

CERGE  
Center for Economics Research and Graduate Education  
Charles University Prague



# Essays on Shadow Banking and Asset Pricing

Martin Kuncil

Dissertation

Prague, April 2014



Martin Kuncł

# Essays on Shadow Banking and Asset Pricing

Dissertation

Prague, April 2014



## **Dissertation Committee**

SERGEY SLOBODYAN (CERGE-EI, Charles University; chair)

MICHAL KEJAK (CERGE-EI, Charles University)

FILIP MATĚJKA (CERGE-EI, Charles University)

MICHAL PAKOŠ (CERGE-EI, Charles University)

## **Referees**

KALIN NIKOLOV (European Central Bank)

FREDERIC MALHERBE (London Business School)



---

# Table of Contents

<b>Abstract</b>	<b>v</b>
<b>Abstrakt</b>	<b>vii</b>
<b>Acknowledgments</b>	<b>ix</b>
<b>Introduction</b>	<b>1</b>
<b>1 Securitization under Asymmetric Information over the Business Cycle</b>	<b>3</b>
1.1 Introduction . . . . .	3
1.2 Literature review . . . . .	6
1.2.1 Securitization and implicit recourse . . . . .	6
1.2.2 Financial intermediation imperfections, information frictions, and business cycles . . . . .	8
1.3 Model . . . . .	10
1.3.1 Model set-up . . . . .	11
1.3.2 Model solution in special cases . . . . .	16
1.3.3 Case of interest: Implicit recourse as a signal of loan quality . . . . .	25
1.4 Dynamics and numerical examples . . . . .	31
1.5 Extensions . . . . .	34
1.5.1 Endogenizing the “skin in the game” . . . . .	34
1.5.2 “Skin in the game” as a policy parameter . . . . .	36
1.6 Empirical analysis . . . . .	37
1.6.1 Hypotheses . . . . .	37
1.6.2 Data description . . . . .	38
1.6.3 Panel regression results . . . . .	39
1.7 Conclusion . . . . .	43
1.8 Acknowledgments . . . . .	44
1.A Appendix 1 . . . . .	45
1.A.1 Proofs . . . . .	45
1.A.2 Derivation of firms’ policy functions . . . . .	58
1.A.3 Calibration of the parameters used in section 1.4 . . . . .	61
1.A.4 Numerical solutions of the fully stochastic dynamic model . . . . .	62

<b>2</b>	<b>Adverse Selection on Re-sale Markets for Securitized Assets</b>	<b>65</b>
2.1	Introduction . . . . .	65
2.2	Model set-up . . . . .	68
2.2.1	Physical set-up . . . . .	69
2.2.2	Household . . . . .	70
2.2.3	Frictions . . . . .	70
2.2.4	Financial firms . . . . .	71
2.2.5	Market clearing conditions . . . . .	77
2.3	Model solution . . . . .	80
2.3.1	Comparative statics . . . . .	80
2.3.2	Methodology for solution of the dynamic model . . . . .	87
2.3.3	Dynamic properties of the model . . . . .	89
2.4	Conclusion . . . . .	92
2.5	Acknowledgments . . . . .	94
2.A	Appendix 2.A: Comparative statics . . . . .	95
2.A.1	Role of the “skin in the game” constraint . . . . .	95
2.A.2	Separating condition without provision of implicit recourse . . . . .	97
2.A.3	Separating condition with provision of informative implicit recourse . . . . .	98
2.A.4	Adverse selection on re-sale markets . . . . .	100
2.B	Appendix 2.B: Markov-switching regimes . . . . .	103
2.B.1	Equilibrium conditions . . . . .	103
2.B.2	Markov regimes’ properties . . . . .	104
<b>3</b>	<b>By Force of Habit: Asset-Pricing Implications of Durable Goods</b>	<b>107</b>
3.1	Introduction . . . . .	107
3.2	Model . . . . .	109
3.2.1	Intratemporal Marginal Rate of Substitution and Technology . . . . .	110
3.2.2	Intertemporal Marginal Rate of Substitution and the Risk-Free Rate . . . . .	112
3.2.3	Choosing the Scaled Sensitivity Function $\tilde{\lambda}(s_t)$ . . . . .	115
3.3	Data . . . . .	116
3.4	Implications of the Intratemporal First-Order Condition . . . . .	117
3.4.1	Cointegration . . . . .	117
3.4.2	Vector Error Correction Model . . . . .	118
3.5	Asset-Pricing Implications . . . . .	118
3.5.1	Time Series Test . . . . .	120
3.5.2	Cross-Sectional Test . . . . .	121
3.6	Conclusion . . . . .	123
3.7	Acknowledgments . . . . .	123
3.A	Appendix 3.A: Derivation of the Mean and Variance of the Stochastic Discount Factor . . . . .	124
3.B	Appendix 3.B: Derivation of the Scaled Sensitivity Function $\tilde{\lambda}(s_t)$ . . . . .	124
3.C	Appendix 3.C: Data construction . . . . .	127
	<b>Bibliography</b>	<b>141</b>



---

# Abstract

This dissertation deals with the topics related to securitization and with pricing of financial assets in general. The topics are analyzed from a macroeconomic perspective using various theoretical and empirical methods.

The first chapter studies the efficiency of financial intermediation through securitization with asymmetric information about the quality of securitized loans. In this theoretical model I show that, in general, by providing reputation-based implicit recourse, the issuer of a loan can credibly signal its quality. However, in boom stages of the business cycle, information on loan quality remains private, and lower quality loans accumulate on balance sheets. This deepens a subsequent downturn. The longer the duration of a boom, the deeper the fall of output in a subsequent recession will be. I present empirical evidence from securitization deal level data consistent with this result. Finally, the model suggests that excessive regulation which requires higher explicit risk-retention by the originators of loans can adversely affect both quantity and quality of investment in the economy.

The second chapter presents a Markov-switching DSGE model which focuses on the adverse selection on re-sale markets for securitized products. The complexity of securitized assets, which make it costly to verify their intrinsic quality, together with the provision of reputation-based implicit recourse, limits adverse selection on re-sale markets in booms or mild recessions. However, in a deep recession, implicit recourse is widely defaulted upon, which causes a serious adverse selection problem, and deepens and prolongs the recession. The adverse selection problem is especially severe when the recession is preceded by a prolonged boom period. Then, securitized loans of high quality may cease being traded altogether.

In the third chapter (co-authored by Michal Pakoš), we propose a model which attempts to explain both time and cross-section variation in the conditional asset pricing moments on the stock market in a unified framework. We combine two strands of literature on asset pricing: external habit formation and literature on asset pricing where a large share of investors' consumption consists of service flows from a stock of durable goods. We develop a tractable theoretical model as a generalization of the seminal Campbell and Cochrane (1999) external habit formation model, where we introduce durable goods and estimate the parameters of the model using the GMM methodology on the average market portfolio and the set of 6 Fama-French portfolios.



Tato disertační práce pojednává o tématech vztahujících se k sekuritizaci a k oceňování finančních aktiv obecně. Témata jsou analyzována z makroekonomické perspektivy za pomoci teoretických a empirických metod.

První kapitola zkoumá efektivitu finančního zprostředkování prostřednictvím sekuritizace při existenci asymetrických informací o kvalitě sekuritizovaných úvěrů. V tomto teoretickém modelu ukazují, že obecně poskytováním implicitního rekursu založeném na reputaci může emitent důvěryhodně signalizovat kvalitu jím sekuritizovaných úvěrů. Avšak během konjunktury hospodářského cyklu informace o kvalitě úvěrů zůstává neveřejná a úvěry nižší kvality se akumulují v rozvahách finančních firem. Toto prohlubuje následný hospodářský pokles. Čím déle trvá hospodářská expanze, tím hlubší bude propad produktu v následné recesi. Představuji výsledky empirických testů na datech z úrovně sekuritizovaných produktů, které jsou konzistentní s teoretickými výsledky. Konečně, model naznačuje, že přehnaná regulace požadující vyšší explicitní zadržování rizika emity úvěrů může negativně ovlivnit množství i kvalitu investic v ekonomice.

Druhá kapitola představuje "Markov-Switching" DSGE model, který se zaměřuje na problém nepříznivého výběru na sekundárních trzích sekuritizovaných produktů. Složitost sekuritizovaných aktiv, která způsobuje vysokou nákladnost ověřování skutečné kvality těchto aktiv, společně s poskytováním implicitního rekursu založeném na reputaci omezují problém nepříznivého výběru na sekundárních trzích během fáze expanze nebo mírné kontrakce hospodářského cyklu. Avšak za hluboké recese implicitní rekurz bývá často neplněn. To pak způsobuje významný problém nepříznivého výběru a prohlubuje i prodlužuje recesi. Problém nepříznivého výběru je zejména významný, když recesi předchází delší fáze expanze hospodářského cyklu. Tehdy sekundární půjčky vysoké kvality mohou přestat být obchodovány kompletně.

Ve třetí kapitole, jejímž spoluautorem je Michal Pakoš, navrhujeme model, který se v jednotném rámci snaží vysvětlit jak časovou, tak i průřezovou variaci v podmíněných momentech oceňování aktiv na akciových trzích. Kombinujeme dvě odvětví literatury o oceňování aktiv: externí vytváření návyků a literaturu o oceňování aktiv, ve které velká část spotřeby investora spočívá v toku služeb ze stavu statků dlouhodobé spotřeby. Vyvíjíme přehledný model jako zobecnění zásadního modelu externího vytváření zvyků (Campbell and Cochrane 1999), do kterého zavádíme statky dlouhodobé spotřeby. Odhadujeme parametry modelu pomocí GMM metodologie na průměrném tržním portfoliu a skupině 6 Fama-French portfolií.



---

# Acknowledgments

I would like to express my gratitude to my supervisor, Sergey Slobodyan, for his continuous support and overall guidance I needed to complete this work.

I also would like to thank Michal Kejak, Filip Matějka, Michal Pakoš, Markus Brunnermeier, Byeongu Jeong and Evangelia Vourvachaki for their valuable help and advice throughout my studies.

Last but not least, I am grateful to Olena Senyuta and my parents for their unconditional support, motivation and encouragement.

Czech Republic, Prague  
April 2014

Martin Kuncel



---

# Introduction

My dissertation focuses on topics at the intersection of macroeconomics and finance. The first two chapters deal with topics related to macroeconomic implications of particular financial innovations and the last chapter is a contribution to the macroeconomic asset pricing literature.

The topic of the first two chapters is motivated by the financial crisis of the late 2000s. It is widely believed that the inefficiency and fragility of the so called “shadow banking” sector was one of the main causes of this financial crisis and its exceptional severity. The “shadow banking” sector encompasses a large range of activities and institutions which perform financial intermediation like traditional banks, but are much less regulated. In the first two chapters, I focus on securitization<sup>1</sup>, which is one of the major tools of the “shadow banking” system. The critiques of securitization point to various related moral hazard and adverse selection problems, and claim that these create systemic risks and inefficiencies in the financial intermediation.

I model the securitization process in the dynamic stochastic general equilibrium models and focus on the macroeconomic implications of various commitment and information frictions. The first chapter studies in detail the role of the asymmetry of information between the issuers of securitized assets and their first buyers on the primary market. The model is able to explain the build-up of low quality assets on the balance sheets of the financial sector in the boom stage of the business cycle, which we could have observed prior to the crisis. The model also predicts a deeper and more prolonged recession after

---

<sup>1</sup>Securitization process is basically the sale of contractual debt. It is described in detail in the literature review of the first chapter.

a prolonged period of economic expansion. In the second chapter, I assume that, due to large complexity and opacity of securitized assets, there may be asymmetric information also on the re-sale (secondary) markets for securitized assets. The model predicts relatively smooth functioning markets in boom periods or mild recessions, where the adverse selection problem is contained despite the fact that traders are ignorant about the quality of the traded assets. However, it also predicts a sudden dramatic increase in the severity of the adverse selection problem in a deeper recession, which is caused by a shock that affects more negatively the performance of assets of lower quality. The results of the second paper may explain why, prior to the recent financial crisis, in spite of the information frictions, the re-sale markets for securitized assets were working well, and why we observed such a sudden dry-up of these markets and dramatic increase in risk premia for securitized assets.

The last chapter of the dissertation, written together with Michal Pakoš, combines two approaches in the asset pricing literature: literature on external habit formation and literature recognizing the importance of durable consumption in the investors' consumption portfolio. Our model is an analytically tractable generalization of Campbell and Cochrane's (1999) model in which investors derive utility from both nondurable goods and service flows from consumer durable goods. We estimate the parameters of this nonlinear model and test the asset pricing model implications using the GMM methodology on the universe of 6 Fama-French portfolios, the risk-free rate and the value-weighted return on NYSE, AMEX and NASDAQ stocks.



## Chapter 1

---

# Securitization under Asymmetric Information over the Business Cycle

## 1.1 Introduction

Securitization has recently attracted a great deal of criticism due to its role in the financial crisis of the late 2000s (e.g. Bernanke 2010). Securitization and the market-based system of financial intermediation generally grew significantly in importance in the decades preceding the crisis (Adrian and Shin 2009). The financial crisis of the late 2000s led to intensified research into the problematic aspects of securitization. New research is often very critical about securitization; consider Shleifer and Vishny (2010), who argue that it creates systemic risks and inefficiencies in financial intermediation. Currently, the regulation of the financial sector is being redrafted and strengthened on national as well as international levels in many developed countries. The new regulation also addresses securitization practices.<sup>1</sup> The agency problems related to securitization to which most of the criticism points are, however, not new. Securitization designs contained tools, such as tranche retention schemes or implicit recourse, that were supposed to limit these negative aspects of securitization. The question is whether these tools worked efficiently in the period prior to the late 2000s financial crisis.

In this paper, I show in a dynamic stochastic general equilibrium model that reputation concerns can allow sponsors of securitized products to credibly signal the quality of

---

<sup>1</sup>Pozsar et al. (2012) describe the role of securitization in shadow banking, and Adrian and Ashcraft (2012) review the proposals for new regulation.

loans by providing implicit recourse, and thus limit the problem of asymmetric information. Implicit recourse is implicit support provided by the issuer of securitized products to the holders of these assets. This support is not contractual and is enforced in a reputation equilibrium.<sup>2</sup> Typically, there are both pooling and separating equilibria in this signaling game. By applying Intuitive Criterion refinement, I can select a unique separating equilibrium, in which the information about loan quality is transferred, and the outcome is therefore efficient. However, there are limits to the degree of commitment based on reputation and thus also to the efficiency of implicit recourse in eliminating the problem of asymmetric information. Following the empirical evidence in Bloom (2009) and Bloom et al. (2012), who find that the second moments of firms' Total Factor Productivity (TFP) in the economy are countercyclical, the relative difference in the productivity of projects' (loans') in this model is also countercyclical. As a result, it turns out that even though the steady state provision of implicit recourse helps to achieve a separating equilibrium, in boom stages of the business cycle the separation equilibrium would require levels of implicit recourse so high that they cannot be enforced through reputation. Therefore, in boom stages of business cycles there are only pooling equilibria, in which the information about the quality of loans remains private and the allocation of investment is inefficient. This has only very moderate effects as long as the economy stays in a boom, where relative difference in the productivity of projects (loans) is low. However, the effect of an accumulated stock of low quality loans becomes more pronounced in a subsequent downturn of the economy, which is thus amplified. Further, the longer the boom, the larger the share of lower quality loans on the balance sheets and the deeper will be the subsequent downturn.

The results of this paper could also have implications for the related macro-prudential policy which requires higher explicit risk-retention for the originators (issuers) of the securitized products (such as in section 941 of the Dodd-Frank reform). Although no frictions in the model are sufficient to rationalize regulation of this sort, the model points to an adverse general equilibrium effect of higher explicit risk-retention. In this model, higher than equilibrium explicit risk-retention, such as the practice of keeping a larger fraction of issued loans on the balance sheet of the issuer, limits the financial intermediation ability of the issuer. Since higher explicit risk-retention restricts the supply of loans, through the general equilibrium effect, it increases equilibrium prices of securitized assets and makes

---

<sup>2</sup>For a review of empirical evidence on implicit recourse, a description of its types, and a discussion of its role in the securitization process, I would like to refer the reader to the literature review.

securitization more profitable. Higher prices mean that even the securitization of lower quality loans is profitable. Therefore, when regulation is excessive, any possible benefits of the regulation, which are not modeled here, can be outweighed by the adverse general equilibrium effect, which lowers both the quantity and the quality of the investment in the economy.

In the empirical section of the paper, I test hypotheses from the theoretical model on the level of securitization deals using data for residential mortgage backed securities issued in Europe. Lagged credit support provided to holders of securitized assets is found to have a positive relation to the loan quality, which is in line with the signaling hypothesis. Further, this effect is smaller and may even be overturned for assets issued in a boom stage of the business cycle. This is in line with the higher likelihood of a pooling equilibrium in a boom which is derived in the theoretical model. The results are especially strong for deals issued in the UK, but are statistically insignificant for deals issued in Spain. The difference could be explained by a significant differences in regulatory framework and securitization practices.

The mechanism presented in this paper can contribute to the understanding of the recent financial crisis as it replicates some of the securitization market outcomes observed prior to and during the crisis. In the period preceding the crisis, many inefficient investments of unknown quality were undertaken. While this was not a problem as long as the economy was performing well, the large amount of low quality loans in the economy contributed to the depth of the financial crisis. Also, during the crisis, the markets for securitized products were severely strained. The paper also points to some unexpected effects of the newly proposed regulation.

The paper is organized in the following way. Section 1.2 reviews the related literature. Section 1.3 introduces the set-up of the model and shows its solution, the effect of assumed financial frictions and the effect of implicit recourse. For analytical tractability, this section focuses on the steady state with only idiosyncratic stochasticity and in which the aggregate variables are deterministic. Section 1.4 shows the results of the full-fledged model with aggregate stochasticity obtained using global numerical methods and focuses on the switching between the separating and pooling equilibria over the business cycle. Section 1.5 develops extensions of the model. In particular, it discusses the policy implications of the model. Section 1.6 describes the empirical testing of hypotheses derived in the theoretical model.

## 1.2 Literature review

My research is broadly related to several strands of literature. In this section, I would like to focus on research related to securitization with implicit recourse and to financial intermediation imperfections, information frictions and business cycles.

### 1.2.1 Securitization and implicit recourse

Securitization is the process of selling cash flows related to the loans issued by the originator (often called the sponsor). The sale of loans is effectuated in a legally separated entity called a special purpose vehicle (SPV) or special purpose entity (SPE). The entity purchases the right to the cash flows with resources obtained by issuing securities in the capital market. The sponsor and the SPV are “bankruptcy remote”, and the sale of loans is officially considered to be complete, i.e., the sponsor should transfer all risks to the buyers of newly emitted securities. Loans are pooled in a portfolio, which is then usually divided into several tranches ordered by seniority, which have a different exposure to risk. Before the crisis, securitization was perceived mainly as a means of dispersing credit risk and allocating it to less risk-averse investors who would be compensated by higher returns, while highly risk-averse investors could invest into the most senior tranches with high ratings. Due to the role securitization played in the late 2000s financial crisis (e.g. Bernanke 2010), it attracted a lot of criticism, and the attention of researchers turned more to the set of agency problems present at different stages of the securitization process (Shin 2009). A detailed review of those agency conflicts has been compiled, for instance, by Paligorova (2009).

Gorton and Pennacchi (1995) were among the first to point to moral hazard problems related to securitization and to address the issue of why securitization takes place despite them. Moral hazard problems stem from the fact that if the risk is transferred with a loan from the originator of the loan to the investor, the bank has a reduced incentive to monitor borrowers to increase loan quality. Gorton and Pennacchi (1995) argue that, before the 1980s, securitization was very limited. In the 1980s several regulatory changes took place that effectively increased the cost of deposit funding. One key factor was the imposition of a binding credit requirement for commercial banks.<sup>3</sup> Banks could

---

<sup>3</sup>“In 1981 regulators announced explicit capital requirements for the first time in U.S. banking history: all banks and bank holding companies were required to hold primary capital of at least 5.5 percent of assets by June 1985” (Gorton and Metrick (2010), p. 10).

avoid increased capital requirements by securitization, which moved some of the risky assets off their balance sheets. This view that an important reason for securitization is regulatory arbitrage is shared by many economists (e.g. Gorton and Pennacchi 1995; Gertler and Kiyotaki 2010; and Gorton and Metrick 2010). Calomiris and Mason (2004) present some evidence suggesting that regulatory arbitrage is effectuated by securitizing banks to increase efficiency of contracting in the situation where capital requirements are unreasonably high, rather than to abuse the safety net. The moral hazard problems and agency problems in general were then alleviated by the practice of keeping part of the loan in the portfolio on the balance sheet of the originator. Fender and Mitchell (2009) study different tranche retention designs and their effect on incentives. However, any loan sale, partial or complete, results in lower incentives to monitor borrowers, which of course affects the price investors are willing to pay for the securitized loan. Loan originators, thus, have an incentive to provide implicit recourse.

Implicit recourse is a particular form of implicit support provided by the issuers of securitized products to the holders of these assets. They represent a certain guarantee of the quality of the loan. The guarantee cannot be explicit since it would then have to abide by regulations and to be kept on the balance sheet of the bank. Nevertheless, much evidence suggests that implicit recourse was frequently used during the securitization process (“As the saying goes, the only securitization without recourse is the last.” [Mason and Rosner 2007, p. 38]). Gorton and Souleles (2006) show in a theoretical model that this mutually implicit collusion between investors and originators of the loans can be an equilibrium result in a repeated game due to the reputation concerns of the originator, who wants to pursue securitization in the future at favorable conditions. Several empirical studies documented concrete cases of implicit recourse or showed indirect evidence of its presence. Higgins and Mason (2004) study 17 discrete recourse events that were directed to an increase in the quality of receivables sponsored by 10 different credit-card banks. The forms of the support provided were, for instance, adding higher quality accounts to the pool of receivables, removing lower quality accounts, increasing the discount on new receivables, increasing credit enhancement, and waiving servicing fees. Higgins and Mason (2004) argue that implicit recourse increases sponsors’ stock prices in the short and long run following the recourse. It also improves their long-run operating performance. Recourse may help to signal to investors that shocks making recourse necessary are only transitory.

Another example showing that the risks were not fully transferred during securitiza-

tion to the SPV is given by Brunnermeier (2009), who argues that when the SPV was subject to liquidity problems, which arise from a maturity mismatch between the SPV's assets and liabilities and a sudden reduced interest in the instruments emitted by the SPV, the sponsor would grant credit lines to it.

In my model, I will concentrate on the relationship between investors and banks, where the latter have better information about the quality of loans, and I will show that, due to reputation concerns, the bank has an incentive to signal this quality. This is in line with the suggestion by Higgins and Mason (2004) that implicit recourse is used as a signaling tool.

## **1.2.2 Financial intermediation imperfections, information frictions, and business cycles**

This paper is related to the volume of literature on financial frictions in macroeconomic models and the role of asymmetric information and reputation in financial intermediation.

In the recent financial crisis, we have witnessed important disruptions of financial intermediation. It became clear that frictions in the financial sector are important and should not be omitted from macroeconomic models. The classical papers that endogenize financial frictions on the side of borrowers include Bernanke and Gertler (1989), Bernanke, Gertler, and Gilchrist (1999), and Kiyotaki and Moore (1997). These papers introduce an agency problem between borrowers and lenders, which give rise to the use of collateral and credit rationing. The resulting endogenous amplification of the effects of the shocks in the economy is denoted as the “financial accelerator”. Some of the recent macroeconomic models with financial frictions directly incorporate securitization. Brunnermeier and Sannikov (2014) find that securitization enables the sharing of idiosyncratic risks but may be amplifying the systemic risk.

In this paper, I will refer often to the Kiyotaki and Moore (2012) model of monetary economy with differences in liquidity among different asset classes. Their model features borrowing and re-saleability constraints and the stochastic uninsurable arrival of idiosyncratic investment shocks among the market participants. I simplify this model, and in order to study the financial intermediation similar to securitization, I introduce asymmetric information and model signaling by the provision of reputation-based implicit recourse.

There is much literature on the adverse selection in lender-borrower relationships

based on asymmetric information, which has developed the original contribution of Akerlof (1970). In Parlour and Plantin (2008), the intensity of adverse selection on the markets for securitized assets (sold loans) depends on the proportion of liquidity sellers and informed sellers who want to sell low quality loans. Kurlat (2013) models a similar adverse selection problem in an extension of the model by Kiyotaki and Moore (2012) and shows that the proportion of sellers of high quality assets is lower in a recession, which can lead to market shutdowns. Martin (2009) shows that the relationship between entrepreneurial wealth and aggregate investment, which is the basis of the already mentioned “financial accelerator”, may not be monotonic. In particular, in states with low entrepreneurial wealth, screening of borrowers using collateral requirements may be too costly, and therefore the economy is in a pooling equilibrium, in which good borrowers cross-subsidize bad borrowers.

Recent papers study the role of asymmetric information on the interbank market. Heider, Hoerova, and Holthausen (2009) show that asymmetric information about counterparty risk can produce market breakdowns. Boissay, Collard, and Smets (2013) explain, in a model with moral hazard and asymmetric information, why interbank market freezes are more likely after a credit boom. While in this paper I focus on securitization markets, I find similar results: The liquidity problems on the securitization markets are more severe in recession especially after a prolonged boom period.

One of the major assumptions in the model is the existence of a dispersion shock, which is inspired by the empirical evidence on countercyclical, cross-sectional variance in the TFP of US firms in Bloom (2009) and Bloom et al. (2012). These authors also build models that assume time-varying variance of idiosyncratic TFP shocks and show that higher variance can cause a recession. Bigio (2013) uses a similar assumption and shows that a dispersion shock due to the existence of asymmetric information will worsen the adverse selection problem and create a recession. Compared to Bigio (2013), my model features reputation-based signaling, which is more effective when the dispersion is larger.

In this paper, the quality of investment decreases in the boom stage of the business cycle. There is much related literature that deals with the evolution of bank lending standards over the business cycle. In an empirical paper, Lown and Morgan (2006) document how bank lending standards in the US deteriorated in the boom stages of the business cycle. In theoretical models with asymmetric information about the quality of borrowers and a costly screening by banks, Dell’Ariccia and Marquez (2006) and Ruckes (2004) suggest the reasons for the countercyclical bank lending standards. In Dell’Ariccia and

Marquez (2006), booms are periods with a lower share of low quality borrowers; therefore, banks, due to competition, decide not to require collateral in those periods. In Ruckes (2004), boom periods are related to lower borrower default probabilities, which induce banks to screen less. This results in lower bank lending standards in the boom, which is similar to the outcome of this paper. However, in this model, the asymmetric information exists among financial firms trading securitized loans, and the adverse selection can be alleviated by reputation-based signaling. Also unlike the mentioned models, my model is fully dynamic and is better suited to study the time dimension of the asymmetric information related effects.

There are also several papers that study the importance of reputation in the lender-borrower relationships. Nikolov (2012) introduces reputation in the model of Kiyotaki and Moore (1997) and shows that reputation represents intangible capital, which is more valuable in the boom stage of the business cycle, and therefore it further strengthens the collateral amplification mechanism. Ordoñez (2012) argues that unregulated banking disciplined only by reputation forces may be efficient due to the saving on regulatory and bankruptcy costs, but is more fragile.

My model is also related to research about the degree of asymmetric information over the business cycle. While some researchers argue that booms are associated with a higher degree of trading and therefore more learning Veldkamp (2005), others argue that information may be lost in boom periods of business cycles. Gorton and Ordoñez (2014) present a model where assets with unknown value can serve as collateral for borrowing. In booms, none of the parties has the incentive to verify the value of an asset, and the economy saves on information acquisition costs and enjoys a “bliss-full ignorance” equilibrium, while in periods with low aggregate productivity lenders have incentives to verify the value of collateral, which leads to underinvestment. In my model, higher productivity will also be associated with less public information, but this would create inefficiencies.

### 1.3 Model

To allow for maximum tractability, the set-up of the model is rather simple. The economy contains a continuum of financial firms which have stochastic investment opportunities. The problem in this model is to transfer resources from firms without investment opportunities or with low quality investment opportunities to firms with the best investment



opportunities. The transfer of funds is possible through securitization, which is modeled as a sale of cash flows from the funded projects.<sup>4</sup>

### 1.3.1 Model set-up

#### 1.3.1.1 Investment projects

There are three types of projects available to financial firms and the allocation of firms to projects is stochastic through an i.i.d. shock:

- $(1 - \pi)$  share of firms (subset  $\mathcal{Z}_t$ ) don't have access to new investment projects;
- $\pi\mu$  share of firms (subset  $\mathcal{H}_t$ ) have access to high quality projects with high gross profit per unit of capital  $r_t^h = A_t^h K_t^{\alpha-1}$ ; and
- $\pi(1 - \mu)$  share of firms (subset  $\mathcal{L}_t$ ) have access to low quality projects with low gross profit per unit of capital  $r_t^l = A_t^l K_t^{\alpha-1}$ .

This shock cannot be insured.

**Assumption 1:** I assume that the relative difference in gross profits from high and low quality projects is countercyclical:

$$\frac{\partial}{\partial A_t} \frac{A_t^h - A_t^l}{A_t^l} < 0, \quad (1.1)$$

where  $A_t$  is the aggregate component of the total factor productivity (TFP) of the projects.

This assumption is inspired by the empirical evidence on countercyclical cross-sectional variance in the TFP of US firms in Bloom (2009) and Bloom et al. (2012).<sup>5</sup> In this model the TFP of the projects has an aggregate component,  $A_t$ , and a type-specific component,  $\Delta_t^h$  and  $\Delta_t^l$  resp.:  $A_t^h = A_t \Delta_t^h$  and  $A_t^l = A_t \Delta_t^l$ . To satisfy the assumption in (1.1) the ratio of type-specific TFP components has to be countercyclical,  $\partial (\Delta_t^h / \Delta_t^l) / \partial A_t < 0$ .

Some of the basic features of the model are inspired by Kiyotaki and Moore (2012). Similarly to Kiyotaki and Moore (2012), agents are subject to an i.i.d. investment shock

---

<sup>4</sup>To keep the model simple, I do not model any alternative means of transferring funds like debt. Elsewhere I presents an extension of this model, where different types of debt, such as deposits or interbank loans, are considered and replicates the main qualitative results of this paper (Kuncl 2013).

<sup>5</sup>Bloom (2009) and Bloom et al. (2012) depart from the empirical evidence and build models that assume a time-varying variance of idiosyncratic TFP shocks and show that higher variance can cause a recession.

and face constant returns to scale, i.e., they take  $r_t^h$ , resp.  $r_t^l$  as given; however, on the aggregate level there are decreasing returns to scale:

$$Y_t = r_t^h H_t + r_t^l L_t = \left( A_t^h \frac{H_t}{K_t} + A_t^l \frac{L_t}{K_t} \right) K_t^\alpha,$$

where  $K_t = H_t + L_t$  and  $H_t (L_t)$  are aggregate holdings of high (low) quality capital.<sup>6</sup>

### 1.3.1.2 Frictions

Two **core frictions** are assumed in the model:

- Investing firms, which sell securitized loans, have to keep “*skin in the game*”, i.e., at least  $(1 - \theta)$  fraction of the investment on their balance sheet. This means they can sell at most  $\theta$  fraction of the current investment and the rest has to be financed from their own resources. For simplicity,  $\theta$  is taken throughout most of the paper as a parameter. However, in section 1.5 this friction is endogenized by the existence of a moral hazard problem.
- There is an *asymmetry of information* about the above-described allocation of investment opportunities among firms. Each firm knows the type of the project it is assigned to in the current period, but it is not aware of the allocation of projects among other firms.

The second friction is motivated by the reality of the securitization market and by the aforementioned criticism of securitization, which takes the asymmetric information as the source of most of the agency problems (for details see the literature review). The first friction can also be observed in reality, but the main reason I include it in this otherwise simple model is that despite the competition among financial firms, a binding “skin in the game” constraint increases equilibrium prices above the costs of investment and, therefore, makes the securitization process profitable. Only when securitization is profitable, does a reputation equilibrium exist with implicit recourse, where the losing of reputation for providing implicit recourse is costly. As I explain later, a firm without the

---

<sup>6</sup>Kiyotaki and Moore (2012) obtain this result by including labor in the production function and requiring a competitive wage to be paid to workers in order to run a project. Here, for simplicity, I omit the workers from the model, but I use the results of constant returns to scale on the individual level and decreasing returns to scale on the aggregate level by assumption.

reputation of providing implicit recourse will be unable to securitize and sell the projects in which they have invested, and therefore, it would lose the profits from securitization.<sup>7</sup>

### 1.3.1.3 Firms' problem

Each financial firm (indexed by  $i$ ) chooses the control variables  $\{c_{i,t+s}, i_{i,t+s}, \{a_{i,j,t+s+1}\}_j, h_{i,t+s+1}^S, l_{i,t+s+1}^S, r_{i,t+s+1}^G, \chi_{i,t+s}\}_{s=0}^\infty$  to maximize the expected discounted utility from the future consumption stream:

$$\sum_{s=0}^{\infty} \beta^s u(c_{i,t+s}),$$

where  $u(c_{i,t+s}) = \log(c_{i,t+s})$ . The budget constraint for all firms is

$$\begin{aligned} c_{i,t} + i_{i,t} (1 - q_{i,t}^G) + \sum_{j \in \mathcal{I}_t} a_{i,j,t+1} q_{j,t}^G + h_{i,t+1}^S q_t^h + l_{i,t+1}^S q_t^l + \chi_{i,t} c_{i,r,t} \\ = \sum_{j \in \mathcal{I}_{t-1}} a_{i,j,t} (r_{j,t}^G + \lambda q_{j,t}) + h_{i,t}^S (r_t^h + \lambda q_t^h) + l_{i,t}^S (r_t^l + \lambda q_t^l) \quad \forall i, \forall t, \end{aligned}$$

and firms with no investment opportunities face an additional constraint  $i_{i,t} = 0 \forall i \in \mathcal{Z}_t$ . This constrained maximization problem describes the following options of firms. The resources of firms consist of stochastic gross profits from projects financed in the past and the market value of a non-depreciated part  $\lambda$  of those projects. They consume the  $c_{i,t}$  part of those resources. If they have an investment opportunity, they can invest at unit costs into new project  $i_{i,t}$ .<sup>8</sup> I denote the subset of firms that decide to invest into new projects (issue new loans) as  $\mathcal{I}_t$ . They can also buy securitized cash flows from newly financed projects on the primary market  $\{a_{i,j,t+1}\}_j$  for prices  $\{q_{j,t}\}_j$  or securitized cash flows from older projects of known high (low) quality on the secondary (re-sale) market  $h_{i,t+1}^S (l_{i,t+1}^S)$  for price  $q_t^h (q_t^l)$ , where  $j \in \mathcal{I}_t$  and superscripts  $h, l$  denote the known quality of the traded asset. Investing firms can securitize and sell cash flows from the newly issued projects. If they sell a part of their investment<sup>9</sup>, they can provide implicit recourse

---

<sup>7</sup>I assume that it is possible to commit to not buying securitized assets from a particular firm and show that such commitment can be credible if the related incentive compatibility constraint holds. However, I assume that it is not possible to prevent a particular firm from buying securitized assets from others, i.e., a threat of complete autarky is not possible. I believe this assumption corresponds to the reality of securitization markets.

<sup>8</sup>Gertler and Kiyotaki (2010) in their study of the interbank market, based on the same modeling approach as Kiyotaki and Moore (2012), refer to investments into projects as loans to entrepreneurs who run those projects. Entrepreneurs are able to offer a perfectly state contingent debt, and since financial firms (banks) have all the bargaining power, they can extract the entire profits from entrepreneurs. Following this approach, I will sometimes refer to the investment into projects as loans too and later calibrate this model on the performance of mortgage-backed securities.

<sup>9</sup>The amount of new loans kept on the balance sheet is the difference between investment  $i_t$  and the next period holdings of assets of firm  $i$  issued by the firm  $i$ :  $a_{i,i,t+1}$ , while  $i_t - a_{i,i,t+1} \geq 0$ .

to buyers of these newly securitized assets in the form of a promise for minimum gross profit per unit of capital next period  $r_{i,t+1}^G$ . An asset with implicit recourse is traded for a market price  $q_{i,t}^G$ , which depends on the information structure in the equilibrium, i.e., on the beliefs of buyers about the type of the sold asset. Each firm can decide whether to default on the implicit recourse from the previous period or not, which is represented by  $\chi_t$ .<sup>10</sup> If a firm honors the implicit recourse, it has to spend part of its resources on covering related costs  $cir_{i,t}$ . The details on the cost of implicit recourse and the choice of default are discussed in detail in sub-section 1.3.2.4. The timing of shocks and choice of controls by firms within each period is shown in Figure 1.1.

Note that since profits (cash flows) are observed and  $\Delta^h, \Delta^l$ , and  $A_t$  are public information, the uncertainty about the quality of financed projects is resolved at latest in the period following the investment in the project. Therefore, depending on the particular equilibrium, the quality of assets traded on the primary market may be either public or private information, and when these assets are traded in the next period on the secondary market, their quality is already public information. Therefore, we can collapse all assets issued in past periods into two categories of high and low quality assets:  $h^S, l^S$ .<sup>11</sup> Laws of motion for high and low quality assets traded on re-sale markets are

$$\begin{aligned} H_{t+1}^S &= \sum_i h_{i,t+1}^S = \sum_i \sum_{j \in \mathcal{H}_{t-1}} \lambda a_{i,j,t} + \sum_i \lambda h_{i,t}^S, \\ L_{t+1}^S &= \sum_i l_{i,t+1}^S = \sum_i \sum_{j \in \mathcal{L}_{t-1}} \lambda a_{i,j,t} + \sum_i \lambda l_{i,t}^S. \end{aligned}$$

Since the uncertainty about project quality lasts only for one period, for simplicity and tractability I also restrict the guarantee on the loan performance to one period after the issuance.

Since utility is logarithmic and budget constraints are linear in individual holdings of assets, the policy functions will also be linear in the individual holdings of wealth. Due to logarithmic utility, all firms will always consume a constant fraction of their current wealth (for derivation see appendix 1.A.2):

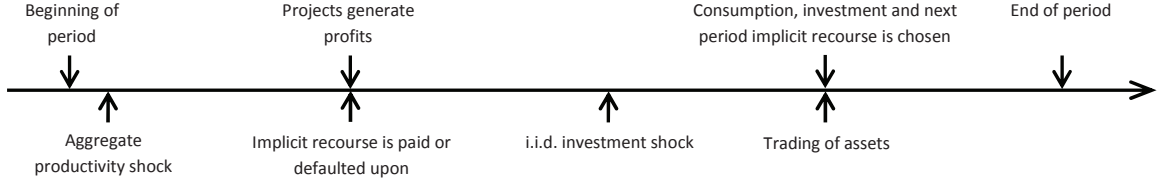
$$c_{i,t} = (1 - \beta) \left( \sum_{j \in \mathcal{I}_{t-1}} a_{i,j,t} (r_{j,t}^{\hat{G}} + \lambda q_{j,t}) + h_{i,t}^S (r_t^h + \lambda q_t^h) + l_{i,t}^S (r_t^l + \lambda q_t^l) \right) \forall i.$$

<sup>10</sup> $\chi_t$  takes the value 1 in case of no-default and 0 in case of default.

<sup>11</sup>In chapter 2 of the dissertation, I relax this assumption and introduce asymmetric information on secondary markets also.

Linear policy functions and i.i.d. investment opportunities enable easy aggregation. An application of the law of large numbers implies that the aggregate quantities and prices do not depend on the distribution of wealth across individual firms.

**Figure 1.1:** Timing of shocks and the choice of a firm's controls within each period



### 1.3.1.4 Goods and asset markets

The model features a market for consumption goods and for capital goods (securitized cash flows from projects). In every period all projects generate gross profits in the form of consumption goods. Consumption goods must be either consumed or converted into capital goods by an investment into new projects. Consumption good markets clear when all current output  $Y_t$  is consumed or invested:  $Y_t = C_t + I_t$ .

Capital goods are traded on asset markets. There is a secondary market on which assets of known quality are traded and a primary market for newly issued assets whose quality is either known or not depending on the type of the equilibrium. As derived in Appendix 1.A.2, the conditions for the clearing of asset markets come from the first order conditions of firms, which buy on asset markets (subset  $\mathcal{S}_t$ ), and which we will call saving firms  $i \in \mathcal{S}_t$ . These conditions imply that the discounted return of all assets traded on markets have to be equal to 1, and that in equilibrium, saving firms will be indifferent between holding different assets.

Asset markets clearing conditions:

$$\begin{aligned}
 E_t \left[ \beta \frac{c_{i,t}}{c_{i,t+1}} \frac{r_{t+1}^{\hat{G}} + \lambda q_{j,t+1}^{\hat{G}}}{q_{j,t}^{\hat{G}}} \right] &= 1 \quad \forall i \in \mathcal{S}_t, \forall j \in \mathcal{I}_t, \\
 E_t \left[ \beta \frac{c_{i,t}}{c_{i,t+1}} \frac{r_{t+1}^h + \lambda q_{t+1}^h}{q_t^h} \right] &= 1 \quad \forall i \in \mathcal{S}_t, \\
 E_t \left[ \beta \frac{c_{i,t}}{c_{i,t+1}} \frac{r_{t+1}^l + \lambda q_{t+1}^l}{q_t^l} \right] &= 1 \quad \forall i \in \mathcal{S}_t.
 \end{aligned}$$

Recall that all assets depreciate over time, so the law of motion for capital (stock of

projects) is  $K_{t+1} = \lambda K_t + I_t$ .<sup>12</sup>

### 1.3.2 Model solution in special cases

To demonstrate the effect of the core frictions in the model, I will first briefly show in this sub-section the behavior and solution of the model without frictions. Then, I will successively introduce a binding “skin in the game” and the asymmetric information. I show that when the “skin in the game” is binding, a reputation equilibrium exists, where implicit recourse can be provided. In the next sub-section, I will show the solution of the model in the case of interest, where both frictions hold and the provided implicit recourse can signal the quality of the securitized cash flows from projects and result in a separating equilibrium, where the inefficiency related to asymmetric information is eliminated.

To show the results analytically, I will, in the next sub-sections, mostly refer to the case with constant aggregate productivity  $A_t = A$ . In section 1.4, I report numerical results from the fully stochastic case.

#### 1.3.2.1 Case with no financial frictions - first best

If none of the two frictions are present, i.e., project allocation is public information and the “skin in the game” constraint is not binding, in equilibrium only firms with high quality investment opportunities will invest, securitize loans, and sell them to firms with low or unproductive investment opportunities. Since there is no asymmetric information and only high quality projects are being financed, there is only one type of asset traded in the economy. When I omit the variables that turn out to be zero in equilibrium, the budget constraints of individual firms with different investment opportunities are:

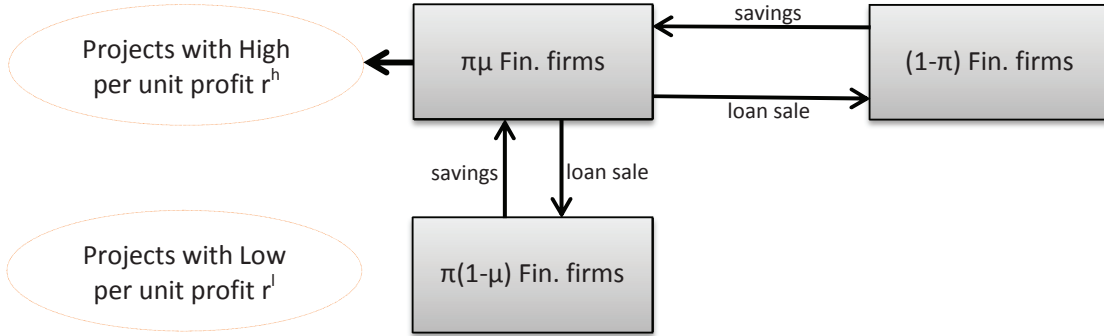
$$\begin{aligned} c_{i,t} + i_{i,t} + (h_{i,t+1} - i_{i,t})q_t^h &= h_{i,t}(r_t^h + \lambda q_t^h) \quad \forall i \in \mathcal{H}_t, \\ c_{i,t} + h_{i,t+1}q_t^h &= h_{i,t}(r_t^h + \lambda q_t^h) \quad \forall i \in \mathcal{L}_t, \\ c_{i,t} + h_{i,t+1}q_t^h &= h_{i,t}(r_t^h + \lambda q_t^h) \quad \forall i \in \mathcal{Z}_t. \end{aligned}$$

Because of competition among firms with high quality investment opportunities, the price of loans is equal to the unit costs of financing the project (issuing the loan),  $q^h = 1$ .

---

<sup>12</sup>Similar laws hold for both types of capital (low quality and high quality):  $H_{t+1} = \lambda H_t + I_t^h$ ,  $L_{t+1} = \lambda L_t + I_t^l$ . Similarly to Kiyotaki and Moore (2012), I assume that the subjective discount factor exceeds the share of capital left after depreciation:  $\beta > \lambda$ .

**Figure 1.2:** Case without frictions - First best case



Note: In the first best case, only firms with access to projects with high profit per unit of capital invest, and they sell some of these projects to remaining firms.

Combining the aggregate consumption function, the goods market clearing condition, and the law of motion for capital, we obtain<sup>13</sup>:

$$r^h + \lambda = \frac{1}{\beta}. \quad (1.2)$$

The current period gross profit per unit of invested capital plus the value of non-depreciated assets is equal to the time preference rate; therefore, the amount of investment is indeed first best.

### 1.3.2.2 Introducing the “skin in the game” constraint

In this section I show that a binding “skin in the game” constraint ( $\theta$  fraction of new loans at most can be sold) increases the equilibrium prices above the replacement rate, which makes securitization profitable. As noted above, only when securitization is profitable, can a reputation equilibrium exist. The “skin in the game” constraint is also a usual practice observed in securitization contracts in the form of tranche retention schemes<sup>14</sup>. This constraint can be motivated and endogenized by a moral hazard problem, which is derived in section 1.5. Section 1.5 also discusses some potential policy implications when making  $\theta$  a policy parameter. In this section, I assume for simplicity a constant  $\theta$ .

By lowering  $\theta$ , we limit the capacity of firms with access to high quality projects to

<sup>13</sup>For details see Appendix 1.A.1.1.

<sup>14</sup>For simplicity, I do not model the existence of different tranches. The “skin in the game” constraint is analogous to keeping a “vertical slice” of all tranches.

issue new investments. When this capacity is lower than the demand for new investments at the zero-profit price  $q^h = 1$ , then the “skin in the game” constraint becomes binding, and the price has to increase above the unit costs of investment to clear the market. Securitization becomes profitable.

If the “skin in the game” is binding in equilibrium for firms with access to high quality projects, i.e., their holdings of newly issued assets represent  $(1 - \theta)$  fraction of their investment  $h_{i,t+1} = a_{i,i,t+1} = (1 - \theta) i_{i,t} \forall i \in \mathcal{H}_t$ <sup>15</sup>, we can rewrite their budget constraint to:

$$c_{i,t} + \frac{(1 - \theta q_t^h)}{(1 - \theta)} h_{i,t+1} = h_{i,t}(r_t^h + \lambda q_t^h) + l_{i,t}(r_t^l + \lambda q_t^l) \forall i \in \mathcal{H}_t. \quad (1.3)$$

Combining these two equations and the consumption function we can find the level of investment of the constrained firm with access to high quality projects:

$$i_{i,t}^h = \frac{\beta (h_{i,t}(r_t^h + \lambda q_t^h) + l_{i,t}(r_t^l + \lambda q_t^l))}{(1 - \theta q_t^h)} \forall i \in \mathcal{H}_t. \quad (1.4)$$

All policy functions are again linear, and therefore can be easily aggregated and, as Appendix 1.A.1.2 shows, we can obtain the following proposition.

**Proposition 1.** *If “skin in the game” is sufficiently large to be binding, i.e.,  $\theta$  is sufficiently low to satisfy*

$$1 - \theta > \frac{\pi\mu}{1 - \lambda},$$

*then in the deterministic steady state:*

- (i) *the price of high quality assets  $q^h$  exceeds 1;*
- (ii) *the steady state level of output and capital is lower than in the first best case.*

The above proposition is analogous to Claim 1 in Kiyotaki and Moore (2012), but for a complete characterization of the model’s steady state, we also need the following proposition.

**Proposition 2.** *Suppose the condition from Proposition 1 holds, then depending on parameter values, deterministic steady state is characterized by one of the following cases:*

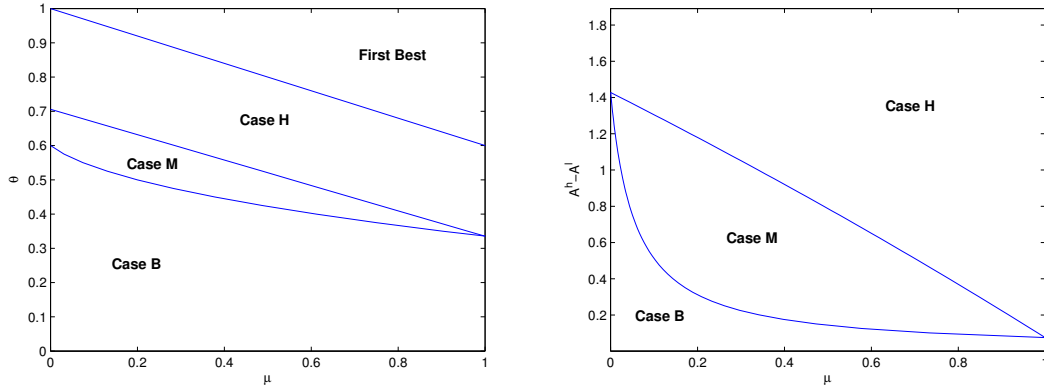
*Case H: Only firms with access to high quality projects issue credit and securitize ( $q^l < 1$ );*

---

<sup>15</sup>I show below that for a subset of parameters, firms with access to low quality projects will be also investing and securitizing loans in equilibrium. They may also face the binding “skin in the game” constraint, i.e.,  $l_{i,t+1}^l = a_{i,i,t+1} = (1 - \theta) i_{i,t}^l \forall i \in \mathcal{L}_t$ .



**Figure 1.3:** Type of deterministic steady state depending on selected parameter values



*Case M:* Firms with access to low quality loans use a mixed strategy and issue credit with probability  $\psi$ , ( $q^l = 1$ );

*Case B:* All firms with access to high and low quality projects issue credit and securitize ( $q^l > 1$ ).

The above cases are ranked from the least restricted ( $q^l < 1$ ), where output and capital levels are relatively the closest to the first best case, to the most restricted ( $q^l > 1$ ), where output and capital are the lowest:

$$Y_{FB} > Y_H > Y_M > Y_B,$$

$$K_{FB} > K_H > K_M > K_B,$$

where subscript FB denotes first-best case, subscript H, M and B denote the above described cases.

Proof of the above propositions are in the appendices (1.A.1.2 and 1.A.1.3).

Figure 1.3 shows the effect of selected parameter values on the type of the steady state. In the left panel we can see that lowering  $\theta$  or  $\mu$  moves the steady state from an unrestricted first-best case to more restricted cases. The right panel shows that lowering the difference in the productivity of the two types makes it more likely that low quality projects would be financed in the steady state.

### 1.3.2.3 Introducing asymmetric information

In this sub-section, I describe the consequences of introducing asymmetric information about the allocation of investment opportunities among firms on the model solution. I

focus on the effect of asymmetric information between issuers of securitized assets and their first buyers; therefore, at this point, I do not consider asymmetric information on re-sale markets.<sup>16</sup>

Unless the difference in qualities is large enough, firms with access to low quality projects mimic firms with access to high quality projects. Since it is not possible to distinguish between the projects, saving firms, which want to diversify their portfolio, buy both high and low quality securitized assets at the rate corresponding to the probabilities of their arrival. This means that in equilibrium a  $\mu$  fraction of investment is allocated to high quality and a  $1 - \mu$  fraction to low quality projects.

**Proposition 3.** *Compared to the public information case, the allocation of capital is generally less efficient (more in favor of low quality projects); therefore, the capital is less productive and, in the steady state, the amount of capital and output is lower.*

For proof see Appendix 1.A.1.4.

The public information case will be equal to the private information case only if the difference in the qualities is large enough. The firm with low quality investment opportunities will avoid mimicking firms with high quality investment opportunities as long as the return from buying high quality assets exceeds the return from mimicking:

$$R \mid \text{buying high loans} > R \mid \text{mimicking}.$$

As shown in Appendix 1.A.1.5, in the steady state this condition implies

$$\frac{A^h}{A^l} > \frac{(1 - \theta) q^h}{1 - \theta q^h} = \frac{(1 - \pi\mu)(1 - \lambda)(1 - \theta)}{\pi\mu\lambda + (1 - \lambda)\theta\pi\mu}. \quad (1.5)$$

If the ratio of the high and low productivity does not satisfy (1.5), the resulting pooling equilibrium will be less efficient than the public information case. The separation condition can also be rewritten as

$$q^l < \frac{1 - \theta q^h}{1 - \theta}. \quad (1.6)$$

Since, by Proposition 1,  $q^h > 1$ , (1.6) implies that a necessary condition for the

---

<sup>16</sup>I assume that past projects are not anonymous; therefore, the quality of all existing projects becomes public information in the period following their securitization. In chapter 2 of the dissertation, I relax this assumption and show that if asymmetric information exists in general between the buyer and seller on the re-sale markets, there can be partial market shutdowns similar to those found by Kurlat (2013).

existence of a separating equilibrium is that the equilibrium price of low quality assets is lower than the costs of investing  $q^l < 1$ .

Note also that increasing the “skin in the game”, i.e., lowering  $\theta$  will only increase the lower bound for the ratio of productivities in the condition 1.5 and, therefore, make mimicking more likely. This result is driven by the general equilibrium effect. A lower  $\theta$  increases the prices in the economy and, therefore, makes mimicking more profitable.

**Proposition 4.** *Under private information, increasing the “skin in the game”, i.e., lowering  $\theta$ , makes pooling equilibrium, in which firms with low quality investment opportunities mimic firms with high quality investment opportunities, more likely.*

#### 1.3.2.4 Introducing implicit recourse and the reputation equilibrium case

Proposition 3 implies that the outcome of a private information case is generally inefficient compared to a public information case. Firms with high quality investment opportunities have incentives to distinguish themselves from low quality investment firms. However, under Proposition 4, we can see that retaining higher “skin in the game” does not lead to a separating equilibrium.

It turns out that by providing **implicit recourse**, a firm with high quality investment opportunities can distinguish itself without restricting its investment potential. Under this strategy, the issuing firm promises minimum gross profit per unit of invested capital  $r_t^G$  to the buyers of securitized loans. Should the actual gross profits in the following period fall below this minimum, the issuing firm would reimburse the difference. This promise is not enforced by any explicit contract; rather, it is a result of collusion between issuers of loans and their buyers<sup>17</sup>. Implicit recourse can be enforced in a reputation equilibrium, where securitizing firms aim to keep their reputation of sticking to the promise, and firms buying securitized projects enforce this promise by punishing the issuing firms in case of default on the implicit recourse. I assume a trigger strategy punishment that prevents a firm without a reputation of honoring implicit recourse from selling securitized assets on the market. The punishment has to be credible; therefore, in this reputation equilibrium, buyers of securitized products with implicit support aim to keep a reputation of being

---

<sup>17</sup>In this paper, I do not compare the advantages of implicit and explicit guarantees. Based on the observed empirical evidence, I model only the implicit guarantee. Reasons for a provision of implicit rather than explicit guarantees can be various. Regulatory arbitrage is probably the major reason. Also, the individual as well as the social costs of default on an implicit guarantee (costs of punishment) can be lower than costs of default on an explicit guarantee, which can be represented by liquidation costs (Ordoñez (2012), mentions the second reason).

“tough investors”, i.e., a reputation of always punishing firms that did not fulfill their promise.

At this point, it is convenient to write the problem recursively:

$$V^{ND}(\bar{s}, w - cir; \bar{S}) = \pi \left( \mu V^{ND,h}(\bar{s}, w - cir; \bar{S}) + (1 - \mu) V^{ND,l}(\bar{s}, w - cir; \bar{S}) \right) + (1 - \pi) V^{ND,z}(\bar{s}, w - cir; \bar{S}), \quad (1.7)$$

$$V^D(\bar{s}, w; \bar{S}) = \pi \left( \mu V^{D,h}(\bar{s}, w; \bar{S}) + (1 - \mu) V^{D,l}(\bar{s}, w; \bar{S}) \right) + (1 - \pi) V^{D,z}(\bar{s}, w; \bar{S}), \quad (1.8)$$

$$V^{ND,k}(\bar{s}, w; \bar{S}) = \max_{c,i,\{a'_j\}_j, h^{S'}, l^{S'}, r^{G'}} [\log(c) + \beta E \left[ \max \left( V^{ND}(\bar{s}', w' - cir'; \bar{S}'), V^D(\bar{s}', w'; \bar{S}') \right) \right]], \quad (1.9)$$

$$V^{D,k}(\bar{s}, w; \bar{S}) = \max_{c,i,\{a'_j\}_j, h^{S'}, l^{S'}} \left[ \log(c) + \beta E V^D(\bar{s}', w'; \bar{S}') \right], \quad (1.10)$$

where  $V^{ND}$  ( $V^D$ ) are the value functions for the firm that never defaulted (has already defaulted) on implicit recourse.  $w$  is individual wealth before deducting the costs of implicit recourse  $cir$ ,  $\bar{s} = \left\{ \{a_j\}_j, h^S, l^S \right\}$  is a vector of individual state variables,  $\bar{S} = \{K, \omega, A\}$  is a vector of aggregate state variables, and superscript  $k$ , which can take values  $\{h, l, z\}$ , represents the type of investment opportunity that the firm faces in the current period.

Equations (1.7) and (1.8) show the investment shock that takes place after the realization of the aggregate productivity shock and the decision on (non)default on implicit recourse from the previous period. After the investment shock, firms optimally choose the level of consumption, the quantity of securitized loans they buy on the primary and secondary market, and if they have an investment opportunity, they choose the optimal level of investment into new projects, the securitization of their cash flows, the fraction of the new investment which is sold, and the implicit recourse they provide.<sup>18</sup> This problem is described by equations (1.9) and (1.10) for firms with a reputation for having never defaulted on implicit recourse and without this reputation, respectively.

The above problem is constrained by budget constraints that take the following form for investing firms for which the “skin in the game” constraint is binding (e.g. in the case where firms have high investment opportunities):

$$c_{i,t} + \frac{(1 - \theta q_{i,t}^G)}{(1 - \theta)} h_{i,t+1} + cir_{i,t} = \sum_{j \in \mathcal{I}_{t-1}} a_{i,j,t} \left( r_{j,t}^{\hat{G}} + \lambda q_{j,t} \right) + h_{i,t}^S (r_t^h + \lambda q_t^h) + l_{i,t}^S (r_t^l + \lambda q_t^l) \quad \forall i \in \mathcal{H}_t,$$

where the price of securitized loans issued by firm  $j$ :  $q_{j,t}^G$  depends on the information

---

<sup>18</sup>Recall that the timing of shocks and the choice of controls by firms within each period is shown in Figure 1.1.

structure, i.e., on the beliefs of buyers about the type of the sold asset  $\varphi_{j,t} \mid r_{j,t}^G$ . When the “skin in the game” is binding, the costs of implicit recourse are given by:

$$cir_{i,t+1} = \theta_{i,t} (r_{i,t}^G - r_t^k) \quad \forall i \notin \mathcal{S}_t, k \in \{h, l\}.$$

The incentive compatible constraints (ICCs), which have to be satisfied in equilibrium for the existence of reputation-based implicit recourse, are the following:

$$V^{ND}(\bar{s}, w - cir; \bar{S}) \geq V^D(\bar{s}, w; \bar{S}); \quad (1.11)$$

$$V^P(\bar{s}; \bar{S}) \geq V^{NP}(\bar{s}; \bar{S}), \quad (1.12)$$

where  $V^P, V^{NP}$  are the value functions for the firm that is always punished for default on implicit recourse, and failed to punish for default, respectively. Condition 1.11 determines the level of implicit recourse that can be credibly provided, i.e., it is not defaulted upon, given the trigger strategy punishment rule. The trigger punishment strategy has to be credible; therefore, the saving firm which observes default on implicit recourse has to be better off punishing the investing firm that defaulted rather than not punishing it. This corresponds to the condition 1.12.<sup>19</sup>

**Definition 1.** A recursive competitive equilibrium consists of prices  $\{q^h(\bar{S}), q^l(\bar{S}), \{q_j^G(\bar{S})\}_j\}$  and gross profits per unit of capital  $\{r^h(\bar{S}), r^l(\bar{S})\}$ , individual decision rules  $\{c(\bar{s}; \bar{S}), h^{S'}(\bar{s}; \bar{S}), l^{S'}(\bar{s}; \bar{S}), r^{G'}(\bar{s}; \bar{S}), \{a'_j(\bar{s}, r_j^G, \varphi_j \mid r_j^G; \bar{S})\}_j, \chi(\bar{s}, r^G; \bar{S})\}$ , value functions  $\{V^{ND}(\bar{s}; \bar{S}), V^{ND,k}(\bar{s}; \bar{S}), V^D(\bar{s}; \bar{S}), V^{D,k}(\bar{s}; \bar{S}), V^{NP}(\bar{s}; \bar{S}), V^P(\bar{s}; \bar{S})\}$ , and the law of motion for  $\bar{S} = \{K, \omega, A, \Sigma\}$  such that: (i) individual decision rules and value functions solve each firm’s problem taking prices, gross profits per unit of capital, and law of motion for  $\bar{S} = \{K, \omega, A\}$  as given; (ii) both asset and good markets clear, and (iii) the law of motion for  $\bar{S} = \{K, \omega, A\}$  is consistent with the individual firms’ decisions.

### 1.3.2.5 Public information case with implicit recourse

Although one might think that the public information case is uninteresting, it is an important benchmark. If issuing firms could coordinate, they wouldn’t be providing implicit recourse in this case, where it does not serve as a tool that would distinguish the firm type. However due to competition, firms tend to out-bet each other.

---

<sup>19</sup>I show that this condition holds in Appendix 1.A.1.6.

Should promises always be credible, the optimal level of implicit recourse would be determined by the following F.O.C. (note that the individual firm ignores the effects of this choice on aggregate variables):

$$\frac{\partial V^{ND}}{\partial r^G} = \frac{\partial V^{ND'}}{\partial (w' - cir')} \frac{\partial (w' - cir')}{\partial r^G} = 0.$$

I show in Appendix 1.A.1.7 that this condition implies that  $q^j = 1$ , which means that as far as there are positive profits from securitization, the competition will drive the level of implicit recourse so high that profits from securitization are zero. However, when profits from securitization are zero, the punishment has zero costs, and the original non-defaulting incentive compatibility constraint (1.11) is not satisfied. This leads us to the following conclusion.

**Proposition 5.** *As long as the implicit recourse is credible, firms find it optimal to increase it up to the level where  $q^j = 1$ . So the level of implicit recourse is defined by the maximum which can be sustained by the no-default condition (1.11).*

For details on the derivation see Appendix 1.A.1.7. The steady state, in this case, is characterized by the following propositions.

**Proposition 6.** *Suppose that the condition from Proposition 1 holds, then depending on parameter values, a deterministic steady state is characterized by one of the following cases:*

*Case 1: Only firms with access to high quality projects issue credit, securitize loans, and provide implicit recourse  $r_{h,cred}^G$  ( $q^h > 1$ ,  $q^l < 1$ ,  $G_{cred}^h \geq r^h$ );*

*Case 2: Firms with access to high quality projects issue credit, securitize loans, and provide implicit recourse  $r_{h,cred}^G$ , and firms with access to low quality projects use a mixed strategy and issue credit with probability  $\psi$  and provide implicit recourse  $r_{l,cred}^G$  ( $q^h > 1$ ,  $q^l = 1$ ,  $r_{h,cred}^G \geq r^h$ ,  $r_{l,cred}^G = r^l$ );*

*Case 3: All firms with access to high and low quality projects issue credit, securitize, and provide implicit recourse  $r_{h,cred}^G$  and  $r_{l,cred}^G$  resp. ( $q^h > 1$ ,  $q^l > 1$ ,  $r_{h,cred}^G \geq r^h$ ,  $r_{l,cred}^G \geq r^l$ ).*

Note that  $r_{k,cred}^G$  is the maximum implicit recourse that can be credibly provided by firms with a  $k \in \{h, l\}$  type of investment opportunity.

**Proposition 7.** *Compared to the public information case without implicit recourse, the amount of capital and output are higher, the allocation of capital is more in favor of high quality projects and wealth is less concentrated inside firms with investment opportunities. This holds in all cases except when the provided implicit recourse has no value ( $r_{h,cred}^G = r^h$ ), and the two cases are identical.*

### 1.3.3 Case of interest: Implicit recourse as a signal of loan quality

In this section, I analyze the case of interest, where the “skin in the game” constraint is binding, where there is asymmetric information about the allocation of firms to investment opportunities, and where the implicit recourse can signal the type of investment opportunity.

As proved in sub-section 1.3.2.4, implicit recourse can be credibly provided in a reputation equilibrium. Under asymmetric information, implicit recourse can be interpreted as a signal of the loan quality. Investing firms (subset  $\mathcal{I}_t$ ) sell securitized cash flows from newly financed projects and provide implicit recourse  $r_{j,t+1}^G \in (0, \infty)$ . The fact that a particular firm sells securitized cash flows and provides  $r_{j,t+1}^G$  is the message that this firm is sending to potential buyers of its securitized cash flows. Saving firms (subset  $\mathcal{S}_t$ ) observing any message sent with positive probability use Bayes’ rule to compute the posterior assessment that the message comes from each type. Without restriction on out-of-equilibrium beliefs (beliefs about the types conditioned on observing messages that are not sent in equilibrium), there is a multiplicity of Perfect Bayesian Equilibria, generally both pooling and separating. I use the Intuitive Criterion (Cho and Kreps 1987) as a refinement to eliminate the dominated equilibria with unreasonable out-of-equilibrium beliefs.

**Pooling Equilibria:** In pooling equilibria, both firms with access to high and low quality investment opportunities choose to provide the same level of implicit recourse given the beliefs of investors. They both provide  $r^{G*}$  with probability 1. Saving firms observe this message and use the Bayes’ rule to compute the posterior assessment that messages are sent by each type:

$$\varphi(j \in \mathcal{H}_t \mid r_j^G = r^{G*}) = \frac{\varphi(j \in \mathcal{H}_t) \cdot 1}{\varphi(j \in \mathcal{H}_t) \cdot 1 + \varphi(j \in \mathcal{L}_t) \cdot 1 + \varphi(j \in \mathcal{Z}_t) \cdot 0} = \frac{\mu\pi}{\mu\pi + (1 - \mu)\pi} = \mu.$$

Under no aggregate stochasticity, there are several candidates for the pooling Perfect Bayesian Equilibria (PBE):

**Case 1:** Firms with access to both high and low quality projects select with probabil-

ity 1:  $r^{G*} = r_{l,cred,p}^G$ , where  $r_{l,cred,p}^G$  is the maximum implicit recourse that can be provided by firms with low quality assets under pooling. Saving firms' out-of-equilibrium beliefs that sustain this equilibrium can be the following:  $\varphi(j \in \mathcal{H}_t \mid r_{l,cred,p}^G < r_j^G < r_{h,cred,s}^G) = 0$  and unrestricted for intervals  $0 < r_j^G < r_{l,cred,p}^G$ , and  $r_j^G > r_{h,cred,s}^G$ .  $r_{h,cred,s}^G$  is the maximum level of implicit recourse that can be promised credibly in a separating equilibrium (see below). In this equilibrium, no firm defaults. None of the firms have the incentive to unilaterally decrease the implicit recourse or increase it.

Note that choosing  $r_j^G < r_{l,cred,p}^G$  is not an equilibrium since both types will have incentives to increase implicit recourse to  $r_j^G = r_{l,cred,p}^G$  due to competition, no matter what the beliefs of investors are, since both types would fulfill the implicit recourse in this interval.

**Case 2:** Firms with access to both high and low quality projects select  $r_j^G = r^{G*}$  s.t.:

$$r_{lb,p}^G \leq r^{G*} \leq \min(r_{minsep}^G, r_{h,cred,p}^G).$$

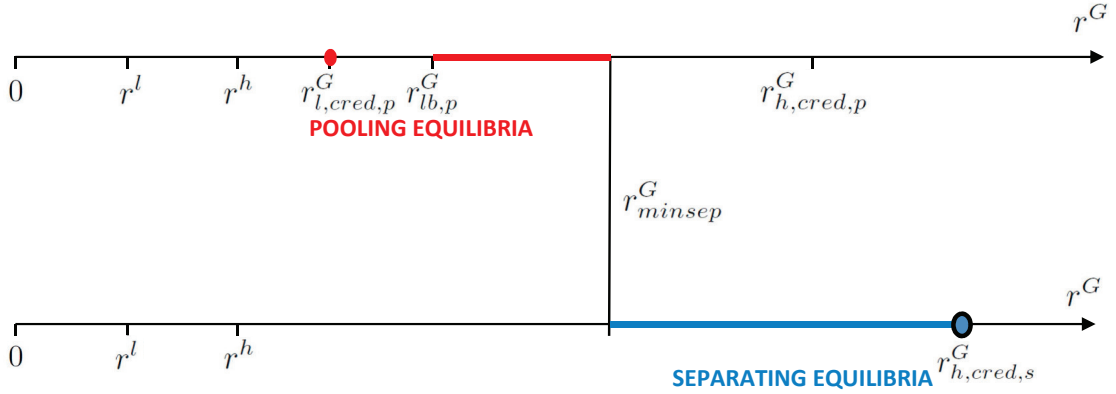
Saving firms' out-of-equilibrium beliefs that sustain this equilibrium can be the following:  $\varphi(j \in \mathcal{H}_t \mid r^{G*} < r_j^G < r_{h,cred,s}^G) = 0$ , and  $\varphi(j \in \mathcal{H}_t \mid 0 < r_j^G < r^{G*}) \leq \mu$  and unrestricted for the interval  $r_j^G > r_{h,cred,s}^G$ .

$r_{minsep}^G$  is the minimum level of implicit recourse, which the low types would not mimic under any beliefs (see derivation in Appendix 1.A.1.9).  $r_{lb,p}^G$  is the lower bound on  $r^G$ , where firms with high quality investments do not have incentives to deviate to  $r_{l,cred,p}^G$ . The fact that for  $r^G$  such that  $r_{l,cred,p}^G < r^G < r_{lb,p}^G$ , both types have incentives to decrease implicit recourse to  $r_j^G = r_{l,cred,p}^G$ , is due to equilibrium defaults on the implicit recourse of firms with low investment, which bring investors lower utility than when  $r^G = r_{l,cred,p}^G$ . This negative effect on price together with potentially higher costs of higher implicit recourse (when  $r^G > r^h$ ) outweighs the positive effect of higher implicit recourse on the price.

**Separating Equilibria:** There is potentially a continuum of separating equilibria, where firms with access to low quality projects save and buy securitized assets from firms with access to high quality projects. Firms with access to high quality projects invest, securitize, and provide implicit recourse  $r^{G*} \in (r_{minsep}^G, r_{h,cred,s}^G)$  with probability 1, where  $r_{minsep}^G$  is the minimum implicit recourse that prevents mimicking by firms with low investment opportunities. Saving firms observe this message and use the Bayes' rule



**Figure 1.4:** The case where the Intuitive Criterion selects a unique Separating Equilibrium



to compute the posterior assessment that message is sent by each type:

$$\varphi(j \in \mathcal{H}_t \mid G_j = G^*) = \frac{\varphi(j \in \mathcal{H}_t) \cdot 1}{\varphi(j \in \mathcal{H}_t) \cdot 1 + \varphi(j \in \mathcal{L}_t) \cdot 0 + \varphi(j \in \mathcal{Z}_t) \cdot 0} = \frac{\mu\pi}{\mu\pi} = 1.$$

Saving firms' out-of-equilibrium beliefs that sustain this equilibrium can be the following:  $\varphi(j \in \mathcal{H}_t \mid r^{G^*} < r_j^G < r_{h,cred}^G) = 0$  and unrestricted for intervals  $0 < r_j^G < r^{G^*}$  and  $r_j^G > r_{h,cred,s}^G$ .

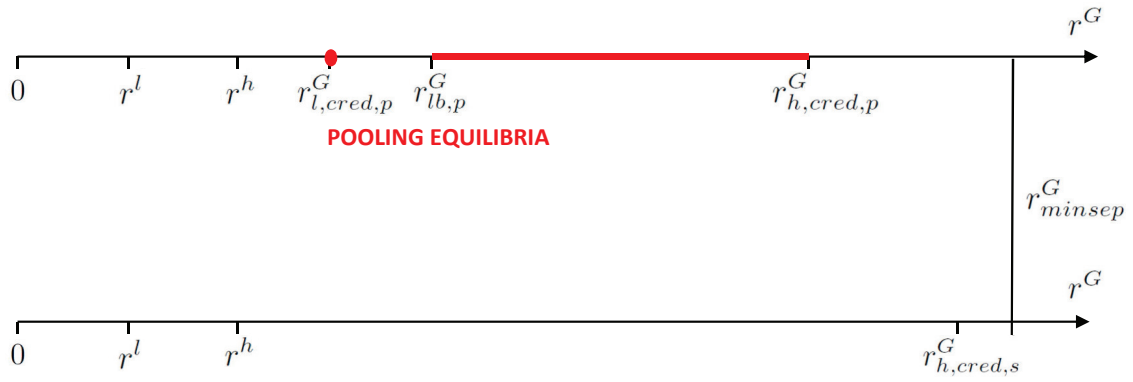
**Application of Intuitive Criterion:** If a separating equilibrium exists, then all pooling equilibria are dominated, and therefore fail the Intuitive Criterion. In particular, due to competition among firms with access to high quality investments, the Intuitive Criterion selects only one separating equilibrium, where firms with access to high quality investments invest, securitize, and provide the maximum credible implicit recourse  $r^{G^*} = r_{h,cred,s}^G$ .<sup>20</sup> So after applying the Intuitive Criterion, there is either one unique separating equilibrium left or one or multiple pooling equilibria.

**The condition for the existence of a separating equilibrium:**

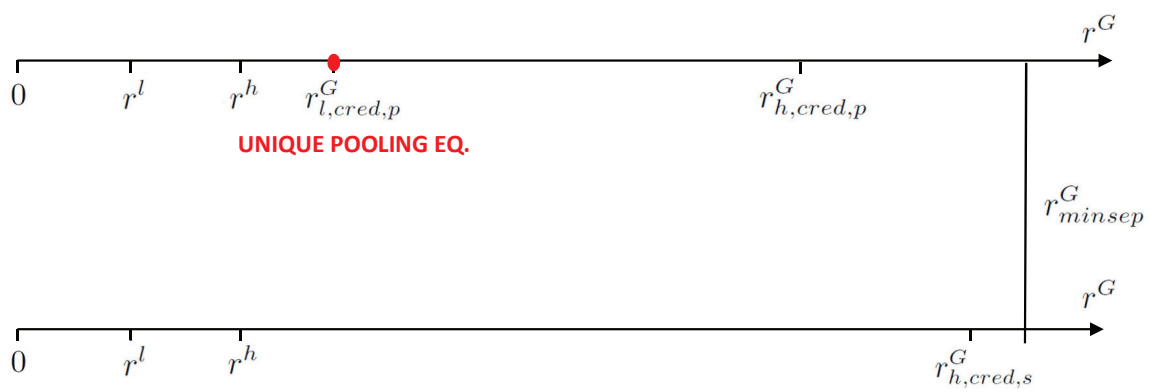
Thanks to Proposition 5, we know that firms have incentives to unilaterally increase the provided implicit recourse up to the maximum credible level. But then, if low quality firms are already at the maximum credible level, where the cost of defaulting and keeping the implicit recourse are equalized, they are better off if they increase the implicit recourse without increasing the cost further but potentially benefiting from being mistaken for a firm with access to high quality projects. Therefore, no separating equilibrium can exist in which firms with low quality investment would provide a different level of implicit

<sup>20</sup>This case is shown in Figure 1.3.

**Figure 1.5:** The case where there is no Separating Equilibrium



**Figure 1.6:** The case with unique Pooling equilibrium



recourse. Firms with low quality investments always prefer mimicking firms with high quality investments to providing a lower implicit recourse and disclosing their quality.

Therefore, separation can take place only when the costs of mimicking become so large that investing into high quality assets is preferred. Under the deterministic case, this condition can be expressed analytically. The implicit recourse  $r^G$  has to be high enough to satisfy:

$$V^l \mid \text{mimicking} < V^l \mid \text{buying high loans}. \quad (1.13)$$

This brings us to one of the main findings in this paper.

**Proposition 8.** *Under asymmetric information, a separating equilibrium is possible in the deterministic steady state if and only if*

$$\frac{A^h}{A^l} > \frac{(1 - \theta B) q^h}{1 - \theta B q^h}, \quad (1.14)$$

where  $B \equiv \frac{q^G}{q^h} = \frac{r^G + \lambda q^h}{r^h + \lambda q^h}$  is the price premium for the equilibrium implicit guarantee. This implies that separating equilibrium:

(i) exists if and only if the level of aggregate productivity does not exceed threshold level  $\bar{A}$ ;

(ii) exists if and only if  $q^l < 1$ ; and

(iii) is more likely in the presence of reputation-based implicit recourse.

In a separating equilibrium, firms with low quality investment projects save and buy securitized assets from firms with high investment opportunities.

**Sketch of proof:** The derivation of (1.14) comes directly from the no-mimicking condition 1.13.<sup>21</sup> Point (i) comes directly from Assumption 1 over the countercyclical relative difference of cash flows from projects of different quality. Since the ratio of TFP on the LHS of (1.14) increases with aggregate TFP  $A$ , the mentioned threshold is defined as  $\Delta^h(\bar{A}) / \Delta^l(\bar{A}) = (1 - \theta B) q^h / (1 - \theta B q^h)$ .

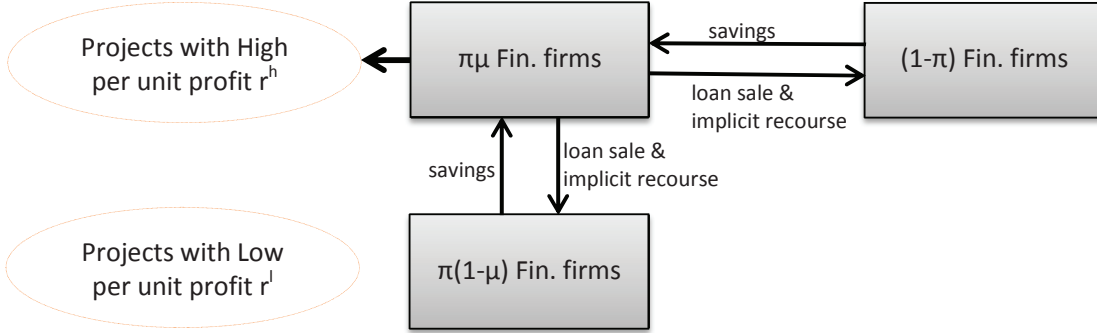
Crucially, as I show in Appendix 1.A.1.8, in a separation equilibrium, both  $q^h$  and  $B$  and, therefore, also the whole RHS of (1.14) are independent of the realizations of aggregate productivity  $A$  and are uniquely determined by the intensity of frictions and the punishment for default on implicit recourse.

After a substitution of the share of TFP by the ratio of prices from the asset market

---

<sup>21</sup>See Appendix 1.A.1.8 for derivation.

**Figure 1.7:** A private information case with implicit recourse: Separating equilibrium



Note: In the separating equilibrium, the implicit recourse provided by the firms with access to high quality projects is high enough so that it is not profitable for firms with access to low quality projects to mimic them. They are better off buying high quality projects.

clearing condition, condition 1.14 can be rewritten as:

$$q^l < \frac{1 - \theta B q^h}{1 - \theta B},$$

which implies that in a separating equilibrium,  $q^l < 1$  since, by Proposition 1,  $q^h > 1$ .

Finally, when comparing the lower bound on the TFP ratio, consistent with the separating equilibrium in cases without implicit recourse (eq. 1.5) and in cases with implicit recourse (eq. 1.14), we can show that the latter is lower. This implies that in the case with implicit recourse, the separation condition (eq. 1.14) is more likely to be satisfied.<sup>22</sup>

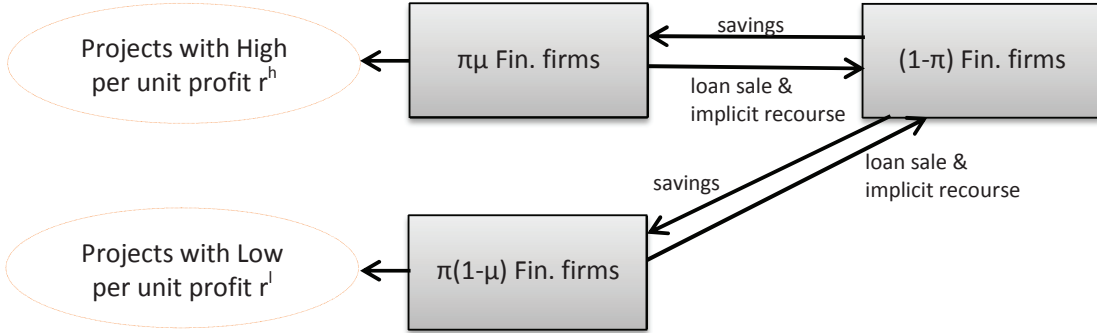
#### **Uniqueness of pooling equilibrium:**

When a separating equilibrium does not exist, there is generally a continuum of pooling equilibria. However, it turns out that for a large set of parameter space, there is only one pooling equilibrium with  $r^{G*} = r_{l,cred,p}^G$ , independent of a specific form of out-of-equilibrium beliefs.<sup>23</sup> I calibrate the model to have only one pooling equilibrium. The advantage of this calibration is not only having a unique equilibrium but also knowing that punishment is never triggered in equilibrium. It still provides the disciplining role, but the dynamic results are not influenced by the exercise of a particular punishment rule.

<sup>22</sup>Complete proof is in Appendix 1.A.1.8.

<sup>23</sup>Figure 1.6 shows this case with a unique pooling equilibrium.

**Figure 1.8:** A private information case with implicit recourse: Pooling equilibrium



Note: In the pooling equilibrium, both firms with access to high and low quality projects provide the same level of implicit recourse. They are indistinguishable, and, therefore, both firms invest into projects and sell them to firms with no investment opportunities.

To obtain such an equilibrium, in general, I have to find values of parameters such that  $r_{lb,p}^G > r_{h,cred,p}^G$  i.e., the minimum level of implicit recourse for which it pays off to provide recourse higher than  $r_{l,cred,p}^G$  is not credible in equilibrium since it exceeds  $r_{h,cred,p}^G$ .

It turns out that this condition is satisfied for a low enough share of high quality investment opportunities,  $\mu$ , and a high enough difference in type-specific TFP in a pooling equilibrium:

$$\mu < \frac{1 - \theta q^l}{q^h - \theta q^l}.$$

For details see Appendix 1.A.1.9.

## 1.4 Dynamics and numerical examples

In this section, I show a solution of the fully stochastic version of the model with asymmetric information, binding “skin in the game” and implicit recourse. The allocation of projects to firms is still driven by an i.i.d. shock. The aggregate productivity for simplicity follows a two-state Markov chain  $A_t \in (A^H, A^L)^{24}$  with a transition matrix  $P = [p, 1 - p; 1 - p, p]$ .<sup>25</sup>

In the analysis of the dynamic properties of the model, I focus on the switching

<sup>24</sup>Note that capital superscripts  $H, L$  refer to the aggregate state of the economy and not to the type of investment opportunity.

<sup>25</sup>The case when  $A_t$  follows a Markov chain is easier to calibrate but is not crucial for the results. An earlier version of this paper works with an AR(1) process for the aggregate TFP.

between the separating and pooling equilibria over the business cycle. Even though in the steady state there is a separating equilibrium, when the aggregate productivity increases and the economy is in the boom stage of a business cycle  $A_t = A^H$ , the separating equilibrium is no longer sustainable, and the economy is in the pooling equilibrium, where both types of firms provide the same level of implicit support and both invest into new projects. This follows directly from Proposition 8. The intuition behind the result is the following. As the aggregate productivity increases, the relative difference in productivity of the two non-zero profit project types is reduced. Therefore, a higher implicit recourse is needed to satisfy the separation condition (1.13). Intuitively, following Proposition 8, the condition says that  $q^l < (1 - \theta B q^h) / (1 - \theta B) < 1$  is necessary for separation, but in a boom even the quality of low type projects is relatively high, and therefore one has to provide high implicit recourse to drive the prices of low quality projects low enough. At some point, the level of implicit recourse required to achieve separation exceeds the maximum level that can be credibly provided, and the economy switches to the pooling equilibrium.

**Calibration of parameters:** Since I extend the model of Kiyotaki and Moore (2012), I use the same level of parameters for:  $\alpha = 0.4$ ,  $\beta = 0.99$ , and  $\pi = 0.05$ . The persistence parameter for the productivity process is  $p = 0.86$ .<sup>26</sup> Parameters  $A^H, A^L$  are chosen to match the annual standard deviation of GDP in the USA, which is 2.8%.<sup>27</sup> The remaining parameters are chosen to replicate the performance (delinquency rates) of securitized assets which has been at the core of recent debates over the efficiency of securitization—subprime residential mortgage backed securities issued in the USA:  $\mu = 0.63$ ,  $\Delta^l(A^H) / \Delta^h(A^H) = 0.94$  and  $\Delta^l(A^L) / \Delta^h(A^L) = 0.71$ .<sup>28</sup> The annual depreciation  $\lambda = 0.78$  is chosen to replicate the weighted average life (WAL) for residential MBS of 54.5 months Centorelli and Peristiani (2012). And finally the fraction of loans that can be sold is set to  $\theta = 0.75$  to allow for the switching between pooling and separating equilibrium over the business cycle, which is supported by the empirical analysis in section 1.6.

**Solution method:** The fully stochastic model is solved using a global numerical approximation method. In particular, I find the price and the value functions by iterating

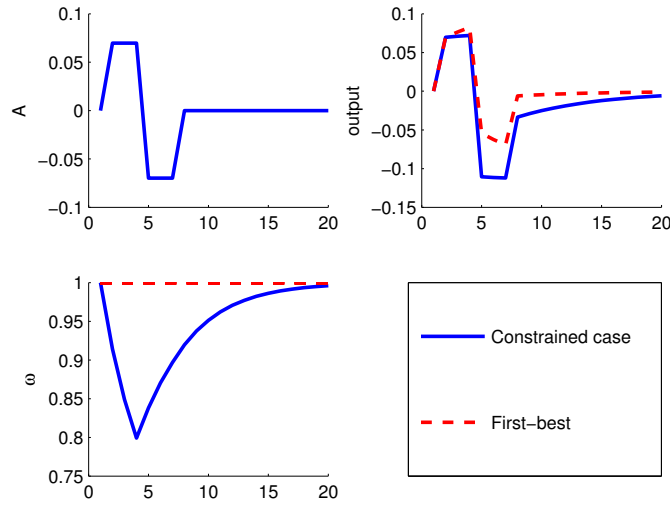
---

<sup>26</sup>This corresponds to an autocorrelation of TFP shocks at the quarterly frequency of 0.95.

<sup>27</sup>A similar approach is used in Nikolov (2012).

<sup>28</sup>For details see Appendix 1.A.3.

**Figure 1.9:** Impulse responses



them on the grid of state variables until convergence.<sup>29</sup>

**Impulse responses:** Figure 1.9 shows how the economy behaves in a particular episode of three periods in a state with high aggregate TFP followed by three periods in a state with low aggregate TFP. Then the productivity shocks are switched off and the economy converges to the steady state.<sup>30</sup> The point of this exercise is to show the switch from separating equilibrium to pooling and back and its effects on output. For comparison on the graph, I report impulse responses<sup>31</sup> of the constrained model under private information, with binding “skin in the game” and with an implicit recourse provision as well as an unconstrained and efficient first-best case. Note that the graph depicts deviations from each model’s steady state. Only the share of high quality assets on the balance sheets ( $\omega$ ) is shown in absolute value. So even though on the graph both the first-best and the constrained cases start at the same point, the first-best case is characterized by higher absolute levels of steady state output and capital.

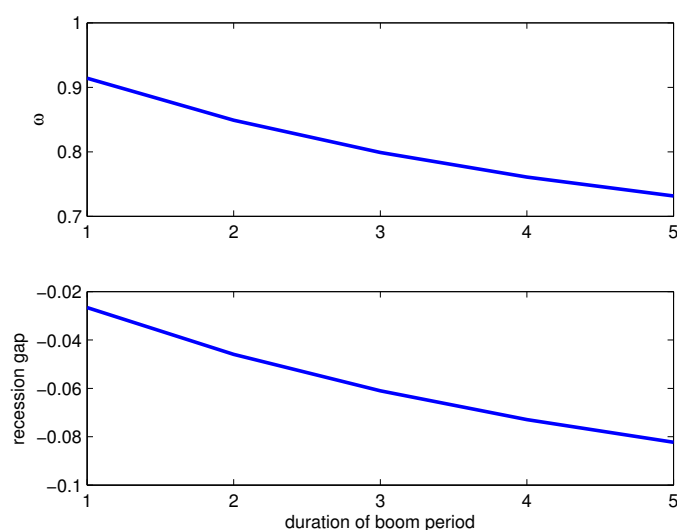
The figure demonstrates that, as the constrained economy moves to the boom stage of the business cycle, the separating equilibrium changes to pooling equilibrium, i.e., the

<sup>29</sup>Details are in Appendix 1.A.4.

<sup>30</sup>In this case with a Markov chain for aggregate productivity, the steady state productivity  $\bar{A}$  is defined as the mean of the ergodic distribution across  $(A^H, A^L)$ , and in this zero-probability steady state, the expectations about the occurrence of either state is set to 50%.

<sup>31</sup>The impulse responses start from a steady state to which they converge after a long period of zero-productivity shocks, i.e. aggregate productivity stays at the steady state productivity  $\bar{A}$ . Then, I introduce the described sequence of productivity shocks after which the shocks are zero again.

**Figure 1.10:** The longer the boom stage, the deeper the subsequent recession



share of high quality projects ( $\omega$ ) decreases, while  $\omega$  remains constant in the first best case at 100%. The lower share of high quality projects in the constrained case slows slightly in the growth of output and the accumulation of capital already in the boom, but the effect is small since in the boom stage, the difference in the two qualities is rather small. However, the inefficiency in allocation of capital continues to accumulate. As the economy exogenously moves to a recession with a higher difference in qualities, one can see that the accumulated inefficiency in the allocation of capital is more pronounced. Therefore, booms have almost the same relative size in a constrained and first-best case, but busts following a boom stage are much deeper in a constrained case.

Figure 1.10 shows the result directly following from the switching property of the model—the fact that the longer the boom period is preceding the recession, the larger the fraction is of low quality assets accumulated in the pooling equilibrium, and the larger the difference in the depth of a recession is compared to the first-best case (a recession gap).

## 1.5 Extensions

### 1.5.1 Endogenizing the “skin in the game”

So far the “skin in the game” (or equivalently, the share of loans that can be sold,  $\theta$ ) has been taken as an exogenous parameter. In this section, I will sketch a simple moral



hazard problem, which would aim to justify the existence of this constraint.

Consider that firms can divert funds from the sale of current period loans needed to cover the unit investment costs. This cannot be immediately verified. To eliminate this problem, investors require the issuing firms to retain a sufficiently large “skin in the game”  $(1 - \theta)$ , i.e., to finance a fraction  $1 - \theta$  of funds in the project from their own resources. The incentive compatible constraint then points down a sufficiently high  $\theta$  that prevents this moral hazard problem<sup>32</sup>:

$$V^D(w\beta R' \mid \textit{diverting funds}) \leq V^{ND}(w\beta R' \mid \textit{investing properly}),$$

where return from diverting funds is  $R' \mid \textit{diverting funds} = \left(\frac{\theta q^G}{1-\theta}\right)^x$ , with  $x$  being the number of times the individual recycles the returns from this operation to issue and sell new “castles-in-the-air” projects. Since I do not restrict the practice of the sequential issuance of loans, which is technically needed even under proper investing, the ICC will always fail unless  $\theta q^G \leq (1 - \theta)$ , which translates to

$$\theta \leq \frac{1}{q^G + 1}. \tag{1.15}$$

Thus, the higher the sale price of loans  $q^G$ , the higher a “skin in the game” level  $(1 - \theta)$  is required to prevent the mentioned moral hazard problem.

Note that in this version of the model I have two sources of asymmetric information. The first is the potential diversion of resources needed to make investment properly, which cannot be immediately observed. The “skin in the game” is found to be an efficient tool to prevent this behavior, while the loss of reputation and subsequent punishment are not so efficient. The second source of information asymmetry is the unobserved allocation of investment opportunities among firms. In this case according to Proposition 4 the “skin in the game” is not an efficient tool, while reputation-based implicit support can overcome the related inefficiencies.

Even with an endogenous “skin in the game”, the main qualitative result of the paper, which is the endogenous switching between the pooling and separating equilibrium, remains unchanged.<sup>33</sup>

---

<sup>32</sup>It is intuitive to assume that if a firm would divert funds, other firms will use at least the same punishment tools as for the case of implicit recourse default.

<sup>33</sup>For the proof see Appendix 1.A.1.10. Also note that the assumption of the moral hazard problem is absolutely essential since without it, the solution would be first-best even under asymmetric information. Under first-best, securitization is not profitable; therefore, firms with access to low quality investment

## 1.5.2 “Skin in the game” as a policy parameter

The “skin in the game” can be considered as a potential policy parameter. For instance, Section 941 of the Dodd-Frank Reform already requires a minimum explicit risk retention of 5%.

If, as in this model, the “skin in the game” is determined endogenously by a moral hazard problem, and securitization is the only means of financial intermediation, policy which tries to increase the “skin in the game” beyond the endogenously determined value would not improve the efficiency of financial intermediation. The reasons are twofold.

First, higher "skin in the game" increases the profits from securitization and lowers the aggregate quantity of investment (this follows from Proposition 1 and 2). Second, higher profits also make the issuance and sale of loans profitable even for firms with lower quality projects, which would otherwise be buyers of high quality projects (this holds both in the symmetric information case from Proposition 2 and under asymmetric information since pooling equilibrium is more likely; see Proposition 4 and Proposition 8). Therefore, both quantity as well as quality of investment are lower with higher "skin in the game" than with the level of this constraint determined by the market.

In contrast to some other models of securitization, such as Gorton and Pennacchi (1995), my model does not feature continuous monitoring or effort level. I only have an option of funds diversion, which is observed only with a time lag. At a high level of abstraction, this can be understood as the analogy to costly monitoring in Gorton and Pennacchi (1995), where the level of monitoring would take only two values (no monitoring or full monitoring). This moral hazard problem indeed points down the optimum level of "skin in the game". Given that everyone is rational, not only is there no reason to increase the "skin in the game" above the level determined by the equilibrium, but increasing it would have negative effects on the economy as described above.<sup>34</sup>

One could possibly introduce additional frictions, which would create benefits of the mentioned regulation. However, those possible benefits can be outweighed by the mentioned adverse general equilibrium effect especially when the regulation is too excessive.

---

do not have any incentives to mimic firms with high quality investments. Therefore, neither reputation equilibria nor implicit recourse would take place.

<sup>34</sup>It can be argued that this model is too simplistic to inform policy recommendations. That is why I reproduce the above results in a richer framework with debt as well as deposit financing and study the optimal mix of macro-prudential policy in Küncl (2013).

## 1.6 Empirical analysis

The main results of the theoretical model are the prediction that providing implicit support can signal the quality of the underlying loans and the prediction that this signaling is less efficient for loans issued in boom stages of the business cycle. This section presents empirical tests of these hypotheses. The results are in line with the model predictions.

Due to the implicit nature of the reputation based support there is no data which would measure directly the level of implicit support. However, when the implicit support is activated for instance in periods of lower than expected cash flows (higher than expected delinquency rates) from the securitized products, it can be observed and often appears in the data.<sup>35</sup> Even using the data on support provided by the originator (credit enhancement) when it is actually explicitly provided, we can test the hypotheses contained in the theoretical model.

The empirical literature on the relationship between credit enhancements and the quality of the loans (typically approximated by the delinquencies on the collateral) is limited. The most relevant paper is the work by Mandel, Morgan, and Wei (2012), where the authors test the signaling and the buffer hypotheses of credit enhancement (credit protection provided to holders of the securitized assets). The signaling hypothesis, which is already described in this paper, predicts a negative correlation of credit enhancements and delinquencies on the collateral. According to the buffer hypothesis credit enhancement does not serve as a signal of high quality of collateral but is rather provided as a buffer against observable risk. In this case securitized assets with poor quality of collateral will need higher credit enhancement, and therefore, it will imply a positive relationship between credit enhancements and delinquency rates.

### 1.6.1 Hypotheses

I perform two tests: first tests the signaling hypothesis (with the alternative being the buffer hypothesis) and the second tests the hypothesis of lower efficiency of signaling (switching to pooling equilibria) when loans are issued in boom periods of the business

---

<sup>35</sup>As anecdotal evidence let me cite the example reported originally by Mandel, Morgan, and Wei (2012) on the increase in credit enhancement by Chase Issuance Trust. The originator of the securitized assets increases credit enhancement on both future issuance as well as all outstanding securitized products. Note that they had no contractual obligation to provide higher credit enhancement on loans products issued in the past, so this is a typical case of implicit support that appears in the data only at the time when the implicit support is activated. Fitch: Chase Increases Credit Enhancement in Credit Card Issuance Trust (CHAIT),” <http://www.reuters.com/article/2009/05/12/idUS260368+12-May-2009+BW20090512>.

cycle.

**H1: Credit enhancement signals the quality of collateral** If the signaling hypothesis is correct, then more support would be positively correlated with the quality of the securitized products. Therefore, this hypothesis would suggest a negative effect of lagged credit enhancements on the delinquency rates of the collateral. If the relationship is opposite then the buffer effect dominates.

**H2: For loans issued in the boom stage of the business cycle a pooling equilibrium is more likely, therefore signaling is less efficient** If the signaling is less strong for assets originated in the boom period of the business cycle as predicted by the model due to higher likelihood of the pooling equilibrium, the positive correlation between credit enhancements and quality of collateral should be smaller or even become negative for this particular subset of products. I construct a dummy for securitized products issued in the boom stages of the business cycle. This hypothesis would suggest that an interaction term of lagged credit enhancements with the dummy for deals issued in the boom should have a positive effect (an increase) on delinquency rates of the collateral.

## 1.6.2 Data description

I use the database Performance Data Services (PDS) provided by Moody's, which contains the data on delinquency rate of collateral in the pool as well as on the credit enhancement provided to back securitized products. I have access to the part of the database which covers Residential Mortgage Backed Securities (RMBS) issued in Europe.<sup>36</sup>

As a proxy for quality of collateral (mortgage loans) which backs the securitized products I use 90plus delinquency rate which is defined as the amount of receivables that are 90 or more days past due divided by the original collateral balance. The support provided to securitized products is captured by credit enhancement, which is the amount of credit protection available to the holders of securitized assets in the form of subordination, over-collateralization, reserve funds, letters of credit, spread accounts, cash collateral accounts and other non-guaranteed funds. The data is available for individual tranches.

Since the quality of collateral is available only on the level of the pool, I need to aggregate credit enhancement data. I aggregate on the level of deals. A deal is typically

---

<sup>36</sup>I would like to thank the European Central Bank for providing me with the access to this part of the PDS database.

backed by a pool of collateral and consists of several tranches. I drop the observations where more pools back the same deal or more deals are backed by the same pool of loans since I do not have the information needed to do proper aggregation. The data on credit enhancement is available on the tranche level; therefore I compute a weighted average. Credit enhancement is expressed as the total amount of credit protection as a fraction of current pool balance. I winsorize both delinquency rates and the credit enhancement rate at the 2.5%-level to account for data errors and limit the effect of potential outliers.

The real output data for the respective countries are obtained from Eurostat. I construct the output gap using the Hodrick-Prescott filter with the smoothing parameter 1600.

### 1.6.3 Panel regression results

I run the following fixed effect regression:

$$\begin{aligned} DelinquencyRate_{i,t} = & \alpha_i + \alpha_t + \beta CERatio_{i,t-1} + \gamma CERatio_{i,t-1} \times D\{boom\}_{i,t} \\ & + \delta CERatio_{i,t-1} \times D\{originated\ in\ boom\}_{i,t} \\ & + \iota Deal\ age_{i,t} + \kappa Output\ gap_{i,t} + \varepsilon_{i,t} \end{aligned}$$

on data with quarterly frequency, where  $CERatio_{i,t-1}$  is the ratio of total credit enhancement to current pool balance lagged one period in time<sup>37</sup>;  $D\{boom\}$  is the dummy variable for the boom periods in the country of issuance;  $D\{originated\ in\ boom\}$  is the dummy variable for deals issued in a boom period of the respective country;  $Deal\ age$  is the number of quarters since the closing date of the deal; and  $Output\ gap = \ln(GDP) - \ln(GDP_{HP})$ , where  $GDP_{HP}$  is the smoothed level of respective real Gross Domestic Product obtained by the HP filter.

Table 1 shows the results for the four largest European countries by securitization activity for residential mortgage loans: the United Kingdom (UK), Netherlands (NL), Spain and Italy. I show results for the whole subset and for the UK and Spain separately. I use fixed effects for deals and time and report Huber-White robust standard errors. Standard errors are clustered by deals. I report the results on the maximum sample

---

<sup>37</sup>Note that I use the variable credit enhancement lagged by one quarter. This is because contemporaneous correlation between credit enhancements and loan quality could be positive due to a trigger of some implicit support in times of temporary distress. However, this does not contradict the signaling hypothesis. In fact it is a part of the signaling story developed in this model. On the other hand if the signaling hypothesis is correct then the lagged credit enhancement should be negatively correlated with current quality of the collateral.

period, but also on the period excluding the recent crisis. The results are consistent for both periods. I also checked the results when initial periods with relatively few observations are excluded and the results are still consistent. Although I do not claim that the relationships found are necessarily causal, I still find that analyzing the magnitude of the relationship is interesting and informative.

For the whole sample of four countries (UK, NL, Spain and Italy) the results are in line with the signaling hypothesis (coefficient of  $CERatio$  is significantly negative), and also in line with the hypothesis that signaling in case of loans issued in periods of boom is much weaker (coefficient of  $CERatio \times D\{originated\ in\ boom\}$  is significantly positive). Finally, the coefficient of  $CERatio \times D\{boom\}$  is significantly negative. This would suggest that the signaling effect is stronger in the boom period for all loans irrespective of the time of issuance. However, I would offer a slightly different interpretation. Following the model presented in the previous sections, since the guaranteed minimum cash flows are not conditional on the state of the economy, implicit support is most likely to be activated and therefore appear in the data in a recession. The lower the quality of the asset the higher the support (additional credit enhancement) needed to keep to the expected implicit obligation. This is an analogue to the buffer effect mentioned in Mandel, Morgan, and Wei (2012). Both signaling and buffer effect are likely to operate all the time. However, in recession the buffer effect might be stronger; that is why the effect of credit enhancements on delinquencies is less negative.

I also analyzed selected countries individually. The UK and Spain had the highest number of observations, so I report these results. In the UK the results are qualitatively the same as for the whole sample. However, in Spain the credit enhancement has no significant effect on delinquencies. I believe that this result is due to a very different regulation of securitization in both countries. Unlike in other countries, in Spain the regulator treated off-balance sheet assets (i.e. all securitized products) in the same way as if they remained on the balance sheet.<sup>38</sup> Therefore, the securitization practice in Spain was very different from other countries. Securitization was not used to transfer risk, but rather to obtain more liquidity. Consistent with this, Almazan, Martín-Oliver, and Saurina (2013) reports that securitization in Spain was used mainly by small banks which had difficulties obtaining debt financing. Following the evidence from Almazan, Martín-Oliver, and Saurina (2013), in Spain securitization was not related to adverse

---

<sup>38</sup>See Acharya and Schnabl (2009) for detailed description of the regulatory practice in different countries.

selection problems, which were so typical of practices in other countries. As a result credit enhancement did not serve as a signaling tool. Consistently with this I cannot find any significant relationship between credit enhancements on the delinquencies on the collateral in Spain.

To conclude, the results of the panel regressions are consistent with the signaling hypothesis as well as the lower efficiency of the signaling for loans issued in a boom period for countries, where securitization was related to a transfer of risk, such as the United Kingdom. However, in countries, such as Spain, where the risk primarily remained on the balance sheet of the originators, no significant relationship between credit enhancement and the quality of loans is found.

**Table 1.1:** Panel Regression Results (Dependent variable: Delinquency rate)<sup>a</sup>

Countries	UK, NL, Spain, Italy	UK, NL, Spain, Italy	UK	UK	Spain	Spain
Time period	1998q3-2013q2	1998q3-2007q2	2000q2-2013q2	2000q2-2007q2	1998q3-2013q2	1998q3-2007q2
$CERatio(-1)$	-0.0191 [4.40]***	-0.0107 [2.17]**	-0.0212 [3.86]***	-0.0118 [2.41]**	0.0031 [0.70]	0.0058 [1.22]
$CERatio(-1) \times$	-0.0039 [4.65]***	-0.0061 [2.79]***	-0.0033 [1.91]*	-0.0052 [1.68]*	-0.0014 [0.67]	-0.0015 [1.72]*
$D_{boom}$	0.0115 [2.31]**	0.0200 [3.63]***	0.0144 [2.31]**	0.0301 [4.83]***	-0.0034 [0.52]	-0.0047 [0.99]
$Deal\ age$	0.0032 [1.57]	-0.0027 [0.33]	-0.042 [4.12]***	-0.068 [5.23]***	0.003 [1.47]	-0.004 [1.88]*
$Output\ gap$	0.15 [0.03]	-5.13 [0.45]	omitted <sup>b</sup>	omitted <sup>b</sup>	omitted <sup>b</sup>	omitted <sup>b</sup>
Observations	15826	4664	4210	1184	5717	1707
Number of deals	747	399	197	129	227	122
$R^2(w/b/o)^c$	0.13/0.19/0.13	0.14/0.08/0.10	0.28/0.04/0.00	0.32/0.00/0.00	0.12/0.01/0.01	0.12/0.04/0.06

<sup>a</sup> Robust t-statistics appear in brackets. Time dummies are not reported. Variables are defined in text. \*\*\*/\*\*/\* - Statistically significant at 1/5/10 percent level.

<sup>b</sup> Ouput gap for individual country varies only over time, so cannot be included due to time fixed effects.

<sup>c</sup> Reports  $R^2$  within/between/overall.



## 1.7 Conclusion

In this paper, I show that, in general, reputation concerns allow sponsors of securitized products to signal the quality of securitized loans by providing implicit recourse and thus they limit the problem of private information typical for securitization. However, there are limits to the efficiency of these particular reputation-based tools, which become more pronounced in boom stages of the business cycles. The level of sufficiently high implicit recourse that would not be mimicked by firms with investment projects of lower quality exceed the level which can be credibly promised. In the resulting pooling equilibrium, the information about the quality of loans is lost, and investment allocation becomes more inefficient. Due to this mechanism, large inefficiencies in the allocation of capital can be accumulated in the boom stage of the business cycle. The accumulated inefficiencies can then amplify a subsequent downturn of the economy. Additionally, the longer the duration of the boom stage of the business cycle the deeper will be the fall of output in a subsequent recession.

The results of this paper also have implications for related macro-prudential policy, which requires higher explicit risk-retention ("skin in the game"). In this model, such requirements restrict the supply of loans and, through the general equilibrium effect, make securitization more profitable. As a result, this regulation lowers both the quantity and the quality (higher likelihood of pooling equilibria) of investment in the economy.

In the empirical section, I test hypotheses from the theoretical model on the level of securitization deals using data for residential mortgage backed securities issued in Europe. Lagged credit support provided to holders of securitized assets is found to have a positive relation to the loan quality, which is in line with the signaling hypothesis. The effect is smaller and may even be overturned for assets that have been issued in a boom stage of the business cycle. This is in line with higher likelihood of a pooling equilibrium in a boom which is derived in the theoretical model. The results are especially strong for deals issued in the UK, however, are not statistically significant for deals issued in Spain. The difference could be explained by significant differences in regulatory framework and practice of securitization.

The mechanism presented in this paper can contribute to the understanding of the recent financial crisis as it describes the experience of securitization markets prior to and during the recent financial crisis. In the period preceding the crisis, many inefficient investments of unknown quality were undertaken. While this was not problematic as

long as the economy was performing well, the large amount of low quality loans in the economy ultimately contributed to the depth of the financial crisis. The paper also points to some unexpected negative effects of the newly proposed regulation.

## 1.8 Acknowledgments

For helpful comments and suggestions, I would like to thank Sergey Slobodyan, Markus Brunnermeier, Byeongu Jeong, Nobuhiro Kiyotaki, Filip Matějka, Benjamin Moll, Kalin Nikolov, Olena Senyuta and seminar participants at PhD Student Macroeconomics Workshop, Princeton University (May 2012); XXI International Conference on Money, Banking and Finance, CASMEF, LUISS Guido Carli University, Rome (Dec 2012); 6th RGS Doctoral Conference, Bochum (Feb 2013); 17th Annual International Conference in Macroeconomic Analysis and International Finance 2013, University of Crete (May 2013); 5th International IFABS Conference, Nottingham (Jun 2013); 28th Annual Congress of the European Economic Association, Gotenburg (Aug 2013); Goethe University, Frankfurt (Oct 2013); European Central Bank, Frankfurt (Oct 2013); Econometric Society European Winter Meeting, Helsinki (Nov 2013); Warwick Business School (Jan 2014); Bank of England, London (Jan 2014); Bank of Spain, Madrid (Jan 2014); Comisiòn Nacional del Mercado de Valores, Madrid (Jan 2014), Vrije Universiteit Amsterdam (Jan 2014) and Bank of Canada, Ottawa (Jan 2014). Remaining errors are solely my own responsibility.

Financial support of the Grant Agency of the Charles University (grant no. 638413), the Global Development Network (RRC 12+69) and the Czech Science Foundation project No. P402/12/G097 DYME Dynamic Models in Economics is gratefully acknowledged.

# 1.A Appendix 1

## 1.A.1 Proofs

### 1.A.1.1 First-best case

Due to logarithmic utility, firms always consume  $1 - \beta$  fraction of their wealth:  $c = (1 - \beta) h (r^h + \lambda)$ . This policy function is linear, so it is trivial to aggregate it across the continuum of firms to obtain the equation describing the evolution of aggregate variables:  $C = (1 - \beta) H (r^h + \lambda)$ .

From the market clearing condition, we know that  $I = Y - C = Hr^h - C$ . From the law of motion for capital, we know that in the steady state  $I = (1 - \lambda) H$ . Combining these two conditions, we obtain:

$$Hr^h - C = (1 - \lambda) H.$$

Substituting for aggregate consumption we get:

$$\begin{aligned} Hr^h - (1 - \beta) H (r^h + \lambda) &= (1 - \lambda) H, \\ r^h + \lambda &= \frac{1}{\beta}. \end{aligned}$$

### 1.A.1.2 Proof of Proposition 1

In the first-best allocation,  $q^h = 1$ . Should the “skin in the game” be binding,  $q^h > 1$ . Let’s consider the least restrictive case where still only the firm with access to high quality loans is issuing credit and securitizes these loans, and the “skin in the game” is not high enough to allow a firm with access to low quality investment opportunities to profitably issue loans  $q^l < 1$ .

Under the binding “skin in the game” constraint, the aggregate investment into a higher quality project will be (obtained as an aggregation of eq. 1.4):

$$I_t^H = \pi \mu \frac{\beta (H_t ((A_t + \Delta^h) K_t^{\alpha-1} + \lambda q_t^h) + L_t ((A_t + \Delta^l) K_t^{\alpha-1} + \lambda q_t^l))}{(1 - \theta q_t^h)}. \quad (1.16)$$

Prices of particular assets are determined from the Euler equations of saving firms. In equilibrium, these firms are indifferent between investing in high or low quality projects:

$$E_t \left[ \frac{\frac{r_{t+1}^h + \lambda q_{t+1}^h}{q_t^h}}{\left( \omega_{t+1} \frac{r_{t+1}^h + \lambda q_{t+1}^h}{q_t^h} + (1 - \omega_{t+1}) \frac{r_{t+1}^l + \lambda q_{t+1}^l}{q_t^l} \right)} \right] = 1 \quad (1.17)$$

$$E_t \left[ \frac{\frac{r_{t+1}^l + \lambda q_{t+1}^l}{q_t^l}}{\left( \omega_{t+1} \frac{r_{t+1}^h + \lambda q_{t+1}^h}{q_t^h} + (1 - \omega_{t+1}) \frac{r_{t+1}^l + \lambda q_{t+1}^l}{q_t^l} \right)} \right] = 1, \quad (1.18)$$

where  $\omega_t$  is the share of high quality projects in the overall assets in the economy  $\omega_t = H_t/K_t$ . The derivation of these conditions can be found in Appendix 1.A.2.

Finally, the goods market clearing condition has to hold, too:

$$Y_t = C_t + I_t. \quad (1.19)$$

**Case 1: Only firms with access to high quality projects give credit and securitize:**

Steady state conditions (1.16, a combination of 1.17 and 1.18, 1.19) in the steady state become the following:

$$\begin{aligned} (1 - \lambda)(1 - \theta q^h) &= \pi\mu\beta(r^h + \lambda q^h), \\ \frac{A^h}{q^h} &= \frac{A^l}{q^l}, \\ r^h &= (1 - \lambda) + (1 - \beta)(r^h + \lambda q^h). \end{aligned}$$

Combining these equations, we can obtain

$$\begin{aligned} q_H^h &= \frac{(1 - \lambda)(1 - \pi\mu)}{(1 - \lambda)\theta + \pi\mu\lambda}, \\ K_H &= \left[ \frac{(1 - \lambda) + \frac{(1 - \beta)\lambda(1 - \lambda)(1 - \pi\mu)}{(1 - \lambda)\theta + \pi\mu\lambda}}{\beta A^h} \right]^{\frac{1}{\alpha - 1}}. \end{aligned} \quad (1.20)$$

As long as  $q^h = 1$ , we would obtain  $K_H = \left[ \frac{1}{A^h} \left( \frac{1}{\beta} - \lambda \right) \right]^{\frac{1}{\alpha - 1}}$ , which is the first-best optimal level of capital (compared with (1.2)). If  $(1 - \lambda)(1 - \pi\mu) > (1 - \lambda)\theta + \pi\mu\lambda$ , then  $q^h > 1$ . The deterministic steady state level of capital is then lower than in the first-best case:

$$K_H = \left[ \frac{(1 - \lambda) + (1 - \beta)\lambda q_H^h}{\beta A^h} \right]^{\frac{1}{\alpha - 1}} < \left[ \frac{(1 - \lambda) + (1 - \beta)\lambda}{\beta A^h} \right]^{\frac{1}{\alpha - 1}} = K_{FB}.$$

### 1.A.1.3 Proof of Proposition 2

Proposition 2 claims that there are three possible types of steady states depending on the parameter values. In the proof of Proposition 1 above, I already described the least restricted case, where only a firm with access to high quality projects will be issuing and securitizing loans. By continuing to tighten the "skin in the game" constraint, we will increase the price of the low quality asset to 1 ( $q^l = 1$ ). At this point, the firms with access to low quality loans will be indifferent between buying high quality securitized assets or issuing and securitizing their own loans. Credit to low quality projects counterweights the effect of tightening the "skin in the game" constraint, and therefore, the price stays at the same levels ( $q^l = 1$ ,  $q^h = A^h/A^l$ ). For an interval of  $\theta$ , there will be a steady state in which firms with access to low quality investment will play a mixed strategy when giving

credit with probability  $\psi$ . As  $\theta$  decreases ("skin in the game" rises),  $\psi$  increases all the way up to 1, where a third type of steady state takes place. In this, firms with access to both high and low quality projects will all be issuing credit and always securitizing.

**Case 2: Firms with access to low quality projects issue credit with probability  $\psi$ :**

Steady state conditions are the following:

$$(1 - \lambda) (1 - \theta q^h) \omega = \pi \mu \beta (\omega (r^h + \lambda q^h) + (1 - \omega) (r^l + \lambda q^l)), \quad (1.21)$$

$$(1 - \lambda) (1 - \theta q^l) (1 - \omega) = \pi (1 - \mu) \psi \beta (\omega (r^h + \lambda q^h) + (1 - \omega) (r^l + \lambda q^l)), \quad (1.22)$$

$$\frac{A^h}{q^h} = \frac{A^l}{q^l}, \quad (1.23)$$

$$q^l = 1, \quad (1.24)$$

$$\omega r^h + (1 - \omega) r^l = (1 - \lambda) + (1 - \beta) (\omega (r^h + \lambda q^h) + (1 - \omega) (r^l + \lambda q^l)). \quad (1.25)$$

Let's define

$$q \equiv \frac{q^h}{A^h} = \frac{q^l}{A^l}, \quad (1.26)$$

and

$$D \equiv \omega A^h + (1 - \omega) A^l. \quad (1.27)$$

Using (1.26), (1.27) and combining equations (1.21), (1.22) and (1.23):

$$(1 - \lambda) (1 - \theta q D) = \pi (\mu + \psi (1 - \mu)) \beta D (K^{\alpha-1} + \lambda q),$$

$$(1 - \lambda) - \pi (\mu + \psi (1 - \mu)) \beta D K^{\alpha-1} = q D [(1 - \lambda) \theta + \pi (\mu + \psi (1 - \mu)) \beta \lambda]. \quad (1.28)$$

We can also rewrite (1.25):

$$\beta D K^{\alpha-1} = 1 - \lambda + (1 - \beta) D \lambda q. \quad (1.29)$$

Combining (1.28) and (1.29), we obtain

$$q_M = \frac{(1 - \lambda) (1 - \pi (\mu + \psi (1 - \mu)))}{(1 - \lambda) \theta + \pi (\mu + \psi (1 - \mu)) \beta \lambda} \frac{1}{D}. \quad (1.30)$$

Substituting (1.30) back into (1.29), we obtain:

$$K_M = \left[ \frac{(1 - \lambda) + \frac{(1 - \beta) \lambda (1 - \lambda) (1 - \pi (\mu + \psi (1 - \mu)))}{(1 - \lambda) \theta + \pi (\mu + \psi (1 - \mu)) \beta \lambda}}{\beta D} \right]^{\frac{1}{\alpha-1}}. \quad (1.31)$$

**Case 3: Firms with access to both high and low quality projects are always**

**giving credit:**

The deterministic steady state is defined by:

$$(1 - \lambda) (1 - \theta q^h) \omega = \pi \mu \beta (\omega (r^h + \lambda q^h) + (1 - \omega) (r^l + \lambda q^l)), \quad (1.32)$$

$$(1 - \lambda) (1 - \theta q^l) (1 - \omega) = \pi (1 - \mu) \beta (\omega (r^h + \lambda q^h) + (1 - \omega) (r^l + \lambda q^l)), \quad (1.33)$$

$$\frac{A^h}{q^h} = \frac{A^l}{q^l}, \quad (1.34)$$

$$\omega r^h + (1 - \omega) r^l = (1 - \lambda) + (1 - \beta) (\omega (r^h + \lambda q^h) + (1 - \omega) (r^l + \lambda q^l)). \quad (1.35)$$

Using (1.26) and (1.27), and combining equations (1.32), (1.33), and (1.34):

$$(1 - \lambda) (1 - \theta q D) = \pi \beta D (K^{\alpha-1} + \lambda q),$$

$$(1 - \lambda) - \pi \beta D K^{\alpha-1} = q D [(1 - \lambda) \theta + \pi \beta \lambda]. \quad (1.36)$$

We can also rewrite (1.35):

$$\beta D K^{\alpha-1} = 1 - \lambda + (1 - \beta) D \lambda q. \quad (1.37)$$

Combining (1.36) and (1.37), we get

$$q_B = \frac{(1 - \lambda) (1 - \pi) 1}{(1 - \lambda) \theta + \pi \lambda D}. \quad (1.38)$$

Substituting (1.38) back into (1.37), we get:

$$K_B = \left[ \frac{(1 - \lambda) + \frac{(1 - \beta) \lambda (1 - \lambda) (1 - \pi)}{(1 - \lambda) \theta + \pi \lambda}}{\beta D} \right]^{\frac{1}{\alpha-1}}. \quad (1.39)$$

The second part of the proposition claims that  $K_H > K_M > K_B$ . To show this lets first focus on the part of the formulae within brackets for capital: Since in Case 1  $q_H^l < 1$ , then  $q_H^h < \frac{A^h}{A^l}$ . And since  $q_M^l = 1$ , then  $\frac{(1 - \lambda) (1 - \pi (\mu + \varphi (1 - \mu)))}{(1 - \lambda) \theta + \pi (\mu + \varphi (1 - \mu)) \lambda} = \frac{D_M}{A^l}$ . The following inequality then holds:

$$\frac{(1 - \lambda) + (1 - \beta) \lambda q_H^h}{\beta A^h} < \frac{(1 - \lambda)}{\beta A^h} + (1 - \beta) \lambda \frac{1}{\beta A^l} < \frac{(1 - \lambda)}{\beta D_M} + (1 - \beta) \lambda \frac{1}{\beta A^l} = \frac{(1 - \lambda) + \frac{(1 - \beta) \lambda (1 - \lambda) (1 - \pi (\mu + \varphi (1 - \mu)))}{(1 - \lambda) \theta + \pi (\mu + \varphi (1 - \mu)) \lambda}}{\beta D_M}.$$

This implies that

$$K_H = \left[ \frac{(1 - \lambda) + (1 - \beta) \lambda q_H^h}{\beta A^h} \right]^{\frac{1}{\alpha-1}} > \left[ \frac{(1 - \lambda) + \frac{(1 - \beta) \lambda (1 - \lambda) (1 - \pi (\mu + \varphi (1 - \mu)))}{(1 - \lambda) \theta + \pi (\mu + \varphi (1 - \mu)) \lambda}}{\beta D_M} \right]^{\frac{1}{\alpha-1}} = K_M.$$

Similarly, we can show that  $K_P > K_B$ . Since  $w_B < w_P$ , then  $D_B < D_P$ . Also  $q_B^l > 1$ , then  $\frac{(1-\lambda)(1-\pi)}{(1-\lambda)\theta+\pi\lambda} > \frac{D_B}{A^l}$ . This implies that

$$\frac{(1-\lambda) + \frac{(1-\beta)\lambda(1-\lambda)(1-\pi(\mu+\psi(1-\mu)))}{(1-\lambda)\theta+\pi(\mu+\psi(1-\mu))\lambda}}{\beta D_M} = \frac{(1-\lambda)}{\beta D_M} + (1-\beta)\lambda \frac{1}{\beta A^l} < \frac{(1-\lambda)}{\beta D_B} + (1-\beta)\lambda \frac{1}{\beta A^l} < \frac{(1-\lambda) + \frac{(1-\beta)\lambda(1-\lambda)(1-\pi)}{(1-\lambda)\theta+\pi\lambda}}{\beta D_B},$$

$$K_M = \left[ \frac{(1-\lambda) + \frac{(1-\beta)\lambda(1-\lambda)(1-\pi(\mu+\psi(1-\mu)))}{(1-\lambda)\theta+\pi(\mu+\psi(1-\mu))\lambda}}{\beta D_M} \right]^{\frac{1}{\alpha-1}} > \left[ \frac{(1-\lambda) + \frac{(1-\beta)\lambda(1-\lambda)(1-\pi)}{(1-\lambda)\theta+\pi\lambda}}{\beta D_B} \right]^{\frac{1}{\alpha-1}} = K_B.$$

### 1.A.1.4 Proof of Proposition 3

Even when the “skin in the game” constraint is not binding enough to influence aggregate quantities and prices, the capital and output levels are lower than in the first-best case due to the inefficient allocation of capital. When the “skin in the game” constraint is not binding, the average gross profit from one unit of invested capital in the economy equals

$$\bar{r} = \mu r^h + (1-\mu)r^l = \frac{1}{\beta} - \lambda.$$

The level of capital  $K_P$  is determined by:

$$K_P = \left[ \frac{1}{\mu A^h + (1-\mu)A^l} \left( \frac{1}{\beta} - \lambda \right) \right]^{\frac{1}{\alpha-1}} < \left[ \frac{1}{A^h} \left( \frac{1}{\beta} - \lambda \right) \right]^{\frac{1}{\alpha-1}} = K_{FB}.$$

Suppose  $(1-\pi)(1-\lambda) > \pi\lambda + (1-\lambda)\theta$ , in which case the "skin in the game" constraint starts to bind in this case of private information. The deterministic steady state conditions then collapse into the two following equations in  $(K, q)$ :

$$(1-\lambda)(1-\theta q) = \pi\beta(\mu r^h + (1-\mu)r^l + \lambda q),$$

$$\mu r^h + (1-\mu)r^l = (1-\lambda) + (1-\beta)(\mu r^h + (1-\mu)r^l + \lambda q),$$

where  $q = \mu q^h + (1-\mu)q^l$ . From this we can easily derive:

$$q = \frac{(1-\pi)(1-\lambda)}{\pi\lambda + (1-\lambda)\theta}, \quad (1.40)$$

$$K = \left[ \frac{(1-\lambda) + (1-\beta)\lambda q}{\beta(\mu A^h + (1-\mu)A^l)} \right]^{\frac{1}{\alpha-1}}.$$

In the proof of Proposition 1 and 2, we already proved that  $K_{FB} > K_H > K_M > K_B$ . To prove Proposition 3, it suffices to prove that  $K_B > K_{private}$ , where  $K_{private}$  is the level of capital under private information about the allocation of investment opportunities. To

obtain  $K_B > K_{private}$ , we need:

$$K_B^{\alpha-1} < K_{private}^{\alpha-1},$$

$$\frac{(1-\lambda) + \frac{(1-\beta)\lambda(1-\lambda)(1-\pi)}{(1-\lambda)\theta+\pi\lambda}}{\beta(\omega A^h + (1-\omega)A^l)} < \frac{(1-\lambda) + \frac{(1-\beta)\lambda(1-\lambda)(1-\pi)}{(1-\lambda)\theta+\pi\lambda}}{\beta(\mu\Delta A^h + (1-\mu)A^l)},$$

$$\omega > \mu.$$

Writing equations (1.32) and (1.33) into a ratio, we obtain:

$$\frac{(1-\lambda)(1-\theta q^h)\omega}{(1-\lambda)(1-\theta q^l)(1-\omega)} = \frac{\pi\mu\beta(\omega(r^h + \lambda q^h) + (1-\omega)(r^l + \lambda q^l))}{\pi(1-\mu)\beta(\omega(r^h + \lambda q^h) + (1-\omega)(r^l + \lambda q^l))}.$$

Since  $q^h > q^l$ , we can obtain:

$$\frac{\omega}{(1-\omega)} = \frac{(1-\theta q^l)}{(1-\theta q^h)} \frac{\mu}{(1-\mu)} > \frac{\mu}{(1-\mu)},$$

and this implies that  $\omega > \mu$ .

#### 1.A.1.5 Proof of proposition 4

Under the private information case, firms with low quality investment opportunities prefer to buy high quality loans rather than to mimic firms with high quality investment opportunities if:

$$\begin{aligned} R \mid \text{mimicking} &< R \mid \text{buying high loans}, \\ \frac{r^l + \lambda q^l}{\frac{1-\theta q^h}{1-\theta}} &< \frac{r^h + \lambda q^h}{q^h}, \\ \frac{(1-\theta)q^h}{1-\theta q^h} &< \frac{r^h + \lambda q^h}{r^l + \lambda q^l} = \frac{q^h}{q^l}, \\ q^l &< \frac{1-\theta q^h}{1-\theta}. \end{aligned}$$

Substituting for  $q$  from (1.20) and using  $\frac{A^h}{q^h} = \frac{A^l}{q^l}$ , we get

$$\frac{A^h}{A^l} > \frac{(1-\pi\mu)(1-\lambda)(1-\theta)}{\pi\mu\lambda + (1-\lambda)\theta\pi\mu}.$$

#### 1.A.1.6 Credibility of the trigger punishment strategy

A necessary condition for the existence of the reputation equilibrium in which implicit recourse is being provided is the credibility of the punishment rule. The saving firm, which observes default on the implicit recourse, has to prefer punishing the defaulting



firm rather than non-punishing the defaulting firm, even ex-post. This is expressed in condition (1.12). I will derive analytically both elements of that inequality in the case of the separating deterministic steady state, where the level of aggregate TFP is constant. In the fully stochastic version, this can be solved numerically. Following the same steps as in Appendix 1.A.1.9, we can find that the value function of the firm that always punished, and therefore has a reputation of being a “tough investor”, is:

$$V^P(w) = \frac{\log[(1-\beta)w]}{1-\beta} + \frac{\beta \log(\beta)}{(1-\beta)^2} + \frac{\beta}{(1-\beta)^2} (\pi\mu \log(R^{h,IR}) + (1-\pi\mu) \log(R^s)),$$

and the value function of the firm that failed to punish and therefore lost its reputation of being a “tough investor” is:

$$V^{NP}(w) = \frac{\log[(1-\beta)w]}{1-\beta} + \frac{\beta \log(\beta)}{(1-\beta)^2} + \frac{\beta}{(1-\beta)^2} (\pi\mu \log(R^{h,IR}) + (1-\pi\mu) \log(R^{s,NP})).$$

If a firm loses its reputation of being a “tough investor”, other firms will expect that this firm will never punish in the future, and as a consequence, they will never again provide implicit support to this firm. So when a firm without the reputation of being a “tough investor” buys assets with implicit support issued in the primary market, its return is  $R^{s,NP} = \frac{r^h + \lambda q^h}{q^G}$ . While firms with a “tough investors” reputation have a return of  $R^{s,NP} = \frac{\hat{r}^G + \lambda q^h}{q^G}$ . If firms without a “tough investor” reputation buy assets without implicit recourse on the secondary (re-sale) markets, they are also in a disadvantageous position. When firms with a “tough investor” reputation sell high quality assets to firms with a reputation, they charge a market price  $q^h$ . However, if firms without the reputation have the outside option of only buying on the primary market, they will be willing to buy a high quality asset even for the price  $q^G$ . The price for which a high quality asset is sold on the secondary market to the firms without a reputation is somewhere on the interval  $q^{h,NP} \in (q^h, q^G)$ , depending on the bargaining power of sellers and buyers. Unless all bargaining power is on the side of firms without reputation, then  $q^{h,NP} > q^h$ . This implies that  $R^{s,NP} < R^s$ , and therefore, saving firms are better-off punishing, and inequality (1.12) would be satisfied.

It is well known that trigger strategies are often not renegotiation-proof. While in this paper I do not address this problem in detail and rule out renegotiation by assumption, it can be shown that for a large set of parameter space and relative bargaining power of different agents in the economy, renegotiation is not optimal. Therefore, a trigger strategy will be robust even in the case when renegotiation is allowed.

Suppose one firm decides to default on the implicit support (which is the case that is relevant for the ICC for non-defaulting, eq. 1.11). Other firms decide whether to punish this firm and face lower returns in the future  $R^{s,NP}$  as shown above or whether not to punish and negotiate for better terms with the defaulted firms, i.e., buy the assets from them for a lower price  $q^{h,RN} < q^h$ , giving it a return  $R^{s,RN} > R^s$ . However, those benefits from renegotiation are limited by the fact that the defaulted firm would be selling the assets only with probability  $\pi\mu$ , and the quantity of assets the firm can sell is limited and proportional to its equity. Even if the quantity of the assets sold by the defaulted firm is

large enough, renegotiation would not be optimal as long as

$$R^s > \pi\mu R^{s,RN} + (1 - \pi\mu) R^{s,NP}.$$

This depends on prices  $q^h, q^{h,NP}, q^{h,RN}$ , which themselves depend upon the relative bargaining power of different agents in the economy.

### 1.A.1.7 Proof of proposition 5

I claimed that if the implicit recourse were to be credible, the optimal level of promise would mean  $q^j = 1$  and therefore zero profit for securitizing firms. The relevant F.O.C. can be transformed in the following way:

Let's consider F.O.C. for firms with high quality investment opportunities. The remaining firms would not invest at all.

$$\begin{aligned} \frac{\partial V^{ND}}{\partial r^G} &= \frac{\partial V^{ND'}}{\partial (w' - cir')} \frac{\partial (w' - cir')}{\partial r^G} = 0, \\ \frac{\partial V^{ND'}}{\partial (w' - cir')} \frac{\partial}{\partial r^G} \frac{(1 - \theta) \beta w (r^{j'} + \lambda q^j) - \theta \beta w (r^G - r^j)}{1 - \theta q^G} &= 0, \\ \frac{\partial V^{ND'}}{\partial (w' - cir')} \frac{\partial}{\partial r^G} \frac{\beta w (r^{j'} + \lambda q^j - \theta (r^G + \lambda q^j))}{1 - \theta q^G} &= 0. \end{aligned}$$

After substituting in this case with constant aggregate productivity  $q^{G,j} = \frac{r^{G'} + \lambda q^j}{r^{j'} + \lambda q^j} q^j$ , this condition implies that

$$\frac{\partial V^{ND'}}{\partial (w' - cir')} \frac{\partial}{\partial r^G} \frac{\beta w (r^{j'} + \lambda q^j) \left(1 - \theta \frac{q^{G,j}}{q^j}\right)}{1 - \theta q^{G,j}} = 0,$$

and since  $\frac{\partial V^{ND'}}{\partial (w' - cir')} > 0$ ,  $\frac{\partial q^{G,j}}{\partial r^G} > 0$ , the above condition simplifies to

$$\frac{\partial}{\partial q^{G,j}} \frac{\left(1 - \theta \frac{q^{G,j}}{q^j}\right)}{1 - \theta q^{G,j}} = \frac{\theta (q^j - 1)}{q^j (1 - \theta q^{G,j})^2} = 0.$$

This implies  $q^j = 1$ .

Note that for when the level of  $r^G$  satisfies this condition, the return from investing and securitizing is equal to the return from investing but not securitizing, i.e., securitization does not increase the return:

$$\begin{aligned} R \mid \text{investing \& securitizing} &= R \mid \text{investing} \\ \frac{(r^j + \lambda q^j - \theta (r^G + \lambda q^j))}{1 - \theta \frac{r^G + \lambda q^j}{r^j + \lambda q^j} q^j} &= \frac{r^j + \lambda q^j}{1}. \end{aligned}$$

When you substitute in the above condition  $q^j = 1$ , the condition is exactly satisfied for

all parameter values.

### 1.A.1.8 Proof of Proposition 8

To complete the proof of Proposition 8 sketched in the main text, I first need to derive from (1.13) the (1.14) and show that the RHS of equation (1.14) is independent of the level of aggregate productivity  $A$ . This means that variables  $B$  and  $q^h$  should be independent of the level of aggregate productivity  $A$ .

Under separation, steady state conditions are the following:

$$(1 - \lambda) (1 - \theta q^{h,IR}) = \pi \mu \beta (r^h + \lambda q^h), \quad (1.41)$$

$$r^h = (1 - \lambda) + (1 - \beta) (r^h + \lambda q^h), \quad (1.42)$$

$$\frac{r^G + \lambda q^h}{q^G} = \frac{(A + \Delta^h) K^{\alpha-1} + \lambda q^h}{q^h}, \quad (1.43)$$

$$V^{ND} (w' - cir') = V^D (w'). \quad (1.44)$$

Using the following property given by the logarithmic utility function:

$$\begin{aligned} V(w) &= \log((1 - \beta)w) + \beta \log((1 - \beta)\beta R w) + \beta^2 \log((1 - \beta)\beta^2 R^2 w) + \beta^3 \log((1 - \beta)\beta^3 R^3 w) \dots \\ &= \frac{1}{1 - \beta} \log(w) + \log((1 - \beta)) + \beta \log((1 - \beta)\beta R) + \beta^2 \log((1 - \beta)\beta^2 R^2) + \beta^3 \log((1 - \beta)\beta^3 R^3) \dots \\ &= \frac{1}{1 - \beta} \log(w) + V(1), \end{aligned}$$

we can transform the no-default condition expressed in (1.44) in the following way:

$$\begin{aligned} V^D (w') &= V^D \left( w \beta \frac{(1 - \theta) (r^h + \lambda q^h)}{(1 - \theta q^G)} \right) = V^D (w) + \frac{1}{1 - \beta} \log \left( \beta \frac{(1 - \theta) (r^h + \lambda q^h)}{(1 - \theta q^G)} \right) \\ V^{ND} (w' - cir') &= V^{ND} \left( w \beta \frac{(1 - \theta) \left( r^h + \lambda q^h - \frac{\theta}{1 - \theta} (r^G - r^h) \right)}{(1 - \theta q^G)} \right) \\ &= V^{ND} (w) + \frac{1}{1 - \beta} \log \left( \beta \frac{(1 - \theta) \left( r^h + \lambda q^h - \frac{\theta}{1 - \theta} (r^G - r^h) \right)}{(1 - \theta q^G)} \right). \end{aligned}$$

For simplicity, let's express the value functions separately from individual wealth in the following way, which is easy to do given the log utility:  $V(w) = V(1) + \frac{1}{1 - \beta} \log(w)$ . We can also find solutions for value functions with wealth normalized to unity, which we can denote simply as  $V = V(1)$ .

$$\begin{aligned} V^{ND} &= \log(1 - \beta) + \beta \left( \pi \mu V^{ND} (\beta R^{h,IR}) + \pi (1 - \mu) V^{ND} (\beta R^l) + (1 - \pi) V^{ND} (\beta R^z) \right) \\ &= \log(1 - \beta) + \beta \left( \frac{\pi \mu \log(\beta R^{h,IR})}{1 - \beta} + \pi (1 - \mu) \frac{\log(\beta R^l)}{1 - \beta} + (1 - \pi) \frac{\log(\beta R^z)}{1 - \beta} + V^{ND} \right) \\ &= \frac{\log(1 - \beta)}{1 - \beta} + \frac{\beta \log(\beta)}{(1 - \beta)^2} + \frac{\beta}{(1 - \beta)^2} \left( \pi \mu \log(R^{h,IR}) + \pi (1 - \mu) \log(R^l) + (1 - \pi) \log(R^z) \right). \end{aligned}$$

$$\begin{aligned} V^D &= \log(1 - \beta) + \beta \left( \pi \mu V^D (\beta R^{h,D}) + \pi (1 - \mu) V^D (\beta R^l) + (1 - \pi) V^D (\beta R^z) \right) \\ &= \log(1 - \beta) + \beta \left( \frac{\pi \mu \log(\beta R^{h,D})}{1 - \beta} + \pi (1 - \mu) \frac{\log(\beta R^l)}{1 - \beta} + (1 - \pi) \frac{\log(\beta R^z)}{1 - \beta} + V^D \right) \end{aligned}$$

$$= \frac{\log(1-\beta)}{1-\beta} + \frac{\beta \log(\beta)}{(1-\beta)^2} + \frac{\beta}{(1-\beta)^2} \left( \pi\mu \log(R^{h,D}) + \pi(1-\mu) \log(R^l) + (1-\pi) \log(R^z) \right).$$

Substituting the above derived conditions into the no-default condition (1.44) and canceling the terms equal for both value functions, we obtain:

$$\log\left(\beta(1-\theta)\left(r^h + \lambda q^h - \frac{\theta}{1-\theta}(r^G - r^h)\right)\right) + \frac{\beta\pi\mu}{1-\beta} \log(R^{h,IR}) = \log\left(\beta(1-\theta)(r^h + \lambda q^h)\right) + \frac{\beta\pi\mu}{1-\beta} \log(R^{h,D}),$$

where LHS shows the utility from consumption when wealth is reduced by repayment of implicit recourse and from the future discounted benefit of having a good reputation. The RHS then shows higher immediate utility from savings on implicit recourse, but the future utility is lower since the firm can no longer issue and sell new loans. This equation can further be simplified using (1.43) and substituting for the returns:

$$\begin{aligned} \log\left(\frac{r^h + \lambda q^h - \theta(r^G + \lambda q^h)}{(1-\theta)(r^h + \lambda q^h)}\right) &= -\frac{\beta\pi\mu}{1-\beta} \log\left(\frac{R^{h,IR}}{R^{h,D}}\right) \\ &= -\frac{\beta\pi\mu}{1-\beta} \log\left(\frac{(1-\theta)\left(r^h + \lambda q^h - \frac{\theta}{1-\theta}(r^G - r^h)\right)}{(1-\theta q^G)} \frac{1}{(r^h + \lambda q^h)}\right) \\ &= -\frac{\beta\pi\mu}{1-\beta} \log\left(\frac{r^h + \lambda q^h - \theta(r^G + \lambda q^h)}{r^h + \lambda q^h - \theta q^h(r^G + \lambda q^h)}\right). \end{aligned}$$

Now let's denote the price premium for the equilibrium implicit guarantee  $B \equiv \frac{q^G}{q^h} = \frac{r^G + \lambda q^h}{r^h + \lambda q^h}$ , then we can express the above equation as follows:

$$\log\left(\frac{1-\theta B}{1-\theta}\right) = \frac{\beta\pi\mu}{1-\beta} \log\left(\frac{1-\theta B q^h}{1-\theta B}\right), \quad (1.45)$$

which is an equation in two unknown endogenous variables  $(B, q^h)$  depending on time preference parameters  $\beta$  and parameters defining the strength of the financing frictions  $(\pi, \mu, \theta)$ .

We can express a second steady state condition in two endogenous variables  $(B, q^h)$  combining two remaining conditions for the steady state (1.41, 1.42):

$$(1-\lambda)(1-\theta B q^h) = \pi\mu(1-\lambda + \lambda q^h). \quad (1.46)$$

Combining the two equations (1.45, 1.46), we can obtain the solution to both the price of the high-quality asset  $q^h$  and the price premium for the equilibrium implicit guarantee  $B$ . Crucially, the solution does not depend on the level of aggregate productivity  $A$ , which is one step we needed to show to complete the proof of Proposition 8.

The second step is to derive (1.14) from (1.13). Note that in the separating equilibrium, selected by the Intuitive Criterion, mimicking firms with access to low quality projects would find it optimal to default on implicit recourse since in a separation equilibrium,  $r^{G*} > r_{l,cred,s}^G$ .

Similarly as with condition 1.44, we can transform the following condition for separation (1.13):

$$V^l(\text{mimicking \& default}) < V^l(\text{buying high loans})$$

$$\begin{aligned}
\log\left(\frac{\beta(1-\theta)(r^l + \lambda q^l)}{(1-\theta q^G)}\right) + \frac{\beta\pi\mu}{1-\beta} \log(R^{h,D}) &< \log\left(\beta\frac{(r^h + \lambda q^h)}{q^h}\right) + \beta\pi\mu \log(R^{h,IR}) \\
-\frac{\beta\pi\mu}{1-\beta} \log\left(\frac{R^{h,IR}}{R^{h,D}}\right) &< \log\left(\frac{(1-\theta q^{h,IR})}{(r^l + \lambda q^l)(1-\theta)}\frac{(r^h + \lambda q^h)}{q^h}\right) \\
\frac{\beta\pi\mu}{1-\beta} \log\left(\frac{1-\theta B q^h}{1-\theta B}\right) &< \log\left(\frac{(1-\theta B q^h)}{(1-\theta)}q^l\right).
\end{aligned}$$

Using (1.45) and the preceding transformations, we can replace LHS to get:

$$\begin{aligned}
\log\left(\frac{1-\theta B}{1-\theta}\right) &< \log\left(\frac{(1-\theta B q^h)}{(1-\theta)}q^l\right) \\
q^l &< \frac{1-\theta B q^h}{1-\theta B}.
\end{aligned} \tag{1.47}$$

If we divide (1.47) by  $q^h$  and substitute the ratio of prices by the steady state asset market clearing condition  $A^h/q^h = A^l/q^l$ , then we obtain:

$$\frac{A^h}{A^l} > \frac{(1-\theta B)q^h}{1-\theta B q^h}.$$

Proposition 8 (iii) also claims that the inequality in (1.5) is less likely to be satisfied than in (1.14). To prove that, let's first rewrite the denominator of (1.5) using (1.20), which says:

$$(1-\theta q^h)(1-\lambda) = \pi\mu(1-\lambda + \lambda q^h),$$

to obtain

$$\frac{A^h}{A^l} > \frac{(1-\theta)(1-\lambda)}{\pi\mu\left(\frac{1-\lambda}{q^h} + \lambda\right)}.$$

Similarly, let's rewrite the denominator of (1.14) using (1.46) to obtain:

$$\frac{A^h}{A^l} > \frac{(1-\theta B)(1-\lambda)}{\pi\mu\left(\frac{1-\lambda}{q^h} + \lambda\right)}.$$

We can show that

$$\frac{1-\lambda}{\pi\mu} = \frac{(1-\theta)(1-\lambda)}{\pi\mu\left(\frac{1-\lambda}{q^h} + \lambda\right)} \Big| \text{no implicit recourse} > \frac{(1-\theta B)(1-\lambda)}{\pi\mu\left(\frac{1-\lambda}{q^h} + \lambda\right)} \Big| \text{implicit recourse},$$

because the price premium for implicit recourse  $B$  is, by definition, higher than one, and  $q^h \Big| \text{no implicit recourse} > q^h \Big| \text{implicit recourse}$ . The latter comes directly from comparing (1.20) and (1.46), which when combined give:

$$\frac{1-\lambda + \lambda q^h}{1-\theta q^h} \Big| \text{no implicit recourse} = \frac{1-\lambda + \lambda q^h}{1-\theta B q^h} \Big| \text{implicit recourse}.$$

Further, this can be satisfied only if  $q^h \Big| \text{no implicit recourse} > q^h \Big| \text{implicit recourse}$ .

### 1.A.1.9 Other derivations from sub-section 1.3.3

#### Conditions for the minimum level of implicit recourse needed for separation

$G_{minsep}$ :

At  $G_{minsep}$ , firms with low quality investments are indifferent between mimicking and separating:

$$\begin{aligned} V^l \mid \text{mimicking \& default} &= V^l \mid \text{buying high loans} \\ \log \left( \frac{\beta (1 - \theta) (r^l + \lambda q^l)}{(1 - \theta q^G)} \right) + \beta \pi \mu \log (R^{h,D}) &= \log \left( \beta \frac{(r^h + \lambda q^h)}{q^h} \right) + \beta \pi \mu \log (R^{h,IR}) \\ -\beta \pi \mu \log \left( \frac{1 - \theta B_{min}}{1 - \theta} \right) &= \log \left( \frac{(1 - \theta B_{min} q^h)}{(1 - \theta)} q^l \right). \end{aligned} \quad (1.48)$$

Combining (1.48) with the following equilibrium investment condition

$$(1 - \lambda) (1 - \theta B_{min} q^h) = \pi \mu (1 - \lambda + \lambda q^h), \quad (1.49)$$

where  $B_{min} \equiv \frac{q^G}{q^h} = \frac{(A + G_{minsep}) K^{\alpha-1} + \lambda q^h}{r^h + \lambda q^h}$ , gives  $\{G_{minsep}, q^h, B_{min}\}$ .

#### Conditions for a unique pooling equilibrium:

A necessary condition for firms to have incentives to increase  $G$  above  $G_{cred,p}^l$  is: it must be considered as profitable to, at least, individually deviate above  $G_{cred,p}^l$ . The following condition should, therefore, be satisfied:

$$\frac{\partial V^{ND}}{\partial r^G} = \frac{\partial V^{ND}}{\partial R^{h,IR}} \frac{\partial R^{h,IR}}{\partial r^G} > 0.$$

Since  $\frac{\partial V^{ND}}{\partial R^{h,IR}} > 0$ , this becomes:

$$\frac{\partial R^{h,IR}}{\partial r^G} = \frac{\partial}{\partial r^G} \frac{((r^h - \frac{\theta}{1-\theta}(r^G - r^h)) + \lambda q^h)(1 - \theta)}{1 - \theta \frac{(\mu r^G + (1-\mu)r^l) + \lambda(\mu q^h + (1-\mu)q^l)}{r^h + \lambda q^h} q^h} > 0.$$

In taking the derivative, we obtain:

$$\begin{aligned} -\theta K^{\alpha-1} \left( 1 - \theta \frac{(\mu r^G + (1-\mu)r^l) + \lambda(\mu q^h + (1-\mu)q^l)}{r^h + \lambda q^h} q^h \right) \\ + \frac{\theta \mu q^h K^{\alpha-1}}{r^h + \lambda q^h} \left( r^h - \frac{\theta}{1-\theta} (r^G - r^h) + \lambda q^h \right) (1 - \theta) > 0, \end{aligned}$$

$$\begin{aligned} \left( r^h - \frac{\theta}{1-\theta} (r^G - r^h) + \lambda q^h \right) (1 - \theta) \mu q^h &> r^h + \lambda q^h - \theta (\mu r^G + (1-\mu)r^l) \\ &+ \lambda (\mu q^h + (1-\mu)q^l) q^h \\ (\mu q^h - 1) (r^h + \lambda q^h) &> \theta q^h (\mu - 1) (r^l + \lambda q^l). \end{aligned} \quad (1.50)$$

As long as  $(\mu q^h - 1) > 0$ , the condition (1.50) always holds since  $\mu < 1$ . When  $(\mu q^h - 1) < 0$ , then we get

$$(r^h + \lambda q^h) < \theta \frac{q^h (1 - \mu)}{(1 - \mu q^h)} (r^l + \lambda q^l),$$

which is not satisfied if:

$$\frac{A^h}{A^l} > \theta \frac{q^h (1 - \mu)}{(1 - \mu q^h)},$$

or when rewritten:

$$\mu < \frac{1 - \theta q^l}{q^h - \theta q^l}.$$

This implies that the share of high quality assets has to be low enough, or in a pooling equilibrium, the relative difference in TFP has to be large enough.

### 1.A.1.10 Endogenizing the "skin in the game"

If we endogenize the "skin in the game" with the moral hazard problem described in section 1.5, we obtain the incentive compatible constraint (1.15). In this sub-section, I would like to show briefly that the main results concerning the provision of implicit recourse and the endogenous switching between the pooling equilibrium and the separating equilibrium hold.

First, we have to check whether firms have the incentive to provide implicit support. The check is equivalent to the proof of Proposition 5 as discussed in section 1.A.1.7 and which boils down to show that

$$\frac{\partial}{\partial q^{G,j}} \left( \frac{1 - \theta \frac{q^{G,j}}{q^j}}{1 - \theta q^{G,j}} \right) = \frac{(q^j - 1)}{q^j (1 - \theta q^{G,j})^2} \frac{\partial \theta q^{G,j}}{\partial q^{G,j}} \geq 0.$$

Since  $\frac{\partial \theta q^{G,j}}{\partial q^{G,j}} = \frac{\partial}{\partial q^{G,j}} \frac{q^{G,j}}{q^{G,j+1}} = \frac{1}{(q^{G,j+1})^2} > 0$ , the above condition corresponds again to  $q^j \geq 1$ . This means that in equilibrium, implicit recourse will be provided.

Given (1.15), the separating equilibrium in the deterministic steady state is defined by:

$$\log \left( \frac{1 - \theta B}{1 - \theta} \right) = \frac{\beta \pi \mu}{1 - \beta} \log \left( \frac{1 - \theta q^h B}{1 - \theta B} \right), \quad (1.51)$$

$$\begin{aligned} (1 - \lambda) (1 - \theta B q^h) &= \pi \mu (1 - \lambda + \lambda q^h) \\ \log \left( \frac{1 - \theta B}{1 - \theta} \right) &= \frac{\beta \pi \mu}{1 - \beta} \log \left( \frac{1 - \theta q^h B}{1 - \theta B} \right) \\ \theta &= \frac{1}{B q^h + 1}. \end{aligned}$$

Which simplifies into two equations, which are independent on the level of TFP  $A$ :

$$(1 - \lambda) \left( \frac{1}{Bq^h + 1} \right) = \pi\mu (1 - \lambda + \lambda q^h)$$

$$\log \left( \frac{B(q^h - 1) + 1}{Bq^h} \right) = \frac{\beta\pi\mu}{1 - \beta} \log \left( \frac{1}{B(q^h - 1) + 1} \right).$$

The conditions for the existence of a separating equilibrium (1.14) becomes:

$$\frac{A^h}{A^l} > q^h (B(q^h - 1) + 1).$$

## 1.A.2 Derivation of firms' policy functions

In this section, I will derive in detail the policy functions of firms in the most general case. It is convenient to rewrite the firm's problem characterized in sub-section 1.3.1.3 in a recursive formulation:

$$\begin{aligned} V^{ND}(\bar{s}, w - cir; \bar{S}) &= \pi (\mu V^{ND,h}(\bar{s}, w - cir; \bar{S}) + (1 - \mu) V^{ND,l}(\bar{s}, w - cir; \bar{S})) \\ &\quad + (1 - \pi) V^{ND,z}(\bar{s}, w - cir; \bar{S}), \\ V^D(\bar{s}, w; \bar{S}) &= \pi (\mu V^{D,h}(\bar{s}, w; \bar{S}) + (1 - \mu) V^{D,l}(\bar{s}, w; \bar{S})) + (1 - \pi) V^{D,z}(\bar{s}, w; \bar{S}), \\ V^{ND,k}(\bar{s}, w; \bar{S}) &= \max_{c,i,\{a'_j\}_j, h^{S'}, l^{S'}, r^{G'}} [\log(c) + \beta E [\max(V^{ND}(\bar{s}', w' - cir'; \bar{S}'), V^D(\bar{s}', w'; \bar{S}'))]], \\ V^{D,k}(\bar{s}, w; \bar{S}) &= \max_{c,i,h',l'} [\log(c) + \beta EV^D(\bar{s}', w'; \bar{S}')], \end{aligned}$$

subject to the budget constraints that take the following form for investing firms for which the “skin in the game” constraint is binding:

$$c_{i,t} + \frac{(1 - \theta q_{i,t}^G)}{(1 - \theta)} h_{i,t+1} + cir_{i,t} = \sum_{j \in \mathcal{I}_{t-1}} a_{i,j,t} (r_{j,t}^{\hat{G}} + \lambda q_{j,t}) + h_{i,t}^S (r_t^h + \lambda q_t^h) + l_{i,t}^S (r_t^l + \lambda q_t^l) \quad \forall i \in \mathcal{H}_t \cap \mathcal{I}_t,$$

$$c_{i,t} + \frac{(1 - \theta q_{i,t}^G)}{(1 - \theta)} l_{i,t+1} + cir_{i,t} = \sum_{j \in \mathcal{I}_{t-1}} a_{i,j,t} (r_{j,t}^{\hat{G}} + \lambda q_{j,t}) + h_{i,t}^S (r_t^h + \lambda q_t^h) + l_{i,t}^S (r_t^l + \lambda q_t^l) \quad \forall i \in \mathcal{L}_t \cap \mathcal{I}_t.$$

The incentive compatible constraints, which have to be satisfied in equilibrium for reputation-based implicit recourse to exist, are the following:

$$\begin{aligned} V^{ND}(\bar{s}, w - cir; \bar{S}) &\geq V^D(\bar{s}, w; \bar{S}), \\ V^P(\bar{s}; \bar{S}) &\geq V^{NP}(\bar{s}; \bar{S}), \end{aligned}$$

where  $V^{ND}$ ,  $V^D$ ,  $V^P$  and  $V^{NP}$  are the value functions if the firm never defaulted, defaulted, always punished a default on implicit recourse and failed to punished, respectively.

From first-order conditions, we can obtain the following Euler equations in cases where



the “skin in the game” is binding for all investing firms:

$$E_t \left[ \beta \frac{c_{i,t}}{c_{i,t+1}} \frac{r_{t+1}^{\hat{G}} + \lambda q_{j,t+1}}{q_{j,t}^G} \right] = 1 \quad \forall i \in \mathcal{S}_t, \forall j \in \mathcal{I}_t, \quad (1.52)$$

$$E_t \left[ \beta \frac{c_{i,t}}{c_{i,t+1}} \frac{r_{t+1}^h + \lambda q_{t+1}^h}{q_t^h} \right] = 1 \quad \forall i \in \mathcal{S}_t, \quad (1.53)$$

$$E_t \left[ \beta \frac{c_{i,t}}{c_{i,t+1}} \frac{r_{t+1}^l + \lambda q_{t+1}^l}{q_t^l} \right] = 1 \quad \forall i \in \mathcal{S}_t, \quad (1.54)$$

$$E_t \left[ \beta \frac{c_{i,t}}{c_{i,t+1}} \frac{r_{t+1}^h + \lambda q_{t+1}^h}{\frac{(1-\theta q_t^G)}{(1-\theta)}} \right] = 1 \quad \forall i \in \mathcal{H}_t \cap \mathcal{I}_t, \quad (1.55)$$

$$E_t \left[ \beta \frac{c_{i,t}^l}{c_{i,t+1}^l} \frac{r_{t+1}^l + \lambda q_{t+1}^l}{\frac{(1-\theta q_t^{G,l})}{(1-\theta)}} \right] = 1 \quad \forall i \in \mathcal{L}_t \cap \mathcal{I}_t. \quad (1.56)$$

I guess and verify that all investing firms provide the same level of implicit support  $r_{j,t+1}^G = r_{t+1}^G \quad \forall j \in \mathcal{I}_t$  (see discussion in section 1.3.3 for details). Then, I guess and verify that policy functions have the following form.

Due to the logarithmic utility function, all firms consume a  $(1 - \beta)$  fraction of their wealth:

$$c_{i,t} = (1 - \beta) \left( \sum_{j \in \mathcal{I}_{t-1}} a_{i,j,t} (r_{j,t}^{\hat{G}} + \lambda q_{j,t}) + h_{i,t}^S (r_t^h + \lambda q_t^h) + l_{i,t}^S (r_t^l + \lambda q_t^l) \right) \quad \forall i.$$

Under binding "skin in the game", firms with access to high quality investment opportunities  $\mathcal{H}_t$  invest all of the non-consumed part of wealth into new projects and sell the maximum fraction of investment  $\theta$  to saving firms:

$$h_{i,t+1} = a_{i,i,t+1} = \frac{\beta \left( \sum_{j \in \mathcal{I}_{t-1}} a_{i,j,t} (r_{j,t}^{\hat{G}} + \lambda q_{j,t}) + h_{i,t}^S (r_t^h + \lambda q_t^h) + l_{i,t}^S (r_t^l + \lambda q_t^l) \right)}{\frac{(1-\theta q_{i,t}^G)}{(1-\theta)}} \quad \forall i \in \mathcal{H}_t \cap \mathcal{I}_t,$$

$$l_{i,t+1} = 0 \quad \forall i \in \mathcal{H}_t \cap \mathcal{I}_t.$$

In the pooling equilibrium, firms with access to low quality investment opportunities  $\mathcal{L}_t$  also invest all of the non-consumed part of wealth into new projects, and if the “skin in the game” constraint is binding, they sell the maximum fraction of the investment  $\theta$  to saving firms:

$$l_{i,t+1} = a_{i,i,t+1} = \frac{\beta \left( \sum_{j \in \mathcal{I}_{t-1}} a_{i,j,t} (r_{j,t}^{\hat{G}} + \lambda q_{j,t}) + h_{i,t}^S (r_t^h + \lambda q_t^h) + l_{i,t}^S (r_t^l + \lambda q_t^l) \right)}{\frac{(1-\theta q_{i,t}^G)}{(1-\theta)}} \quad \forall i \in \mathcal{L}_t \cap \mathcal{I}_t,$$

$$h_{i,t+1} = 0 \quad \forall i \in \mathcal{L}_t \cap \mathcal{I}_t.$$

If the economy is in a separating equilibrium, the intersection  $\mathcal{L}_t \cap \mathcal{I}_t = \emptyset$  is an empty

set, and firms with access to low quality investment opportunities  $\mathcal{L}_t$  are not investing into new projects, but rather are buying securitized assets from other firms  $\mathcal{L}_t \subset \mathcal{S}_t$ .

Saving firms  $\mathcal{S}_t$  are in equilibrium indifferent between investing into different types of assets. All of them try to diversify their investment, so I guess and verify that in equilibrium, all will allocate the same fraction of wealth into different types of assets:

$$h_{i,t+1}^S = \frac{\zeta^{hS} \beta \left( \sum_{j \in \mathcal{I}_{t-1}} a_{i,j,t} \left( r_{j,t}^{\hat{G}} + \lambda q_{j,t} \right) + h_{i,t}^S (r_t^h + \lambda q_t^h) + l_{i,t}^S (r_t^l + \lambda q_t^l) \right)}{q_t^h} \quad \forall i \in \mathcal{S}_t,$$

$$l_{i,t+1}^S = \frac{\zeta^{lS} \beta \left( \sum_{j \in \mathcal{I}_{t-1}} a_{i,j,t} \left( r_{j,t}^{\hat{G}} + \lambda q_{j,t} \right) + h_{i,t}^S (r_t^h + \lambda q_t^h) + l_{i,t}^S (r_t^l + \lambda q_t^l) \right)}{q_t^l} \quad \forall i \in \mathcal{S}_t,$$

$$\begin{aligned} h_{i,t+1}^P &= \sum_{j \in \mathcal{H}_t \cap \mathcal{I}_t} a_{i,j,t+1} \\ &= \frac{\zeta^{hP} \beta \left( \sum_{j \in \mathcal{I}_{t-1}} a_{i,j,t} \left( r_{j,t}^{\hat{G}} + \lambda q_{j,t} \right) + h_{i,t}^S (r_t^h + \lambda q_t^h) + l_{i,t}^S (r_t^l + \lambda q_t^l) \right)}{q_t^G} \quad \forall i \in \mathcal{S}_t, \end{aligned}$$

$$\begin{aligned} l_{t+1}^P &= \sum_{j \in \mathcal{L}_t \cap \mathcal{I}_t} a_{i,j,t+1} \\ &= \frac{\zeta^{lP} \beta \left( \sum_{j \in \mathcal{I}_{t-1}} a_{i,j,t} \left( r_{j,t}^{\hat{G}} + \lambda q_{j,t} \right) + h_{i,t}^S (r_t^h + \lambda q_t^h) + l_{i,t}^S (r_t^l + \lambda q_t^l) \right)}{q_t^G} \quad \forall i \in \mathcal{S}_t, \end{aligned}$$

where  $\zeta^{hS} + \zeta^{lS} + \zeta^{hP} + \zeta^{lP} = 1$ .

The consumption of the firms in the following period depends on the return from their investment:

$$\begin{aligned} c_{i,t+1} &= (1 - \beta) [h_{i,t+1}^S (r_{t+1}^h + \lambda q_{t+1}^h) + l_{i,t+1}^S (r_{t+1}^l + \lambda q_{t+1}^l) \\ &\quad + h_{i,t+1}^P (r_{t+1}^{\hat{G},h} + \lambda q_{t+1}^h) + l_{t+1}^P (r_{t+1}^{\hat{G},l} + \lambda q_{t+1}^l)] \quad \forall i \in \mathcal{S}_t, \\ c_{i,t+1} &= (1 - \beta) (h_{i,t+1} (r_{t+1}^h + \lambda q_{t+1}^h)) \quad \forall i \in \mathcal{H}_t \cap \mathcal{I}_t, \\ c_{i,t+1} &= (1 - \beta) (l_{i,t+1} (r_{t+1}^l + \lambda q_{t+1}^l)) \quad \forall i \in \mathcal{L}_t \cap \mathcal{I}_t. \end{aligned}$$

Using these guesses and substituting in (1.55) and (1.56), we can see that these conditions always hold.

The remaining Euler equations (1.53), (1.54), and (1.52) after substitutions, can be rewritten into:

$$E_t \left[ \frac{\frac{r_{t+1}^h + \lambda q_{t+1}^h}{q_t^h}}{\Xi_{t+1}} \right] = 1,$$

$$E_t \left[ \frac{\frac{r_{t+1}^l + \lambda q_{t+1}^l}{q_t^l}}{\Xi_{t+1}} \right] = 1,$$

$$E_t \left[ \frac{\frac{r_{t+1}^{\hat{G}} + \lambda q_{t+1}^{\hat{G}}}{q_t^{\hat{G}}}}{\Xi_{t+1}} \right] = 1,$$

where  $\Xi_{t+1} = \zeta^{hS} \frac{r_{t+1}^h + \lambda q_{t+1}^h}{q_t^h} + \zeta^{lS} \frac{r_{t+1}^l + \lambda q_{t+1}^l}{q_t^l} + \zeta^{hP} \frac{r_{t+1}^{\hat{G},h} + \lambda q_{t+1}^{\hat{G},h}}{q_t^{\hat{G},h}} + \zeta^{lP} \frac{r_{t+1}^{\hat{G},l} + \lambda q_{t+1}^{\hat{G},l}}{q_t^{\hat{G},l}}$ .

The allocation of saving firms (those with zero-profit projects) between high and low investment projects have to satisfy the market clearing conditions on both primary and secondary markets for high and low projects:

$$\begin{aligned} \lambda H_t &= \zeta^{hS} \beta \sum_{i \in \mathcal{S}_t} \left( \sum_{j \in \mathcal{I}_{t-1}} a_{i,j,t} (r_{j,t}^{\hat{G}} + \lambda q_{j,t}) + h_{i,t}^S (r_t^h + \lambda q_t^h) + l_{i,t}^S (r_t^l + \lambda q_t^l) \right), \\ \lambda L_t &= \zeta^{lS} \beta \sum_{i \in \mathcal{S}_t} \left( \sum_{j \in \mathcal{I}_{t-1}} a_{i,j,t} (r_{j,t}^{\hat{G}} + \lambda q_{j,t}) + h_{i,t}^S (r_t^h + \lambda q_t^h) + l_{i,t}^S (r_t^l + \lambda q_t^l) \right), \\ \theta &= \frac{\beta \sum_{i \in \mathcal{H}_t \cap \mathcal{I}_t} \left( \sum_{j \in \mathcal{I}_{t-1}} a_{i,j,t} (r_{j,t}^{\hat{G}} + \lambda q_{j,t}) + h_{i,t}^S (r_t^h + \lambda q_t^h) + l_{i,t}^S (r_t^l + \lambda q_t^l) \right)}{(1 - \theta q_t^{\hat{G}})} \\ &= \frac{\zeta^{hP} \sum_{i \in \mathcal{S}_t} \left( \sum_{j \in \mathcal{I}_{t-1}} a_{i,j,t} (r_{j,t}^{\hat{G}} + \lambda q_{j,t}) + h_{i,t}^S (r_t^h + \lambda q_t^h) + l_{i,t}^S (r_t^l + \lambda q_t^l) \right)}{q_t^{\hat{G}}}, \\ \theta &= \frac{\beta \sum_{i \in \mathcal{L}_t \cap \mathcal{I}_t} \left( \sum_{j \in \mathcal{I}_{t-1}} a_{i,j,t} (r_{j,t}^{\hat{G}} + \lambda q_{j,t}) + h_{i,t}^S (r_t^h + \lambda q_t^h) + l_{i,t}^S (r_t^l + \lambda q_t^l) \right)}{(1 - \theta q_t^{\hat{G}})} \\ &= \frac{\zeta^{lP} \sum_{i \in \mathcal{S}_t} \left( \sum_{j \in \mathcal{I}_{t-1}} a_{i,j,t} (r_{j,t}^{\hat{G}} + \lambda q_{j,t}) + h_{i,t}^S (r_t^h + \lambda q_t^h) + l_{i,t}^S (r_t^l + \lambda q_t^l) \right)}{q_t^{\hat{G}}}. \end{aligned}$$

And the goods market clears, too:  $Y_t = C_t + I_t$ .

### 1.A.3 Calibration of the parameters used in section 1.4

In section 1.4, I explain the choice of most of the model parameters. Here I would like to specifically comment on the choice of the share of high quality investment opportunities  $\mu$  and the dispersion of the type-specific component of high and low quality projects in the two states  $\Delta^l(A^H) / \Delta^h(A^H)$ ,  $\Delta^l(A^L) / \Delta^h(A^L)$ .

I choose these parameters to replicate the performance (delinquency rates) of securitized assets, which has been at the core of recent debates over the efficiency of securitization—subprime residential mortgage backed securities. Demyanyk and Van Hemert (2011) study the delinquency rates of subprime mortgage loans. In Figure 1.11, which is taken from Demyanyk and Van Hemert (2011), they report the actual delinquency rates of these loans in the left panel and in the right panel the delinquency rates adjusted by the effect of various observable characteristics of the loans and the economy. They conclude that the quality of the loans measured by the adjusted delinquency rates has

deteriorated significantly since 2004. This finding is consistent with the switching mechanism presented in this paper. As you can see in the left panel of the Figure 1.12, the U.S. emerged from a recession in 2003, and in 2004, the output again reached its potential. The model predicts that as the economy moves to the boom stage of a business cycle, the equilibrium in the signaling game becomes pooling, and as a consequence, low quality loans start to be financed. As shown in the right panel of the Figure 1.11, the boom period of 2004-2007 is associated with lower quality loans, and the economic downturn of 2001-2003 is associated with higher quality loans.

I used the reported delinquency rates by Demyanyk and Van Hemert (2011) to calibrate the model parameters.<sup>39</sup> I particularly want to match the delinquency rate of high quality loans after 12 months in the low state to the delinquency of the 2001 vintage, which is 12.5%; the delinquency rate of high quality loans after 12 months in the high state to the average of the delinquency of the 2002 and 2003 vintage which is approx. 7%; the delinquency rate of a mix of high and low quality loans after 12 months in the high state to the delinquency of the 2005 vintage, which is 9.5%; and the delinquency rate of the mix of high and low quality loans after 12 months in the low state to the delinquency of the 2007 vintage, which is 22.5%. This gives me:  $\Delta^l(A^H)/\Delta^h(A^H) = 0.94$  and  $\Delta^l(A^L)/\Delta^h(A^L) = 0.71$ .

Calibration of the share of high quality investment opportunities  $\mu$  is more complicated since I do not have disaggregated data for the USA. However, assuming the growth in the volume of subprime mortgage loans between 2003 and 2004 was driven mainly by the entry of firms with access to low quality loans into the market, we would obtain  $\mu = 0.6$ . Since this estimate is rather rough, I use loan level data from Moody's PDS database for the UK, which according to the empirical analysis in section 1.6 seems to be in line with the model predictions. When we compare the delinquency rates of the collateral of the RMBS in the period with the lowest output gap, i.e., in the period 2009q3, on one hand for loans issued in previous boom stages of the business cycle, i.e., in 2005q3-2008q1 (left panel), and on the other hand for loans issued in previous recessions, i.e., in periods 2001Q3-2003Q2 and 2004Q3-2005Q2, we find a significant difference. In particular, it seems that we can distinguish two relatively clear cut groups in the subset of RMBS issued in the boom period. One has very low delinquency rates (below 4%) and the other has, at times, much higher delinquency rates. When I use the threshold delinquency rate of 4% to identify high and low quality assets and combine the reported frequency with volumes, I find the share of high quality investment opportunities  $\mu = 0.63$ . This is approximately consistent with my initial guess for the subprime mortgage loans in the USA, so I use this parameter level.

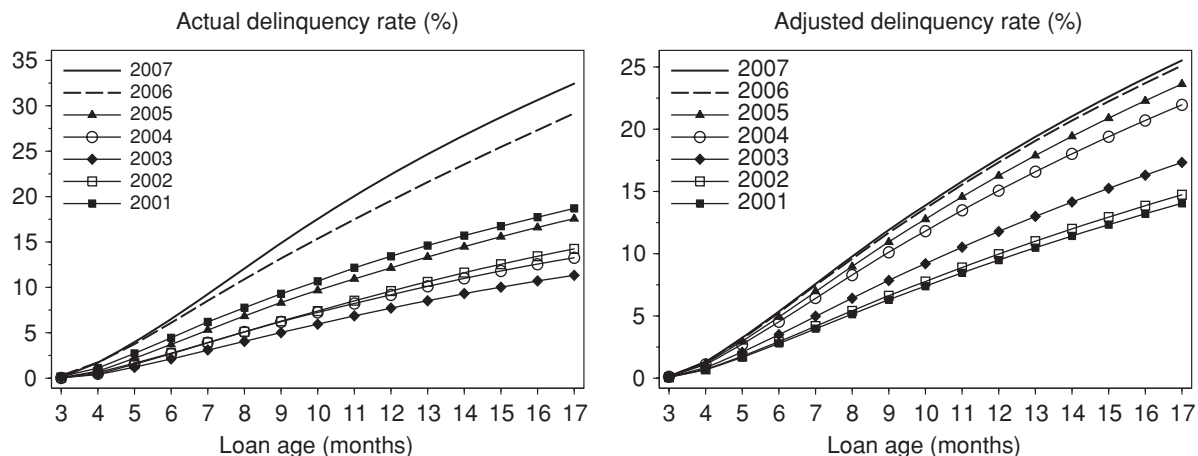
#### 1.A.4 Numerical solutions of the fully stochastic dynamic model

To solve the fully stochastic dynamic model, I use global numerical approximation methods. Since, depending on the state variables, the economy is switching between separating and pooling equilibria, I use global approximation methods. In particular, I

---

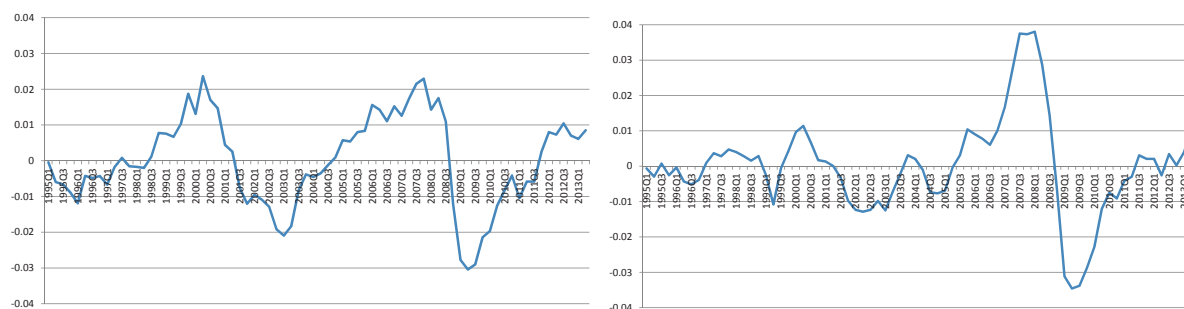
<sup>39</sup>The model presented in this paper does not model loan repayments explicitly. If I assume that a delinquent fraction of loans/projects do not generate cash-flows in the current period, then I can compute the ratio of gross profits in the two types of projects.

**Figure 1.11:** Actual and adjusted delinquency rates for subprime mortgages by Demyanyk and Van Hemert (2011).



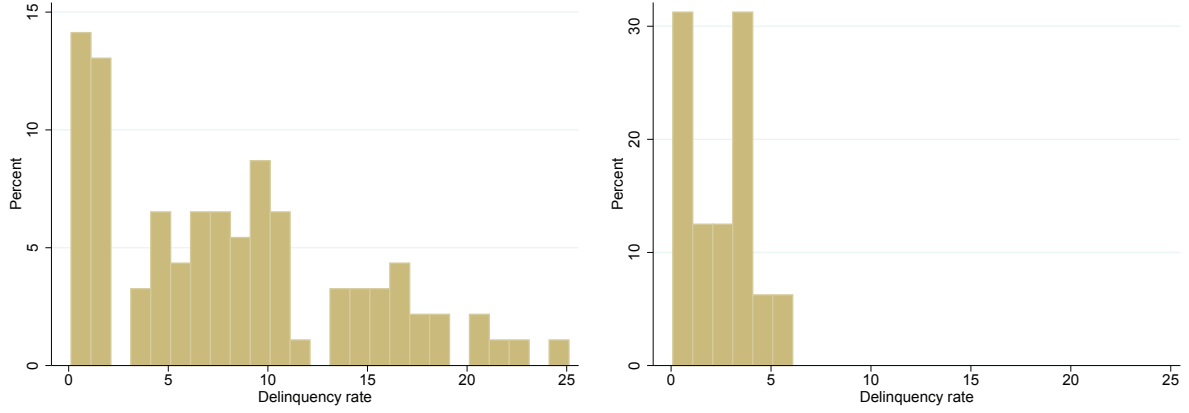
Note: On p.1, Demyanyk and Van Hemert (2011) describe their figure: “The figure shows the age pattern in the actual (left panel) and adjusted (right panel) delinquency rate for the different vintage years. The delinquency rate is defined as the cumulative fraction of loans that were past due 60 or more days, in foreclosure, real-estate owned, or defaulted, at or before a given age. The adjusted delinquency rate is obtained by adjusting the actual rate for year-by-year variation in FICO scores, loan-to-value ratios, debt-to-income ratios, missing debt-to-income ratio dummies, cash-out refinancing dummies, owner- occupation dummies, documentation levels, percentage of loans with prepayment penalties, mortgage rates, margins, composition of mortgage contract types, origination amounts, MSA house price appreciation since origination, change in state unemployment rate since origination, and neighborhood median income.”

**Figure 1.12:** Log of the output gap in the USA (left panel) and the UK (right panel)



Note: Data are from Eurostat for the UK and from FRED (St.Louis FED) for the USA. I construct the output gap using the Hodrick-Prescott filter with the smoothing parameter 1600.

**Figure 1.13:** Histograms of delinquency rates for collateral of the RMBS issued in the UK in 2009q3 for loans issued in the boom (left panel) and for loans issued in the bust (right panel)



Note: The figure shows histograms of the delinquency rates of the collateral for the RMBS, which are defined as the amount of receivables that are 90 or more days past due divided by the original collateral balance (in %). The source of the data is Moody's PDS database. The left panel shows the delinquency rate for the subset of RMBS issued in the boom periods 2005q3-2008q1 and the right panel RMBS issued in recessions in periods 2001Q3-2003Q2 and 2004Q3-2005Q2.

look for the values of the following functions:

$$\begin{aligned} q_t^h &= \Gamma_1(A_t, K_t, \omega_t), \\ q_t^l &= \Gamma_2(A_t, K_t, \omega_t), \\ V^{ND} - V^D &= \Gamma_3(A_t, K_t, \omega_t), \end{aligned}$$

I construct a grid for the three aggregate states  $A$ ,  $K$ , and  $\omega$  and start with a guess equal to steady-state values. Then, I iterate using a set of equilibrium conditions to find the updated values of  $(\Gamma_1, \Gamma_2, \Gamma_3)$  until the updated values are close to previous guesses:

$$\begin{aligned} &|q_t^h(ite\text{r}) - q_t^h(ite\text{r} - 1)| + |q_t^l(ite\text{r}) - q_t^l(ite\text{r} - 1)| \\ &+ |V^{ND}(ite\text{r}) - V^{ND}(ite\text{r} - 1)| + |V^D(ite\text{r}) - V^D(ite\text{r} - 1)| < \varepsilon. \end{aligned}$$

During iteration at each point on the grid, it is evaluated whether the economy is in a separating or pooling equilibrium. The values of  $(\Gamma_1, \Gamma_2, \Gamma_3)$  out of the grid are obtained by trilinear interpolation.

## Chapter 2

---

# Adverse Selection on Re-sale Markets for Securitized Assets

## 2.1 Introduction

The first chapter of the dissertation dealt with the efficiency of financial intermediation through securitization. It showed that asymmetric information between the issuers of securitized assets and their first buyers may prevail especially in a boom, which leads to a build-up of inefficient investment and then deepens and prolongs the subsequent recession. The first chapter studied the asymmetric information on the primary market for securitized assets. In this chapter, I focus on the problem of asymmetric information on the re-sale markets for securitized assets.

During the financial crisis of the late 2000s, we observed a severe drop in volumes and increase of spreads on the markets for securitized products. Brunnermeier (2009) reports the drying up of the asset-backed commercial paper (ABCP) market in 2007-08. These were the assets issued by the SPVs to back loans such as mortgages.<sup>1</sup> Brunnermeier (2009) also shows that, during this time, the spreads for different securitized assets such as ABCP or mortgage backed securities (MBS) increased dramatically. This of course stems from the burst in the housing market bubble and the subsequent increased delinquencies on mortgages, especially on those of lower quality. However, the build-up of the bubble, the supposed underestimation of risks of securitized assets as well as the extreme market dry-up may seem to contradict the rational expectations paradigm. Indeed, economists

---

<sup>1</sup>More details on securitization processes can be found in the literature review of the first chapter.

such as Shleifer and Vishny (2010) or Gennaioli, Shleifer, and Vishny (2013) refer to animal spirits, market sentiment and irrationality to explain the mentioned phenomena. Some market sentiment, underestimation of risks or simply too little experience with the new types of assets was probably present on the markets. However, in this chapter, I show that those phenomena can be explained in a purely rational expectations framework by a varying degree of asymmetric information and the induced adverse selection on the re-sale markets for securitized products. This chapter presents a theoretical model which predicts that, in booms or mild recessions, the degree of asymmetry of information and the adverse selection problem is limited and the re-sale markets for securitized assets work well. However, in a deep recession, the problem of asymmetric information and the implied adverse selection become suddenly severe and may lead to partial market shutdowns. This further exacerbates the depth and persistence of the recession. Such findings are in line with the empirical evidence found by Jordà, Schularick, and Taylor (2013) suggesting that financial crisis recessions are deeper and longer than normal recessions.

The whole chapter is based on the idea that there may be a potential information asymmetry on the re-sale markets for securitized products. This asymmetry may stem from the fact that securitized assets have been very complex and hard to price. For instance Arora et al. (2012) show that, for some derivatives, it may be prohibitively costly to find their intrinsic quality and price them correctly. Securitized assets have often been very complex bundles of various assets. An extreme example of such assets were the Collateralized Debt Obligations Squared (CDO Squared), which were assets backed by cash-flows from other CDOs, which themselves were backed by various ABCPs. Additional opacity was caused by the fact that there was little standardization in the securitized assets, little information about the performance of these assets and the trading was mainly over the counter (OTC). Based on these observations I assume in this model that the holder of a securitized asset may privately observe its cash-flows and if they are informative, find privately the intrinsic quality of the asset, which remains unobservable for a potential buyer on the re-sale markets. This assumption leads to a standard adverse selection problem in the spirit of Akerlof (1970).

There is a large literature studying the adverse selection problem in lender-borrower relationships.<sup>2</sup> The main innovation in this paper is the study of the interaction of the adverse selection problem on re-sale markets with the switching between pooling and

---

<sup>2</sup>A more detailed survey of relevant literature on adverse selection can be found in the literature review of the first chapter.



separating equilibria on the primary market for securitized assets over the business cycle proposed in the first chapter<sup>3</sup>, and the interaction with the provision of reputation based implicit recourse for securitized assets by their originators.

I show that the adverse selection is more severe, the larger is the stock of low quality assets on the balance sheets and the larger is the difference in the cash-flows of the assets. Since it is in the boom, when due to the existence of a pooling equilibrium on the primary market, the stock of low quality assets is build-up, and the recession is the period when there are larger differences in cash-flows generated by the assets, it is intuitive that the most severe adverse selection problem on the re-sale markets are predicted for recessions which are preceded by a prolonged boom period. A similar result is found in Boissay, Collard, and Smets (2013), which studies asymmetric information on the interbank markets and finds that interbank market freezes are more likely after a credit boom.

The provision of implicit recourse also affects the adverse selection problems on the re-sale markets. In this chapter, the implicit recourse is provided for the whole infinite lifetime of the asset. Further, to avoid the detection of the implicit recourse by regulators suspecting regulatory arbitrage, the implicit recourse is provided in a way that mimics the cash-flows of high quality assets. In booms or mild recessions, when the relative dispersion in asset qualities is low, issuers keep providing previously issued implicit recourse to maintain their reputation. However, during a deep recession, when the dispersion in asset qualities increases significantly, there are widespread defaults on the previously issued implicit recourse. The above dynamics have two-fold implications on the adverse selection and the re-sale market price.

First, as long as the implicit recourse is being provided, cash flows of high and low quality assets do not differ. Therefore, even if there is some level of adverse selection, the effect on the market price is limited. Second, as the implicit recourse is being provided in a manner so as not to be easily identified by the regulators, even holders of the assets observing high cash-flows cannot easily differentiate between low quality assets with implicit guarantees and high quality assets. Therefore, there is no asymmetric information and the adverse selection problem is contained. The second point is related to the “bliss-full ignorance” equilibrium introduced in Gorton and Ordoñez (2014), in which both sellers and buyers ignore the intrinsic value of the asset, but this is welfare improving since there is no adverse selection.

---

<sup>3</sup>In the remaining of the paper, I will refer to the first chapter as Kuntl (2014).

Therefore, provision of implicit recourse may both limit the scope of adverse selection and limit its effect, since it narrows the difference in effective cash-flows of the traded assets. However, implicit recourse is by definition not a contractual agreement, and therefore, when the costs of the recourse increase, a default on the recourse occurs. In the model, an economy-wide default indeed takes place in a deep recession in which low quality assets perform substantially worse than high quality assets. This leads to a sudden surge in adverse selection on the re-sale markets with significant negative effects on the market price, and may lead to partial market shutdowns when high quality assets cease being traded altogether.

The chapter is organized in the following way. Section 2.2 introduces the set-up of the model. Section 2.3 shows the main properties of the model and the effects of model assumptions analytically in a static framework and then introduces the methodology for the solution of the dynamic model in a Markov regime switching set-up. Finally, the dynamic properties of the model are described based on the solution of the Markov regime switching model.

## 2.2 Model set-up

This model presents a non-trivial extension of the financial intermediation model with securitization from the first chapter of this dissertation, i.e., from Kuncl (2014). Therefore, it replicates the results of Kuncl (2014), which include the build-up of low quality assets on firms' balance sheets in the boom period and the resulting subsequent prolonged and deeper recession. Compared to Kuncl (2014), this model is based on a more general set-up in the representative household framework inspired by Gertler and Karadi (2011) and Gertler and Kiyotaki (2010). The model introduces information frictions on the re-sale markets for securitized products, which may lead to asymmetric information and adverse selection problem. The model also features the reputation based implicit recourse. But unlike in Kuncl (2014), the recourse is provided for the whole lifetime of the asset and the model features equilibrium defaults on the recourse. The model shows how the provision of reputation-based implicit guarantees can interact with this adverse selection problem on the re-sale markets.

## 2.2.1 Physical set-up

There is a continuum of projects available separately on a continuum of islands. Each project can produce output using capital as input. The production function has constant returns to scale on the level of the individual project, but decreasing returns to scale on the aggregate.<sup>4</sup> Capital is not mobile across islands. Each period, an i.i.d. shock makes projects on  $\pi\mu$  fraction of islands highly productive, projects on  $\pi(1 - \mu)$  fraction of islands less productive and  $1 - \pi$  fraction of islands do not feature any new productive projects in this period. The production function for projects on the island with high and low production technology, respectively, is the following:

$$\begin{aligned} y_t^h &= r_t^h k_t = A_t \Delta_t^h K_t^\alpha k_t, \\ y_t^l &= r_t^l k_t = A_t \Delta_t^l K_t^\alpha k_t, \end{aligned}$$

where  $y_t^i$  is the amount of output of project with productivity  $i$ ,  $A_t$  is the aggregate level of total factor productivity (TFP),  $\Delta_t^i$  is the type component of TFP,  $K_t$  is the aggregate level of capital used in production and  $k_t$  is the level of capital used in this particular project.

The type component of TFP are a function of  $A$ . In particular, following the evidence from Bloom (2009) and Bloom et al. (2012), the cross-sectional variance of TFP across firms is counter-cyclical. Therefore,

$$\frac{\partial (\Delta_t^h - \Delta_t^l)}{\partial A_t} < 0. \quad (2.1)$$

Capital on islands increases with new investment and depreciates over time with a constant depreciation rate  $(1 - \lambda)$ . Therefore, the law of motion for the aggregate level of capital is:

$$K_{t+1} = I_t + \lambda K_t,$$

where  $I_t$  is the aggregate level of investment in period  $t$ .

---

<sup>4</sup>Kiyotaki and Moore (2012) assume a Cobb-Douglas production function with capital and labor as inputs. Due to competitive labor markets, they find that returns to capital are decreasing on aggregate, while constant on the level of individual firm. In this model, for simplicity this result is taken as an assumption.

## 2.2.2 Household

There is a representative household with a continuum of members and the size normalized to one. Within the household, there is perfect consumption insurance. The household is composed of financial firms. Financial firms manage all wealth in the economy  $N_t$ . The consumption is financed from non-negative dividends distributed by firms back to the households.

The household maximizes the objective function:

$$E_t \sum_{s=0}^{\infty} \beta^s \log(C_{t+s}),$$

where  $C_t$  is the household consumption. The budget constraint for the household is:  $C_t = \Pi_t$ , where  $\Pi_t$  are the distributed dividends from financial firms.

In this model, in order to enforce a reputation based implicit recourse, loss of reputation has to lower the value of equity. Therefore, the marginal value of equity should exceed its unitary costs.<sup>5</sup> Therefore, following Gertler and Kiyotaki (2010), I assume exogenous exit of financial firms. In particular, I assume that with a probability  $(1 - \sigma)$  a financial firm exits, and transfers all equity to the household. An exiting firm is replaced by a new firm, which receives limited start-up funds from the household, in particular  $\xi / (1 - \sigma)$  fraction of equity of exiting firms such that  $\beta > \sigma + \xi$ . Therefore the distributed dividends are equal to:

$$\Pi_t = N_t (1 - \sigma - \xi), \quad (2.2)$$

where  $N_t$  is the net wealth of all financial firms in the economy before the exit shock (see Figure 2.1 for the timing of shocks within each period).

## 2.2.3 Frictions

There are **three major frictions** (two similar to Küncl (2014) and one additional introducing a possibility for asymmetric information on the re-sale markets):

- The quality of the new projects can be observed only by the firm located on this island - there is **asymmetric information** about the quality of new projects.

---

<sup>5</sup>Should the value of equity be optimal, i.e.,  $E_t (\Lambda_{t,t+1} R_{t+1}^N) = 1$ , then the marginal value of equity would be equal to one. Any firm after losing its reputation would simply be liquidated. However, this means that there will be no costs of losing reputation making it non-valuable.

- Investing firms, which decide to securitize part of their investment, have to keep a “**skin in the game**”, i.e., they have can sell at most  $\theta$  fraction of the current investment.<sup>6</sup>
- It is prohibitively **costly to verify the quality of securitized assets bought on the re-sale market**. The holders of the assets can identify their quality only when their observed cash flows are informative, i.e., are distinct from cash flows of other types of assets.

The first friction is supposed to model the main criticism of securitization which argues that the asymmetry of information on the primary markets between the issuers of these assets and their first buyers is the main source of the problems on securitization markets. When the second friction is binding, then despite competitive markets, securitization becomes profitable, and therefore, I can construct signaling through reputation based guarantees.

The third friction concerns the information structure on resale markets for securitized products. The idea that it is hard to find the intrinsic value of the asset is supposed to model the high complexity of those assets in reality which made their pricing very costly. Also these opaque assets have been traded often on the OTC markets and little information was available for their potential buyers. Under those conditions, a holder of the asset, who can observe its cash flows, may have an information advantage, and therefore, adverse selection problems may arise on the re-sale markets. However, as long as the cash-flows of projects of different quality are equal, e.g. due to provision of implicit support, even the holders of those assets remain ignorant about their quality, and the information about the asset quality remains symmetric.

## 2.2.4 Financial firms

Each financial firm (indexed by  $i$ ) maximizes its distributed profit function by choosing its control variables  $\{i_{i,t+s}, \{a_{i,j,t+s}^p\}_j, a_{i,t+s}^s, \{r_{i,t+s,t+s+k}^G\}_{k=0}^\infty, \varphi_{i,t+s}, z_{i,t+s}\}_{s=0}^\infty$ . The return on equity exceeds its unitary costs:

$$E_t (\Lambda_{t,t+1} R_{t+1}^N) > 1, \quad (2.3)$$

---

<sup>6</sup>For simplicity  $\theta$  is taken as a parameter. Kuncil (2014) shows that this friction can be endogenized by the existence of a moral hazard problem. Fixing  $\theta$  does not alter the qualitative results of the paper.

where

$$\Lambda_{t,t+1} \equiv \beta \frac{C_t}{C_{t+1}}$$

is the stochastic discount factor and  $R_{t+1}^N$  is the return on firms' equity.<sup>7</sup>

Therefore, as in Gertler and Karadi (2011), financial firms maximize the following value function:

$$V_t(n_t; S_t) = \max E_t \sum_{s=0}^{\infty} (1 - \sigma) \sigma^s \Lambda_{t,t+s} n_{t+s},$$

where  $n_t$  is the equity of the individual financial firm and  $S_t$  is the set of all state variables.

Each financial firm is situated on an island and has exclusive access to the projects on this island. Given the investment shock to the productivity described above, the financial firm has either a high quality investment opportunity with probability  $\pi\mu$  (subset  $\mathcal{H}_t$  of firms), a low quality investment opportunity with probability  $\pi(1 - \mu)$  (subset  $\mathcal{L}_t$  of firms), or has no access to any new productive projects this period with probability  $1 - \pi$  (subset  $\mathcal{Z}_t$  of firms). In every period, each financial firm chooses whether and how much to invest in a new investment project  $i_{i,t}$  available on this island. I will denote the subset of firms which decide to invest  $\mathcal{I}_t$  and the subset of firms which do not invest, i.e., only save  $\mathcal{S}_t$ . When firms invest, they choose how much of this investment to securitize and sell to other firms ( $i_t - a_{i,i,t}^p$ ) for the price  $q_{i,t}^p$ . All firms also choose how many securitized projects to buy from the current issuers (indexed by  $j$ )  $\{a_{j,i,t}^p\}_j$  for prices  $\{q_{j,t}^p\}_j$ , how many projects to buy on the secondary markets  $a_{i,t}^s$  for the price  $q_t^s$  and which projects to keep further on their balance sheet (since the firm may privately find information about those projects, these quantities are  $a_{t+1}^{hG}$ ,  $a_{t+1}^{lG}$  and  $a_{t+1}^{mG}$  for projects of high, low and unknown quality with implicit recourse<sup>8</sup>, and  $a_{t+1}^h$ ,  $a_{t+1}^l$ , and  $a_{t+1}^m$  for projects of high, low and unknown quality without implicit recourse, respectively). When they sell the securitized part of the current investment, they may decide to provide an implicit recourse (an implicit guarantee on the minimum cash flows from the project)  $\{r_{i,t,t+k}^G\}_{k=0}^{\infty}$  and in the case that such guarantees were provided in the past, they decide whether to default

---

<sup>7</sup>Using (2.2), you can obtain  $C_{t+1} = (1 - \sigma - \xi)(\sigma + \xi) N_t R_{t+1}^N$  and substituting this into (2.3), you obtain  $E_t (\Lambda_{t,t+1} R_{t+1}^N) = \frac{\beta}{\sigma + \xi}$ , which exceeds one by assumption.

<sup>8</sup>Given the regulatory limitations on implicit recourse, which are discussed in the next paragraph, the relevant recourse which remains hidden from the regulator can take only the value  $r_{i,t,t+k}^G = r_{i,t+k}^G = r_{i,t+k}^h \forall s \forall k \in (0, \infty)$ . Alternatively, the recourse may not be provided at all, i.e.,  $r_{i,t,t+k}^G = r_{i,t+k}^G \leq r_{i,t+k}^l \forall s \forall k \in (0, \infty)$ . That is why I do not have to keep detailed track of levels of previous implicit guarantees and differentiate the assets accordingly.

on those guarantees  $\varphi_{i,t}$ .<sup>9</sup> Financial firms may also use the storage technology and keep part of the consumption good till the next period  $z_{i,t+1}$ .

The budget constraint of the financial firm is:

$$i_{i,t} (1 - q_{i,t}^p) + \sum_{j \in \mathcal{I}_t} a_{i,j,t+1}^p q_{j,t}^p + a_{i,t+1}^s q_t^s + a_{i,t+1}^h q_t^h + a_{i,t+1}^l q_t^l + a_{i,t+1}^m q_{t+1}^m + z_{i,t+1} + \pi_{i,t} = n_{i,t} \quad \forall i, \forall t,$$

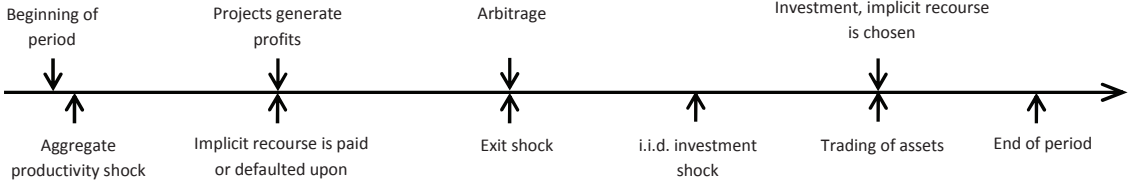
where  $n_{i,t}$  is the firm's equity after repayment of all obligations but before the redistribution of dividends, which is defined for a firm that decides not to sell its assets:

$$\begin{aligned} n_{i,t} = & z_{i,t} + a_{i,t+1}^{hG} (r_t^G + \lambda q_t^{hG}) + a_{i,t+1}^{lG} (r_t^G + \lambda q_t^{lG}) + a_{i,t+1}^{mG} (r_t^G + \lambda q_t^{mG}) \\ & + a_{i,t+1}^h (r_t^h + \lambda q_t^h) + a_{i,t+1}^l (r_t^l + \lambda q_t^l) + a_{i,t+1}^m (r_t^m + \lambda q_t^m) - \varphi_{i,t} cir_{i,t}, \end{aligned}$$

where  $cir_{i,t}$  are the current period costs of honoring the issued implicit recourse guarantees and which are related to the the stock of implicit recourse obligations of this particular firm.

Under asymmetric information on the re-sale market, when the firm decides to sell its assets, they receive for them the re-sale market price  $q_t^s$ .

**Figure 2.1:** Timing of events withing each period



**Implicit recourse.** Financial firms can provide the implicit support in order to increase the quality of the assets sold and potentially signal the quality. Küncl (2014) discusses in detail the role of signaling through provision of reputation based implicit recourse in the form of a promise of minimum gross profits from the projects. The guarantee is not explicit.<sup>10</sup> The implicit recourse is enforced by a threat of punishment in the case of default on the recourse. The punishment doesn't allow financial firms to sell securitized assets in the future. I will describe the equilibria with a trigger strategy punishment. Such a punishment is the most efficient in enforcing the recourse.

<sup>9</sup> $\varphi_{i,t} = 0$  in case of default on implicit recourse or  $\varphi_{i,t} = 1$  when the recourse is honored.

<sup>10</sup>Though not modeled here, the advantage of an implicit guarantee as opposed to explicit may be in reality regulatory arbitrage and lower costs of bankruptcy.

The incentive compatible constraints (ICCs) which have to be satisfied at least for some states in the following period  $t + 1$  for the existence of reputation based implicit recourse are:

$$V_{t+1}^{ND} (n_{t+1}^{ND}; \bar{S}_{t+1}) \geq V_{t+1}^D (n_{t+1}^{DD}; \bar{S}_{t+1}) \quad (2.4)$$

$$V_{t+1}^P (n_{t+1}; \bar{S}_{t+1}) \geq V_{t+1}^{NP} (n_{t+1}; \bar{S}_{t+1}), \quad (2.5)$$

where the equity of a firm which has not defaulted on the implicit recourse is  $n_{t+1}^{ND} = n_{t+1} \mid (\varphi_{i,t} = 1)$ , the equity of a firm that used to honor the implicit obligations but has just defaulted for the first time  $n_{t+1}^{DD} = n_{t+1} \mid (\varphi_{i,t} = 0)$  and  $V_{t+1}^P, V_{t+1}^{NP}$  are the value functions for the firm that has a reputation for punishing for defaults on implicit recourse, and for a firm which failed to punish for default in the past and suffers the negative consequences, respectively.  $V_{t+1}^{ND}, V_{t+1}^D$  are the value functions of the firm when it has a reputation of not defaulting on implicit recourse and when it has defaulted already in the past and suffers the punishment, respectively. Those conditions have to be satisfied in some future states of the world otherwise no reputation based implicit recourse can be credibly provided. The condition 2.4 determines whether the implicit recourse provided is not defaulted upon in some future state, given the trigger strategy punishment rule. If the condition is satisfied, the implicit recourse provided is credible. Also the trigger punishment strategy has to be credible, therefore, in the same period  $t + 1$  state of the world, when (2.4) is satisfied, (2.5) has to be satisfied too, i.e., the saving firm observing a default on the implicit recourse has to be better off punishing the investing firm that defaulted rather than not punishing it.

In some states of the world, the condition (2.4) is not satisfied. Some firms find it unilaterally beneficial to default on the implicit recourse even when the punishment is triggered. In some states of the world, the condition (2.5) may not be satisfied too. Following the discussion in Kuncl (2014), the punishment is chosen even ex-post due to expectations that a firm which failed to punish will fail to punish in the future. This implies that all firms will stop providing implicit guarantees to such a firm. However, a firm which failed to punish may have preferential terms of trade with the defaulting firm when such a firm has access to a profitable investment opportunity as discussed in Kuncl (2014). Intuitively, when a single infinitesimally small firm defaults on the implicit recourse, the benefits of preferential trade with such a firm are low due to the limited supply of assets subject to the investment shock. However, when a larger fraction of firms



find it optimal to default on implicit recourse, the benefits from preferential trade with them are higher since, due to the law of large numbers the supply of assets is positive in all states.

Therefore, when, due to a large negative aggregate exogenous shock, a fraction of firms find it optimal to default on the implicit recourse, the punishment is not triggered. Since the punishment is not triggered, all remaining firms will default on the implicit recourse. In this case, a large negative aggregate exogenous shock coordinates an economy wide default on implicit recourse but the punishment is not triggered. After such an event, the economy may stay in equilibrium without reputation and implicit recourse, or alternatively the economy may move to a new reputation equilibrium where the newly issued assets may carry credible implicit recourse. I will consider the latter case in my infinite horizon model.

As already mentioned, one of the main reasons for provision of implicit guarantees as opposed to explicit was the regulatory arbitrage. For this reason this practice was relatively concealed by the issuers. For simplification, I assume that the originators try to conceal implicit guarantee also. Therefore, the increased cash flows from the asset should mimic cash flows of some other existing asset, which would make it impossible to detect which asset is the one with naturally higher cash flows and which with artificially higher cash flows due to the existence of the implicit support. This assumption introduces some natural limit to the size of the implicit support<sup>11</sup> and simplifies the tractability of the aggregation of infinite horizon implicit guarantees. However, the model is solvable even without this assumption, when the level of implicit guarantee is determined by the strictly binding condition (2.4).

The above assumption implies that the level of implicit support is  $r_{i,t+s,t+k}^G = r_{i,t+k}^G = r_{i,t+k}^h \forall s \forall k \in (0, \infty)$  or  $r_{i,t+s,t+k}^G = r_{i,t+k}^G \leq r_{i,t+k}^l \forall s \forall k \in (0, \infty)$ . Note that the latter case is equivalent to the case where implicit recourse is not provided, which is how I will refer to this case. This assumption also limits the number of potential Perfect Bayesian Equilibria compared to the case of Kuncl (2014). Similarly to Kuncl (2014), I use the Intuitive Criterion by Cho and Kreps (1987), to obtain a single separating equilibrium as long as a separating equilibrium exists. As I will prove below for some combinations of parameters, a separating equilibrium is achieved only thanks to the signaling by provision of the implicit recourse  $r_{i,s,t+k}^G = r_{i,t+k}^G = r_{i,t+k}^h \forall s \forall k \in (0, \infty)$ .

---

<sup>11</sup>If the projects would represent loans with delinquency rates differing among loans of different quality, such a natural limit would be zero delinquency.

When there is no separating equilibrium, we obtain a single pooling equilibrium, in which both firms with access to high and low quality investment opportunities provide the same level of implicit recourse,  $r_{i,s,t+k}^G = r_{i,t+k}^G = r_{i,t+k}^h \forall s \forall k \in (0, \infty)$ . In this case, the aggregate costs of providing implicit recourse<sup>12</sup> issued in period  $t$  is

$$\begin{aligned} IR_t &= \sum_{i \in \mathcal{L}} i_t E_t \left( \sum_{s=1}^{\infty} \beta^s \Lambda_{t,t+s} \lambda^{s-1} \left( \prod_{j=1}^s (1 - \varphi_{i,t+j}) \right) A_{t+s} (\Delta_{t+s}^h - \Delta_{t+s}^l) K_{t+s}^{\alpha-1} \right) \\ &= P_t \sum_{i \in \mathcal{L}} i_t, \end{aligned}$$

where  $\prod_{j=1}^s (1 - \varphi_{i,t+j})$  is the probability that between period  $t$  and  $t + s$ , the firm  $i$  has defaulted on the provided implicit recourse and  $P_t$  is the cost of providing implicit recourse per unit of investment, which can be written also recursively

$$\begin{aligned} P_t &= \sum_{s=1}^{\infty} \sigma^s \Lambda_{t,t+s} \lambda^{s-1} \left( \prod_{j=1}^s (1 - \varphi_{i,t+j}) \right) A_{t+s} (\Delta_{t+s}^h - \Delta_{t+s}^l) K_{t+s}^{\alpha-1} \\ &= \sigma \Lambda_{t,t+1} (1 - \varphi_{i,t+1}) [A_{t+1} (\Delta_{t+1}^h - \Delta_{t+1}^l) K_{t+1}^{\alpha-1} + \lambda P_{t+1}]. \end{aligned}$$

On the aggregate, there is an outstanding stock of implicit recourse obligations

$$SIR_t = \sum_i sir_{i,t} = (1 - f_t^{NIR}) (1 - \omega_t) K_t P_t,$$

where  $\omega_t$  is the share of low quality securitized assets in the total stock of capital  $K_t$  and  $f_t^{NIR}$  is the share of low quality securitized assets which does not bear implicit recourse either because they were not provided or they were defaulted upon in the past.

**Arbitrage.** Due to the provision of infinite-horizon implicit support, the solution of the model may potentially require keeping track of the distribution of firms' stock of implicit recourse obligations as well as firms' equity. Therefore, to keep the tractability of the model, I make an assumption in the spirit of Gertler and Kiyotaki (2010). In their island economy, to prevent keeping track of the distribution of equity across islands they allow for arbitrage at the beginning of each period. In particular at the beginning of each period "a fraction of banks on islands where the expected returns are low can move to

---

<sup>12</sup>Note that I sum all the new investment carried out in this period by all the issuers with access to low quality projects and not only the sold part of their investment. This is because the "skin in the game" constraint holds only for one period. In the following periods, the remaining part of the investment can be sold, but it still has to carry the implicit guarantee.

islands where they are high" (Gertler and Kiyotaki 2010, p.13). This arbitrage equalizes ex ante expected rates of return to intermediation.

In this model, a similar arbitrage would imply an equal level of equity as well as an equal stock of provided implicit obligations across islands. Uniform distribution of equity as well as implicit support obligations across islands maximizes the ex ante return given the i.i.d. nature of the investment shock. Firms with a higher than average stock of implicit obligations would be at a disadvantage compared with others. Therefore, it is optimal for them to equalize the ratio of stock of implicit obligations to equity too.

The process works as follows. A fraction of firms from islands with high level of equity move to islands with a low level of equity. On entry to the island, they can privately observe the stock of implicit obligations still kept on the island<sup>13</sup>. If the ratio of this stock to equity is higher than the average in the economy, they will decide not to enter. Such islands would remain with a low level of equity compared to others, which would reveal to everyone that there is a high stock of implicit obligations on the island, and would hinder the ability on the island located firm to sell securitized assets and exploit potential investment opportunities. Anticipating such a development, firms find it optimal to pay for the transfer of some of their stock of implicit obligations to other firms, or accept payment for receiving some additional stock of implicit obligations prior to the redistribution of equity.

## 2.2.5 Market clearing conditions

There are two types of goods in the model: consumption goods produced by productive projects and capital goods.

**Consumption goods market** clears if the consumption goods produced in the current period are all either consumed, converted into capital goods, i.e., invested into new projects, or stored till the next period:

$$Y_t + Z_t = C_t + I_t + Z_{t+1},$$

where  $Y_t = (\omega_t r_t^h + (1 - \omega_t) r_t^l) K_t$  is the output from all existing projects in the economy and  $Z_t$  is the aggregate storage in the economy from period  $t - 1$ .

**Capital goods markets** clearing conditions are derived from the optimization of

---

<sup>13</sup>Note that the stock of implicit obligations cannot be observed publicly, otherwise, the distribution of investment opportunities would also become public information.

the financial firms in the economy. In equilibrium, firms which are buying various types of assets have to be marginally indifferent among them.

To obtain the respective market clearing condition, let's first rewrite the firm's value functions recursively:

$$\begin{aligned}
V_t^{ND}(n_t; S_t) &= \max E_t \{ (1 - \sigma) n_t + \sigma \Lambda_{t,t+1} [\varphi_{i,t} V_{t+1}^{ND}(n_{t+1}^{ND}; S_{t+1}) \\
&\quad + (1 - \varphi_{i,t}) p_{i,t} V_{t+1}^D(n_{t+1}^D; S_{t+1}) + (1 - \varphi_{i,t}) (1 - p_{i,t}) V_{t+1}^D(n_{t+1}^D; S_{t+1}) \}, \\
V_t^{ND}(n_t; S_t) &= \max E_t \{ (1 - \sigma) n_t + \sigma \Lambda_{t,t+1} V_{t+1}^D(n_{t+1}; S_{t+1}) \}, \tag{2.7}
\end{aligned}$$

for the firm with a reputation of not-defaulting on implicit recourse and for the firm which has defaulted already in the past and suffers the trigger punishment. Note that when the firm is punished for defaulting,  $p_{i,t} = 1$ , and when the firm is not punished after defaulting,  $p_{i,t} = 0$ . I guess and verify that  $V_t^{ND} = n_t \nu_t^{ND}$  and  $V_t^D = n_t \nu_t^D$ . From this guess, obtain

$$\begin{aligned}
\nu_t^{ND} &= E_t \left\{ (1 - \sigma) + \sigma \Lambda_{t,t+1} \left[ \varphi_{i,t} \frac{n_{t+1}^{ND}}{n_t} \nu_{t+1}^{ND} + (1 - \varphi_{i,t}) p_{i,t} \frac{n_{t+1}^{DD}}{n_t} \nu_{t+1}^D \right. \right. \\
&\quad \left. \left. + (1 - \varphi_{i,t}) (1 - p_{i,t}) \frac{n_{t+1}^{DD}}{n_t} \nu_{t+1}^{ND} \right] \right\}, \tag{2.8}
\end{aligned}$$

$$\nu_t^{ND} = E_t \left\{ (1 - \sigma) + \sigma \Lambda_{t,t+1} [\varphi_{i,t} R_{t+1}^{n,ND} \nu_{t+1}^{ND} + (1 - \varphi_{i,t}) p_{i,t} R_{t+1}^{n,DD} \nu_{t+1}^D \right. \tag{2.9}$$

$$\left. + (1 - \varphi_{i,t}) (1 - p_{i,t}) R_{t+1}^{n,DD} \nu_{t+1}^{ND} \right\}, \tag{2.10}$$

for the value of equity of a firm with a reputation of not-defaulting on implicit recourse and

$$\begin{aligned}
\nu_t^D &= E_t \left\{ (1 - \sigma) + \sigma \Lambda_{t,t+1} \frac{n_{t+1}}{n_t} \nu_{t+1}^D \right\}, \\
\nu_t^D &= E_t \left\{ (1 - \sigma) + \sigma \Lambda_{t,t+1} R_{t+1}^{n,D} \nu_{t+1}^D \right\}, \tag{2.11}
\end{aligned}$$

where  $R_{t+1}^{n,ND}$ ,  $R_{t+1}^{n,DD}$ ,  $R_{t+1}^{n,D}$  is the next period return on equity for a firm which does not default on implicit recourse, which has defaulted and would suffer the consequences for the first time and a firm which defaulted in the past and suffers the negative consequences. However, note that during the arbitrage, firms may move the equity across islands in order to equalize the ex ante returns and since the value of equity of a firm which suffers the punishment for defaulting is lower than equity in a firm without punishment ( $\nu_t^D < \nu_t^{ND}$ ), firms move equity from firms which suffer the punishment. Those firms without equity

will never produce in the future. It is optimal then for the household to liquidate such a firm and establish a new one. Therefore, the effective value of equity of firms which suffer the punishment is one  $\bar{\nu}_t^D = 1$ .

To derive the capital goods market clearing condition, we maximize the above value function conditional on observed realization of the i.i.d. investment shock. In this case, the return on equity of an individual firm may differ depending on the investment opportunity. However, due to arbitrage, the next period marginal value of equity will be equal across firms  $\nu_{i,t+1}^{ND} = \bar{\nu}_{t+1}^{ND}$  and  $\nu_{i,t+1}^D = \bar{\nu}_{t+1}^D$ , where  $\bar{\nu}_{t+1}^{ND}$  and  $\bar{\nu}_{t+1}^D$  denote the value of equity for the aggregate sector of financial firms of the respective type.

As explained in the previous section, in equilibrium, there will be either no defaults observed or economy-wide defaults on implicit obligations. When the economy-wide defaults take place firms fail to impose the punishment. Taking this equilibrium behavior into account, we can rewrite (2.8) into

$$\nu_t^{ND} = E_t \left\{ (1 - \sigma) + \sigma \Lambda_{t,t+1} \bar{\nu}_{t+1}^{ND} \left( (1 - \chi_{D,t+1}) R_{t+1}^{n,ND} + \chi_{D,t+1} R_{t+1}^{n,DD} \right) \right\},$$

where  $\chi_{D,t+1} = 1$  when the state in which all firms due to sufficiently negative large aggregate shock default and  $\chi_{D,t+1} = 0$  when such a state does not occur. Further, due to logarithmic utility function, we can show that  $E_t (\bar{\nu}_{t+1}^{ND})$  is a constant. To demonstrate this, we can compute  $\bar{\nu}_t^{ND}$  from (2.9) but taking the expectations before the arrival of the i.i.d. investment shock when the expected return on equity  $R_{t+1}^{ND}$  is equal across firms:

$$\begin{aligned} \bar{\nu}_t^{ND} &= \bar{E}_t \left\{ (1 - \sigma) + \sigma \Lambda_{t,t+1} R_{t+1}^{ND} \bar{\nu}_{t+1}^{ND} \right\}, \\ &= \bar{E}_t \left\{ (1 - \sigma) + \frac{\beta \sigma}{\sigma + \xi} \bar{\nu}_{t+1}^{ND} \right\}, \\ &= \frac{1 - \sigma}{1 - \frac{\beta \sigma}{(\sigma + \xi)}}. \end{aligned}$$

Maximizing such a transformed value function with respect to the choice of various capital goods, we obtain standard Euler equations. In this paper we will be interested in the case when both markets both primary as well as secondary (re-sale) securitization markets are working, which require their expected return to be equal and not lower than the return on storage. Similarly, to have new investment being undertaken, the return from taking advantage of the investment opportunity should not be lower than buying

assets on the re-sale markets. Therefore, we obtain

$$E_t [\Lambda_{t,t+1} R_{t+1}^i] \geq E_t [\Lambda_{t,t+1} R_{t+1}^p] = E_t [\Lambda_{t,t+1} R_{t+1}^s] \geq E_t [\Lambda_{t,t+1} R_{t+1}^z],$$

where  $R_{t+1}^i$  is the return from investing,  $R_{t+1}^p$  is the return from buying on the re-sale markets,  $R_{t+1}^s$  is the return from buying on the re-sale markets and  $R_{t+1}^z$  is the return from storage. When the return from storage is equal to the return from buying assets on the primary or secondary markets, there will be a positive level of storage in the economy.

## 2.3 Model solution

### 2.3.1 Comparative statics

In this section, I derive analytically the behavior of the model and the effects of the above introduced frictions in the steady state. The subsequent sections show the numerical results for the fully dynamic model in the case where all frictions are binding.

#### 2.3.1.1 Effect of the “skin in the game” constraint and asymmetric information on the primary market

The basis of the model is similar to Kuncl (2014). When none of the frictions is binding, only high quality projects are being financed and, due to competition, their market price equals the unitary costs of financing  $q^h = 1$ , and storage is not used in equilibrium  $z = 0$ . However, unlike in Kuncl (2014), due to the binding exit shock, i.e.,  $\sigma + \xi < \beta$ , there is underinvestment in the economy and the return to investment is higher than in the first best case.<sup>14</sup>

$$r^h + \lambda = \frac{1}{\sigma + \xi} > \frac{1}{\beta}.$$

The introduction of a **binding “skin in the game” constraint** restricts the supply of securitized assets on the primary market, which despite perfect competition drives their price above the unitary investment costs  $q^h > 1$ . Kuncl (2014) shows in Proposition 1 that the “skin in the game” constraint is binding as long as it exceeds the ratio of the probability of arrival of high quality projects and the fraction of non-depreciated projects

$$1 - \theta > \frac{\pi\mu}{1 - \lambda}.$$

---

<sup>14</sup>See the Appendix 2.A.1 for the derivation.

Even lower  $\theta$  is needed for a positive level of storage in the steady state. Storage is positive in equilibrium iff<sup>15</sup>

$$1 - \theta > \frac{(\sigma + \xi) \pi \mu + 1 - \sigma - \xi}{1 - \lambda} > \frac{\pi \mu}{1 - \lambda}.$$

Similarly, if  $\theta$  is sufficiently low, even the price of low quality projects can exceed one  $q^l \geq 1$  and in this case low quality projects will be financed in the steady state too, even under public information about the quality of projects as suggested by the Proposition 2 in Kuncl (2014).

Introducing **asymmetric information on the primary market** can lead to the existence of a pooling equilibrium in which projects of both qualities are being financed, but they are indistinguishable to the buyers. Therefore, the allocation of investment is inefficiently skewed more in favor of low quality projects. However, there the economy may still be in a separating equilibrium in which only high quality assets are being financed since firms with access to low quality investment opportunities prefer to buy high quality projects to investing and mimicking firms with access to high quality investment opportunities:

$$R_i | \textit{buying high assets} \geq R | \textit{mimicking}.$$

This condition is satisfied if the difference in TFP between high and low quality projects is large enough. In particular, as derived in Appendix 2.A.2, the following inequality has to be satisfied in the economy without equilibrium use of storage technology. Such a pooling equilibrium is possible only if the difference in the TFP between the high and low quality projects is large enough, in particular when:

$$\frac{A^h}{A^l} \geq \frac{(1 - \pi \mu) (1 - \lambda) (1 - \theta)}{\pi \mu \lambda + (1 - \lambda) \theta \pi \mu}, \quad (2.12)$$

and with the use of storage technology in equilibrium

$$\frac{A^h}{A^l} \geq \frac{(\sigma + \xi) \pi \mu + 1 - \sigma - \xi}{(\sigma + \xi) \pi \mu}. \quad (2.13)$$

Note that when the economy is more constrained, achieving the separating equilibrium would require a larger difference in TFP. The RHS of (2.12) increases with lower  $\pi$ ,  $\mu$ ,  $\theta$

---

<sup>15</sup>This equation holds in the case that the difference between TFP of high and low quality projects are large enough so that only high quality projects are financed in equilibrium. Derivations can be found in Appendix 2.A.1.

or lower  $\lambda$ , which constrain the supply of securitized assets more than the demand for those assets, and therefore increase the return and prices of both high and low quality projects, thus making pooling equilibrium more likely. The RHS of (2.13) increases with lower  $\pi, \mu, \sigma$  or lower  $\xi$  increases the price of both types of assets. Other parameters in this case influence the size of the storage rather than the investment into low quality assets.

### 2.3.1.2 Reputation equilibria with the implicit recourse

The inefficiencies related to the existence of asymmetric information on the primary market can be alleviated by signaling through provision of the implicit recourse. This result is similar to Kuncl (2014) despite non-trivial differences in the provision of implicit recourse. Similarly to Kuncl (2014) implicit recourse is enforced in a reputation equilibrium, in which conditions (2.4) and (2.5) have to be satisfied. The main difference is that the implicit recourse is provided for the whole lifetime of the asset, i.e., it is an infinite horizon. The second difference is the introduction of limits to the size of the implicit recourse. Those are motivated by the fact that in reality regulators try to detect and limit the implicit recourse because they consider it a means of regulatory arbitrage. To conceal the provision of implicit recourse, it is possible only to improve the cash flows of the project to the level of another existing asset. In this set-up, it means that the only implicit recourse which has the potential to affect the equilibrium guarantees cash flows on the level of a high quality asset is  $r_{i,t,t+k}^G = r_{i,t+k}^G = r_{i,t+k}^h \forall k \in (0, \infty)$ .

The provision of implicit recourse, which is more costly for the issuers of low quality assets makes the separating equilibrium more likely. In particular, a separating equilibrium exists iff

$$\frac{A^h}{A^l} \geq \frac{(1 - \pi\mu)(1 - \lambda)(1 - \theta)(1 + B)}{\pi\mu\lambda + (1 - \lambda)\theta\pi\mu + B(1 - \pi\mu)(1 - \lambda)(1 - \theta)} \quad (2.14)$$

in the case without usage of storage technology and

$$\frac{A^h}{A^l} \geq \frac{((\sigma + \xi)\pi\mu + 1 - \sigma - \xi)(1 + B)}{(\sigma + \xi)\pi\mu + B((\sigma + \xi)\pi\mu + 1 - \sigma - \xi)} \quad (2.15)$$

in the case with usage of storage technology. The RHS of those conditions are lower than in conditions (2.12) and (2.13), respectively.<sup>16</sup> Therefore, an even smaller dispersion in

---

<sup>16</sup>For proof, see the Appendix 2.A.3.



TFP may still result in a separating equilibrium when implicit recourse is being provided.

### 2.3.1.3 Asymmetric information on the re-sale market

So far, we have considered the asymmetry of information on the primary market, i.e., between the originators of securitized assets and buyers of these assets. The results of these frictions have been similar to those in Kuncl (2014) despite several differences. However, the focus of this paper is the asymmetry of information on the re-sale market.

In this section, I describe the effects of the third main friction, i.e., the difficult verification of quality of a securitized asset. Only holders of the asset may privately observe its quality provided that its cash flow is informative. This assumption may lead to asymmetric information between the seller and the buyer on the re-sale market, which causes a typical adverse selection. The new results in this paper come from the interaction of the adverse selection on re-sale markets with the switching between pooling and separating equilibria over the business cycle and with the provision of the implicit recourse.

**Case without provision of implicit guarantees.** To demonstrate the effect of switching between the pooling and separating equilibria on the adverse selection problem, let's consider first the case without the provision of implicit guarantees.

The assumption of asymmetric information on re-sale markets has the following impact on the model behavior. First, when an asset is re-sold, there is a unique price that is independent of the quality of this asset  $q_t^s$ . If an asset is not re-sold, the owner who knows its quality will value high quality asset  $q_t^h$  and low quality asset  $q_t^l$ , but these are not the market prices. Second, prices depend on the share of high quality assets sold on the re-sale market.<sup>17</sup> In every period, there are liquidity and informed sellers on the market. Firms with access to profitable investment opportunities may decide to sell even high quality assets to finance the costs of the investment. I will refer to these sellers as liquidity sellers. In every period, all holders of the assets observe the cash-flows from the projects on their balance sheet. Without the provision of the implicit recourse, they will be able to identify the low quality projects and sell all of them on the re-sale market. These sellers are called informed sellers.

Therefore, when the “skin in the game” constraint makes securitization profitable such that all investing firms sell all of their holdings to cover the costs of investment, the share

---

<sup>17</sup>See Appendix 2.A.4.1 for details.

of high quality assets on the re-sale market is

$$f_t^h = \frac{\pi\mu\omega_t}{\pi\mu + (1 - \pi\mu)(1 - \omega_t)} \quad (2.16)$$

in the case of a separating equilibrium, where  $(1 - \pi\mu)(1 - \omega_t)(\sigma + \xi)K_t$  are the low quality assets sold by informed traders and  $\pi\mu(\sigma + \xi)K_t$  are the assets sold by the liquidity traders. In a pooling equilibrium this condition becomes

$$f_t^h = \frac{\pi\omega_t}{\pi + (1 - \pi)(1 - \omega_t)}. \quad (2.17)$$

If in the steady state, there is a separating equilibrium, then  $\omega = 1$  and obviously only high quality assets are being traded on the re-sale markets too, i.e.,  $f^h = 1$  and  $q^s = q^h$ . However, if there is a pooling equilibrium in the steady state, then  $\omega = \mu$ ,

$$f^h = \frac{\pi\mu}{\pi + (1 - \pi)(1 - \mu)} < 1,$$

and  $q^l < q^s < q^h$ . Therefore, due to the adverse selection, liquidity traders sell high quality assets for too low a price and informed sellers sell low quality assets for an overvalued price. There is inefficient cross-subsidization of informed traders by liquidity traders, which reduces the investment and output of the economy.

If, due to the adverse selection, the price of assets on the re-sale market drops low enough, even firms which sell assets for liquidity reasons will cease selling high quality assets. The price is so low that the return from taking advantage of the investment opportunity would not compensate for the cost of selling a valuable asset at a low market price. In a deterministic steady state, this situation takes place if:

$$V_i(\textit{keeping high projects}) \geq V_i(\textit{selling high projects}) \quad \forall i \in \mathcal{I}.$$

As shown in Appendix 2.A.4.2, this condition implies that the share of high quality assets traded on the re-sale market has to be low enough to satisfy:

$$f^h \leq \frac{1 - \theta\mu q^h - (1 - \theta\mu)q^l}{(1 - \theta)(q^h - q^l)}. \quad (2.18)$$

This condition is satisfied when the difference in qualities is large enough (i.e., for sufficiently large difference  $q^h - q^l$ ). Note that there will never be complete market

shutdowns since low quality assets would still be sold at a fair price, but the volume of sales would diminish by the absence of high quality assets, and the level of overall investment in the economy would also be significantly lower.<sup>18</sup>

The dynamic implications are demonstrated in greater detail in the next sections, but the basic intuition can be shown on the above derivations. The prices on the re-sale market  $q_t^s$  depend positively on the share of high quality assets  $f_t^h$  and negatively on the dispersion of qualities between the two assets, which both determine the expected value of assets sold on the re-sale market. The share of high quality assets  $f_t^h$  in turn depends positively on the share of high quality assets in the economy  $\omega_t$  as shown in (2.16) and (2.17). Therefore, since recessions are characterized by a larger dispersion in qualities, intuitively the adverse selection is more important in a recession than in a boom. Further, since low dispersion between the qualities in the boom leads to the occurrence of pooling equilibria, the longer the boom period is, which precedes the recession, the larger is the share of low quality loans on the market and the more acute the adverse selection issue becomes. If adverse selection is strong enough, securitized loans of high quality cease being traded on the re-sale markets altogether, which further deepens the recession.

**Case with provision of implicit guarantees.** The provision of infinite horizon implicit guarantees influences the problem of adverse selection on re-sale markets in two ways.

The **first effect** of implicit recourse provision is on **the lower effective difference between the value of high quality assets and low quality assets with implicit recourse**. Since low quality assets with implicit recourse will have the same cash-flows as high quality assets, the market price on the re-sale market is much less negatively influenced by the presence of the low quality assets with implicit recourse. Indeed, it is the presence of low quality assets without implicit recourse which significantly negatively influences the re-sale market price  $q^S$ .<sup>19</sup> Therefore, as long as all low quality assets bear implicit recourse making their cash-flows equal to high quality assets, the re-sale market works relatively well. However, after a potential default on implicit recourse, low quality assets with low cash-flows will appear on the re-sale market and negatively influence its

---

<sup>18</sup>In the dynamic solution of the model, I do not have partial market shutdowns, since such nonlinearities and their duration are hard to endogenously establish in the model, however, I show the varying degree of adverse selection.

<sup>19</sup>Note that even in the steady state, there are low quality assets without the implicit recourse. This is due to the exit shock. Exiting firms of course do not provide implicit recourse in the future periods.

price. This becomes especially pronounced when such a default is widespread in the economy. In the next sections I will show that this is the case in a deep recession.

The **second effect** of implicit recourse provision is related to its effect on **the degree of asymmetric information on the re-sale market**. I have assumed that implicit recourse is costly to detect, and therefore, holders of an asset may find its quality only based on the cash-flows it generates. As long as the implicit recourse is being provided, holders cannot distinguish between high quality assets and low quality assets with implicit recourse. However, when implicit recourse is being defaulted upon, low quality assets are easily privately identified and a large quantity of informed sellers appear on the re-sale market. As I show in the next section, the default on implicit recourse is limited to the exiting firms in boom times or mild recessions, but they are widespread in deep recessions, when the difference in qualities becomes too large to continue providing implicit recourse. This implies that in booms and mild recessions, the problem of asymmetric information, and therefore, of adverse selection on re-sale markets is marginal, but becomes very severe in a deep recession.

I show in Appendix 2.A.4.3 that the prices on the re-sale market  $q_t^s$  are negatively affected by the fact that, in the following period, the share  $f_{t+1}^{NIR} (1 - f_t^h)$  of assets sold on the re-sale market will generate only low cash-flows, where  $f_{t+1}^{NIR}$  is the share of low quality assets without implicit recourse (out of all low quality assets), and the share of high quality assets is given by

$$f_t^h = \frac{\pi \omega_t}{\pi + f_t^{NIR} (1 - \pi) (1 - \omega_t)}. \quad (2.19)$$

Liquidity traders sell  $\pi$  fraction of capital, out of which  $\omega_t$  is the share of high quality assets, and informed traders sell  $f_t^{NIR} (1 - \pi) (1 - \omega_t)$  fraction of capital on the re-sale market.<sup>20</sup>

In this case with implicit recourse, we can again observe the positive effect of the share of high quality assets  $f^h$  on the re-sale market price  $q^s$ . Moreover, we can observe the effect of the share of low quality assets without implicit recourse  $f_t^{NIR}$ . A higher  $f_t^{NIR}$  which due to persistence implies higher  $f_{t+1}^{NIR}$  in the next period lowers the cash-flows from assets bought today on the re-sale market. Moreover, a higher  $f_t^{NIR}$  increases the

---

<sup>20</sup>Note that I assume that, between periods, any potential information about the asset quality is lost and has to be learned again. This assumption is not crucial for the results but simplifies the solution and rules away the adverse selection by the original issuers of low quality assets who might decide to hold the skin in the game only for one period. In reality, the skin in the game is held longer, but for tractability, I do not want to make such a restriction and I rather assume the loss of information between periods.

share of informed traders on the re-sale markets, and therefore, lowers the share of high quality assets sold on the market  $f_t^h$ . Both of these effects make the adverse selection more important and depress the market price.

Compared to the case without implicit recourse, the adverse selection is milder, since the share of high quality assets on the re-sale markets in (2.17) is lower than in (2.19).

### 2.3.2 Methodology for solution of the dynamic model

This section presents the methodology which is used to solve the fully dynamic model. The model is too complex to be computed by global numerical approximation methods as in Kuncl (2014). In particular, it contains four state variables  $(A_t, K_t, \omega_t, f_t^D)^{21}$ , which make the iteration on the grid of state variables challenging. Therefore, I use a perturbations method, i.e., I find the linear approximations of the policy functions around the steady state which determine the laws of motion for the model variables.

The equilibrium conditions of the model are very different for various combinations of state variables. Standard perturbation methods cannot capture this non-linearity. Therefore, to solve this model, I use perturbation method for Markov-switching DSGE models using the methodology introduced by Foerster et al. (2013).

Foerster et al. (2013) propose an algorithm which can provide first- and second-order approximation for policy functions for Markov-switching rational expectations models where some parameters follow a discrete Markov chain process indexed by  $s_t$ . The Markov chain has a state-independent transition matrix  $\mathcal{P} = (p_{s,s'})$ .

The model equilibrium conditions can be written in a general form as

$$E_t f(y_{t+1}, y_t, x_{t+1}, x_t, \chi_{t+1}, \chi_t) = 0_{n_x+n_y}, \quad (2.20)$$

where  $y_t$  is an  $n_y \times 1$  vector of non-predetermined (control) variables,  $x_t$  is an  $n_x \times 1$  vector of predetermined (state) variables, which are known already at time  $t - 1$ , and  $\chi_t$  is the vector of Markov switching parameters. In our case, there are 4 state variables  $x_t = (A_t, K_t, \omega, f_t^D)$ , i.e.,  $n_x = 4$ . Markov-switching parameters  $\chi_t$  can influence the values of the steady state. To compute a unique steady state Foerster et al. (2013) propose the use of the mean of their ergodic distribution across Markov regimes  $\bar{\chi}_t = \sum_s p_s \chi_s$ , where

---

<sup>21</sup>  $f_t^D$  is the share of low quality assets without the implicit recourse at the end of the period which is more convenient state variable in the recursive formulation of the model than  $f_t^{NIR}$ . The relation between  $f_t^D$  and  $f_t^{NIR}$  is explained in detail in the appendix 2.B.1.

$p_s$  is the unconditional probability of occurrence of Markov regime  $s$  ( $s \in \{1, \dots, n_s\}$ ).

The solution of the recursive model (2.20) is

$$\begin{aligned} y_t &= g(x_t, \psi, s_t), \\ y_{t+1} &= g(x_{t+1}, \psi, s_{t+1}), \\ x_{t+1} &= h(x_t, \psi, s_t), \end{aligned}$$

where  $\psi$  is the perturbation parameter. I do not know the explicit functional form for  $g$  and  $h$  and therefore, I do a first-order Taylor expansion around the steady state. The first order approximations  $g^{first}$  and  $h^{first}$  are

$$\begin{aligned} g^{first}(x_t, \psi, s_t) - y_{ss} &= Dg_{ss}(s_t) \mathbf{S}_t, \\ h^{first}(x_t, \psi, s_t) - x_{ss} &= Dh_{ss}(s_t) \mathbf{S}_t, \end{aligned}$$

where  $\mathbf{S}_t = \left[ (x_t - x_{ss})^T \ \psi \right]^T$  and  $\{Dg_{ss}(s_t), Dh_{ss}(s_t)\}_{s=1}^{n_s}$  are the unknown matrices. Foerster et al. (2013) use the method of successive differentiation to find these unknown matrices. They show that this problem can be reduced to finding a solution to a system of quadratic equations. Finally, Foerster et al. (2013) check the stability of the solution using the concept of mean square stability (MSS) defined in Costa, Frago, and Marques (2005).

The algorithm works only with constant transition probabilities, while our model predicts that the change between different regimes endogenously depends on the four state variables  $(A_t, K_t, \omega_t, f_t^D)$ . Only the level of TFP ( $A_t$ ) is exogenous in this model and  $K_t, \omega_t, f_t^D$  are endogenous variables. It is the  $A_t$  together with the dispersion between TFP of high and low quality projects, which is related to  $A_t$  by (2.1), that is the main determinant of the switch between a pooling equilibrium and a separating equilibrium and a default on implicit guarantees. Therefore, I construct a Markov process for  $A_t$  and the related  $\Delta_t^h, \Delta_t^l$  such that for a subset of endogenous state variables  $K_t, \omega_t, f_t^D$  around the steady state the endogenous conditions for the existence of a separating or pooling equilibrium and for default or non-default on implicit support predict the same type of equilibrium for the particular Markov regime. This reconciles to some extent the need for constant transition probabilities in the used algorithm for solution and the endogenous conditions for the change in the above mentioned regimes.

The exogenously switching regimes, which satisfy the endogenous conditions, have the following properties for this subset of state variables:

**Regime 1 - Expansion:** high aggregate TFP ( $A_1 = A_H$ ) and lowest dispersion in type specific TFP ( $\Delta_1^h - \Delta_1^l$ ) make this a pooling equilibrium;

**Regime 2 - Mild Recession:** low aggregate TFP ( $A_2 = A_L$ ) and higher dispersion of type specific TFP ( $\Delta_2^h - \Delta_2^l > \Delta_1^h - \Delta_1^l$ ) is sufficient to make this a separating equilibrium but implicit recourse is still being honored; and

**Regime 3 - Deep Recession:** the low level of aggregate TFP ( $A_3 = A_L$ ) and the highest dispersion of type specific TFP ( $\Delta_3^h - \Delta_3^l > \Delta_2^h - \Delta_2^l$ ) not only make this a separating equilibrium, but also all firms, upon arrival to this regime, find it optimal to default on their outstanding implicit recourse obligations.

I also assume some particular properties of the transition matrix  $\mathcal{P}$ . First, I assume that the economy typically switches between the expansion and mild recession, while rarely the expansion is followed by a deep recession so  $p_{1,2} \gg p_{1,3}$  and  $p_{2,3} = 0$ . Since the defaults on implicit guarantees take place only upon entry to Regime 3, and therefore, the equilibrium conditions would be different for the first period in Regime 3 and compared to the subsequent periods, I assume that  $p_{3,3} = 0$ .

### 2.3.3 Dynamic properties of the model

In this section, I show the results of the dynamic fully stochastic model with the above introduced three Markov regimes to illustrate the dynamic implications of the model with the focus on the effects of the adverse selection on the re-sale markets.

**Parametrization of the model.** In this section, I focus on the case when all the three frictions introduced in section 2.2.3 bind. As demonstrated in the preceding steady state derivations, this restricts some of the parameters. Furthermore, to reconcile the methodology by Foerster et al. (2013), which requires exogenous transition probabilities between Markov regimes, with the endogenous model conditions for a significant subset of state variables, I need significant differences in some of the parameters across the regimes. Following Kiyotaki and Moore (2012), I set  $\alpha = 0.4$  and  $\beta = 0.99$ . The persistence parameter for the productivity process is set to  $p_{1,1} = p_{2,2} = p_{3,2} = 0.86$ .<sup>22</sup> I assume

---

<sup>22</sup>This corresponds to an auto-correlation of TFP at a quarterly frequency of 0.95. Note that I have assumed that  $p_{3,3} = 0$ . Therefore, by persistence in the case of Regime 3, I mean the persistence of the recession (i.e. either Regime 2 or 3).

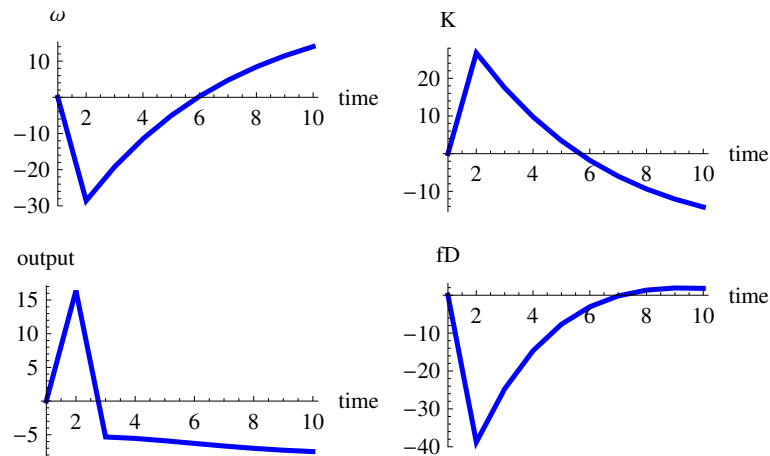
that deep recession can only follow an expansion period, i.e.,  $p_{2,3} = 0$ . The probability of a deep recession is set to be very low compared to mild recession:  $p_{1,3} = 0.005$  and  $p_{1,2} = 1 - 0.86 - 0.005$ . The deep recession is characterized by the same level of TFP as Regime 2 ( $A_L$ ) but by higher dispersion in type specific components of TFP. The ratio of aggregate components of TFP is  $A_H/A_L = 1.05$  and the ratios of type specific TFP are  $\Delta_1^l/\Delta_1^h = 1$ ,  $\Delta_2^l/\Delta_2^h = 0.65$  and  $\Delta_3^l/\Delta_3^h = 0.6$ . The depreciation rate  $1 - \lambda$  is set to 0.18, which is supposed to match the Weighted Average Life (WAL) of securitized assets, which is reported to be on average 5.6 years by Efung and Hau (2013) (p.11). The probability of firms' survival  $\sigma = 0.979$  is set such that the ratio of storage to capital in the steady state is 6% which is comparable to the level calibrated in Kiyotaki and Moore (2012). Parameters  $\pi = 0.1$  and  $\theta = 0.37$  are set such that the endogenous conditions for pooling, separation and default fit the properties of Markov regimes for a subset of state variables around the steady state.

**Impulse responses.** The switching between the pooling in the expansion (Regime 1) and the separating equilibrium on the primary market in recession (Regime 2 and 3) is the property shared with Kuncl (2014). Therefore, the main results of Kuncl (2014) are reproduced here. In particular, the longer the economy stays in the boom, the higher will be the share of the low quality assets accumulated on its balance sheet and the deeper will be the subsequent downturn. Figure 2.2 shows the evolution of endogenous variables for an economy which moves to the expansion (Regime 1) for one period and then to a mild recession (Regime 2). First, due to higher productivity of both high and especially low quality projects, investment, capital and output increase dramatically. Due to lower dispersion in qualities, the economy moves to the pooling equilibrium, therefore the share of high quality assets  $\omega$  decreases. But the subsequent downturn is deeper due to the accumulation of low quality assets on financial firms' balance sheets.

The main focus of this paper is the effect of asymmetric information on the re-sale markets over the business cycle. Section 2.3.3 explains that as long as the implicit recourse is provided, the problem of adverse selection on the re-sale market is limited. This is due to two reasons. When implicit recourse is provided, the cash flows from low quality assets are high. Moreover, it is harder to identify low quality assets and therefore, there are fewer informed sellers on the re-sale markets. Those positive effects suddenly disappear when the implicit recourse is defaulted upon. This takes place in Regime 3. Figure 2.3 shows the effect of defaults on implicit recourse. It compares two cases of the economies,



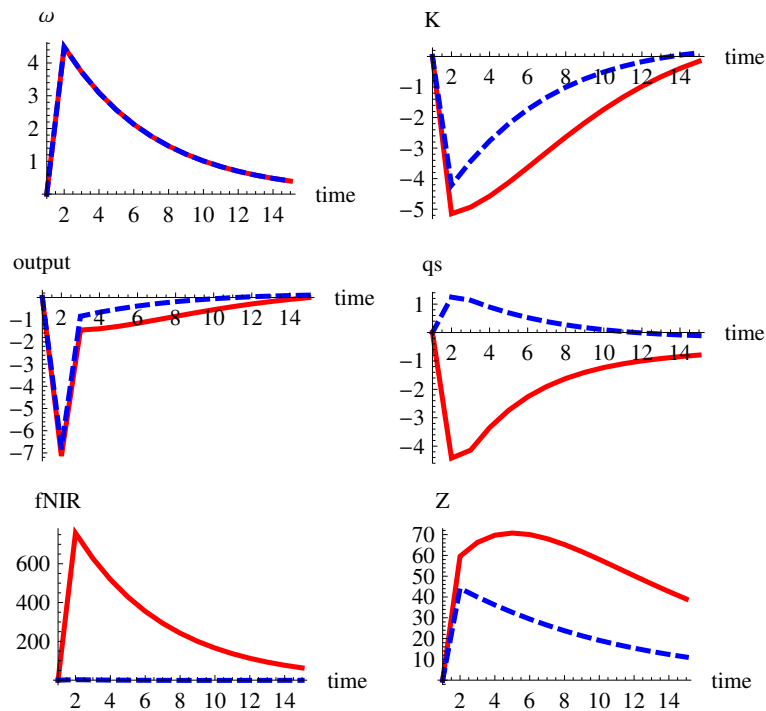
**Figure 2.2:** Economy switches to a pooling equilibrium in boom



Note: Impulse responses shows the percentage deviations of endogenous variables from their steady state level for an economy which moves for one period to the Expansion Regime and then to Mild Recession.

both moving from the steady state to the Deep Recession (Regime 3) for one period and then back to the steady state. In the first case (red full curves), the optimizing firms choose to default on the implicit recourse. In the second case (dashed blue curves), the economy is affected by the same shocks, but as a surprise, I do not allow firms to default on the implicit recourse, even though otherwise they would choose to default. Therefore, the difference between the two cases is given by the default on implicit recourse. In the case where default is allowed, all firms default and the share of low quality assets without implicit recourse increases to 100% ( $f_t^{NIR} = 1$ ). The market price on the resale market  $q_t^s$  drops due to a severe adverse selection problem while in the case of no default the price on the re-sale market slightly increases, which is related among others to higher  $\omega_t$ . Indeed, the economy switched to the separating equilibrium, and therefore, one positive development in the economy is that new low quality assets are not being issued. In the case of default, a low re-sale market price reduces the resources that the investing firms can obtain for selling their assets. Adverse selection causes an outflow of resources from liquidity sellers (investors) to informed sellers. This reduces the investment and the level of capital in the economy drops further. Due to a low supply of new securitized assets, investing firms decide to store more resources rather than to buy securitized assets. All those effects combined have a negative effect on the output of the economy. For the sake of clarity, the Figure 2.4 depicts the difference in the model variables between these two cases. It is clear that due to the default on the implicit recourse and the implied adverse selection problem the re-sale market price is depressed, which reduces the level of capital

**Figure 2.3:** Effect of defaults on implicit recourse on adverse selection



Note: Impulse responses shows the percentage deviations of endogenous variables from their steady state level for an economy which moves for one period to the Deep Recession Regime and then moves back to the steady state. The red full line shows the case when optimizing firms default on the implicit recourse and the blue line shows shows the case when, by surprise, such defaults cannot take place.

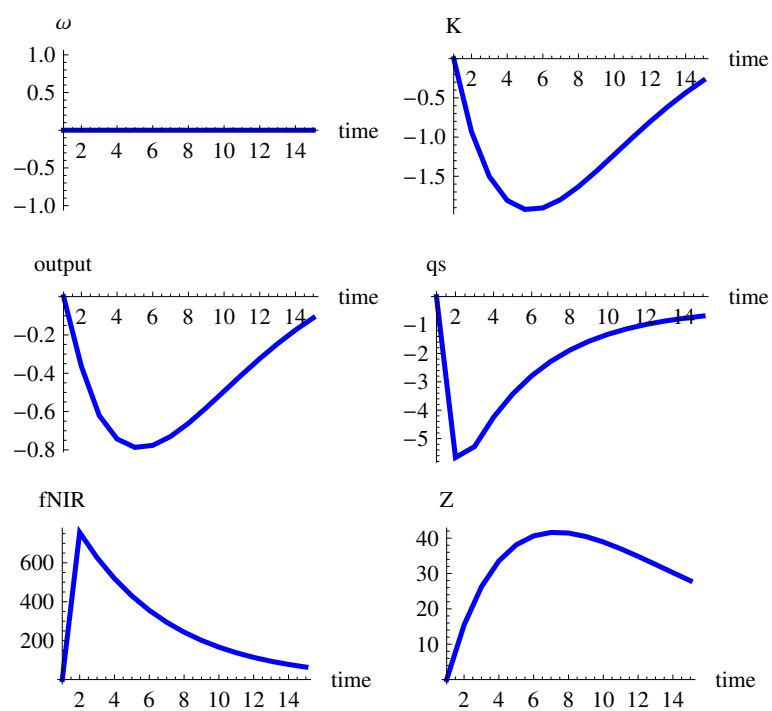
and output, but increases the level of storage. Note that this effect is highly persistent.

## 2.4 Conclusion

This chapter tries to explain the great boom of the market for securitized products in the period prior to the recent financial crisis and the following collapse of this market characterized by low volumes and high spreads. In a theoretical model, I propose a mechanism within the rational expectations framework which is based on information frictions and asymmetries.

Following Küncl (2014), I model the securitization process with its peculiarities such as the provision of implicit recourse by originators of securitized assets. Similarly as in Küncl (2014), the model features a switching between a pooling and separating equilibrium on the primary market for securitized products, over the business cycle, which leads to a build-up of inefficient investment in the boom stages of the business cycle. This chapter introduces information frictions on the re-sale market. These may lead to information

**Figure 2.4:** Effect of defaults on implicit recourse on adverse selection (cont.)



Note: These impulse responses show the difference between the case with defaults and without defaults on implicit recourse from the previous Figure 2.3. The difference is reported in percentages relative to the steady state level.

asymmetries between the sellers and buyers of securitized assets on this market and to the adverse selection problem. The main contribution of this paper is the study of the interaction of the severity of the adverse selection problem with the switching between the pooling and separating equilibrium on the primary market for securitized assets, and the interaction with the provision of the infinite horizon implicit recourse.

The model shows that the adverse selection is contained in boom periods and mild recessions. This is due to low dispersion in cash flows generated by the securitized assets supported with the implicit recourse. Moreover, due to the provided implicit recourse, it is harder to find the intrinsic quality of the assets, and therefore, there are very few informed traders on the re-sale markets. The model also predicts a sudden dramatic increase in adverse selection after a larger dispersion shock, which lowers the cash flows generated by low quality assets. This makes the provision of implicit recourse too costly and there is a widespread default on these reputation based guarantees. As a result, the effective dispersion in cash flows generated by different types of securitized assets increases dramatically and the proportion of informed traders on the market also increases. Both effects exacerbate the effects of the adverse selection problem. The price of the assets sold on the re-sale markets, investment and output of the economy are persistently depressed. Finally, the model predicts that these adverse selection problems are most severe when the recession is preceded by a prolonged boom period during which a large quantity of low quality assets is accumulated on the balance sheets of financial firms.

## 2.5 Acknowledgments

Part of this work has been carried out during my traineeship at the European Central Bank. For helpful comments and suggestions, I would like to thank Sergey Slobodyan, Christoffer Kok, Dawid Zochowski and Tao Zha. I also would like to thank the authors of the methodology for perturbation methods for Markov-switching DSGE models, Andrew Foerster and Tao Zha, for providing me their benchmark Mathematica code. Remaining errors are solely my own responsibility.

## 2.A Appendix 2.A: Comparative statics

### 2.A.1 Role of the “skin in the game” constraint

When the **“skin in the game” constraint is not binding**, then, due to competition, prices of high quality assets are equal to the unitary costs of financing high quality projects  $q_t^h = 1$ . Firms do not make profits from securitizing part of their investment. Therefore, firms with access to low quality projects do not have incentives to mimic firms with high quality investment opportunities. In the steady state, the consumption goods market clearing condition  $Y_t = I_t + C_t$  becomes

$$\begin{aligned} r^h &= (1 - \lambda) + (1 - \sigma - \xi) (r^h + \lambda), \\ r^h + \lambda &= \frac{1}{\sigma + \xi} > \frac{1}{\beta}. \end{aligned}$$

Due to the binding exit shock, where by assumption  $\sigma + \xi < \beta^{23}$ , the return to investment is higher than in the first-best, and therefore, there is underinvestment. In this case, only high quality loans are being financed  $\omega = 1$  and storage technology is not used  $Z = 0$ .

When the **“skin in the game” constraint becomes binding**, the supply of securitized cash flows from projects becomes limited, which drives their market price above the unitary costs of refinancing. The outcome with only this friction binding is analogous to Kuncl (2014), where you can find a precise definition of steady state under different levels of the parameter  $\theta$ . Here, I derive only the condition, which makes the “skin in the game” constraint binding, so that the prices of high quality projects exceeds the unitary costs but low quality projects are still not financed in equilibrium.

The level of aggregate investment becomes determined by the constraint:

$$I_t = \frac{\pi\mu(\sigma + \xi) K_t (r_t^h + \lambda q_t^h)}{(1 - \theta q_t^h)},$$

which in the steady state becomes  $(1 - \lambda) (1 - \theta q^h) = \pi\mu(\sigma + \xi) (r^h + \lambda q^h)$ . The consumption goods market clearing condition in the steady state takes the form

$$r^h = (1 - \lambda) + (1 - \sigma - \xi) (r^h + \lambda q^h). \quad (2.21)$$

Combining these equations, we can obtain the expression for the steady state price of high quality assets

$$q^h = \frac{(1 - \lambda) (1 - \pi\mu)}{(1 - \lambda) \theta + \pi\mu\lambda}. \quad (2.22)$$

Should the price exceed one, we can derive from (2.22) that we need a large enough skin in the game

$$1 - \theta > \frac{\pi\mu}{(1 - \lambda)}. \quad (2.23)$$

---

<sup>23</sup>Note that if  $\sigma + \xi \geq \beta$ , then the exit shock would not be binding since households would decide to distribute more dividends than those obtained by the exit shock.

A binding “skin in the game” is a precondition for the **use of storage technology**. When (2.23) is not binding, then  $q^h = 1$  and the profits from investment are equally shared by all firms. Binding (2.23) increases the returns for investing and securitizing firms and lowers the return for saving firms. But even when  $q^h > 1$ , storage would not be used if the “skin in the game” constraint does not exceed the level needed to bring the return from buying securitized loans to the unit return from storage:

$$\begin{aligned} R^h &> R^z, \\ \frac{r^h + \lambda q^h}{q^h} &> 1. \end{aligned}$$

This condition can be rewritten using (2.21) and (2.22) to

$$\begin{aligned} \frac{r^h + \lambda q^h}{q^h} &= \frac{(1 - \lambda)}{(\sigma + \xi) q^h} + \frac{\lambda}{(\sigma + \xi)} > 1, \\ \frac{(1 - \lambda)\theta + \lambda}{(1 - \pi\mu)(\sigma + \xi)} &> 1, \\ (1 - \lambda)(1 - \theta) &< (\sigma + \xi)\pi\mu + 1 - \sigma - \xi. \end{aligned} \quad (2.24)$$

Since  $\sigma + \xi < 1$ , then  $\pi\mu < 1 - (1 - \pi\mu)(\sigma + \xi)$ , and therefore, there is a non-empty interval of parameter  $\theta$  such that both (2.23) and (2.24) are satisfied. In other words, the “skin in the game” constraint consistent with positive amount of storage, i.e., not satisfying the condition (2.24), has to be stricter than condition (2.23).

If the condition (2.24) is not satisfied, then  $Z > 0$  and the market clearing conditions become

$$\begin{aligned} r^h + z &= (1 - \lambda) + (1 - \sigma)\lambda(r^h + \lambda q^h + z) + z, \\ \frac{r^h + \lambda q^h}{q^h} &= 1, \end{aligned}$$

for the consumption goods market and the capital goods market, respectively. Note that  $z \equiv Z/K$  is the ratio of the level of storage to capital. The investment function becomes

$$(1 - \lambda)(1 - \theta q^h) = \pi\mu(\sigma + \xi)(r^h + \lambda q^h + z).$$

Combining the two market clearing conditions, we obtain

$$(\sigma + \xi - \lambda)q^h = 1 - \lambda + (1 - \sigma - \xi)z,$$

and combining the investment function with the capital goods market clearing condition, we obtain

$$(1 - \pi\mu)(1 - \lambda) = (\theta(1 - \lambda) + \lambda\pi\mu)q^h + \pi\mu z.$$

From this system of 2 equations with 2 unknowns, we obtain

$$q^h = \frac{(\sigma + \xi)\pi\mu + 1 - \sigma - \xi}{(\sigma + \xi)\pi\mu + (1 - \sigma - \xi)\theta},$$

and

$$z = \frac{(\sigma + \xi)(1 - \pi\mu) - \lambda - \theta - \lambda\theta}{(\sigma + \xi)\pi\mu + (1 - \sigma - \xi)\theta}.$$

Note that for lower  $\theta$ ,  $\pi$ ,  $\mu$  or  $\lambda$  the economy is more constrained, and therefore both  $q^h$  and  $z$  would increase in the steady state.

## 2.A.2 Separating condition without provision of implicit recourse

When we introduce asymmetric information on the primary market for securitized products, there may still be a separating equilibrium, in which firms with access to low quality projects find it optimal to buy high quality projects rather than investing and securitizing cash flows from the low quality projects and selling these for the best possible price, i.e., for the market price for high quality projects. The condition for the existence of such a separating equilibrium is in the steady state:

$$\begin{aligned} V(\text{buying high projects}) &\geq V(\text{mimicking}), \\ R | \text{buying high projects} &\geq R | \text{mimicking}. \end{aligned} \quad (2.25)$$

When the “skin in the game” is not binding, then this condition ( $r^h + \lambda \geq r^l + \lambda q^l$ ) is always satisfied. Note that using the market clearing condition for capital goods markets  $A^h/q^h = A^l/q^l$ , this condition can be rewritten to

$$\frac{A^h}{A^l} > 1.$$

When the “skin in the game” is binding, condition (2.25) becomes

$$\begin{aligned} \frac{r^h + \lambda q^h}{q^h} &\geq \frac{r^l + \lambda q^l}{\frac{1 - \theta q^h}{1 - \theta}}, \\ \frac{r^h + \lambda q^h}{r^l + \lambda q^l} = \frac{q^h}{q^l} &\geq \frac{(1 - \theta) q^h}{1 - \theta q^h}. \end{aligned}$$

If the condition (2.24) is satisfied, i.e., storage is not used in equilibrium, substituting for  $q^h$  from (2.22), the separating condition becomes

$$\frac{A^h}{A^l} \geq \frac{(1 - \pi\mu)(1 - \lambda)(1 - \theta)}{\pi\mu\lambda + (1 - \lambda)\theta\pi\mu}. \quad (2.26)$$

When storage is not used in equilibrium, a higher share of high quality projects  $\pi\mu$ , a lower skin in the game  $(1 - \theta)$  or smaller depreciation rate  $(1 - \lambda)$  would decrease the RHS of (2.26), and therefore, it will be easier to satisfy the separating condition.

If storage is used in equilibrium substituting for  $q^h$  from (2.22), the separating condition becomes

$$\frac{A^h}{A^l} \geq \frac{(\sigma + \xi)\pi\mu + 1 - \sigma - \xi}{(\sigma + \xi)\pi\mu}. \quad (2.27)$$

In this case, the higher share of high quality projects  $\pi\mu$ , the higher rate of survival

of financial firms  $\sigma$  or higher equity share of new firms  $\xi$ , the lower the RHS of (2.27) is and the more likely it would be to satisfy the separating condition.

### 2.A.3 Separating condition with provision of informative implicit recourse

Separating equilibrium condition when the provided implicit recourse is informative is

$$V(\text{buying high projects}) \geq V(\text{mimicking without defaulting}). \quad (2.28)$$

When implicit recourse is being provided, there are two outcomes possible. Either the condition (2.4) is satisfied, then

$$V(\text{mimicking without defaulting}) \geq V(\text{mimicking and defaulting}),$$

and the signal in the form of implicit recourse is informative, or (2.4) is not satisfied, then

$$V(\text{mimicking without defaulting}) < V(\text{mimicking and defaulting}),$$

and the signal is not informative. We concentrate on the prior case of informative signal otherwise the existence of separating equilibrium condition collapses to (2.12).

Separating equilibrium conditions when the provided implicit recourse is informative is in the steady state

$$\begin{aligned} V(\text{buying high projects}) &\geq V(\text{mimicking without defaulting}), \\ R | \text{buyin high projects} &\geq R | \text{mimicking without defaulting}, \\ \frac{r^h + \lambda q^h}{q^h} &\geq \frac{r^l + \lambda q^l - \frac{1}{1-\theta}P}{\frac{1-\theta q^h}{1-\theta}}. \end{aligned}$$

After substituting for the steady state cost of keeping the steady state promise  $P = \beta\sigma(r^h - r^l + \lambda P) = \beta\sigma(r^h - r^l) / (1 - \beta\sigma\lambda)$ , we obtain

$$\begin{aligned} \frac{1 - \theta q^h}{(1 - \theta) q^h} &\geq \frac{r^l + \lambda q^l - \frac{\beta\sigma(r^h - r^l)}{(1-\theta)(1-\beta\sigma\lambda)}}{r^h + \lambda q^h}, \\ \frac{1 - \theta q^h}{(1 - \theta) q^h} &\geq \frac{q^l}{q^h} - \frac{\beta\sigma}{(1 - \theta)(1 - \beta\sigma\lambda)} \frac{\frac{r^h}{q^h} \left(1 - \frac{q^l}{q^h}\right)}{\frac{r^h + \lambda q^h}{q^h}}. \end{aligned} \quad (2.29)$$

To simplify the above expression, we need to find the expression for  $r^h/q^h$ . In the case when the “skin in the game” constraint is not sufficiently binding to have positive storage in equilibrium, then by combining the relevant steady state market clearing condition and the investment function, we obtain

$$\begin{aligned} \frac{r^h}{q^h} &= \frac{1 - \lambda}{q^h(\sigma + \xi)} + \frac{(1 - \sigma - \xi)}{(\sigma + \xi)} \lambda, \\ &= \frac{\theta(1 - \lambda) + \lambda(1 - \sigma - \xi) + \pi\mu\lambda(\sigma + \xi)}{(1 - \pi\mu)(\sigma + \xi)}. \end{aligned}$$



Substituting this and the expression for  $q^h$  from (2.22) into (2.29), we obtain

$$\frac{\pi\mu\lambda + (1-\lambda)\theta\pi\mu}{(1-\pi\mu)(1-\lambda)(1-\theta)} \geq \frac{q^l}{q^h} - B \left(1 - \frac{q^l}{q^h}\right), \quad (2.30)$$

where

$$B = \frac{\beta\sigma(\theta + \lambda(1-\theta - (1-\pi\mu)(\sigma + \xi)))}{(1-\theta)(1-\beta\sigma\lambda)(\theta + \lambda(1-\theta))}.$$

The inequality (2.30) after substitution of  $A^h/q^h = A^l/q^l$ , becomes

$$\begin{aligned} \frac{A^h}{A^l} &\geq \frac{1+B}{\frac{\pi\mu\lambda+(1-\lambda)\theta\pi\mu}{(1-\pi\mu)(1-\lambda)(1-\theta)} + B} \\ \frac{A^h}{A^l} &\geq \frac{(1-\pi\mu)(1-\lambda)(1-\theta)(1+B)}{\pi\mu\lambda + (1-\lambda)\theta\pi\mu + B(1-\pi\mu)(1-\lambda)(1-\theta)}. \end{aligned} \quad (2.31)$$

In the case when there is a positive level of storage in equilibrium, we can depart from the capital asset market clearing condition  $(r^h + \lambda q^h)/q^h = 1$  to transform (2.29) into

$$\frac{(\sigma + \xi)\pi\mu}{(\sigma + \xi)\pi\mu + 1 - \sigma - \xi} \geq \frac{q^l}{q^h} - B \left(1 - \frac{q^l}{q^h}\right),$$

where

$$B = \frac{\beta\sigma(1-\lambda)}{(1-\theta)(1-\beta\sigma\lambda)},$$

which becomes

$$\frac{A^h}{A^l} \geq \frac{((\sigma + \xi)\pi\mu + 1 - \sigma - \xi)(1+B)}{(\sigma + \xi)\pi\mu + B((\sigma + \xi)\pi\mu + 1 - \sigma - \xi)}. \quad (2.32)$$

When  $B = 0$ , conditions (2.31) and (2.32) collapse to (2.12) and (2.13), respectively. To prove that (2.31) is less strict than (2.12), we have to prove that the RHS of (2.31) is increasing with  $B$ :

$$\begin{aligned} &\frac{\partial}{\partial B} \frac{(1-\pi\mu)(1-\lambda)(1-\theta)(1+B)}{\pi\mu\lambda + (1-\lambda)\theta\pi\mu + B(1-\pi\mu)(1-\lambda)(1-\theta)} \\ &= \frac{\pi\mu\lambda + (1-\lambda)\theta\pi\mu - (1-\pi\mu)(1-\lambda)(1-\theta)}{\frac{(\pi\mu\lambda+(1-\lambda)\theta\pi\mu+B(1-\pi\mu)(1-\lambda)(1-\theta))^2}{(1-\pi\mu)(1-\lambda)(1-\theta)}} < 0. \end{aligned}$$

The last inequality comes from the fact that  $q^h > 1$ , and therefore, the RHS of (2.12) also exceeds 1. Similarly, we can show that the RHS of (2.32) is increasing with  $B$ :

$$\begin{aligned} &\frac{\partial}{\partial B} \frac{((\sigma + \xi)\pi\mu + 1 - \sigma - \xi)(1+B)}{(\sigma + \xi)\pi\mu + B((\sigma + \xi)\pi\mu + 1 - \sigma - \xi)} \\ &= \frac{(\sigma + \xi)\pi\mu - ((\sigma + \xi)\pi\mu + 1 - \sigma - \xi)}{\frac{((\sigma + \xi)\pi\mu + ((\sigma + \xi)\pi\mu + 1 - \sigma - \xi))^2}{((\sigma + \xi)\pi\mu + 1 - \sigma - \xi)}} < 0. \end{aligned}$$

## 2.A.4 Adverse selection on re-sale markets

### 2.A.4.1 Case without implicit recourse: Prices depend on the share of high quality assets

We derive the pricing conditions from the F.O.C. of saving firms (subset  $\mathcal{S}_t$ ) in a pooling equilibrium. The value of a high quality asset  $q_t^h$  reflects the expected gross profit next period and the value of the asset next period, which is  $q_{t+1}^h$  if the firm has no investment opportunities and keeps the asset on the balance sheet, or  $q_{t+1}^s$  if the firm has an investment opportunity and sells the asset. The Euler condition below shows the marginal indifference of the saving firm between keeping a high quality asset or buying an asset on the primary market:

$$E_t \left[ \Lambda_{t,t+1} \frac{r_{t+1}^h + \lambda \pi q_{t+1}^s + \lambda (1 - \pi) q_{t+1}^h}{q_t^h} \right] = E_t \left[ \Lambda_{t,t+1} R_{t+1}^p \right],$$

where the expected return of an asset bought on the primary market is

$$E_t \left( \frac{\mu (r_{t+1}^h + \lambda (\pi q_{t+1}^s + (1 - \pi) q_{t+1}^h)) + (1 - \mu) (r_{t+1}^l + \lambda q_{t+1}^s)}{q_t^p} \right).$$

An asset bought on the primary market in the pooling equilibrium is, with probability  $\mu$ , of high quality and with probability  $1 - \mu$  of low quality.

The value of the low quality asset reflects the expected next period gross profits and the expected next period resale price since low assets are always sold on the re-sale market

$$E_t \left[ \Lambda_{t,t+1} \frac{r_{t+1}^l + \lambda q_{t+1}^s}{q_t^l} \right] = E_t \left[ \Lambda_{t,t+1} R_{t+1}^p \right].$$

The price of an asset sold on the re-sale market satisfies:

$$E_t \left[ \Lambda_{t,t+1} \frac{f_t^h (r_{t+1}^h + \lambda (\pi q_{t+1}^s + (1 - \pi) q_{t+1}^h)) + (1 - f_t^h) (r_{t+1}^l + \lambda q_{t+1}^s)}{q_t^s} \right] = E_t \left[ \Lambda_{t,t+1} R_{t+1}^p \right],$$

where  $f_t^h$  is the share of high quality assets sold on the re-sale market in this period, which is in the case of a pooling equilibrium

$$f_t^h = \frac{\pi \omega_t}{\pi + (1 - \pi) (1 - \omega_t)}.$$

### 2.A.4.2 Case without implicit recourse: Conditions for no trade in high quality assets

Investing firms prefer to keep their high quality loans rather than to sell them and invest the obtained liquidity if the following condition is satisfied in the steady state:

$$\begin{aligned} V(\text{keeping high projects}) &\geq V(\text{selling high projects}), \\ R(\text{keeping high projects}) &\geq R(\text{selling high projects}), \end{aligned}$$

$$\begin{aligned}
r^h + \lambda\pi\mu q^s + \lambda(1 - \pi\mu)q^h &\geq q^s \frac{r^h + \lambda\pi\mu q^s + \lambda(1 - \pi\mu)q^h}{\frac{1 - \theta q^p}{1 - \theta}}, \\
\frac{1 - \theta q^p}{1 - \theta} &\geq q^s = f^h q^h + (1 - f^h)q^l, \\
1 - \theta(\mu q^h + (1 - \mu)q^l) &\geq (1 - \theta)(f^h q^h + (1 - f^h)q^l), \\
1 - \theta\mu q^h - (1 - \theta\mu)q^l &\geq f^h(1 - \theta)(q^h - q^l), \\
f^h &\leq \frac{1 - \theta\mu q^h - (1 - \theta\mu)q^l}{(1 - \theta)(q^h - q^l)}.
\end{aligned}$$

### 2.A.4.3 Case with implicit recourse: Prices depend on the share of assets without implicit recourse

We derive the pricing conditions from the F.O.C. of saving firms in a pooling equilibrium. In contrast to the case without implicit recourse, the prices depend on the share of low quality assets without implicit recourse  $f^{NIR}$ . The shadow value of a high quality asset remains the same

$$E_t \left[ \Lambda_{t,t+1} \frac{r_{t+1}^h + \lambda\pi q_{t+1}^s + \lambda(1 - \pi)q_{t+1}^h}{q_t^h} \right] = E_t [\Lambda_{t,t+1} R_{t+1}^p],$$

where the expected return of an asset bought on the primary market is  $R_{t+1}^p = E_t \left( \frac{x_{t+1}^p}{q_t^p} \right)$ , where

$$\begin{aligned}
x_{t+1}^p &= \mu \left( r_{t+1}^h + \lambda \left( \pi q_{t+1}^s + (1 - \pi)q_{t+1}^h \right) \right) + (1 - \mu) \left( \chi_{D,t+1} r_{t+1}^l + (1 - \chi_{D,t+1}) r_{t+1}^h \right) \\
&\quad + (1 - \mu) \lambda \left( (1 - \pi)(1 - \chi_{D,t+1})q_{t+1}^l + (1 - (1 - \pi)(1 - \chi_{D,t+1}))q_{t+1}^s \right).
\end{aligned}$$

An asset bought on the primary market in the pooling equilibrium is with probability  $\mu$  of high quality and with probability  $1 - \mu$  of low quality. The implicit recourse on the low quality asset may be provided and then the asset generates cash flow  $r_{t+1}^h$ , or recourse may be defaulted upon and then the asset generates cash flow  $r_{t+1}^l$ . In a pooling equilibrium, assets will be sold on the re-sale market in order to take advantage of the investment opportunity with probability  $\pi$ . If the implicit recourse is defaulted upon ( $\chi_{D,t+1} = 1$ ), holders will be able to identify the low quality assets and will sell them on the re-sale markets. Otherwise assets are kept on the balance sheet (high quality assets with probability  $1 - \pi$  and low quality assets with probability  $(1 - \pi)(1 - \chi_{D,t+1})$ ) and valued by their shadow price  $q^h$  or  $q^l$ .

As already mentioned, the low quality assets are either non-identified or without implicit recourse. The value of the low quality asset without implicit recourse is  $q_t^s$  since it is never kept on the balance sheet till the next period and it is immediately sold in the re-sale market. The shadow value of the low quality assets which are a non-identified part of the firm's portfolio with implicit recourse is determined by:

$$E_t \left[ \Lambda_{t,t+1} \frac{x_{t+1}^l}{q_t^l} \right] = E_t [\Lambda_{t,t+1} R_{t+1}^p],$$

where

$$\begin{aligned} x_{t+1}^l &= \chi_{D,t+1} r_{t+1}^l + (1 - \chi_{D,t+1}) r_{t+1}^h \\ &\quad + \lambda (1 - \pi) (1 - \chi_{D,t+1}) q_{t+1}^l + \lambda (1 - (1 - \pi) (1 - \chi_{D,t+1})) q_{t+1}^s. \end{aligned}$$

The price of an asset sold on the re-sale market satisfies:

$$E_t \left[ \Lambda_{t,t+1} \frac{x_{t+1}^s}{q_t^s} \right] = E_t \left[ \Lambda_{t,t+1} R_{t+1}^p \right],$$

where

$$\begin{aligned} x_{t+1}^s &= f_t^h (r_{t+1}^h + \lambda (\pi q_{t+1}^s + (1 - \pi) q_{t+1}^h)) \\ &\quad + (1 - f_t^h - f_t^{l,IR} (1 - \chi_{D,t+1})) r_{t+1}^l + f_t^{l,IR} (1 - \chi_{D,t+1}) r_{t+1}^h \\ &\quad + \lambda (1 - \pi) f_t^{l,IR} (1 - \chi_{D,t+1}) q_{t+1}^l + \lambda \left[ (1 - \pi) (1 - f_t^h - f_t^{l,IR} (1 - \chi_{D,t+1})) + \pi \right] q_{t+1}^s, \end{aligned}$$

where  $f_t^h$  is the share of high quality assets sold on the re-sale market in this period, which is in the case of a pooling equilibrium:

$$f_t^h = \frac{\pi \omega_t}{\pi + f_t^{NIR} (1 - \pi) (1 - \omega_t)},$$

and  $f_t^{l,IR}$  is the share of low quality assets on the re-sale markets which bears implicit recourse

$$f_t^{l,IR} = \frac{\pi (1 - \omega_t) (1 - f_t^{NIR})}{\pi + f_t^{NIR} (1 - \pi) (1 - \omega_t)}.$$

In the steady state with informative implicit recourse, firms default only when they exit, i.e., with probability  $\sigma$ . The share of low quality assets without implicit recourse (out of all low quality assets) after the decisions on the default on implicit recourse  $f_t^{NIR}$  is then given by

$$f_t^{NIR} = f_t^D,$$

where  $f_{t+1}^D$  is the share of low quality assets with defaulted implicit recourse at the end of the period  $t$  is

$$f_{t+1}^D = (f_t^{NIR} + 1 - \sigma) \frac{\lambda K_t}{\lambda K_t + I_t}.$$

This gives us the steady state level  $f^{NIR} = \lambda (1 - \sigma) / (1 - \lambda)$ . In the next section, we will show that when economy moves to a deep recession, there will be systemic default on implicit recourse given and  $f_t^{NIR} = 1$ .

## 2.B Appendix 2.B: Markov-switching regimes

### 2.B.1 Equilibrium conditions

The investment function:

$$I_t = \frac{\chi_{1,t}(\sigma + \xi) [(\omega_t r_t^h + (1 - \omega_t) r_t^l + \lambda q_t^s) K_t + Z_t]}{1 - \theta q_t^p}.$$

Consumption function:

$$C_t = (1 - \sigma - \xi) [(\omega_t r_t^h + (1 - \omega_t) r_t^l) K_t + \lambda K_t (\omega_t q_t^h + (1 - f_t^{NIR}) (1 - \omega_t) q_t^l + f_t^{NIR} (1 - \omega_t) q_t^s) + Z_t].$$

The consumer goods market clearing condition:

$$(\omega_t r_t^h + (1 - \omega_t) r_t^l) K_t + Z_t = I_t + C_t + Z_{t+1}.$$

Law of motion for capital

$$K_{t+1} = \lambda K_t + I_t,$$

and law of motion for the share of high quality assets

$$\omega_{t+1} = \frac{H_{t+1}}{K_{t+1}} = \frac{\lambda H_t + I_t^H}{\lambda K_t + I_t} = \frac{\omega_t \lambda K_t + \chi_{2,t} I_t}{\lambda K_t + I_t} = \frac{\omega_t \lambda + \chi_{2,t} I_t / K_t}{\lambda + I_t / K_t}.$$

Capital goods market clearing conditions: I calibrate the model such that there is a positive amount of storage as well as investment in the economy. Since the return on storage is  $R_{t+1}^Z = 1$ , the market clearing conditions are the following:

$$E_t \left[ \Lambda_{t,t+1} \frac{x_{t+1}^h}{q_t^h} \right] = E_t \left[ \Lambda_{t,t+1} \frac{x_{t+1}^l}{q_t^l} \right] = E_t \left[ \Lambda_{t,t+1} \frac{x_{t+1}^p}{q_t^p} \right] = E_t \left[ \Lambda_{t,t+1} \frac{x_{t+1}^s}{q_t^s} \right] = E_t [\Lambda_{t,t+1}],$$

where the next period cash flows and values of these assets are defined

$$\begin{aligned} x_{t+1}^h &= r_{t+1}^h + \lambda \chi_{1,t+1} q_{t+1}^s + \lambda (1 - \chi_{1,t+1}) q_{t+1}^h, \\ x_{t+1}^l &= \pi_{D,t+1} r_{t+1}^l + (1 - \pi_{D,t+1}) r_{t+1}^h \\ &\quad + \lambda (1 - \chi_{1,t+1}) (1 - \pi_{D,t+1}) q_{t+1}^l + \lambda (1 - (1 - \chi_{1,t+1}) (1 - \pi_{D,t+1})) q_{t+1}^s, \\ x_{t+1}^p &= \chi_{2,t} \left( r_{t+1}^h + \lambda (\chi_{1,t+1} q_{t+1}^s + (1 - \chi_{1,t+1}) q_{t+1}^h) \right) \\ &\quad + (1 - \chi_{2,t}) \left( \pi_{D,t+1} r_{t+1}^l + (1 - \pi_{D,t+1}) r_{t+1}^h \right) \\ &\quad + (1 - \chi_{2,t}) [\lambda (1 - \chi_{1,t+1}) (1 - \pi_{D,t+1}) q_{t+1}^l \\ &\quad + \lambda (1 - (1 - \chi_{1,t+1}) (1 - \pi_{D,t+1})) q_{t+1}^s], \\ x_{t+1}^s &= f_t^h \left( r_{t+1}^h + \lambda (\chi_{1,t+1} q_{t+1}^s + (1 - \chi_{1,t+1}) q_{t+1}^h) \right) \\ &\quad + \left( 1 - f_t^h - f_t^{l,IR} (1 - \chi_{D,t+1}) \sigma \right) r_{t+1}^l + f_t^{l,IR} (1 - \chi_{D,t+1}) \sigma r_{t+1}^h \\ &\quad + \lambda (1 - \chi_{1,t+1}) f_t^{l,IR} (1 - \chi_{D,t+1}) \sigma q_{t+1}^l \\ &\quad + \lambda \left[ (1 - \chi_{1,t+1}) \left( 1 - f_t^h - f_t^{l,IR} (1 - \chi_{D,t+1}) \sigma \right) + \chi_{1,t+1} \right] q_{t+1}^s. \end{aligned}$$

The probability of defaults on implicit recourse in the next period conditional on the assets still bearing an implicit recourse

$$\pi_{D,t+1} = (1 - \chi_{D,t+1})(1 - \sigma) + \chi_{D,t+1}.$$

The share of high quality assets on the re-sale markets

$$f_t^h = \frac{\chi_{1,t}\omega_t}{\chi_{1,t} + f_t^{NIR}(1 - \chi_{1,t})(1 - \omega_t)}.$$

The share of low quality assets on the resale markets which bear implicit recourse

$$f_t^{l,IR} = \frac{\chi_{1,t}(1 - \omega_t)(1 - f_t^{NIR})}{\chi_{1,t} + f_t^{NIR}(1 - \chi_{1,t})(1 - \omega_t)}.$$

The share of low quality assets without implicit recourse

$$f_t^{NIR} = (1 - \chi_{D,t})f_{t-1}^D + \chi_{D,t}.$$

The share of low quality assets with defaulted implicit recourse at the end of the period

$$f_t^D = (f_t^{NIR} + 1 - \sigma) \frac{\lambda K_t}{\lambda K_t + I_t}.$$

The costs of providing implicit recourse

$$P_t = \sigma \Lambda_{t,t+1} (1 - \chi_{D,t+1}) [A_{t+1} (\Delta_{t+1}^h - \Delta_{t+1}^l) K_{t+1}^{\alpha-1} + \lambda P_{t+1}].$$

The outstanding stock of implicit recourse obligations

$$SIR_t = (1 - f_t^{NIR})(1 - \omega_t) K_t P_t.$$

When a firm defaults on the implicit recourse, the marginal value of its equity becomes  $\nu_t^D$ , which is defined as

$$\nu_t^D = (1 - \sigma) + \sigma E_t \Lambda_{t,t+1} R_{t+1}^D \nu_{t+1}^D,$$

where  $R_{t+1}^D$  is the return on firm's equity when the firm loses the reputation and cannot sell on the securitization markets:

$$\begin{aligned} E_t R_{t+1}^D &= \pi \mu E_t (x_{t+1}^h) + (1 - \chi_s) \pi (1 - \mu) E_t (r_{t+1}^l + \lambda q_{t+1}^s) \\ &\quad + (\chi_s \pi (1 - \mu) + (1 - \pi)) E_t \frac{(x_{t+1}^p)}{q_t^p}. \end{aligned}$$

## 2.B.2 Markov regimes' properties

The Markov switching parameters  $\bar{\chi}$  take the following values in different states. The parameter determining the share of investing firms  $\chi_{1,t}$  takes the value  $\pi$  in a pooling equilibrium and  $\pi\mu$  in a separating equilibrium, therefore,  $\chi_1(1) = \pi$  and  $\chi_1(2) = \chi_1(3) = \pi\mu$ . The parameter determining the share of high quality assets available on the primary

market  $\chi_{2,t}$  takes the value 1 in a separating equilibrium and  $\mu$  in a pooling equilibrium; therefore,  $\chi_2(1) = \mu$  and  $\chi_2(2) = \chi_2(3) = 1$ . The parameter determining an economy-wide default on implicit recourse  $\chi_{D,t}$  takes the value 0 in all non-default states and value 1 in the default state; therefore,  $\chi_D(1) = \chi_D(2) = 0$  and  $\chi_D(3) = 1$ . The parameter determining the existence of a separating equilibria  $\chi_{s,t}$  takes the value 1 in a separating equilibrium and 0 in a pooling equilibrium; therefore,  $\chi_s(1) = 0$  and  $\chi_s(2) = \chi_s(3) = 1$ .

Parametrization of the model has to satisfy the endogenous conditions for the existence of a separating equilibrium or a pooling equilibrium and conditions for the equilibrium provision of implicit recourse and default on this recourse are satisfied according to the definitions of the 3 Markov regimes for a relevantly large subset of state variables combinations. These conditions are the following.

### 2.B.2.1 Pooling and separating equilibria conditions

The Markov Regime 1 with the high productivity and lowest dispersion should be a pooling equilibrium. Therefore, firms with access to low quality investment opportunities have to prefer mimicking firms with high quality investment opportunities rather than buying assets on the markets:

$$\begin{aligned} V_{i,t}(\text{buying projects}) &\leq V_{i,t}(\text{mimicking \& no default}) \quad \forall i \in (\mathcal{L}_t \cap \mathcal{I}_t), s_t = 1, \\ E_t(\Lambda_{t,t+1} R_{t+1}^n n_t \nu_{t+1}) | \text{buying projects} &\leq E_t(\Lambda_{t,t+1} R_{t+1}^n n_t \nu_{t+1}) | \text{mimicking \& no default}, \end{aligned}$$

$$E_t \left( \Lambda_{t,t+1} \frac{x_{t+1}^h}{q_t^h} \right) \leq E_t \left( \Lambda_{t,t+1} \frac{x_{t+1}^l - (1 - \chi_{D,t+1}) \frac{1}{1-\theta} ((r_{t+1}^h - r_{t+1}^l) + \lambda P_{t+1})}{\frac{1-\theta q_t^p}{1-\theta}} \right).$$

While in Regime 2 and 3 the above inequality have to be exactly opposite for all firms with access to low quality investment opportunities

$$V_{i,t}(\text{buying projects}) \geq V_{i,t}(\text{mimicking \& no default}) \quad \forall i \in (\mathcal{L}_t \cap \mathcal{I}_t), s_t \in \{2, 3\}.$$

### 2.B.2.2 Default on implicit recourse conditions

For implicit recourse to have some value, it should not be defaulted upon at least in some states of the economy. According to the specification of the Markov regimes, any implicit recourse provided in a pooling equilibrium in Regime 1 should not be defaulted upon as long as the economy stays in Regime 1 or Regime 2. However, in Regime 3, all firms should find it optimal to default on the implicit recourse. For this to hold, we have to check the following conditions.

When the economy moves from Regime 1 to Regime 2, then for a significant subset of state variables, we should find in Regime 2 in period  $t + 1$  that all firms including those that had, in period  $t$ , access to low quality investment opportunities and which mimicked firms with high quality investment opportunities, will find it more profitable not to default on the existing implicit guarantees:

$$E_{t+1} V_{i,t+1}(\text{not defaulting}) \geq E_{t+1} V_{i,t+1}(\text{defaulting}) \quad \forall i \in (\mathcal{L}_t \cap \mathcal{I}_t), s_t = 1, s_{t+1} = 2,$$

$$\begin{aligned}
& \left( \frac{(1-\theta) \left( x_{t+1}^l - \frac{1}{1-\theta} (r_{t+1}^h - r_{t+1}^l + \lambda P_{t+1}) \right)}{1 - \theta q_t^p} \right) \\
& - \frac{(1 - f_t^{NIR}) (1 - \omega_t) \lambda (r_{t+1}^h - r_{t+1}^l + \lambda P_{t+1})}{R_t^N} + \frac{(1 - f_t^D) (1 - \omega_{t+1}) \lambda P_{t+1}}{R_{t+1}^N} \bar{v}_{t+1}^{ND} \quad (2.33) \\
& \geq \frac{(1 - \theta) (r_{t+1}^l + \lambda q_{t+1}^s)}{1 - \theta q_t^{IR}} \bar{v}_{t+1}^D. \quad (2.34)
\end{aligned}$$

This condition is sufficient to claim that implicit recourse is not defaulted upon for the respective subset of state variables as long as the economy stays in Regime 1, Regime 2 or Regime 3. This is because transferring from Regime 1 to Regime 2 implies the highest relative costs for honoring the implicit recourse.

Similarly, when an economy switches to Regime 3, all firms should find it optimal to default on their implicit guarantees. As discussed previously, due to the limited enforceability of the punishment rule, all firms find it optimal to default if a subset of firms default. Since Regime 3 can follow only after Regime 1, we again check the condition (2.33) but with an inverted inequality sign

$$E_{t+1}V_{i,t+1}(\text{not defaulting}) < E_{t+1}V_{i,t+1}(\text{defaulting}) \quad \forall i \in (\mathcal{L}_t \cap \mathcal{I}_t), \quad s_t = 1, s_{t+1} = 3.$$



## Chapter 3

---

# By Force of Habit: Asset-Pricing Implications of Durable Goods

Co-authored by Michal Pakoš

### 3.1 Introduction

The time-separable expected-utility frameworks of Lucas (1978) and Mehra and Prescott (1985) have immense difficulty in accounting for the large and persistent variation in asset prices. In response to one of the suggestions in Mehra and Prescott, a large literature has developed with the aim of relaxing the rather restrictive preference specification. Motivated by Stigler and Becker (1977), Constantinides (1990) explores the asset-pricing implications of internal habit formation and is able to account for the large equity premium puzzle with a small coefficient of relative risk aversion. In addition, Campbell and Cochrane (1999) broaden his research agenda; their setting of external habit formation is able to account for a broad variety of dynamic asset pricing phenomena in the time-series. The intuition is that as the macroeconomy falls into recession and the surplus consumption ratio falls, the investors' capacity for further reductions in their consumption become more limited. The concomitant rise in aversion to risk drives risk premiums higher and hence asset prices move in pro-cyclical fashion. Unfortunately, the model's performance in pricing the cross-section of expected return is rather weak and, in addition, the level of necessary aversion to gamble over wealth (the relative risk aversion coefficient) is perhaps too large.

It is also well-known that investors' consumption portfolios consist in a large part of the service flow from durable consumer goods such as housing, cars, furniture and so on (see Pakoš (2005, Pakoš (2011) for recent evidence). This is suggestive of the estimation of investors' Euler equation by using solely nondurable goods and services is likely to be misspecified. In fact, Yogo (2006) employs the Epstein-Zin preference specification, defined over the service flow from durable goods and nondurable goods (plus services) and investigates the asset-pricing implications in both the time series and the cross-section (i.e., 25 Fama-French portfolios sorted on size and book-to-market ratio). Unfortunately, his results are rather mixed. In fact, the linearized version of Yogo's (2006) model, wherein his stochastic discount factor is an affine function of nondurables growth rate and durables growth rate separately, seems to present fairly solid evidence in favor of the model - value stocks are riskier exactly because they co-vary so much more with durable goods growth rate. However, the point estimates of preference parameters which are based on the Euler equations (where the discount factor is exactly the marginal rate of substitution) are rather extreme. That is to say, given his model, one needs to believe that the coefficient of the relative risk aversion is about 200 to justify the level of observed risk premiums.

We blend these two strands of literature by building an analytically tractable generalized version of Campbell and Cochrane's (1999) model which allows investors to derive the utility from consuming both nondurable goods and service flow from consumer durable goods. We estimate the parameters of this nonlinear model and test the asset pricing model implications using the GMM methodology on the universe of 6 Fama-French portfolios (described above), the risk-free rate and the value-weighted return on NYSE, AMEX and NASDAQ stocks. Since the durables are smoother than nondurable consumption, their presence in the utility function worsens the "equity premium puzzle". Nevertheless, thanks to the presence of habit, the value of the relative risk aversion is lower than for instance in the model with durable goods by Yogo (2006).

The remainder of the paper is organized as follows: section 3.2 describes the model, section 3.3 contains the data description, section 3.4 deals with the cointegration among model variables implied by the intratemporal first-order condition, and section 3.5 describes the estimation methodology and results from the GMM estimations.

## 3.2 Model

Following Campbell and Cochrane (1999), we endow the representative investor with the following time non-separable preferences

$$U_t = E_t \left( \sum_{\tau=t}^{\infty} \beta^{\tau-t} (C_{\tau} - X_{\tau})^{1-\gamma} / (1-\gamma) \right),$$

where  $C_t$  denotes the investor's consumption and  $X_t$  is the level of accumulated consumption experience; we refer to it as habit level. In addition, the parameter  $\beta$  equals the subjective discount factor and  $\gamma$  is a measure of the marginal utility curvature.

Following Stigler and Becker (1977), the investor produces his final consumption stream using the time- and state- independent household production function

$$C_t = AN_t^{\alpha} D_t^{1-\alpha}, \quad (3.1)$$

where  $N_t$  denotes the consumption of nondurable goods whereas  $D_t$  equals the stock of the consumer durable goods at the beginning of the period  $t$ . The stock enters the investors' preferences because we assume that the service flow is proportional to the corresponding stock. Furthermore, without loss of generality, we set  $A = 1$ . Note that the original model of Campbell and Cochrane (1999) is nested as a special case for  $\alpha = 1$ .

The household stock of consumer durable goods at the beginning of period  $t + 1$  is given by the non-depreciated stock of durables from the previous period  $(1 - \delta_t) D_t$ , where  $\delta_t$  is the depreciation rate, augmented by the expenditures on durables  $E_t$ :

$$D_{t+1} = (1 - \delta_t) D_t + E_t. \quad (3.2)$$

Therefore, the stock of durables is a backward looking variable.

With hindsight, it is convenient to define the surplus consumption ratio

$$S_t = \frac{(C_t - X_t)}{C_t}. \quad (3.3)$$

The relationship between consumption and habit is then defined by the law of motion for the surplus consumption ratio. We denote consumption series in logs by lower-case letters and assume that the law of motion (in logs) for the surplus consumption ratio is

given by the following first-order autoregressive process

$$\begin{aligned} s_{t+1} &= (1 - \phi) \bar{s} + \phi s_t + \lambda(s_t) (E_{t+1} - E_t) \Delta c_{t+1} \\ &= (1 - \phi) \bar{s} + \phi s_t + \lambda(s_t) (\alpha (E_{t+1} - E_t) \Delta n_{t+1} + (1 - \alpha) (E_{t+1} - E_t) \Delta d_{t+1}), \end{aligned} \quad (3.4)$$

where the nonlinear sensitivity function of the log surplus consumption ratio to the weighted forecast errors in consumption series  $\lambda = \lambda(s_t)$  is specified later. Since  $0 < \phi < 1$  the process for log surplus consumption ratio is stationary and mean reverting. The long-term mean is its steady state value  $\bar{s}$ . The process has a time-varying volatility dependent on the state  $s_t$ .

As in Campbell and Cochrane (1999), we use an external habit specification, where habit responds to the average level of consumption across all agents in the model. But since, in our model, all agents are identical, we do not need to differentiate between the average and individual variables. The advantage of the Campbell and Cochrane (1999) habit specification is that the habit level depends on the history of aggregate consumption rather than on individual consumption. This eliminates the effect of today's individual consumption choice on the level of future habit from the first order conditions of rational investors. Since we defined the relationship of habit to the history of consumption through the law of motion for the surplus consumption ratio, the habit adjusts only slowly and nonlinearly to changes in past consumption. This ensures that the habit is always below consumption level and that marginal utilities are always positive and finite.

### 3.2.1 Intratemporal Marginal Rate of Substitution and Technology

The marginal utility of nondurable consumption and of the stock of consumer durable goods are:

$$\begin{aligned} U_N(N_t, D_t, X_t) &= (C_t - X_t)^{-\gamma} \alpha N_t^{\alpha-1} D_t^{1-\alpha}, \\ U_D(N_t, D_t, X_t) &= (C_t - X_t)^{-\gamma} (1 - \alpha) N_t^\alpha D_t^{-\alpha}. \end{aligned}$$

Marginal utilities depend not only on the level of nondurable consumption and stock of durables but also on the level of habit.

The intratemporal first-order condition states that marginal utility per unit of cur-

rency spent must be the same for both consumption goods:

$$\frac{U_N(N_t, D_t, X_t)}{1} = \frac{U_D(N_t, D_t, X_t)}{RC_t},$$

where  $RC_t$  stands for the rental costs of consumer durables in terms of nondurables, which are a numeraire good in our model. We also know that the no arbitrage condition relates the rental costs to the relative price of durables in terms of nondurables  $Q_t$  by the means of the following formula:

$$RC_t = Q_t - (1 - \delta_t) E_t(M_{t+1}Q_{t+1}),$$

where  $M_t$  is the stochastic discount factor. This no arbitrage condition intuitively states that rental costs should be the net present value (NPV) of purchasing one unit of durable good at price  $Q_t$  and selling the non-depreciated part  $(1 - \delta_t)$  for price  $Q_{t+1}$  in the following period.

Combining the intratemporal first-order condition with the no arbitrage condition we obtain:

$$\frac{N_t(1 - \alpha)}{D_t} = Q_t - (1 - \delta_t) E_t(M_{t+1}Q_{t+1}).$$

Our model, therefore, implies a certain long run relationship between between nondurable consumption  $N_t$ , the stock of durables  $D_t$  and the relative price of durables in terms of nondurables  $Q_t$ . To demonstrate that let us refer to a well known fact in empirical macroeconomics that ratios of asset prices to income flows such as price-dividend ratios of stocks are stationary i.e.  $I(0)$ , while both asset prices and income flows are usually integrated of order one  $I(1)$ . When applied to durable goods, this would lead us to a conjecture that the log of durable consumption price and the log of rental costs of durable goods are cointegrated. The intratemporal first-order condition together with this conjecture would imply that the log of nondurable consumption  $n_t$ , the log of stock of durable consumption goods  $d_t$  and the log of durable goods price expressed in terms of nondurables  $q_t$  share a common stochastic trend.

Similarly, from the law of motion for the stock of durables (3.2), we obtain:

$$\frac{D_{t+1}}{D_t} = 1 - \delta_t + \frac{E_t}{D_t}.$$

Since the growth of durables  $D_{t+1}/D_t$  as well as  $\delta_t$  are stationary processes, this implies

that non-stationary processes: expenditures on durables  $E_t$  and their stock  $D_t$  as well as their logs, have to be integrated. Based on these predictions we test the cointegration relationship among  $(n_t, e_t, q_t)'$  in the empirical section.

Having in mind the cointegrated relationship among  $(n_t, e_t, q_t)'$ , we can proceed to the specification of the consumption and price processes. We assume an endowment economy, where log growth of nondurable consumption, log growth of the expenditures on durables and the change in the log relative price of durables can be defined by the vector error correction model (VECM), which takes into account the long-term relationship of these variables.

The representative rational investor who populates our model knows the specification of consumption and price processes and forms expectations based on the prediction of the vector error correction model. Forecast errors from the VECM model  $(\varepsilon_t^n, \varepsilon_t^e, \varepsilon_t^q)$  have an approximately normal distribution. Therefore, nondurable consumption growth, growth of expenditures on durables as well as the growth of the relative price of durables follow a log-normal processes.

Under these assumptions about the consumption processes and since the stock of durables is a predetermined variable, the law of motion for the surplus consumption ratio becomes

$$s_{t+1} = (1 - \phi) \bar{s} + \phi s_t + \lambda(s_t) \alpha \varepsilon_{t+1}^n,$$

and therefore is affected only by forecast error in the log growth of nondurable consumption. The surplus consumption ratio  $S_t$  is, therefore, a lognormal process with time-varying volatility.

### 3.2.2 Intertemporal Marginal Rate of Substitution and the Risk-Free Rate

The intertemporal marginal rate of substitution is

$$\begin{aligned} M_{t+1} &= \beta \frac{(C_{t+1} - X_{t+1})^{-\gamma}}{(C_t - X_t)^{-\gamma}} \left( \frac{N_{t+1}}{N_t} \right)^{\alpha-1} \left( \frac{D_{t+1}}{D_t} \right)^{1-\alpha} \\ &= \beta \left( \frac{S_{t+1}}{S_t} \right)^{-\gamma} \left( \frac{N_{t+1}}{N_t} \right)^{(1-\gamma)\alpha-1} \left( \frac{D_{t+1}}{D_t} \right)^{(1-\gamma)(1-\alpha)}. \end{aligned}$$

Since the consumption processes and the evolution of the surplus consumption ratio is defined in logs, it is convenient to take logs of the intertemporal marginal rate of

substitution:

$$m_{t+1} = \log(\beta) - \gamma \Delta s_{t+1} + ((1 - \gamma)\alpha - 1) \Delta n_{t+1} + (1 - \gamma)(1 - \alpha) \Delta d_{t+1}. \quad (3.5)$$

The intertemporal marginal rate of substitution, which corresponds exactly to the stochastic discount factor, depends on the growth of nondurable consumption, the growth of durables and the growth of the surplus consumption ratio. All these processes are lognormal, therefore, the stochastic discount factor is also lognormal. The mean and variance of the log of the stochastic discount factor are:

$$E_t(m_{t+1}) = \log(\beta) + ((1 - \gamma)\alpha - 1) \mu_t^n + (1 - \gamma)(1 - \alpha) \mu_t^d - \gamma(1 - \phi)(\bar{s} - s_t), \quad (3.6)$$

$$\begin{aligned} \text{Var}(m_{t+1}) &= ((1 - \gamma)\alpha - 1 - \gamma\alpha\lambda(s_t))^2 \sigma_n^2 \\ &= \gamma^2 (1 + \tilde{\lambda}(s_t))^2 \sigma^2, \end{aligned} \quad (3.7)$$

where  $\mu_t^n \equiv E_t \Delta n_{t+1}$ ,  $\mu_t^d \equiv E_t \Delta d_{t+1}$  are the conditional expectations of the log growth of nondurables and log growth of the stock of durables,  $\sigma^2 = \frac{1}{\gamma^2} ((1 - \gamma)\alpha - 1)^2 \sigma_n^2$  is the scaled variance of forecast errors from the VECM equation for the log growth of nondurables  $\sigma_n^2$ , and  $\tilde{\lambda}(s_t) = \frac{\alpha\gamma}{(\gamma-1)\alpha+1} \lambda(s_t)$  is the scaled sensitivity function in the law of motion of surplus consumption ratio. The introduction of new notation  $\sigma^2, \tilde{\lambda}(s_t)$  makes the set-up more easily comparable to Campbell and Cochrane (1999). Note that, in case of zero weight on durables in the utility function ( $\alpha = 1$ ), we obtain  $\sigma^2 = \sigma_n^2$  and  $\tilde{\lambda}(s_t) = \lambda(s_t)$  and the model collapses to Campbell and Cochrane (1999). The derivation of the above formulas is in the Appendix 3.A.

Knowing the specification of the intertemporal marginal rate of substitution, we can proceed to the asset-pricing implications.

**Slope of the Mean Standard Deviation Frontier:** Following Hansen and Jagannathan (1991) we know that the moment condition  $E_t(M_{t+1} R_{t+1}^{i,e}) = 0$  for an  $i$  asset excess return over the risk-free rate  $R^{i,e}$  implies that its Sharpe ratio should satisfy

$$\frac{E_t(R_{t+1}^{i,e})}{\sigma_t(R_{t+1}^{i,e})} \leq \frac{\sigma_t(M_{t+1})}{E_t(M_{t+1})}.$$

In our model the stochastic discount factor  $M_{t+1}$  is lognormal, so similarly to Campbell and Cochrane (1999), we can express the maximum possible Sharpe ratio an asset can

achieve as:

$$\max_i \frac{E_t (R_{t+1}^{i,e})}{\sigma_t (R_{t+1}^{i,e})} = \frac{\sigma_t (M_{t+1})}{E_t (M_{t+1})} = \sqrt{\exp(\gamma^2 (1 + \lambda(s_t))^2 \sigma^2) - 1} \approx \gamma (1 + \tilde{\lambda}(s_t)) \sigma.$$

This implies that the maximum possible Sharpe ratio is proportional to the marginal utility curvature  $\gamma$ , scaled standard deviation of forecast errors on the log growth of non-durables  $\sigma$  and the scaled sensitivity function of log surplus consumption ratio to forecast errors  $\tilde{\lambda}(s_t)$ . We know that the conditional Sharpe ratio changes over time, in particular, it is countercyclical. Since the conditional mean of the stochastic discount factor equals the inverse of the risk-free rate, which varies only negligibly over the business cycle, the variation should come from the time-varying conditional volatility of the stochastic discount factor. To satisfy this in our model, where the marginal utility curvature  $\gamma$  and scaled standard deviation of forecast errors  $\sigma$  are constant, the sensitivity function  $\tilde{\lambda}(s_t)$  has to be dependent on the state  $s_t$ . In particular it should be higher in recessions when  $s$  declines.

**Risk-Free Interest Rate:** From the standard Euler equation we find that the risk-free interest rate is the reciprocal of the expected stochastic discount factor

$$R_t^f = \frac{1}{E_t (M_{t+1})}. \quad (3.8)$$

Combining equations (3.5), (3.8) and taking into account the lognormality of the stochastic discount factor we get the expression for the log risk-free rate

$$r_t^f = -E_t (m_{t+1}) - \frac{1}{2} Var_t (m_{t+1}).$$

Substituting the expression for mean and variance of the stochastic discount factor from (3.6) and (3.7) the risk-free interest rate can be expressed as

$$\begin{aligned} r_t^f = & -\log(\beta) + ((\gamma - 1)\alpha + 1)\mu_t^n + (\gamma - 1)(1 - \alpha)\mu_t^d \\ & -\gamma(1 - \phi)(s_t - \bar{s}) - \frac{\gamma^2 \sigma^2}{2} (1 + \tilde{\lambda}(s_t))^2. \end{aligned} \quad (3.9)$$

The term  $s_t - \bar{s}$  reflects the effect of habit through intertemporal substitution. The lower the log surplus consumption ratio is, the higher the marginal utility of consumption, and the lower are savings which drive the equilibrium interest rate up. Terms with expected



growth of nondurables consumption and growth of stock of durables have a positive effect on the risk-free rate (in the case of durable stock only for  $\gamma > 1$ ). Higher expected growth of consumption makes consumers, who prefer to smooth their consumption profile, less willing to save, which creates upward pressure on the risk-free rate. The last term can be interpreted as a precautionary savings term. Higher volatility of non-forecasted innovations to non-durable consumption increases precautionary savings and, therefore, lowers the equilibrium risk-free interest rate. Since the historical data suggest that the volatility of the risk-free interest rate is rather low, we will follow Campbell and Cochrane (1999) in lowering the risk-free rate volatility by assuming the risk-free rate to be independent of the level of habit. In particular the variation in the habit term  $s_t - \bar{s}$  would be exactly offset by the variation in the precautionary savings term.

### 3.2.3 Choosing the Scaled Sensitivity Function $\tilde{\lambda}(s_t)$

We already mentioned some characteristics that the sensitivity function should satisfy in order to imply countercyclical Sharpe ratios. Similarly to Campbell and Cochrane (1999), we specify the functional form of  $\tilde{\lambda}(s_t)$  such that it meets also the following three conditions. First, we reduce the volatility of the risk-free rate by making it independent of the surplus consumption ratio  $s_t$ :  $r_t^f(\mu_t^n, \mu_t^d) = r_t^f(\mu_t^n, \mu_t^d, s_t) \forall s_t$ . Second, we restrict the habit to move non-negatively with both consumption of nondurables and the stock of durables:  $\frac{dx}{dn} \geq 0$ ,  $\frac{dx}{dd} \geq 0$ . Third, the habit is predetermined at the steady state.

The motivation for these assumptions is similar to that in Campbell and Cochrane (1999). We want to reduce the volatility of the risk-free rate, since in the historical data for the U.S., only a limited variation has been observed. Also we would like to explain stock market excess return by variation in risk aversion rather than by the variation in the risk-free rate. The constraint on positive comovement of habit and both consumption streams seems to be a natural property of a consumption habit. Finally, the last condition is in line with the notion of habit which only slowly follows the consumption growth. This condition also helps to prevent situations where habit would exceed the consumption index.

Since the stock of durables is a backward looking variable, and therefore, predetermined, the necessary condition for a predetermined habit at the steady state is  $\frac{dx}{dn} |_{s=\bar{s}} = 0$  and  $\frac{d}{ds} \left( \frac{dx}{dn} \right) |_{s=\bar{s}} = 0$ . The later condition also implies that the habit moves non-negatively with consumption of nondurables since the function  $\frac{dx}{dn}$  is convex in  $s$ , and therefore,

attains the minimum at the steady state and  $\frac{dx}{dn} \geq 0 \forall s_t$ . The habit always moves non-negatively with the stock of durables as  $\frac{dx}{dd} = 1 - \alpha \forall s$ .

As shown in Appendix 3.B, these conditions point to a unique solution to the functional form of the scaled sensitivity function  $\tilde{\lambda}(s_t)$  and the steady state level of surplus consumption ratio  $\bar{S}$ :

$$\begin{aligned}\tilde{\lambda}(s_t) &= \frac{1}{\sigma} \sqrt{\frac{1-\phi}{\gamma} (\psi - 2(s_t - \bar{s}))} - 1, & s_t \leq s_{max} \\ &= 0 & s_t \geq s_{max},\end{aligned}$$

where

$$\bar{S} = \frac{\alpha\gamma\sigma}{(\gamma-1)\alpha+1} \sqrt{\frac{\gamma}{1-\phi}} \psi.$$

The value  $s_{max}$  is the value of log surplus consumption ratio, where the scaled sensitivity function  $\tilde{\lambda}(s_t)$  attains zero:

$$s_{max} = \bar{s} + \frac{1}{2} \left( \psi - \sigma^2 \frac{\gamma}{1-\phi} \right).$$

We show in Appendix 3.B that if the weight of durables in the utility function is zero, i.e.,  $\alpha = 1$ , then  $\psi = 1$  and we obtain exactly the same functional forms as in Campbell and Cochrane (1999). We can therefore easily compare the predictions of our model with those of Campbell and Cochrane (1999).

### 3.3 Data

We retrieve the personal consumption expenditure (PCE) data from the U.S. National Income and Product Accounts as provided by the Bureau of Economic Analysis.

Our measure of nondurable consumption includes food and beverages purchased for off-premises consumption, clothing and footwear, and gasoline and other energy goods. The corresponding seasonally adjusted quarterly quantity index for the sample period 1952:I–2011:IV is from line 8 of Table 2.3.3. (Real Personal Consumption Expenditures by Major Type of Product).

A detailed description of our measure of the stock of consumer durable goods can be found in the Appendix 3.C

The relative price of consumer durable goods is constructed as the ratio of the PCE price index for durable goods from line 3 over the PCE price index for nondurable goods from line 8 of Table 2.3.4 (Price Indexes for Personal Consumption Expenditures by Major Type of Product). Note that the BEA reports only the annual series of the net stock of consumer durable goods. The quarterly series must be interpolated by assuming that the depreciation rate is constant within the year and finding its implied value that is consistent both with the annual stocks of net consumer durables at the beginning as well as the end of the year, and with quarterly series of PCE expenditures on durable goods.<sup>1</sup> The U.S. population measure used to calculate per-capita quantities covers the period 1952–2011 and may be retrieved from the Bureau of Labor Statistics. The quarterly and annual returns on the short-term nominal interest rate for the sample period 1952:I–2011:IV are from the online dataset of the Fama-French Factors. Because nondurable consumption is the numéraire in our analysis we deflate the nominal short-term riskless rate with the PCE price index for nondurable goods to obtain real quantities.

The 6 Fama-French portfolios are constructed from all NYSE, AMEX and NASDAQ stocks. They are at the intersection of 2 portfolios defined on size (market equity) and 3 portfolios defined on the ratio of book equity to market equity (BE/ME). Portfolios for year  $t$  are formed based on market equity and BE/ME recorded in June of the year  $t$ . The breakpoint for size is the median of NYSE market equity. Breakpoints for BE/ME are 30th and 70th percentiles at NYSE. The data are obtained from Kenneth French’s web page, where a more detailed description is also available. As before, because nondurable consumption is the numéraire in our analysis we deflate the nominal rates of return with the PCE price index for nondurable goods to obtain real quantities.

## 3.4 Implications of the Intratemporal First-Order Condition

### 3.4.1 Cointegration

The HQ and BIC criteria suggest that the vector of time series  $(n_t, e_t, q_t)$ , containing the log of nondurable consumption  $n_t = \log N_t$ , the log of expenditures on durables

---

<sup>1</sup>The law of motion of the consumer durable goods  $K_{t+1} = (1 - \delta_t) K_t + I_t$  yields after four iterations the equation  $K_{t+4} = (1 - \delta)^4 K_t + (1 - \delta)^3 I_t + (1 - \delta)^2 I_{t+1} + (1 - \delta) I_{t+2} + I_{t+3}$  that implicitly defines the depreciation rate  $\delta$  for the given year.

consumption goods  $e_t = \log E_t$  and the log of the relative price of durables in terms of nondurables  $q_t = \log Q_t$ , follows a vector autoregression of order 2, VAR(2). Using Splus commercial software (library FinMetrics) we test for cointegration between the series  $(n_t, e_t, q_t)$ . Table 3.2 presents the estimates of the cointegrated vector along with their standard errors.

### 3.4.2 Vector Error Correction Model

Since the vector of time series  $(n_t, e_t, q_t)$  follows a cointegrated VAR(2), the vector error correction model has the lag  $p = 2 - 1 = 1$ . Table 3.3 presents estimates of VECM(1) along with the standard errors.

We specify the evolution of nondurable consumption and the evolution of expenditure on durable consumption according to the estimated VECM model. In particular we also assume that the representative investor in our model forms expectations about the future level of nondurable consumption according to this model. Recall, that the stock of durables is a backward looking variable. Therefore, it is the forecast errors of this VECM model for the growth of nondurable consumption that affects the evolution of log surplus consumption ratio in our model (equation 3.4).

## 3.5 Asset-Pricing Implications

Having specified the exogenous evolution of nondurable consumption and expenditures on durables according to the VECM model we can proceed in this sub-section to the parameter estimation of our durable consumption model with habit formation. We estimate the parameters and test the asset pricing implications of the model using the GMM methodology. First, we estimate the model on the data for the average market return ( $N = 1$ ) and the risk-free rate. Then we also perform a cross-sectional test using the data for the set of 6 Fama-French portfolios ( $N = 6$ ).

We use an approach similar to Hansen and Singleton (1982). Our durable consumption model with habit formation described in section 3.2 defines a set of moment restrictions:

$$E \left[ \left( M_{t+1} R_t^f - 1 \right) z_t \right] = 0, \quad (3.10)$$

$$E \left[ M_{t+1} \left( R_{t+1}^i - R_t^f \right) z_t \right] = 0, \quad (3.11)$$

$$E \left[ \left( 1 - \frac{U_D}{U_N P_t} - (1 - \delta_t) M_{t+1} \frac{P_{t+1}}{P_t} \right) \right] = 0, \quad (3.12)$$

where  $z_t$  is the  $I \times 1$  vector of instrumental variables known at time  $t$ . The equation (3.10) represents  $I$  restrictions imposed by the Euler equation on the risk free rate. The equation (3.11) represents  $NI$  restrictions imposed by the Euler equation for  $N$  risky portfolio returns. In the time-series test, we use only the average market return ( $N = 1$ ), while in the cross-sectional test, we use 6 Fama-French portfolios sorted by size (market equity) and book to market equity ratio (BE/ME), i.e.,  $N = 6$ . Finally, the equation (3.12) represents the restriction imposed by the intratemporal first-order condition. Note that we do not scale the equation (3.12) by the vector of instruments. The reason is that we prefer to interpret the use of instruments as a creation of new assets managed with a linear rule according to the evolution of the instrument  $z_t$  similarly to Cochrane (2005). This intuition works well for managing portfolios of stocks or TBills, but cannot be so easily applied to the decision on the substitution between nondurable consumption and stock of durables. Furthermore, the stock of durables is a backward looking variable, evolving according to a well specified law of motion (3.2).

We assume that the surplus consumption ratio starts at its steady state at the beginning of the sample period. Therefore, we estimate only 4 parameters of the model  $(\alpha, \beta, \gamma, \phi)$  using the GMM methodology from a total of  $(N + 1)I + 1$  restrictions, which leaves us with  $(N + 1)I - 3$  overidentifying restrictions which can be tested by the J-test Hansen (1982). By assuming a particular aggregator of nondurables and stock of durables in the utility function (equation 3.1) and then including the intratemporal moment restriction in the estimation, we might be influencing the resulting estimate of the share of nondurables  $\alpha$ , therefore, we also run GMM estimations without the intratemporal moment condition (3.12).

We report the estimates from the standard two stage GMM (2SGMM) with the spectral density matrix used as the efficient weighting matrix in the second stage. Following the argumentation in Cochrane (2005), we also report the estimates from the first-stage GMM, in which we use a fixed weighting matrix. This has an advantage that it “avoids the potential trap of blowing up the standard errors instead of improving pricing errors”, and in general, leads to “parameter estimates that are more robust to small model misspecification” (Cochrane (2005), p.210). In particular, we use as a weighting matrix a diagonal matrix with inverses of variances of the respective returns on the diagonal. While this approach might lose some of the asymptotic efficiency, it focuses on economically more

significant measures of model fit than the standard 2SGMM.

### 3.5.1 Time Series Test

In the time series test, we estimate the model using the data on returns from the Average market Portfolio and the 3-month TBill. We report the first-stage GMM estimates in Table 3.4 and second-stage GMM estimates in Table 3.5. For both methods we report several estimations. We report the estimation which excludes the intratemporal moment condition. In this case, there are only 2 moment conditions for the two assets, and therefore, we use two instruments in order to identify the 4 estimated parameters. The instruments are a constant, the ratio of expenditures on durables and the stock of durables  $E_t/D_t$ , and the price-dividend ratios for the stock market  $P_t/DV_t$ . In Figure 3.2, we show the time evolution of these instruments. In this case, there are 4 parameters to be estimated from 6 moment restrictions, leaving us with 2 degrees of freedom. We also report the estimation which includes the non-scaled intratemporal moment condition. This additional condition allows us to use only two instruments. For the sake of completeness, we report three combinations of the above mentioned instruments.

When the intratemporal condition is not included among the moment conditions, the estimated weight of durables in the utility function is zero, i.e.,  $\alpha = 1$ . The reason for this is that the log growth of consumer durables is a more persistent and less volatile process (see Figure 3.3) which correlates less with the asset returns. In the absence of the intratemporal moment condition, which determines marginal rate of substitution between nondurables and durables, the estimation puts maximum weight on the more volatile nondurable consumption process. Therefore, these results correspond to the estimation of the original Campbell and Cochrane (1999) model.

However, it is a fact that a large share of an investor's consumption is derived from durable goods and therefore, a consumption based asset pricing model should take this into account. In the estimations with the intratemporal moment condition the parameter  $\alpha$  is estimated to be 0.92 in both first- and second- stage GMM estimations for all combinations of instruments.

The estimate of the subjective discount factor  $\beta$  is in the interval 0.9-0.92 with a relatively large standard error in the first-stage GMM estimation, while in the second-stage GMM estimation, it is 0.97. The reason for the difference is a higher estimate of the parameter  $\gamma$  in the first-stage. As you can see in Figure 3.4, which depicts the

Hansen-Jagannathan bounds, higher  $\gamma$  increases the mean and standard deviation of the stochastic discount factor. Higher gamma increases the precautionary saving term in (3.9) and puts downward pressure on  $\beta$ . The exception to the above point estimates of  $\beta$  is the case with intratemporal condition and instruments  $E_t/D_t$  and a constant. In this case the estimate of the persistence of the log surplus consumption process is lower. As shown in Figure 3.4, this increases the mean of the stochastic discount factor  $E(M)$  and creates further downward pressures on  $\beta$ .<sup>2</sup>

The estimates of the marginal utility curvature parameter  $\gamma$  is in the interval 10-13 and 1.4-2.3 in the first-stage GMM and second-stage GMM, respectively. Thanks to a time-varying surplus consumption ratio, we have a time-varying relative risk aversion (both are plotted in Figure 3.1). In the first-stage GMM estimation with all three instruments, the steady state relative risk aversion  $r\bar{r}a = \gamma/\bar{S}$  is 74 and 80 in cases without intratemporal condition and with this condition, respectively. In the second-stage GMM estimation, the steady state relative risk aversion is 32 and 34, respectively. Despite the inclusion of durable consumption goods in the utility function, which is much more persistent than the nondurable consumption, the increase in the relative risk aversion needed to fit the model is small.

The estimate of the persistence of the surplus consumption  $\phi$  is very high, in the interval 0.97-0.98, with the exception of the already mentioned estimation with intratemporal condition and instruments  $E_t/D_t$  and a constant. The estimates of the parameters  $\beta, \gamma, \phi$  in the case with durables and without durables are comparable.

### 3.5.2 Cross-Sectional Test

In the cross-sectional test, we estimate the model using the data on returns of 6 Fama-French portfolios and the 3-month TBill. We report the first-stage GMM estimates in Table 3.6 and second-stage GMM estimates in Table 3.7. With a higher number of assets, the number of moment restrictions is higher. Therefore, we do not need to use instruments to estimate the 4 parameters of the model. Again, we report the estimation which excludes the intratemporal moment condition and estimation including the intratemporal moment condition. There are 7 moment conditions in the former case, and 8 moment conditions in the latter.

---

<sup>2</sup>As shown in the Appendix 3.B, the sum of the habit term  $-\gamma(1-\phi)(s_t - \bar{s})$  and the precautionary savings term  $-\frac{\gamma^2\sigma^2}{2}\left(1 + \tilde{\lambda}(s_t)\right)^2$  in equation (3.9), can be expressed as  $\gamma(1-\phi)\psi/2$ . Higher  $\gamma$  or lower  $\phi$  increase this term and therefore depress  $\beta$ .

Similarly to the time series test, when the intratemporal condition is not included among the moment conditions, the estimated weight of durables in the utility function is zero, i.e.,  $\alpha = 1$ . This can be explained again by higher persistence and lower volatility of the log growth of consumer durables. In this case the results correspond to the estimation of the original Campbell and Cochrane (1999) model. In the estimations with the intratemporal moment condition, the parameter  $\alpha$  is estimated to be 0.92 in both first- and second- stage GMM consistently with the results in the time series test.

The estimate of the subjective discount factor  $\beta$  is in the interval 0.68-0.81 with large standard errors. These point estimates are much lower than in the time series test. The reason for this difference in the point estimate for  $\beta$  is a significantly higher estimate of the parameter  $\gamma$  in the first-stage GMM and a significantly lower estimate of  $\phi$  in the second-stage GMM compared to results in the time-series test.

The estimate of the parameter  $\gamma$  is in the interval 26.1-29.8 in the first-stage GMM with a steady state relative risk aversion 132-147, but is significantly lower for second-stage GMM: 1.2 for the case with intratemporal condition and even 0.173 in the case without this condition, implying the steady state relative risk aversion of 73 and 27 respectively. Such low levels of marginal utility curvature reduce the steady state relative risk aversion needed to fit the data. However, they have to be accompanied by lower persistence of the surplus consumption process, which allows for occasional extreme spikes in relative risk aversion that help to fit the data despite the low steady state relative risk aversion. Comparison of the relative risk aversion constructed on the parameter estimates from the first-stage and second-stage GMM is reported in Figure 3.5. The existence of extreme spikes in relative risk aversion is not plausible, therefore, in the case of the cross-sectional test, we believe that the above mentioned critique by Cochrane applies and we consider the estimation results from the first-stage GMM more plausible.

From the first-stage estimates, we can again conclude that the inclusion of a persistent stock of durables in the utility function requires a slightly higher curvature of the marginal utility  $\gamma$  and slightly higher level of the steady state relative risk aversion. In the cross-sectional test, we require a significantly higher curvature of the marginal utility  $\gamma$  and a higher level of the steady state relative risk aversion compared to the time-series test. The estimates of  $\alpha$  and  $\phi$  is comparable to the time-series test. The point estimates of the subjective discount factor  $\beta$  are lower than in the time-series test, which is due to a larger precautionary saving motive (with higher  $\gamma$ ). However, the standard error is very large so using the standard t-test, we cannot reject the hypothesis that the parameter  $\beta$



is the same in the time-series test and the cross-sectional test.

## 3.6 Conclusion

We derived an analytically tractable model which blends two prominent strands of literature on asset pricing: literature on external habit formation and literature stressing the importance of durable consumption in the investors' consumption portfolio. Our model is a generalization of Campbell and Cochrane's (1999) model in which investors derive utility from both nondurable goods and service flows from consumer durable goods.

We estimate this highly nonlinear model first without the intratemporal moment condition, which collapses to the original model of Campbell and Cochrane (1999), then we estimate our generalized model with a positive weight of durables in the utility function. We report results of the first- and second-stage GMM estimations for the average market portfolio as well as for the universe of 6 Fama-French portfolios and the risk-free rate. The inclusion of more persistent and less volatile durables in the utility function requires only slightly higher relative risk aversion to fit the data, which is lower than in the model of durables without habit by Yogo (2006). The estimated weight of the durables in the utility function is consistently estimated at 0.08 in both time-series and cross-sectional tests.

## 3.7 Acknowledgments

The financial support of the Czech Science Foundation (grant no. P403/12/1394) is gratefully acknowledged. Remaining errors are solely our own responsibility.

### 3.A Appendix 3.A: Derivation of the Mean and Variance of the Stochastic Discount Factor

The log stochastic discount factor is:

$$m_{t+1} = \log(\beta) + ((1-\gamma)\alpha - 1)\mu_t^n + (1-\gamma)(1-\alpha)\mu_t^d - \gamma(1-\phi)(\bar{s} - s_t) + ((1-\gamma)\alpha - 1 - \gamma\alpha\lambda(s_t))\varepsilon_n.$$

It is straightforward to take expectations and obtain the mean of the stochastic discount factor:

$$E_t(m_{t+1}) = \log(\beta) + ((1-\gamma)\alpha - 1)\mu_t^n + (1-\gamma)(1-\alpha)\mu_t^d - \gamma(1-\phi)(\bar{s} - s_t)$$

and the formula for the variance of the stochastic discount factor:

$$\begin{aligned} Var_t(m_{t+1}) &= ((1-\gamma)\alpha - 1 - \gamma\alpha\lambda(s_t))^2 \sigma_n^2 \\ &= ((1-\gamma)\alpha - 1)^2 \left(1 + \frac{\alpha\gamma\lambda(s_t)}{(\gamma-1)\alpha + 1}\right)^2 \sigma_n^2 \\ &= \gamma^2 \left(1 + \tilde{\lambda}(s_t)\right)^2 \sigma^2, \end{aligned}$$

where  $\sigma^2 = \frac{1}{\gamma^2} ((1-\gamma)\alpha - 1)^2 \sigma_n^2$  and  $\tilde{\lambda}(s_t) = \frac{\alpha\gamma}{(\gamma-1)\alpha + 1} \lambda(s_t)$ . Note that this can be considered as a generalization of Campbell and Cochrane (1999). If the weight of durables in the utility function is set to zero ( $\alpha = 1$ ), we obtain  $\sigma^2 = \sigma_n^2$  and  $\tilde{\lambda}(s_t) = \lambda(s_t)$  and the above formulas collapse to their form in Campbell and Cochrane (1999).

### 3.B Appendix 3.B: Derivation of the Scaled Sensitivity Function $\tilde{\lambda}(s_t)$

We use similar assumptions to Campbell and Cochrane (1999) to specify the functional form of the scaled sensitivity function  $\tilde{\lambda}(s_t)$ :

1. We reduce the volatility of the risk-free rate by making it independent on the surplus consumption ratio  $s_t$

$$r_t^f(\mu_t^n, \mu_t^d) = r_t^f(\mu_t^n, \mu_t^d, s_t) \quad \forall s_t;$$

2. Habit moves non-negatively with both consumption of nondurables and stock of durables

$$\frac{dx}{dn} \geq 0, \quad \frac{dx}{dd} \geq 0;$$

3. The habit is predetermined at the steady state

$$\frac{dx}{dn} \Big|_{s=\bar{s}} = 0; \text{ and}$$

$$\frac{d}{ds} \left( \frac{dx}{dn} \right) \Big|_{s=\bar{s}} = \frac{d}{ds} \left( \frac{dx}{dd} \right) \Big|_{s=\bar{s}} = \frac{d}{ds} \left( \frac{dx}{dc} \right) \Big|_{s=\bar{s}} = 0.$$

Those three conditions uniquely determine the functional form for the scaled sensitivity function  $\tilde{\lambda}(s_t)$  and for the steady state surplus consumption ratio  $\bar{S}$ .

### Condition 1

For the risk-free rate to be independent on the surplus consumption ratio, we require

$$\begin{aligned} r_t^f(\mu_t^n, \mu_t^d) &= r_t^f(\mu_t^n, \mu_t^d, s_t) \quad \forall s_t \Leftrightarrow \gamma(1-\phi)(\bar{s}-s_t) - \frac{1}{2}\gamma^2(1+\tilde{\lambda}(s_t))^2\sigma^2 = const \\ (1+\tilde{\lambda}(s_t))^2 &= \frac{2}{\gamma^2\sigma^2}[-const - \gamma(1-\phi)(s_t-\bar{s})] \\ &= \frac{(1-\phi)}{\gamma\sigma^2}[\psi - 2(s_t-\bar{s})], \text{ where } \psi = -\frac{2 \cdot const}{\gamma(1-\phi)}. \end{aligned}$$

This leads us to the specification of the scaled sensitivity function  $\tilde{\lambda}(s_t)$

$$\begin{aligned} \tilde{\lambda}(s_t) &= \frac{1}{\sigma} \sqrt{\frac{1-\phi}{\gamma}(\psi - 2(s_t-\bar{s}))} - 1, \quad s_t \leq s_{max} \\ &= 0 \quad \quad \quad s_t \geq s_{max}. \end{aligned} \tag{3.13}$$

This expression satisfies the additional condition  $\lambda(s_t) \geq 0$ . To this end  $s_{max}$  is defined as the value of  $s_t$  for which  $\lambda(s_t) = \tilde{\lambda}(s_t) = 0$ :

$$s_{max} = \bar{s} + \frac{1}{2} \left( \psi - \sigma^2 \frac{\gamma}{1-\phi} \right).$$

Note that for  $\psi = 1$ , which, as shown later, arises if  $\alpha = 1$ , we obtain the same functional form as in Campbell and Cochrane (1999).

### Condition 2

In order for habit to move non-negatively with both consumption of durables and non-durables  $\left( \frac{dx_{t+1}}{dn_{t+1}} \geq 0, \quad \frac{dx_{t+1}}{dd_{t+1}} \geq 0 \right)$ , which would automatically imply that habit moves non-negatively with the aggregated final consumption stream  $\left( \frac{dx_{t+1}}{dc_{t+1}} \geq 0 \right)$ , we require

$$\frac{dx_{t+1}}{dn_{t+1}} = \alpha \left( 1 - \frac{\lambda(s_t)}{e^{-s_{t+1}} - 1} \right) \geq 0, \tag{3.14}$$

and

$$\frac{dx_{t+1}}{dd_{t+1}} = 1 - \alpha \geq 0. \tag{3.15}$$

The condition (3.15) is always satisfied since  $\alpha \in (0, 1)$ . Condition (3.14) is satisfied if Condition 3 holds. This is due to the fact that function  $\frac{dx}{dn}$  is convex in  $s$ , therefore, if we set its minimum to the steady state, where the effect of nondurables on habit is zero, then habit will move non-negatively with the consumption of nondurables for all defined levels of log surplus consumption ratio  $s_t$ .

### Condition 3

For habit to be predetermined we require that  $\frac{dx}{dn} |_{s=\bar{s}} = 0$ . This condition applied to (3.14) implies that

$$\lambda(s_t) = \frac{1}{\bar{S}} - 1. \quad (3.16)$$

We also require that the effect of both non-durable and durable consumption on habit attains a minimum at the steady state:

$$\frac{d}{ds} \left( \frac{dx}{dn} \right) |_{s=\bar{s}} = \frac{d}{ds} \left( \frac{dx}{dd} \right) |_{s=\bar{s}} = \frac{d}{ds} \left( \frac{dx}{dc} \right) |_{s=\bar{s}} = 0. \quad (3.17)$$

Since  $\frac{d}{ds} \left( \frac{dx}{dd} \right) |_{s=\bar{s}} = 0$  and  $\frac{d}{ds} \left( \frac{dx}{dc} \right) |_{s=\bar{s}} = 0$  is the linear combination of  $\frac{d}{ds} \left( \frac{dx}{dn} \right) |_{s=\bar{s}}$  and  $\frac{d}{ds} \left( \frac{dx}{dd} \right) |_{s=\bar{s}}$ , to satisfy the condition (3.17) it is sufficient to require

$$\frac{d}{ds} \left( \frac{dx}{dn} \right) |_{s=\bar{s}} = \frac{d}{ds} \left( \alpha + \frac{\alpha \lambda(s_t)}{1 - e^{-s_t+1}} \right) |_{s=\bar{s}} = 0,$$

which implies

$$\lambda'(\bar{s}) \left( 1 - \frac{1}{\bar{S}} \right) = \lambda(\bar{s}) \frac{1}{\bar{S}}. \quad (3.18)$$

Further transforming this condition we obtain the functional form for the steady state level of surplus consumption ratio  $\bar{S}$ . Taking the derivative of equation (3.16) and (3.13), we obtain  $\lambda'(\bar{s}) = -\frac{1}{\bar{S}}$ , and  $\tilde{\lambda}'(\bar{s}) = -\frac{1}{\sigma} \sqrt{\frac{1-\phi}{\gamma\psi}}$  and  $\lambda'(\bar{s}) = -\frac{(\gamma-1)\alpha+1}{\sigma\gamma} \sqrt{\frac{1-\phi}{\gamma\psi}}$ , respectively. Substituting those equations into (3.18), we obtain

$$\frac{(\gamma-1)\alpha+1}{\sigma\alpha\gamma} \sqrt{\frac{1-\phi}{\gamma\psi}} = -\frac{1}{\bar{S}},$$

which gives us

$$\bar{S} = \frac{\alpha\gamma\sigma}{(\gamma-1)\alpha+1} \sqrt{\frac{\gamma}{1-\phi}} \psi.$$

Again note that for  $\psi = 1$  and  $\alpha = 1$  we would obtain the same condition as in Campbell and Cochrane (1999), i.e.,  $\bar{S} = \sigma \sqrt{\frac{\gamma}{1-\phi}}$ .

The scaled sensitivity function  $\tilde{\lambda}(s_t)$  can be rewritten also in terms of  $\bar{S}$

$$\begin{aligned} \tilde{\lambda}(s_t) &= \frac{\alpha\gamma\sqrt{\psi}}{(\gamma-1)\alpha+1} \frac{1}{\bar{S}} \sqrt{\psi - 2(s_t - \bar{s})} - 1, \quad s_t \leq s_{max} \\ &= 0 \quad s_t \geq s_{max}. \end{aligned}$$

Finally, we can derive the constant  $\psi$  by combining the condition (3.16) and (3.13):

$$\lambda(\bar{s}) = \frac{1}{\bar{S}} - 1 = \frac{(\gamma-1)\alpha+1}{\alpha\gamma} \left( \frac{1}{\sigma} \sqrt{\frac{1-\phi}{\gamma}} \psi - 1 \right)$$

$$\frac{\alpha\gamma}{(\gamma-1)\alpha+1} \left( \frac{(\gamma-1)\alpha+1}{\alpha\gamma\sigma} \sqrt{\frac{1-\phi}{\gamma\psi}} - 1 \right) = \frac{1}{\sigma} \sqrt{\frac{1-\phi}{\gamma}} \psi - 1$$

$$\frac{1}{\sigma} \sqrt{\frac{1-\phi}{\gamma\psi}} - \frac{\alpha\gamma}{(\gamma-1)\alpha+1} = \frac{1}{\sigma} \sqrt{\frac{1-\phi}{\gamma}} \psi - 1.$$

This can be transformed into a quadratic function in  $\sqrt{\psi}$

$$\psi^{\frac{2}{\sigma}} - \frac{(1-\alpha)\sigma}{(\gamma-1)\alpha+1} \sqrt{\frac{\gamma}{1-\phi}} \psi^{\frac{1}{\sigma}} - 1 = 0.$$

Since  $\sqrt{\psi} > 0$ , there is only one admissible solution

$$\psi = \left( \frac{-B + \sqrt{B^2 + 4}}{2} \right)^2,$$

where  $B = -\frac{(1-\alpha)\sigma}{(\gamma-1)\alpha+1} \sqrt{\frac{\gamma}{1-\phi}} = -\frac{1-\alpha}{\alpha\gamma} \bar{S} > 0$ . Note that if  $\alpha = 1$ , then we obtain  $B = 0$ , and therefore,  $\psi = 1$  and we would obtain the formulas equal to those in the Campbell and Cochrane (1999).

### 3.C Appendix 3.C: Data construction

Our measure of the stock of consumer durable goods includes:

- motor vehicles (autos, light trucks, motor vehicle parts and accessories);
- furnishings and durable household equipment (furniture and furnishings such as furniture, clocks, lamps, lighting fixtures, and other household decorative items, carpets and other floor coverings, window coverings, household appliances such as major household appliances and small electric household appliances except built-in appliances (which are classified as part of residential structures), glassware, tableware, and household utensils (such as dishes, flatware, non-electric cookware and tableware tools and equipment for house and garden));
- recreational goods (video, audio, photographic, and information processing equipment and media, sporting equipment, supplies, guns, and ammunition, recreational books, musical instruments);
- sports and recreational vehicles such as motorcycles, bicycles and accessories, pleasure boats, aircraft, and other recreational vehicles);
- and other durable goods (jewellery and watches, therapeutic appliances and equipment, educational books, luggage and similar personal items, telephone and facsimile equipment).

The corresponding annual quantity index for the period 1952–2011 is from line 1 of Table 8.2 (Chain-Type Quantity Indexes for Net Stock of Consumer Durable Goods).

**Table 3.1:** Tests for the Cointegration Rank <sup>a</sup>

Panel A: Cointegration between $\log D_t$ and $\log E_t$							
Cointegration Rank	Test Statistic and Critical Values						
	Eigenvalue	Trace	95% CV	99% CV	Max	95% CV	99% CV
$H(0)^{+++}$	0.192	50.417	15.410	20.040	50.400	14.070	18.630
$H(1)$	0.000	0.016	3.760	6.650	0.016	3.760	6.650

Panel B: Cointegration between $\log C_t$ , $\log E_t$ and $\log Q_t$							
Cointegration Rank	Test Statistic and Critical Values						
	Eigenvalue	Trace	95% CV	99% CV	Max	95% CV	99% CV
$H(0)^{+++}$	0.106	37.235	29.680	35.650	26.560	20.970	25.520
$H(1)$	0.043	10.675	15.410	20.040	10.326	14.070	18.630
$H(2)$	0.002	0.349	3.760	6.650	0.349	3.760	6.650

<sup>a</sup> Trace tests significant at the 5% level are flagged by "+". Trace tests significant at the 1% level are flagged by "++". Max Eigenvalue tests significant at the 5% level are flagged by "\*". Max Eigenvalue tests significant at the 1% level are flagged by "\*\*". Sample period 1952:I–2011:IV.

**Table 3.2:** Estimated Cointegrating Vectors

	Cointegrating Vectors		
	$\log Q_t$	$\log E_t$	$\log E_t$
Cointegrated VAR(1)			
$\log C_t$	-0.456	-1.035	-1.036
(S.E.)	(0.031)	(0.008)	(0.007)
		Cointegrated VAR(2)	
		$\log D_t$	
		(S.E.)	(S.E.)

<sup>a</sup> Maximum number of lags allowed in each test is 12. None of the tests is significant at 5 percent level. Sample period 1952:I–2011:IV.

**Table 3.3:** Estimated Vector Error Correction Model

Time Series	Intercept	$\log C_t - \theta \log Q_t - \eta \log E_t$	$\Delta \log C_t$	$\Delta \log E_t$	$\Delta \log Q_t$	$R^2$
$\Delta \log C_{t+1}$	-0.208	-0.042	0.121	0.024	0.077	0.084
(S.E.)	(0.091)	(0.018)	(0.069)	(0.016)	(0.043)	
[t-stats]	[-2.282]	[-2.313]	[1.767]	[1.480]	[1.791]	
$\Delta \log E_{t+1}$	0.610	0.119	1.267	-0.082	0.285	0.101
(S.E.)	(0.397)	(0.078)	(0.300)	(0.070)	(0.188)	
[t-stats]	[1.536]	[1.514]	[4.219]	[-1.173]	[1.518]	
$\Delta \log Q_{t+1}$	0.397	0.079	-0.206	0.059	0.281	0.140
(S.E.)	(0.133)	(0.026)	(0.100)	(0.024)	(0.063)	
[t-stats]	[2.990]	[3.013]	[-2.051]	[2.513]	[4.484]	

<sup>a</sup> Asymptotic standard errors are in parentheses. The log-likelihood function attained is 2099.746. Sample period 1952:I–2011:IV.



**Table 3.4:** First-Stage GMM: Time Series Test <sup>a</sup>

Moment conditions	No Intratemporal Cond.		Intratemporal Cond. (Not Scaled)	
	$\left\{1, \frac{E_t}{D_t}, \frac{P_t}{DV_t}\right\}$ Estimate (S.E.)	$\left\{1, \frac{E_t}{D_t}, \frac{P_t}{DV_t}\right\}$ Estimate (S.E.)	$\left\{1, \frac{E_t}{D_t}\right\}$ Estimate (S.E.)	$\left\{1, \frac{P_t}{DV_t}\right\}$ Estimate (S.E.)
Preference Parameter				
$\alpha$	1.000 (0.011)	0.920 (0.002)	0.922 (0.003)	0.920 (0.002)
$\beta$	0.903 (0.103)	0.907 (0.100)	0.628 (0.002)	0.920 (0.074)
$\gamma$	13.273 (0.0004)	11.499 (0.004)	13.064 (5.821)	9.988 (0.0003)
$\phi$	0.979 (0.020)	0.976 (0.022)	0.929 (0.013)	0.976 (0.019)
Specification Test				
$J_T$	8.682 (0.013)	9.165 (0.027)	2.371 (0.124)	7.935 (0.005)

<sup>a</sup> The first-stage GMM weighting matrix is diagonal with the elements equal to the inverse of the variances of tested asset returns. HAC standard errors with ARMA(1,1) prewhitening used. The test assets used are the Average Market Portfolio and the 3-month TBill. Sample period is 1952:IV–2011:IV.

**Table 3.5:** Second-Stage GMM: Time Series Test <sup>a</sup>

Moment conditions	No Intratemporal Cond.		Intratemporal Cond. (Not Scaled)	
	$\left\{1, \frac{E_t}{D_t}, \frac{P_t}{DV_t}\right\}$ Estimate (S.E.)	$\left\{1, \frac{E_t}{D_t}, \frac{P_t}{DV_t}\right\}$ Estimate (S.E.)	$\left\{1, \frac{E_t}{D_t}\right\}$ Estimate (S.E.)	$\left\{1, \frac{P_t}{DV_t}\right\}$ Estimate (S.E.)
Preference Parameter				
$\alpha$	1.000 (6.103)	0.921 (0.002)	0.923 (0.007)	0.922 (0.002)
$\beta$	0.968 (0.032)	0.968 (0.016)	0.646 (0.729)	0.970 (0.011)
$\gamma$	1.560 (4.113)	1.510 (4.634)	2.337 (4.876)	1.378 (10.052)
$\phi$	0.966 (0.069)	0.967 (0.071)	0.757 (0.258)	0.978 (0.153)
Specification Test				
$J_T$	22.521 (0.000)	22.871 (0.000)	2.032 (0.154)	25.739 (0.000)

<sup>a</sup> HAC standard errors with ARMA(1,1) prewhitening used. The test assets used are the Average Market Portfolio and the 3-month TBill. Sample period is 1952:IV–2011:IV.

**Table 3.6:** First-Stage GMM: Cross-Sectional Test

Moment conditions	No Intratemporal Cond.		Intratemporal Cond.	
	Estimate	(S.E.)	Estimate	(S.E.)
Preference Parameter				
$\alpha$	1.000	(0.016)	0.921	(0.003)
$\beta$	0.677	(0.285)	0.689	(0.312)
$\gamma$	26.078	(0.0005)	29.844	(0.0007)
$\phi$	0.966	(0.032)	0.969	(0.030)
Specification Test				
$J_T$	30.916	(0.000)	30.881	(0.000)

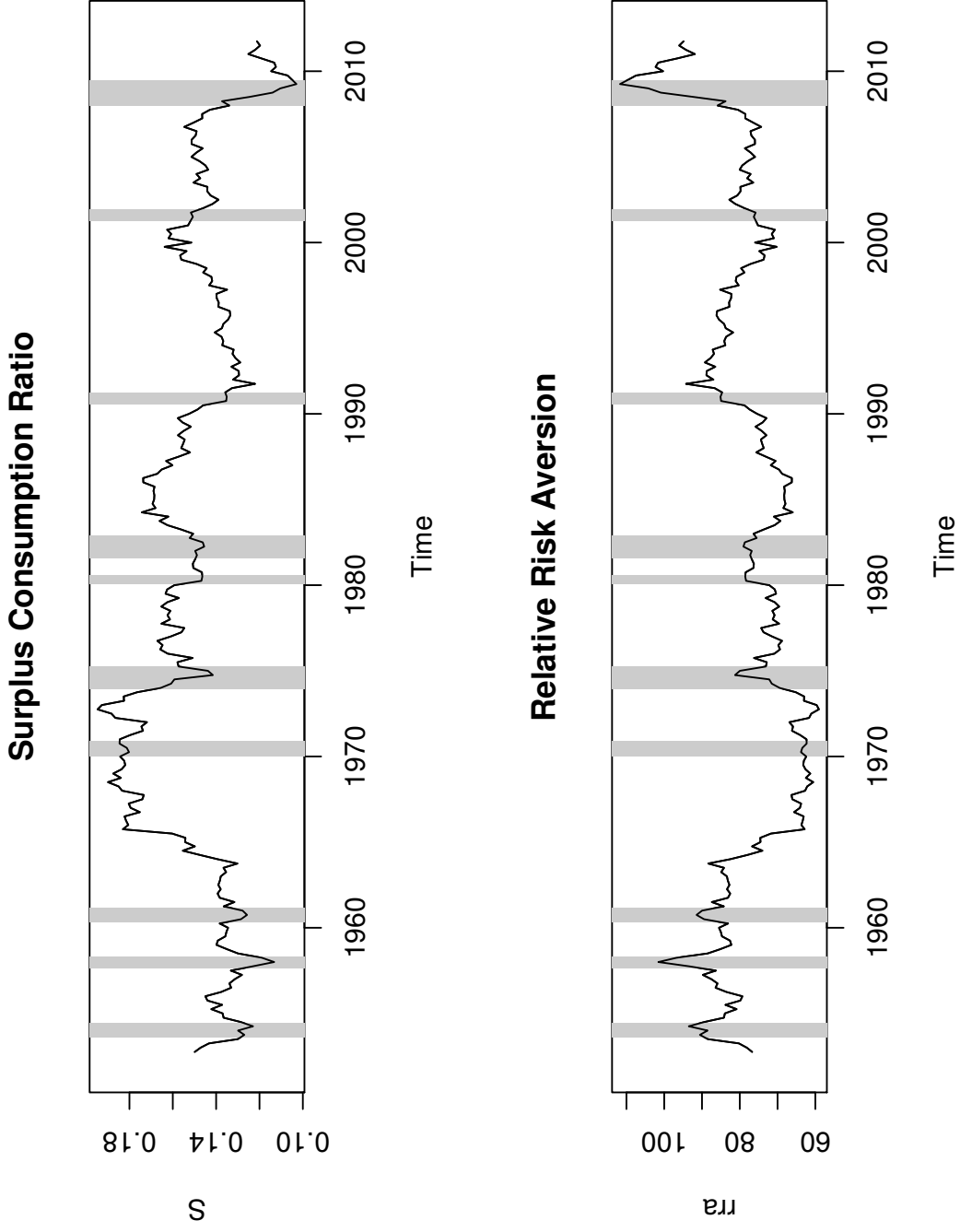
<sup>a</sup> The first-stage GMM weighting matrix is diagonal with the elements equal to the inverse of the variances of tested asset returns. HAC standard errors with ARMA(1,1) prewhitening used. The test assets used are the 6 Fama-French Portfolios and the 3-month TBill. Sample period is 1952:IV–2011:IV.

**Table 3.7:** Second-Stage GMM: Cross-Sectional Test <sup>a</sup>

Moment conditions	No Intratemporal Cond.		Intratemporal Cond.	
	Estimate	(S.E.)	Estimate	(S.E.)
Preference Parameter				
$\alpha$	1.000	(0.155)	0.923	(0.002)
$\beta$	0.809	(0.021)	0.757	(0.092)
$\gamma$	0.173	(0.014)	1.171	(1.884)
$\phi$	0.783	(0.028)	0.804	(0.139)
Specification Test				
$J_T$	132.22	(0.000)	36.999	(0.000)

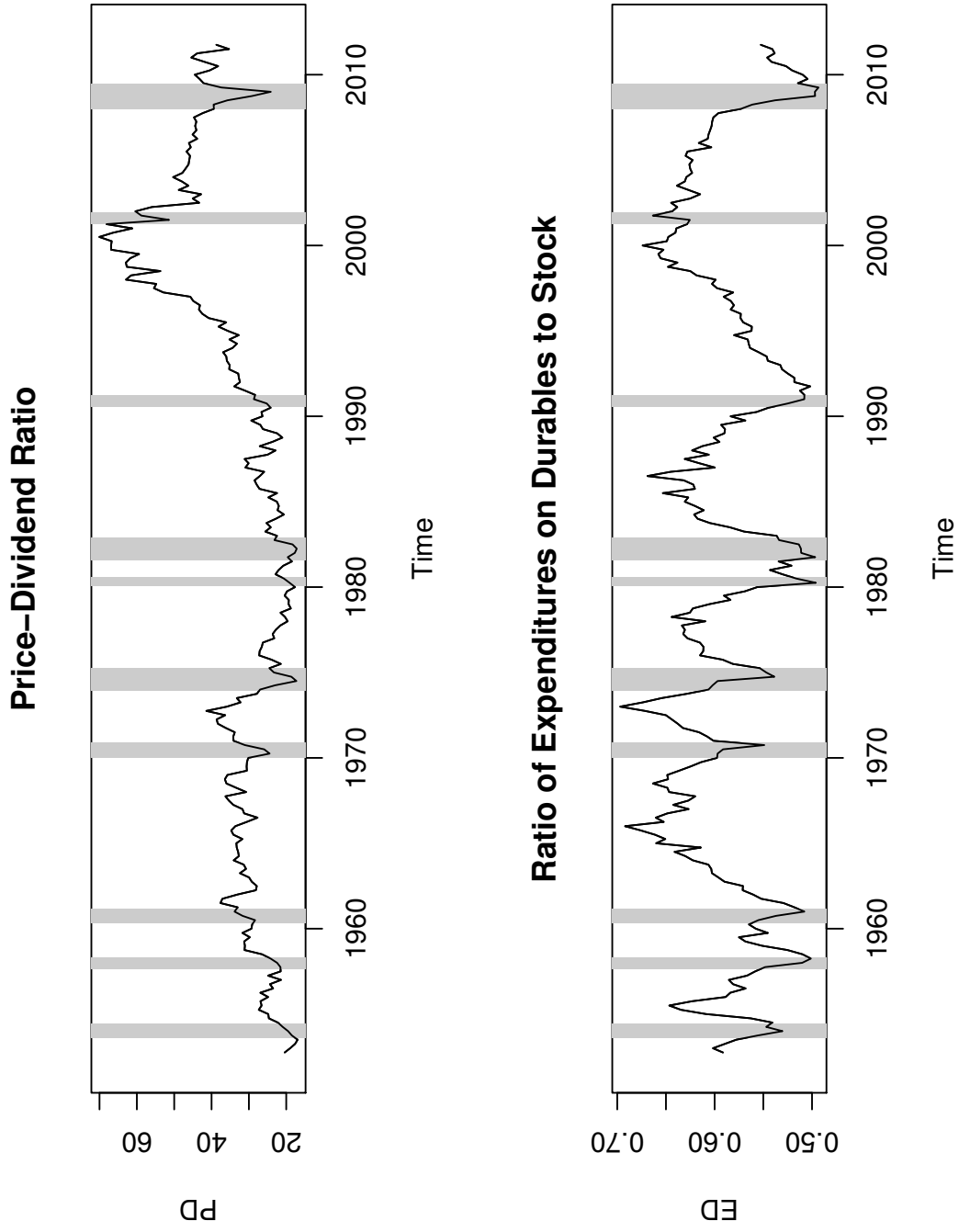
<sup>a</sup> HAC standard errors with ARMA(1,1) prewhitening used. The test assets used are the 6 Fama-French Portfolios and the 3-month TBill. Sample period is 1952:IV–2011:IV.

Figure 3.1: Surplus Consumption Ratio and Relative Risk Aversion



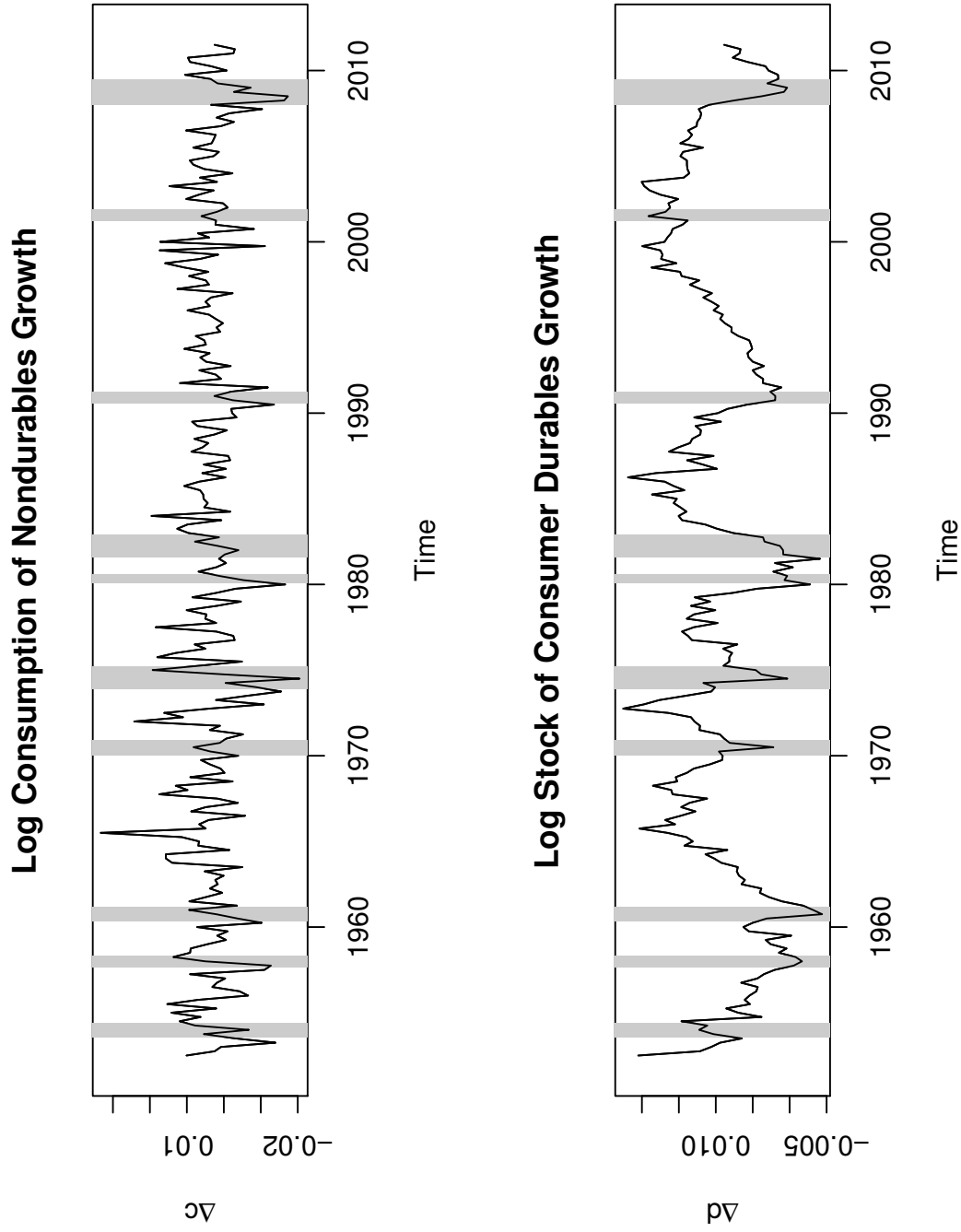
Notes: The plotted surplus consumption ratio  $S_t$  is constructed based on first-stage GMM estimates of parameters from the time series test reported in Table 3.4. Relative risk aversion computed by  $rra_t = \gamma/S_t$ . Sample period is 1952.IV – 2011.IV. The shaded time intervals refer to recessions in the U.S. as identified by the National Bureau of Economic Research.

Figure 3.2: Instruments



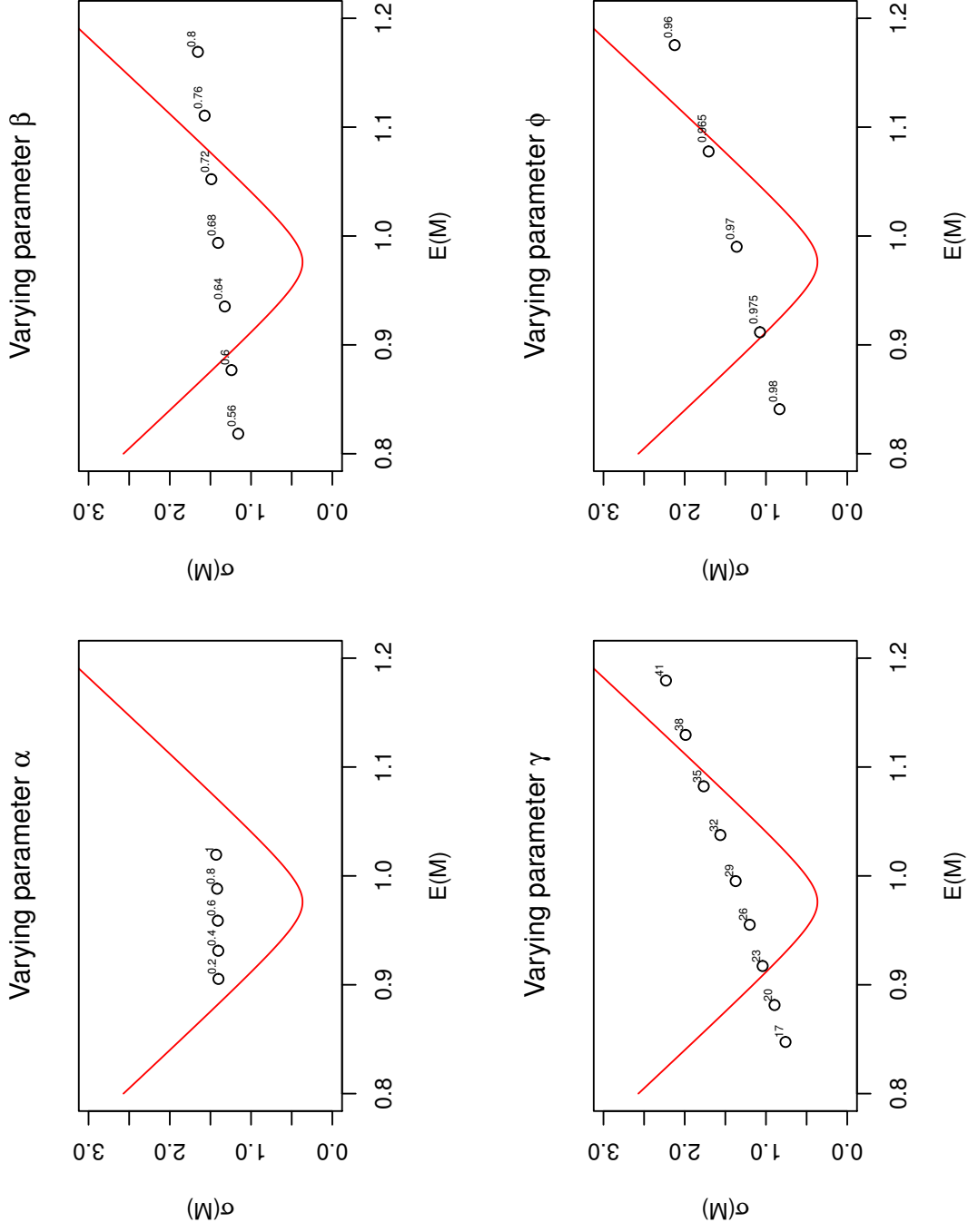
Notes: The graph shows the time evolution of the instruments used in a subset of estimations. Sample period is 1952.IV – 2011.IV. The shaded time intervals refer to recessions in the U.S. as identified by the National Bureau of Economic Research.

**Figure 3.3:** Log Consumption of Nondurables and Log Stock of Consumer Durables



Notes: Sample period is 1952:IV – 2011:IV. The shaded time intervals refer to recessions in the U.S. as identified by the National Bureau of Economic Research.

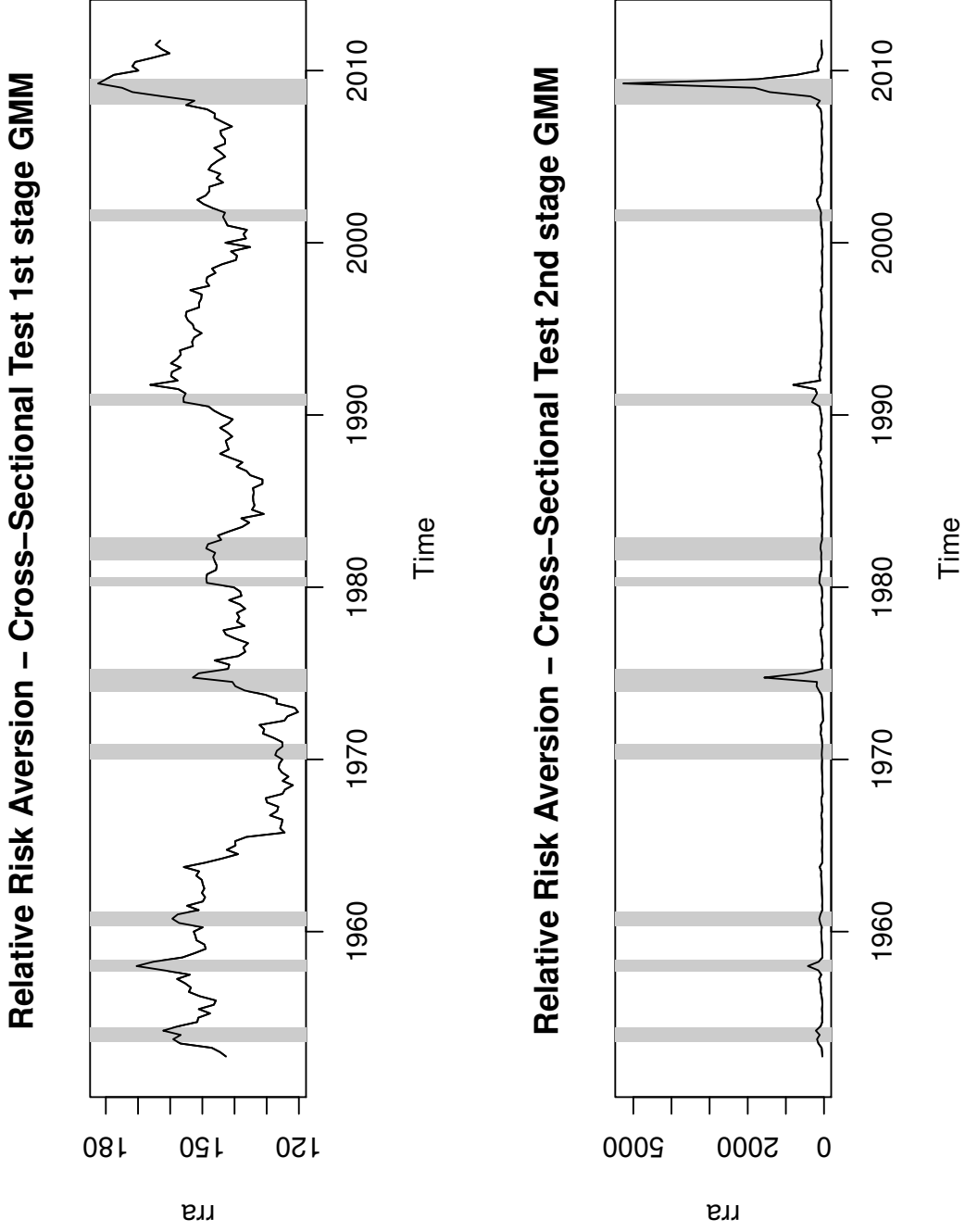
Figure 3.4: Hansen-Jagannathan Bounds



Notes: Graphs show the Hansen-Jagannathan bounds constructed for the 6 Fama-French portfolios. We report the effect of changing individual parameters on the mean and standard deviation of the stochastic discount factor. During this exercise only a single parameter is being changed, other parameters are kept at the second stage GMM point estimates of the Cross-Sectional test reported in Table 3.6.



Figure 3.5: Relative Risk Aversion in the Cross-Sectional Test



Notes: Graphs show the relative risk aversion constructed for estimated parameters in the cross-sectional test. The upper panel corresponds to parameters reported in Table 3.6 in case of the estimation with the intratemporal condition. The lower panel corresponds to the parameters reported in Table 3.7 also in case of the estimation with the intratemporal condition.



---

## Bibliography

- Acharya, V., and P. Schnabl. 2009. “Do Global Banks spread Global Imbalances? The Case of Asset-Backed Commercial Paper during the Financial Crisis of 2007-09.” *Working Paper, New York University*.
- Adrian, T., and A. Ashcraft. 2012. “Shadow Banking Regulation.” *Annual Review of Financial Economics* 4 (1): 99–140.
- Adrian, T., and H. Shin. 2009. “Money, liquidity, and monetary policy.” *American Economic Review: Papers and Proceedings* 99 (2): 600–605.
- Akerlof, G. 1970. “The market for “lemons”: Quality uncertainty and the market mechanism.” *The Quarterly Journal of Economics* 84 (3): 488–500.
- Almazan, A., A. Martín-Oliver, and J. Saurina. 2013. Loan securitization, access to funds and banks’ capital structures. Mimeo.
- Arora, S., B. Barak, M. Brunnermeier, and R. Ge. 2012. Computational Complexity and Information Asymmetry in Financial Products. Mimeo, Princeton University.
- Bernanke, B. 2010. Economic challenges: Past, present and future. A speech at the Dallas Regional Chamber, Dallas, Texas, April 7, 2010.
- Bernanke, B., and M. Gertler. 1989. “Agency costs, net worth, and business fluctuations.” *American Economic Review* 79 (1): 14–31.
- Bernanke, B., M. Gertler, and S. Gilchrist. 1999. Chapter 21 of *The financial accelerator in a quantitative business cycle framework*, Volume 1 of *In: Taylor, J. and Woodford, M. (Eds.), Handbook of Macroeconomics*, 1, 1341–1393. Elsevier.
- Bigio, S. 2013. Endogenous liquidity and the business cycle. Mimeo. Columbia Business School.
- Bloom, N. 2009. “The impact of uncertainty shocks.” *Econometrica* 77 (3): 623–685.
- Bloom, N., M. Floetotto, N. Jaimovich, I. Sapora-Eksten, and S. Terry. 2012. “Really uncertain business cycles.” *NBER Working Papers*, no. 18245.
- Boissay, F., F. Collard, and F. Smets. 2013. “Booms and systemic banking crises.” *Working Paper Series 1514, European Central Bank*.
- Brunnermeier, M. 2009. “Deciphering the liquidity and credit crunch 2007-2008.” *Journal of Economic Perspectives* 23 (1): 77–100.

- Brunnermeier, M., and Y. Sannikov. 2014. "A macroeconomic model with a financial sector." *American Economic Review*, no. forthcoming.
- Calomiris, C., and J. Mason. 2004. "Credit card securitization and regulatory arbitrage." *Journal of Financial Services Research* 26 (1): 5–27.
- Campbell, John Y., and John H. Cochrane. 1999. "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior." *Journal of Political Economy* 107 (2): 205–251.
- Centorelli, N., and S. Peristiani. 2012. "The Role of Banks in Asset Securitization." *FRBNY Economic Policy Review* 18 (2): 47–63.
- Cho, I.-K., and D. M. Kreps. 1987. "Signaling Games and Stable Equilibria." *The Quarterly Journal of Economics* 102 (2): 179–222.
- Cochrane, John H. 2005. *Asset Pricing*. Princeton University Press, Revised Edition, Princeton NJ.
- Constantinides, George M. 1990. "Habit Formation: A Resolution of the Equity Premium Puzzle." *Journal of Political Economy* 98 (3): 519–543.
- Costa, O. L. V., M. D. Fragoso, and R.P. Marques. 2005. *Discrete-time Markov jump linear systems*. Springer.
- Dell’Ariccia, G., and R. Marquez. 2006. "Lending booms and lending standards." *The Journal of Finance* 61 (5): 2511–2546.
- Demyanyk, Y., and O. Van Hemert. 2011. "Understanding the subprime mortgage crisis." *Review of Financial Studies* 24 (6): 1848–1880.
- Efing, M., and H. Hau. 2013. "Structured Debt Ratings: Evidence on Conflicts of Interest." *Swiss Finance Institute Research Paper*, no. 13-21.
- Fender, I., and J. Mitchell. 2009. "Incentives and tranche retention in securitization: A screening model." *BIS Working Paper* 289:1–41.
- Foerster, A., J. Rubio-Ramírez, D. F. Waggoner, and T. Zha. 2013. "Perturbation methods for Markov-switching DSGE models." *Federal Reserve Bank of Atlanta Working Paper Series*, no. 2013-1.
- Gennaioli, N., A. Shleifer, and R. W. Vishny. 2013. "A Model of Shadow Banking." *The Journal of Finance* 68 (4): 1331–1363.
- Gertler, M., and P. Karadi. 2011. "A model of unconventional monetary policy." *Journal of Monetary Economics* 58 (1): 17–34.
- Gertler, M., and N. Kiyotaki. 2010. Chapter 11 of *Financial intermediation and credit policy in business cycle analysis*, Volume 3 of *In: Friedman, B. and Woodford, M. (Eds.), Handbook of Monetary Economics*, 1, 547–599. Elsevier.
- Gorton, G., and A. Metrick. 2010. "Regulating the shadow banking system." *Brookings Papers on Economic Activity* 41 (2): 261–312.
- Gorton, G., and G. Ordoñez. 2014. "Collateral crises." *American Economic Review* 104 (2): 343–378.

- Gorton, G., and G. Pennacchi. 1995. "Banks and loan sales: Marketing nonmarketable assets." *Journal of Monetary Economics* 35 (3): 389–411.
- Gorton, G., and N. Souleles. 2006. Pages 549–602 in *Special purpose vehicles and securitization*, In: Carey, M. and Stulz, R. (Eds.), The risks of financial institutions. University of Chicago Press, Chicago.
- Hansen, Lars Peter. 1982. "Large sample properties of generalized method of moments estimators." *Econometrica* 50:1029–1054.
- Hansen, Lars Peter, and Ravi Jagannathan. 1991. "Implications of Security Market Data for Models of Dynamic Economies." *Journal of Political Economy* 99 (2): 225–262.
- Hansen, Lars Peter, and Kenneth J. Singleton. 1982. "Generalized instrumental variables estimation of nonlinear rational expectations models." *Econometrica* 50:1269–1286.
- Heider, F., M. Hoerova, and C. Holthausen. 2009. "Liquidity hoarding and interbank market spreads: The role of counterparty risk." *Working Paper Series 1126, European Central Bank*.
- Higgins, E., and J. Mason. 2004. "What is the value of recourse to asset-backed securities? A clinical study of credit card banks." *Journal of Banking and Finance* 28 (4): 875–899.
- Jordà, Òscar, Moritz HP Schularick, and Alan M Taylor. 2013. "Sovereigns versus banks: credit, crises, and consequences." Technical Report, National Bureau of Economic Research.
- Kiyotaki, N., and J. Moore. 1997. "Credit cycles." *Journal of Political Economy* 105 (2): 211–248.
- . 2012. "Liquidity, business cycles and monetary policy." *NBER Working Papers*, no. 17934.
- Kuncl, M. 2013. Securitization under asymmetric information and macroprudential regulation. Mimeo. European Central Bank.
- . 2014. "Securitization under asymmetric information over the business cycle." *CERGE-EI Working Paper Series*, no. 506.
- Kurlat, P. 2013. "Lemons Markets and the Transmission of Aggregate Shocks." *American Economic Review* 103 (4): 1463–1489.
- Lown, C., and D. Morgan. 2006. "The Credit Cycle and the Business Cycle: New Findings using the Loan Officer Opinion Surveys." *Journal of Money, Credit and Banking* 38 (6): 1575–1597.
- Lucas, Robert E. 1978. "Asset Prices in an Exchange Economy." *Econometrica* 46 (6): 1429–1445.
- Mandel, B. H., D. Morgan, and C. Wei. 2012. "The role of bank credit enhancements in securitization." *FRBNY Economic Policy Review* 18 (2): 35–46.
- Martin, A. 2009. "A model of collateral, investment and adverse selection." *Journal of Economic Theory* 144 (4): 1572–1588.

- Mason, J., and J. Rosner. 2007. "Where did the risk go? How misapplied bond ratings cause mortgage backed securities and Collateralized Debt Obligation market disruptions." *SSRN Working Paper series*, pp. 1–87.
- Mehra, Rajnish, and Edward C. Prescott. 1985. "The Equity Premium: A Puzzle." *Journal of Monetary Economics* 15:145–161.
- Nikolov, K. 2012. "A model of borrower reputation as intangible collateral." *Working Paper Series 1490, European Central Bank*.
- Ordoñez, G. 2012. Confidence banking. Mimeo. Yale University.
- Pakoš, Michal. 2005. "Asset Pricing With Durable Goods and Nonhomothetic Preferences." Ph.D. diss., Graduate School of Business, The University of Chicago, Chicago, Illinois, United States.
- . 2011. "Estimating Intertemporal and Intratemporal Substitutions When Both Income and Substitution Effects Are Present: The Role of Durable Goods." *Journal of Business and Economic Statistics* 29 (3): 439–454.
- Paligorova, T. 2009. "Agency conflicts in the process of securitization." *Bank of Canada Review*, no. Autumn:33–47.
- Parlour, C., and G. Plantin. 2008. "Loan sales and relationship banking." *The Journal of Finance* 63 (3): 1291–1314.
- Pozsar, Z., T. Adrian, A. Ashcraft, and H. Boesky. 2012. "Shadow Banking." *Federal Reserve Bank of New York Staff Reports*, no. 458.
- Ruckes, M. 2004. "Bank Competition and Credit Standards." *The Review of Financial Studies* 17 (4): 1073–1102.
- Shin, H. 2009. "Securitization and financial stability." *The Economic Journal* 119 (536): 309–332.
- Shleifer, A., and R. Vishny. 2010. "Unstable banking." *Journal of Financial Economics* 97 (3): 306–318.
- Stigler, George J., and Gary S. Becker. 1977. "De Gustibus Non Est Disputandum." *American Economic Review* 67 (2): 76–90.
- Veldkamp, L. 2005. "Slow boom, sudden crash." *Journal of Economic Theory* 124 (2): 230–257.
- Yogo, Motohiro. 2006. "A Consumption-Based Explanation of Expected Stock Returns." *Journal of Finance* 61 (2): 539–580.