of non–proportional slacks when searching for efficiency improvements. As mentioned in the methodology section, this information can be used to predict additional performance improvement. To specify sources of this improvement, we present summary statistics of non–proportional slacks in Table 3.6. From this summary it follows that adjustments in store size can be the most important driver of possible performance improvements.

These results, as presented above imply that retail network costs can be reduced more if retail units were to emulate the "best practice" rather than trying to adjust for scale eff -

ciency of operations. These results are in line with other retail studies (Athanassopoulos and Ballantine 1995, Donthu and Yoo 1998) which conclude that reductions in cost arising from the realization of economies of scale are less important than the costs saved when a retailing network is undertakes improvements in technical efficiency.

A summary of RTS identification by the DEA method is presented in Table 3.7. These results reveal that a majority of retail units appears to be operating in the decreasing returns to scale region of the production possibility set when the input reduction objective is imposed. This conclusion indicates the presence of economies of scale in the operation of individual retail stores.

The RTS identification results obtained by the DEA are supported by tests of the hypothesis that retail units employ CRS retail technology. To do this, we employed a parametric (COLS and SFA) technique. Two one–output models of production frontier were estimated. As mentioned above, we used the same outputs as in the one–output DEA models; both inputs and outputs are expressed in terms of logarithms (output was defined in terms of log of revenues and log of number of SIMs acquired, respectively). Tables 3.8 and 3.9 present the results of this estimation, while Table 3.10 summarizes the technical efficiency scores estimated by the parametric method. The estimated input elasticities do not significantly vary across the parametric methods. However, based on a log–likelihood ratio test we have to accept the hypothesis of no presence of an ineffi - ciency component for the SFA model. As Schmidt and Sickles (1984) note, estimation of the cross–sectional stochastic frontier model is based on strong distributional assumptions of statistical noise and inefficiency components of the error term. Therefore, we attribute the failure to identify the asymmetric inefficiency component under the SFA model in this sample to the negligible skewness of inefficiency distribution.

Figure 3.3 illustrates the distribution of technical efficiency and compares parametric (COLS) and non–parametric (DEA) methods. The density estimates reveal patterns typical for efficiency scores from the DEA approach, where the peak close to unity is due to eff cient DMUs that are used to identify the production possibility frontier. Therefore, it is more appropriate to compare rankings of technical efficiency scores that result from parametric and non–parametric methods. These results are presented in Table 3.11.

Table 3.12 reports the results of testing for the prevailing type of returns to scale among retail units. Based on the Wald test, the null hypothesis of CRS was rejected. The sum of the elasticities of output with respect to inputs generated an estimated scale elasticity. The values of elasticities sums were less than one, which supports the DEA results regarding the presence of decreasing returns to scale.

The fact that the DRS property prevails for the majority of retail stores suggests that further expansion of units' operation size would lead to a less than proportional increase in outputs and that units may became even less effective. In this case, contraction in the size of retail store operations may increase their efficiency levels at the cost of a less than proportional reduction of achieved output levels.

To test the sensitivity of results to the presence of the busiest units, Table 3.13 and Table 3.14 present results of the parametric approach when a reduced sample of 38 observations is used. With this reduced sample, based on the log–likelihood ratio test we were able to reject the hypothesis of no presence of the technical inefficiency component in the SFA model.

Table 3.15 summarizes the results of technical efficiency estimation using reduced sample under the DEA, COLS and SFA approach. Estimated average technical efficiency range from 0.5253 to 0.6986. We report only the results for technical efficiency scores under the VRS assumption because according to test results presented in Table 3.16 we were able to reject the hypothesis of CRS technology. Figure 3.4 shows estimates of distributions of technical eff ciency scores. Again, to assess the consistency of technical efficiency results, we use ranking correlations. The ranking correlations for reduced sample ranks are summarized in Table 3.11. Both extent of rank correlation (Spearman rank correlation ranges from 0.67 to 0.98) and the fact that we were able to reject the hypothesis of rank independence for all cases (at 1% signif cance level) lead us to conclude that the technical efficiency rankings are robust with respect to the choice of the frontier approach.

Further, when comparing results for the reduced sample with results from the full sample presented in Tables 3.3 and 3.10, we observe an upward shift in the average and minimal technical efficiency score. This fact is consistent with the nature of technical efficiency estimates used in this analysis. To analyze the sensitivity of rankings, we compare rankings for the full and reduced sample by use of ranking correlation coeff - cients. The ranking correlation coefficients for both samples are summarized in Table 18. These results allow us to consider our results as robust with respect to the choice of the productivity frontier approach and to model specif cation across the full and reduced samples.

Assessing Figure 3.5 and Figure 3.6, where type of returns to scale is drawn against the level of input, we conclude that the overall DRS property of DMUs results from the DRS property of size of store. This conclusion supports the ratio analysis results indicating that sales per square meter decrease with size of the store. However, we cannot make straightforward conclusion about the prevailing returns to scale property of labor input characterized by number of employees.

Finally, the relation between levels of inputs used for the production and efficiency scores is illustrated in Figures 3.7 and 3.8. We observe that the highest efficiency scores are attained by units with a relatively small size (Figure 3.7). Similarly, as in the case of the relation of the RTS type to number of employees, we cannot draw a straightforward conclusion about the effect the number of employees has on the efficiency of retail store 3.8.

Figures 3.9 and 3.10 illustrate the ratio analysis of performance evaluation and links it to the DEA approach to efficiency evaluation. These figures show DMUs in the space of the output per unit of input: SIMs sold per employee and SIMs per square meter in Figure 3.9, and revenue generated per employee and revenue generated per square meter in Figure 3.10. To link the ratio analysis to performance evaluation with the DEA results, we labelled data points in Figure 3.9 and Figure 3.10 with the DEA efficiency scores. Figure 3.9

and Figure 3.10 reveal that units considered highly performing according to ratio analysis also show high DEA efficiency scores. Efficiency growth in this space is indicated by the thick arrow on the bottom of each graph. Productivity ratios (output per unit of input) are highly correlated with efficiency scores.

In general, the higher the output per square meter and/or output per employee, the higher the efficiency score. Lines that divide the graph area into four parts are meant to separate the units that are in the upper quartile of maximal level of output per unit of input. Based on ratio analysis, units in the outer regions are considered units with high efficiency. This reasoning is confirmed by the high DEA eff ciency score of these outer units.

3.4 Conclusion

The main objective of this paper was to demonstrate the use of a complex framework for performance analysis by estimating technical and scale efficiency of individual stores in the retail network of a mobile network operator. This framework allowed us to overcome the shortcomings arising from the univariate nature of ratio analysis. The goal of this technical efficiency study was to facilitate the optimization of resource allocation so that retail units consume their inputs in an optimal mix to provide retail services.

We quantified the possible efficiency improvements of inefficient retail stores, using an equi–proportional input reduction approach. Efficiency improvements can be driven by improvements in better operational practices (improvements in headcount planning, adjusting for variation in sales over time) or in adjustments of the production mix (size of sales area per employee). We also identified the returns to scale type of analyzed units as additional information for the adjustment of production mix size. The information on the RTS type allows managers to decide on future expansion or on contraction in size of operations after the unit adopts the practice of efficient production mix. Analysis of non–proportional slacks indicated that reduction in store size can yield substantial improvements in technical (and likely also in cost efficiency) for some of the larger stores. Robustness of the DEA results is supported by results from parametric frontier methods (COLS and SFA).

In the performance evaluation of the retail units, managers can put too much emphasis on measuring output levels alone. However, there may be well managed stores whose performance can be negatively affected by exogenous factors, or and poorly managed store helped by favorable environmental factors. Low levels of outputs, therefore, are not sufficient to judge on retail unit efficiency; to make decisions about units we hence assessed the efficiency level and output levels together as two key performance measures.

To assess these performance measures together, we placed the retail units in an outputs–efficiency space as displayed in Figure 3.11 and Figure 3.12. In both f igures the space is divided into four quadrants at mean values. Retail units located in the "Stars" quadrant are those with the highest efficiency scores and which are probably operating in a favorable economic environment. Opposite to this is the “Cows” quadrant, which contains low efficiency units probably located in an unfavorable environment (area with low sales potential). The “Dogs” quadrant contains efficiently operated retail units with lower levels of outputs, likely due to being located in low sales potential area. The “Sleepers” quadrant contains retail units that show high levels of outputs, but this has more to do with favorable environmental conditions than with good management. Units located in the “Sleepers” quadrant are candidates for efficiency improvements that may lead to even greater profits. Managers should attempt to increase the efficiency of stores in these locations.

From a comparison of the extent of scale inefficiency and pure technical inefficiency, we conclude that managers should implement the operational practices of the technically efficient units rather than exploit economies of scale to improve retail network performance. This conclusion is supported by the results of RTS identif cation, which indicates that the majority of retail units is operating in the DRS region.

We argue that the DEA and parametric production frontier study permits a more thorough and complex understanding of the assessment of retail store performance than does simple ratio analysis. Finally, the use of a "best practice" approach to predicting the operations of retail stores allows managers to set realistic and individual goals based on store–specific profile.

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3.A Tables and Figures

Variables	Obs.	Mean	STD. Dev.	Min	Max
Inputs					
Size (m^2)	42	38.95	28.43	5	122
Employees	42	17.73	4.10	3	27
Outputs					
SIM cards sold	42	2507.16	1428.56	793	6869
Sold SIM cards revenue	42	31.8033	19.6507	8.5255	92.6930

Table 3.1: Input–output summary

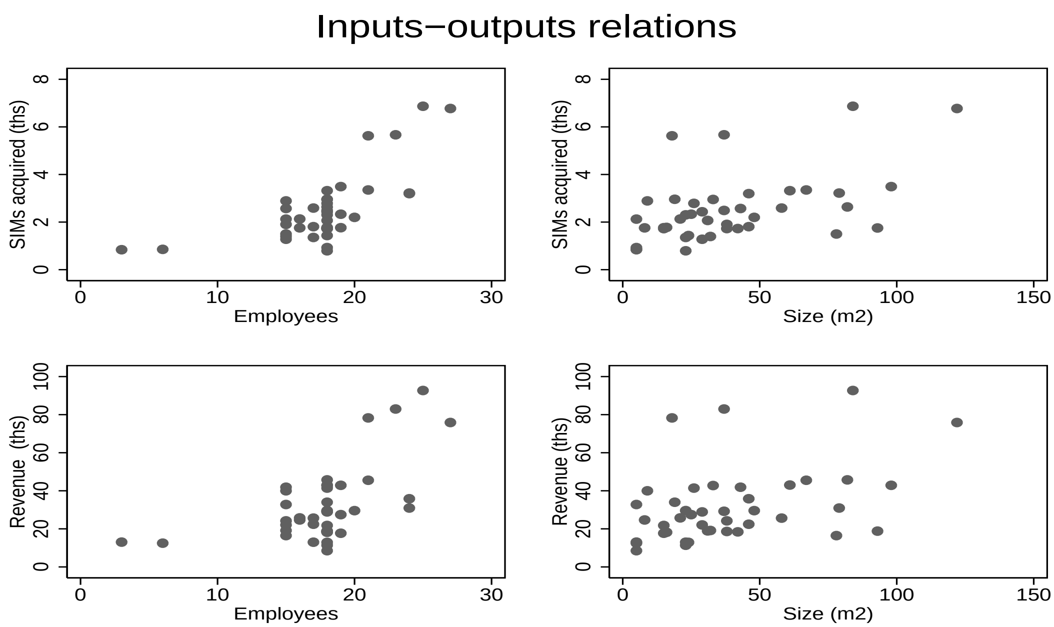

Figure 3.1: Inputs–outputs relations

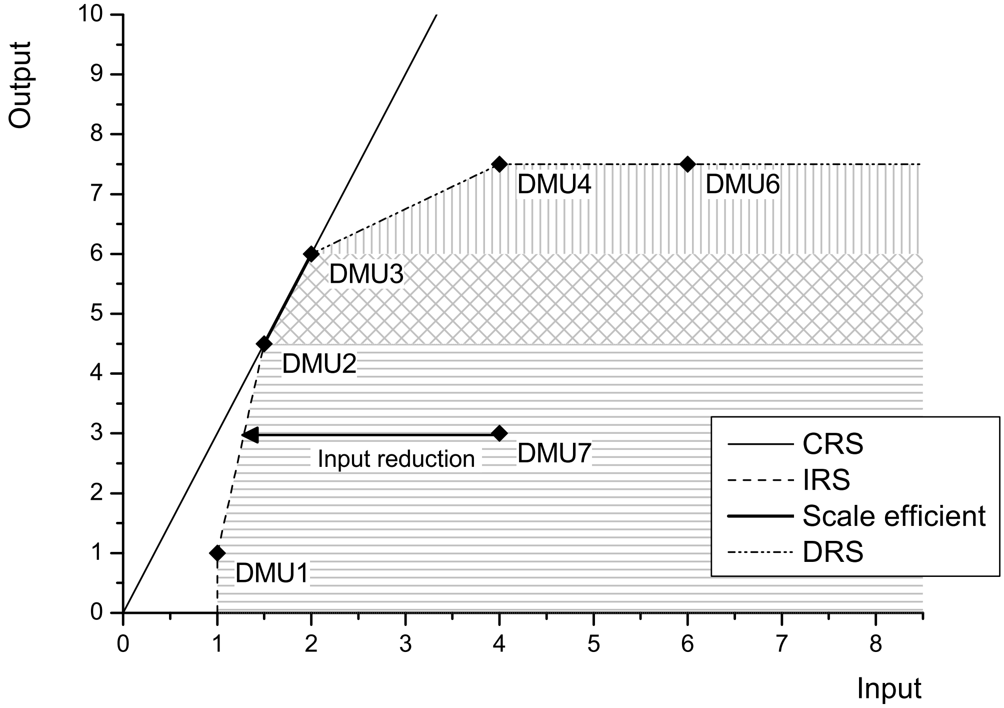

Figure 3.2: Production frontier and returns to scale types

Rank	SIMs per m2	SIMs per empl.	Revenue per m2	Revenue per empl.
Top 4	DMU39	DMU33	DMU39	DMU33
	DMU4	DMU31	**DMU4**	**DMU4**
	DMU6	**DMU4**	DMU6	DMU31
	DMU12	DMU24	DMU12	DMU30
Middle 4	DMU31	DMU13	**DMU13**	DMU14
	DMU22	DMU29	DMU5	DMU17
	DMU25	**DMU22**	DMU25	**DMU13**
	DMU9	DMU17	DMU22	DMU12
Bottom 4	**DMU2**	DMU1	DMU27	DMU1
	DMU32	DMU37	DMU29	DMU37
	DMU8	DMU35	DMU8	DMU2
	DMU15	**DMU2**	DMU15	DMU35
Rank correlation	0.28		0.47	

Table 3.2: Ratio scores ranking

Model		Obs.	Mean	Std.Dev.	Min	Max
SIMs	χ–CCR	42	.4474	.2235	.1510	1
	θ–CCR	42	.5341	.2196	.1606	1
	χ–BCC	42	.5394	.2312	.1962	1
	θ–BCC	42	.5736	.2380	.2174	1
	Scale eff ciency	42	.9433	.1175	.4372	1
Revenue						
	χ–CCR	42	.3855	.2410	.1039	1
	θ–CCR	42	.4555	.2358	.1560	1
	χ–BCC	42	.4161	.2976	.0966	1
	θ–BCC	42	.5253	.2675	.2049	1
	Scale eff ciency	42	.8830	.1261	.2600	1
SIMs&Revenue						
	χ–CCR	42	.3856	.2410	.1040	1
	θ–CCR	42	.5377	.2193	.1606	1
	χ–BCC	42	.4799	.2728	.1550	1
	θ–BCC	42	.5841	.2414	.2174	1
	Scale eff ciency	42	.9333	.1152	.4372	1

Table 3.3: Efficiency scores (χ) and θ summary statistics

Model		SIMs		Revenue		SIMs&Revenue	
		CCR	BCC	CCR	BCC	CCR	BCC
SIMs	CCR	1.0000					
	BCC	0.8802	1.0000				
Revenue	CCR	0.9115	0.8436	1.0000			
	BCC	0.8815	0.7979	0.9092	1.0000		
SIMs&Revenue	CCR	0.9109	0.8439	0.9994	0.9089	1.0000	
	BCC	0.8457	0.9436	0.8350	0.8246	0.8353	1.0000

Note: All coeff cients are signif cantly different from 0 at 1% level.

Table 3.4: Spearman rank correlation

Model	SIMs				Revenue				SIMs&Revenue			
Unit	θ_{CCR}	θ_{BCC}	SE–score	χ_j–BCC	θ_{CCR}	θ_{BCC}	SE–score	χ_j–BCC	θ_{CCR}	θ_{BCC}	SE–score	χ_j–BCC
DMU1	0.5540	0.6020	0.9738	0.6020	0.5462	0.6215	0.8788	0.4334	0.5862	0.6215	0.9432	0.4334
DMU2	0.3660	0.4387	0.8341	0.4387	0.2622	0.3333	0.7867	0.3301	0.3659	0.4387	0.8341	0.2965
DMU3	0.6060	0.6082	0.9967	0.6082	0.4855	0.4871	0.9967	0.4831	0.6062	0.6082	0.9967	0.4937
DMU4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
DMU5	0.3620	0.4402	0.8230	0.4402	0.3285	0.3867	0.8495	0.3867	0.3623	0.4402	0.8230	0.3747
DMU6	0.2900	0.2907	0.9959	0.2907	0.1742	0.2049	0.8502	0.2027	0.2895	0.2907	0.9959	0.1776
DMU7	0.4670	0.4712	0.9917	0.4712	0.4073	0.4200	0.9698	0.3898	0.4673	0.4712	0.9917	0.4236
DMU8	0.3800	1.0000	0.4372	0.6220	0.2600	1.0000	0.2600	1.0000	0.4372	1.0000	0.4372	1.0000
DMU9	0.5380	1.0000	0.5381	0.9462	0.5648	1.0000	0.5648	0.9588	0.5648	1.0000	0.5648	0.9785
DMU10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
DMU11	0.2510	0.5309	0.9862	0.3810	0.5842	0.6669	0.8760	0.2342	0.5842	0.6669	0.8760	0.2341
DMU12	0.6010	1.0000	0.9825	1.0000	0.8523	1.0000	0.8523	1.0000	0.9825	1.0000	0.9825	1.0000
DMU13	0.8840	0.9117	0.9695	0.9117	0.8377	1.0000	0.8377	1.0000	0.8839	1.0000	0.8839	1.0000
DMU14	0.2980	0.4870	0.9850	0.4649	0.2964	0.3304	0.8971	0.1596	0.4797	0.4870	0.9850	0.3236
DMU15	0.4020	0.6695	0.9851	0.6293	0.5494	0.6252	0.8788	0.2811	0.6595	0.6695	0.9851	0.5909
DMU16	0.2850	0.6682	0.9847	0.4481	0.5196	0.5913	0.8787	0.1887	0.6580	0.6682	0.9847	0.3998
DMU17	0.3630	0.5791	0.9850	0.5688	0.4986	0.5691	0.8761	0.2651	0.5704	0.5791	0.9850	0.5606
DMU18	0.4390	0.5168	0.9783	0.5168	0.3818	0.4237	0.9011	0.2890	0.5056	0.5168	0.9783	0.4300
DMU19	0.4340	0.9131	0.9825	0.7209	0.6460	0.7520	0.8590	0.2526	0.8971	0.9131	0.9825	0.5985
DMU20	0.3310	0.5520	0.9866	0.5002	0.3469	0.3807	0.9112	0.1869	0.5446	0.5520	0.9866	0.3552
DMU21	0.4360	0.4895	0.9741	0.4895	0.3435	0.3868	0.8881	0.2678	0.4768	0.4895	0.9741	0.3924
DMU22	0.5600	0.5704	0.9825	0.5704	0.5523	0.6007	0.9194	0.4903	0.5604	0.6007	0.9329	0.4902
DMU23	0.2950	0.3510	0.9846	0.3510	0.2398	0.2540	0.9441	0.1904	0.3456	0.3510	0.9846	0.2629
DMU24	0.3670	0.4316	0.8501	0.4316	0.2686	0.3237	0.8298	0.3237	0.3669	0.4316	0.8501	0.3000
DMU25	0.4470	0.4516	0.9889	0.4516	0.3540	0.3670	0.9646	0.3364	0.4466	0.4516	0.9889	0.3728
DMU26	0.4850	0.4893	0.9914	0.4893	0.3962	0.4064	0.9749	0.3824	0.4851	0.4893	0.9914	0.4125
DMU27	0.2720	0.3844	0.9893	0.3828	0.3035	0.3287	0.9233	0.1964	0.3803	0.3844	0.9893	0.3325
DMU28	0.1510	0.3519	0.9895	0.2100	0.2403	0.2549	0.9427	0.0966	0.3482	0.3519	0.9895	0.1574
DMU29	0.3420	0.4598	0.9865	0.4598	0.3710	0.4049	0.9163	0.2503	0.4536	0.4598	0.9865	0.4099
DMU30	0.4860	0.4965	0.9778	0.4965	0.3746	0.4093	0.9152	0.3329	0.4855	0.4965	0.9778	0.4152
DMU31	0.6070	0.6823	0.8895	0.6823	0.5813	0.6370	0.9126	0.6370	0.6069	0.6823	0.8895	0.6273
DMU32	0.3080	0.3997	0.9832	0.3997	0.3399	0.3776	0.9002	0.2323	0.3930	0.3997	0.9832	0.3796
DMU33	0.2750	0.3493	0.9865	0.3493	0.2361	0.2498	0.9452	0.1758	0.3446	0.3493	0.9865	0.2589
DMU34	0.4030	0.4205	0.9774	0.4205	0.2417	0.2566	0.9419	0.2199	0.4110	0.4205	0.9774	0.2691
DMU35	0.1540	0.3608	0.9911	0.2051	0.2522	0.2625	0.9608	0.1033	0.3576	0.3608	0.9911	0.1550
DMU36	0.2860	0.3364	0.9884	0.3364	0.2936	0.3122	0.9404	0.2340	0.3325	0.3364	0.9884	0.3150
DMU37	0.9150	0.9525	0.9610	0.9525	0.8809	0.9105	0.9675	0.9105	0.9154	0.9525	0.9610	0.9048
DMU38	0.4250	0.6220	0.9854	0.6220	0.6418	0.7293	0.8800	0.3743	0.6418	0.7293	0.8800	0.3742
DMU39	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
DMU40	0.2750	0.3082	0.9893	0.3082	0.3382	0.3656	0.9251	0.2760	0.3382	0.3656	0.9251	0.2759
DMU41	0.1610	0.2174	0.7387	0.1962	0.1560	0.2174	0.7176	0.1938	0.1606	0.2174	0.7387	0.1938
DMU42	0.2900	0.2905	0.9976	0.2905	0.1861	0.2174	0.8560	0.2140	0.2898	0.2905	0.9976	0.1885

Table 3.5: θ and scale efficiency scores

Relative slacks	Obs.	Mean	Std. Dev.	Min	Max
Employees – BCC	42	0.0009	0.0060	0	0.0395
Employees – CCR	42	0.0017	0.0112	0	0.0727
Size – BCC	42	0.0626	0.1281	0	0.5280
Size – CCR	42	0.1114	0.1547	0	0.5663

Table 3.6: Relative non–proportional slacks summary

	RTS type		
Model	IRS	Scale eff cient	DRS
SIMs	8	3	31
Revenue	10	3	29
SIMs&Revenue	8	3	31

Table 3.7: Returns to scale summary

COLS–SIMs

```
      Source |       SS       df       MS              Number of obs =      42
-------------+------------------------------           F(  2,    39) =   14.43
       Model |  4.55676971     2  2.27838485           Prob > F      =  0.0000
    Residual |   6.1590294    39  .157923831           R-squared     =  0.4252
-------------+------------------------------           Adj R-squared =  0.3958
       Total |  10.7157991    41  .261360954           Root MSE      =   .3974
------------------------------------------------------------------------------
    lsimsacq |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        lfte |    .664571   .2136263     3.11   0.003      .232471    1.096671
         lm2 |   .1633477   .0889871     1.84   0.074    -.0166456    .3433411
       _cons |   5.263163   .5035711    10.45   0.000     4.244594    6.281732
------------------------------------------------------------------------------
```

COLS–revenue

```
      Source |       SS       df       MS              Number of obs =      42
-------------+------------------------------           F(  2,    39) =    7.13
       Model |  3.36684954     2  1.68342477           Prob > F      =  0.0023
    Residual |    9.211201    39  .236184641           R-squared     =  0.2677
-------------+------------------------------           Adj R-squared =  0.2301
       Total |  12.5780505    41   .30678172           Root MSE      =  .48599
------------------------------------------------------------------------------
    lrevenue |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        lfte |   .4894449   .2612503     1.87   0.069    -.0389837    1.017873
         lm2 |   .1776836    .108825     1.63   0.111    -.0424358     .397803
       _cons |   11.62867   .6158328    18.88   0.000     10.38303    12.87431
------------------------------------------------------------------------------
```

Table 3.8: COLS – results for full sample

SFA–SIMs

```
Stoc. frontier normal/half-normal model          Number of obs   =         42
                                                 Wald chi2(2)    =      31.07
Log likelihood = -19.280665                      Prob > chi2     =     0.0000
------------------------------------------------------------------------------
    lsimsacq |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        lfte |    .664576   .2058572     3.23   0.001     .2611033    1.068049
         lm2 |   .1633458   .0857516     1.90   0.057    -.0047243    .3314159
       _cons |   5.279347   1.060924     4.98   0.000     3.199975    7.358719
-------------+----------------------------------------------------------------
     sigma_v |    .382747   .0476542                      .2998695      .48853
     sigma_u |   .0203664   1.184192                      6.55e-52    6.33e+47
      sigma2 |     .14691   .0443624                      .0599613    .2338587
      lambda |    .053211   1.207792                     -2.314017    2.420439
------------------------------------------------------------------------------
Likelihood-ratio test of sigma_u=0: chibar2(01) = 0.00   Prob>=chibar2 = 1.000
```

SFA–revenue

```
Stoc. frontier normal/half-normal model          Number of obs   =         42
                                                 Wald chi2(2)    =      15.35
Log likelihood = -27.733185                      Prob > chi2     =     0.0005
------------------------------------------------------------------------------
    lrevenue |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        lfte |   .4894459    .251747     1.94   0.052    -.0039691     .982861
         lm2 |   .1776832   .1048665     1.69   0.090    -.0278513    .3832177
       _cons |   11.63776   .9571444    12.16   0.000     9.761789    13.51373
-------------+----------------------------------------------------------------
     sigma_v |   .4682589   .0517822                       .377013    .5815884
     sigma_u |   .0114784   .9416534                      1.70e-72    7.76e+67
      sigma2 |   .2193982    .049804                      .1217841    .3170123
      lambda |   .0245129     .95139                     -1.840177    1.889203
------------------------------------------------------------------------------
Likelihood-ratio test of sigma_u=0: chibar2(01) = 0.00   Prob>=chibar2 = 1.000
```

Table 3.9: SFA – results for full sample

Model	Obs	Mean	Std. Dev.	Min	Max
COLS–SIMs	42	0.4479	0.1807	0.1500	1.0000
COLS–Revenue	42	0.3960	0.1976	0.1474	1.0000
SFA–SIMs	42	0.9839	0.0003	0.9828	0.9848
SFA–Revenue	42	0.9909	0.0001	0.9907	.9911

Table 3.10: Parametric methods: Technical efficiency summary

Ranking	COLS–SIMs	COLS–Rev.	DEA–Rev. VRS	DEA–SIMs VRS
COLS–SIMs	1.0000			
COLS–Revenue	0.9316	1.0000		
DEA–Revenue VRS	0.6018	0.6562	1.0000	
DEA–SIMs VRS	0.8550	0.8128	0.8071	1.0000

Note: All coeff cients are signif cantly different from 0 at 1%.

Table 3.11: Spearman rank correlation coeff cients COLS–DEA (42 obs.)

Model	Sum of elasticities	F(1, 39)	Prob > F
COLS–SIMs	0.8279	0.9300	0.3416
COLS–Revenue	0.6671	2.3200	0.1359
Model	Sum of elasticities	chi2(1)	Prob > chi2
SFA–SIMs	0.8279	1.0000	0.3178
SFA–Revenue	0.6671	2.5000	0.1140

Note: Reduced sample

Table 3.12: Wald test for hypothesis H0: CRS production function

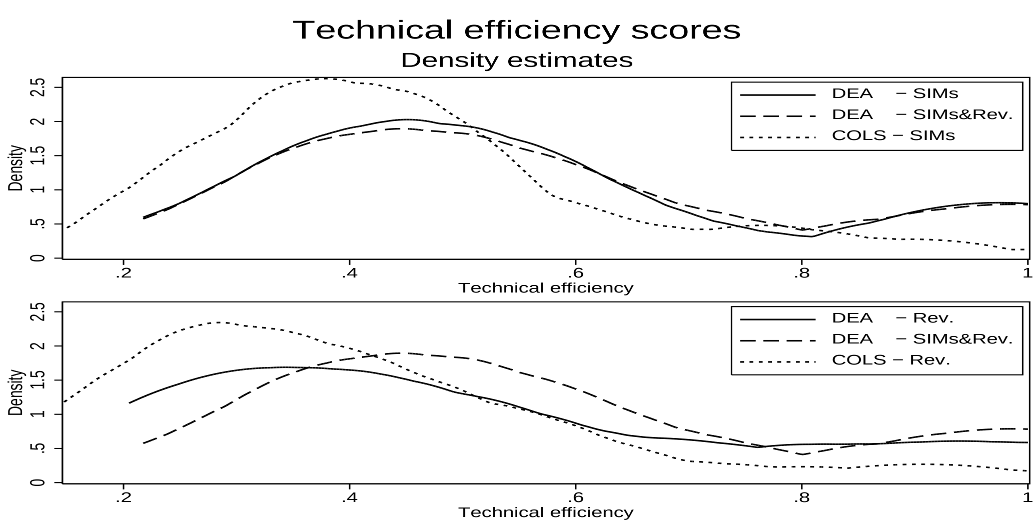

Figure 3.3: Comparison of density estimates (42 obs.)

COLS–SIMs

```
      Source |       SS       df       MS              Number of obs =      38
-------------+------------------------------           F(  2,    35) =   11.33
       Model |  2.32361108     2  1.16180554           Prob > F      =  0.0002
    Residual |  3.59035357    35  .102581531           R-squared     =  0.3929
-------------+------------------------------           Adj R-squared =  0.3582
       Total |  5.91396465    37  .159836882           Root MSE      =  .32028
------------------------------------------------------------------------------
    lsimsacq |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        lfte |   .4473402     .17792     2.51   0.017     .0861433    .8085371
         lm2 |   .1535561   .0747587     2.05   0.048     .0017879    .3053244
       _cons |   5.827332   .4230969    13.77   0.000     4.968399    6.686264
------------------------------------------------------------------------------
```

COLS–revenue

```
      Source |       SS       df       MS              Number of obs =      38
-------------+------------------------------           F(  2,    35) =    4.61
       Model |  1.49073594     2   .74536797           Prob > F      =  0.0167
    Residual |  5.66231177    35  .161780336           R-squared     =  0.2084
-------------+------------------------------           Adj R-squared =  0.1632
       Total |  7.15304771    37  .193325614           Root MSE      =  .40222
------------------------------------------------------------------------------
    lrevenue |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        lfte |   .2320428    .223436     1.04   0.306    -.2215564    .6856419
         lm2 |   .1732971   .0938837     1.85   0.073    -.0172969     .363891
       _cons |    12.2748   .5313346    23.10   0.000     11.19614    13.35347
------------------------------------------------------------------------------
```

Table 3.13: COLS – results (38 obs.)

SFA–SIMs

```
Stoc. frontier normal/half-normal model          Number of obs   =         38
                                                 Wald chi2(2)    =      46.78
Log likelihood = -6.9128045                      Prob > chi2     =     0.0000
------------------------------------------------------------------------------
    lsimsacq |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        lfte |   .5718647   .1427028     4.01   0.000     .2921724     .851557
         lm2 |   .0786142   .0637068     1.23   0.217    -.0462489    .2034772
       _cons |   6.126882   .2915422    21.02   0.000      5.55547    6.698295
-------------+----------------------------------------------------------------
     sigma_v |    .094448    .059624                      .0274055    .3254973
     sigma_u |   .4983721   .0871752                      .3537216    .7021759
      sigma2 |   .2572952   .0806131                      .0992963     .415294
      lambda |   5.276685   .1319612                      5.018046    5.535325
------------------------------------------------------------------------------
Likelihood-ratio test of sigma_u=0: chibar2(01) = 4.36   Prob>=chibar2 = 0.018
```

SFA–revenue

```
Stoc. frontier normal/half-normal model          Number of obs   =         38
                                                 Wald chi2(2)    =  7.850e+08
Log likelihood = -13.671618                      Prob > chi2     =     0.0000
------------------------------------------------------------------------------
    lrevenue |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        lfte |   .3801373   .0000816  4658.74   0.000     .3799774    .3802972
         lm2 |   .0293778   6.84e-06  4294.70   0.000     .0293644    .0293912
       _cons |   12.90372    .000208        .   0.000     12.90331    12.90413
-------------+----------------------------------------------------------------
     sigma_v |   9.10e-09   2.37e-06                      7.6e-231    1.1e+214
     sigma_u |   .6934945   .0795493                      .5538645    .8683255
      sigma2 |   .4809346    .110334                       .264684    .6971853
      lambda |   7.62e+07   .0795493                      7.62e+07    7.62e+07
------------------------------------------------------------------------------
Likelihood-ratio test of sigma_u=0: chibar2(01) = 8.15   Prob>=chibar2 = 0.002
```

Table 3.14: SFA – results (38 obs.)

Model	Obs	Mean	Std. Dev.	Min	Max
DEA–SIMs	38	0.6800	0.2369	0.2174	1.0000
DEA–Revenue	38	0.5921	0.2825	0.2083	1.0000
DEA–SIMs&Rev.	38	0.6986	0.2485	0.2174	1.0000
COLS–SIMs	38	0.5777	0.1666	0.2189	1.0000
COLS–Revenue	38	0.5263	0.1985	0.2262	0.9999
SFA–SIMs	38	0.6979	0.1767	0.2727	0.9578
SFA–Revenue	38	0.6110	0.2293	0.2023	0.9999

Table 3.15: Parametric methods: Technical efficiency summary

Model	Sum of elasticities	F(1, 35)	Prob > F
COLS–SIMs	0.6008	6.9500	0.0124
COLS–Revenue	0.4053	9.7800	0.0035
Model	Sum of elasticities	chi2(1)	Prob > chi2
SFA–SIMs	0.6504	10.3100	0.0013
SFA–Revenue	0.4095	6.2e+07	0.0000

Table 3.16: Wald test for hypothesis H0: CRS production function (38 obs.)

Ranking	DEA–SIMs	DEA–Rev.	DEA–SIMs&Rev.	COLS–SIMs	COLS–Rev.	SFA–SIMs	SFA–Rev.
DEA–SIMs	1.0000						
DEA–Rev.	0.6695	1.0000					
DEA–SIMs&Rev.	0.7859	0.9176	1.0000				
COLS–SIMs	0.7583	0.8108	0.7863	1.0000			
COLS–Rev.	0.6336	0.9188	0.8650	0.9011	1.0000		
SFA–SIMs	0.7236	0.8193	0.7921	0.9845	0.8947	1.0000	
SFA–Rev.	0.5517	0.8972	0.8374	0.8822	0.9691	0.8997	1.0000

Note: In all cases the hypothesis of rank independence was rejected at 1% signif cance level.

Table 3.17: Spearman rank correlation coefficients COLS–DEA (38 obs.)

Ranking		42			38		
		DEA–SIMs	COLS–SIMs	SFA–SIMs	DEA–SIMs	COLS–SIMs	SFA–SIMs
42	DEA–SIMs	1.0000					
	COLS–SIMs	0.8130	1.0000				
	SFA–SIMs	0.8569	0.7934	1.0000			
38	DEA–SIMs	0.7158	0.7236	0.9564	1.0000		
	COLS–SIMs	0.7409	0.7583	0.9690	0.9845	1.0000	
	SFA–SIMs	0.8565	0.7941	1.0000	0.9560	0.9687	1.0000
Ranking		42			38		
		DEA–Rev.	COLS–Rev.	SFA–Rev.	DEA–Rev.	COLS–Rev.	SFA–Rev.
42	DEA–Rev.	1.0000					
	COLS–Rev.	0.6575	1.0000				
	SFA–Rev.	0.6580	0.9702	1.0000			
38	DEA–Rev.	0.6575	0.8972	0.9201	1.0000		
	COLS–Rev.	0.5100	0.9188	0.9529	0.9691	1.0000	
	SFA–Rev.	0.6580	0.9702	1.0000	0.9201	0.9529	1.0000

Note: In all cases the hypothesis of rank independence was rejected at 1% signif cance level.

Table 3.18: Spearman rank correlation coefficients COLS–DEA

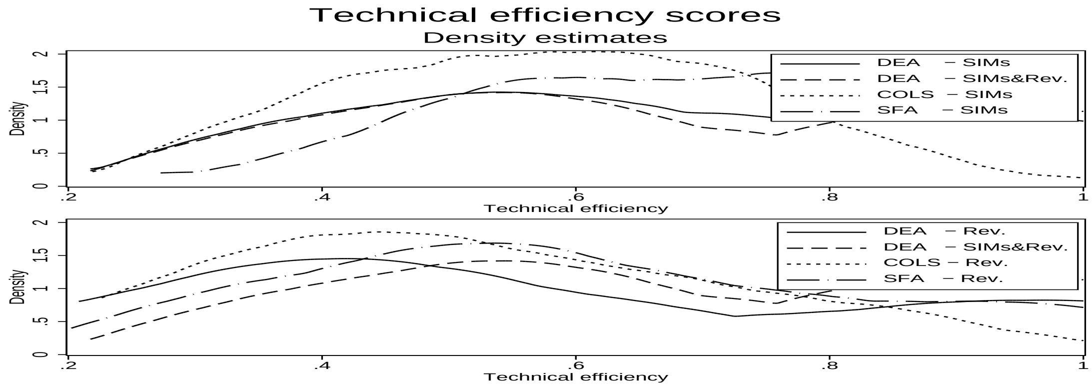

Figure 3.4: Comparison of density estimates (38 obs.)

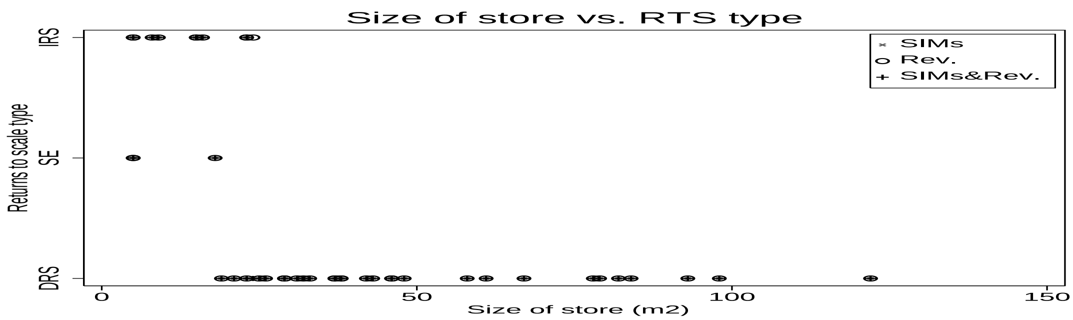

Figure 3.5: Returns to scale type and size of store

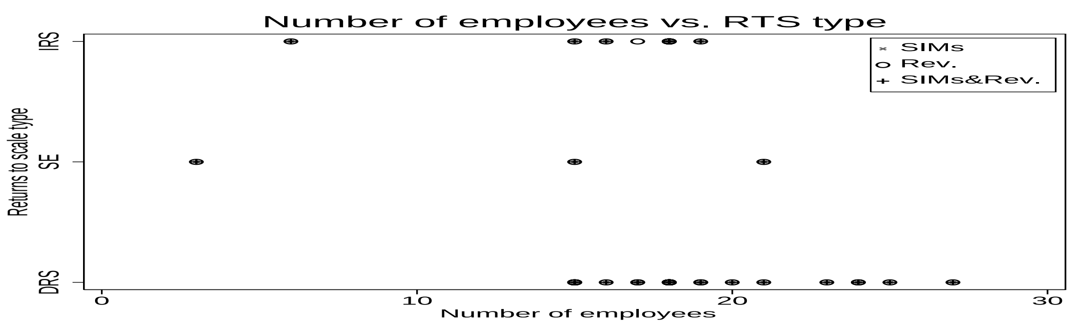

Figure 3.6: Returns to scale type and number of employees

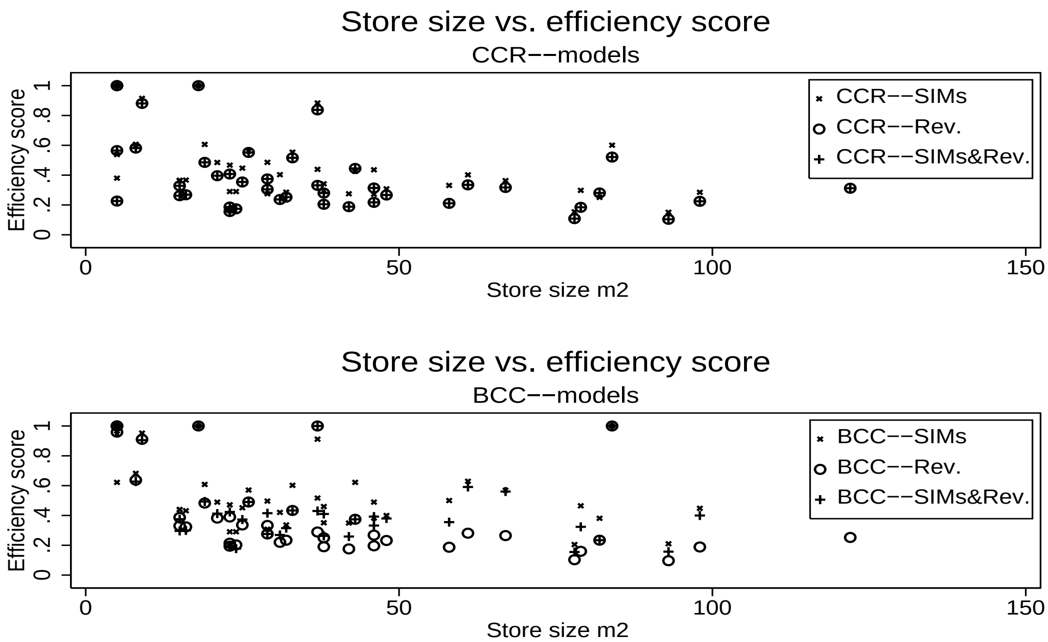

Figure 3.7: Size of store and efficiency score

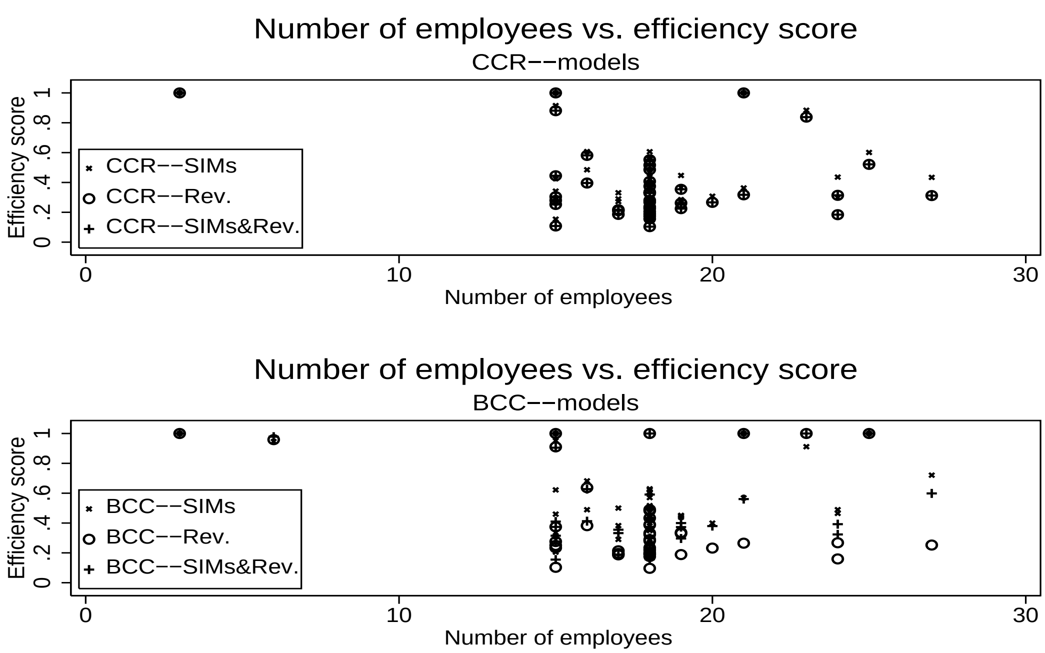

Figure 3.8: Number of employees and efficiency score

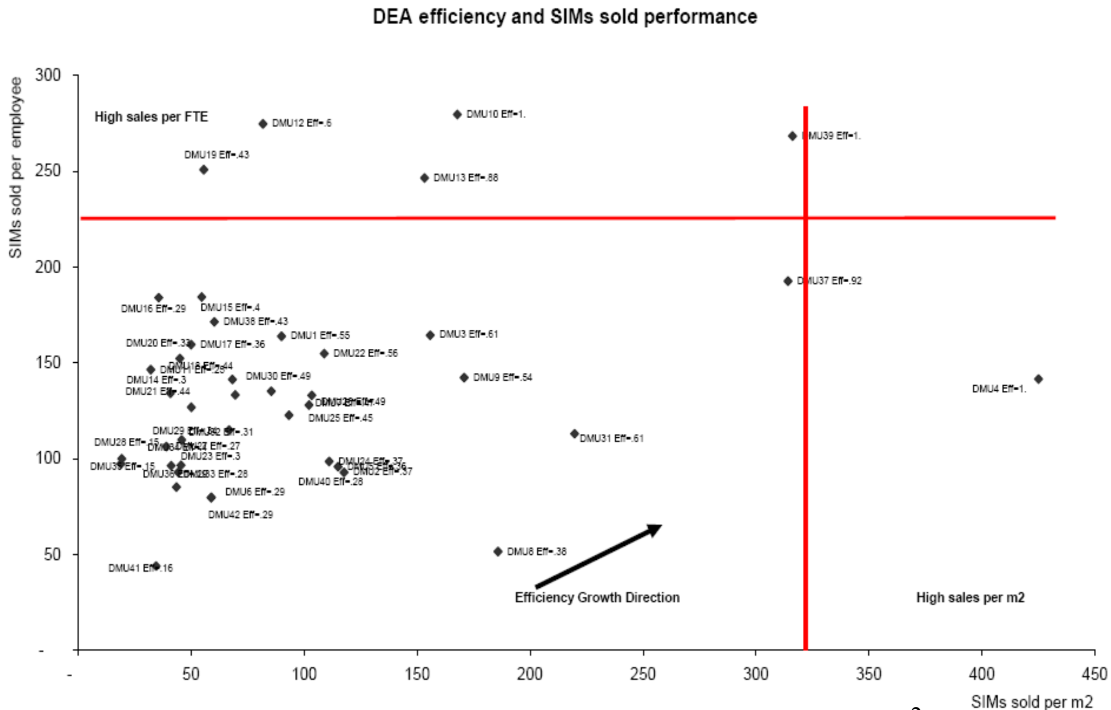

Figure 3.9: SIM efficiency measure in per employee–per m^2 space

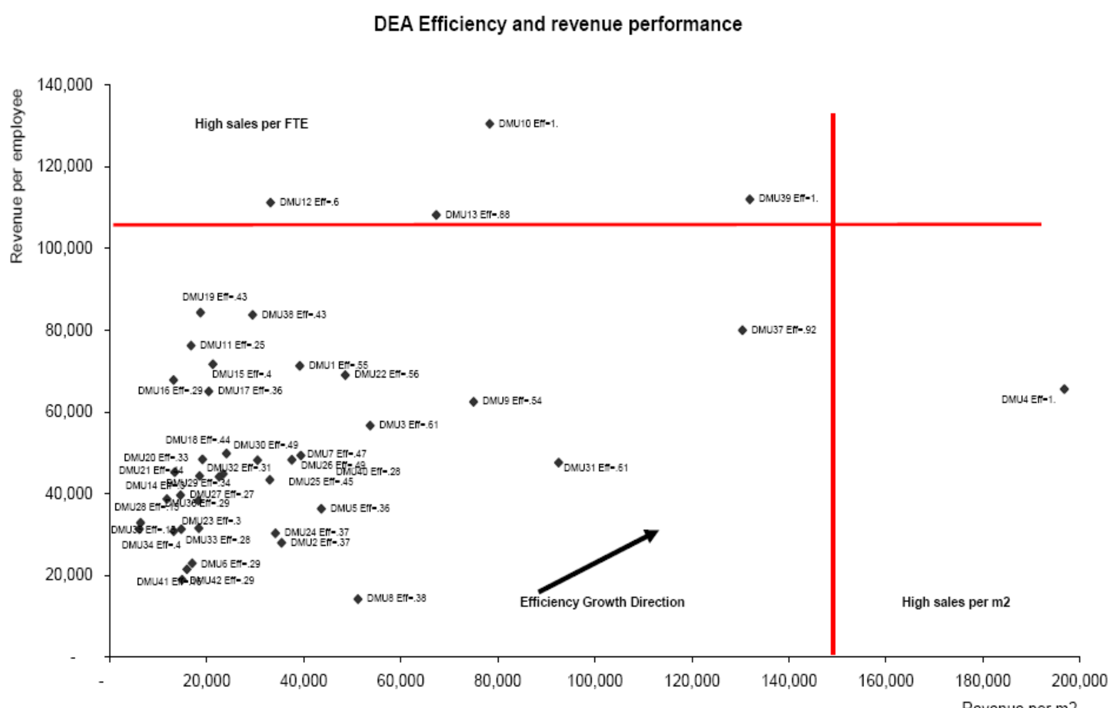

Figure 3.10: Revenue efficiency measure in per employee–per m^2 space

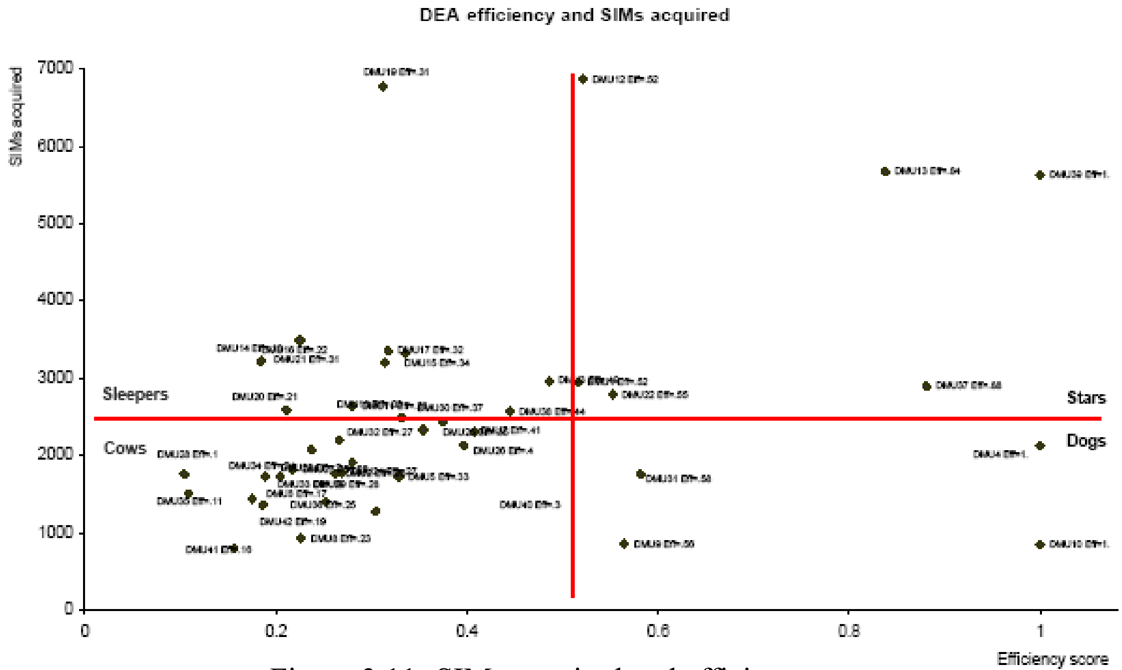

Figure 3.11: SIMs acquired and efficiency score

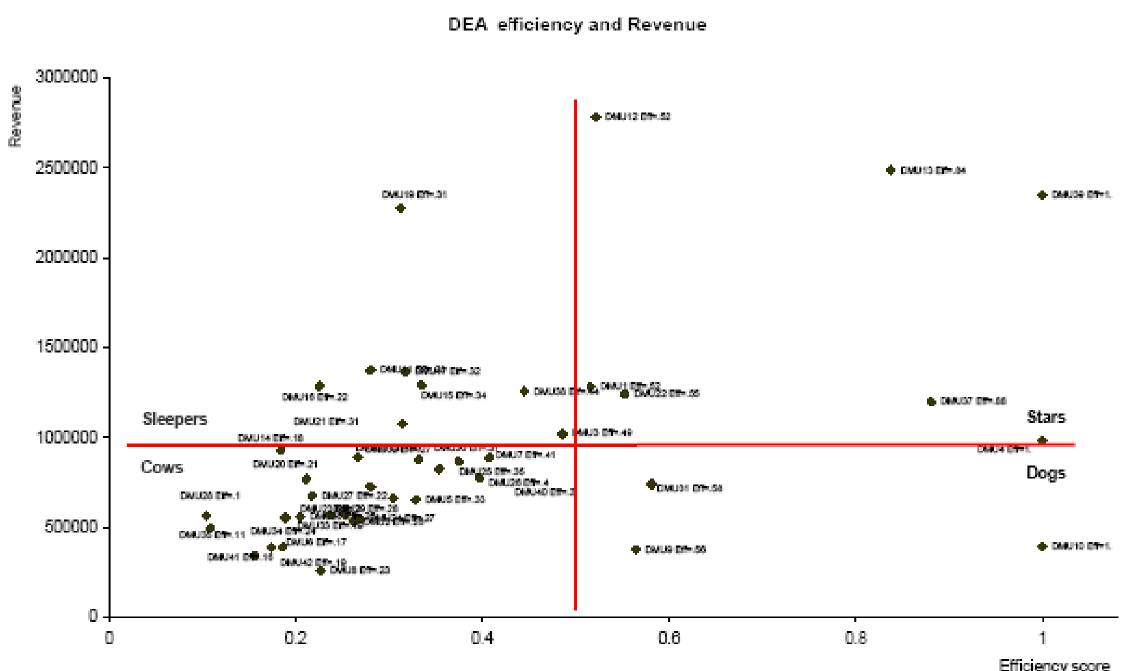

Figure 3.12: Revenue and efficiency score