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# Essays on the Effective Market Dynamics

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**Abstract**

In the first chapter, I employ high frequency data to study extreme price changes (i.e., price jumps) in the Prague, Warsaw, Budapest, and Frankfurt stock market indexes from June 2003 to December 2010. I use the price jump index and normalized returns to analyze the distribution of extreme returns. The comparison of jump distributions across different frequencies, periods, up and down moves, and markets suggests a possible relationship with market micro-structure. I also show that the recent financial crisis resulted in an overall increase in volatility; however, this was not translated into an increase in the absolute number of jumps. In the second paper, I empirically analyze the price jump behavior of heavily traded US stocks during the recent financial crisis. Namely, I test the hypothesis that the collapse of Lehman Brothers caused no change in the price jump behavior. To accomplish this, I employ data on realized trades for 16 stocks and one ETF from the NYSE database. These data are at a 1-minute frequency and span the period from January 2008 to the end of July 2009. I employ five model-independent and three model-dependent price jump indicators to robustly assess the price jump behavior. The results confirm an increase in overall volatility during the recent financial crisis—after the collapse of Lehman Brothers; however, the results cannot reject the hypothesis that there was no change in price jump behavior in the data during the financial crisis. This implies that the uncertainty during the crisis was scaled up but the structure of the uncertainty seems to be the same. Finally, in the third chapter, I perform an extensive simulation study to compare the relative performance of many price-jump indicators with respect to false positive and false negative probabilities. I simulated twenty different time series specifications with different intraday noise volatility patterns and price-jump specifications. The double McNemar (1947) non-parametric test has been applied on constructed artificial time series to compare fourteen different price-jump indicators that are widely used in the literature. The results suggest large differences in terms of performance among the indicators, but I was able to identify the best-performing indicators. In the case of false positive probability, the best-performing price-jump indicator is the one based on thresholding with respect to centiles. In the case of false negative probability, the best indicator is based on bipower variation.

**Abstract**

V první kapitole studuji za použití vysokofrekvenčních dat extrémní pohyby, neboli cenové skoky, hlavních akciových indexů z Prahy, Varšavy, Budapešti a Frankfurtu pro období mezi červnem 2003 a prosincem 2010. Používám index cenových skoků a normalizované výtěžky k analýze extrémních výtěžků. Porovnání distribucí skoků napříč frekvencemi, délkami paměti, směrem pohybu a trhy naznačuje možný vztah s mikro-strukturou trhů. Rovněž ukazuji, že stávající finanční krize způsobila vzrůst ve volatilitě, ale nikoli nárůst cenových skoků. Ve druhé kapitole empiricky analyzuji chování cenových skoků likvidních amerických akcií během současné finanční krize. Jmenovitě testuji hypotézu, že kolaps Lehman Brothers nezpůsobil žádnou změnu v chování titulů vzhledem k chování cenových skoků. K tomuto účelu použiji obchodní data 16 akcií a jednoho ETF z databáze NYSE. Tato data jsou na 1-minutové frekvenci a pokrývají období od ledna 2008 do konce července 2009. Dále použiji pět, na modelu nezávislých, a tři, na podkladovém modelu závislých, indikátorů cenových skoků k robustnímu odhadu chování skoků. Výsledky potvrzují zvýšené úrovně volatility po pádu Lehman Brothers, ale výsledky nemohou vyloučit hypotézu, že nedošlo ke změně chování cenových skoků po této události. Toto naznačuje, že se celý cenový proces zvětšil, aniž by došlo ke změně jeho struktury. Nakonec v třetí kapitole provádím extensivní simulační studii na porovnání relativní výkonnosti mnoha cenových indikátorů vzhledem k falešným pozitivním a negativním pravděpodobnostem. Simuluji dvacet rozdílných časových řad s různými vzorci intradenní volatility a specifikací cenových skoků. Dále použiji dvojitý neparametrický McNemarův test (McNemar, 1947) k porovnání výkonnosti čtrnácti různých indikátorů často používaných v literatuře. Výsledky naznačují velké rozdíly ve výkonnosti indikátorů, mezi nimiž jsem ale byl schopný identifikovat nejlepší z nich. V případě falešné negativní pravděpodobnosti je nejlepším indikátorem ten, který je založený na centilech. V případě falešné pozitivní pravděpodobnosti je nejlepší indikátor postavený bipower varianci.

# Preface

Properties of the price process of various financial assets has occupied in the attention of financial econometricians since the time of Bachelier's paper (Bachelier et al., 2006). With the emergence of information and telecommunication technologies, the financial markets have become more closely correlated on a global scale due to increase in the speed of communication and complexity of the financial network rocketed up. Thus, financial markets are unavoidably becoming unpredictable and uncertain. Uncertainty or volatility means that when we observe the price process for any financial instrument, we see that the price process follows a stochastic-like path. It is thus of great interest for financial practitioners, policy makers and regulators to understand as many aspects of the volatility as possible.

The volatility of financial assets, thankfully, carries some regularities, as was suggested by Merton (1976). It can be decomposed into two components: the first component correspondes to the regular white noise, and the second to irregular but very extreme price movements, or price jumps. The first component is relatively easy to handle analytically and all the calculations including pricing of various financial assets and estimating the market risk are technically possible. Unfortunately, it is not enough to work with the regular noise, as was shown by Andersen, Bollerslev, and Dobrev (2007), and one has to include price jump—building elements which are hard to describe analytically—to truthfully assess the risk, to price the financial assets and to understand the emergence of catastrophic phenomena like market crashes. Though, without considering price jumps properly, portfolia or financial markets are prone for instabilities, big loses and irrational panic.

Price jumps, despite their reported significance, are still not fully resolved in the literature. There is neither a clear agreement on what is the best technique to identify them in real price time series, nor is there a consensus on what is the main source causing them. Many authors dispute whether the main price jump source lies in the released news announcements or whether they emerge endogenously as a consequence of positive feedback. This further stipulates the need for deep theoretical and empirical study of price jumps. The literature also does not shed much light on the correlation of price jumps across different financial assets.

In this dissertation, I provide three studies which extend the understanding of price jumps in financial markets in several ways. This dissertation was partially motivated by the financial turmoil observed on the global financial markets starting in mid-2008. Events three years later, however, suggest that the study does not only serve to provide better understanding of what happened during the financial crisis but also to prepare for the next one, which may be very soon.

In the first paper titled "Price Jumps in Visegrad-Country Stock Markets: An Empirical Analysis," I perform an empirical analysis of the price jump behavior of the Visegrad stock markets using high frequency data. According to my best knowledge, this is the first study which focuses on the Visegrad stock markets and compare them with the mature market in Germany. I employ 5-minute data of the main stock market indexes for the period June 2003 to December 2010. Descriptive statistics show a significant deviation from the Gaussian distribution, which supports the presence of jumps. I employ the price jump index and normalized returns to measure jumpiness of the above mentioned markets. In particular, I study the distribution of price jumps across different frequencies, periods, and up and down movements. The results suggest a close relation of the price jump behavior to the market micro-structure. Namely, low turnover and very tolerant margin trading makes the behavior of the Prague Stock market index PX significantly deviate from the other three stock indices. Finally, I also show that the recent financial crisis was not transmitted to increased number of price jumps, or jumpiness. On the other hand, the recent financial crisis caused the stock market to behave in a more similar manner since the discrepancies in the price jump behavior among them significantly decreased during the financial turmoil.

In the second part of the thesis named "The Impact of the Lehman Brothers Collapse: Were Stocks More Jumpy?," I answer the question whether the collapse of Lehman Brothers—an event considered by many people as a trigger of the recent financial crisis—caused more price jumps or market panic on the financial markets. Namely, I test the hypothesis that the collapse of Lehman Brothers caused no change in the price jump behavior. I employ 1-minute data on realized trades for 16 major stocks traded on the NYSE and one ETF tracking the performance of the S&P 500 index. The data spans from January 2008 to the end of July 2009 with Lehman Brother's collapse standing in the middle of the data set. I employ five model-independent and three model-dependent price jump indicators to robustly assess the price jump behavior. The results confirm a well-accepted fact that financial problems caused a significant increase in volatility; however, the results cannot reject the hypothesis of no change in the price jump behavior. This is counter-intuitive since many policy makers as well as investors characterize turmoil periods by increased volatility and increased number of extreme price movements. The results thus bring a new perception of the financial crises and provides hints how to prepare for future crises.

Finally, in the part titled "The Identification of Price Jumps," I target the problem which appears throughout the literature: which price jump indicator is the most suitable for identification of real price jumps. The literature so far provides a broad range of various price jump indicators based on different theoretical grounds; however, they are usually tested using different methodology and thus one cannot directly compare them. The lack of direct comparison raises a question about the findings used with a particular price jump indicator. To fill this gap, I performed an extensive simulation study to compare the relative performance of many price-jump indicators with respect to false positive and false negative probabilities. I simulated twenty different time series specifications with different intraday noise volatility patterns and price-jump specifications, and applied the double McNemar (1947) non-parametric test on constructed artificial time series to compare fourteen different price-jump indicators that are widely used in the literature. Surprisingly, the results suggest large differences in terms of performance among the indicators. In addition to finding the discrepancies, I identified the best-performing indicators. In the case of false positive probability, the best-performing price-jump indicator is based on thresholding with respect to centiles. In the case of false negative probability, the best indicator is based on bipower variation.

The three chapters of this dissertation give together a wide and robust picture on the problematic of price jumps in financial markets. The three papers extend the literature in several ways and suggest many different future applications based on this research. Among others, the first chapter suggests that policy makers, who plan, for example to introduce pension reform in the Czech Republic, should not use the existing historical data for calculating pension fund performance and risk analysis since the operation of pension funds may change the properties of the markets. The second paper, on the other hand, helps financial regulators to prepare for financial crises. The intuitive treatment of financial crisis as a period with increased volatility and higher number of price jumps is not supported by the data. Finally, the third chapter provides a recipe how to perform a meta-analysis over existing papers by defining two optimality criteria. The findings of three chapters, however, open up more research questions to exploit, which I plan to address in my future research.

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## Part I

# Price Jumps in Visegrad-Country Stock Markets: An Empirical Analysis[1]


**Abstract**

I employ high frequency data to study extreme price changes (i.e., price jumps) in the Prague, Warsaw, Budapest, and Frankfurt stock market indexes from June 2003 to December 2010. I use the price jump index and normalized returns to analyze the distribution of extreme returns. The comparison of jump distributions across different frequencies, periods, up and down movements, and markets suggests a possible relationship with the market micro-structure. I also show that the recent financial crisis resulted in an overall increase in volatility; however, this was not translated into an increase in the absolute number of jumps.


[1]An earlier version of this work has been published in Novotny, J., 2010. "Price Jumps in Visegrad Country Stock Markets: An Empirical Analysis", CERGE-EI Working Paper Series, 2010, No. 412, 33 pages. Work was presented at the 6th CSE Biennial Conference, Prague, Czech Republic 11/2010, and at Mathematical Methods in Economics, Kostelec upon Black Woods, Czech Republic, 9/2009. This study is supported by a GAČR grant (402/08/1376) and by grant No. 271111 of the Grant Agency of Charles University. All errors remaining in this text are the responsibility of the author.

# 1 Literature Review and Motivation

The volatility of financial markets—or, in other words, the uncertainty of the price process for various financial instruments—is a deeply studied phenomenon in the financial literature (see, e.g., Gatheral, 2006). However, most of the attention has been focused on the part of volatility known as regular noise, which can be described by a standard Gaussian distribution. The remaining component of volatility, known as price jumps, involves irregular but abrupt price changes. See Merton (1976) for an early reference or the recent discussion of how to decompose volatility into two parts by Giot et al. (2010). Price jumps substantially differ from regular noise and are more difficult to explicitly define and handle mathematically (Broadie and Jain, 2008; Johannes, 2004; Nietert, 2001; Pan, 2002).

Price jumps, i.e., irregular and extreme price movements, are associated with various interesting market phenomena. Price jumps can be connected to important issues in the market micro-structure, such as the efficiency of price formation, the provision of liquidity or the interaction between market players.[2] From a practical point of view, traders are also interested in analyzing price jumps since they are a part of volatility and therefore associated with significantly large losses and gains. Thus, understanding price jumps helps to avoid big losses, improves portfolio performance and creates better hedge positions. Finally, knowledge of price jumps is needed by financial regulators; see Becketti and Roberts (1990) or Tinic (1995). Price jumps can be also used as a proxy to study market (in)efficiency, information flows across markets including market panic or changes in the information-driven trading. Overall, the study of price jumps can shed more light on a broad class of market phenomena and significantly extend the existing knowledge.

One of the major problems associated with price jumps is the lack of evidence of, and very different views about, their origins presented in the literature. For example, one stream in the literature claims that price jumps primarily originate in news announcements. This stream is represented by Lee and Mykland (2008) or Lahaye et al. (2011), where the authors confirm in their framework that the main source of price jumps are corporate statements or macroeconomic news announcements. In addition, several authors, e.g., Hanousek et al. (2009), claim that news announcements cannot be perceived absolutely, but rather only relative with respect to market

[2] See the survey in Madhavan (2000).

expectations. Therefore relative departures from market expectations would be a source for price jumps. Other authors, however, oppose the explanation that revealed news are a primary source of price jumps and state that the main source of price jumps is the lack of liquidity on either the bid or ask side.[3] Further, the literature on behavioral finance suggests another explanation for price jump origins. For example, Schiller (2005) suggests that jumps are caused by the market participants themselves, who create a fragile environment, which ends up in extreme reactions manifested as price jumps. This is further supported by Taleb (2007), who connects extreme price movements with the systemic properties of complex systems. The lack of a theoretical explanation of price jumps means that empirical analysis is currently the only tool one can employ.

In addition to the main streams of literature, security trading itself provides several additional explanations. Sudden price movements could be attributed to traders' responses to changes in market mood. When the market mood changes, events are perceived differently and could result in price jumps (see Andersen et al., 2007). For example, when a market bottoms out, it is not affected by negative news, and even less-negative-than-usual news can cause an upswing. Such an upswing could be further fueled by herd behavior. Market mood is also closely connected with a phenomenon known from international finance: in bad times markets are more correlated with each other than in good times.[4] Therefore, from a behavioral point of view, a change in market mood can affect the way markets are correlated with the underlying economic fundamentals, supporting the strong relation between price jumps and market mood.

Price jumps can also be a useful tool to study information spillover in financial markets. The spread of price jumps can be perceived as a transfer of important information also known as information flow. In particular, an especially interesting case is a potential link between price jumps and the revelation of insider information.[5] Price jumps and their distribution in particular could also reflect the inefficiency of financial markets. Efficient financial markets, as Fama (1970) puts it, should reflect all the available information and, therefore, a market with more price jumps should be less efficient.

In this paper, I empirically estimate a broad range of price jump properties for Central and

[3]For example, Joulin et al. (2008) and Bouchaud et al. (2006) study excess liquidity and its impact on the formation of price jumps.

[4]See, for example, Erb et al. (1994); Ribiero and Veronesi (2002); and Knif et al. (2008).

[5]The insider trading dimension of price jumps was studied by Cornell and Sirri (1992) and Kennedy et al. (2006). It causes problems for policy makers and other market participants.

Eastern European (CEE) emerging markets in a discrete-time framework, which is suitable for markets with low and irregular frequency of trades. I use high frequency data for the main stock indexes[6] from the countries of the Visegrad region[7]—small emerging economies regionally and culturally close to each other. Further, as a benchmark I include the stock index from Germany as the most important trading partner with mature financial markets. The data spans from June 2003 to the end of 2010 and thus cover the period before the recent financial crisis, the phase before the crisis as well as part of the recovery phase. To my best knowledge, this is the first study of price jumps for small emerging markets that includes economic and financial interference and discusses the impact of a financial crisis on extreme price movements. The rest of the paper is structured as follows: Section 2 gives a short overview and classification of various price jump indicators, Section 3 briefly describes my methodology, in particular how I use non-parametric measures to compare stock market jumpiness across time and markets, Section 4 describes the data, Section 5 is devoted to my results, and finally Section 6 concludes the paper.

## 2 Price Jump Indicators

The volatility of financial instruments can generally be decomposed into two parts: regular noise and the remainder. Regular noise is characterized by an underlying Gaussian distribution that was first identified by Bachelier, see Bachelier's original work Bachelier (1900) or the book devoted to this work (Bachelier et al., 2006) and ever since it has been extensively discussed in the literature. The remaining part, known as price jumps, includes irregular but extreme price movements, and has not been deeply studied in the economic literature. It is believed that these extreme price movements are rather stemming from a Levy walk (Fama, 1965), which leads in distributions with infinite moments; see Levy (1925). This is in stark contrast to the Gaussian distribution where all moments are finite. Despite the fact that there exist models taking into account this component (e.g., Merton, 1976), a deeper theoretical understanding of price jumps is still missing.

One of the fundamental problems of price jumps is choosing an appropriate identification tech-

[6]In contrast to the existing studies, the indexes included in this work are not directly traded, which can have consequences for the properties of their price process, namely, serial auto-correlation can have a slower decay.

[7]The Visegrad region actually consists of four countries: the Czech Republic, Poland, Hungary and Slovakia. The sample does not include Slovakia since its financial market has very low capitalization and extremely low turnover, and therefore it is not suitable for the high-frequency statistical analysis applied in this paper.

nique. The literature contains a number of different jump indicators based on different assumptions. Generally, I can divide the price jump indicators into two main categories: indicators aiming to exactly identify the moments or periods when the price jump occurred and indicators evaluating the statistical propensity of a given time series to undergo price jumps.

The first group of indicators are those employing higher moments like bipower variance and swap variance. The bipower variance-based indicators deal with statistics coming from the difference between the realized variance and the bipower variance: the measure of variance, which, as opposed to the realized variance, is not sensitive to non-normal price movements.[8] The swap variance-based indicators were developed by Jiang and Oomen (2008) and rely on the difference between the realized variance and the swap variance. The authors claim that that swap variance is more sensitive to price jumps than bi-power variance and thus the indicator is more efficient, which they support by a Monte Carlo simulation study.

The second group of price jump indicators targets a measure of jumpiness, not the individual jumps. One of the streams of indicators belonging to this group are those developed by Ait-Sahalia along with various co-authors;[9] however, these indicators are better suited to investigate the properties of ultra-high-frequency data. Another stream of indicators measuring price jumpiness, which will be employed in this paper, are the price jump index and normalized returns.[10]

The price jump index was introduced by Joulin et al. (2008) and is defined as an absolute value of a return normalized by a moving average of the same quantity over a given window:

$$j_T(t) = \frac{|r(t)|}{< |r(t)| >_T}, \tag{2.1}$$

where $< |r(t)| >_T$ denotes the equally weighted moving average of $T$ values of absolute log-returns, including the current value, i.e.,

[8] Bipower variance-based indicators were employed by Barndorff-Nielsen et al. (2006, 2008); Barndorff-Nielsen and Shephard (2004), and Barndorff-Nielsen and Shephard (2006), Lee and Mykland (2008) further elaborated this method and derived an indicator that tests for the exact moments when a price jump occurred. Bipower variance is defined as a sum of absolute values of return with its lag value, while realized variance is just a sum of squares of returns. Swap variance is construction, which takes into account both returns and prices. Proper definitions follow later on in the subsequent chapters.

[9] See the papers by Ait-Sahalia (2004); Ait-Sahalia and Jacod (2009a,b), and Ait-Sahalia et al. (2009).

[10] These indicators belong to the field of statistical finance, see e.g., Stanley and Mategna (2000) for further references.

$$< |r(t)| >_T = \frac{1}{N_T} \sum_{i=0}^{T-1} |r(t-i)| \ , \tag{2.2}$$

and $N_T$ stands for the actual number of observations in the time window to control for missing observations.

The authors also employed a large data set of US stocks and showed that the price jump index has a distribution with tails following power-law behavior, i.e., its probability distribution is proportional to $r^{-\alpha}$, with a characteristic coefficient corresponding to an inverse cubic distribution, or $\alpha \approx 4$. The prominent role of the inverse cubic distribution in the financial high-frequency data was confirmed by other studies.[11] On the other hand, the inverse cubic distribution is close to a distribution describing Levy-like behavior with infinite volatility.

Normalized returns, on the other hand, are defined as centered returns normalized by their standard deviation:[12]

$$r_T^n(t) = \frac{r(t) - < r(t) >_T}{\sigma_T(t)} \ , \tag{2.3}$$

where $< r(t) >_T$ is defined in a similar way as in equation (2.2), and $\sigma_T(t)$ is the standard deviation—or the realized volatility with respect to the $L_2$ measure—calculated from the same set of $T$ returns. Such a definition allows me both to compare the different price time series and to compare periods with different market volatility.

Eryigit et al. (2009) study price jumps for a broad range of stock market indexes with the help of normalized returns. They focus mainly on the functional form of the tail distribution and test a broad range of possible tails of price jump distributions. The big Chinese emerging markets are studied by Jiang et al. (2009) using normalized returns, where the authors show that power-law behavior is valid only for long-term moving averages, while for a short-term history, the tail behavior behaves in a more exponential-like manner. Novotny (2010) employed these indicators, as well as several other price jump indicators, to study the change in the price jump behavior during the recent financial crisis and found no significant change in the price jump behavior due to the financial crisis

[11] Plerou et al. (1999) and Gopikrishnan et al. (1999), among others.

[12] Plerou et al. (1999) and Gopikrishnan et al. (1999) employed normalized returns to study price jumpiness for US stocks and the S&P 500 index using high-frequency data. They confirm that the tail distribution of normalized returns for US stocks follows a distribution close to the inverse of the cubic one, i.e., they do not behave either as a pure Gaussian or as a Levy-like distribution with infinite variance.

for the most capitalized US stocks.

In this paper I am interested in measuring to what extent the jumpiness of the Visegrad markets has been affected by the recent financial crisis and how price jump indexes depend on a chosen frequency, as well as analyzing the (a)symmetry of jump distribution. In such a setup, the primary concern lies in the measure of price jumps per a certain period, or, in other words, in the measure for propensity of indexes to undergo a jump. Therefore, I would rather focus on the overall propensity of financial markets to be jumpy than to focus on the more rigorous hypothesis of testing a given period for the presence of one or more price jumps.[13] Hence, in my comparison I will use two main price jump indicators from the second group: the price jump index and normalized returns.

# 3 Methodology

I employ two measures for price jumps: the price jump index introduced in equation (2.1) and normalized returns defined by equation (2.3). The two definitions of the price jump indicators are close to each other but still they are significantly different. The similarity lies in the normalization with respect to recent history expressed by dividing the returns by the realized volatility. The moving average of the absolute returns and standard deviation represent two different definitions of the realized volatility. It is easy to show that standard deviation is relatively more sensitive to extreme returns compared to the average of absolute returns. Hence, normalized returns are on average more suppressed.

In this section I will show what parameters I opted to use for particular price jump indicators (and why), how I estimated the characteristic coefficient $\alpha$ describing a parameter for the tail behavior, and how the whole comparative analysis was conducted.

## 3.1 Memory of Market History

Both definitions of price jump indicators require a choice of parameter $T$ driving the length of the history to which I compare the recent returns. With this I am able to estimate the propensity of price jumps, i.e., the rate and size of price jumps with respect to the recent market situation. Parameter $T$ is chosen relatively and is defined as a number of time steps. Hence, the same value

[13] For the most extensive overview of the broad class of price jump indicators available in the literature and an analysis of their performance see Hanousek et al. (2011).

of $T$ has a different absolute length in minutes for different frequencies. The length of the time window defines the filtering property of the price jump indicator, i.e., it is related to the frequency of processes that will be captured by the chosen indicator. Generally, the longer the moving average, the more sensitive the indicator is with respect to low frequency processes (cycles), and the more insensitive the indicator would be to high frequency events. On the other hand, a very short time window does not take into account slowly varying processes and considers only fast and abrupt changes.

When using the price jump indicator I will, therefore, employ a wide range of values for $T$, which allows me to capture processes at all time scales. The literature suggests that price jump properties are filter dependent. For example, Plerou et al. (1999) show that the longer the time window $T$, the higher the probability of the occurrence of extreme events.[14] In addition, the authors also provide empirical evidence that for a short time window $T$, the behavior of the tail distribution for normalized returns is rather exponential and thus different from power-law behavior, which was observed for long time windows by many authors.[15]

To capture the views and recommendations made by various authors I therefore employ the following sequence of time windows: $T = 12$, 24, 100, 1000, 2000, and 5000 time steps. Naturally, the length in minutes depends also on the frequency used. I applied steps $T = 12$ and 24 to focus on the immediate effects during the same trading day. Time widows $T = 100$ and longer are taken to study the behavior of long-term averages, as suggested in the above-mentioned literature.

### 3.2 Estimating Tail Behavior

Joulin et al. (2008), among others, show that the tail distribution of the price jump index $j_T(t)$ for various financial assets should behave similarly as $\propto s^{-\alpha_T^{(f)}}$, i.e.,

$$P(j_T > s) \sim s^{-\alpha_T^{(f)}}, \tag{3.1}$$

where $\alpha_T^{(f)}$ depends both on the frequency of the data and the length of the time window $T$. For the sake of simplicity, in the following formulas and expressions, I will omit the frequency index. The index for the time window $T$ will be kept explicit in all expressions. Let me note that the

[14]This is also supported by the theoretical and empirical evidence presented by Kleinert (2009).

[15]Except the previously mentioned authors, Joulin et al. (2008) do not reject the presence of power-law behavior.

values of the estimated parameter $\alpha_T^{(f)}$, or $\alpha$, are associated with certain distributions. For example several authors claim that the characteristic parameter $\alpha$ tends to be around 4 for a large set of US stocks, which corresponds to the inverse cubic distribution.[16] On the other hand, the value of $\alpha \leq 3$ indicates Levy-like behavior with infinite volatility, see Kleinert (2009). It is clear that the following rule holds: the lower $\alpha$ is, the more likely extreme price jumps will be observed.

The characteristic coefficient $\alpha$, as defined in the previous relation (3.1), is estimated for both the price jump index and normalized returns. To obtain an equation suitable for the estimation procedure, I first linearize relation (3.1) into the form

$$\ln P(j > s) \propto -\alpha \ln s \,. \tag{3.2}$$

Assuming that the tail part follows such a power-law behavior, I can formulate the linear equation as

$$\ln P(j > s) = \alpha_0 - \alpha \ln s + \nu \,, \tag{3.3}$$

where $\alpha_0$ is an intercept and $\nu$ is assumed to be a homogenous Gaussian noise stemming from the statistical nature of the data. This equation holds $j$ above some tail threshold values $s_j$.

In the next step, I employ the following algorithm to estimate the characteristic coefficient: Using OLS, I estimate $\alpha$ in equation (3.3) for various tail intervals and the resulting value of $\alpha$ is from the OLS regression with the highest $R^2$. Such an algorithm is both simple and corresponds to the linearization of the tail distribution.[17] Using this algorithm, I perform two steps at once: identifying the linear part of the tail—implicit estimation of the tail threshold $s_j$—and estimating its properties—estimation of $\alpha$.

### 3.3 Comparative Analysis

The idea of my comparative analysis is based on a very simple and intuitive approach. I aim to estimate the characteristic coefficients $\alpha$ over various sub-samples representing different phases of the market, different time frequencies, etc. Associated non-parametric tests comparing the underlying

[16] This was also confirmed by Plerou et al. (1999) and Gopikrishnan et al. (1999).

[17] An alternative approach would rely on using the MLE or Principal Component Analysis as in Vaglica et al. (2008).

distributions (or the equality of $\alpha$ coefficients) would actually test if the propensity to jump of the price indexes has changed. I can therefore test if the jumpiness of stock market indexes is different in different time periods like during the recent financial crisis; I can test the (a)symmetry of jumps up and down, as well as the sensitivity of $\alpha$ to particular high frequencies. This relatively simple setup allows me to study the jump component of the price generating process in more detail.

For the above-mentioned comparison I need to use a test that will be distribution-free, i.e., a non-parametric test with a null hypothesis that two or more sets of estimated parameters have the same distribution. The non-parametric feature of the test is necessary since I cannot assume any underlying distribution of the estimated parameters. Moreover, I also require that the chosen test(s) would have good finite sample properties even for small samples. Taking all of these considerations together. I opted for two non-parametric tests: the Wilcoxon rank-sum test and the Kruskal-Wallis test.

**The Wilcoxon Rank-sum Test**[18] is a non-parametric statistical test used to compare whether two samples drawn from the independent populations have equally large values. The test itself goes in two steps. In the first step, all observations are ranked together according to their value no matter what sample they belong to. The sum of all the ranks assigned to all observations is equal to $N(N+1)/2$, where $N$ is the number of observations in both samples together. When both samples are equally distributed, the sum of ranks tends to be equal. Therefore, in the second step, two statistics are constructed:

$$U_i = \sum_{i \in Sample_i} rank_i - \frac{n_i(n_i+1)}{2}, \tag{3.4}$$

where $n_i$ is the number of observations in sample $i$ and $rank_i$ is the rank of observation $i$.

For large samples, the sum of the two later statistics, $U = U_1 + U_2$, is asymptotically equal to a normal distribution.[19] For convenience, I employ standardized statistics

$$z = \frac{U - m_U}{\sigma_U}, \tag{3.5}$$

[18] For the original reference, see Wilcoxon (1945). A modification of the test is known as the Mann-Whitney test; see Mann and Whitney (1947).

[19] For small values, statistics have to be compared with significance tables.

with mean $m_U = \frac{n_1 n_2}{2}$ and standard deviation $\sigma_U = \sqrt{\frac{n_1 n_2 (n_1+n_2+1)}{12}}$, which is asymptotically equal to $z \sim N(0,1)$.

**The Kruskal-Wallis Test**[20] is a non-parametric statistical test used to test the null hypothesis that $K$ independent samples were drawn from distributions with the same median. The test is a direct generalization of the Mann-Whitney U test for two samples. The test follows a similar strategy to the later test of Wilcoxon and goes in two steps. In the first step, all observations are ranked together according to their value no matter what sample they belong to. In the second step, the Kruskal-Wallis statistics is calculated according to the following formula:

$$KW = (N-1) \frac{\sum_{i=1}^{K} n_i \left(\bar{r}_i - \bar{r}\right)^2}{\sum_{i=1}^{K} \sum_{j \in Sample_i} \left(r_j - \bar{r}\right)^2}\,, \tag{3.6}$$

where $n_i$ is the number of observations in sample $i$, $\bar{r} = (N+1)/2$ is the average rank of the entire sample, $\bar{r}_i = \left(\sum_{j \in Sample_i} r_j\right)/n_i$, $N$ is total number of observations, and $K$ is the number of independent samples. The $KW$ statistics is for large values of all $n_i$ asymptotically equal to $KW \sim \chi^2_{K-1}$. The $KW$ statistics for a small size of the sample is presented in statistical tables.[21]

## 4 Data and Descriptive Statistics

I use 5-minute-frequency data for the main indexes from the Prague Stock Exchange (PSE, the PX index), the Budapest Stock Exchange (BSE, the BUX index), the Warsaw Stock Exchange (WSE, the WIG20 index), and the Frankfurt Stock Exchange (FSE, the DAX index); the data spans from June 2003 to December 2010. The period included in the sample starts just before the three emerging markets joined the European Union and goes until the end of 2010. The sample covers the period before the recent financial crisis, its beginning phases at the end of 2007 and beginning of 2008, the full emergence of financial crisis in 2008 and 2009 and, finally, a certain period of the recovery until the end of 2010. It thus describes the market evolution of three culturally, historically, geographically, and economically connected countries along with Germany, representing an EU benchmark: a country with more matured financial markets and institutions as well the major

[20]For the reference see Kruskal and Wallis (1952).

[21]Statistical software like Stata contains these values and automatically checks whether one can use asymptotic values or exact values.

EU trading partner of the other countries.

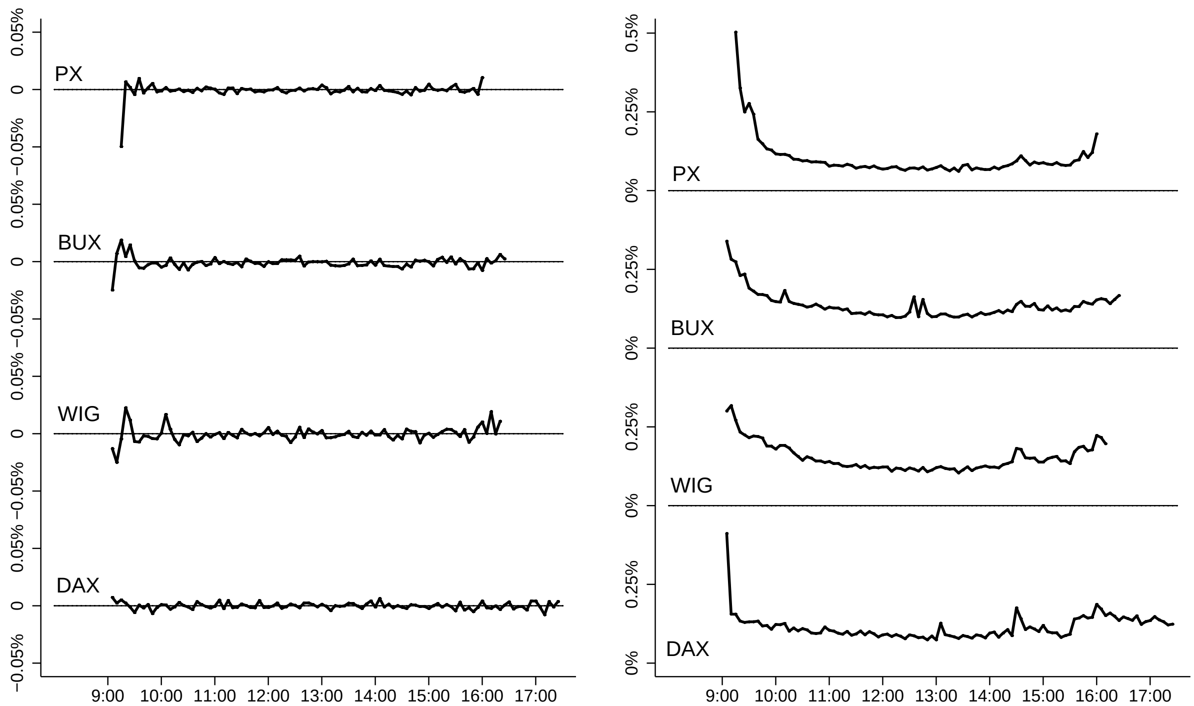
Since the main purpose of this paper is to study market dynamics and especially the propensity of each particular market to extreme price changes (i.e., jumps), I have cut off the very beginning and the very end of each trading day. The cut-off at the beginning of the trading day is performed due to the different construction of the market indexes used. Generally, any market index can be either dividend-included or dividend-excluded. In my case, the DAX and BUX indexes are dividend-included, while the PX and WIG indexes are dividend-excluded. This obviously causes different behavior at the market opening due to the ex-dividend day effects.[22] Moreover, the beginning-of-day cut-off could have a negative effect on the observed propensity to sizable price moves. This is, first, because I remove the period close to the opening, when markets react to overnight events. Second, the cut-off data could show even an opposite and smoother reaction to the (overnight) events since markets could in the very first moments over-react negatively to negative events, and after this abrupt overshooting, they could positively and smoothly move up to adjust the previous (cut-off) price change. These effects have to be taken into account when I make financial implications. The cut-off at the end of the trading day is performed for similar reasons: some markets have a different final stage (such as a final auction), so cutting off the very end also avoids a possible bias in the data.[23] Nevertheless, the cut-off occurs long enough after US markets open, therefore I do not lose significant market moves associated with the start of US trading.

Cutting off the initial phase of the market is also necessary when one wants to treat markets in a panel-like manner. Nevertheless I do not use a panel-like approach for these CEE markets. The reason stems from the fact that the stock exchanges open and close at different times. When markets open they usually accommodate information that happened overnight. Thus, comparing markets at the beginning/end of the day can result in a situation where one of the markets is just in the opening/closing stage while the others are relatively far from the boundaries of their trading days. This could produce some false signals and lower market correlation.

[22] Dividend-excluded market indexes measure the price performance of markets without including dividends. This means that on any given day, the price return of an index captures the sum of its constituents' free-float-weighted market capitalization returns. For a description of the dividend structures and the particular composition of the stock market indexes, see the official web pages for the four stock exchanges: www.bcpp.cz, www.bse.hu, www.gpw.pl, and www.deutsche-boerse.com. For an illustration of how the dividend process influences the price process (for the dividend-included DAX index), see Fengler et al. (2007).

[23] For example, the Prague Stock Exchange has a closing auction from 4:20 p.m. to 4:27 p.m. Traders may postpone some trades from the trading hours to the closing auction and thus the trading hours may not fully capture the trading activity.

Figure 4.1: Distribution of returns (left panel) and standard deviation of returns (right panel) over the trading day.




Note: The left panel of the figure describes the distribution of returns over a trading day using a 5-minute frequency. Plotted are distributions for all four indexes: PX, BUX, WIG, and DAX. The right panel of the figure captures the distribution of the standard deviation over a trading day using the same 5-minute frequency for the four indexes depicted in the same order. The initial double peak for the WIG index is caused by the fact that the stock exchange changed its operating hours in the middle of the sample from 10:00 to 9:00.

In order to provide a valid inference, I first summarize in Figure 4.1 the distribution of returns and the standard deviation over the entire trading day (without any cut-off) for all four indexes at a 5-minute frequency.[24] The figure on the left side shows that the three emerging markets have on average negative returns during the opening period, i.e., all three markets drop during the opening phase. On the contrary, the mature market does the opposite, i.e., it slightly increases in value. In addition, PX has the most abrupt changes (likely also fueled by the fact that PX is a dividend-excluded index), which is further supported by the figure on the right, where I present the distribution of the standard deviation. Besides the well-known U-shape distribution during the trading day, which is again the strongest for the PX index and less pronounced for the German market, I can see a small increase in volatility during the lunch period and during the opening of the US markets.

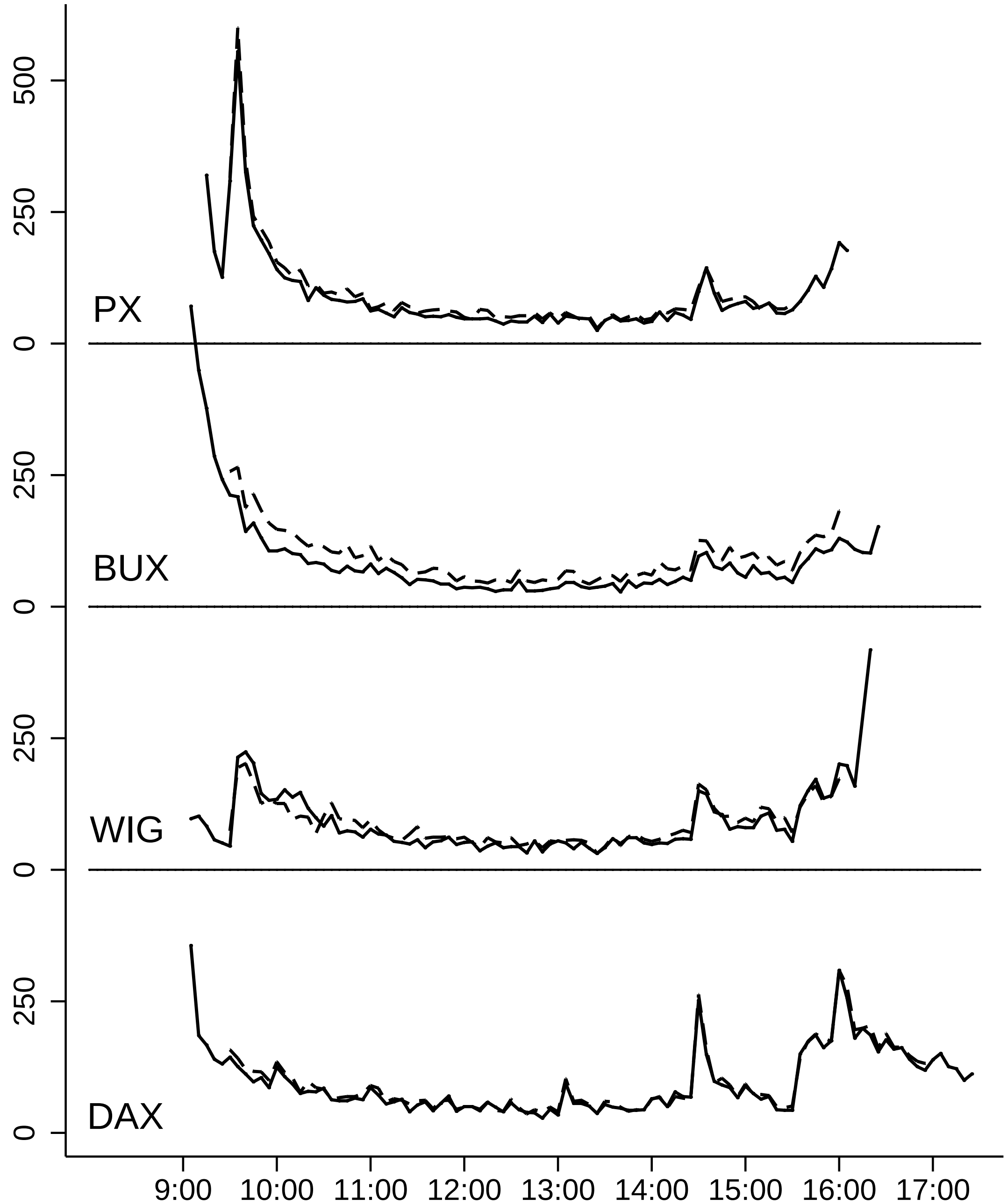
More specifically, the stock exchanges studied here have the following trading hours: PX (the Czech Republic) from 9:15 to 16:00, DAX (Germany) from 9:00 to 17:30, BUX (Hungary) from 9:00 to 17:00, and WIG (Poland) from 9:00 to 16:20.[25] As discussed earlier, because of possible sensitivity to (the size of) the cut-off trading periods at the beginning and end of the trading day, I consider various cut-offs running from 10 to 30 minutes and present the sensitivity analysis in Appendix A. For the purpose of my analysis I will present the results associated with the largest cut-off (30 minutes). In the following pictures, graphs and computations, I distinguish between two types of time: clock time and trading time. Trading time skips the cut-off in the early morning and late afternoon phases and the relevant variables are stacked into one time series with no gaps, i.e., the last minute at the end of the trading period is followed by the first minute of the next trading period.

The effect of cutting off the very first and very last moments of the trading period on the distribution of extreme movements is depicted in Figure 4.2. The figure shows the distribution of the number of extreme returns over the trading day for both the entire trading day without any cut-off (solid line), and for the day with the cut-off (dashed line). Extreme returns are defined as

[24] In addition, for sensitivity tests I have constructed all the relevant variables also on three lower frequencies: 10, 15, and 30 minutes.

[25] The PX opening hours were originally 9:30 to 16:00 but changed on June 30, 2008 to 9:15 to 16:00. BUX originally operated from 9:00 to 16:30 and on December 2, 2010, trading hours were prolonged until 17:00. WSE originally operated from 10:00 to 16:00, but from October 3, 2005, exchange trading started at 9:30 and closed at 16:10 and this was further modified on September 1, 2008, when market operations were extended from 9:00 to 16:10.

Figure 4.2: Distribution of extreme returns over the trading day.




Note: Shown is the distribution of extreme returns over a trading day using a 5-minute frequency. Extreme returns are defined as returns below the 2.5th centile or above the 97.5th centile, calculated over the entire period. The solid line takes into account the entire trading day, while the dashed line refers to the trading day with the beginning and end cut off. The double peak for the WIG index is caused by the fact that the stock exchange changed its opening hours in the middle of the sample from 10:00 to 9:00.

Table 4.1: Basic statistics of returns.

| Index | $f$ | $N$ | $\mu$ | $\sigma$ | $S$ | $K$ |
|---|---|---|---|---|---|---|
| PX | 5 | 135 893 | -2.515e-06 | 0.932e-03 | -0.284 | 31.13 |
| | 10 | 68 100 | -4.363e-06 | 1.444e-03 | -0.493 | 26.12 |
| | 15 | 45 693 | -5.087e-06 | 1.887e-03 | -0.550 | 26.95 |
| | 30 | 22 517 | -1.189e-05 | 2.736e-03 | -0.580 | 19.45 |
| BUX | 5 | 148 248 | -1.545e-05 | 1.277e-03 | 0.025 | 63.45 |
| | 10 | 75 057 | -2.681e-05 | 1.869e-03 | -0.398 | 39.66 |
| | 15 | 50 668 | -3.801e-05 | 2.326e-03 | -0.888 | 34.20 |
| | 30 | 26 271 | -7.282e-05 | 3.469e-03 | -0.560 | 18.57 |
| WIG | 5 | 137 238 | -7.646e-06 | 1.428e-03 | 0.040 | 12.74 |
| | 10 | 69 218 | -1.369e-05 | 2.017e-03 | -0.025 | 11.67 |
| | 15 | 46 509 | -1.866e-05 | 2.447e-03 | 0.070 | 11.34 |
| | 30 | 23 765 | -4.083e-05 | 3.523e-03 | 0.047 | 12.26 |
| DAX | 5 | 173 382 | -1.889e-06 | 1.114e-03 | -0.129 | 24.52 |
| | 10 | 87 632 | -3.978e-06 | 1.576e-03 | -0.111 | 22.66 |
| | 15 | 59 055 | -5.020e-06 | 1.923e-03 | -0.106 | 17.83 |
| | 30 | 30 460 | -6.160e-06 | 2.837e-03 | -0.211 | 25.12 |

Note: The table summarizes the standard statistics of log-returns $r(t)$ for all four market indexes used in this study; in brackets is the corresponding stock exchange: PX (Prague Stock Exchange), BUX (Budapest Stock Exchange), WIG (Warsaw Stock Exchange), and DAX (Frankfurt Stock Exchange). All four frequencies were used: 5-, 10-, 15-, and 30-minute. The table shows: frequency ($f$), the number of observations ($N$), mean returns ($\mu$), standard deviations ($\sigma$), skewness ($S$), and kurtosis ($K$). I have employed Jarque-Bera statistics to test the deviation from normality. Since in all cases the Jarque-Bera statistics had the $p$-value $< 0.0001$, I reject a normal distribution of returns for all indexes and frequencies.

those that are below the 2.5th centile or above the 97.5th centile, calculated over the entire sample. The two lines tend to coincide for all four indexes, except for a small deviation for the BUX index. The coincidence of the two lines combined with the information in Figure 4.1 means that the initial and/or final periods do not contain significantly more price jumps. The overall pattern of the data depicted in the left panel of Figure 4.1 suggests that the initial moments consist of returns with the same sign rather than being dominated by extreme downward movements. However, the right panel of Figure 4.1 still shows that the spread of returns tends to be higher in the initial period.

## 4.1 Descriptive Statistics

The descriptive statistics of returns provide the first hints about the possible properties of price jumps. The first four centered moments can be found in Table 4.1.

Table 4.1 shows that the means of returns are shifted toward negative values. The standard deviation is increasing with decreasing frequency, or with increasing sampling intervals $\Delta t$, and is roughly in agreement with the known scaling law; see Stanley and Mategna (2000),

$$\sigma_{\Delta t} \propto \sqrt{\Delta t}\,. \tag{4.1}$$

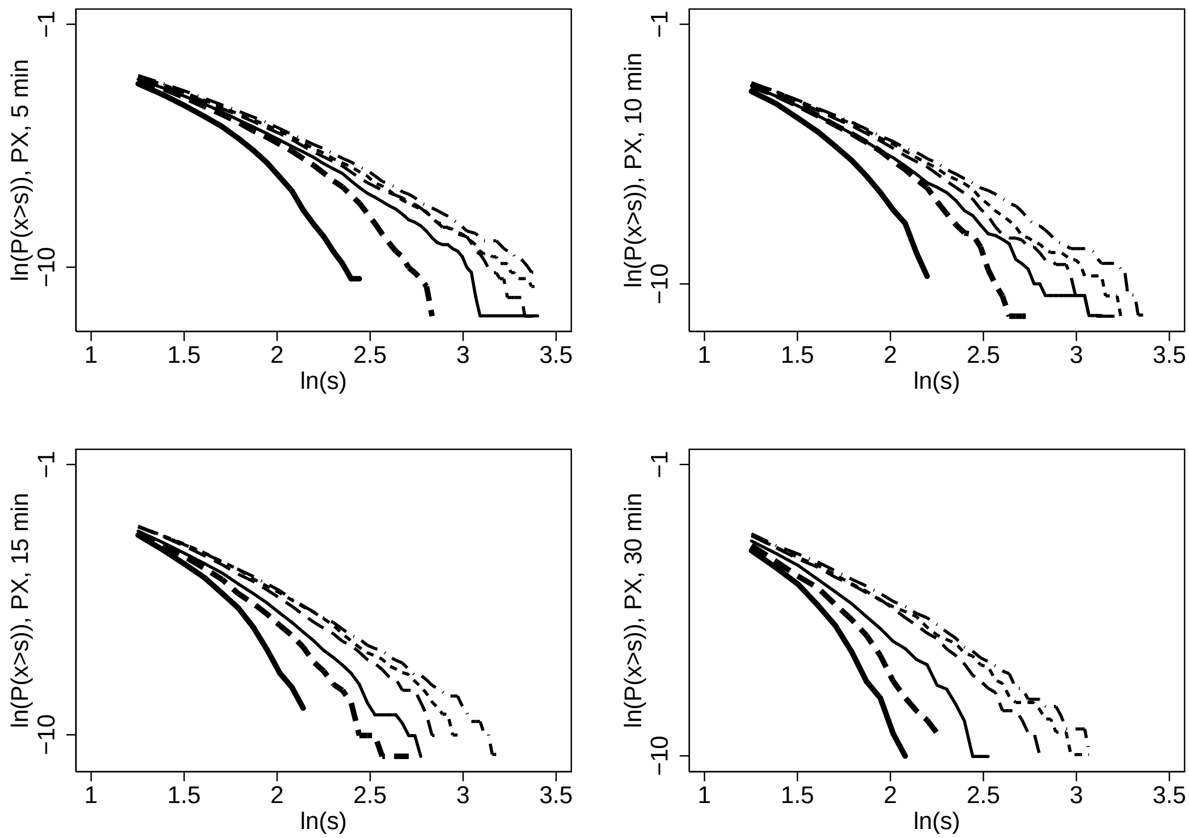
A further measure reported in Table 4.1 is skewness, which is a measure of the asymmetry of the distribution. The results show that the PX, BUX, and DAX indexes have negative skewness and thus their distributions have longer negative tails. On the other hand the WIG index shows positive skewness over all the frequencies, which supports the claim that the WIG index is dominated by positive jumps. The next column with reported kurtosis shows that all time series are leptokurtic, which supports the presence of a fat-tail distribution, or in other words, the presence of extreme price jumps not coming from a Gaussian distribution. This fact is also verified by very low $p$-values for the Jarque-Bera statistics, which rejects for all reported cases the null hypothesis that data come from an i.i.d. Gaussian distribution.

# 5 Results

I employ two price jump indicators to assess the price jump properties of four stock market indexes from the Visegrad region and Germany using high frequency data.[26] I employ returns at a 5-minute frequency, accompanied by returns at three lower frequencies (10, 15, and 30 minutes) for the sake of robustness. To fully explore the filtering properties, I take time windows equal to $T = 12$, 24, 100, 1000, 2000, and 5000 time steps and plot the distribution of the price jump index for all six time windows and all four frequencies. Since the literature suggests a deviation from power-law behavior,[27] in Figure 5.1 I plot the linearized version of equation (3.3) for the PX index.[28]

[26]The body of the main text presents the main results. Many complementary results are further presented in Appendix B.
[27]See Plerou et al. (1999) and Kleinert (2009).
[28]The other indexes show similar patterns and detailed results are available upon request.

Figure 5.1: Log transformed version of the tail part of the price jump index distribution for the PX index.




Note: The distribution was calculated using all four frequencies and six different time windows $T$. The two short-term windows have a more suppressed occurrence of extreme events compared to the four long-term windows. The symbols used in this table are: $T = 12$ (thick solid) , $T = 24$ (thick dash) , $T = 100$ (solid), $T = 1000$ (dash), $T = 2000$ (short dash), and $T = 5000$ (dash dot).

The figure clearly shows that the longer the time window $T$, the higher the probability of the occurrence of extreme events and that a short time window produces a non-linear distribution. Both observations are in agreement with the literature. Since the main scope of this paper is the domain of power-law behavior, I employ in the following the longest time window $T = 5000$ and estimate and present here characteristic coefficients $\alpha$ solely for this filter.

Table 5.1: Estimated characteristic coefficient $\alpha_T$ for the price jump index.

| Index | T | 5-minute $\alpha_T(\sigma_\alpha)$ | $t$ | 10-minute $\alpha_T(\sigma_\alpha)$ | $t$ | 15-minute $\alpha_T(\sigma_\alpha)$ | $t$ | 30-minute $\alpha_T(\sigma_\alpha)$ |
|---|---|---|---|---|---|---|---|---|
| PX | 5000 | 3.716 (0.035) | $\overset{a}{>}$ | 3.408 (0.030) | $\overset{a}{<}$ | 3.654 (0.030) | $\overset{a}{<}$ | 3.939 (0.077) |
| | 2000 | 4.021 (0.043) | $\overset{a}{<}$ | 4.337 (0.067) | $\overset{a}{>}$ | 4.085 (0.055) | $\overset{a}{<}$ | 4.411 (0.111) |
| | 1000 | 3.640 (0.035) | $\overset{a}{<}$ | 4.712 (0.097 | $\overset{a}{>}$ | 4.372 (0.067) | $\overset{a}{<}$ | 5.259 (0.129) |
| BUX | 5000 | 3.879 (0.032) | $\overset{a}{>}$ | 3.341 (0.028) | $\approx$ | 3.407 (0.030) | $\approx$ | 3.503 (0.062) |
| | 2000 | 4.814 (0.075) | $\overset{a}{>}$ | 3.739 (0.034) | $\approx$ | 3.692 (0.041) | $\approx$ | 3.678 (0.057) |
| | 1000 | 4.757 (0.066) | $\overset{a}{>}$ | 3.925 (0.075) | $\approx$ | 3.875 (0.059) | $\overset{b}{>}$ | 4.039 (0.050) |
| WIG | 5000 | 5.236 (0.087) | $\overset{a}{>}$ | 4.776 (0.092) | $\overset{a}{>}$ | 4.489 (0.097) | $\overset{a}{>}$ | 3.913 (0.067) |
| | 2000 | 5.202 (0.124) | $\approx$ | 5.405 (0.228) | $\overset{b}{>}$ | 5.395 (0.133) | $\overset{a}{>}$ | 4.009 (0.070) |
| | 1000 | 6.683 (0.248) | $\overset{c}{<}$ | 7.612 (0.415) | $\approx$ | 6.052 (0.179) | $\overset{a}{>}$ | 4.232 (0.124) |
| DAX | 5000 | 4.221 (0.043) | $\overset{a}{>}$ | 3.906 (0.030) | $\overset{a}{>}$ | 3.617 (0.054) | $\overset{a}{>}$ | 3.091 (0.027) |
| | 2000 | 4.723 (0.059) | $\overset{a}{>}$ | 4.285 (0.043) | $\overset{a}{>}$ | 3.718 (0.067) | $\overset{a}{>}$ | 3.126 (0.044) |
| | 1000 | 4.526 (0.044) | $\overset{a}{<}$ | 4.738 (0.048) | $\overset{a}{>}$ | 4.077 (0.080) | $\overset{a}{>}$ | 2.583 (0.068) |

Note: The estimation was done for all four indexes—PX (Prague Stock Exchange), BUX (Budapest Stock Exchange), WIG (Warsaw Stock Exchange), and DAX (Frankfurt Stock Exchange)—all four frequencies—5-, 10-, 15-, and 30-minute—and for time windows $T = 5000$, $T = 2000$, and $T = 1000$. The value in the brackets is the standard deviation. The higher the standard deviation, the worse the estimation of the characteristic coefficient was found. Column $t$ denotes the result of the $t$-test with the null hypothesis that the two estimated coefficients are equal. The inequality sign illustrates the relation between the estimated coefficients, when I can reject the null hypothesis of no difference between them. Superscripts $a$ , $b$, and $c$ denote the significance level at which I reject the null hypothesis: $a$ for 99%, $b$ for 95%, and $c$ for 90%. Asymptotic normal distributions were used.

## 5.1 Scaling of Price Jumps

I estimate the characteristic coefficients $\alpha$ for both price jump indicators using the linearized equation (3.3) and the algorithm in which I use the OLS regression maximizing the $R^2$. First, I report in Table 5.1 the estimated characteristic coefficients for the price jump index using all four indexes and all four frequencies. Comparing the characteristic coefficients for the highest frequency, the PX index has the lowest significant value of all the indexes. This suggests the presence of extreme price jumps on the PX index. At the other pole stands the WIG index.

The smaller frequencies also reveal another important pattern that further distinguishes the behavior of the PX index compared to the other three indexes. For the other three indexes, the characteristic coefficient decreases with decreasing frequency. This implies that more extreme events are present for lower frequencies, which is in agreement with general understanding. In the case of the

PX index; however, I observe significantly different behavior. Namely, the characteristic coefficient for the 30-minute frequency is higher than the characteristic coefficient for the 5-minute frequency, even exceeding the 95% confidence interval. This suggests the presence of specific determinants on the Prague Stock Exchange that cause a deviation from the rest of the group. I denote this deviation from the standard behavior as the "PX Puzzle".

There are several possible market-specific explanations for the PX Puzzle. First, it could be the role of dividends, since the PX index is a dividend-excluded index and a number of the major components of the PX index offer a dividend yield well above 5%. Therefore, the decline of the PX index due to the dividend day could send false signals, which are initially followed by extreme price movements; however, they are smoothed away very soon. If this were so, I would also observe a "WIG puzzle," but I do not. Second, one can further argue that the explanation of the PX Puzzle lies in the small turnover and liquidity of the exchange itself.[29] The price of stocks traded at such an exchange could easily be influenced, especially over a short period of time. This explanation uses the parallel between the volume of traded assets and the mass in dynamics. The heavier an object is (i.e., more liquid trading), the more effort has to be expended to make it move. Consequently, fast movements, when viewed from a longer perspective, are averaged out and the movements are not so jumpy. The capitalization of the PSE is not extremely high; some stocks have a very low free-float and a low frequency of trades. Such a combination can contribute to this phenomenon as well.

Finally, the market micro-structure point of view offers a complementary argument for the presence of the PX Puzzle. Namely, the representative investor at the Prague Stock Exchange is different from the one in Warsaw. The PSE is dominated by foreign investors, while WSE possesses a large number of domestic institutional investors, namely pension funds, which are obliged to invest in domestic assets. Further, the regulatory differences may also explain the observed PX Puzzle. The PSE has much weaker regulatory requirements especially on the side of margins, where investors may enjoy much higher leverage when compared to the other three exchanges.[30]

[29] Several of the major stocks listed at the PSE are actually cross-listed abroad on more mature and bigger stock exchanges. Sometimes for a short period local prices depart significantly and then—likely using arbitrage trading—the prices are quickly driven closer to the stock prices on the main (abroad) stock exchange.

[30] Fortune (2001) discusses the positive correlation between the rate of margin lending and market volatility.

Table 5.2: Up/down asymmetry for the price jump index and normalized returns.

| | 5-minute | | 10-minute | | 15-minute | | 30-minute | | |
|---|---|---|---|---|---|---|---|---|---|
| | PJI | NR | PJI | NR | PJI | NR | PJI | NR | $\sum$ |
| PX | $-^b$ | $+^a$ | $-^a$ | $+^a$ | $+^a$ | $+^a$ | $+^a$ | $+^a$ | $+^a$ |
| BUX | $-$ | $+^a$ | $-$ | $-^a$ | $-^a$ | $-^a$ | $-^a$ | $-^a$ | $-^a$ |
| WIG | $+^a$ | $+^a$ | $+^a$ | $+^a$ | $+^a$ | $+^a$ | $+^a$ | $+^a$ | $+^a$ |
| DAX | $+$ | $+^a$ | $+^c$ | $+^a$ | $+$ | $+^a$ | $+$ | $+^a$ | $+^a$ |
| $\sum$ | 0 | $+^a$ | $+^c$ | $+^a$ | $+^a$ | $+^a$ | $+^a$ | $+^a$ | |

Note: The estimation was done for all four indexes—PX, BUX, WIG, and DAX—and all four frequencies—5-, 10-, 15-, and 30-minute. The length of the time window is $T = 5000$. In each entry, the characteristic coefficients for positive and negative price jumps were compared. When computing the actual significance level, I used the asymptotic normality of estimated coefficients $\alpha$. The symbol + means that $\alpha_T^+$ is lower than $\alpha_T^-$, i.e., more price jumps are observed in the upward direction and similarly for the symbol $-$. Superscripts $a$ , $b$, and $c$ denote the significance level at which I reject the null hypothesis: $a$ for 99%, $b$ for 95%, and $c$ for 90%. In addition, the table presents the marginal effects with respect to frequencies and stock market indices.

## 5.2 Is There an Up/Down Asymmetry?

Intuitively, the distribution of extreme positive and negative price movements can be different. To assess this intuition on quantitative grounds, I estimate the characteristic coefficients for positive and negative price movements separately. For the normalized returns this modification comes naturally from the definition. In the case of the price jump index, I estimate the characteristic coefficients separately for positive and negative movements, while the average of absolute returns is composed of a given history no matter what the sign of the returns was. I focus on the quantitative comparison between price jumps up and down. Table 5.2 summarizes the results of a battery of pair-wise comparisons, with null hypothesis $H_0$: Means of positive and negative jumps are same, i.e., $\alpha_T^{(+)} = \alpha_T^{(-)}$. [31] A positive or negative mark in the table cells denotes whether I observe more price jumps up or down for a given indicator, index, and frequency, i.e., if a given cell contains the + symbol, the coefficient $\alpha_T^{(+)}$ is smaller than $\alpha_T^{(-)}$ and, thus, more extreme price jumps occur in the upward direction. In addition, the significance level of the test is denoted using superscripts $a$, $b$, and $c$ (1%, 5%, and 10%); no superscript means that the difference is not statistically significant.

Intuitively, one would expect that I will observe more negative extreme price movements than positive ones. In terms of the symbols used in Table 5.2, I should observe substantially more $-$

[31] For every index and every frequency, I calculated the characteristic coefficient separately for negative and positive movements. Then I conducted a test using the mean and standard errors of $\alpha_T^{(+)}$ and $\alpha_T^{(-)}$ using asymptotic normality.

than $+$. However, the reverse is true: $+$ dominates in the table in all rows and columns with the exception of the BUX index. If I can say that I observe an asymmetry, then CEE markets (with the exception of BUX) show significantly larger positive than negative extreme price movements. This result is robust since both price jump indicators show very similar patterns.

One can speculate that the fact that I observe larger positive extreme price movements can be caused by the data cut-offs at the beginning and end of the trading days. In other words, returns driving the intuitive asymmetry should occur mainly in the truncated period, i.e., shortly after the open and/or shortly before the close I should see significant drops in returns. Although the data shows some negative trends in the cut-off periods (see Figure 4.1), the downward movements at the beginning of trading days are not dominated by extreme price jumps but rather smooth adjustments with the same downward orientation (see Figure 4.2).

## 5.3 Stability of Results – Analysis by Quarters

The previous results were produced using the entire sample. However, the presence of business cycles with repeating peaks and troughs or the recent financial crisis may suggest that the price generating process is not stable over time, and I can thus expect a variation in extreme price movements. That is, $\alpha$ coefficients may not be stable over time. Therefore, I divide the data set into smaller sub-samples, repeat the computations on sub-samples, and test the stability of the characteristic coefficients $\alpha$ over time. Since the estimation of the characteristic coefficient requires a large amount of data, the shortest time period for computing $\alpha$'s and therefore for testing their stability is three months. For all stock market indexes, frequencies, up and down movements, and price jump indicators, I estimate the characteristic coefficients and perform a battery of tests, comparing the obtained results.

First, I repeat the analysis of the asymmetry between movements up and down using the quarterly estimated characteristic coefficients. I run the Wilcoxon rank-sum test described above to compare the sizes of the characteristic coefficients up and down for every stock market index and both price jump indicators (i.e., $H_0$: Ranks of the values of positive and negative jumps are the same, so $\tilde{\alpha}_T^{(+)} = \tilde{\alpha}_T^{(-)}$). I was not able to reject the hypothesis of the stability of the characteristic coefficients in a single case. Therefore, I conclude that the propensity of the studied price indexes to show price jumps are stable over time.

Table 5.3: Comparing indexes pair-wise.

| $F$ | $D$ | PJI | NR |
|---|---|---|---|
| 5 | $\uparrow$ | $\text{PX}\overset{c}{<}\text{DAX}\approx\text{WIG}\approx\text{BUX}$ | $\text{PX}\approx\text{DAX}\approx\text{WIG}\approx\text{BUX}$ |
| | $\uparrow$ | $\text{PX}\overset{a}{<}\text{WIG; DAX}\overset{c}{<}\text{BUX; PX}\overset{a}{<}\text{BUX}$ | $\text{PX}\overset{c}{<}\text{WIG; DAX}\overset{c}{<}\text{BUX; PX}\overset{b}{<}\text{BUX}$ |
| | $\downarrow$ | $\text{PX}\overset{b}{<}\text{DAX}\approx\text{BUX}\overset{b}{<}\text{WIG}$ | $\text{PX}\overset{c}{<}\text{BUX}\approx\text{DAX}\approx\text{WIG}$ |
| | $\downarrow$ | $\text{PX}\overset{b}{<}\text{BUX; DAX}\overset{a}{<}\text{WIG; PX}\overset{a}{<}\text{WIG}$ | $\text{PX}\overset{b}{<}\text{DAX; BUX}\overset{c}{<}\text{WIG; PX}\overset{a}{<}\text{WIG}$ |
| 10 | $\uparrow$ | $\text{PX}\approx\text{BUX}\approx\text{WIG}\approx\text{DAX}$ | $\text{PX}\approx\text{BUX}\approx\text{WIG}\approx\text{DAX}$ |
| | $\uparrow$ | $\text{PX}\overset{c}{<}\text{WIG; BUX}\approx\text{DAX; PX}<^{b}\text{DAX}$ | $\text{PX}\overset{c}{<}\text{WIG; BUX}\approx\text{DAX; PX}\overset{b}{<}\text{DAX}$ |
| | $\downarrow$ | $\text{PX}\overset{b}{<}\text{BUX}\approx\text{DAX}\approx\text{WIG}$ | $\text{PX}\approx\text{BUX}\approx\text{DAX}\approx\text{WIG}$ |
| | $\downarrow$ | $\text{PX}\overset{a}{<}\text{DAX; BUX}\approx\text{WIG; PX}\overset{a}{<}\text{WIG}$ | $\text{PX}\overset{b}{<}\text{DAX; BUX}\overset{b}{<}\text{WIG; PX}\overset{a}{<}\text{WIG}$ |
| 15 | $\uparrow$ | $\text{PX}\approx\text{BUX}\approx\text{WIG}\approx\text{DAX}$ | $\text{PX}\approx\text{BUX}\approx\text{WIG}\approx\text{DAX}$ |
| | $\uparrow$ | $\text{PX}\approx\text{WIG; BUX}\approx\text{DAX; PX}\overset{c}{<}\text{DAX}$ | $\text{PX}\approx\text{WIG; BUX}\approx\text{DAX; PX}\approx\text{DAX}$ |
| | $\downarrow$ | $\text{PX}\overset{c}{<}\text{WIG}\approx\text{DAX}\approx\text{BUX}$ | $\text{PX}\approx\text{BUX}\approx\text{DAX}\approx\text{WIG}$ |
| | $\downarrow$ | $\text{PX}\approx\text{DAX; WIG}\approx\text{BUX; PX}\overset{c}{<}\text{BUX}$ | $\text{PX}\overset{b}{<}\text{DAX; BUX}\approx\text{WIG; PX}\overset{b}{<}\text{WIG}$ |
| 30 | $\uparrow$ | $\text{PX}\approx\text{BUX}\approx\text{WIG}\approx\text{DAX}$ | $\text{PX}\approx\text{WIG}\approx\text{BUX}\approx\text{DAX}$ |
| | $\uparrow$ | $\text{PX}\approx\text{WIG; BUX}\approx\text{DAX; PX}\overset{b}{<}\text{DAX}$ | $\text{PX}\overset{c}{<}\text{BUX; WIG}\approx\text{DAX; PX}\overset{b}{<}\text{DAX}$ |
| | $\downarrow$ | $\text{PX}\approx\text{DAX}\approx\text{BUX}\approx\text{WIG}$ | $\text{PX}\approx\text{DAX}\approx\text{WIG}\approx\text{BUX}$ |
| | $\downarrow$ | $\text{PX}\approx\text{BUX; DAX}\approx\text{WIG; PX}\overset{b}{<}\text{WIG}$ | $\text{PX}\approx\text{WIG; DAX}\approx\text{BUX; PX}\approx\text{BUX}$ |

Note: I estimate the characteristic coefficients $\alpha^{\pm}$ for every quarter and every index. I compare the order of indexes using the (pair-wise) Wilcoxon rank-sum test. The test was performed for every frequency $F$ and both directions $D$: up $\uparrow$ and down $\downarrow$. The letter in the superscript of the inequality mark denotes the significance level at which I can reject the null hypothesis (i.e., the equality of coefficients): $a$ stands for 99%, $b$ for 95%, and $c$ for 90%.

Then, I reverse the approach to the data and study the difference between stock market indexes using all frequencies and both price jump indicators with separate directions. Table 5.3 depicts the pair-wise mutual ordering of indexes for every frequency, both price jump indicators and both directions. The pair-wise comparison was performed using a battery of Wilcoxon rank-sum tests (see Mann and Whitney, 1947). If the difference is significant, I denote the inequality sign with the significance level $a$ for 99% , $b$ for 95%, and $c$ for 90%. Otherwise, I use the symbol $\approx$ for indexes with (statistically) equal $\alpha$. Let me note that the results may show that all four indexes are pair-wise indistinguishable even though the difference between those on the different sides of the relation is statistically significant.

The results show that major differences are observed on higher frequencies, something I would expect intuitively. Another pattern, which is consistent across all frequencies, both jump price indexes, and up and down movements, is the fact that PX was clearly the most jumpy index (i.e., smallest $\alpha$) while WIG presented the smallest propensity to jump.

### 5.4 The Effect of Financial Crisis on Price Jumps

We can follow the strategy of pair-wise comparisons and ordering the indexes by their jumpiness or more precisely by their characteristic coefficient $\alpha$ also for periods before, during, and after the economic crisis. For that purpose I define the financial crisis as the period that started at 2009/Q1 and lasted until 2010/Q4, thus eight quarters in total.[32] The above ordering of the indexes and a comprehensive summary of all non-parametric comparisons are presented in Table 5.4.

I used the Wilcoxon test to compare the characteristic coefficients before and during the crisis for every stock market index, every frequency and both price jump indicators with both directions separately. I have also included the difference between the characteristic coefficient up and down, or $\Delta = \alpha^{Up} - \alpha^{Down}$.

Table 5.5 contains the Wilcoxon statistics for the test between the period before the crisis and the period during the crisis. The results suggest that there was no prevalent change in the price jump component of the price-generating process. The difference between the characteristic coefficients up and down further strengthens the findings and suggests, except in three cases, no change before or during the crisis. This result is in agreement with Novotny (2010) and suggests that there was either no change in the underlying price generating process at all or the entire price generating process was scaled up in such a way that the distribution of extreme price movements was similar.[33] Using Figure 5.2, which depicts the standard deviation of returns for all four stock market indexes at a 5-minute frequency, I can conclude that the latter explanation is the case, i.e., the overall price generating process scaled up during the crisis, but the rate of price jumps remained untouched.

To illustrate the behavior of the characteristic coefficients more closely. Figure 5.3 contains the estimated characteristic coefficient quarterly for the price jump index and at a 5-minute frequency. The figure further contains $\pm\sigma$ bands of the estimated coefficient. The figure suggests several quarters with unusually high values of the estimated coefficient, which rather appear to be outliers

[32]For robustness I also consider several different specifications, for example with the crisis starting at 2008/Q4 (nine quarters of financial crisis). The results were very similar, therefore I do not present them here but they are available upon request.

[33]The absolute size of the extreme price movements depends on the current overall volatility; therefore, in the latter case, the absolute size of price jumps would be different.

Table 5.4: Comparing indexes pair-wise: Before and during the crisis.

| $F$ | $P$ | $D$ | PJI | NR |
|---|---|---|---|---|
| 5 | $B$ | ↑ | PX≈DAX$\overset{c}{<}$BUX≈WIG | PX≈DAX≈BUX≈WIG |
| | | ↑ | PX$\overset{a}{<}$BUX; DAX$\overset{b}{<}$WIG; PX$\overset{a}{<}$WIG | PX$\overset{b}{<}$BUX; DAX$\overset{b}{<}$WIG; PX$\overset{a}{<}$WIG |
| | | ↓ | PX$\overset{b}{<}$DAX≈BUX$\overset{c}{<}$WIG | PX$\overset{c}{<}$DAX≈BUX$\overset{c}{<}$WIG |
| | | ↓ | PX$\overset{b}{<}$BUX; DAX$\overset{a}{<}$WIG; PX$\overset{a}{<}$WIG | PX$\overset{c}{<}$BUX; DAX$\overset{b}{<}$WIG; PX$\overset{a}{<}$WIG |
| | $C$ | ↑ | WIG≈PX≈DAX≈BUX | WIG≈PX≈DAX≈BUX |
| | | ↑ | WIG$\overset{c}{<}$DAX; PX≈BUX; WIG$\overset{b}{<}$BUX | WIG≈DAX; PX≈BUX; WIG$\overset{a}{<}$BUX |
| | | ↓ | PX≈BUX≈DAX≈WIG | PX≈BUX≈WIG≈DAX |
| | | ↓ | PX≈DAX; BUX≈WIG; PX$\overset{c}{<}$WIG | PX≈WIG; BUX≈DAX; PX$\overset{c}{<}$DAX |
| 10 | $B$ | ↑ | PX≈BUX≈DAX≈WIG | PX≈BUX≈DAX≈WIG |
| | | ↑ | PX$\overset{b}{<}$DAX; BUX≈WIG; PX$\overset{b}{<}$WIG | PX$\overset{b}{<}$DAX; BUX≈WIG; PX$\overset{b}{<}$WIG |
| | | ↓ | PX$\overset{b}{<}$DAX≈BUX≈WIG | PX≈BUX≈DAX≈WIG |
| | | ↓ | PX$\overset{b}{<}$BUX; DAX≈WIG; PX$\overset{a}{<}$WIG | PX$\overset{c}{<}$DAX; BUX≈WIG; PX$\overset{a}{<}$WIG |
| | $C$ | ↑ | WIG≈PX≈BUX≈DAX | BUX≈PX≈WIG≈DAX |
| | | ↑ | WIG≈BUX; PX≈DAX; WIG$\overset{b}{<}$DAX | BUX≈WIG; PX≈DAX; BUX≈DAX |
| | | ↓ | PX≈BUX≈WIG≈DAX | BUX≈PX≈WIG≈DAX |
| | | ↓ | PX$\overset{c}{<}$WIG; BUX$\overset{b}{<}$DAX; PX$\overset{b}{<}$DAX | BUX≈WIG; PX$\overset{c}{<}$DAX; BUX$\overset{b}{<}$DAX |
| 15 | $B$ | ↑ | PX≈BUX≈DAX≈WIG | PX≈BUX≈DAX≈WIG |
| | | ↑ | PX≈DAX; BUX≈WIG; PX≈WIG | PX≈DAX; BUX≈WIG; PX≈WIG |
| | | ↓ | PX≈DAX≈WIG≈BUX | PX$\overset{c}{<}$WIG≈DAX≈BUX |
| | | ↓ | PX$\overset{c}{<}$WIG; DAX≈BUX; PX$\overset{b}{<}$BUX | PX$\overset{b}{<}$DAX; WIG≈BUX; PX$\overset{c}{<}$BUX |
| | $C$ | ↑ | PX≈WIG≈BUX≈DAX | PX≈WIG≈BUX≈DAX |
| | | ↑ | PX≈BUX; WIG≈DAX; PX$\overset{c}{<}$DAX | PX≈BUX; WIG≈DAX; PX≈DAX |
| | | ↓ | BUX≈PX≈WIG≈DAX | BUX≈PX≈DAX≈WIG |
| | | ↓ | BUX≈WIG; PX≈DAX; BUX≈DAX | BUX≈DAX; PX≈WIG; BUX≈WIG |
| 30 | $B$ | ↑ | PX≈ BUX≈DAX≈WIG | PX≈DAX≈WIG≈BUX |
| | | ↑ | PX$\overset{c}{<}$DAX; BUX≈WIG; PX$\overset{c}{<}$WIG | PX≈WIG; DAX≈BUX; PX≈BUX |
| | | ↓ | PX≈DAX≈BUX≈WIG | PX≈WIG≈DAX≈BUX |
| | | ↓ | PX≈BUX; DAX≈WIG; PX$\overset{c}{<}$WIG | PX≈DAX; WIG≈BUX; PX≈BUX |
| | $C$ | ↑ | BUX≈PX≈WIG≈DAX | PX≈WIG≈BUX≈DAX |
| | | ↑ | BUX≈WIG; PX≈DAX; BUX≈DAX | PX≈BUX; WIG≈DAX; PX$\overset{c}{<}$DAX |
| | | ↓ | BUX≈DAX≈PX≈WIG | DAX≈WIG≈PX≈BUX |
| | | ↓ | BUX≈PX; DAX≈WIG; BUX≈WIG | DAX≈PX; WIG≈BUX; DAX≈BUX |

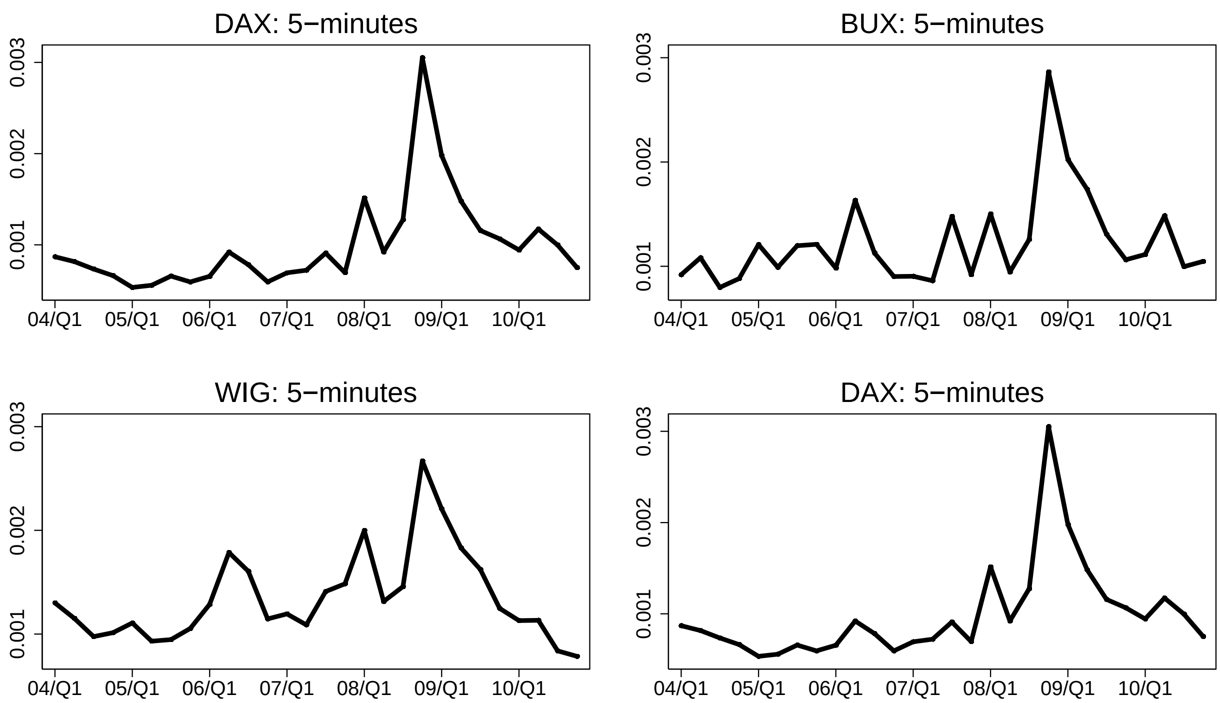
Note: I estimate the characteristic coefficients $\alpha^{\pm}$ for every quarter and every index. I employ the Wilcoxon test to compare the order of indexes pair-wise. The test was performed for every frequency $F$, both directions $D$: up ↑ and down ↓, and both phases $P$: before the crisis $B$ and during the crisis $C$. The financial crisis is defined as the period starting at 2009/Q1 and lasting until the end of the sample at 2010/Q4. A letter denotes the significance level at which I can reject the null hypothesis: $a$ for 99%, $b$ for 95%, and $c$ for 90%. This is the significance at which I can say that the indexes are different.

Table 5.5: Financial crisis defined as 2009/Q1 – 2010/Q4.

| Index | $D$ | 5-minutes PJI | 5-minutes NR | 10-minutes PJI | 10-minutes NR | 15-minutes PJI | 15-minutes NR | 30-minutes PJI | 30-minutes NR |
|---|---|---|---|---|---|---|---|---|---|
| PX | $\uparrow$ | -2.38$^b$ | -2.00$^b$ | -1.38 | -1.05 | 0.44 | -0.11 | -1.16 | -0.22 |
| | $\downarrow$ | -2.22$^b$ | -1.33 | -1.27 | -1.55 | -1.22 | -1.22 | -1.38 | -1.05 |
| | $(\uparrow - \downarrow)$ | -0.83 | -1.69$^c$ | -0.56 | -0.72 | 1.10 | 0.24 | -0.13 | 0.78 |
| BUX | $\uparrow$ | -1.55 | -0.50 | -1.33 | -0.44 | -0.55 | -0.33 | -0.44 | -0.44 |
| | $\downarrow$ | -0.50 | -0.94 | 0.50 | 0.44 | -0.16 | 0.05 | -0.11 | -0.94 |
| | $(\uparrow - \downarrow)$ | -0.61 | -0.40 | -1.53 | -1.10 | -0.67 | -1.05 | -0.08 | 0.29 |
| WIG | $\uparrow$ | 2.11$^b$ | 1.88$^c$ | 1.77$^c$ | 0.22 | 0.66 | 0.44 | -0.33 | -0.11 |
| | $\downarrow$ | 0.22 | 0.38 | 0.16 | 0.33 | -1.44 | -1.22 | -1.16 | -0.38 |
| | $(\uparrow - \downarrow)$ | 0.72 | 1.21 | 1.15 | -0.83 | 1.26 | 1.80$^c$ | 0.35 | -0.45 |
| DAX | $\uparrow$ | -1.61 | -1.72$^c$ | -1.88$^c$ | -1.61 | -1.33 | -1.38 | -1.00 | -2.27$^b$ |
| | $\downarrow$ | -1.50 | -2.00$^b$ | -1.55 | -1.27 | -1.44 | -1.50 | 0.16 | 0.00 |
| | $(\uparrow - \downarrow)$ | 1.26 | 1.42 | -0.51 | -0.18 | 0.24 | 0.40 | -1.42 | -1.96$^b$ |

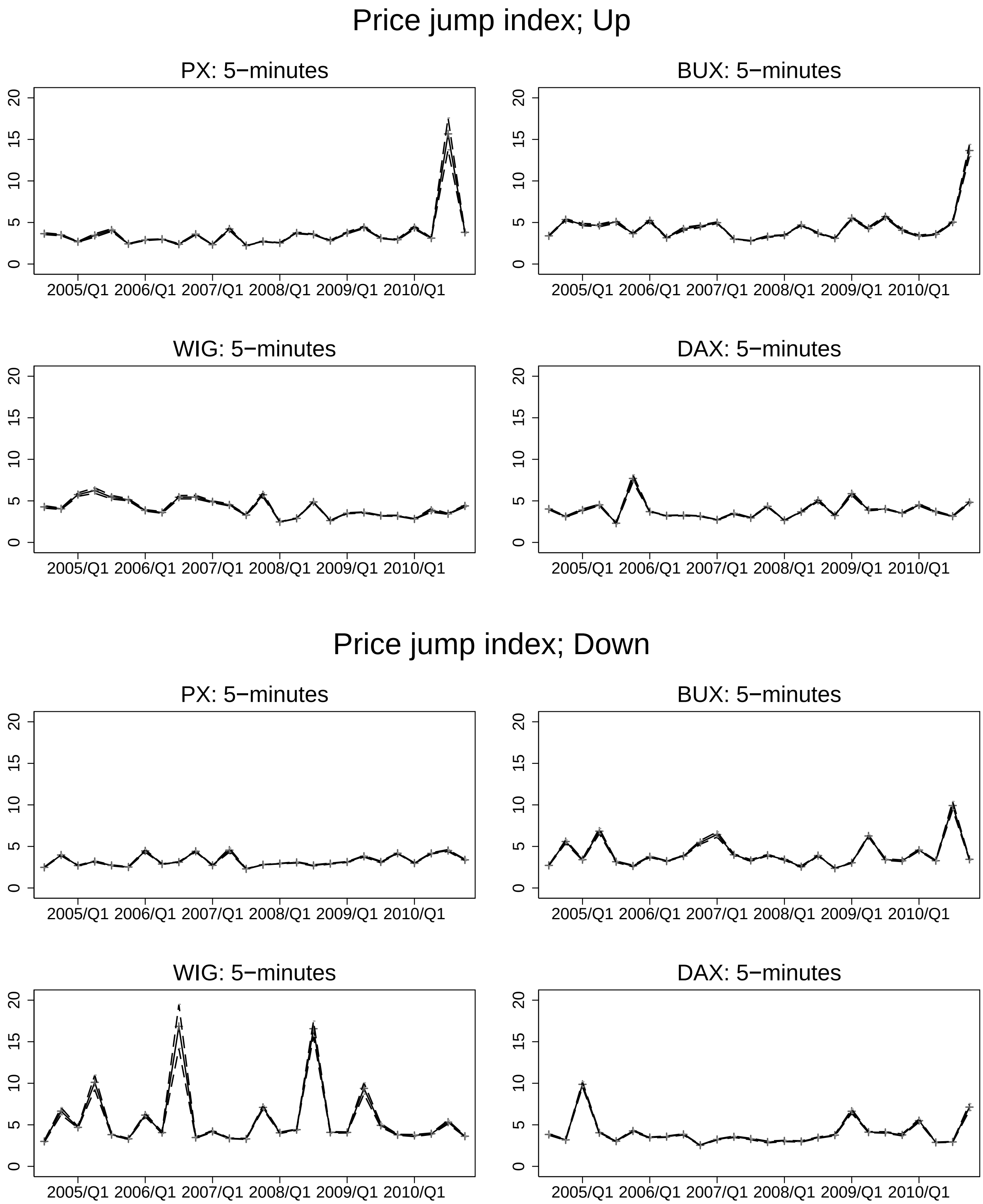
Note: I estimate the characteristic coefficients $\alpha^{\pm}$ for every quarter and every index. I employ the Wilcoxon test to compare the characteristic coefficients for the periods before the financial crisis and during the financial crisis. The crisis is defined as the period starting at 2009/Q1 and lasting until 2010/Q4. The directions of the price jumps $D$ are as follow: symbol $\uparrow$ stands for price movements up, the symbol $\downarrow$ for movements down, and symbol $(\uparrow - \downarrow)$ stands for the difference between them. A letter denotes the significance level at which I can reject the null hypothesis: $a$ for 99%, $b$ for 95%, and $c$ for 90%. That is the significance at which I can say that the indexes are different. A positive (negative) value in a cell means that the median of the distribution for the characteristic coefficients $\alpha$ before the crisis is greater (smaller) than the median for $\alpha$ during the crisis.

Figure 5.2: Standard deviation for returns by quarter.




Note: Plotted is the standard deviation by quarters for 5-minute returns.

Figure 5.3: Estimated characteristic coefficient.




Note: Plotted is the characteristic coefficient with $\pm\sigma$ bands.

in the estimation. The results do not confirm the intuitive hypothesis that during the financial crisis, the estimated characteristic coefficients are significantly and in this case visibly lower than before the crisis.

## 6 Conclusion

I performed an extensive analysis of price jumps using high-frequency data (5-, 10-, 15-, and 30-minute frequencies) for three emerging stock market indexes (PX, BUX, and WIG20) from the CEE Visegrad region. As a benchmark representing a geographically close and mature EU market, I use the German DAX index. The time period of the data is June 2003 to December 2010. For my analysis I employed two different indicators of price jumps: the price jump index and normalized returns. The analysis of returns revealed that the data substantially deviates from a Gaussian distribution and tends to support the presence of price jumps. I also analyze the intuitive asymmetry to observe more larger negative extreme price movements compared to positive ones. However, the reverse is true and the intuitive asymmetry favoring negative price jumps does not hold, moreover, this result was robustly confirmed by both indicators.

Further, the Prague Stock Exchange differs with respect to the presence of price jumps when lower frequencies are used. Based on the theory, one would assume that the lower the frequency, the more price jumps will be observed. However, the PX index reveals almost the opposite behavior, supporting the hypothesis that the behavior of the PX index significantly differs from the remaining three market indexes. One can speculate that this difference could be explained by the composition of the PX index: a small number of components, a relatively high number (and weight) of stocks with dual trading, prices determined in other exchanges, and some components not being traded with high frequency. Simply, a relatively small number of trades with a few stocks could have a large impact on the entire PX index. These explanations, however, would need additional analysis and the market micro-structure perspective should be tested across the markets, which is beyond the scope of this study.

I have estimated the price jump properties quarter by quarter. This allows me to compare the estimated characteristic coefficients across stock market indexes and over time. I have thus employed quarterly estimates and the Wilcoxon test and shown that there is no significant difference in the distributions of the characteristic coefficients up with respect to those moving down. Further, I have answered the question whether the price-generating process, or its price jump component, differs for all stock market indices. The results of the Kruskal-Wallis test used with quarterly estimates suggests a deviation among the indexes for high-frequency returns. A detailed pair-wise comparison using the Wilcoxon test revealed that it is the PX index that causes the disagreement and the results thus further support the presence of the PX Puzzle. Another pattern which is consistent across all frequencies for both price jump indexes and up and down movements show that the PX was clearly the most jumpy index while WIG had the smallest propensity to jump. This calls for further research, suggesting a link between the market micro-structure and jump propensity. In particular, higher market volatility and also higher propensity to jump is explained by differences in the population of investors (Prague is dominated by foreign investors, while Warsaw is dominated by strong domestic institutional investors, namely pension funds), differences in the regulatory framework in Prague, (where there are much weaker margin regulatory requirements and much higher leverage possibilities).

Finally, I tested for the stability of the price jump component over time and in particular during the recent financial crisis. The statistical tests suggest that the price jump component is stable before and during the financial crisis, although there are a few cases when the processes were different. These disagreements occurred especially for high-frequency data.

Overall, I aim to cast light on the issue of extreme price movements which frighten both market practitioners and financial regulators in the environment of small emerging markets. A quantitative understanding of price jumps can obviously help to decrease the risk connected with irregular but abrupt price changes and can be used to develop various financial models. The empirical analysis presented in this study can also serve as a starting point for a larger study of the integration of financial markets, including the role of market micro structure and the regulation of price jumps.

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## Appendix A – Stability of the Results with Respect to Cut-offs

The estimations of the characteristic coefficients $\alpha$ intuitively depend on the cut-off of the data at the beginning and end of the trading day. The intuition stems from the well-known $U$-shape in the intraday volatility. which is also confirmed by Figure 4.2. Therefore, I perform a sensitivity test of the estimation of characteristic coefficients with respect to the cut-off at the beginning and end of the trading day.

Namely, I estimate the characteristic coefficient for the year 2010 using all four stock market indexes, four frequencies, and $T = 5000$ with three different cut-offs: First, I cut-off 30 minutes at the beginning and end of the trading day. Second, I cut-off 20 minutes at both sides of the trading and, finally, I cut-off 10 minutes. I employ the OLS algorithm described in the main section and estimate characteristic coefficients. Then, I employ a $t$-test to compare in a pair-wise manner the estimated characteristic coefficients estimated from different cut-offs but using the same stock market index and frequency. The null hypothesis of such a test states that both estimated characteristic coefficients–corresponding to two different cut-offs–are statistically the same. The alternative hypothesis states the opposite.

The OLS algorithm itself decides what part of the tail is optimal for estimation, thus the number of points in every estimation is in general different. Therefore, the asymptotic distribution of such a test changes for every estimation and is equal to $\chi^2_{n_1+n_2-2}$, where $n_1$ corresponds to the number of points for the estimation of the first characteristic coefficient and $n_2$ for the second characteristic coefficient.

The results of the sensitivity test are summarized in Table 6.1. For every stock market index and every frequency, I present three results corresponding to pair-wise comparisons of 30-minute vs. 20-minute cut-offs, 20-minute vs. 10-minute cut-offs, and 10-minute vs. 30-minute cut-offs. The entry in the table contains $t$-statistics and level of significance at which I can reject the null hypothesis about the equality of the two estimates. In addition, the sign of the $t$-statistic suggests the direction of the inequality. Namely, a positive sign suggests that the characteristic coefficient corresponding to the first member of the pair is bigger than the other one, and vice versa for a negative sign.

The results does not suggest dramatic sensitivity of the estimation with respect to the length of

Table 6.1: Sensitivity test with respect to the cut-off period.

| | | 30 vs. 20 | 20 vs. 10 | 10 vs. 30 |
|---|---|---|---|---|
| PX | 5-minute | $4.244^a$ | $-0.646$ | $-3.193^a$ |
| | 10-minute | $-0.237$ | $0.580$ | $-0.176$ |
| | 15-minute | $0.641$ | $6.896^a$ | $-4.350^a$ |
| | 30-minute | $-1.195$ | $-0.694$ | $13.062^a$ |
| BUX | 5-minute | $-1.155$ | $1.266$ | $-0.070$ |
| | 10-minute | $-0.908$ | $-0.550$ | $1.462$ |
| | 15-minute | $2.268^b$ | $-4.153^a$ | $0.836$ |
| | 30-minute | $0.577$ | $-2.581^b$ | $2.162^c$ |
| WIG | 5-minute | $-3.002^b$ | $-0.159$ | $2.765^b$ |
| | 10-minute | $-1.011$ | $-1.169$ | $2.202^c$ |
| | 15-minute | $-1.167$ | $1.115$ | $0.343$ |
| | 30-minute | $2.353^c$ | $1.505$ | $-3.269^b$ |
| DAX | 5-minute | $-2.039^c$ | $0.172$ | $1.697$ |
| | 10-minute | $0.050$ | $-1.165$ | $1.611$ |
| | 15-minute | $2.157^c$ | $-4.643^a$ | $0.389$ |
| | 30-minute | $-0.254$ | $-3.468^b$ | $3.476^a$ |

Note: Each entry contains $t$-statistics for the null hypothesis that the two estimated characteristic coefficients $\alpha$ corresponding to two different cut-offs are the same. Due to the nature of the OLS algorithm, the asymptotic distribution of the $t$-tests is equal to $\chi^2_{n_1+n_2-2}$, where $n_1$ corresponds to the number of points for the estimation of the first characteristic coefficient and $n_2$ for the second characteristic coefficient. Letters state at which significance level the null hypothesis can be rejected, namely: $a$ for 99%, $b$ for 95%, and $c$ for 90%. In addition, the positive sign of the $t$-statistics suggests that the characteristic coefficient corresponding to the first member of the pair is bigger than the other one, and vice versa for the negative sign.

the cut-off although there are significant deviations, i.e., there are entries where $t$-statistics suggests that one can reject the null hypothesis and rather accept the alternative hypothesis. The results suggest that most of the differences can be found for a pair of 10-minute vs. 30-minute cut-offs. This is intuitive since these two pairs corresponds to two time series which are the most different with each other.

The biggest discrepancy is for the Prague Stock Exchange, and 30-minutes frequency for a pair of 10-minute and 30-minute cut-offs. The $t$-statistic of 13.062 suggests that the characteristic coefficient for a 10-minute cut-off is bigger than the one for the 30-minute cut-off. This further strengthens the presence of the PX Puzzle of the decreasing probability to observe a price jump for lower frequencies for the year 2010.

Table 6.2: Estimated characteristic coefficient $\alpha_T^{\pm}$ for the price jump index.

| $\alpha_T^{\pm}(\sigma_\alpha)$ | | 5-minute | 10-minute | 15-minute | 30-minute |
|---|---|---|---|---|---|
| PX | + | 3.739 (0.033) | 3.729 (0.062) | 3.363 (0.048) | 3.462 (0.086) |
| | - | 3.624 (0.041) | 3.128 (0.033) | 3.854 (0.067) | 3.782 (0.065) |
| BUX | + | 3.919 (0.045) | 3.384 (0.025) | 3.746 (0.074) | 3.688 (0.081) |
| | - | 3.848 (0.035) | 3.354 (0.041) | 3.300 (0.059) | 3.397 (0.061) |
| WIG | + | 4.305 (0.078) | 4.261 (0.086) | 3.959 (0.084) | 3.171 (0.091) |
| | - | 7.396 (0.368) | 5.225 (0.187) | 4.827 (0.191) | 4.096 (0.159) |
| DAX | + | 3.942 (0.038) | 3.885 (0.049) | 3.396 (0.056) | 2.875 (0.044) |
| | - | 4.409 (0.054) | 4.004 (0.040) | 3.771 (0.069) | 3.348 (0.045) |

Note: The estimation was done for all four indexes—PX, BUX, WIG, and DAX—using all four frequencies—5-, 10-, 15-, and 30-minute—and the time window $T = 5000$. The characteristic coefficient is calculated separately for upward movements (+) and downward movements ($-$). The value in the bracket is the standard deviation.

## Appendix B – Complementary Estimations

This appendix comprises further complementary results which do not fit into the main body.

### Addendum: Is There an Up/Down Asymmetry?

Intuitively, the distribution of extreme positive and negative price movements can be different. To assess this intuition on quantitative grounds, I estimate the characteristic coefficients for positive and negative price movements separately. For normalized returns, this modification comes naturally from the definition. In the case of the price jump index, I estimate the characteristic coefficients separately for positive and negative movements, while the average of absolute returns is composed of a given history no matter what the sign of the returns was.

Table 6.2 contains estimates using the price jump index, while Table 6.3 contains the estimates using normalized returns. Characteristic coefficients for positive and negative movements estimated separately are presented here for all four indexes using all four frequencies and the longest time window.

### Stability of Results – Analysis by Quarters

The previous results were obtained using the entire sample. However, the presence of business cycles with ever repeating peaks and troughs or the emergence of the recent financial crisis rather suggests that the price generating process is not stable over time and one can thus expect a variation

Table 6.3: Estimated characteristic coefficient $\alpha_T^{\pm}$ for normalized returns.

| $\alpha(\sigma_\alpha)$ | | 5-minute | 10-minute | 15-minute | 30-minute |
|---|---|---|---|---|---|
| PX | + | 3.731 (0.015) | 3.741 (0.027) | 3.768 (0.048) | 3.945 (0.052) |
| | - | 3.949 (0.023) | 3.877 (0.030) | 4.125 (0.031) | 3.975 (0.048) |
| BUX | + | 3.815 (0.023) | 4.008 (0.029) | 3.895 (0.028) | 3.663 (0.046) |
| | - | 5.015 (0.046) | 3.310 (0.023) | 3.387 (0.050) | 3.505 (0.033) |
| WIG | + | 4.681 (0.033) | 4.530 (0.041) | 3.821 (0.043) | 3.093 (0.034) |
| | - | 7.103 (0.172) | 5.321 (0.074) | 5.564 (0.098) | 4.946 (0.140) |
| DAX | + | 4.144 (0.026) | 3.398 (0.036) | 3.915 (0.049) | 2.447 (0.023) |
| | - | 4.850 (0.036) | 4.711 (0.039) | 4.139 (0.029) | 3.360 (0.029) |

Note: The estimation was done for all four indexes: PX, BUX, WIG, and DAX. The length of the time window is $T = 5000$. The value in the bracket is the standard deviation. The estimated characteristic coefficients are for both negative ($-$) and positive ($+$) sides of the normalized returns. Parameters were estimated using the standard OLS algorithm.

in the extreme price movements. Therefore, I divide the sample into shorter sub-samples and do the analysis on these sub-samples. Since the estimation of the characteristic coefficient requires large statistics, the shortest suitable period available for this purpose are quarters. Therefore, for every stock market index, every frequency and both price jump indicators, I estimate characteristic coefficients corresponding to price movements up and down separately.

First, I repeat the analysis of the asymmetry between movements up and down using the quarterly estimated characteristic coefficients. I run the Wilcoxon rank-sum test described above to compare the medians of the characteristic coefficients up and down for every stock market index and both price jump indicators.

Table 6.4 contains the $z$-values of the Wilcoxon statistics. The results do not suggest the rejection of the null hypothesis stating that there is no difference between the two samples of the characteristic coefficients. This, however, does not directly answers the question about asymmetry since I explicitly broke the link between two characteristic coefficients corresponding to the same quarter. The results suggest that the difference, or asymmetry, between characteristic coefficients up and down is rather subtle and may potentially switch quarter by quarter.

Then, I change the way how I approach the data and study the difference between stock market indexes using all frequencies and both price jump indicators with separated up/down directions. I employ the Kruskal-Wallis non-parametric test and test the null hypothesis that the four samples of characteristic coefficients obtained for all quarters independently come from the same distribution.

Table 6.4: Up/down asymmetry for the price jump index and normalized returns calculated quarterly.

| | 5-minute | | 10-minute | | 15-minute | | 30-minute | |
|---|---|---|---|---|---|---|---|---|
| | PJI | NR | PJI | NR | PJI | NR | PJI | NR |
| PX | -0.01 | -0.43 | 0.01 | 0.20 | -0.29 | -0.60 | -0.05 | 0.21 |
| BUX | -1.28 | -1.48 | 0.01 | -0.05 | 0.45 | -0.20 | -0.38 | 0.05 |
| WIG | 1.35 | 0.95 | 0.53 | -0.16 | 0.16 | 0.05 | 0.18 | -0.32 |
| DAX | -0.20 | 0.58 | 0.51 | 0.01 | 0.01 | -0.34 | -1.33 | -1.42 |

Note: I have estimated characteristic coefficients $\alpha^{\pm}$ for every quarter and every index and employed the Wilcoxon test to compare the distributions of the characteristic coefficient up and down. The table captures the $z$-value for the price jump index and for normalized returns. The $H_0$ states there is no asymmetry. Superscripts $a$ , $b$, and $c$ denote the significance level at which I reject the null hypothesis: $a$ for 99%, $b$ for 95%, and $c$ for 90%. The positive/negative value means that the median of the distribution for characteristic coefficients $\alpha^{+}$ is greater/smaller than the median for $\alpha^{-}$. This means that price jumps down are more/less likely than price jumps up.

Table 6.5: Comparing indexes using characteristic coefficients.

| | 5-minute | | 10-minute | | 15-minute | | 30-minute | |
|---|---|---|---|---|---|---|---|---|
| | PJI | NR | PJI | NR | PJI | NR | PJI | NR |
| Up | $12.97^{a}$ | $7.69^{c}$ | 5.86 | $6.27^{c}$ | 2.84 | 2.60 | 4.27 | 4.65 |
| Down | $19.09^{a}$ | $12.05^{a}$ | $13.40^{a}$ | $11.51^{a}$ | 4.76 | 6.18 | 4.64 | 1.63 |

Note: I have estimated characteristic coefficients $\alpha^{\pm}$ for every quarter and every index. I have employed the Kruskal-Wallis test to compare the distributions of characteristic coefficients for all four stock market indexes, where $H_0$ states that all four indexes have characteristic coefficients drawn from the same distribution. Entry in the table corresponds to the $\chi^2_3$-statistics of this test. The letter denotes the significance level at which I reject the null hypothesis: $a$ for 99%, $b$ for 95%, and $c$ for 90%, i.e., significance at which I can say that the indexes are different.

Table 6.6: Comparing indexes using rankings.

| | 5-minute | | 10-minute | | 15-minute | | 30-minute | |
|---|---|---|---|---|---|---|---|---|
| | PJI | NR | PJI | NR | PJI | NR | PJI | NR |
| Up | $16.22^a$ | $8.62^b$ | $11.19^b$ | $12.05^a$ | 3.82 | 3.71 | $7.08^c$ | 3.82 |
| Down | $25.37^a$ | $12.74^a$ | $15.48^a$ | $14.39^a$ | $6.74^c$ | 3.02 | 3.65 | 1.19 |

Note: I have estimated characteristic coefficients $\alpha^{\pm}$ for every quarter and every index. Then, I have sorted out stock market indexes for every quarter according to the estimated characteristic coefficient. I have employed the Kruskal-Wallis test to compare the distributions of rankings, where $H_0$ states that all four indexes have characteristic coefficients drawn from the same distribution. Entry in the table corresponds to the $\chi^2_3$-statistics of this test. The letter denotes the significance level at which I reject the null hypothesis: $a$ for 99%, $b$ for 95%, and $c$ for 90%, i.e., significance at which I can say that the indexes are different.

Table 6.5 contains the Kruskal-Wallis test statistics for every frequency, every price jump indicator and every direction. The statistics follow asymptotically $\chi^2_3$ distribution. The results show that for higher frequencies, I can reject the null hypothesis that stock market indexes have characteristic coefficients coming from the same distribution, or, put in plain English, the indexes have different underlying price generating processes. The lower frequencies, on the other hand, do not allow me to reject the null hypothesis. I can therefore conclude that differences between the stock exchanges are prevalent at higher frequencies and for lower frequencies, the stock markets seems to be closer to each other.

I further deepen the previous analysis and focus on ordering of the stock market indexes with respect to the estimated characteristic coefficient at every quarter using all four frequencies and both price jump indicators with both directions separately. Thus, I translated the characteristic coefficients into rankings having values between 1 and 4.[34]

Table 6.6 contains the Kruskal-Wallis statistics for distribution of rankings performed in a similar manner as the test with characteristic coefficients. The results agree with those based on the characteristic coefficient and the disagreement between the stock market indexes is even more pronounced at higher frequencies–higher confidence levels at which I can reject the null hypothesis. In addition, there is also a small disagreement at lower frequencies. Hence, by combining both results, I can conclude that the price jump component part of the underlying price generating process is different for higher frequencies and vanishes with decreasing the sampling frequency.

[34]The convention is that 1 is assigned to the stock market index with the lowest characteristic coefficient for this quarter using a particular price jump indicator and direction. If two or more stock market indexes have the same value os characteristic coefficients, I assign them the lowest rank.

Table 6.7: Comparing indexes pair-wise.

| | | 5-minute | | 10-minute | | 15-minute | | 30-minute | |
|---|---|---|---|---|---|---|---|---|---|
| | | PJI | NR | PJI | NR | PJI | NR | PJI | NR |
| $\frac{PX}{BUX}$ | U | $-3.32^{a}$ | $-2.37^{b}$ | -1.44 | -0.86 | -0.91 | -0.95 | -1.22 | $-1.70^{c}$ |
| | D | $-2.15^{b}$ | $-1.77^{c}$ | $-1.99^{b}$ | -0.84 | $-1.83^{c}$ | -1.13 | -0.91 | -1.46 |
| $\frac{PX}{WIG}$ | U | $-2.70^{a}$ | $-1.79^{c}$ | $-1.77^{c}$ | $-1.93^{c}$ | -1.31 | -1.18 | -1.48 | -1.20 |
| | D | $-4.02^{a}$ | $-3.09^{a}$ | $-3.33^{a}$ | $-3.18^{a}$ | $-1.86^{c}$ | $-2.28^{b}$ | $-1.99^{b}$ | -0.42 |
| $\frac{PX}{DAX}$ | U | $-1.84^{c}$ | -1.46 | $-2.39^{b}$ | $-1.99^{b}$ | $-1.64^{c}$ | -1.51 | $-1.99^{b}$ | $-2.03^{b}$ |
| | D | $-2.06^{b}$ | $-2.17^{b}$ | $-2.94^{a}$ | $-2.39^{b}$ | -1.61 | $-2.25^{b}$ | -1.02 | -0.43 |
| $\frac{BUX}{WIG}$ | U | 0.29 | 0.98 | -0.07 | -1.40 | -0.43 | -0.49 | -0.34 | 0.31 |
| | D | $-2.39^{b}$ | $-1.63^{c}$ | -1.37 | $-1.97^{b}$ | 0.23 | -0.67 | -1.18 | 0.45 |
| $\frac{BUX}{DAX}$ | U | $1.75^{c}$ | $1.72^{c}$ | -0.56 | -1.40 | -0.43 | -0.51 | -0.80 | -0.54 |
| | D | 0.03 | -0.73 | -1.02 | -1.61 | -0.09 | -0.58 | -0.07 | 0.71 |
| $\frac{WIG}{DAX}$ | U | 1.17 | 0.73 | -0.60 | -0.21 | -0.07 | -0.21 | -0.21 | -0.62 |
| | D | $2.67^{a}$ | 1.53 | -0.27 | -0.10 | -0.20 | 0.00 | 1.40 | 0.40 |

Note: I have estimated characteristic coefficients $\alpha^{\pm}$ for every quarter and every index. I have employed Wilcoxon test to compare the order of indexes pairwise. Financial crisis is defined as period starting at 2009/Q1 and lasting till the end of the sample 2010/Q4. Symbol $U$ stands for price movements Up and symbol $D$ for movements Down. The positive/negative value means that median of the distribution for characteristic coefficients $\alpha$ corresponding to the first index is bigger/smaller than the median for $\alpha$ of the second index. The letter denotes the significance level at which I reject the null hypothesis: $a$ for 99%, $b$ for 95%, and $c$ for 90%, i.e., significance at which I can say that the indexes are different.

Table 6.6 contains the Kruskal-Wallis statistics for the distribution of rankings performed in a similar manner as the test with characteristic coefficients. The results agrees with those based on the characteristic coefficient, and the disagreement between the stock market indexes is even more pronounced at higher frequencies–higher confidence levels at which I can reject the null hypothesis. In addition, there is also a small disagreement at lower frequencies. Hence, by combining both results, I can conclude that the price jump component part of the underlying price generating process is different for higher frequencies and venishes with decreasing sampling frequency.

Finally, the results do not give me an answer to whether the disagreement among stock market indexes is equally distributed nor whether there is only one outsider which is the cause of the violation for the overall test. To answer this question, I perform a pair-wise comparison of the characteristic coefficients for all stock market indexes using the Wilcoxon statistics.

Table 6.7 contains Wilcoxon $z$-values for the pairwise test of the characteristic coefficients of the stock market indexes. The results clearly shows that the PX index deviates from the rest of the group at high frequencies, i.e., at 5-, and 10-minutes frequencies. At some cases, the difference prevails even for lower frequencies; however, the disagreement is not so strong. The results thus supports the existence of the "PX Puzzle" as was advocated in the main text and suggest that it is the high-frequency phenomena, which is the cause of the Puzzle and the deviation of the PX index from the rest of the group.

### Stability of Results – The Effect of Crisis

I further extend the above performed analysis and extend it by a dimension of the financial crisis. Namely, I repeat the previous analysis but for two separated periods: before the financial crisis period and during the financial crisis period. The analysis is performed with quarterly sub-samples and the financial crisis is assumed to last from 2009/Q1 until 2010/Q4.

Table 6.8 contains the pair-wise comparison of the stock market indexes using the Wilcoxon statistics for the period of the financial crisis, as well as for the period before the financial crisis. The results clearly suggests that before the financial crisis, there was big disagreement among the stock market indexes, while during the financial crisis, the disagreement disappeared. This can be explained by the fact that during the crisis the stock market indices were collectively driven down by the global market panic, which made them behave in very similar ways. During the crisis, most of the shocks which hit the markets were common for the entire financial world and thus the stock markets were more close to each other with respect to properties of extreme price movements. On the other hand, before the crisis, the stock markets had more independence to evolve independently and one could easily face more idiosyncratic shocks, e.g., revealing the performance of national economy, which affected only one of the stock market indices. The analysis thus suggests that financial crisis caused a change in the collective behavior of financial markets, where the differences among them significantly decreased.

Table 6.8: Comparing indexes pair-wise.

| | | 5-minute | | 10-minute | | 15-minute | | 30-minute | |
|---|---|---|---|---|---|---|---|---|---|
| | | PJI | NR | PJI | NR | PJI | NR | PJI | NR |
| | | During the crisis | | | | | | | |
| $\frac{PX}{BUX}$ | U | -1.19 | -0.92 | -0.48 | -0.22 | -0.92 | -0.39 | 0.30 | -0.66 |
| | D | -0.57 | -0.83 | -0.04 | -0.22 | 0.04 | 0.13 | 0.39 | -0.57 |
| $\frac{PX}{WIG}$ | U | 0.66 | -1.28 | -0.04 | -0.13 | -0.48 | -0.30 | -0.04 | -0.39 |
| | D | -1.72$^c$ | -0.13 | -1.72$^c$ | -1.45 | -0.39 | -1.54 | -0.83 | 0.13 |
| $\frac{PX}{DAX}$ | U | -1.01 | -0.57 | -1.01 | -0.83 | -1.89$^c$ | -1.01 | -0.75 | -1.81$^c$ |
| | D | -0.83 | -1.81$^c$ | -2.42$^b$ | -1.72$^c$ | -1.01 | -1.28 | 0.30 | 0.48 |
| $\frac{BUX}{WIG}$ | U | 2.07$^b$ | 2.78$^a$ | 1.45 | -0.39 | 0.39 | 0.04 | -0.13 | 0.30 |
| | D | -1.54 | -0.92 | -1.45 | -1.45 | -0.66 | -1.54 | -1.36 | 0.39 |
| $\frac{BUX}{DAX}$ | U | 0.83 | 1.10 | -0.66 | -1.10 | -0.48 | -1.01 | -1.28 | -1.36 |
| | D | -0.75 | -0.83 | -2.07$^b$ | -2.07$^b$ | -1.28 | -1.45 | -0.22 | 0.66 |
| $\frac{WIG}{DAX}$ | U | -1.89$^c$ | -1.45 | -1.98$^b$ | -0.92 | -1.45 | -1.28 | -0.66 | -1.36 |
| | D | 0.22 | -0.92 | -1.28 | -0.83 | -0.48 | 0.30 | 1.36 | 0.48 |
| | | Before the crisis | | | | | | | |
| $\frac{PX}{BUX}$ | U | -3.08$^a$ | -2.15$^b$ | -1.18 | -1.15 | -0.53 | -1.05 | -1.53 | -1.46 |
| | D | -2.18$^b$ | -1.84$^c$ | -2.15$^b$ | -1.32 | -2.29$^b$ | -1.67$^c$ | -1.36 | -1.46 |
| $\frac{PX}{WIG}$ | U | -3.66$^a$ | -2.91$^a$ | -2.39$^b$ | -2.35$^b$ | -1.18 | -1.32 | -1.80$^c$ | -1.05 |
| | D | -3.66$^a$ | -2.80$^a$ | -2.91$^a$ | -2.84$^a$ | -1.73$^c$ | -1.70c | -1.94$^c$ | -0.63 |
| $\frac{PX}{DAX}$ | U | -1.63 | -1.53 | -2.25$^b$ | -2.04$^b$ | -0.70 | -0.87 | -1.94$^c$ | -1.05 |
| | D | -2.04$^b$ | -1.84$^c$ | -2.04$^b$ | -1.70$^c$ | -1.39 | -2.01$^b$ | -1.42 | -0.94 |
| $\frac{BUX}{WIG}$ | U | -1.46 | -0.53 | -1.22 | -1.39 | -1.11 | -0.94 | -0.25 | 0.12 |
| | D | -1.91$^c$ | -1.77$^c$ | -0.91 | -1.49 | -0.80 | 0.43 | -0.49 | 0.36 |
| $\frac{BUX}{DAX}$ | U | 1.67$^c$ | 1.18 | -0.36 | -0.80 | -0.08 | 0.08 | -0.29 | 0.25 |
| | D | 0.84 | -0.01 | 0.25 | -0.53 | 0.67 | 0.43 | 0.15 | 0.36 |
| $\frac{WIG}{DAX}$ | U | 2.39$^b$ | 2.01$^b$ | 0.67 | 0.43 | 0.87 | 0.43 | 0.32 | 0.32 |
| | D | 2.70$^a$ | 2.01$^b$ | 0.77 | 0.53 | 0.01 | -0.18 | 0.67 | 0.15 |

Note: I have estimated characteristic coefficients $\alpha^{\pm}$ for every quarter and every index. I have employed the Wilcoxon test to compare the order of indexes pairwise. The financial crisis is defined as the period starting at 2009/Q1 and lasting till the end of the sample 2010/Q4. Symbol $U$ stands for price movements Up and symbol $D$ for movements Down. The positive/negative value means that the median of the distribution for characteristic coefficients $\alpha$ corresponding to the first index is bigger/smaller than the median for $\alpha$ of the second index. The letter denotes the significance level at which I reject the null hypothesis: $a$ for 99%, $b$ for 95%, and $c$ for 90%, i.e., significance at which I can say that the indexes are different.

# Part II

# The Impact of the Lehman Brothers Collapse: Were Stocks More Jumpy?[35]

**Abstract**

This paper empirically analyzes the price jump behavior of heavily traded US stocks during the recent financial crisis. Namely, I test the hypothesis that the collapse of Lehman Brothers caused no change in the price jump behavior. To accomplish this, I employ data on realized trades for 16 stocks and one ETF from the NYSE database. These data are at a 1-minute frequency and span the period from January 2008 to the end of July 2009, where the recent financial crisis is generally understood to have begun with the plunge of Lehman Brothers shares on September 9, 2008. I employ five model-independent and three model-dependent price jump indicators to robustly assess the price jump behavior. The results confirm an increase in overall volatility during the recent financial crisis triggered by the Lehman Brothers' fall; however, the results cannot reject the hypothesis that there was no change in price jump behavior in the data during the financial crisis. This implies that the uncertainty during the crisis was scaled up but the structure of the uncertainty seems to be the same.

[35]This part was published as "Were Stocks during the Crisis More Jumpy?: A Comparative Study", CERGE-EI Working Paper Series, 2010, No. 416, 57 pages. In addition, this work was further presented at the following conferences: The 29th International Conference MME 2011, Janska Dolina, Slovakia, 09/201; Warsaw International Economic Meeting, Warsaw, Poland, 07/2011; MFS Annual Conference, Rome, Italy, 06/2011; Prague Economic Meeting, Prague, Czech Republic, 06/2011; and RCMFI Workshop, Crete, Greece, 06/2011. This study is supported by a GAČR grant (402/08/1376) and by grant No. 271111 of the Grant Agency of Charles University. All errors remaining in this text are the responsibility of the author.

# 1 Motivation

Financial markets are uncertain even where there is no crisis. Uncertainty means that when I observe the price process for any financial instrument, I see that the price process follows a stochastic-like path. This path can be with or without a deterministic drift; however, the price process is in any case disturbed by noise movements. The noise movements, known as market volatility, make the price unpredictable. However, the unpredictability of the price movements is not a priori a negative feature; it is rather the nature of financial markets since many different interests meet there. Unpredictability, though, can carry important information when the markets are working properly and no one has an inappropriate informative advantage. Thus, it is of great interest to describe the noise movements as accurately as possible (Gatheral, 2006). Such a description can then be used both in the financial industry to minimize risk and in theoretical economics, where various models of financial behavior are proposed. In addition, a deeper empirical understanding of market volatility during the recent financial crisis can shed some light on the crisis itself and thus helps to deal with future crises. In this work, I contribute to this field by studying the behavior of the extreme noise movements of high-frequency stock returns.

The literature suggests that financial markets reveal a striking characteristic of noise price movements. These noise movements can be decomposed into two components, see e.g., Giot, Laurent, and Petitjean (2010), which are very different in nature. The first component, termed regular noise, represents noise that is frequent but does not bring any abrupt changes. Regular noise stems from the statistical nature of the markets, where markets are simply a result of the interplay of many different market players with different incentives and different financial constraints. This interaction of many different agents can be mathematically described as a standard Gaussian distribution. It is its Gaussian nature that makes the first component it easy to deal with in mathematical models of the price processes of financial instruments. Hence, various characteristics of financial instruments can be established and expectations can be calculated.

The second component, known as price jumps, is rare but very abrupt price movements. Price jumps do not fit into the description of the first noise component and thus have to be treated on their own; see e.g., Merton (1976). However, the mathematical description of price jumps cannot be easily handled. Therefore, the calculations of various market characteristics in the presence of price jumps are very difficult (Pan, 2002; Broadie and Jain, 2008). The serious problems in the mathematical description of price jumps are very often the reason why price jumps are wrongly neglected. In addition, it is still not clear what the main source of price jumps is.

A possible explanation of the source of these jumps says that they originate in the herd behavior, or irrationality, of market participants (Cont and Bouchaud, 2000; Hirshleifer and Teoh, 2003). An illustration of such behavior is a situation when a news announcement is released, and every market participant has to accommodate the impact of that announcement. However, this herding behavior can provide an arbitrage opportunity and can be thus easily questioned. Another explanation is that the source of price jumps can lie in hidden liquidity problems (Bouchaud et al., 2004; Joulin et al., 2008). A hidden liquidity problem is when either the supply or the demand side faces a lack of credit and thus is not able to prevent massive price changes. Both of the presented explanations are very different in nature. Thus, it is impossible to predict a priori what the change would be in price jump behavior in the recent financial crisis.

The two components of the noise movements together contribute to the volatility of the market. In this paper, I focus on both components of market volatility separately and study the change of each of them over time, with an explicit focus on the period of the recent financial crisis. It is widely accepted that periods of financial turbulence cause higher volatility on the market as investors become more nervous and tend to over-react to bad signals (Andersen et al., 2007a). However, it is still not well described empirically how the two components of market volatility change during the crisis. Thus, this study focuses on this issue. Let us assume that a ratio between the two components during the not-so-bad times varies in some specific range. The question would be how would the same ratios vary during bad times, namely, how would the ratio of price jump volatility to regular noise volatility change during the recent financial crisis.

The goal of my paper is to explicitly answer two questions. First, I ask whether an overall increase in market volatility during the recent financial crisis occurred. Second, I focus on the part corresponding to price jump volatility and ask whether the behavior of price jumps changed during the recent financial crisis. To answer these questions, I employ 16 highly traded stocks and one Exchange Traded Fund (ETF) from the North American exchanges found in the TAQ database. These highly traded stocks represent a significant portion of the traded financial assets. Data from the TAQ database are originally at the tick level; thus, I have to integrate them to a 1-minute frequency. The data set spans from January 2008 to July 2009. It is found that the overall volatility significantly increased in September 2008 when Lehman Brothers filed for Chapter 11 bankruptcy protection. In addition, the periods immediately after this announcement reveal significantly higher levels of volatility. However, the ratio between the regular noise and price jump components of volatility does not change significantly during the crisis. The results suggest individual cases where the ratio increases as well as decreases. Thus, it is not possible to draw any industry-dependent conclusions.

This paper contributes to the understanding of market volatility in several ways. In addition to confirming the increase in volatility during the recent financial crisis, namely during the period after the Lehman Brothers announced collapse, I extend the discussion of the decomposition of volatility into two components, which has not been well developed in the literature. I employ various technical indicators to estimate the rate of price jumps, i.e., the second component of volatility. This shows that my approach has several advantages. First, such an approach makes results more robust. Second, many papers focus on one of the indicators employed in my work and thus a direct comparative analysis is not possible. A comparative analysis, however, is one of the outcomes of my paper because I use several indicators on the same data. Third, I employ both model-dependent and model-independent indicators of price jumps on the same data. The same data set containing real prices used for both kinds of indicators is the reason why a comparison of the results can shed light on the validity of the underlying models, which are tacitly assumed to be valid when the model-dependent indicators are derived.

# 2 Literature Review

## 2.1 Price Jumps

The literature contains a broad range of ways to classify volatility. Each classification is suitable for an explanation of a different aspect of volatility or an explanation of volatility from a different point of view; see e.g., Harris (2003) where the volatility is discussed from the financial practitioners' points of view. In the context of my work, the most important aspect is to separate the Gaussian-like component from price jumps. This separation can be seen in the first pioneering papers dealing with price jumps (see e.g., Merton, 1976, or a summary in Gatheral, 2006). Recently, the division in the Gaussian-like component and price jumps was used by Giot et al. (2010). Despite the fact that the motivation for this separation can be purely mathematical, it can be advocated by financial intuitions.

The first reason lies in the primary cause of price jumps. The literature supports two main explanations of the source of price jumps. Bouchaud et al. (2004) and Joulin et al. (2008) advocate jumps are mainly caused by a local lack of liquidity on the market or what they call relative liquidity. In addition, the two papers also claim that an effect of news announcements on the emergence of price jumps can be neglected. On the contrary, Lee and Mykland (2008) and Lahaye et al. (2010) conclude that news announcements are a significant source of price jumps. They also show a connection between macroeconomic announcements and price jumps on developed markets.

Price jumps, understood as an abrupt price change over a very short time, are also related to a broad range of market phenomena that cannot be connected to the noisy Gaussian distribution. For example the inefficient provision of liquidity caused by an imbalanced market micro-structure can cause extreme price movements (see the survey in Madhavan, 2000). Price jumps can also reflect moments when some signal hits the market or a part of the market. Therefore, they can serve as a proxy for these moments and be utilized as tools to study market efficiency (Fama, 1970) or phenomena like information-driven trading; see e.g., Cornell and Sirri (1992) or Kennedy, Sivakamur, and Vetzal (2006). An accurate knowledge of price jumps is necessary for financial regulators to implement the most optimal policies; see Becketti and Roberts (1990) or Tinic (1995). Finally, the non-Gaussian price movements influence the models employed in finance to estimate the performance of various financial vehicles (Heston, 1993; Bates, 1996; Scott, 1997; Gatheral, 2006).

## 2.2 Review of the Price Jumps Empirics

Generally, a price jump is understood as an abrupt price movement that is much larger when compared to the current market situation. The advantage of this definition is that it is model-independent: it does not require any specific form of an underlying price-generating process. On the other hand, this definition is too general and hard to explicitly define and test. The best way to treat this definition is to define the indicators for price jumps that fulfills the intuitive definition. The indicators are by definition parametrized. These parameters govern, for example, the length of the history to which returns are referred or a certain threshold.

Alternatively, price jumps can be defined in such a way where some specific form of the underlying price process is assumed. The most frequent approach in the literature is based on the assumption that the price of asset $S_t$ follows a stochastic differential equation, where the two components contributing to volatility, i.e., regular noise and price jumps, are modeled as

$$dS_t = \mu_t dt + \sigma_t dW_t + Y_t dJ_t \,, \tag{2.1}$$

where $\mu_t$ is a deterministic trend, $\sigma_t$ is time-dependent volatility, $dW_t$ is standard Brownian motion and $Y_t dJ_t$ corresponds to the Poisson-like jump process (see e.g., Merton, 1976). The term $\sigma_t dW_t$ corresponds to the regular noise component, while the term $Y_t dJ_t$ corresponds to price jumps. Both terms together form the volatility of the market. Based on this assumption for the underlying process, one can construct price jump indicators and theoretically assess their efficiency. Their efficiency, however, deeply depends on the assumption that the underlying model holds. Any deviation of the true underlying model from the assumed model can have serious consequences on the efficiency of the indicators.

The remaining part of this section discusses the price jump indicators based on both approaches: the model-independent price jump indicators and the model-dependent price jump indicators.

### Model-independent Indicators

The model-independent price jump indicators do not require any specific form of underlying price process. This paper introduces the following indicators to measure the rate of price jumps in financial markets: extreme returns, temperature, $p$-dependent realized volatility, the price jump index, and the wavelet filter.

#### Extreme Returns

Price jumps are intuitively understood as very high or very low returns. This intuitive understanding of price jumps gives rise to the definition of an extreme returns indicator testing for the presence of a price jump at a given particular time $t$. Hence, a price jump is present at time $t$ if the return at time $t$ is above some threshold. The threshold value can be selected in either of two ways. It can be selected globally, where there is one threshold value for the entire sample, e.g., the threshold is a given centile of the distribution of returns over the entire data set. Or, it can be selected locally, i.e., some sub-samples have different threshold values. A global definition of the threshold allows us to compare the behavior of returns over the entire sample. However, the distribution of returns can vary, e.g., the width of the distribution can change due to changes in market conditions, and thus the global definition of the threshold is not suitable to directly compare price jumps over periods with different market conditions.

There are three versions of the extreme returns indicator. The first definition gives rise to absolute returns $|r_\tau|$. In this case, a price jump occurs at time $\tau$ if the absolute return exceeds the $(100 - X)$-th centile of the entire distribution of absolute returns. This definition assumes a symmetric distribution centered around zero.

Second, the assumption about centering the distribution around zero is omitted. Then, centered absolute returns can be defined as $|r_\tau - \langle r_\tau \rangle_S|$. Hence, a price jump occurs at time $\tau$ if the centered absolute return exceeds the $(100 - X)$-th centile of the entire distribution of centered absolute returns. In this definition, $< X >_S$ stands for the mean taken over the entire sample.

Third, the extreme price jump indicator can be defined generally without any assumption about the specific symmetry of the underlying distribution. In this case, a price jump occurs at time $\tau$ if the return is either below the $X/2$-th centile or above the $(100 - X/2)$-th centile, where centiles are calculated from the entire sample.

### Temperature

Kleinert (2009) shows that high-frequency returns at a 1-minute frequency for the S&P 500 and the NASDAQ 100 indices have the property that they have purely an exponential behavior for both the positive as well as negative sides.[36] The distribution can fit the Boltzmann distribution,

$$B(r) = \frac{1}{2T} \exp\left(\frac{-|r|}{T}\right) , \tag{2.2}$$

where $T$ is the parameter of the distribution conventionally known as the temperature, and $r$ stands for returns. The distribution is assumed to be symmetrically centered around zero. The parameter $T$ governs the width of the distribution; the higher the temperature of the market, the higher

[36] There is a small exception at the tails of the distribution.

the volatility. This follows from the fact that the second centered moment for this distribution is $\sigma_T^2 = 2T^2$. Silva, Prange, and Yakovenko (2004); Kleinert and Chen (2007); Kleinert (2009) and Kleinert (2009) document that this parameter varies slowly, and its variation is connected to the situation on the market.

### $p$-dependent Realized Volatility

Realized volatility can be calculated in a standard way as the second centered moment in a given sample. This definition is a special case in the general definition of the $p$-dependent realized volatility

$$pRV_T^p(t) = \left( \sum_{\tau=t-T+1}^{t} |r_\tau|^p \right)^{1/p} , \tag{2.3}$$

where the sample over which the volatility is calculated is represented by a moving window of length $T$ (see e.g., Dacorogna, 2001). The interesting property of this definition is that the higher the $p$ is, the more weight the outliers have. Since price jumps are simply extreme price movements, the property of realized volatility can be translated into the following statement: The higher the $p$ is, the more price jumps are stressed. Naturally, the ratio of two realized volatilities with different $p$ can be thus used as an estimator of price jumps.

### Price Jump Index

The price jump index $j_T(t)$ at time $t$ (as employed by Joulin et al., 2008) is defined as

$$j_{T,t} = \frac{|r_t|}{< |r_t| >_T} , \tag{2.4}$$

where the history is simply calculated as $< |r_t| >_T = \frac{1}{T} \sum_{i=0}^{T-1} r_{t-i}$ and $T$ is the market history employed.

The distribution of the price jump index $j_{T,t}$ for extreme price movements shows fat tails, i.e.,

$$P(j_T > s) \propto s^{-\alpha_T^{(f)}} , \tag{2.5}$$

where $\alpha_T$ is usually called the characteristic coefficient and explicitly depends on the length of the time window $T$, and $s$ is a threshold value for the price jump index. It generally holds that the lower the $\alpha$, the more jumpy the time series is on average. The characteristic coefficient serves as a measure of the jumpiness of the data.

**Wavelet Filter**

The Maximum Overlap Discrete Wavelet Transform (MODWT) filter represents a technique that is used to filter out effects at different scales. In the time series case, the scale is equivalent to the frequency, thus, the MODWT can be used to filter out high frequency components of time series. This can be also described as the decomposition of the entire time series into high- and low-frequency component effects (see e.g., Gencay et al., 2002).[37] The MODWT technique projects the original time series into a set of other time series, where each of the time series captures effects at a certain frequency scale.

Applying the MODWT technique, the original time series $\{X_t\}$ is deconstructed as $\{X_t\} = \sum_{i=1}^{N}\{\tilde{W}_{i,t}\} + \{\tilde{V}_{2,t}\}$. The time series $\left\{\tilde{W}_{1,t}\right\}$ consists of the fastest effects. The time series $\left\{\tilde{W}_{i,t}\right\}$ with a higher index $i$ capture effects at lower frequencies. Finally, the $\left\{\tilde{V}_{N,t}\right\}$ is a time series after filtering out the effects captured by the $N$ previous time series $\left\{\tilde{W}_{i,t}\right\}$. The construction of the MODWT filter for $N = 2$ is defined as

$$\tilde{W}_{1,t} = \sum_{l=0}^{L-1} \tilde{h}_{1,l} X_{t-l \mathrm{mod} N} \text{ and, } \tilde{W}_{2,t} = \sum_{l=0}^{L-1} \tilde{h}_{1,l} \tilde{V}_{1,t-2l \mathrm{mod} N} \tag{2.6}$$

and

$$\tilde{V}_{1,t} = \sum_{l=0}^{L-1} \tilde{g}_{1,l} X_{t-l \mathrm{mod} N} \text{ and, } \tilde{V}_{2,t} = \sum_{l=0}^{L-1} \tilde{g}_{1,l} \tilde{V}_{1,t-2l \mathrm{mod} N}\, , \tag{2.7}$$

where $\tilde{h}_l$ and $\tilde{g}_l$ are coefficients defining a given wavelet filter.

The most straightforward way to study the contribution of processes at certain scales is to calculate the energy decomposition of the price time series. The energy decomposition of the time series for $N = 2$ is defined as $\|X\|^2 = \left\|\tilde{W}_1\right\|^2 + \left\|\tilde{W}_2\right\|^2 + \left\|\tilde{V}_2\right\|^2$, where $\|X\|$ is the standard $L^2$ norm.

[37] The high-frequency component is intuitively connected to price jumps.

### Model-dependent Indicators

Model-dependent indicators assume a specific form of the underlying price process. The remaining part of this section follows the main stream in the literature and assumes that the price process is governed by equation (2.1), where we assume that price increments can be explicitly written as a Gaussian noise plus some a non-homogenous price jump process. Three indicators are introduced in this paper: the integral and differential indicators based on the difference between the bi-power variance and standard deviation, and the bi-power statistics for the identification of price jumps.

#### The Difference between Bi-power Variance and Standard Deviation

Barndorff-Nielsen and Shephard (2004) discuss the role of the standard variance—the second centered moment—in the models where the underlying process follows equation (2.1). In such a case, the standard variance captures the contribution from both the noise and the price jump process. In addition, the authors show that a definition exists for the realized variance, which does not take into account the term with price jumps. Such a definition is called the realized bi-power variance. The difference between the standard and the bi-power variance can be used to define indicators that assess the jumpiness of the market. Generally, there are two ways to employ bi-power variance: the differential approach and the integral approach.

**The Differential Approach** The standard variance is defined as

$$\hat{\sigma}_t^2 = \frac{1}{T-1} \sum_{\tau=t-T}^{t-1} (r_\tau - < r_{\tau'} >_T)^2 \tag{2.8}$$

with $< r_{\tau'} >_T = \frac{1}{T} \sum_{i=0}^{T-1} r_{t-i}$.

The bi-power variance is defined according to Barndorff-Nielsen and Shephard (2004) as

$$\hat{\hat{\sigma}}_t^2 = \frac{1}{T-2} \sum_{\tau=t-T+2}^{t-1} |r_\tau| \, |r_{\tau-1}| \ . \tag{2.9}$$

The ratio between the two variances, defined as $R_t^{S/BP} = \hat{\sigma}_t^2 / \hat{\hat{\sigma}}_t^2$, satisfies by definition $R_t^{S/BP} \geq 1$. The higher the ratio, the more jumps are contained in the past $T$ time steps back. This method is called a differential since it treats the jumpiness of the markets at every time step.

**The Integral approach** The integral approach is motivated by the work of Pirino (2009). The integral approach employs the two cumulative estimators for the total volatility over a given period. The first one is the cumulative realized volatility estimator defined as

$$RV_{Day} = \sum_{Day} (r_\tau)^2 \ , \tag{2.10}$$

where the sum runs over all prices inside a given period.

The second estimator is the bi-power cumulative volatility estimator defined in an analogous way to equation (2.9):

$$BPV_{Day} = \frac{\pi}{2} \sum_{Day} |r_\tau| \, |r_{\tau-1}| \ , \tag{2.11}$$

where the sum runs over all entries inside a given period and $\pi/2$ is a normalization constant. This estimator does not take into account the contribution of price jumps. Analogously to the previous case, the ratio of the two cumulative estimators defined as $R_{Day}^{RV/BPV} = RV_{Day}/BPV_{Day}$ serves as a measure of the relative contribution of price jumps to the overall volatility over the particular period.

### Bi-power Test Statistics

The bi-power variance can be used to define the proper statistics for the identification of price jumps one by one. This means testing every time step for the presence of a price jump as defined in equation (2.1). These statistics were developed by Andersen et al. (2007b) and Lee and Mykland (2008) and are defined as $\mathcal{L}_t = r_t/\hat{\hat{\sigma}}_t$, where all the symbols are in agreement with the previous definitions. Following Lee and Mykland, the variable $\xi$ is defined as

$$\frac{\max_{\tau \in A_n} |\mathcal{L}_\tau| - C_n}{S_n} \to \xi \, , \tag{2.12}$$

where $A_n$ is the tested region with $n$ observations and the employed parameters are $C_n = \frac{(2\ln n)^{1/2}}{c} - \frac{\ln\pi + \ln(\ln n)}{2c(2\ln n)^{1/2}}$, $S_n = \frac{1}{c(2\ln n)^{1/2}}$ and $c = \sqrt{2}/\sqrt{\pi}$.

In the presence of no price jumps the variable $\xi$ has the cumulative distribution function $P(\xi \leq x) = \exp\left(e^{-x}\right)$. The knowledge of the underlying distribution can be used to determine the critical value $\xi_{CV}$ at a given significance level. Whenever $\xi$ is higher than the critical value $\xi_{CV}$, the hypothesis of no price jump is rejected, and such a price movement is identified as a price jump. In contrast, when $\xi$ is below the critical value, we cannot reject the null hypothesis of no price jump. Such a price movement is then treated as a noisy price movement. These statistics can be used to construct a counting operator for the number of price jumps in a given sample.

# 3 Data and Descriptive Statistics

## 3.1 Data Selection

I employ a set of 16 stocks and one ETF from the Trade and Quote database (TAQ) established by the NYSE. The data ranges from the beginning of January 2008 to the end of July 2009. The selected time span covers the critical period of the Lehman Brothers collapse in September 2008 and long periods before and after this event. Table 3.1 summarizes the selected stocks, where stocks are ordered alphabetically according to their tickers.

The stocks used for this analysis accord to several criteria. First, all the stocks are heavily traded with a large intraday stock flow. This fact is important for the derivation of the homogeneous time series, which is extensively described in the following section. Second, the stocks and the ETF selected for this study are of the high market importance. Market importance can be judged in several ways. The most obvious is the market capitalization of the company and its inclusion into the main stock market indices. Therefore I include stocks that are a substantial part of the S&P 500 index. Since the S&P 500 index is a capitalization-weighted stock market index, the larger its weight, the more capitalized a company is. I also include stocks with a large weight in the Dow Jones Industrial Average index.[38] This index is price-weighted, and therefore a large share in the index is taken by companies whose shares have the highest price. In addition, this index is considered to be a representative benchmark of the industrial performance of the US economy. Therefore selecting companies with a large weight in this index enables me to track changes in US industrial performance.

In addition to the companies selected due to their weights in the two main indexes, I have also included Citigroup, Inc. This stock was badly hit during the recent financial crisis and its value

[38]The exact composition of the Dow Jones Industrial Average index is discussed in the Appendix.

Table 3.1: List of stocks and ETF used in this analysis.

| ID | Ticker | Name | Sector | Reason/Index | Avg. Daily Vol. Traded 11/09 |
|---|---|---|---|---|---|
| 1 | APPL | Apple Inc. | Information Technology | S&P500 | 15,856,500 |
| 2 | BAC | Bank of America Corp. | Financials | S&P500; DJI | 163,920,700 |
| 3 | C | Citigroup. Inc. | Financials | see Note A | 300,960,700 |
| 4 | CVX | Chevron Corp. | Energy | S&P500; DJI | 9,415,900 |
| 5 | GE | General Electric Co. | Industrials | S&P500; DJI | 80,429,200 |
| 6 | GOOG | Google Inc. | Information Technology | S&P500 | 2,131,500 |
| 7 | HPQ | Hewlett-Packard | Information Technology | S&P500; DJI | 15,752,200 |
| 8 | IBM | Intl. Business Machines Corp. | Information Technology | S&P500; DJI | 5,852,500 |
| 9 | JNJ | Johnson & Johnson | Health Care | S&P500; DJI | 12,211,600 |
| 10 | KO | Coca Cola Company | Consumer Staples | S&P 500; DJI | 8,671,900 |
| 11 | MSFT | Microsoft Corp. | Information Technology | S&P500; DJI | 53,121,400 |
| 12 | PFE | Pfizer | Health Care | S&P 500; DJI | 50,452,700 |
| 13 | PG | Procter & Gamble | Consumer Staples | S&P500; DJI | 12,742,300 |
| 14 | SPY | S&P 500 ETF | ETF | Note B | 179,587,000 |
| 15 | T | AT&T Inc. | Telecommunications Services | DJI | 26,854,300 |
| 16 | WFC | Wells Fargo & Co. | Financials | S&P500 | 38,866,300 |
| 17 | XOM | Exxon Mobil Corp. | Energy | S&P500; DJI | 18,893,700 |

Note: Stocks and ETF are sorted in alphabetical order by the TICKER name. The column Reason/Index captures the main reason for the stock to be included. S&P500 or DJI means that the stock has a substantial weight in the S&P500 index or the Dow Jones Industrial Average index, respectively. The reason to include Citigroup is its very high daily volume traded on exchanges. In addition, this stock is quite specific. Despite its very low nominal value, price movements of this stock influence other stocks. The abrupt changes in this stock are very often taken as a signal for changes in the financial sector. The SPY Exchange Traded Fund by was chosen because it tracks the performance of the S&P 500 index, very often considered as a benchmark for the performance of US stocks. The last column shows the average daily volume traded at US exchanges in November 2009. Data were taken from Yahoo! Finance (http://finance.yahoo.com) dramatically declined. However, this company traditionally has a large impact on financial markets, which advocates its inclusion.

Finally, I have also included an Exchange Traded Fund (ETF) which tracks the performance of the S&P 500 index.[39] This ETF serves as a vehicle—in reality it is quite popular and highly traded—for those who want to be exposed to the S&P 500 index performance as a whole. The ETF represents the benchmark for the performance of the US economy, which is why it is important for this study. Therefore, this ETF reflects the overall trends on the market since the excess movements by any single stock is smoothed out by other stocks.

In conclusion, the stocks selected for this analysis are important representatives of the US stock markets. They cover the markets from a market capitalization point of view as well as from an industrial point of view. However, the selection is still small and thus enables me to keep track of each stock during the analysis.

## 3.2 Data Frequency

The TAQ database contains two separate databases: realized trades and quotes. Data with quotes are useful to calculate the depth of the market, to study the market micro-structure or to estimate a fair price at a given tick, but a database with realized trades cannot be used on its own for any estimation of the price process on the tick-level or to study the market micro-structure. In this work, I derive the data at a 1-minute frequency from the database with realized trades. The data at a 1-minute frequency are defined as an equally weighted average over all trades inside a given minute. Such an average captures fully the trading activity over the entire period. In addition, this method smooths out the possible discrepancies in the data as well as the known problem with the bid-ask bounce (Huang and Stoll, 1997; Hasbrouck, 2002).

The equally weighted average of realized trades requires a sufficient amount of tick data in every minute. The selection of stocks, as described above, assures this requirement. To illustrate this, Table 3.1 contains the average daily traded volumes for March 2010. The very large volumes demonstrate that the selected stocks and the ETF have very large intraday activity.[40]

[39] The ETF does not track the S&P 500 index precisely since the value of the ETF includes maintenance fees.

[40] The small number of realized trades per minute does not smooth out the bid-ask bounce. This consequently leads to a wrong estimate of the price in this particular minute. Therefore, I have included a formal check, which counts the number of realized trades per minute. Whenever the number of realized trades is less than 15—an empirically chosen threshold—the price for this minute is obtained by interpolation. This check assures that no spurious price

## 3.3 Data Filtering

Following the official data description provided by the NYSE (see the NYSE official website), I discard observations with the CORR flag—an indicator denoting an ex post correction of the given tick—different from zero as well as all entries with the COND flag equal to Z—COND with a value of Z denotes delayed entries. Following the paper by Brownlees and Gallo (2006), I then test the data for the presence of significant outliers. These outliers also have to be carefully discarded from the data. However, when the activity is high, the net effect of the outlier is averaged out when taking the equally weighted average over a given minute; therefore, I employ the condition used in Brownlees and Gallo (2006)

$$|p(t) - p_k(t)| < 3\sigma_k(t) + \gamma\,, \tag{3.1}$$

where $p_k(t)$ is an average calculated for the moving window running $\pm k$ periods around time $t$, and $\sigma_k(t)$ is a standard deviation calculated on the same time window. Based on Brownlees and Gallo (2006), I have chosen $\gamma = 0.005$ and $k = 5$.

## 3.4 Trading Hours

The data from the database come in tape time from 4:00:00 to 19:59:59. The trading hours for the exchanges included in the database are from 9:30:00 to 15:59:59. The trades realized before the official trading hours are in what is known as pre-opening market hours, while the trades realized after the official trading hours are in after-market hours. During the trading hours, I calculate the price of the stock at a given minute as an average of prices for all valid realized trades in this minute.

In this work, I study the main trading period and therefore completely discard after-market hours. The pre-opening hours, however, cannot be easily discarded. This comes from the fact that in a few cases, I need to estimate the current situation on the market, which is simply some statistics over a moving window going a given number of time steps back. Naturally, this can cause some problems for the initial moments of the trading day, where no data in the trading hours are available. Therefore, I employ the data from the pre-opening period to estimate the situation on the market before the official opening occurs.

jumps will be created.

Since the pre-opening period is not heavily traded, I have introduced the following empirical rule to estimate the situation on the market in the pre-opening period: I separate the two hours preceding the opening hours into 10-minute blocks, where each block will have a separate price. The price in the block is calculated as an average over all trades in the block. If the activity in the block is not high enough, if the number of trades is less than 50, the price for a given block is taken as the same as the price in the first minute following the block. The prices are estimated in a backward direction starting at the immediate moments preceding the opening of the markets.[41] This procedure utilizes more information for a given trading day compared to the case where the pre-opening hours would be completely cut off.

### 3.5 Descriptive Statistics

The descriptive statistics of returns provide the first insight into the behavior of price jumps. Returns are defined in a standard way as $r_t = \log(R_t/R_{t-1})$, where $R_t$ is the average price of the stock (or ETF) for time $t$. Time is measured in minutes. Figure 3.1 depicts the first four moments of the distribution of returns—the measures of mean, standard deviation, skewness and kurtosis—calculated daily, i.e., every trading day.

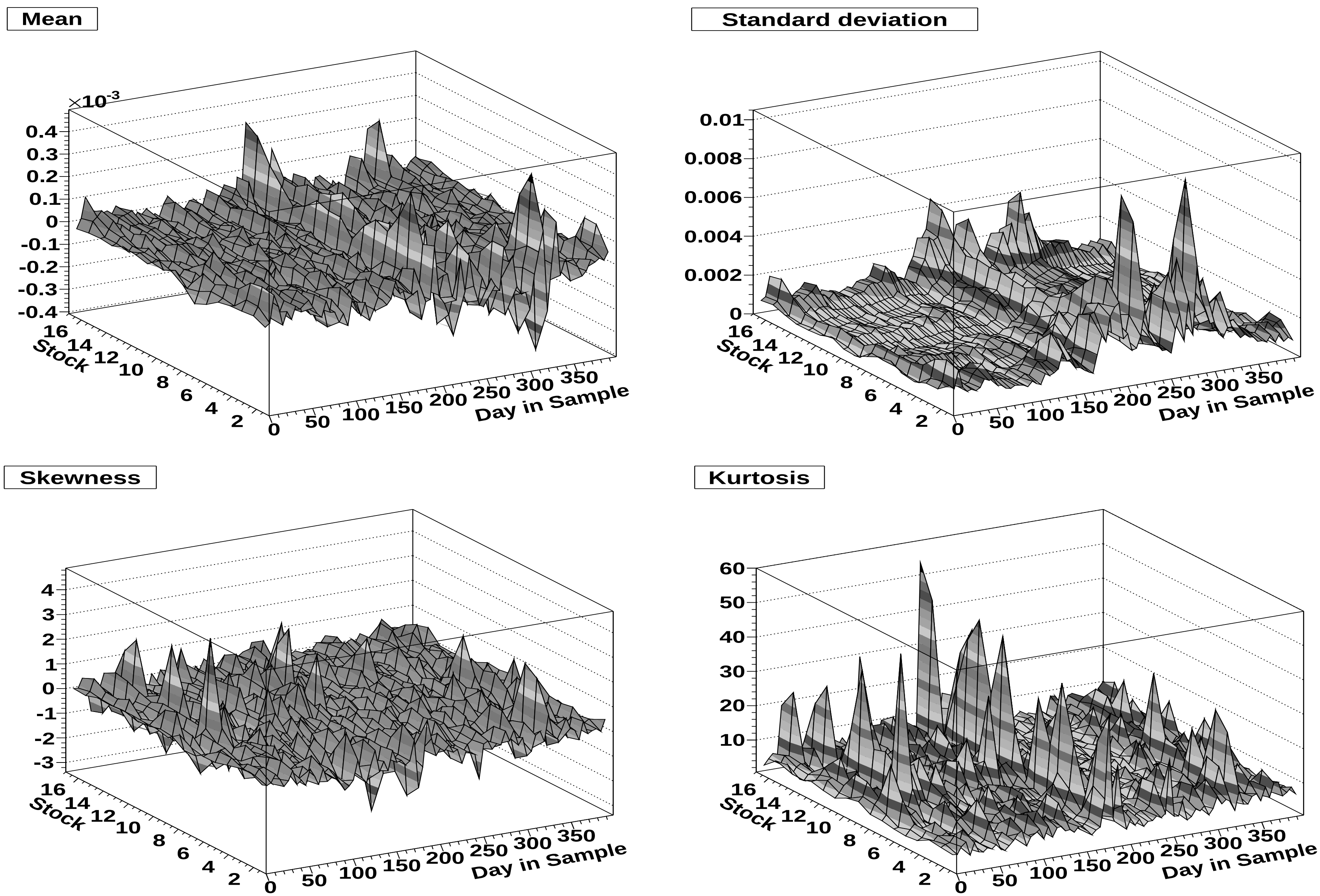
The results show that the mean fluctuates around zero. The rate of fluctuations has increased during the crisis. The first swing does not come directly after Lehman Brothers filing for bankruptcy protection.[42] However, it took some time for the markets to realize the oncoming problems. One month after the plunge of Lehman Brothers' shares, the markets were in crisis. At this time, we observed a big swing in the fluctuations of shares. In addition, mainly the stocks from the banking sector (Bank of America, Citigroup and Wells Fargo) experienced other significant turbulent periods starting in January 2009 and continuing in the first three months of 2009. The excessive movements in the means of returns can be explained by the market mood changing every day and stocks soaring one day and falling the next day.

Similar patterns can be concluded from the figure with standard deviation. In this case, however, the period with increased volatility started directly after the Lehman Brothers' problems. The

[41] First, I estimate the price for the period 9:20 to 9:29. If there is a low number of trades, the price is taken as the price at 9:30. Second, I estimate the price for the period 9:10 to 9:19. If there is a low number of trades, I take the price at 9:20, which is the price of the entire first block.

[42] The big plunge of Lehman Brothers shares occurred on September 9, 2008. This date corresponds to the Day174 in the Sample.

Figure 3.1: The first four moments of returns calculated day by day for every stock and ETF.



problems escalated and the volatility reached towards new heights. In addition, the banking sector and oil industry showed strong increases in volatility in the first months of 2009.

Skewness, on the other hand, does not reveal any striking systematic difference during the crisis. The measure of skewness oscillates but without any systematic pattern or without any change of rate in oscillations during the crisis. The measure of kurtosis, on the other hand, reaches very high values at the beginning of the crisis. After these heights, the kurtosis seems to be significantly lower. This means that the underlying distribution of returns was at the beginning of the crisis highly leptokurtic, i.e., with fatter tails and thus with a higher rate of price jumps, while after the first weeks of crisis, at the end of October 2008, the kurtosis reaches pre-crisis levels and the values show lower variance. This suggests "slimmer" tails on average with a low rate of price jumps.

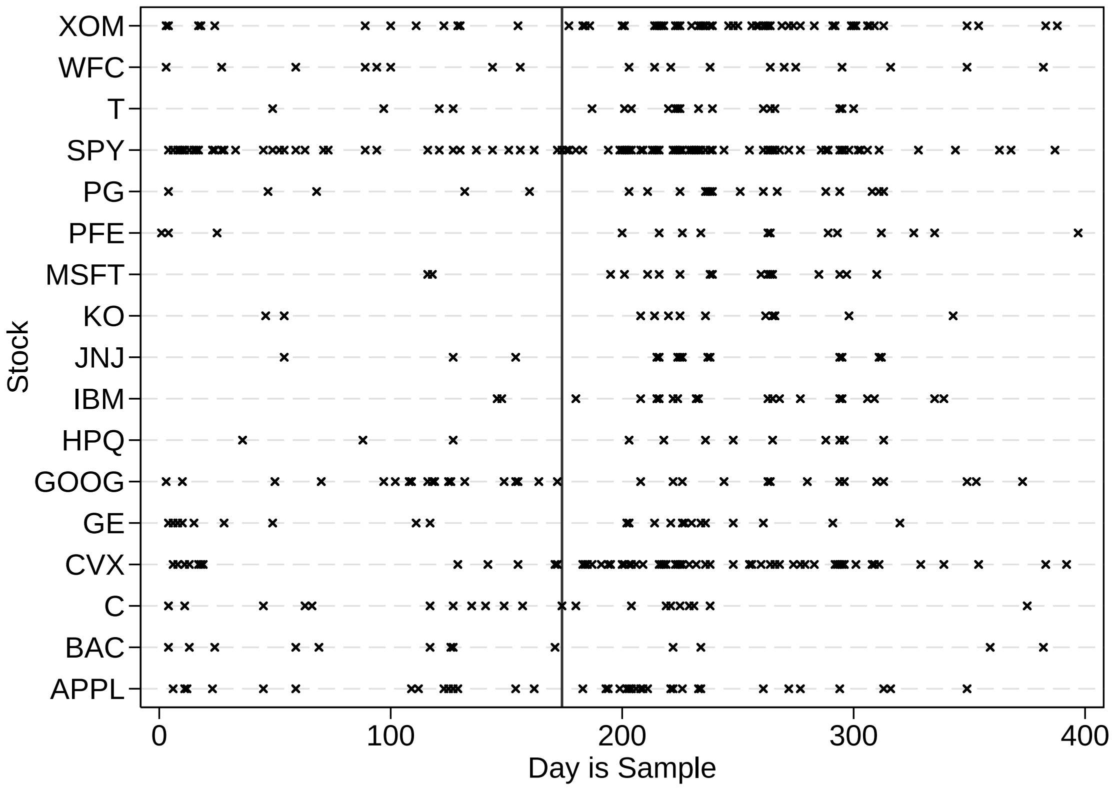
### 3.5.1 Jarque-Bera statistics

In addition to the first four moments, a more subtle test is needed to test for the non-normality of returns and, thus, for the presence of price jumps. A standard test to address this question is to employ the Jarque-Bera statistics (Jarque and Bera, 1980) defined as $JB = \frac{N}{6}\left(S^2 + \frac{(K-3)^2}{4}\right)$, with $S$ being the measure of skewness, $K$ the measure of kurtosis and $N$ the number of observations. The test is asymptotically equal to $\chi^2_2$ and specifies the null hypothesis that data are iid and come from a Gaussian distribution. The alternative hypothesis means either a deviation from a Gaussian distribution or a non-iid feature of the underlying generating process.

Figure 3.2 depicts the result of the Jarque-Bera test. Namely, the Jarque-Bera statistics is calculated for every stock on a daily basis. Every day, the Jarque-Bera statistics is compared to the critical value of the $\chi^2_2$ distribution at the 95% confidence level. For every stock and every trading day, there are two possible outcomes: the test statistics is either equal to or below the critical value or it exceeds the critical value. In the former case, we fail to reject the null hypothesis and tend to accept the fact that the underlying process is iid and follows a Gaussian distribution. In the latter case, we reject the null hypothesis and accept the alternative hypothesis. The situation where the Jarque-Bera statistics did not exceed the critical value is marked by a cross. In addition, the figure contains a vertical line denoting the day when the Lehman Brothers problems occurred.

An eye check of the results confirms the observations inferred from the previous figure measuring a kurtosis. After the emergence of the crisis in October, there is a significant period of time where

Figure 3.2: Jarque-Bera statistics for returns.




Note: The Jarque-Bera statistics was calculated for every stock and every trading day separately. Then, the statistics were compared with the critical value at the 95% confidence level. The figure contains 17 lines–one for each stock. Whenever there is a cross, the Jarque-Bera statistics did not exceed the critical value and therefore the null hypothesis of returns to come from iid Gaussian distribution cannot be rejected. The vertical line corresponds to September 9th, 2009–the day when the Lehman Brothers problems started.

the Jarque-Bera statistics are rather low and even below the critical value. This, unsurprisingly, corresponds to the period with low levels of kurtosis. In addition, a visual inspection suggests that the ETF behaves according to the iid Gaussian distribution more often than other stocks. This comes from the fact that the ETF mimics the composition of the S&P 500 index and is thus composed of many underlying stocks, where extremes coming from a single stock are averaged out and only those extremes which occurred at the same time remain.

# 4 Methodology

I first summarize the price jump indicators used for this study and then outline the procedure for how the indicators were employed.

## 4.1 Indicators

The Literature Review section contains an extensive overview of the price jump indicators. To summarize, I shall employ the following set of price jump indicators:

1. Model-independent indicators
   (a) Extreme returns
   (b) Temperature
   (c) $p$-dependent realized volatility
   (d) The price jump index
   (e) The wavelet filter
2. Model-dependent indicators
   (a) The difference between bi-power variance and standard deviation
      (i) The differential approach
      (ii) The integral approach
   (b) Bi-power test statistics

The indicators, as they are explained in the previous sections, are by construction very different. Besides the obvious division of model-independent and model-dependent indicators, they can also be divided from another point of view: whether they aim to exactly identify price jumps or rather to assess the jumpiness of the markets. The jumpiness of the markets is understood as a measure of the rate of price jumps occurring during a specified period without counting price jumps explicitly. In both cases, I shall refer tothem as price jump measures.

The price jump indicators that identify price jumps explicitly are extreme returns and bi-power test statistics. The rest of the indicators can be utilized to construct exact price jump indicators; however, they are employed as a measure of jumpiness throughout this work. Therefore, whenever the two periods are compared with respect to price jumps, they are either compared by counting the number of price jumps or by comparing the measure of price jumps, i.e., the jumpiness.

## 4.2 Definition of the Financial Crisis – Collapse of Lehman Brothers

The main goal of this work is to answer the question whether the current financial turmoil caused any change in the price jump behavior in the financial markets using high-frequency data. I approach the problem by dividing the entire sample into sub-samples corresponding to individual trading days. For every day, I assess the number of price jumps or the measure for jumpiness. Then I compare the statistics of these measures for days before the crisis with the statistics for days during the crisis.

Deciding when the crisis started and how long it lasted is not clear and cannot be done explicitly. I rather focus on the situation on the financial markets around the collapse of Lehman Brothers. This event is clearly significant, causing the panic on the financial markets among professionals as well as general public and thus it is considered as one of the events characterizing the financial turmoil starting in mid-2008. Therefore, I consider the plunge of the shares of Lehman Brothers on September 9, 2008 as the first main event triggering the financial crisis. Based on this event, I define the financial crisis as a structural break in the behavior of financial markets. I employ two different versions of the breaking scheme:

- **The Permanent Break (PB):** The crisis started on September 9, 2008 and lasted until the end of the sample.
- **The Temporary Break (TB):** The crisis started on September 9, 2008 and lasted 30 trading days or for 1.5 months.

The first scheme is intuitive and aims to describe the permanent change on the financial markets due to the collapse of big bank. Thus, in this scheme, I assume that the effect of the financial crisis was permanently present on financial markets at least until the end of July 2009. The second scheme, however, focuses solely on the most problematic days following the plunge of shares. The period of 30 working days was chosen based on the news and the behavior of financial markets. The two schemes thus provide different pictures. The first scheme answers the question about a permanent change in the behavior of financial markets, while the second scheme rather focuses on the immediate panic that spread through the financial markets and affected the trading habits of market participants.

The estimation of price jumps, or jumpiness, itself is done by employing the battery of tests described above. The tests were developed in the literature and very often require a fine-tuning process to obtain unbiased results. The fine-tuned indicators then allow us to measure the number of price jumps, or the jumpiness, at absolute levels. This defines the cardinal measure of price jumps. Having in hand the cardinal measure, the absolute numbers of price jumps can be interpreted on their own as well as the price jumps being identified with particular events at given moments. Such a formulation is, however, too strong to answer the main question: how can we compare the days relatively.

The weak formulation of employing the indicators is to use them as an ordinal measure. The indicators used in this way are not required to be absolutely unbiased. The bias in the number of price jumps can be present as soon as it is proportional to the number of price jumps. This still allows me to compare days with respect to the number of price jumps truthfully, i.e., to assess which of the days, or general periods, were more jumpy.

In the remaining part of this section, I employ the battery of tests described in the preceding sections. I explicitly test the following hypothesis: The recent financial crisis caused no statistically significant change in the price jump properties of the price time series.

## 4.3 The Trading Days

The sample of price times series employed in this work covers the period spanning from January 2, 2008 to July 31, 2009. I divide the entire sample into sub-samples, each corresponding to one trading day. On every sub-sample, I apply the price jump indicators. Then I test for differences across days. The length of the sub-sample was chosen intuitively. This enabled me to obtain reasonable statistics within a day as well as between days.

Days in the sample are denoted in "Day in Sample" (DiS) units. The advantage of these units compared to calendar days is it makes the figures smooth, without gaps corresponding to weekends and holidays. The seeming disadvantage is that calendar days cannot be easily identified. Therefore, Table 4.1 provides a frame of reference for a conversion between DiS and calendar days. In addition, some important dates are mentioned in the table explicitly.

Table 4.1: Conversion table for “Day in Sample” units and calendar days.

| DiS | Calendar | DiS | Calendar | DiS | Calendar |
|---|---|---|---|---|---|
| 1 | Jan 2nd, 2008 | 148 | Aug 1st, 2008 | 254 | Jan 2nd, 2009 |
| 22 | Feb 1st, 2008 | 169 | Sep 2nd, 2008 | 274 | Feb 2nd, 2009 |
| 42 | Mar 3rd,2008 | 174 | Sep 9th, 2008 | 293 | Mar 2nd, 2009 |
| 62 | Apr 1st, 2008 | 190 | Oct 1st, 2008 | 315 | April 1st, 2009 |
| 84 | May 1st, 2008 | 203 | Oct 20th, 2008 | 336 | May 1st, 2009 |
| 105 | Jun 2nd, 2008 | 213 | Nov 3rd, 2008 | 356 | Jun 1st, 2009 |
| 126 | Jul 1st, 2008 | 232 | Dec 1st, 2008 | 378 | Jul 1st, 2009 |

Note: The table includes September 9th, 2008, when the Lehman Brothers' shares plunged, the beginning of the financial crisis. The table also includes October 20th, 2008. This day is used to define the end of the temporary break.

## 4.4 Hypotheses to Test

The indicators employed in this work measure the jumpiness of the financial markets on a daily basis. The indicators can be divided into two groups according to the way the daily measure is achieved. First, there are indicators that by construction estimate one number for every day. The second group of indicators gives an estimate of jumpiness for every tick. Then, the measure of jumpiness per day is obtained based on these tick estimates. These two different groups of indicators also imply different hypotheses to test with different meanings.

Thus, I form four different hypotheses in this work, two for each of the two groups of price jump indicators.

**Group I: One Number per Trading Day** The first group of indicators gives exactly one number per trading day. I divide the sample of trading days into two sub-samples. These two sub-samples are formed by trading days occurring during the crisis or not during the crisis. The period of the financial crisis is defined above.

**Hypothesis I-A:** The null hypothesis of this test says that the two sub-samples come from the same distribution. The main scope of this test is to compare whether the estimated price jump measures changed during the crisis.

**Test:** I employ the two-sample Wilcoxon test (see the Appendix) and test whether the estimated price jump measures for the two sub-samples come from the same distribution.

**Hypothesis I-B** The null hypothesis states that the variance of the two sub-samples, i.e., during and not during the crisis, are the same. This tests whether the trading days in either of the two sub-samples were on average more heterogeneous. In other words, this procedure tests the heterogeneity of the trading days between the sub-samples.

**Test:** I employ the standard $F$-test and compare whether the variance of the estimated price jump measures changed during the crisis. The $F$-test is defined as

$$\frac{S_C^2}{S_{No-C}^2} \sim F_{(N_C - 1, N_{No-C} - 1)}, \tag{4.1}$$

where $S^2$ is the standard deviation of the characteristic coefficient calculated during the crisis "$C$" and outside the crisis "$No - C$". The $N_C$ is the number of days the crisis lasts and $N_{No-C}$ is the complement to the total number of days in the sample.

**Group II: One Number per Tick** The second group of price jump indicators gives one number per tick, in my case one number per minute. Having in hand these numbers, I calculate the mean and variance of these numbers per trading day. Analogously to the previous case, I divide the sample into two sub-samples.

**Hypothesis II-A** The null hypothesis of this test says that the two sub-samples composed of daily means come from the same distribution. The main scope of this test is to compare whether the estimated price jump measures changed during the crisis.

**Test:** I employ the two-sample Wilcoxon test and test whether the daily means of the estimated price jump measures for the two sub-samples come from the same distribution.

**Hypothesis II-B** The null hypothesis of this test says that the two sub-samples composed of daily variances come from the same distribution. The main scope of this test is to question whether the heterogeneity inside the trading days changed during the financial crisis.

**Test:** I employ the two-sample Wilcoxon test and test whether the daily variances of the estimated price jump measures for the two sub-samples come from the same distribution.

# 5 Results

This section summarizes the results when all the price jump indicators are employed. The price jump indicators are described in the same order as above.

## 5.1 Model-independent Indicators

### 5.1.1 Extreme Returns

Extreme returns define price jumps globally. Figure 5.1 shows the number of absolute returns per day above the 90th, 95th, and 99th centile calculated from the distribution of the same quantity over the entire period. Figure 5.2 shows the number of absolute centered returns per day above the 90th, 95th, and 99th centile calculated from the distribution of the same quantity over the entire period. Figure 5.3 shows the number of returns per day below/above the 5/95th, 2.5/97.5th, and 0.5/99.5th centiles calculated from the distribution of the same quantity over the entire period, respectively.

The figures suggest that the period following the plunge of Lehman Brothers' shares is characterized by an increase in extreme returns. However, this does not directly respond to the question about the behavior of price jumps when price jumps are understood as much extreme movements bigger compared to the current market situation. Rather, the increased levels of the extreme returns indicator suggests a rise in market volatility. In addition to the period following the problems of Lehman Brothers, the turmoil period also appeared in the beginning of 2009, when extreme returns also sky rocketed.

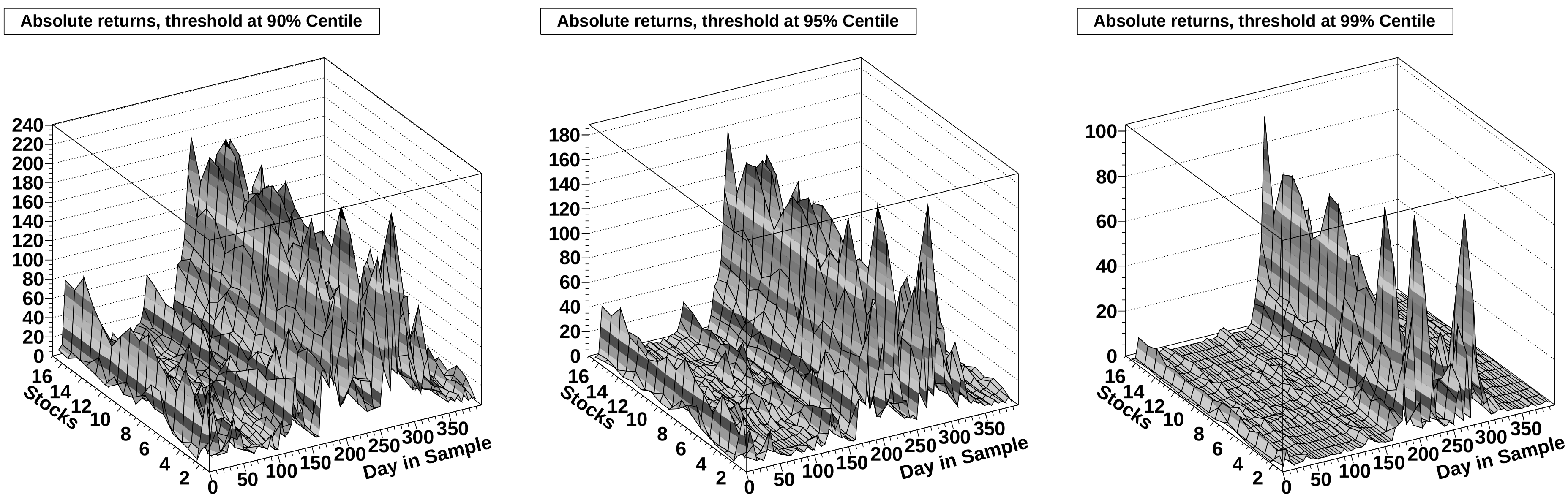
### 5.1.2 Temperature

The temperature $T$ is estimated according to equation (2.2). The equation is non-linear, however, a practical way to approach it is to log-linearize it and then apply the standard least squares method. The linearized equation (2.2) reads

$$\ln B(r) = \ln \frac{1}{2T} - \frac{1}{T} r \,. \tag{5.1}$$

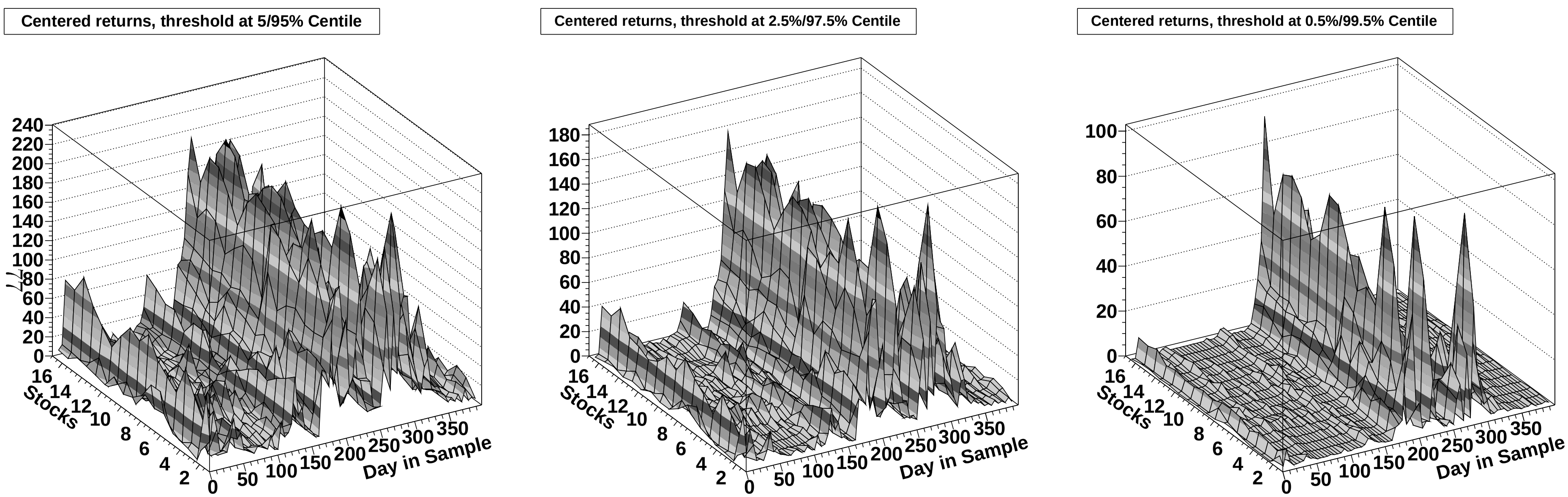
To estimate the parameter $T$, we assume that the previous equation, due to the finiteness of the

Figure 5.1: Price jump indicator based on absolute returns.



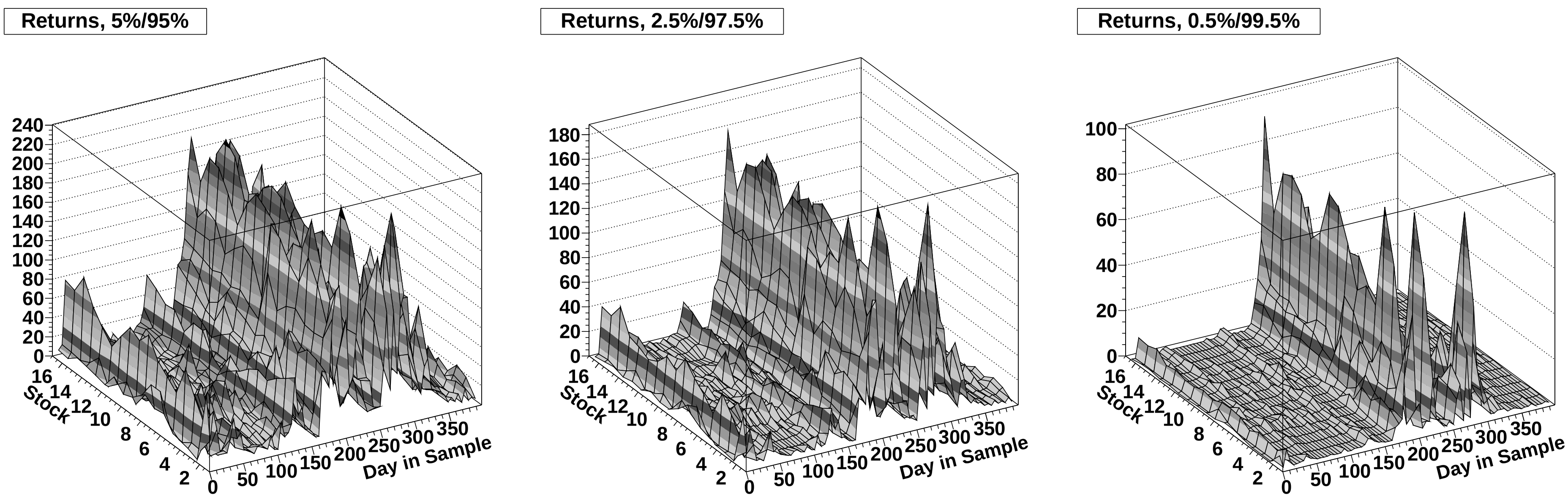

Note:Centiles are taken over the entire sample. The number of absolute returns above the threshold is calculated for every trading day and every stock. The stocks are numbered according to Table 3.1.

Figure 5.2: Price jump indicator based on absolute centered returns.



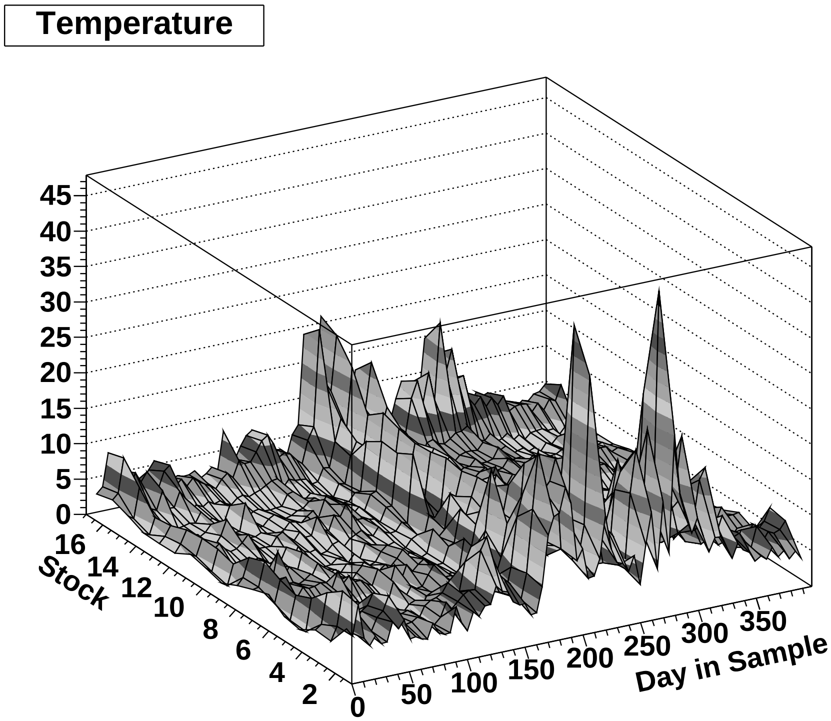

Note: Centiles are taken over the entire sample. The number of centered absolute returns above the threshold is calculated for every trading day and every stock. The stocks are numbered according to Table 3.1.

Figure 5.3: Price jump indicator based on extreme returns.




Note: Centiles are taken over the entire sample. The number of returns below/above the threshold is calculated for every trading day and every stock. The stocks are numbered according to Table 3.1.

Figure 5.4: Temperature.




Note: Temperature is estimated for returns for every stock and every trading day based on eq. (2.2). The stocks are numbered according to Table 3.1.

sample, has the form

$$\ln B(r) = \ln \frac{1}{2T} - \frac{1}{T} r + \nu \,, \tag{5.2}$$

where $\nu$ is the homogeneous Gaussian noise. From this equation, the inverse of the temperature can be directly estimated. When the estimation is carried out, returns are by definition assumed to be symmetrical with respect to the origin.

Figure 5.4 shows the estimated temperature for every stock calculated day by day. The temperature does not distinguish price jumps. Therefore, these results support the same conclusion as those obtained from the extreme returns indicator. The period after Lehman Brothers' problems emerged is characterized by increased market volatility. In addition, the banking sector (Bank of America, Citigroup, and Wells Fargo) shows significantly higher market volatility at the beginning of 2009.

### 5.1.3 $p$-dependent Realized Volatility

I employ equation (2.3) for two particular values of $p = 1$ and $p = 4$. The realized volatility with $p = 4$ is relatively more sensitive to price jumps compared to the realized volatility with $p = 1$. I employ this difference between them to construct the following price jump indicator defined as

$$pRV_T^{p/p'}(t) \equiv pRV_T^{p}(t)/pRV_T^{p'}(t)\,, \tag{5.3}$$

where $pRV_T^p(t)$ is defined by equation (2.3), and the two parameters governing the sensitivity to price jumps are $p = 4$ and $p' = 1$. In addition, this price jump indicator is defined for each time and takes into account the history of the $T$ preceding time steps, including the current time. I employ two time windows for history: $T = 60$ and $T = 120$. The history at the beginning of the trading day is calculated from the pre-opening period, as is extensively discussed in the preceding sections.

This indicator captures the change in price jumps in the following way: If there is an extreme movement in either of the $T$ time steps of the preceding window, the indicator will be higher compared to the situation without any price jump. The indicator keeps its higher value until the price jump is present in the moving window. The first occurrence of the high value of the indicator suggests the occurrence of a price jump.

I will explicitly test Hypothesis II-A and Hypothesis II-B, i.e., whether the means and the variances of the estimated ratio defined by equation (5.3) come from the same distribution during and not during the crisis. I do not report figures in this case since they do not provide any strong visible hints about the behavior of this indicator.

**Hypothesis II-A:** Table 5.1 shows the result of the Wilcoxon statistics for this hypothesis. The table contains the $z$-value of the test. The stars denote at what level of significance we can reject the null hypothesis, stating that means are the same over the entire period. In addition, the excessively high $z$-value of the Wilcoxon statistics corresponds to a situation when the median of means during the crisis is smaller than the median of means outside the crisis. The excessively low $z$-values mean the opposite. For illustration, the case of Apple using $T = 60$, and the financial crisis defined as a Permanent Break gives a $z$-value equal to $-3.332$, which means that we can reject the null hypothesis at the 99% confidence level. In addition, the negative $z$-value suggests that the median of means after the emergence of Lehman Brothers' problems is higher compared to the

Table 5.1: Result of the two-sample Wilcoxon test for the mean of $pRV_T^{p/p'}(t)$.

| ID/Ticker | Permanent break | | Temporary break | |
|---|---|---|---|---|
| | 60 | 120 | 60 | 120 |
| 1 APPL | -3.332*** | -4.362*** | 1.019 | 0.828 |
| 2 BAC | 4.538*** | 3.283*** | -0.118 | -1.506 |
| 3 C | -3.110*** | -0.828 | 0.195 | 0.775 |
| 4 CVX | 6.021*** | 5.666*** | 1.754* | 1.030 |
| 5 GE | 2.543** | 0.071 | -1.310 | -2.187** |
| 6 GOOG | -1.956* | -1.427 | 0.511 | 0.205 |
| 7 HPQ | 5.184*** | 4.102*** | 0.954 | 0.724 |
| 8 IBM | 5.006*** | 3.633*** | 0.459 | 1.078 |
| 9 JNJ | 5.442*** | 5.243*** | 0.457 | 0.110 |
| 10 KO | 5.752*** | 5.329*** | -0.248 | -0.931 |
| 11 MSFT | 2.133** | 1.209 | 2.128** | 1.578 |
| 12 PFE | 1.821* | 1.482 | 0.214 | -0.294 |
| 13 PG | 5.101*** | 4.967*** | -0.055 | -0.092 |
| 14 SPY | 1.259 | 3.590*** | 3.307*** | 3.005*** |
| 15 T | 4.265*** | 3.408*** | 1.129 | 0.503 |
| 16 WFC | 4.133*** | 0.787 | -3.455*** | -3.160*** |
| 17 XOM | 4.904*** | 2.913*** | -0.034 | -1.298 |

Note: The mean was calculated for every stock and every trading day. I have used the two definitions of the financial crisis, Permanent Break and Temporary Break, and two time windows, $T = 60$ and $T = 120$ minutes. The table captures the $z$-statistics for the test. The additional stars denote whether we can reject $H_0$ that the two samples come from the same distribution and the corresponding confidence levels: 90% (*), 95% (**) and 99% (***). The overall positive/negative value of the $z$-statistics suggests that the median of the means is lower/higher during the financial crisis.

previous period. This means that Apple stocks were more jumpy during the crisis.

To summarize, the table shows that when the financial crisis is defined through the Permanent Break, the distributions of the mean are more likely to be different. This suggests that no matter how turbulent the days were following the plunge of Lehman Brothers' shares, the crisis emerged in the subsequent months. In addition, the $z$-values tend to be positive, which suggests that the median of means for the $p$-ratio is lower during the crisis. This means that the rate of price jumps decreased during the crisis, or alternatively, price jumps were overwhelmed by the overall increase in the magnitude of returns.

**Hypothesis II-B:** Table 5.2 shows the results of the Wilcoxon test. The results are in agreement with the previous test in several aspects. First, the case of a Temporary Break does not lead to a situation where we can reject the null hypothesis about the agreement of the distributions of variance

Table 5.2: Result of the two-sample Wilcoxon test for the variance of $pRV_T^{p/p'}(t)$.

| ID/Ticker | Permanent Break | | Temporary Break | |
|---|---|---|---|---|
| | 60 | 120 | 60 | 120 |
| 1 APPL | -4.844*** | -5.467*** | 0.888 | 1.899* |
| 2 BAC | 5.742*** | 4.933*** | -0.218 | -0.839 |
| 3 C | 6.705*** | 7.286*** | 1.078 | 1.628 |
| 4 CVX | 4.100*** | 3.991*** | 0.908 | 0.821 |
| 5 GE | 1.579 | -0.510 | -0.148 | -0.503 |
| 6 GOOG | -1.533 | -0.807 | 0.307 | -0.113 |
| 7 HPQ | 2.653*** | 2.239** | 1.303 | 0.930 |
| 8 IBM | 2.098** | 1.526 | 2.168** | 1.879* |
| 9 JNJ | 3.466*** | 3.443*** | 1.140 | 0.485 |
| 10 KO | 3.099*** | 3.382*** | -0.059 | -0.428 |
| 11 MSFT | 0.685 | 0.494 | 2.115** | 1.637 |
| 12 PFE | -0.840 | -0.683 | -0.526 | -0.875 |
| 13 PG | 4.761*** | 4.808*** | 1.060 | 0.701 |
| 14 SPY | -1.219 | 2.933*** | -0.661 | 0.916 |
| 15 T | 1.999** | 1.979** | 0.714 | 0.436 |
| 16 WFC | 1.723* | -0.674 | -0.614 | -1.825* |
| 17 XOM | 0.483 | -0.241 | -1.423 | -1.654* |

Note: The variance was calculated for every stock and every trading day. I have used two definitions of the financial crisis, Permanent Break and Temporary Break, and two time windows, $T = 60$ and $T = 120$ minutes. The table captures the $z$-statistics for the test. The additional stars denote whether we can reject $H_0$ of the two samples come from the same distribution and the corresponding confidence levels: 90% (*), 95% (**) and 99% (***). The overall positive/negative value of the $z$-statistics suggests that the median of variances is lower/higher during the financial crisis.

during and not during the crisis. Second, the stocks that had significantly different distributions of the ratio $pRV_T^{p/p'}(t)$ also tend to have significantly different distributions of the estimated variances.

### 5.1.4 The Price Jump Index

I estimate the characteristic coefficient $\alpha$ introduced in equation (2.5). I estimate the coefficient for every trading day in the sample. I use two time windows, namely $T = 60$ and $T = 120$ time steps back. The estimation of the characteristic coefficient was done for the log-linearized version of equation (2.5). Then I employed OLS for the tail parts of the distribution to estimate $\alpha$.[43] The price jump index captures the behavior of extreme price movements normalized by the current market situation and thus assesses the jumpiness of the financial markets. This indicator gives one number

[43]The other methods to estimate $\alpha$ are discussed, for example, by Vaglica, Lillo, Moro, and Mantegna (2008), who use MLE for estimation.

per trading day, thus I shall test Hypothesis I-A and Hypothesis I-B.

**Hypothesis I-A:** The results of the Wilcoxon test are in Table 5.3. The results show that the difference in the price jump behavior is more likely to occur when the financial crisis is defined using the Permanent Break. In addition, the results suggest that the banking sector was hit hard by the problems of Lehman Brothers. This observation follows from the fact that the three banks show a difference in the characteristic coefficient for both definitions of the financial crisis. In every case, the $z$-ratio is positive for the banking industry, which means that the median of the distribution of the characteristic coefficient is lower during the crisis, and therefore, returns for the banking industry's stocks were more jumpy. In addition, the results show that the price jump index captures different aspects of extreme price movements when compared to the previous indicator.

Table 5.3: Result of the two-sample Wilcoxon test for the means of the price jump index.

| ID/Ticker | Permanent Break | | Temporary Break | |
|---|---|---|---|---|
| | 60 | 120 | 60 | 120 |
| 1 APPL | -1.543 | -2.534** | 1.433 | 2.704*** |
| 2 BAC | 3.736*** | 6.006*** | 2.324** | 2.018** |
| 3 C | 5.889*** | 7.057*** | 3.211*** | 2.601*** |
| 4 CVX | 3.211*** | 3.428*** | 0.080 | 0.209 |
| 5 GE | 2.645*** | 2.073** | 0.999 | 1.206 |
| 6 GOOG | -1.880* | -1.740* | 1.203 | 1.061 |
| 7 HPQ | 0.967 | 0.987 | 1.563 | 0.391 |
| 8 IBM | 0.494 | 0.773 | 3.442*** | 2.655*** |
| 9 JNJ | 2.174** | 2.912*** | 1.293 | 0.151 |
| 10 KO | 1.976** | 2.718*** | 1.335 | 0.704 |
| 11 MSFT | 1.610 | 0.257 | 2.228** | 1.470 |
| 12 PFE | 1.106 | 0.852 | 1.086 | -0.100 |
| 13 PG | 1.719* | 2.058** | 1.764* | 1.163 |
| 14 SPY | 0.263 | 0.423 | 0.533 | 1.562 |
| 15 T | 0.710 | 1.489 | -0.502 | -0.383 |
| 16 WFC | 5.561*** | 4.610*** | 4.173*** | 2.805*** |
| 17 XOM | 2.975** | 2.538** | 1.758* | 1.488 |

Note: The characteristic coefficient was calculated for every stock and every trading day. I have used two definitions of the financial crisis, Permanent Break and Temporary Break, and two time windows, $T = 60$ and $T = 120$ minutes. The table captures the $z$-statistics for the test. The additional stars denote at what confidence level we can reject $H_0$ of the two samples come from the same distribution. Notation for the confidence levels is as follows: 90% (*), 95% (**) and 99% (***). The overall positive/negative value of the $z$-statistics suggests that the median of characteristic coefficients is lower/higher during the financial crisis.

**Hypothesis I-B:** Table 5.4 shows the $F$-statistics defined above. The results clearly show that the characteristic coefficients for the price jump index tend to have different variances during both definitions of the financial crisis when the length of the moving window is rather short. Generally, the value of an $F$-statistic higher/lower than one suggests that the variance during the crisis was higher/lower relative to the variance outside the crisis, respectively.

The implication of this claim brings another interesting insight. There are stocks for which the $F$-statistic is significantly lower using one definition of the financial crisis and significantly higher for the other definition. This is the case, for example, for Bank of America stocks. In this case, the variance for the Permanent Break is lower during the crisis, while it is significantly higher for the Temporary Break. The period immediately following the plunge of Lehman Brothers shares was then dominated by huge movements in the characteristic coefficient. This means that a short,

Table 5.4: Result of the two-sided $F$-test for the variance of the characteristic coefficients of the price jump index.

| ID/Ticker | Permanent Break | | Temporary Break | |
|---|---|---|---|---|
| | 60 | 120 | 60 | 120 |
| 1 APPL | 1.887*** | 1.112 | 1.160** | 1.077 |
| 2 BAC | 0.581*** | 0.859 | 2.021*** | 0.653 |
| 3 C | 1.039 | 2.084*** | 1.345 | 1.071 |
| 4 CVX | 2.874*** | 1.839*** | 0.973 | 0.437*** |
| 5 GE | 1.541*** | 1.526*** | 1.077 | 0.941 |
| 6 GOOG | 0.661*** | 0.858 | 0.928 | 0.685 |
| 7 HPQ | 1.165 | 1.197 | 0.845 | 0.844 |
| 8 IBM | 1.624*** | 0.985 | 2.113*** | 1.055 |
| 9 JNJ | 1.296* | 1.242 | 1.029 | 0.801 |
| 10 KO | 1.169 | 1.657*** | 2.586*** | 1.949*** |
| 11 MSFT | 1.057 | 0.720** | 0.259*** | 0.422*** |
| 12 PFE | 0.466*** | 0.501*** | 0.595* | 0.602* |
| 13 PG | 1.108 | 1.158 | 0.780 | 0.988 |
| 14 SPY | 1.204 | 0.933 | 2.594*** | 1.805** |
| 15 T | 1.499*** | 1.116 | 0.479** | 0.714 |
| 16 WFC | 2.173*** | 1.811*** | 1.361 | 1.299 |
| 17 XOM | 1.216 | 1.586*** | 1.573* | 1.488 |

Note: The null hypothesis says that the variances during and not during the crisis match. Stars denote at what confidence level we can reject the null hypothesis: 90% (*), 95% (**) or 99% (***). In addition, the value of $F$-statistics higher/lower than one means that variance of the characteristic coefficient during the crisis was higher/lower when compared to the period not during the crisis. The two $F$-distributions are $F_{225,172}$ for the Permanent Break and $F_{29,368}$ for the Temporary Break.

volatile period was followed by a long period with a rather stable characteristic coefficient, which causes a decrease of volatility.

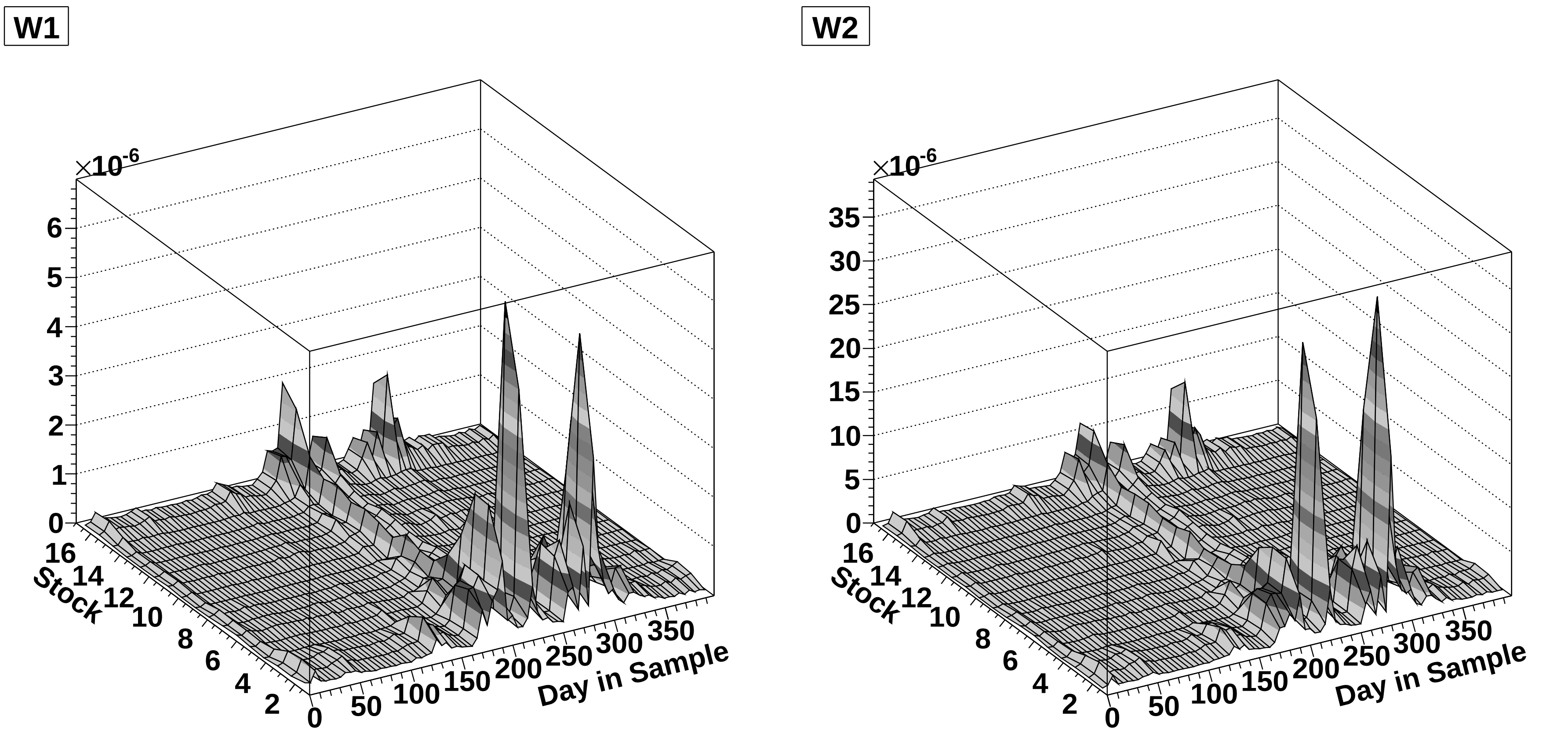
The opposite is true, for example, for Chevron Mobil stocks, where the short period following the plunge of Lehman Brothers shares was dominated by a rather stable characteristic coefficient, which turns out to be more volatile in the long term.

### 5.1.5 Wavelets

I employ the Daubechies LA wavelet filter with length $L = 8$. The length of the filter is sufficient to compensate for the possible non-stationarity in the price time series (see Gencay, Selcuk, and Whitcher, 2002). The non-stationarity in the data is usually treated by taking first differences, while for the MODWT analysis, price levels are employed directly. I perform the MODWT decomposition using the first two levels as described above. As a measure of jumpiness, I perform an energy decomposition for trading days. Then, I calculate the ratio of the total energy for a given day corresponding to each of the two levels of MODWT decomposition: $\left\|\tilde{W}_1\right\|^2 / \left\|X\right\|^2$ and $\left\|\tilde{W}_2\right\|^2 / \left\|X\right\|^2$. The higher the first ratio, the more high-frequency processes the time series contains. Therefore, a high ratio suggests an increased period of price jumps; however, this indicator can also reach high values even for non-jumpy periods. The increased ratio thus suggests an increased level of volatility caused by a high-frequency process, which does not necessarily coincide with the intuitive definition of price jumps.

Figure 5.5 contains two sub-figures: on the LHS they are depicted as $\left\|\tilde{W}_1\right\|^2 / \left\|X\right\|^2$ , while on the RHS they are depicted as $\left\|\tilde{W}_2\right\|^2 / \left\|X\right\|^2$. The figures show an increase in the energy corresponding to the high-frequency processes after the emergence of the financial crisis. Since the high-frequency processes do not correspond solely to price jumps, the increased ratio of the energy corresponding to high-frequency processes can also be caused by the increased rate of noise. This ratio thus presents a necessary condition for the presence of price jumps. In addition, the ratio serves as an indicator for an increase in high-frequency volatility rather than solely for the identification of price jumps.

Figure 5.5: The relative energy of the first MODWT level (left panel) and the second MODWT level (right panel).




The relative energy is defined as $\left\|\tilde{W}_i\right\|^2 / \|X\|^2$, where $\{\tilde{W}_i\}$, with $i = 1, 2$, stands for the first-level and second-level wavelet coefficients and $\|X\|^2$ is the total energy over a trading day. The ratio reveals the first significant increase at the beginning of September 2008, where the Lehman Brothers' problems started. We can also clearly recognize a big increase in the ratio for all the big banks (Bank of America, Citi and Wells Fargo) and for both oil companies (Chevron and Exxon Mobile).

Table 5.5: The two-sided Wilcoxon statistics for the mean of $R_t^{S/BP}$.

| ID/Ticker | Permanent Break | | Temporary Break | |
|---|---|---|---|---|
| | 60 | 120 | 60 | 120 |
| 1 APPL | -1.818* | -0.166 | 1.435 | 0.999 |
| 2 BAC | -3.087*** | -5.482*** | -1.167 | -1.155 |
| 3 C | -1.698* | -3.147*** | -0.179 | -0.347 |
| 4 CVX | 1.495 | 1.322 | 0.370 | 0.691 |
| 5 GE | -4.460*** | -6.036*** | -0.694 | -0.865 |
| 6 GOOG | 1.814* | 2.127** | -0.004 | -0.237 |
| 7 HPQ | 1.026 | 0.984 | 0.324 | 0.355 |
| 8 IBM | 0.093 | 0.162 | 1.353 | 1.501 |
| 9 JNJ | 1.349 | 0.989 | -0.757 | -0.854 |
| 10 KO | 1.900* | 1.604 | 0.069 | 0.120 |
| 11 MSFT | 2.010** | 1.206 | 0.752 | 0.408 |
| 12 PFE | -2.337** | -2.167** | -0.837 | -1.028 |
| 13 PG | 2.743*** | 2.234** | 0.404 | 0.237 |
| 14 SPY | 2.195** | 3.175*** | -1.397 | -1.233 |
| 15 T | 1.070 | 1.165 | 1.323 | 1.422 |
| 16 WFC | -6.037*** | -7.405*** | -1.977** | -1.850* |
| 17 XOM | -1.820* | -2.085** | -1.939* | -1.998** |

Note: The mean was calculated for every stock and every trading day. I have used two definitions of the financial crisis, Permanent Break and Temporary Break, and two time windows, $T = 60$ and $T = 120$ minutes. The table captures the $z$-statistics for the test. The additional stars denote at what confidence level we can reject $H_0$ of the two samples come from the same distribution. Notation for the confidence levels is as follows: 90% (*), 95% (**) and 99% (***). The overall positive/negative value of the $z$-statistics suggests that the median of means is lower/higher during the financial crisis.

## 5.2 Model-dependent Indicators

In this part, I present the results for model-dependent indicators as they were introduced in the previous sections.

### 5.2.1 The Difference between Bi-power Variance and the Standard Deviation: A Differential Approach

First, I calculate the ratio $R_t^{S/BP} = \hat{\sigma}_t^2/\hat{\hat{\sigma}}_t^2$. The two variances in the ratio require certain time windows. I chose $T = 60$ and $T = 120$. The ratio is defined for every time step $t$, where the history at the beginning of the trading day is calculated from the pre-opening period. Since $\hat{\sigma}_t$ is the variance, which also takes into account the price jumps, a high level of the ratio means the

Table 5.6: The two-sided Wilcoxon statistics for the variance of $R_t^{S/BP}$.

| ID/Ticker | Permanent Break | | Temporary Break | |
|---|---|---|---|---|
| | 60 | 120 | 60 | 120 |
| 1 APPL | -2.677*** | -0.141 | 1.560 | 1.032 |
| 2 BAC | 1.254 | -3.471*** | -0.632 | -0.963 |
| 3 C | 0.424 | -1.840* | 0.411 | -0.228 |
| 4 CVX | 2.323** | 2.083** | 0.826 | 0.841 |
| 5 GE | 0.100 | -3.635*** | 1.191 | 0.467 |
| 6 GOOG | 0.345 | 0.960 | -0.465 | -0.602 |
| 7 HPQ | 1.822* | 1.604 | 0.571 | 0.658 |
| 8 IBM | 1.851 | 1.347 | 1.330 | 1.183 |
| 9 JNJ | 2.732*** | 2.230** | -0.107 | -0.503 |
| 10 KO | 2.266** | 2.216** | -0.123 | -0.363 |
| 11 MSFT | 2.845 | 1.788* | 2.131** | 1.570 |
| 12 PFE | -1.205 | -1.558 | 0.788 | -0.019 |
| 13 PG | 3.874*** | 3.718*** | 1.070 | 0.767 |
| 14 SPY | 1.482 | 2.050** | -1.781* | -1.870* |
| 15 T | 1.103 | 1.318 | 1.473 | 1.707* |
| 16 WFC | -0.040 | -3.940 | 0.044 | -0.505 |
| 17 XOM | 0.783 | -0.266 | -1.680* | -1.988** |

Note: The variance was calculated for every stock and every trading day. I have used two definitions of the financial crisis, Permanent Break and Temporary Break, and two time windows, $T = 60$ and $T = 120$ minutes. The table captures the $z$-statistics for the test. The additional stars denote at what confidence level we can reject $H_0$ of the two samples come from the same distribution. Notation for the confidence levels is as follows: 90% (*), 95% (**) and 99% (***). The overall positive/negative value of the $z$-statistics suggests that the median of variances is lower/higher during the financial crisis.

presence of price jumps. In addition, the ratio remains at increased levels as long as the price jump is contained in the history of up to $T$ time steps back.

Since the ratio is by its nature very similar to the ratio constructed using $p$-dependent realized volatility, I shall test Hypothesis II-A and Hypothesis II-B.

**Hypothesis II-A:** The results of the test are summarized in Table 5.5. The test shows that in the case of the Temporary Break, we cannot reject the null hypothesis stating that the means of $R_t^{S/BP}$ come from the same distribution. In the case of the Permanent Break, three titles show more significant differences: Bank of America, General Electric and Wells Fargo. In addition, in all three cases the ratio is negative, which means that median of means during the crisis is higher. This also means that these three assets had the most significant increase in price jumps during the crisis. Since there is no immediate increase in the ratio during the initial days of the crisis, the increase in

Table 5.7: The two-sided Wilcoxon statistics for the ratio $R_{Day}^{RV/BPV}$.

| ID/Ticker | Permanent Break | Temporary Break |
|---|---|---|
| 1 APPL | 0.355 | 0.395 |
| 2 BAC | 0.037 | 1.177 |
| 3 C | 1.472 | 0.215 |
| 4 CVX | 0.729 | 1.116 |
| 5 GE | -0.948 | -1.845* |
| 6 GOOG | -0.812 | 0.994 |
| 7 HPQ | 0.858 | -0.075 |
| 8 IBM | 1.348 | 1.873* |
| 9 JNJ | 1.134 | 0.029 |
| 10 KO | 2.048** | -0.266 |
| 11 MSFT | 1.031 | -1.292 |
| 12 PFE | 2.087** | 0.345 |
| 13 PG | 1.249 | 0.525 |
| 14 SPY | 2.101** | 2.461** |
| 15 T | 2.900*** | 1.558 |
| 16 WFC | -0.142 | -0.418 |
| 17 XOM | -1.301 | 1.392 |

Note: The ratio $R_{Day}^{RV/BPV}$ was calculated for every stock and every trading day. I have used two definitions of the financial crisis, Permanent Break and Temporary Break, and two time window, $T = 60$ and $T = 120$ minutes. The table captures the $z$-statistics for the test. The additional stars denote whether we can reject $H_0$ of the two samples come from the same distribution and the corresponding confidence levels: 90% (*), 95% (**) and 99% (***). The overall positive/negative value of the $z$-statistics suggests that the median of ratios is lower/higher during the financial crisis.

price jumps appear on the long term horizon. In addition, the ETF shows the opposite behavior, i.e., a decrease in the jumpiness after the emergence of the financial crisis.

**Hypothesis II-B:** Table 5.6 shows the result of the test. In the case of the Temporary Break, the variance is rather stable. On the other hand, for the Permanent Break, there are a few cases where the variance varies during the financial crisis. The most striking difference is in Procter and Gamble shares, where the variance decreased during the financial crisis, and thus, the trading activity was on average more uniform over the trading day.

### 5.2.2 The Difference between Bi-power Variance and the Standard Deviation: An Integral Approach

For the next step, I employ the ratio $R_{Day}^{RV/BPV} = RV_{Day}/BPV_{Day}$. I calculate the ratio for every trading day and every stock. Then, I test Hypothesis I-A and Hypothesis I-B.

Table 5.8: Results of the two-sided $F$-test for the variance of the ratio $R_{Day}^{RV/BPV}$.

| ID/Ticker | Permanent Break | Temporary Break |
|---|---|---|
| 1 APPL | 0.834 | 0.974 |
| 2 BAC | 5.518*** | 0.195*** |
| 3 C | 1.090 | 0.829 |
| 4 CVX | 0.605*** | 0.697 |
| 5 GE | 1.467*** | 1.177 |
| 6 GOOG | 1.247 | 2.368*** |
| 7 HPQ | 0.952 | 4.330** |
| 8 IBM | 1.018 | 0.636 |
| 9 JNJ | 0.450*** | 1.662** |
| 10 KO | 0.374*** | 1.035 |
| 11 MSFT | 1.184 | 0.738 |
| 12 PFE | 1.041 | 1.767** |
| 13 PG | 1.009 | 4.952*** |
| 14 SPY | 1.000 | 1.089 |
| 15 T | 0.912 | 0.597* |
| 16 WFC | 1.116 | 1.116 |
| 17 XOM | 0.915 | 1.094 |

Note: The null hypothesis says that the variances of the ratio $R_{Day}^{RV/BPV}$ during and not during the crisis match. Stars denote at what confidence level we can reject the null hypothesis: 90% (*), 95% (**) or 99% (***). In addition, the value of $F$-statistics higher/lower than one means that variance of the ratio $R_{Day}^{RV/BPV}$ during the crisis was higher/lower when compared to the period not during the crisis. The two $F$-distributions are $F_{225,172}$ for the Permanent Break and $F_{29,368}$ for the Temporary Break.

**Hypothesis I-A:** The result of the two-sample Wilcoxon test is summarized in Table 5.7. The results show that the only asset that has significantly non-zero $z$-values for both definitions of the financial crisis is the one of the ETF. On the other hand, the assets for the banking industry show no significant deviation and therefore suggest no change in the price jump behavior.

**Hypothesis I-B:** The results of the $F$-test are summarized in Table 5.8. The table suggests that the most significant difference in the variance is for the stocks of Bank of America. In this, the Permanent Break definition of the financial crisis gives a significantly higher variance during the crisis. On the other hand, in the Temporary Break case, the variance during the financial crisis is significantly lower. This favors the claim that after the emergence of Lehman Brothers' problems, the trading days were rather uniform with the same rate of market panic. On the other hand, in the long term, the trading days became more heterogeneous.

Table 5.9: The two-sided Wilcoxon statistics for the counting indicator.

| ID/Ticker | Permanent Break | | Temporary Break | |
|---|---|---|---|---|
| | 60 | 120 | 60 | 120 |
| 1 APPL | -0.666 | -0.141 | 0.099 | -0.156 |
| 2 BAC | 6.286*** | 5.416*** | 0.526 | 0.298 |
| 3 C | 1.732* | 3.312*** | -1.334 | -1.446 |
| 4 CVX | 2.251** | 0.004 | 0.433 | -0.045 |
| 5 GE | 7.561*** | 6.082*** | 1.722* | 0.567 |
| 6 GOOG | -2.411** | -2.644*** | 1.623 | 2.016** |
| 7 HPQ | 1.607 | 2.143** | -1.706* | -1.991** |
| 8 IBM | 0.659 | 1.656* | -1.333 | -1.418 |
| 9 JNJ | -0.824 | -1.337 | 0.675 | -0.566 |
| 10 KO | -1.142 | -1.869* | 1.578 | 1.948* |
| 11 MSFT | 0.163 | -0.405 | 0.624 | -0.702 |
| 12 PFE | 1.427 | 1.668* | -0.028 | 0.280 |
| 13 PG | -2.989*** | -2.674*** | -3.673*** | -2.289** |
| 14 SPY | -0.198 | -0.128 | -1.598 | -0.781 |
| 15 T | 0.754 | 0.2400 | -1.861* | -2.186** |
| 16 WFC | 5.144*** | 4.786*** | 0.120 | 0.032 |
| 17 XOM | 2.293** | 2.308** | -1.022 | -0.848 |

Note: The mean of the counting indicator was calculated for every stock and every trading day. I have used two definitions of the financial crisis, Permanent Break and Temporary Break, and two time windows, $T = 60$ and $T = 120$ minutes. The table captures the $z$-statistics for the test. The additional stars denote at what confidence level we can reject $H_0$ of the two samples come from the same distribution. Notation for the confidence levels is as follows: 90% (*), 95% (**) and 99% (***). The overall positive/negative value of the $z$-statistics suggests that the median of means of the counting indicator is lower/higher during the financial crisis.

However, the opposite observation is true for the returns of Johnson and Johnson stocks. In this case, the days following the Lehman Brothers' problems were, on average, very different from each other. The difference subsequently smoothed out in the long term. In addition, the sectors do not share the same characteristics. For example, companies from the sensitive banking sector show very different behavior, as can be illustrated by Bank of America and Citigroup.

### 5.2.3 Bi-power Test Statistics

Finally, I employ the test statistics developed by Lee and Mykland (2008) and introduced above. The test statistics require choosing a moving window, as can be seen in equation (2.12). Lee and Mykland (2008) suggest using $T = 270$ time steps back for a 5-minute frequency. However, such a moving window cannot be satisfied in my framework since I do not allow for overlap between trading days.

Table 5.10: Results of the two-sided $F$-test for the variance of the counting indicator.

| ID/Ticker | Permanent Break | | Temporary Break | |
|---|---|---|---|---|
| | 60 | 120 | 60 | 120 |
| 1 APPL | 1.217 | 1.281* | 1.315 | 1.855** |
| 2 BAC | 0.956 | 0.997 | 1.582* | 2.245*** |
| 3 C | 0.823 | 0.890 | 1.229 | 1.619** |
| 4 CVX | 0.704** | 0.765* | 0.948 | 0.698 |
| 5 GE | 1.277* | 1.054 | 1.253 | 1.213 |
| 6 GOOG | 0.902 | 0.933 | 1.637** | 0.901 |
| 7 HPQ | 0.873 | 1.083 | 0.971 | 1.557* |
| 8 IBM | 0.936 | 1.048 | 1.189 | 1.510* |
| 9 JNJ | 0.982 | 1.028 | 1.291 | 1.049 |
| 10 KO | 0.951 | 0.975 | 1.217 | 1.506* |
| 11 MSFT | 0.998 | 0.832 | 1.562* | 1.091 |
| 12 PFE | 1.004 | 0.905 | 1.019 | 1.231 |
| 13 PG | 1.033 | 1.036 | 1.477 | 1.103 |
| 14 SPY | 0.976 | 0.946 | 1.235 | 1.514* |
| 15 T | 0.900 | 0.759* | 1.188 | 1.107 |
| 16 WFC | 1.224 | 1.034 | 0.875 | 1.169 |
| 17 XOM | 1.248 | 1.112 | 1.043 | 0.735 |

Note: The null hypothesis says that the variances of the mean of the counting indicator during and not during the crisis match. Stars denote at what confidence level we can reject the null hypothesis: 90% (*), 95% (**) or 99% (***). In addition, the value of $F$-statistics higher/lower than one means that variance of the mean of the counting indicator during the crisis was higher/lower when compared to the period not during the crisis. The two $F$-distributions are $F_{225,172}$ for the Permanent Break and $F_{29,368}$ for the Temporary Break.

I chose instead two moving windows, $T = 60$ and $T = 120$, which are the lengths used in the previous cases. The possible bias stemming from this choice of moving windows is again compensated for by considering the relative differences of the number of jumps. For the purpose of test statistics, I consider the 95% confidence level. The test statistics enable me to identify price jumps exactly and thus construct a counting indicator for the number of price jumps. I shall explicitly test Hypothesis I-A and Hypothesis I-B.

**Hypothesis I-A:** The results of the test are summarized in Table 5.9. In the case of the Permanent Break, there are several cases where the number of price jumps differs during the crisis. All the banks, General Electric and Exxon Mobil are characterized by positive $z$-values and thus by a lower number of price jumps during the crisis. On the other hand, Google and Procter and Gamble show a higher number of price jumps. In addition, Procter and Gamble is the only one that shows the same change of price jumps also for the Temporary Break. This suggests that the short period immediately after Lehman Brothers' problems was dominated by a huge increase in price jumps. In the case of the remaining stocks, there are no agreements between the different number of price jumps using the Permanent Break and the different number of price jumps using the Temporary Break. This means that the main change in the number of price jumps occurred in the long-time horizon.

**Hypothesis I-B:** The results of the $F$-test are in Table 5.10. In the case of the Permanent Break, the variance in the number of price jumps is not present. On the other hand in the case of the Temporary Break, the difference in the variance is present, namely for Bank of America where the $F$-statistics are higher than one. This suggests that the variance was higher during the crisis, i.e., the days were very different during the crisis than they were not during the crisis.

# 6 Conclusion

I performe an extensive technical analysis of price jumps using high-frequency market data (1-minute frequency) covering 16 major traded stocks and one ETF traded on the main North American stock exchanges. The data spans the period from January 2009 until the end of July 2009. The main question of this paper is whether the behavior of price jumps, understood as extreme and irregular price movements different in their nature from regular Gaussian noise, changed during the recent financial crisis.

The paper provides a broad range of model-dependent as well as model-independent price jump indicators. Using these indicators. I measure the number of price jumps, or the jumpiness, of every trading day for every stock. Then, I compare the days of the financial crisis with those not during the crisis. I define a financial crisis as a structural break. First, I define it as a permanent break starting the day when Lehman Brothers' shares plunged. Second, I define the financial crisis as a temporary break starting the same day but lasting only 30 trading days. Having in hand such tools, I test the hypothesis that the days during the financial crisis are the same with respect to price jump properties as those not during the crisis.

First of all, the results support the claim that volatility increased during the financial crisis. The volatility soars after the Lehman Brothers problems were announced and the peak lasts until mid-October. Then the volatility decreases but keeps above its pre-crisis level. In the first two months of 2009, the volatility increases again, mainly for the banking industry. The increased levels of volatility are in agreement with the general knowledge since they reflect the increase in overall market impatience. The results, however, do not show an increase in price jumps. An overall increase in price jumps would mean a higher rate of market panic and more irrational behavior. A rather stable rate of price jumps, on the other hand, suggests that the proportion of market panic with respect to general impatience remained the same. However, there are some individual cases where the rate of price jumps increased and decreased during the crisis. In addition, it is not possible to draw any industry-dependent conclusions, which is surprising for the banking industry.

Finally, this paper also proves that different price jump indicators measure price jumps very differently. The difference in sensitivity between the indicators, however, is not so easy to describe; this would require a detailed numerical analysis. Such an analysis would be worth performing since the exact quantitative connection between the various price jump indicators would enable us to perform a meta-analysis of the results from various papers that use different indicators. The synergy obtained from such a study would draw a more complex picture about the market mechanisms governing the spread of information. Such mechanisms play a key role when market panic is forming. In addition, this would enable us to better quantitatively describe the irrational behavior of financial markets and thus, hopefully, understand them more deeply.

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# Appendix A

## A.1 Wilcoxon test

The Wilcoxon test is a non-parametric test comparing whether the two observed samples come from the same distribution (Mann and Whitney, 1947; Wilcoxon, 1945). The observations in each of the two samples are ranked and then compared. Finally, the $z$-statistics is defined based on the results of comparison between the two samples. This $z$-statistics follows for large samples[44] a standard normal distribution. The null hypothesis of the test states that both observed samples come from the same distribution. When the calculated $z$-statistics exceeds the critical value, we reject the null hypothesis. In addition, the sign of the $z$-statistics can suggest something about the position of medians of the two compared samples.

## A.2 Composition of the Dow Jones Industrial Average index

Table 6.1 shows the composition of the Dow Jones Industrial Average index including weights. The stocks included in this work are in bold.

[44] Usually above 20; Mann and Whitney (1947) and Wilcoxon (1945).

Table 6.1: Composition of the Dow Jones Industrial Average index.

| Company | Ticker | Weight | Company | Ticker | Weight |
|---|---|---|---|---|---|
| **International Business Machines Corp.** | IBM | 9.27% | American Express Co. | AXP | 3.01% |
| **Chevron Corp.** | CVX | 5.69% | Merck & Co. Inc. | MRK | 2.63% |
| 3M Co. | MMM | 5.62% | E.I. DuPont de Nemours & Co. | DD | 2.51% |
| **Exxon Mobil Corp.** | XOM | 5.47% | Verizon Communications Inc. | VZ | 2.27% |
| United Technologies Corp. | UTX | 4.97% | Walt Disney Co. | DIS | 2.20% |
| McDonald's Corp. | MCD | 4.63% | **Microsoft Corp.** | MSFT* | 2.16% |
| **Procter & Gamble Co.** | PG | 4.54% | Home Depot Inc. | HD | 1.99% |
| **Johnson & Johnson** | JNJ | 4.53% | Kraft Foods Inc. Cl A | KFT | 1.98% |
| **Coca-Cola Co.** | KO | 4.21% | **AT&T Inc.** | T | 1.94% |
| Caterpillar Inc. | CAT | 4.20% | Cisco Systems. Inc. | CSCO*? | 1.73% |
| Wal-Mart Stores Inc. | WMT | 3.95% | Intel Corp. | INTC* | 1.40% |
| The Travelers Companies. Inc. | TRV | 3.83% | **Pfizer Inc.** | PFE | 1.34% |
| Boeing Co. | BA | 3.81% | **Bank of America Corp.** | BAC | 1.18% |
| **Hewlett-Packard Co.** | HPQ | 3.69% | **General Electric Co.** | GE | 1.16% |
| JPMorgan Chase & Co. | JPM | 3.13% | Alcoa Inc. | AA | 0.94% |

Note: The table contains the weights of stocks included in the Dow Jones Industrial Average index. The stocks whose ticker is marked by (*) are traded on NASDAQ. The rest of the stocks are traded on NYSE. Figures are taken from November 2009. Bold names are those included in this study.

## Part III

# The Identification of Price Jumps[45]


**Abstract**

We performed an extensive simulation study to compare the relative performance of many price-jump indicators with respect to false positive and false negative probabilities. We simulated twenty different time series specifications with different intraday noise volatility patterns and price-jump specifications. The double McNemar (1947) non-parametric test has been applied on constructed artificial time series to compare fourteen different price-jump indicators that are widely used in the literature. The results suggest large differences in terms of performance among the indicators, but we were able to identify the best-performing indicators. In the case of false positive probability, the best-performing price-jump indicator is based on thresholding with respect to centiles. In the case of false negative probability, the best indicator is based on bipower variation.


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# 1 Motivation and relevant literature

Discontinuities in price evolution have been recognized as an essential part of the price time series generated on financial markets. Many studies, from the seminal work of Merton (1976) to Andersen et al. (2002), demonstrate that continuous-time models have to incorporate the discontinuous component known in the literature as price jumps. The presence of price jumps has serious consequences for financial risk management and pricing. Nyberg and Wilhelmsson (2009) discuss the inevitable importance to include event risk as is recommended by Basel II accord, which suggests employing the Var model with continuous component and price jumps representing event risks. Andersen et al. (2007) conclude that most of the standard approaches in the financial literature on pricing assets assume a continuous price path. Since this assumption is clearly violated in most cases the results tend to be heavily biased.[46] Before a price jump can be accounted for in an estimation stage, it first has to be identified. Surprisingly, the literature up to now does not offer a consensus on how to identify price jumps properly. Jumps are identified with various techniques that yield different results. The value-added of this paper is that we perform an extensive and detailed non-parametric study that employs a wide variety of price-jump indicators to identify the superior techniques. Specifically, we have employed the double McNemar (1947) test and compared the fourteen different price-jump indicators most frequently used in the literature employing simulated time series.

Researchers agree on the presence of price jumps, but they disagree about the source. One branch of the literature considers new information as a primary source of price jumps (see e.g. Merton, 1976; Lee and Mykland, 2008; and Lahaye et al., 2010). Joulin et al. (2008) and Bouchaud et al. (2004) conclude that price jumps are usually caused by a local lack of liquidity on the market. They also claim that news announcements have a negligible effect on the origin of price jumps. The behavioral finance literature provides other explanations for price jumps. Shiller (2005) claims that price jumps are caused by market participants who themselves create an environment that tends to cause extreme reactions and thus price jumps. Finally, price jumps can be viewed as a

[46] For illustration, Jarrow and Rosenfeld (1984), Nietert (2001) and Pan (2002) study pricing in the presence of jumps and all of them confirm the presence of the jump risk premium. Pricing with jumps using continuous-time diffusion equations has been studied by Broadie and Jain (2008), where the authors consider the pricing of volatility and variance swaps. They conclude that the pricing of swaps significantly differs when jumps are taken into account, thus one cannot appropriately price the risk connected with jumps while ignoring the jumps. Carr and Wu (2010) use a jump-diffusion model to simultaneously price the stock options and credit default swaps and find a significant presence of an interplay between credit and market risks. A similar confirmation of the change in the pricing mechanism was also shown by Duffie et al. (2000), Liu et al. (2003) and Johannes (2004).

manifestation of Black Swans, as discussed by Taleb (2007), where the jumps are rather caused by complex systemic interactions that cannot be easily tracked down. In this view, the best way to understand jumps is to be well aware of them and be ready to react to them properly, instead of trying to forecast them.

The key role price jumps play in financial engineering has triggered interest in the financial econometrics literature, especially how to identify price jumps. Several different approaches have evolved over the recent years. Generally, we can identify in the literature four groups of econometric price-jump indicators.

The first group is represented by the work of Ait-Sahalia (2004), Ait-Sahalia et al. (2009), Ait-Sahalia and Jacod (2009a), and Ait-Sahalia and Jacod (2009b) using proper statistical methods to derive and analyze the jump statistics based on different analytic models. The indicators have well-defined analytic properties; however, they do not identify price jumps one by one but rather measure the jumpiness of the given period. These methods are more suitable to assess the jumpiness of ultra-high-frequency data, even though they were also employed in previous studies for high-frequency time series.

The second group of price-jump indicators comprises indicators based on bipower variation and is promoted in a series of papers: Barndorff-Nielsen and Shephard (2004), Barndorff-Nielsen and Shephard (2006), Barndorff-Nielsen et al. (2004), and Barndorff-Nielsen et al. (2006). The method is based on two distinct measures of overall volatility, where the first one takes into account the entire price time movement while the second one ignores the contribution of the model-dependent price-jump component. The papers above also illustrate a broad range of application. This method has been further improved by Lee and Mykland (2008), who developed a statistics for the exact identification of moments when particular price jumps occur and employ it for high-frequency stock returns. However, the main disadvantage of bipower variation-based methods lie in the sensitivity of the intraday volatility patterns, which leads to a high rate of jump mis-identification.

The third group is represented by a test developed by Jiang and Oomen (2008). This test relies on the difference between the swap variance and the realized variance. The authors claim that their test is better than the one based on bipower variation since it amplifies the discontinuities to a larger extent, as they show with a comparative analysis using Monte Carlo simulation. The amplification of discontinuities tends, according to the authors, to suppress the effects of intraday volatility patterns.

Finally, the fourth group of price jump-indicator techniques has its roots in Econophysics. Such techniques are based on the scaling properties of time series known in physics; see e.g. Stanley and Mantegna (2000), Gopikrishnan et al. (1999), Eryigit et al. (2009), Jiang et al. (2009), and Joulin et al. (2008), who take normalized price time series—the normalization differs across these papers—and define the scaling properties of the tails of the distributions. Then, the scaling index enables them to define the jumpiness of the market purely based on how much of the weight lies in the tails and how this weight is distributed.

As mentioned above, there is still no clear consensus in the literature on how to identify price jumps properly. Bajgrowicz and Scaillet (2010) treat the problem of the spurious identification of price jumps by adaptive thresholds in the testing statistics. The problem with most of the price-jump indicators lies in what model they are built upon. This study illustrates the need for robustness when dealing with price jumps.

The question of the intraday patterns of overall volatility mentioned above is deeply studied in the literature. The work of Wood et al. (1985) documents a U-shaped pattern in the intraday equity volatility. Bollerslev et al. (2008) confirm this effect more robustly. In addition, Novotny (2010) employed many price-jump indicators and studied the difference in price-jump properties during the recent financial crisis using stocks from the NYSE.

Still, to our best knowledge, the literature lacks a deep non-parametric study based on a wide variety of price jump indicators. The literature suggests that identification techniques vary a lot, therefore direct comparison of different papers is not easy. We have focused on this gap in the research and perform a detailed Monte Carlo simulation study to compare price-jump indicators. We have compared price-jump indicators with respect to the false positive and false negative probabilities. We have simulated twenty different kinds of time series with various intraday noise volatilities and different price-jump specifications. Using these simulated time series, we have employed the double McNemar test and compared fourteen different price-jump indicators most frequently used in the literature.

our analysis revealed significant differences among the indicators. It was very often the case that one type of indicator clearly dominated the others with respect to the given criterion. Namely, the comparison with respect to the false positive probability was significantly dominated by the indicator based on thresholding with respect to centiles. On the other hand, the comparison with respect to the false negative probability yielded results in which the bipower variation-based indicator dominated. The differences in indicators is very often significant at the highest significance level, which further supports the initial suspicion that the results obtained using different price-jump indicators are not comparable.

## 2 Price Jump Indicators

We employ a set of price-jump indicators divided into four groups as outlined in the introduction. These indicators are widely used in the empirical literature but the results of two or more indicators are rarely compared with respect to a single string of data. Hence, the results derived in different papers are hard to compare. In our study we perform a non-parametric comparison of the set of price-jump indicators whose details are outlined in the following section. In this section we first introduce the four groups of price-jump indicators. The technical details of the indicators are further elaborated in Appendix A.

### 2.1 Group 1: Ait-Sahalia

The first class of indicators follows the same set of assumptions as Ait-Sahalia (2004), Ait-Sahalia and Jacod (2009a) and Ait-Sahalia and Jacod (2009b). The price process is assumed to be decomposed into the Gaussian component—corresponding to normal (white) noise—and the non-homogenous Poisson component—corresponding to price jumps. Under certain assumptions it holds that whenever a significant price jump appears, the price increment is dominated by the non-homogenous Poisson component. On the other hand, when the price movements are governed solely by Gaussian noise, the average and/or maximum magnitudes of such increments can be estimated (at a given confidence level). Therefore, one can invert such an argument and set a threshold value that will effectively distinguish the two components.

In practical cases, however, the proper threshold values require a knowledge of what should be estimated. Thus we employ an empirical approach and set the threshold by calculating certain threshold levels, or certain percentiles, of the distribution of returns observed over the entire sample. In addition, the financial time series often have intraday volatility patterns, i.e., the average absolute returns systematically differ over the trading day. To reflect this phenomenon, we further divide every trading day into several blocks and calculate percentiles over these blocks separately.

### 2.2 Group 2: Bipower Variation

The second group is based on bipower variation, as in Barndorff-Nielsen and Shephard (2004) and Barndorff-Nielsen and Shephard (2006). Specifically, it is based on the difference between the two measures of variation: realized variation and bipower variation. Assuming the price generating process can be decomposed into two components—regular white noise and price jumps—the realized variation measures the variation in the prices coming from both the white noise and the price jumps, while the bipower variation measures the variation coming from the white noise only. This measure can be applied in two different ways.

The first approach, proposed by the above mentioned authors and further elaborated by Huang and Tauchen (2005), involves the construction of a statistics whose purpose is to determine the presence of price jumps over a given time window, i.e., to test the hypothesis that a given time window contains price jumps at all. This statistics, known as the Max-Adjusted statistics, can be thus employed to identify the exact moment when a price jump occurs. Namely, we fix the length of the testing window, and for every time step we test a given window ending at that time step for the presence of price jumps. Then, we say that a price jump occurs at that moment if the window ending at that moment contains a price jump while the window ending at the preceding time step does not.

A problem occurs for consecutive price jumps. If two price jumps are separated by an interval shorter than the given window, the second price jump cannot be identified. Hence, we modify the technique in such a way that after we identify a price jump, we replace it with an average calculated over a moving window of the same length. Since these observations by definition do not contain a price jump, their average also excludes price jumps.

The second approach, constructed by Lee and Mykland (2008), also employs bipower variation. However, it is by definition constructed as a statistics to identify price jumps and the moments when they occurred. The statistics compares the current price movement with the bipower variation calculated over a moving window with a given number of preceding observations, excluding the current one.

### 2.3 Group 3: Jiang-Oomen Statistics

Jiang and Oomen (2008) proposed another statistics to test for the presence of price jumps over a certain time window based on Swap Variance. It is claimed that this test amplifies the contribution of price jumps to a larger extent than bipower variation indicators and thus are less sensitive to intraday volatility patterns. Since the Jiang and Oomen statistics is constructed as a test statistics for a certain time window, the price-jump indicator is analogous to the one following the Berndorff-Nielsen and Shephard method: For every time step, we define a moving time window of a given length ending at the time step and test for the presence of price jumps over the window. Then, we identify a certain moment as the one when the price jump occurred if the window ending at the current time step contains a price jump and the one ending at the previous time step does not. In addition, we define an analogous improved indicator, which involves replacing the identified price jumps with moving averages and thus allows for identification of consecutive price jumps.

### 2.4 Group 4: Statistical Finance

The last group of identification techniques comes from the field of statistical finance as it is called by Bouchaud (2002), although it is also known as Econophysics. This group of indicators relies on the scaling properties of price movements. We employ the price-jump index defined by Joulin et al. (2008). The price index is defined as the absolute returns normalized with respect to the $L_1$ variance, i.e., the variance defined as an average of absolute returns over a certain moving window. The price-jump index has, as the literature confirms (Joulin et al., 2008), certain scaling properties of the tail part of its distribution. Thus, we define price jumps as those returns for which the price-jump index exceeds a certain empirically determined threshold.

## 3 Test to compare the performance of the different price-jump indicators

Here we introduce the procedure to compare the performance of the different price-jump indicators. The procedure itself is based on the double McNemar (1947) test, which is a non-parametric method used on nominal data. The intuition behind employing this method lies in the fact that, based on extensive simulations, we want to compare the price-jump indicators with each other rather than study their finite sample properties. Hence, the comparison will be based on the prediction accuracy of the indicators. This means that price-jump indicator A dominates indicator B if A has a significantly better accuracy of jump prediction. This strategy leads to a test procedure to compare the proportions of correctly and incorrectly predicted jumps. The main idea for this approach is natural: if the price-jump indicators are not different in terms of the accuracy of the prediction, it is hard to judge whether one indicator dominates the other in other ways (for binary models see Hanousek, 2000).

Since the jump process $\{Y_t\}$ could be understood as a binary process (0-1), 1 being associated with a jump, studying the prediction accuracy would lead to the following binary outcomes with the following probabilities:

$$\begin{aligned}
p_{11} = &\quad Pr\,(1|1) = Pr\left(\hat{Y} = 1|Y = 1\right) \quad \text{, i.e., the probability of correct prediction when } Y = 1; \\
p_{22} = &\quad Pr\,(0|0) = Pr\left(\hat{Y} = 0|Y = 0\right) \quad \text{, i.e., the probability of correct prediction when } Y = 0; \\
p_{12} = &\quad Pr\,(1|0) = Pr\left(\hat{Y} = 1|Y = 0\right) \quad \text{, i.e., the probability of wrong prediction when } Y = 0; \\
p_{21} = &\quad Pr\,(0|1) = Pr\left(\hat{Y} = 0|Y = 1\right) \quad \text{, i.e., the probability of wrong prediction when } Y = 1.
\end{aligned} \tag{3.1}$$

In diagnostics terminology the above probabilities are usually called *sensitivity* ($p_{11}$), *selectivity* ($p_{22}$), *false positive* ($p_{12}$) and *false negative* ($p_{21}$) probabilities. It is clear that in different situations one might prefer different treatments of misclassification by giving to $Pr\,(1|0)$ and $Pr\,(0|1)$ different subjective weights. For the sake of simplicity let us consider the case when both misclassifications have equal weights, i.e., we concentrate on the standard case in which the probability of correct prediction is maximized and where

$$Pr(\text{correct prediction}) = Pr\,(0|0) + Pr\,(1|1) = p_{11} + p_{22}. \tag{3.2}$$

Using a complementary approach, one can minimize the probability of incorrect prediction:

$$Pr(\text{incorrect prediction}) = Pr\,(0|1) + Pr\,(1|0) = p_{21} + p_{12}. \tag{3.3}$$

Table 3.1: Pair-wise comparison of the prediction accuracy of two jump indicators.

| | | $I_1$ jump prediction | | $\Sigma$ (total) |
|---|---|---|---|---|
| | | correct | incorrect | |
| $I_2$ jump prediction | correct | $n_{11}$ | $n_{12}$ | $n_{1\bullet}$ |
| | incorrect | $n_{21}$ | $n_{22}$ | $n_{2\bullet}$ |
| $\Sigma$ (total) | | $n_{\bullet 1}$ | $n_{\bullet 2}$ | $n$ |

The above approach could be used to search for and study the "best" price-jump indicator in terms of the accuracy of prediction. However, the optimization procedures conducted on simulated data (potentially) might not capture the relationship as well as the difference (in prediction) between the relationships between the studied indicators, and therefore it is better to use this approach for a pair-wise comparison of jump indicators.

In terms of any pair-wise comparison/test we can assume the following. We have the available outcomes of price-jump prediction given by two jump indicators denoted as $I_1$ and $I_2$. The combination of their outcomes in terms of the accuracy of price-jump prediction can be summarized by Table 3.1.

In this table, $n$ is the total number of simulated returns, $n_{11}$ denotes the number of cases when both indicators correctly identify a price jump, $n_{12}$ is the number of cases when the $I_2$ correctly identifies a price jump and $I_1$ does not, $n_{21}$ is the number of cases when the $I_1$ correctly identifies a price jump and $I_2$ does not. Finally, $n_{22}$ denotes the number of cases when both indicators do not correctly identify a price jump. In other words, Table 3.1 is a contingency table summarizing the outcomes of the two binary variables $I_1$ and $I_2$ using the accuracy of prediction as the additional classification dimension. Therefore we adopt standard notation for contingency tables, and a dot used in a subscript indicates the corresponding marginal distribution; for example, $n_{1\bullet}$ stands for the number of price jumps correctly identified by $I_1$.

The statistical inference of whether jump indicator $I_1$ dominates $I_2$ in prediction accuracy can be assessed by testing the null hypothesis $H_0$: $n_{1\bullet} = n_{\bullet 1}$ , or equivalently $H_0$: $n_{12} = n_{21}$. This approach directly leads to the well-known McNemar (1947) test, whose underlying test statistics is $\chi^2_1 = \frac{(n_{12}-n_{21})^2}{n_{12}+n_{21}}$ and is distributed asymptotically as $(n_{12}+n_{21} \geq 8)$. For smaller values of $n_{12}+n_{21}$ one can construct an exact test using probabilities in multinomial distribution.[47]

[47] It is also recommended to conduct an exact test if 20% of $n \cdot n_{\bullet i} \cdot n_{j\bullet}$ is less than 5, or if any of $n \cdot n_{\bullet i} \cdot n_{j\bullet}$ is smaller than one (see for example Gibbons, 1997).

In Table 3.1 we can set various criteria for prediction accuracy, for example the classification used in the table. The example above used the approach where misclassifications $Pr(1|0)$ and $Pr(0|1)$ have the same weight in selecting price-jump indicators. We compare the correct identification of the price jump only with the incorrect identification in which we combine both types of misclassification. Since in reality those misclassifications have different impacts, we test and compare the price-jump indicators using only false negative and false positive classifications. This means that we can treat misclassification only when an indicator predicts a jump but there was no jump (false positive) or when the indicator does not predict a jump, but we observe a jump (false negative). If we minimize the false positive criterion, the winning indicator would identify fewer returns as false price jumps and would potentially miss some true price jumps. A similar logic is valid for the false negative criterion.

The testing procedure in the simulation framework is applied as follows:

**Step 1:** In the first step, we simulate 100 trading days and compare the indicators pair-wise.[48] As the prediction criteria, we use

a. the number of correctly identified price jumps and

b. the number falsely identified price jumps.

We conduct the McNemar-type test described above, and we count number of cases when indicator $I_1$ dominates $I_2$ (90%, 95% and 99% significance levels).

**Step 2:** In the second step, we repeat each simulation 100 times.[49] The results from the test procedure (the first step) are used as the input for the second step. The second test is again the McNemar-type test, where we compare the number of cases when one indicator dominates the other at a given confidence level. For both tests, we use three confidence levels—90%, 95% and 99%.

To summarize, first we use the test for a given (simulated) window of trading days to analyze if one jump indicator dominates the other in terms of the accuracy of the prediction of the price jump. The second step analyzes the results of repeated simulations using the same time window.

[48]We actually simulate 105 trading days and then cut off the first five trading days.

[49]The number of repetitions should be theoretically infinite. However, in practical calculations, we have to restrict ourselves to some finite number of repetitions. This finite number has to produce stable results—mean of the measured variables over simulated sample should be stable—and the simulation itself has to run in a meaningful time. In this, case, we have performed stability checks and chosen 100 repetition as a optimal number with respect to both above mentioned criteria.

# 4 Data Generation of Artificial Time Series

The goal of this part is to compare the price-jump indicators to find the one that performs best. For that purpose, we perform an extensive simulation study with simulated data. We simulate the price of a virtual asset during a trading day: every trading day lasts seven trading hours or 420 trading minutes. The price time series is simulated at a 1-minute frequency as a discrete process generally defined using the Euler scheme:

$$p_t - p_{t-1} = F_t \,, \tag{4.1}$$

where $F_t$ is the time-dependent price generator. Generally, the drift is insignificant for high-frequency data.

## 4.1 Normal price movements

The most intuitive price generation process uses an iid normal distribution:

$$p_t - p_{t-1} = \sigma Z_t \,, \tag{4.2}$$

where $Z_t = Z \sim N\left(0, 1\right)$ and $\sigma$ is a constant. This is the first intraday volatility pattern we employ.

## 4.2 Intraday Volatility Patterns

The flat intraday volatility pattern is, however, not close to observed data. Therefore, we mimic the well-known $U$-shaped volatility pattern, which says that the price time series show a significant increase in volatility at the beginning and end of the trading day. We implement three different intraday volatility patterns. The purpose is to test the behavior of the indicators under these intraday volatility patterns as well as to compare them over the broadest possible range of situations. The four different specifications for intraday volatility patterns further serve as a testing ground for a proper understanding of price-jump indicators.

### 4.2.1 Step function I

The second intraday volatility pattern is based on the assumption that volatility undergoes a two-regime switching process, where one regime is at the beginning and end of the trading day, while the other regime is at the middle of the trading day. Namely, we assume a price-generating process given as

$$p_t - p_{t-1} = \sigma_t Z_t \,, \tag{4.3}$$

where the volatility $\sigma_t$ governs the two-regime process and is defined as

$$\sigma_t = \begin{cases} \sigma_{high} & t \in [0, \alpha \cdot Day) \\ \sigma_{low} & t \in [\alpha \cdot Day, \beta \cdot Day) \\ \sigma_{high} & t \in [\beta \cdot Day, Day] \end{cases} \,, \tag{4.4}$$

where $\sigma_{low} < \sigma_{high}$ and $\alpha$ and $\beta$ are parameters governing the periods with the different volatility regime. Compared to the previous case, there is an artificial "jump" in volatility at the moment where volatility changes from $\sigma_{low}$ to $\sigma_{high}$.

### 4.2.2 Step function II

The third intraday volatility pattern is an extension of the previous one. We employ a four-level volatility regime to mimic the $U$-shaped volatility smile in a more subtle way. Such a definition also partially gets rid of the artificial jumps at the corners where the volatility regimes change. Namely, we assume the price generating process is given as

$$p_t - p_{t-1} = \sigma_t Z_t \,, \tag{4.5}$$

with

$$\sigma_t = \begin{cases} 3\sigma_{high} & t \in [0, \alpha \cdot Day) \\ 2\sigma_{high} & t \in [\alpha \cdot Day, \beta \cdot Day) \\ 1\sigma_{high} & t \in [\beta \cdot Day, \gamma \cdot Day) \\ \sigma_{low} & t \in [\gamma \cdot Day, \delta \cdot Day) \\ 1\sigma_{high} & t \in [\delta \cdot Day, \epsilon \cdot Day) \\ 2\sigma_{high} & t \in [\epsilon \cdot Day, \phi \cdot Day) \\ 3\sigma_{high} & t \in [\phi \cdot Day, Day] \end{cases}, \tag{4.6}$$

where $\sigma_{low} < \sigma_{high}$ and parameters $\alpha$, $\beta$, $\gamma$, $\delta$, $\epsilon$ and $\phi$ define the periods with different volatility regimes. In this case, the volatility pattern is smoother and mimics the empirical patterns better.

### 4.2.3 Linear-like smooth smile

The fourth volatility pattern mimics the $U$-shaped volatility smile more closely. We use three linear functions which ensure a smooth transition in volatility between different parts of the trading day. Namely, we assume the price-generating process is given as

$$p_t - p_{t-1} = \sigma_t Z_t \,, \tag{4.7}$$

with

$$\sigma_t = \begin{cases} 3\sigma_{high} - \frac{(3\sigma_{high} - \sigma_{low})}{\alpha \cdot Day}(t) & t \in [0, \alpha \cdot Day) \\ \sigma_{low} & t \in [\alpha \cdot Day, \beta \cdot Day) \\ \sigma_{low} + \frac{(3\sigma_{high} - \sigma_{low})}{(1-\beta) \cdot Day}(t - \beta \cdot Day) & t \in [\beta \cdot Day, Day] \end{cases}, \tag{4.8}$$

and $\sigma_{low} < \sigma_{high}$. The parameters define the periods with different volatility; $3 \cdot \sigma_{high}$ was chosen to be able to compare this pattern with the previous one.

## 4.3 Volatility Specifications

We employ the four different intraday volatility patterns defined above with the parameters as follows.

**Volatility Pattern A:**

The first type of intraday volatility pattern consists of a basic homogenous iid normal process, namely

$$p_t - p_{t-1} = \sigma Z_t \,, \tag{4.9}$$

where we use $\sigma = 0.0004$, which corresponds to the values observed in the real data (used in the literature and based on the annual realized volatility).

**Volatility Pattern B:**

The second intraday volatility pattern is given as

$$p_t - p_{t-1} = \sigma_t Z_t \,, \tag{4.10}$$

with

$$\sigma_t = \begin{cases} \sigma_{high} & t \in [0, Day/4) \\ \sigma_{low} & t \in [Day/4, 3Day/4) \\ \sigma_{high} & t \in [3Day/4, Day] \end{cases} , \tag{4.11}$$

using$\sigma_{low} = 0.0001$ and $\sigma_{high} = 0.0004$.

**Volatility Pattern C:**

The third intraday volatility pattern is defined as

$$p_t - p_{t-1} = \sigma_t Z_t \,, \tag{4.12}$$

with volatility defined as

$$\sigma_t = \begin{cases} 3\sigma_{high} & t \in [0, 45min) \\ 2\sigma_{high} & t \in [45min, 90min) \\ 1\sigma_{high} & t \in [90min, 135min) \\ \sigma_{low} & t \in [135min, 285min) \\ 1\sigma_{high} & t \in [285min, 330min) \\ 2\sigma_{high} & t \in [330min, 375min) \\ 3\sigma_{high} & t \in [375min, 420min] \end{cases} . \tag{4.13}$$

The 45-minute step corresponds approximately to $Day/9$, thus the trading day has three periods of approximately the same duration: the first at the beginning of the day with decreasing volatility, the second at the middle of the day with increasing volatility and the third at the end of the day with increasing volatility. We use $\sigma_{low} = 0.0001$ and $\sigma_{high} = 0.0002$.

**Volatility Pattern D:**

This pattern prevents a possible criticism that could emerge in the previous cases: whenever we change the volatility regime we introduce an artificial jump of average size $\sigma_{high}$. This can have a negative effect on the performance of some indicators; therefore we make the transition smoother. Thus, volatility is defined as

$$\sigma_t = \begin{cases} 3\sigma_{high} - \frac{(3\sigma_{high} - \sigma_{low})}{135min}(t) & t \in [0, 135min) \\ \sigma_{low} & t \in [135min, 285min) \\ \sigma_{low} + \frac{(3\sigma_{high} - \sigma_{low})}{135min}(t - 285) & t \in [285min, 420min] \end{cases} , \tag{4.14}$$

where $\sigma_{low} < \sigma_{high}$. We use $\sigma_{low} = 0.0001$ and $\sigma_{high} = 0.0002$.

## 4.4 Price-Jump Specification

This study focuses on price jumps, so we extend the price movements defined above with non-normal price jumps. The Euler scheme for price evolution with price jumps is defined as

$$p_t - p_{t-1} = F_t = \sigma_t Z_t + J \cdot j_t \,, \tag{4.15}$$

where $\sigma_t Z_t$ is the term defined above and $J \cdot j_t$ is the term generating price jumps. We conveniently define $j_t$ as a Poisson process with a rate of price jumps arrival $\lambda_j$:

$$j_t = \begin{cases} 0 & p_j \\ 1 & 1 - p_j \end{cases} \,, \tag{4.16}$$

where $p_j = e^{-\lambda_j}$ and parameter $J$ governs the size of the jumps. For single-size price jumps $J = \pm J_{param}$, where both signs have the same probability of occurring. In the most sophisticated cases, parameter $J$ can have a value from any statistical distribution.

Due to the independence of increments, the probability to observe $n$ jumps at a time step is given as

$$P\left(\text{No. of jumps}{=}n\right) = \frac{e^{-\lambda}\left(\lambda\right)^n}{n!} \,. \tag{4.17}$$

By definition, we assume that only one price jump per time step can occur and thus we define first the probability that no price jump will occur as $P\left(\text{No. of jumps}{=}0\right) = e^{-\lambda}$ and the probability that one price jump will occur as a complement value $P\left(\text{No. of jumps}{=}1\right) = 1 - e^{-\lambda}$.

We employ five different specifications of price jumps. These five specifications are combined with the four different groups of indicators. Thus we will have twenty different price time series (excluding four different time series without price jumps).[50]

**Price Jumps 1–3:**

The first three price-jump specifications have the same rate of arrival and a constant size of jump $J = \pm const$. Both signs occur with the same probability.

[50] An alternative approach to estimate price jumps is to assume that the error term follows a given stochastic distribution and combine it with a non-homogenous Poisson term describing a jump. Then, Score Method of Moments or Simulated Method of Moments could be employed to estimate the parameters of the model. See, for example, Jiang and Oomen (2007) who estimate an affine jump diffusion model for a series of returns from the S&P 500 index. However, this approach relies on the proper specification of the underlying model, including the intraday volatility pattern as well as the distribution. Thus, this approach is not appropriate for our analysis.

**Price Jump 1:** We employ combinations of $J = 5\sigma_{jump}$ and $\lambda = 5/N_{Day}$.

**Price Jump 2:** We employ combinations of $J = 7\sigma_{jump}$ and $\lambda = 5/N_{Day}$.

**Price Jump 3:** We employ combinations of $J = 9 \cdot \sigma_{jump}$ and $\lambda = 5/N_{Day}$.

The parameter $\sigma_{jump} = 0.0004$ and $N_{day}$ means the number of minutes per trading day.

**Price Jumps 4–5:**

The next two price-jump specifications use a uniform distribution to select the size of price jumps. We select price jumps from a given distribution, with $0 < a < b$, and both signs occur with the same probability. We use the following specifications.

**Price Jump 4:** $J \sim \pm U\left(5\sigma_{jump}, 9\sigma_{jump}\right)$ and $\lambda_j = 5/N_{Day}$.

**Price Jump 5:** $J \sim U\left(5\sigma_{jump}, 9\sigma_{jump}\right)$ and $\lambda_j = 15/N_{Day}$.

The parameter for volatility is chosen as $\sigma_{jump} = 0.0004$ and $N_{day}$ is defined above.

# 5 Comparison Strategy

The goal of the simulation procedure is to compare price-jump indicators with each other, understand their properties and select the most appropriate indicator for real data.

## 5.1 Price Jump Indicators

We employ the following extensive list of price-jump indicators that are defined in Appendix A.

1. Centiles as defined in A1.1: The price jump is identified as those returns below the 0.5th centile or above the 99.5th centile. Centiles are calculated for the entire sample.

2. Block-centiles as defined in A1.2: The price jump is identified as those returns below the 0.5th centile or above the 99.5th centile. Every trading day is divided into 15-minute blocks and centiles are calculated separately for every block for the entire sample.

3. $Z_{RJ,TP}$ as defined in A2.1 with a 99% confidence interval (CI) and length of moving window $n = 60$.

4. $Z_{RJ,TP}$ as defined in A2.1 with a 99% CI and $n = 120$.

5. Improved $Z_{RJ,TP}$ as defined in A2.2 with a 99% CI and $n = 60$.

6. Improved $Z_{RJ,TP}$ as defined in A2.2 with a 99% CI and $n = 120$.

7. $\xi$-statistics as defined in A2.3 with a 99% CI and $n = 60$.

8. $\xi$-statistics as defined in A2.3 with a 99% CI and $n = 120$.

9. $JO_{Ratio}$ as defined in A3.1 with a 99% CI and $n = 60$.

10. $JO_{Ratio}$ as defined in A3.1 with a 99% CI and $n = 120$.

11. Improved $JO_{Ratio}$ as defined in A3.2 with a 99% CI and $n = 60$.

12. Improved $JO_{Ratio}$ as defined in A3.2 with a 99% CI and $n = 120$.

13. Price-jump index as defined in A4.1: The price jump is identified as those returns with $pji > 4$ and $n = 120$.

14. Price-jump index as defined in A4.1: The price jump is identified as those returns with $pji > 4$ and $n = 420$.

## 5.2 Artificial Time Series

We employ a Monte Carlo simulation technique to simulate an artificial time series with price jumps.[51] We simulate all the combinations of four different intraday volatility patterns (specified above) and five different price-jump specifications (specified above), thus there are 20 different time series in total. Every trading day is sampled at a one-minute frequency, starting at 9:01 and ending at 16:00; seven hours in total, which gives 420 trading minutes per trading day. We further match

[51] We have used Stata, which contains subroutines for Monte Carlo simulation analysis. These subroutines assures that random generators are called appropriately without unwanted repetitions of random number sequences. In addition, the length of the simulation didn't require employment of more complex random number generators nor treatment for to treat for simulations on parallel platforms.

Table 6.1: Summary of the analysis based on false positive and false negative probabilities.

| Indicator No. | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| False positive | No. of dominances | **13** | 3 | 0 | 0 | 0 | 0 | 1 | 2 | 0 | 0 | 0 | 0 | 0 | 0 |
| False negative | No. of dominances | 2 | 0 | 0 | 0 | 0 | 0 | 10 | **15** | 0 | 0 | 0 | 0 | 0 | 0 |

the end of the trading day with the beginning of the next trading day and thus produce a continuous time series.

We simulate 105 trading days and define price-jump indicators. Then, we cut off the first five days, which serve to settle down the simulation as well as produce the necessary observations for the moving windows. In addition, the Jiang-Oomen statistics-based indicators require absolute levels. For that purpose, we set an initial value $p_0 = 100$ and produce price levels instead of returns.

### 5.3 Relative Comparison of Price Jump Indicators

In the last step we perform an extensive comparison of the performance of the different price-jump indicators. We follow the methodology outlined in the previous sections based on the McNemar (1947) test.

## 6 Results

We compared 14 different price-jump indicators with respect to false positive and false negative probabilities. We present all the details of the comparison in Appendix B. In Table 6.1, we present a summary of our results: the number of cases when a given price-jump indicator dominates the other indicators with respect to both false positive and false negative probabilities. Several times there were two indicators that were dominating the other indicators. In such a case, we counted both indicators as dominating the given simulated specification.

In the case of the false positive probability—the false identification of non-jump cases—the best indicator seems to be indicator No. 1 based on centiles. This indicator dominates others the most often. In addition, there are other indicators which perform in some specifications well, namely No. 2—the one based on block-centiles—and Nos. 7 and 8 based on the ξ-statistics with 99% CI and $n = 60$ or $n = 120$, respectively.

The other case, false negative probability—jumps that occur are not identified—shows that the best performing statistics is indicator No. 8, the $\xi$-statistics with 99% CI and $n = 120$. In addition, the analysis shows that even the version with time window $n = 60$ performs well since these two statistics are in many cases statistically indistinguishable.

The analysis further reveals that the performance of price-jump indicators is not homogeneously distributed among all the indicators but rather their performance is dominated by a few best indicators. This can be further seen in the results, where most of the time when one indicator dominates another, it dominates it at the highest significance level.

## 7 Conclusion

We performed an extensive simulation study to compare the relative performance of a broad class of price-jump indicators with respect to false positive and false negative probabilities. We simulated twenty different time-series specifications with different intraday noise volatility patterns and price-jump specifications and using these artificial time series. We employed the double McNemar test and compared fourteen different price-jump indicators that are widely used in the literature. We compared them with respect to false positive and false negative probabilities. The results suggest large differences among the indicators in terms of their performance. In the case of false positive probability, the best-performing price-jump indicator is the one based on thresholding with respect to centiles. In the case of false negative probability, the best indicator is the one based on bipower variation. Significant differences among the indicators further confirms the fact that any meta-analysis based on different price-jump indicators is not possible since the indicators tend to perform in very different ways.

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# Appendix A: Price-Jump Indicators – The Details

In this section, we provide technical details for all of the price-jump indicators we tested.

## A1 Group 1: Ait-Sahalia

This class of indicator assumes that the underlying price increment process is given as $\Delta S = \sigma\Delta X + \Delta J$, where the price increment means $\Delta S = S_t - S_{t-\Delta t}$, where we assume that we observe the realization of prices in equidistant time steps $\Delta t$, i.e., $\Delta S$ denotes a price change over the time interval $\Delta t$. In this definition, $X$ corresponds to the Brownian motion and $J$ is a $\beta$-stable process. The increments of the two components can be expressed as $\Delta X = (\Delta t)^{1/2} X_1$ and $\Delta J = (\Delta t)^{1/\beta} J_1$, where the equalities are equalities in distribution. In this specification, $X$ corresponds to the Brownian motion and $J$ is a $\beta$-stable process.

The different magnitudes in the two components can be used to discriminate between the noise components and the big price jumps coming solely from the $J$-process.[52] The big price jumps cause $\Delta S = \Delta J$ (in distribution), while in the presence of no big price jumps, which is most of the time, $\Delta S = \sigma (\Delta t)^{1/2} X_1$. Therefore, we can, for a given $\Delta t$, choose a threshold value equal to $\alpha (\Delta t)^{\gamma}$, with $\alpha > 0$ and $\gamma \in (0, 1/2)$, such that if $\Delta S > \alpha (\Delta t)^{1/2}$ then $\Delta S$ is at a given moment dominated by $J$ with a certain probability.

This argument can be inverted. Assuming that we know the rate of the arrival of big jumps, we can easily construct a threshold based on the centile value. Therefore we will use centiles as a threshold to discriminate price jumps from the noise. Using centiles, however, can produce biased results due to the intraday volatility patterns. The intraday volatility pattern means that $\sigma = \sigma (t)$. In addition, the $J$-process can also be different either across different phases of the trading day or across different trading days. To account for the former, we divide every trading day into several trading blocks and assume that inside every trading block the price process is constant no matter the trading day. In this case, we can apply the same logic block by block. Namely, we calculate the centiles for the same block over different trading days and threshold price jumps for every trading block separately.

[52] The $J$-process contributes to a large amount of small price jumps; however, we want to focus on big price jumps only. The goal is not to completely determine the properties of the $J$-process but rather to determine how to distinguish extreme price movements.

### A1.1 Global Centiles

We define price jumps as those returns that are higher/lower than a given upper/lower centile. Centiles are calculated based on the observation of the entire sample, where we use the 99.5th centile as the upper threshold and the 0.5th centile as the lower threshold.

### A1.2 Centiles over Block-Windows

To compensate for intraday volatility, we divide every trading day into several 15 minute-long blocks. Then, we apply the procedure defined above for every trading block separately, i.e., we calculate the upper/lower threshold for every trading block independently and then define the price jumps as those price movements that are higher/lower than the corresponding threshold values.

## A2 Group 2: Bipower Variation

The two different measures for variation, as defined by Barndorff-Nielsen and Shephard (2004) and Barndorff-Nielsen and Shephard (2006), are Realized Variation defined as $RV_t = \sum_{i=2}^{n} r_i^2$ and Bipower Variation defined as $BV_t = \mu_1^{-2}\left(\frac{n-1}{n-2}\right)\sum_{i=3}^{n}|r_i|\,|r_{i-1}|$, with $\mu_\alpha = E\left(|Z|^\alpha\right)$ for $Z \sim N(0,1)$, or generally $\mu_\alpha = 2^{\alpha/2}\Gamma\left(\frac{\alpha+1}{2}\right)/\sqrt{\pi}$.

### A2.1 The Max-adjusted Statistics

The difference between the two variations is the key ingredients; however, one also needs to estimate the conditional standard deviation $\int \sigma^4$. There are at least two possible ways to estimate this Andersen et al. (2007) introduced tripower quarticity

$$TP = n\mu_{4/3}^{-3}\frac{n-1}{n-3}\sum_{i=4}^{n}|r_i|^{4/3}\,|r_{i-1}|^{4/3}\,|r_{i-2}|^{4/3} \to \int \sigma^4\,, \tag{7.1}$$

to measure the conditional standard deviation, while Barndorff-Nielsen and Shephard (2004) and Barndorff-Nielsen and Shephard (2006) used Quadpower Quarticity

$$QP = n\mu_1^{-4}\frac{n-1}{n-4}\sum_{i=5}^{n}|r_i|\,|r_{i-1}|\,|r_{i-2}|\,|r_{i-3}| \to \int \sigma^4\,. \tag{7.2}$$

The same authors then proposed several different asymptotically equal statistics to estimate the presence of price jumps.

According to Huang and Tauchen (2005), the best statistics is $Z_{RJ,TP}$, defined as

$$Z_{RJ,TP} = \frac{RJ}{\sqrt{\left(\left(\frac{\pi}{2}\right)^2 + \pi - 5\right)\left(\frac{1}{n}\right)\max\left(1, \frac{TP}{BV^2}\right)}}, \tag{7.3}$$

with $RJ = (RV - BV)/RV$. The null hypothesis states that there is no jump in a given period. If the statistics exceeds the critical value $\Phi^{-1}(\alpha)$, then we reject the null hypothesis of no price jump at confidence level $\alpha$.

Realized Variation and Bipower Variation are forward-looking; however, we need a backward-looking specification re-defined as

$$RV_j = \sum_{i=j-n+2}^{j} r_i^2, \tag{7.4}$$

$$BV_j = \mu_1^{-2}\left(\frac{n-1}{n-2}\right)\sum_{i=j-n+3}^{j} |r_i|\,|r_{i-1}|\,. \tag{7.5}$$

The statistics thus refer to a window of length $n$ ending at time step $j$. Thus, observing a significant jump at time step j means that somewhere in the window of length n ending at time step $j$ was at least one significant jump. Thus, the change between periods with no price jump and periods with a price jump can serve as an indicator for the moments when jumps happened the first time. This also assumes that the average time between two jumps will be much larger than the window used in this statistics. On the other hand, a very short time window skews the results with a small-sample bias. Since we assume more than one price jump per day, we employ $n = 60$ and $n = 120$.

The indicator for a price jump is defined as follows: price jumps are those prices for which $Z_{t-1} \leq \Phi^{-1}(\alpha)$ and $Z_t > \Phi^{-1}(\alpha)$. By definition, the indicator cannot distinguish two consecutive steps, otherwise we would have to work with the absolute levels of the statistics.

### A2.2 Max-adjusted Statistics: Improved Identification Method

The improvement works as described in the main section, namely returns identified as price jumps are replaced by the average value calculated over the same length as was used for identification. The replaced value is clearly not a price jump; otherwise, the price jump would not be identified as a price jump. Therefore, we define a pair of improved indicators based on the above-defined Max-adjusted statistics with $n = 60$ and $n = 120$.

### A2.3 Lee-Mykland

The statistics of Lee and Mykland (2008) is based on bipower variation and is given as

$$\mathcal{L}\left(i\right) = \frac{r_i}{\hat{\sigma}\left(i\right)}\,, \tag{7.6}$$

with $\hat{\sigma}^2\left(i\right) = \frac{1}{n-2}\sum_{j=i-n+2}^{i-1} |r_i|\,|r_{i-1}|$. Then

$$\frac{max_i\left|\mathcal{L}\left(i\right)\right| - C_n}{S_n} \rightarrow \xi\,, \tag{7.7}$$

where $\xi$ has a cumulative distribution function $P\left(\xi \leq x\right) = \exp\left(-e^{-x}\right)$, and the two constants are given as $C_n = \frac{(2\log n)^{1/2}}{c} - \frac{\log\pi + \log(\log n)}{2c(2\log n)^{1/2}}$, $S_n = \frac{1}{c(2\log n)^{1/2}}$ and $c = \sqrt{2}/\sqrt{\pi}$. Whenever the $\xi$-statistics exceeds the critical value $\xi_{CV}$, we reject the null hypothesis of no price jump at time $t_i$.

Lee and Mykland recommend $n_{15-min} = 156$ and $n_{5-min} = 270$. In our analysis, we are using $n = 60$ and $n = 120$.

## A3 Group 3: Jiang-Oomen Statistics

The Jiang and Oomen (2008) statistics is based on Swap Variance defined as

$$SwV = 2\sum_{i=2}^{n}\left(R_i - r_i\right)\,, \tag{7.8}$$

with $R_i = \frac{P_i - P_{i-1}}{P_i}$, where $P_i = \exp\left(p_i\right)$ and $r_i = p_i - p_{i-1}$. The authors claim that employing swap variance further amplifies the contribution coming from price jumps and thus makes the estimator less sensitive to intraday variation.

### A3.1 Jiang-Oomen Statistics-based Price-Jump Indicator

The Jiang-Oomen statistics is defined as

$$JO_{Ratio} = \frac{nBV}{\sqrt{\Omega_{SwV}}} \left( 1 - \frac{RV}{SwV} \right) , \tag{7.9}$$

where the Realized Variation $RV$ and the Bipower Variation $BV$ are defined as above. The statistics is asymptotically equal to $z \sim N(0,1)$ and tests the null hypothesis that a given window does not contain any price jump. The indicator for a price jump is defined as those price movements for which $JO_{t-1} \leq \Phi^{-1}(\alpha)$ and $JO_t > \Phi^{-1}(\alpha)$. The same comments as for the Max-adjusted statistics apply. We use two price-jump indicators with $n = 60$ and $n = 120$.

### A3.2 Jiang-Oomen Statistics: Improved Identification Method

We use the same improvement technique as in section A2.1 and define two improved indicators based on the Jiang-Oomen statistics with $n = 60$ and $n = 120$.

## A4 Group 4: Statistical Finance

The scaling properties of returns can be studied using different techniques (see Stanley and Mantegna, 2000, and references therein), where we employ the price-jump index as defined by Joulin et al. (2008) for this study.

### A4.1 Price-Jump Index

The price-jump index is defined as

$$pji_i = \frac{|r_i|}{\frac{1}{n} \sum_{j=i-n+1}^{i} |r_i|} , \tag{7.10}$$

where $n$ governs the length of the moving window over which we normalize the absolute returns at a given time moment. The empirical observations suggest (Joulin et al., 2008) that the scaling properties behave as $P\,(pji > s) \sim s^{-\alpha}$; therefore, we define a price jump as those price returns which the price-jump index exceeds a given threshold $\hat{s}$. In our analysis, we choose $\hat{s} = 4$ and $n = 120$ and $n = 420$.

## Appendix B: Simulation Results

This appendix summarizes all the results from the simulations. First, we compare the price-jump indicators with respect to the false positive probability and then with respect to the false negative probability. We simulate 20 different combinations of intraday volatility patterns and price jumps, as they are defined in the preceding sections. Table 7.1 contains the notation for the combinations used in the tables below.

Table 7.1: Notation for combinations of different intraday volatility patterns and price jumps

| | Vol. Pattern A | Vol. Pattern B | Vol. Pattern C | Vol. Pattern D |
|---|---|---|---|---|
| Price Jumps 1 | A1 | B1 | C1 | D1 |
| Price Jumps 2 | A2 | B2 | C2 | D2 |
| Price Jumps 3 | A3 | B3 | C3 | D3 |
| Price Jumps 4 | A4 | B4 | C4 | D4 |
| Price Jumps 5 | A5 | B5 | C5 | D5 |

Note: In the tables below, the different price-jump indicators are denoted by the numbers introduced in the Comparison Strategy Section.

Table 7.1 presents the results of every type of false probability and every combination of intraday volatility pattern and price-jump specification. To eliminate the necessity of having the same note describe the contents of each table, we describe how the tables should be interpreted here. Each table presents a pair-wise comparison of price jumps as denoted above. The price-jump indicator corresponding to a row is denoted by A, the price-jump indicator corresponding to a column is denoted as B. Therefore, whenever the entry in the table contains A, the row indicator dominates in performance the one in the column, and vice versa for B. We use the conventional * for 90%, ** for 95% and *** for 99% confidence levels. The equals symbol (=) means that we cannot reject the null hypothesis that both indicators are equal with respect to a given false probability.

## B1 False positive probability: Indicator predicts a jump that does not exist

Table 7.2: Combination A1

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | | | | | | | | | |
| 5 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $B^{***}$ | $=$ | $B^{***}$ | $=$ | | | | | | | | | |
| 7 | $B^{**}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $A^{**}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | |
| 13 | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | |
| 14 | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{**}$ | |

Table 7.3: Combination B1

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | | | | | | | | | |
| 5 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $=$ | | | | | | | | | |
| 7 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $=$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $=$ | $A^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{**}$ | $B^{***}$ | $B^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | |
| 13 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | |
| 14 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $=$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $=$ | $B^{***}$ | $B^{***}$ | $A^{**}$ | $B^{***}$ | $A^{***}$ | |

Table 7.5: Combination D1

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $=$ | | | | | | | | | | | |
| 5 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{**}$ | | | | | | | | | |
| 7 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $A^{**}$ | $A^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{*}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | |
| 13 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | |
| 14 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | |

Table 7.4: Combination C1

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $=$ | | | | | | | | | | | |
| 5 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $=$ | | | | | | | | | |
| 7 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{**}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | |
| 13 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | |
| 14 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | |

Table 7.6: Combination A2

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | | | | | | | | | |
| 5 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $=$ | | | | | | | | | |
| 7 | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $=$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{**}$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | |
| 13 | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | |
| 14 | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{**}$ | |

Table 7.7: Combination B2

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | | | | | | | | | |
| 5 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $=$ | | | | | | | | | |
| 7 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{**}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $B^{***}$ | $B^{***}$ | $A^{**}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{*}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $=$ | $B^{***}$ | $B^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{*}$ | $A^{***}$ | $A^{***}$ | $=$ | $A^{***}$ | $B^{**}$ | $B^{***}$ | $A^{***}$ | | | |
| 13 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | |
| 14 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | |

Table 7.8: Combination C2

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | | | | | | | | | |
| 5 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $=$ | | | | | | | | | |
| 7 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | |
| 13 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | |
| 14 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | |

Table 7.9: Combination D2

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | | | | | | | | | |
| 5 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | |
| 7 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $B^{**}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $=$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | |
| 13 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | |
| 14 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | |

Table 7.10: Combination A3

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | | | | | | | | | |
| 5 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $=$ | | | | | | | | | |
| 7 | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $=$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{**}$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{**}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $=$ | $B^{***}$ | $A^{***}$ | | | |
| 13 | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | |
| 14 | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{**}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{**}$ | |

Table 7.11: Combination B3

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | | | | | | | | | |
| 5 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | |
| 7 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{**}$ | $A^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $=$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $=$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | |
| 13 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | |
| 14 | $B^{***}$ | $B^{***}$ | $A^{**}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $=$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{*}$ | $A^{***}$ | |

Table 7.12: Combination C3

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | | | | | | | | | |
| 5 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | |
| 7 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{*}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{**}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | |
| 13 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | |
| 14 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | |

Table 7.13: Combination D3

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | | | | | | | | | |
| 5 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | |
| 7 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{*}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{**}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $A^{**}$ | $A^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $=$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | |
| 13 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | |
| 14 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | |

Table 7.14: Combination A4

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | | | | | | | | | |
| 5 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $=$ | | | | | | | | | |
| 7 | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $=$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | |
| 13 | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | |
| 14 | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{*}$ | |

Table 7.15: Combination B4

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | | | | | | | | | |
| 5 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{*}$ | | | | | | | | | |
| 7 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $=$ | $B^{***}$ | $B^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $=$ | $A^{***}$ | $A^{***}$ | $A^{**}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | |
| 13 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | |
| 14 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | |

Table 7.16: Combination C4

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | | | | | | | | | |
| 5 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{*}$ | | | | | | | | | |
| 7 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $=$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | |
| 13 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | |
| 14 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | |

Table 7.17: Combination D4

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | | | | | | | | | |
| 5 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | |
| 7 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{**}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $=$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $=$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | |
| 13 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | |
| 14 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | |

Table 7.18: Combination A5

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | = | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | | | | | | | | | |
| 5 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{*}$ | | | | | | | | | |
| 7 | = | = | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | = | = | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | = | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | |
| 13 | $B^{**}$ | $B^{**}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | = | $B^{*}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | |
| 14 | = | = | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | = | = | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | = | |

Table 7.19: Combination B5

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | = | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | | | | | | | | | |
| 5 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | | | | | | | |
| 7 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | = | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | = | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | |
| 13 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | |
| 14 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | |

Table 7.20: Combination C5

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | = | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | | | | | | | | | |
| 5 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | | | | | | | |
| 7 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $A^{*}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | |
| 13 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | |
| 14 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | |

Table 7.21: Combination D5

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | = | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | | | | | | | | | |
| 5 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | | | | | | | |
| 7 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | | | |
| 13 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | |
| 14 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | |

### Conclusion

Table 7.22 summarizes how many times a particular price jump indicator dominats other price jump indicators. The indicator with the least number of false positive identifications is the first one, i.e., the one based on global centiles. This indicator, however, intuitively depends on the given level of centile and thus taking threshold levels based on centiles that are too tolerant, this indicator can easily lose its power.

Table 7.22: Summary of the false positive analysis.

| Indicator | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| No. of dominances | 13 | 3 | 0 | 0 | 0 | 0 | 1 | 2 | 0 | 0 | 0 | 0 | 0 | 0 |

It is also demonstrated that improved identification techniques, namely price jump indicators No. 5 and 6 and No. 11 and 12 are worse than the non-improved versions with respect to this kind of error as was intuitively anticipated.

## B2 False negative probability: Indicator does not predict existing jump

Table 7.23: Combination A1

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | | |
| 5 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | | | |
| 7 | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | | | |
| 13 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | |
| 14 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | |

Table 7.24: Combination B1

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | | |
| 5 | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | | | |
| 7 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | | | |
| 13 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | |
| 14 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | |

Table 7.26: Combination D1

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | | |
| 5 | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | | | |
| 7 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | | | |
| 13 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | |
| 14 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | |

Table 7.25: Combination C1

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | | |
| 5 | $B^{***}$ | $B^{*}$ | $A^{***}$ | $A^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | | | |
| 7 | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | | | |
| 13 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | |
| 14 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | |

Table 7.27: Combination A2

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | | |
| 5 | $A^{*}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | | | |
| 7 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $=$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{**}$ | $A^{***}$ | $B^{***}$ | | | |
| 13 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | |
| 14 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $=$ | |

Table 7.28: Combination B2

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | | |
| 5 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | | | |
| 7 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $=$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $=$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $=$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $=$ | $A^{***}$ | $B^{***}$ | | | |
| 13 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | |
| 14 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $=$ | |

Table 7.29: Combination C2

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | | |
| 5 | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $=$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | | | |
| 7 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | | | |
| 13 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | |
| 14 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $=$ | |

Table 7.30: Combination D2

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | | |
| 5 | $A^{**}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | | | |
| 7 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{**}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $=$ | $A^{***}$ | $B^{***}$ | | | |
| 13 | $B^{***}$ | $B^{***}$ | $=$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | |
| 14 | $B^{***}$ | $B^{***}$ | $=$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $=$ | |

Table 7.31: Combination A3

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | | |
| 5 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | | | |
| 6 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | | | |
| 7 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $=$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{*}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | |
| 13 | $B^{***}$ | $B^{***}$ | $=$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | |
| 14 | $B^{***}$ | $B^{***}$ | $=$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $=$ | |

Table 7.32: Combination B3

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | | |
| 5 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | | | |
| 6 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | | | |
| 7 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $=$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | |
| 13 | $B^{***}$ | $B^{***}$ | $=$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | |
| 14 | $B^{***}$ | $B^{***}$ | $=$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $=$ | |

Table 7.33: Combination C3

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | | |
| 5 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | | | |
| 7 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $=$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{**}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $=$ | $A^{***}$ | $B^{***}$ | | | |
| 13 | $B^{***}$ | $B^{***}$ | $=$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | |
| 14 | $B^{***}$ | $B^{***}$ | $=$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $=$ | |

Table 7.34: Combination D3

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | | |
| 5 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | | | |
| 6 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | | | |
| 7 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $=$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | |
| 13 | $B^{***}$ | $B^{***}$ | $=$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | |
| 14 | $B^{***}$ | $B^{***}$ | $=$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $=$ | |

Table 7.35: Combination A4

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | | |
| 5 | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | | | |
| 7 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{**}$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | | | |
| 13 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | |
| 14 | $B^{***}$ | $B^{***}$ | $B^{**}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | = | |

Table 7.36: Combination B4

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | | |
| 5 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | | | |
| 7 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{**}$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $B^{***}$ | = | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | = | $A^{***}$ | $B^{***}$ | | | |
| 13 | $B^{***}$ | $B^{***}$ | = | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | |
| 14 | $B^{***}$ | $B^{***}$ | = | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | = | |

Table 7.37: Combination C4

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | | |
| 5 | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $B^{**}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | | | |
| 7 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $=$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{**}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | | | |
| 13 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | |
| 14 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | |

Table 7.38: Combination D4

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | | |
| 5 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | | | |
| 7 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $=$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $=$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | | | |
| 13 | $B^{***}$ | $B^{***}$ | $=$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | |
| 14 | $B^{***}$ | $B^{***}$ | $A^{**}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{**}$ | |

Table 7.39: Combination A5

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | | |
| 5 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | | | |
| 6 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | | | |
| 7 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $B^{*}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $=$ | $A^{***}$ | $B^{***}$ | | | |
| 13 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | |
| 14 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | |

Table 7.40: Combination B5

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | | |
| 5 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | | | |
| 7 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{*}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $=$ | $A^{***}$ | $B^{***}$ | | | |
| 13 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | |
| 14 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | |

Table 7.41: Combination C5

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | | |
| 5 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $A^{**}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | | | |
| 7 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $=$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{**}$ | $A^{***}$ | $B^{***}$ | | | |
| 13 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | |
| 14 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | |

Table 7.42: Combination D5

| A\B | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | |
| 2 | $B^{***}$ | | | | | | | | | | | | | |
| 3 | $B^{***}$ | $B^{***}$ | | | | | | | | | | | | |
| 4 | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | | | | | | |
| 5 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | | | |
| 6 | $B^{***}$ | $=$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | | | | | | | | | |
| 7 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | | |
| 8 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | | | | | | |
| 9 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | | |
| 10 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | | | | | |
| 11 | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | | | | |
| 12 | $B^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $B^{***}$ | $=$ | $A^{***}$ | $B^{***}$ | | | |
| 13 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | | |
| 14 | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $B^{***}$ | $A^{***}$ | $B^{***}$ | $B^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | $A^{***}$ | |

**Conclusion**

Table 7.43 summarizes how many times a particular price jump indicator dominates other price jump indicators. The indicator with the least number of false negative identifications is the one based on the $\xi$-statistics with 99% CL and $N = 120$, i.e., No. 8.

Table 7.43: Summary of the false negative analysis.

| Indicator | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| No. of dominances | 2 | 0 | 0 | 0 | 0 | 0 | 10 | 15 | 0 | 0 | 0 | 0 | 0 | 0 |