Essays on Mathematical Methods for Economics

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To my parents.

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Abstract

In the first chapter, by introduction of output augmentation and input reduction I extend additive models for stochastic data envelopment analysis (SDEA), which were developed by Li (1998) to handle the noise in the data. Applying the linearization procedure by Li (1998) the linearized versions of models are derived. In the empirical part of this chapter, the efficiency scores of West Java rice farms are computed. The computed scores are compared to the stochastic frontier approach scores by Druska and Horrace (2004) and weak ranking consistency with results of stochastic frontier method is observed.

The objectives of the second chapter are to evaluate technical and scale efficiency of rice farms in West Java and to identify determinants affecting farms' efficiency. Further, the farm size–productivity relation is investigated. Data Envelopment Analysis is used for estimation of technical efficiency scores. Additionally, Tobit regression is used to explain the variation in the efficiency scores by farm–specific factors. I conclude that the farm size is one of the most important factors of farm technical efficiency and that high land fragmentation was the main source of farm inefficiency during the final period of intensification era, known as Green Revolution.

In the last, chapter I examine macroeconomic stability and the properties of business cycles in the model with an announced change of the monetary regime type. Further, I solve for the optimal monetary policies over the transition towards the pegged exchange rate with respect to alternative loss function specification for the monetary authority and to transition length. The subject of my study is the Czech Republic. The results of calibrated experiment show that monetary policy should be more concerned about demand type shocks when announcing a switch towards the exchange rate peg.

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Introduction

This work collects three applications of mathematical methods that covers operations research, development and monetary economics.

The first chapter is focused on the theoretical development of the models used in the operations research. Results of data envelopment analysis sensitively respond to stochastic noise in the data. Therefore, I propose an inclusion of the stochastic factor in the oriented model for the non-parametric method of the production frontier estimation, know as the Data Envelopment Analysis. Further, the results obtained the with the stochastic version of oriented models are compared to results of stochastic frontier method.

The second chapter presents the results of the efficiency analysis of the rice farms in the West Java. Using the combination of non-parametric and parametric methods, I identify the size of the farming plot as an important factor of the rice farming. This analysis shows that the merging of the plots may be beneficial for increase of the output.

In the third chapter, I propose a theoretical framework for modeling of the announced switch of the monetary regime. In this chapter, the analyze the synchronization of the business cycles over the transition period. Also, an optimal policies for the various lengths and specifications of monetary authority loss function are computed.

Chapter 1

Oriented stochastic data envelopment models

Data envelopment analysis (DEA) involves an non–parametric principle for extracting information about observations of a population of production mixes, so called decision making units (DMUs), that are described by the same quantitative characteristics. The primary objective of this chapter is to extent the work of Huang and Li (2001) and Li (1998) on additive stochastic DEA models (SDEA) by derivation of SDEA models that allow for proportional input reduction and output augmentation – oriented SDEA models. The empirical part of this chapter is motivated by Horrace and Schmidt's (1996) comparison of methods and by Mortimer's (2002) conclusion, that more comparative studies for the DEA and stochastic frontier approach are needed to evaluate the consistency of results with respect to method choice.

Data envelopment analysis, developed by Charnes, Cooper, and Rhodes (1978), involves an alternative approach to stochastic frontier analysis (SFA) that was developed at the same time by Aigner, Lovell, and Schmidt (1977), for efficiency evaluation of the decision process observations. The DEA approach is a nonparametric approach to production frontier estimation and requires specification of the production possibility set properties rather than the production function form that is required when the stochastic frontier approach is used. In contrast to parametric approaches for information extraction, the objective of the DEA is to identify the smallest set that satisfies production possibility properties.

The general model of production function is defined as: $y_j = f(x_j, \beta) + e_j$, where x_j represents inputs, β unknown parameters of production function $f(x_j, \beta)$ and y_j represents output of the DMU_j. The aggregate error term e_j is considered as extent of inefficiency in the DEA approach. In the SFA approach [e.g. Aigner, Lovell, and Schmidt (1977); Meeeusen and van den Broeck (1977)] the error component e_j is decomposed into a stochastic random component and a true technical efficiency component. Therefore, together with the extreme point nature of the DEA, the noise in data may lead to bias in the DEA technical efficiency measure. The dilemma of the efficiency evaluation approach depends on the trade off between the minimal specification of production function form that favors the DEA approach and the handling of stochastic error in measuring efficiency that favors the SFA approach. To compete with the SFA in error handling, the stochastic data envelopment analysis (SDEA) approach was developed by considering the used levels of inputs and outputs as random variables in the DEA model specification.

The theoretical part of this chapter extends the work on derivation of almost 100% confidence SDEA models by Li (1998) and Huang and Li (2001) by specification of the performance improvement direction, so called model orientation. Further, assumptions to simplify the disturbance structure are taken and using linearization methods the linear deterministic equivalents of these models are derived. This is utilized in the application section where it allows for the use of the linear programming method to solve SDEA problems. These SDEA results are compared to SFA results, so the consistency of results across frontier estimation methods can be assessed.

The following literature review section presents details of the motivation for the SDEA. In the second and third section, notation and definitions used to construct SDEA models are presented. Subsequently, the derivation of Huang and Li's (2001) additive models is summarized and in the fifth section I introduce input reduction and output augmentation directions for efficiency measure definition. In the sixth and following sections, I derive oriented models and their linearized forms. The ninth section describes numerical methods used to solve derived linearized versions of the oriented SDEA models. In the tenth section, I evaluate the SDEA, DEA and SFA efficiency scores consistency assessing the results of the Indonesian rice farms efficiency evaluation, as in Horrace and Schmidt (1996). The comparison of methods reveals inconsistency between efficiency rankings acquired by the SFA approach and SDEA approach. All figures and tables that I reference to, are included in the appendix.

1.1 Literature review

As Charnes et al. (1994) explain in their introduction, the story of data envelopment analysis began with Edwardo Rhodes's dissertation, which was the basis for the later published paper by Charnes, Cooper, and Rhodes (1978). In his dissertation, Rhodes used the production efficiency concept by Farrell (1957) to analyze the educational program for disadvantaged students in the USA. Rhodes compared the performance of students from schools participating and not participating in the program. Students' performance was recorded in terms of inputs and outputs, e.g. "increased self–esteem" (measured by psychological tests) as one of the outputs and "time spent by mother reading with child" as one of the inputs. The subsequent work on efficiency evaluation of multiple inputs and outputs technology led to Charnes, Cooper, and Rhodes's (1978) model (CCR model).

The introduced CCR model is suitable for analysis of the technological process under the constant returns to scale assumption. This fact is reflected in the shape of the production possibility frontier when the frontier is formed by a single half–ray and the DMU identified as efficient is an element of the production possibility frontier set up by this half–ray. To handle the variable returns to scale, introduced by Farrell and Fieldhouse (1962) in the SFA framework, the CCR model was reformulated by Banker, Charnes, and Cooper (1984) (BCC model). Since the production possibility frontier of the BCC model is a piecewise linear set, they defined weak efficiency (a weakly efficient DMU has nonzero slacks) and efficiency (an efficient DMU has zero slacks). To review the DEA models Table 1.1 summarizes a generalized versions of the aforementioned DEA models. The generalized versions of the DEA models collapse to the CCR model (constant returns to scale) for $\varphi = 0$ and for $\varphi = 1$ it matches the form of the BCC model (variable returns to scale).

As many applications suggest, the capability of handling multiple inputs–outputs and the fact that the specification of production function form is not required, make the DEA a powerful tool that is applied in various industries [e.g. in air transportation, Land, Lovell, and Thore (1993); fishing, Walden and Kirkley (2000); banking, Ševčovič, Halická, and Brunovský (2001); health care, Byrnes and Valdmanis (1989) where 123 US hospitals were covered; and in Halme and Korhonen (1998) dental care units were assessed] for technical efficiency evaluation. The expanding number of papers using the DEA approach helped to identify the limitations that an analyst should keep in mind when choosing whether or not to use the approach.

It is worth noting that the DEA approach performs very well when estimating the "relative" efficiency but it is not such a powerful technique when estimating "absolute" efficiency. In other words, the DEA reveals how well the considered DMU is doing compared to the DMU's peers but not compared to a "theoretical maximum". Figure 1.1 illustrates this situation as the difference between the true production frontier and the estimated production frontier. This difference results from the analyst's limitation in knowledge of the true production function.

A more remarkable limitation originates from the extreme point nature of the DEA approach which makes computed technical efficiency measure sensitive to changes in data. Therefore, noise (even symmetrical noise with zero mean) such as measurement error can cause significant problems. The literature on recent developments for noise incorporation in the DEA identifies three approaches: mixture of the DEA and SFA approaches, bootstrapping, and taking inputs and outputs as random variables.

Gstach (1998) proposes using the DEA technique to estimate a pseudo–production frontier (non–parametric production possibility set estimation) to select the efficient DMUs that identify the production possibility frontier. After this selection, he applies a maximum likelihood–technique to estimate the scalar value in production frontier form, by which this pseudo–frontier must be shifted downward to get the true production frontier (frontier location estimation), using the DEA–estimated efficiencies. Simar (2003) described the iterative bootstrapping method for improving the performance of the deterministic DEA frontier estimation. However, this bootstrapping approach is suitable only for cases where noise to signal ratio is low.

In this chapter, I focus on the approaches were the noise is introduced by considering DMUs as realizations of random variables. These theoretical attempts are based on Land, Lovell, and Thore's (1993) paper, where the authors use improved models to examine the efficiency of the same schooling program for disabled scholars as in Charnes, Cooper, and Rhodes (1978). Land, Lovell, and Thore (1993) offer the prospect of stochastic data envelopment analysis and constructed their own model (LLT model). The LLT model is derived as a chance constrained version of the BCC output oriented model in envelopment form. Further, they transform these chance constrained problems to their deterministic non–linear equivalents, which allow them to determine the efficient DMUs.

Olesen and Petersen (1995) present a different approach to incorporating the stochastic component into the DEA and their model (OP model) originates from the multiplier formulation of the BCC model. They assume that the inefficiency term of the considered DMU can be decomposed into true inefficiency and disturbance term as in the SFA approach. Further, Olesen (2002) compares the approaches of the models by Olesen and Petersen (1995) and Land, Lovell, and Thore (1993) and identifies weaknesses of both model types. The LLT model is criticized because it does not account for all the correlations that can occur in disturbances. Olesen (2002) criticizes the OP model because it ignores correlations between DMUs. A related weakness is the omission of the fact that a convex combination of two DMUs can have a lower variance than the DMUs considered solely. A straightforward remedy for the OP model is to take the union of confidence regions for any linear combination of the stochastic vectors themselves rather than using a piecewise linear envelopment of the confidence regions. Olesen (2002) implements this idea and derives the combined chance constrained model.

The approach that will be extended in this chapter, originates from work by Huang and Li (2001), where inputs and outputs are introduced as random variables and the relation of stochastic efficiency dominance is defined. Huang and Li (2001) define the efficiency dominance of a DMU via joint probabilistic comparisons of inputs and outputs with other DMUs which are evaluated by solving a chance constrained programming problem. By utilizing the theory of chance constrained programming, deterministic equivalents are obtained for both situations of multivariate symmetric random disturbances and a single random factor in production relationships. Under the assumption of the single random factor, Huang and Li (2001) obtain linear deterministic equivalent to stochastic programming problems via linear programming theory. In this chapter, I propose the oriented form of the additive SDEA models derived by Huang and Li (2001). Further, by use of Huang and Li's (2001) linearization approach I linearize the proposed oriented SDEA models.

In the empirical part of this chapter, I compare the results of the different methods to productivity evaluation as in Horrace and Schmidt (1996). This comparison is motivated by Mortimer's (2002) comparative study of recent literature that summarizes the results from SFA and DEA studies to identify the amount of correlation between scores in SFA and DEA comparative studies. Mortimer (2002) calls for more studies that will compare efficiency scores correlation across production efficiency approaches because the present comparative studies show either strong [e.g. Ferro–Luzzi et al. (2003)] or very weak [e.g. Lan and Lin (2002), Wadud and White (2000a)] correlation of obtained efficiency rankings.

The major problems associated with solving the DEA models are the analysis of a large set of DMUs and interpretation of the optimal solutions with zero elements. The analysis of a large data set leads to large size optimization problems that can be costly to solve. The solutions that contain many zero elements can make the results of the analysis questionable because the elements of optimal solutions are interpreted as shadow prices of inputs and outputs. Gonzales-Lima, Tapia, and Thrall (1996) present the primal–dual interior–points computational methods as the methods that significantly improve the reliability of the solution in comparison to simplex methods. The interior–points methods maximize the product of the positive components in the optimal solutions, so they identify optimal solution with the minimal number of zero components. Due to this property of the optimal solution it is easier to interpret the DEA models results. Therefore, as part of my theoretical work the interior point method solver is constructed.

1.2 Notation

In this section, the notation used to construct the oriented stochastic DEA models is introduced. Additional notation will be introduced in the following section to describe the considered error structure. In contrast to the deterministic approach to envelopment analysis, where DMUs are observations of decision realization, the DMUs in the stochastic approach are characterized by random variables and the technology realizations are observations of these random variables. The notation in this chapter coincides with the notation usually found in data envelopment analysis literature [e.g. Charnes et al. (1994),Cooper et al. (1998), and Huang and Li (2001)].[1] The task is to analyze the set of DMU_j, where $1 \leq j \leq n$. Each of the DMUs is described by a random vector $\tilde{x}_j$, $\tilde{x}_j = (\tilde{x}_{1j}, \ldots, \tilde{x}_{mj})^T$ of m input amounts (random variables) that are used to produce s outputs in amounts described by random vector $\tilde{y}_j$, $\tilde{y}_j = (\tilde{y}_{1j}, \ldots, \tilde{y}_{sj})^T$. These vectors are aggregated to matrices of random vectors of inputs and outputs, so the following matrix notation will be used:

[1] In this chapter, the random variables are denoted by ˜ and means of these variables are denoted by an upper bar.

matrix of inputs random vectors	$\tilde{X} = (\tilde{x}_1, \ldots, \tilde{x}_n)$
i^{th} row of "input" matrix $\tilde{X}$	${}_i\tilde{x} = (\tilde{x}_{i1}, \ldots, \tilde{x}_{in}),\ i = 1, \ldots, m$
$m \times n$ matrix of expected inputs	$\bar{X} = (\bar{x}_1, \ldots, \bar{x}_n)$
i^{th} row of expected "input" matrix $\bar{X}$	${}_i\bar{x} = (\bar{x}_{i1}, \ldots, \bar{x}_{in}),\ i = 1, \ldots, m$
matrix of outputs random vectors	$\tilde{Y} = (\tilde{y}_1, \ldots, \tilde{y}_n)$
r^{th} row of "output" matrix $\tilde{Y}$	${}_r\tilde{x} = (\tilde{y}_{r1}, \ldots, \tilde{y}_{rn}),\ r = 1, \ldots, s$
$s \times n$ matrix of expected outputs	$\bar{Y} = (\bar{y}_1, \ldots, \bar{y}_n)$
r^{th} row of expected "output" matrix $\bar{Y}$	${}_r\bar{y} = (\bar{y}_{r1}, \ldots, \bar{y}_{rn}),\ r = 1, \ldots, s.$

1.3 Stochastic efficiency dominance

In this section, the efficiency dominance relation and derivation of additive almost 100% chance constrained models by Huang and Li (2001) is reviewed. These theorems and definitions form the basis for derivation of the oriented SDEA derived in the following sections.

Definition 1. General stochastic production possibility set $T \subset \mathbb{R}_+^{m+s}$ is defined as: $T = \{(\tilde{x}, \tilde{y}) \mid \text{ outputs } \tilde{y} \text{ can be produced using inputs } \tilde{x}\}$.[2]

This definition of the stochastic production possibility set relates to random vectors that characterize DMUs and it means that all DMUs are required to be an element of the stochastic production possibility set but not all observations of DMUs are required to be in the stochastic production possibility set. As mentioned in the literature review, the function form is not known, therefore the estimate of the production possibility set is identified by the properties that the production possibility set should fulfill.

Almost 100% confidence production possibility set T constructed from the set of $\text{DMU}_j,\ j = 1, \ldots, n$ should fulfill the following properties:

Property 1. Convexity: If $(\tilde{x}_j, \tilde{y}_j) \in T$, $j = 1, \ldots, n$ and $\lambda \in \mathbb{R}_+^n$, $\Rightarrow (\tilde{X}\lambda, \tilde{Y}\lambda) \in T$.

Property 2. Inefficiency property: If $(\bar{x}, \bar{y}) \in T$ and $x \geq \bar{x}$, then $(x, \bar{y}) \in T$.
If $(\bar{x}, \bar{y}) \in T$ and $y \leq \bar{y}$ then $(\bar{x}, y) \in T$.

Property 3. Minimum extrapolation: T is the intersection of all sets satisfying convexity and inefficiency property and subject to each of the observed random vectors $(\tilde{x}_j, \tilde{y}_j) \in T, j = 1, \ldots, n$.

[2]Here, $\mathbb{R}_+$ means set of positive real numbers and $\mathbf{1}$ is column vector of ones.

From the first two properties follows that less output can be produced with the same amount of inputs. This reflects the situation when some portion of inputs is wasted in the production process. The parametric production possibility set T_φ; $T_\varphi = \{(\tilde{x}, \tilde{y}) \mid \tilde{x} \geq \tilde{X}\lambda,\ \tilde{y} \leq \tilde{Y}\lambda,\ \varphi(\mathbf{1}^T\lambda) = \varphi,\ \lambda \geq 0\}$, where $\varphi \in \{0, 1\}$, satisfies all aforementioned properties. T_0 is the stochastic generalization of the production possibility set under the assumption of the constant returns to scale production function as used by Charnes, Cooper, and Rhodes (1978) in the derivation of the CCR model. Similarly, the stochastic generalization of the production possibility set T_1 will be used to derive models with variable returns to scale as in a case of the BCC model by Banker, Charnes, and Cooper (1984).

The concept of efficiency in the DEA (based on the following relative efficiency definition) is used to define the α–stochastic efficiency dominance.

Definition 2. Relative Efficiency: A DMU is to be identified as efficient on the basis of available evidence if and only if the performances of other DMUs does not show that some of its inputs or outputs can be improved without worsening some of its other inputs or outputs.

The efficient point of the production possibility set is identified if there is no other production point that produces more output without consuming more input, or consumes less input without producing less output. This leads to the following efficiency domination definition of the production possibility set element:

Definition 3. Efficiency dominance relation: The point (x, y) is not dominated in the sense of efficiency if $\nexists\ (x^*, y^*)$ in the production possibility set such that $x^* \leq x$ or $y^* \geq y$ with at least one strict inequality for input or output components.

This definition demonstrates the efficiency concept of the DEA and is used to derive the deterministic models with no possibility of a violation of the production possibility set properties or efficiency dominance. In the deterministic environment, the non–dominated DMUs are elements of the production possibility set frontier. Figure 1.1 illustrates this situation where the set of DMUs is divided into efficient (DMU1, DMU2 and DMU3) and inefficient DMUs (DMU4 and DMU5). The efficient DMUs – points that dominate in efficiency the other elements of the production possibility set – are used to identify the production possibility frontier.

In the stochastic framework, where efficiency dominance can be violated due to random errors, the efficiency dominance violations are allowed with the probability α, $0 \leq$

$\alpha \leq 1$. In chance constrained programming methodology the term $1-\alpha$ is interpreted as the modeler's confidence level and α is interpreted as the modeler's risk (the extent of conditions violations). In the almost 100% confidence approach, the production possibility constraints are almost certainly not violated and the efficiency dominance can be violated with probability α. For the case of the almost 100% confidence chance constrained approach, Li (1998) and Huang and Li (2001) define the α–stochastically efficiency of point as:

Definition 4. α–stochastic efficiency of point in set T_φ: $(\tilde{x}^*, \tilde{y}^*) \in T_\varphi$ is called α–stochastically efficient point associated with $T_\varphi \Leftrightarrow$ if the analyst is confident that $(\tilde{x}^*, \tilde{y}^*)$ is efficient with probability $1-\alpha$ in the set T_φ.

Definition 4 means that point $(\tilde{x}^*, \tilde{y}^*)$, considered as α–stochastically efficient may be dominated (in the sense of efficiency dominance) by any other point in T_φ with a probability less or equal to α. For the DMU_j associated with this point this definition is used to evaluate the α–stochastic efficiency of DMU_j.

This definition and the aforementioned properties of the set T_φ straightforwardly imply that for the efficient DMU_j and for any $\lambda_j \in \mathbb{R}^n_+$ such that $\varphi(\mathbf{1}^T\lambda_j) = \varphi$, $\lambda \geq 0$ the expression $Prob(\tilde{X}\lambda_j \leq \tilde{x}_j, \tilde{Y}\lambda_j \geq \tilde{y}_j) \leq \alpha$ holds with at least one strict inequality in input–output constraints.

To illustrate the DEA and almost 100% confidence SDEA approach, Figure 1.1 illustrates the relation of the deterministic frontier to the possible true production possibility frontier. The solid piecewise linear line is the possible true production possibility frontier and the dashed line is the DEA estimate of this production possibility frontier. In Figure 1.2 the expected values of DMUs (same values as the observations in Figure 1.1) are pictured and the set of α–efficiency dominant elements is presented as a grey shaded area. A comparison of Figures 1.1 and 1.2 shows that for the almost 100% confidence SDEA approach, the deterministic production possibility set frontier is a subset of the stochastic possibility set frontier. Due to this fact more DMUs can be identified as efficiency dominant in the stochastic framework than in the deterministic.

1.3.1 Stochastic model

In this subsection, the derivation of the almost 100% confidence chance constrained problem is reviewed. The reviewed stochastic model for assessing efficiency of DMU_j is the equivalent to the additive DEA model and serves as the basis for the further theoretical development of SDEA models. In the following subsection, specific assumptions about the error structure in the data are made and the stochastic model is transformed into its deterministic equivalent.

Now, from the set properties for the virtual peers $(\tilde{X}\lambda, \tilde{Y}\lambda)$ that are used for evaluation of efficiency of DMU$_j$ follows that

$$\{\tilde{X}\lambda \leq \tilde{x}_j, \tilde{Y}\lambda \geq \tilde{y}_j\} \quad \subset \quad \{\mathbf{1}^T(\tilde{X}\lambda - \tilde{x}_j) + \mathbf{1}^T(\tilde{y}_j - \tilde{Y}\lambda) < 0\} \tag{1.1}$$

and using the probability properties the following inequality is derived:[3]

$$Prob(\tilde{X}\lambda \leq \tilde{x}_j, \tilde{Y}\lambda \geq \tilde{y}_j) \leq Prob(\mathbf{1}^T(\tilde{X}\lambda - \tilde{x}_j) + \mathbf{1}^T(\tilde{y}_j - \tilde{Y}\lambda) < 0).$$

Therefore, for $\lambda \in \mathbb{R}^n_+$ such that $\varphi(\mathbf{1}^T\lambda) = \varphi$ and $\lambda \geq 0$ the condition

$$Prob(\mathbf{1}^T(\tilde{X}\lambda - \tilde{x}_j) + \mathbf{1}^T(\tilde{y}_j - \tilde{Y}\lambda) < 0) \leq \alpha$$

is a necessary condition for the DMU$_j$ to be α–stochastically efficient. Using the necessary condition for α–stochastic efficiency of the DMU$_j$, the following almost 100% confidence chance constrained problem (in matrix notation) for the technical efficiency evaluation of the DMU$_j$, $j = 1, \ldots, n$ is constructed (Cooper et al. (1998), Li (1998) and Huang and Li (2001))

$$\begin{aligned} \max_{\lambda_j} \quad & Prob(\mathbf{1}^T(\tilde{X}\lambda_j - \tilde{x}_j) + \mathbf{1}^T(\tilde{y}_j - \tilde{Y}\lambda_j) < 0) - \alpha \\ s.t. \quad & Prob({}_i\tilde{x}\lambda_j < \tilde{x}_{ij}) \geq 1 - \epsilon, \qquad i = 1, \ldots, m; \\ & Prob({}_r\tilde{y}\lambda_j > \tilde{y}_{rj}) \geq 1 - \epsilon, \qquad r = 1, \ldots, s; \\ & \varphi(\mathbf{1}^T\lambda_j) = \varphi, \\ & \lambda_j \geq 0, \end{aligned} \tag{1.2}$$

where ϵ is a non–Archimedean infinitesimal quantity.[4] The optimal solution of problem 1.2 is related to the stochastic efficiency of the DMU$_j$ by following two theorems which

[3] The inequality type change is due to the additional restriction that $\{\tilde{X}\lambda \leq \tilde{x}_j, \tilde{Y}\lambda \geq \tilde{y}_j\}$ holds with at least one strict inequality. The accuracy of this simplification is closely discussed in Ruszczynski and Shapiro (2003).

[4] This means that ϵ is a very small positive number such that $\sum_{i=1}^n \epsilon < 1$ no matter how large is n. According to the chapter "Computational Aspects of DEA" in Charnes et al. (1994), $\epsilon < \min_{j=1,\ldots,n} 1/(\sum_{i=1}^m x_{ij})$ is selected in the calculations of these models.

are direct corollaries of Theorem 3 by Cooper et al. (1998):[5]

Theorem 1. *Let the DMU_j be α–stochastically efficient. The optimal value of the objective function in the chance constrained programming problem 1.2 is less than or equal to zero.*

Theorem 2. *If the optimal value objective functional of problem 1.2 is greater than zero, then DMU_j is not α–stochastically efficient.*

Theorem 2 implies that if the maximum value of the chance functional $Prob(\mathbf{1}^T(\tilde{X}\lambda_j - \tilde{x}_j) + \mathbf{1}^T(\tilde{y}_j - \tilde{Y}\lambda_j) < 0)$ exceeds α, then the considered DMU_j is not α–stochastically efficient. The value of the chance functional of the additive SDEA model represented by problem 1.2 can be used as the simplest efficiency measure when interpreted as the sum of input excess and output slack. In the section on derivation of the oriented SDEA models, I introduce measures based on possible proportional input reduction or output augmentation.

1.3.2 Error structure

In this subsection, the error structure that allows the transformation of the model from a chance constrained problem to a linear deterministic equivalent is introduced and the linearization approach by Cooper et al. (1998) is summarized. The following structure of m inputs and s outputs of the DMU_j, for $j = 1, \dots, n$ with noise driven by normally distributed shocks is considered

$$\begin{aligned} \tilde{x}_{ij} &= \bar{x}_{ij} + a_{ij}\zeta_{ij} \qquad i = 1, \dots, m; \\ \tilde{y}_{ij} &= \bar{y}_{ij} + b_{ij}\xi_{rj}, \qquad r = 1, \dots, s; \end{aligned} \tag{1.3}$$

where it is assumed $E(\zeta_{ij}) = E(\xi_{rj}) = 0$, $j = 1, \dots, n$ and the following variance–covariance structure of errors for all DMUs is assumed:[6]

$$Var(\zeta_{ij}) = Var(\xi_{rj}) = \sigma_\varepsilon^2 \qquad 1 \le i \le m;\ 1 \le r \le s;\ 1 \le j \le n;$$

[5] See Theorem 3 and its proof in Cooper et al. (1998).

[6] For linearization procedure the standard normal distribution $N(0,1)$ can be assumed. The scaling of the measurement units is used when numerical problems with tiny diagonals of the input–output variance matrices occurs, therefore the more general assumption of $N(0, \sigma_\varepsilon^2)$ is used. This simplifying assumption also reduces the number of parameters to be estimated for efficiency evaluation to $2n(m+s)$. Without simplifying assumption $[n^2(m+s)^2 + 3n(m+s)]/2$ parameters are needed to be estimated.

$$
\begin{aligned}
&Cov(\zeta_{ij}, \zeta_{kl}) = 0 \qquad 1 \le i, k \le m;\ 1 \le j, l \le n; \\
&Cov(\xi_{rj}, \xi_{kl}) = 0 \qquad 1 \le r, k \le s;\ 1 \le j, l \le n; \\
&Cov(\xi_{rj}, \zeta_{il}) = 0 \qquad 1 \le r \le s;\ 1 \le i \le m;\ , 1 \le j, l \le n.
\end{aligned}
$$

Under this error structure follows that inputs and outputs are normally distributed with $E(\tilde{x}_{ij}) = \bar{x}_{ij},\ E(\tilde{y}_{rj}) = \bar{y}_{rj}$ and variance $Var(\tilde{x}_{ij}) = (a_{ij}\sigma_\varepsilon)^2,\ Var(\tilde{y}_{rj}) = (b_{rj}\sigma_\varepsilon)^2$.

When assessing the production processes it is also reasonable to consider the case of log–normally distributed variables. In the case of log–normality of inputs and outputs with disturbances driven by normal random variables, the following structure of inputs and outputs can be considered:

$$
\begin{aligned}
\tilde{x}_{ij}^{log} &= exp(\bar{x}_{ij} + a_{ij}\zeta_{ij}) \qquad i = 1, \ldots, m; \\
\tilde{y}_{ij}^{log} &= exp(\bar{y}_{ij} + b_{ij}\xi_{rj}), \qquad r = 1, \ldots, s.
\end{aligned}
\tag{1.4}
$$

The log–normal input–output structure can be transformed to normal input–output structure by taking logs, therefore in the following text I assume only the input–output structure with normally distributed input and output variables.

Additionally, when assuming $\varepsilon = \xi_{ij} = \xi_{kl} = \zeta_{rj} = \zeta_{il}$, for $1 \le r \le s;\ \ 1 \le i \le m;$ $1 \le j, l \le n$ then the assumed error structure collapses to a single factor symmetric error structure where ε follows normal distribution with $E(\varepsilon) = 0, Var(\varepsilon) = \sigma_\varepsilon^2$. To simplify this notation, the vectors

$$
\begin{aligned}
&a_j = (a_{1j}, \ldots, a_{mj})^T, \quad b_j = (b_{1j}, \ldots, b_{sj})^T, \quad j = 1, \ldots, n; \\
&{}_ia = (a_{i1}, \ldots, a_{in}), \qquad {}_rb = (b_{r1}, \ldots, b_{rn}), \qquad i = 1, \ldots, m,\ r = 1, \ldots, s;
\end{aligned}
$$

are introduced and these vectors are aggregated to construct the following matrices of input and output variations $A_{m\times n} = (a_1, \ldots, a_n), B_{s\times n} = (b_1, \ldots, b_n)$. Using the properties of normal distribution it is derived that ${}_i\tilde{x}\lambda_j - \tilde{x}_{ij}$ is distributed according to $N({}_i\bar{x}\lambda_j - \bar{x}_{ij}; ({}_ia\lambda_j - a_{ij})^2\sigma_\varepsilon^2)$ and $({}_r\tilde{y}\lambda_j - \tilde{y}_{rj})$ is normally distributed according to $N({}_r\bar{y}\lambda_j - \bar{y}_{rj}; (b_{rj} - {}_rb\lambda_j)^2\sigma_\varepsilon^2)$. Applying the inverse cumulative distribution function $\Phi^{-1}(\alpha)$, the constraints and objective function in the almost 100% confidence chance constrained problem 1.2 can be rewritten as in Cooper et al. (1998) or Huang and Li (2001) and the following deterministic equivalent of problem 1.2 is derived:

$$
\min_{\lambda_j \in \mathbb{R}_+^{m+s}} \ \mathbf{1}^T(\bar{X}\lambda_j - \bar{x}_j) + \mathbf{1}^T(\bar{y}_j - \bar{Y}\lambda_j) + \mid \mathbf{1}^T(A\lambda_j - a_j) + \mathbf{1}^T(b_j - B\lambda_j) \mid \sigma_\varepsilon \Phi^{-1}(\alpha) \tag{1.5}
$$

$$
\begin{aligned}
s.t. \quad & {}_i\bar{x}\lambda_j \leq \bar{x}_{ij} + \mid {}_ia\lambda_j - a_{ij} \mid \sigma_\varepsilon \Phi^{-1}(\epsilon), \quad i = 1, \ldots, m, \\
& \bar{y}_{rj} \leq {}_r\bar{y}\lambda_j + \mid b_{rj} - {}_rb\lambda_j \mid \sigma_\varepsilon \Phi^{-1}(\epsilon), \quad r = 1, \ldots, s, \\
& \varphi(\mathbf{1}^T\lambda_j) = \varphi, \\
& \lambda_j \geq 0.
\end{aligned}
$$

Applying the linearization procedure, new variables $q_{1r}, q_{2r}, h_{1i}, h_{2i}$ and the cumulative term $\epsilon(\sum_{r=1}^{s}(q_{1r}+q_{2r})+\sum_{i=1}^{m}(h_{1i}+h_{2i}))$ introduced into the objective function allows for the decomposition of the absolute value terms and to linearize the constraints in problem 1.5.[7] Moreover, this modification does not affect the optimal solutions of problem 1.5 and this problem is equivalent to the following problem with linear constraints:

$$
\min_{\lambda_j, q_{kr}, h_{ki}} \mathbf{1}^T(\bar{X}\lambda_j - \bar{x}_j) + \mathbf{1}^T(\bar{y}_j - \bar{Y}\lambda_j) + \tag{1.6}
$$

$$
+ \mid \mathbf{1}^T(A\lambda_j - a_j) + \mathbf{1}^T(b_j - B\lambda_j) \mid \sigma_\varepsilon \Phi^{-1}(\alpha) + \epsilon(\sum_{r=1}^{s}(q_{1r} + q_{2r}) + \sum_{i=1}^{m}(h_{1i} + h_{2i}))
$$

$$
\begin{aligned}
s.t. \quad & {}_i\bar{x}\lambda_j \leq \bar{x}_{ij} + (h_{1i} + h_{2i})\sigma_\varepsilon \Phi^{-1}(\epsilon), & \\
& {}_ia\lambda_j - a_{ij} = h_{1i} - h_{2i}, & i = 1, \ldots, m, \\
& \bar{y}_{rj} \leq {}_r\bar{y}\lambda_j + (q_{1r} + q_{2r})\sigma_\varepsilon \Phi^{-1}(\epsilon), & \\
& b_{rj} - {}_rb\lambda_j = q_{1r} - q_{2r}, & r = 1, \ldots, s, \\
& \varphi(\mathbf{1}^T\lambda_j) = \varphi, & \\
& \lambda_j \geq 0, q_{kr} \geq 0, h_{ki} \geq 0, & k = 1, 2.
\end{aligned}
$$

In the following step, the absolute value from the objective function is removed. The inverse of cumulative distribution function $\Phi(\alpha)$ takes a positive or negative values; to account for this factor let's define δ such that

$$
\delta = \begin{cases} -1 & \text{if } \alpha < 0.5; \\ 0 & \text{if } \alpha = 0.5; \\ 1 & \text{if } \alpha > 0.5. \end{cases}
$$

The absolute value term in the objective function is the sum of the absolute value terms in the constraints of problem 1.6; therefore, the decomposition that was used in these constraints is just substituted in the objective function. Thus as in used literature [e.g. Li (1998) and Huang and Li (2001)], the absolute value terms are eliminated from the

[7] For simplicity of notation, in the following text the index j is omitted in the terms q_{1r}, q_{2r}, h_{1i}, h_{2i} that are used to replace the absolute value term.

objective function and the following problem with a linear objective function is obtained:

$$\min_{\lambda_j, q_{kr}, h_{ki}} \mathbf{1}^T(\bar{X}\lambda_j - \bar{x}_j) + \mathbf{1}^T(\bar{y}_j - \bar{Y}\lambda_j) + \tag{1.7}$$

$$+\delta(\mathbf{1}^T(A\lambda_j - a_j) + \mathbf{1}^T(b_j - B\lambda_j))\sigma_\varepsilon\Phi^{-1}(\alpha) + \epsilon(\sum_{r=1}^{s}(q_{1r} + q_{2r}) + \sum_{i=1}^{m}(h_{1i} + h_{2i}))$$

$$\begin{aligned} s.t. \quad & {}_i\bar{x}\lambda_j \leq \bar{x}_{ij} + (h_{1i} + h_{2i})\sigma_\varepsilon\Phi^{-1}(\epsilon), & \\ & {}_ia\lambda_j - a_{ij} = h_{1i} - h_{2i}, & i = 1, \ldots, m, \\ & \bar{y}_{rj} \leq {}_r\bar{y}\lambda_j + (q_{1r} + q_{2r})\sigma_\varepsilon\Phi^{-1}(\epsilon), & \\ & b_{rj} - {}_rb\lambda_j = q_{1r} - q_{2r}, & r = 1, \ldots, s, \\ & \varphi(\mathbf{1}^T\lambda_j) = \varphi, & \\ & \lambda_j \geq 0, q_{kr} \geq 0, h_{ki} \geq 0, & k = 1, 2. \end{aligned}$$

Problem 1.7 is known as the envelopment formulation of the DEA model, because the optimal solution identifies the projected point on to the envelopment surface for DMU_j. Using Li's (1998) definition of the dual problem, the dual problem 1.8 to primal problem 1.7 is restated as:

$$\max_{\mu,\nu,\eta,\omega,\psi_j} \mu^T\bar{y}_j - \nu^T\bar{x}_j - \eta^Tb_j - \omega^Ta_j - \varphi\psi_j \tag{1.8}$$

$$\begin{aligned} s.t. \quad & \mu^T\bar{y}_l - \nu^T\bar{x}_l - \eta^Tb_l - \omega^Ta_l - \varphi\psi_j \leq 0, \; l = 1, \ldots, n; \\ & -\sigma_\varepsilon\Phi^{-1}(\varepsilon)\mu + \eta \geq -\sigma_\varepsilon(\Phi^{-1}(\varepsilon) + \varepsilon)\mathbf{1} - \delta\sigma_\varepsilon\Phi^{-1}(\alpha)\mathbf{1}, \\ & -\sigma_\varepsilon\Phi^{-1}(\varepsilon)\mu - \eta \geq -\sigma_\varepsilon(\Phi^{-1}(\varepsilon) + \varepsilon)\mathbf{1} + \delta\sigma_\varepsilon\Phi^{-1}(\alpha)\mathbf{1}, \\ & -\sigma_\varepsilon\Phi^{-1}(\varepsilon)\nu - \omega \geq -\sigma_\varepsilon(\Phi^{-1}(\varepsilon) + \varepsilon)\mathbf{1} - \delta\sigma_\varepsilon\Phi^{-1}(\alpha)\mathbf{1}, \\ & -\sigma_\varepsilon\Phi^{-1}(\varepsilon)\nu + \omega \geq -\sigma_\varepsilon(\Phi^{-1}(\varepsilon) + \varepsilon)\mathbf{1} + \delta\sigma_\varepsilon\Phi^{-1}(\alpha)\mathbf{1}, \\ & \mu \geq \mathbf{1} \\ & \nu \geq \mathbf{1}, \\ & \eta, \omega, \psi_j \text{ unconstrained.} \end{aligned}$$

For the DMU$_j$ represented by point $(\tilde{x}_j, \tilde{y}_j)$, the following stochastic hyperplane $Prob(c^T\tilde{x}_j + d^T\tilde{y}_j + f_j \leq 0) = 1 - \epsilon$ is the supporting hyperplane for T_φ at $(\tilde{x}_j, \tilde{y}_j)$ if and only if

$$c^T\tilde{x}_j + d^T\tilde{y}_j + f_j + \Phi^{-1}(\epsilon)\sigma_\varepsilon \mid c^Ta_j + d^Tb_j \mid = 0 \tag{1.9}$$

$$\text{and for } \forall\, (\tilde{x}, \tilde{y}) \in T_\varphi: \; c^T\tilde{x} + d^T\tilde{y} + f_j + \Phi^{-1}(\epsilon)\sigma_\varepsilon \mid c^Ta_j + d^Tb_j \mid \geq 0. \tag{1.10}$$

The dual problem 1.8 is known as the multiplier problem because the optimal solutions $(\mu_j^*, \nu_j^*, \eta_j^*, \omega_j^*, \psi_j^*)$, for $j = 1, \ldots, n$, set up the supporting hyperplanes that are used to construction the production possibility frontier. If there is an unique optimal solution $(\mu_j^*, \nu_j^*, \eta_j^*, \omega_j^*, \psi_j^*)$ to problem 1.8 that satisfies

$$\mu_j^{*T}(b_j - b_k) + \nu_j^{*T}(a_j - a_k) - \Phi^{-1}(\epsilon)\sigma_\varepsilon(\mid \mu_j^{*T} b_j - \nu_j^{*T} a_j \mid - \mid \mu_j^{*T} b_k - \nu_j^{*T} b_k \mid) \geq 0,$$

for $k = 1, \ldots, n$, then the optimal solution $(\mu_j^*, \nu_j^*, \eta_j^*, \omega_j^*, \psi_j^*)$ identifies the following stochastic hyperplane $Prob(\mu_j^{*T}\tilde{y}_j - \nu_j^{*T}\tilde{x}_j + f_j^* \leq 0) = 1 - \epsilon$, where $f_j^* = -\eta_j^{*T} b_j - \omega_j^{*T} a_j - \varphi\psi_j^* + \Phi^{-1}(\epsilon)\sigma_\varepsilon \mid \mu_j^{*T} b_j - \nu_j^{*T} a_j \mid$. This almost 100% confidence hyperplane is the supporting hyperplane to T_φ at the DMU_j. Further, in the section on returns to scale, the sign of f_j is related to the returns to scale type and these relations are summarized in Table 1.2. In a case without a unique optimal solution to problem 1.8, the supporting hyperplane for T_φ at $(\tilde{x}_j, \tilde{y}_j)$ is not uniquely identified.

1.4 Efficiency measure

In this section, by introducing the input reducing and output augmenting direction for projection into the data envelopment I derive the extension to the reviewed additive models. As explained in the previous section, the optimal solution to the envelopment problem 1.7 for the DMU$_j$ identifies the point $(\hat{x}_j, \hat{y}_j) = (\bar{X}\lambda_j^*, \bar{Y}\lambda_j^*)$ and the optimal solution of the multipliers problem 1.8 identifies the supporting hyperplane assigned to the DMU_j. Therefore, the simplest inefficiency measure can be defined by the distance measure of a discrepancy between the projected and expected point as: $|(\hat{x}_j, \hat{y}_j) - (\bar{x}_j, \bar{y}_j)|$. This discrepancy measure expresses the difference between the efficient frontier represented by the projected point $(\hat{x}_j, \hat{y}_j)$ and the present position of the DMU_j. Starting from $(\bar{x}_j, \bar{y}_j)$, various projection paths on the corresponding part of the envelopment surface can be followed as is illustrated by Figure 1.3. Figure 1.3 illustrates directions of inputs reduction and augmentation in outputs. I will use these two directions to derive the input and output oriented efficiency measures that are used to state the oriented SDEA models.

First, for inputs of the DMU$_j$ let's denote $e_{ij} \in \mathbb{R}_+$, $e_{ij} = \bar{x}_{ij} - {}_i\bar{x}\lambda_j$, $i = 1, \ldots, m$ and define the column vector of inputs excess $e_j \in \mathbb{R}_+^m$, $e_j = (e_{1j}, \ldots, e_{mj})^T$. If the following inequality $Prob({}_i\tilde{x}\lambda_j < \tilde{x}_{ij}) > 1 - \epsilon$ holds there must exist $e_{ij} > 0$, $i \in \{1, \ldots, m\}$ such that $Prob(e_{ij} \leq \tilde{x}_{ij} - {}_i\tilde{x}\lambda_j) = 1 - \epsilon$. Therefore, for inputs of the DMU_j, by following the path $-e_j$ the inputs can be decreased and the projected point is moved towards the production possibility frontier. This projection direction is given in Figure 1.3 as the input reduction direction and the point DMU5i is the input oriented projection of the DMU#5.

Similarly, the DEA output oriented model is derived using the column vector of output slacks $s_j \in \mathbb{R}^s_+, s_j = (s_{1j}, \ldots, s_{sj})^T$, $s_{rj} = {}_r\bar{y}\lambda_j - \bar{y}_{rj}$, $r = 1, \ldots, s$. For $r \in \{1, \ldots, s\}$ such that $Prob({}_r\tilde{y}\lambda_j > \tilde{y}_{rj}) > 1 - \epsilon$ exists $s_{rj} > 0$ for which the following equality holds: $Prob({}_r\tilde{y}\lambda_j - \tilde{y}_{rj} \geq s_{rj}) = 1 - \epsilon$. The path s_j projects the DMU$_j$ on to the production possibility frontier in an outputs augmenting direction and the projected point is shown in Figure 1.3 as the DMU5o.

Next, to determine the maximal scale effects in inputs reduction or outputs augmentation, the projection paths s_j, e_j are decomposed to a proportional increase (decrease) of output (input) and residual as follows: $s_j = \rho_j\bar{y}_j + \delta^j_s$, $e_j = \gamma_j\bar{x}_j + \delta^j_e$, where a proportional increase of outputs ρ_j and proportional decrease of inputs γ_j for $j = 1, \ldots, n$ are defined as

$$\begin{aligned} \rho_j = &\quad \min_{r=1,\ldots,s} \quad \frac{\hat{y}_{rj} - \bar{y}_{rj}}{\bar{y}_{rj}} \geq 0, \\ \gamma_j = &\quad \min_{i=1,\ldots,m} \quad \frac{\bar{x}_{ij} - \hat{x}_{ij}}{\bar{x}_{ij}} \geq 0, \end{aligned}$$

and $\delta^j_e \geq 0, \delta^j_s \geq 0,\ j = 1, \ldots, n$.[8]

Next as in Ali and Seiford (1993), the new variables for the output oriented model are defined as $\phi_j = 1 + \rho_j$ and for the input oriented model $\theta_j = 1 - \gamma_j$. From the construction of the scaling parameters, the θ_j satisfies $0 < \theta_j \leq 1$ and for ϕ_j in the output problem we have $\phi_j \geq 1$. The maximal output scale effect is identified by optimal value ϕ^*_j and the maximal input reduction is identified by the optimal value of θ^*_j.

For the identification of possible proportional scaling of inputs or outputs and efficiency evaluation of the DMU$_j$, two stage models are constructed. In the first model stage, the maximal ϕ_j or minimal θ_j is found to identify the maximal equi–proportional effect. In the second stage of modelling, the identified scale effect is utilized to evaluate the efficiency of the DMU$_j$ with optimally reduced levels of inputs (augmented levels of outputs, in case of the output oriented model). These two stage models are summarized in Table 1.3. The optimal solution to the first stage for the DMU$_j$ is denoted as $\hat{\theta}_j$ and

[8] Note that at least one component of each δ is zero because of the projection on to the production possibility frontier.

in the case of the output oriented model $\hat{\phi}_j$. The second stage of almost 100% confidence problem is constructed by replacing $\bar{x}_j$ (in output oriented model: $\bar{y}_j$) with $\hat{\theta}_j\bar{x}_j$ (respectively for input model with: $\hat{\phi}_j\bar{y}_j$) in constraints and objective function of problem 1.2 as presented in Table 1.3.

When the two stage models are used, the inefficiency of the DMU$_j$ can be evaluated by use of values of $\hat{\phi}_j^{-1}$ or $\hat{\theta}_j$. The major drawback of use of $\hat{\phi}_j^{-1}$ and $\hat{\theta}_j$ as inefficiency measures of the DMU$_j$ is that these measures do not uniquely identify efficient points. This shortage is present because for $\hat{\phi}_j = 1$ ($\hat{\theta}_j = 1$) the DMU$_j$ is the boundary point of T_φ but the positive non–proportional slacks can be present. The elements of production possibility set with $\hat{\phi}_j = 1$ ($\hat{\theta}_j = 1$) and positive non–proportional slacks are usually referred to as weakly efficient points. Due to the aforementioned shortage, the identification of efficiency of the DMU$_j$ has to be done in two stages. Therefore, the DMU$_j$ is identified as efficient if the proportional scaling parameter equality $\hat{\phi}_j = 1$ ($\hat{\theta}_j = 1$) holds and the second stage model identify the DMU$_j$ as α–stochastically efficient. The additional condition on slacks is referred to as the sum of slacks and for α–stochastic efficiency it is required that it holds with probability $1-\alpha$.

1.5 Oriented SDEA models

In both stages the objective function optimization is subject to the same constraints, the only difference being the objective function, therefore the two stage oriented SDEA models can be merged into a one–stage model. To merge these stages in one optimization problem, the non–Archimedean ϵ is used as a weight for the second stage objective function. The choice of non–Archimedean ϵ as the weight guarantees that proportional movement towards the frontier pre–empts the additive slacks optimization.

Output oriented model The one stage model for evaluation of efficiency of the DMU$_j$ is derived from the two stages optimization model presented in Table 1.3 and can be stated as:

$$\begin{aligned} \max_{\lambda_j,\phi_j} \quad & \phi_j + \epsilon(Prob(\mathbf{1}^T(\tilde{X}\lambda_j - \tilde{x}_j) + \mathbf{1}^T(\phi_j\tilde{y}_j - \tilde{Y}\lambda_j) < 0) - \alpha) && (1.11)\\ s.t. \quad & Prob({}_i\tilde{x}\lambda_j < \tilde{x}_{ij}) \geq 1-\epsilon, && i = 1,\ldots,m;\\ & Prob({}_r\tilde{y}\lambda_j > \phi_j\tilde{y}_{rj}) \geq 1-\epsilon, && r = 1,\ldots,s;\\ & \varphi(\mathbf{1}^T\lambda_j) = \varphi; && \end{aligned}$$

$$\lambda_j \geq 0.$$

After the same linearization procedure that was applied to problem 1.2 and reviewed in the fourth section of this chapter, the following linear model is derived:

$$\max_{\lambda_j, q_{kr}, h_{ki}, \phi_j} \phi_j - \epsilon[\mathbf{1}^T(\bar{X}\lambda_j - \bar{x}_j) + \mathbf{1}^T(\phi_j \bar{y}_j - \bar{Y}\lambda_j) + \tag{1.12}$$
$$+\delta(\mathbf{1}^T(A\lambda_j - a_j) + \mathbf{1}^T(\phi_j b_j - B\lambda_j))\sigma_\varepsilon \Phi^{-1}(\alpha)] + \epsilon(\sum_{r=1}^{s}(q_{1r} + q_{2r}) + \sum_{i=1}^{m}(h_{1i} + h_{2i}))$$
$$\begin{aligned} s.t. \quad & {}_i\bar{x}\lambda_j \leq \bar{x}_{ij} + (h_{1i} + h_{2i})\sigma_\varepsilon \Phi^{-1}(\epsilon), & \\ & {}_i a\lambda_j - a_{ij} = h_{1i} - h_{2i}, & i = 1, \ldots, m, \\ & \phi_j \bar{y}_{rj} \leq {}_r\bar{y}\lambda_j + (q_{1r} + q_{2r})\sigma_\varepsilon \Phi^{-1}(\epsilon), & \\ & \phi_j b_{rj} - {}_r b\lambda_j = q_{1r} - q_{2r}, & r = 1, \ldots, s, \\ & \varphi(\mathbf{1}^T\lambda_j) = \varphi, & \\ & \lambda_j \geq 0, q_{kr} \geq 0, h_{ki} \geq 0, & k = 1, 2. \end{aligned}$$

Input oriented model Similarly, as for the output oriented model, the almost 100% confidence chance constrained input oriented model for efficiency evaluation of the DMU$_j$ is derived as:

$$\min_{\lambda_j, \theta_j} \theta_j - \epsilon(Prob(\mathbf{1}^T(\tilde{X}\lambda_j - \theta_j \tilde{x}_j) + \mathbf{1}^T(\tilde{y}_j - \tilde{Y}\lambda_j) < 0) - \alpha) \tag{1.13}$$
$$\begin{aligned} s.t. \quad & Prob({}_i\tilde{x}\lambda_j < \theta_j \tilde{x}_{ij}) \geq 1 - \epsilon, & i = 1, \ldots, m; \\ & Prob({}_r\tilde{y}\lambda_j > \tilde{y}_{rj}) \geq 1 - \epsilon, & r = 1, \ldots, s; \\ & \varphi(\mathbf{1}^T\lambda_j) = \varphi; & \\ & \lambda_j \geq 0. & \end{aligned}$$

Finally, the linearized form of the almost 100% confidence chance constrained input oriented model is stated as:

$$\min_{\lambda_j, q_{kr}, h_{ki}, \theta_j} \theta_j + \epsilon[\mathbf{1}^T(\bar{X}\lambda_j - \theta_j \bar{x}_j) + \mathbf{1}^T(\bar{y}_j - \bar{Y}\lambda_j) + \tag{1.14}$$
$$+\delta(\mathbf{1}^T(A\lambda_j - \theta_j a_j) + \mathbf{1}^T(b_j - B\lambda_j))\sigma_\varepsilon \Phi^{-1}(\alpha)] + \epsilon(\sum_{r=1}^{s}(q_{1r} + q_{2r}) + \sum_{i=1}^{m}(h_{1i} + h_{2i}))$$

$$
\begin{aligned}
s.t.\quad & {}_i\bar{x}\lambda_j \leq \theta_j \bar{x}_{ij} + (h_{1i} + h_{2i})\sigma_\varepsilon \Phi^{-1}(\epsilon), & \\
& {}_i a\lambda_j - \theta_j a_{ij} = h_{1i} - h_{2i}, & i = 1, \ldots, m, \\
& \bar{y}_j \lambda_j \leq {}_r\bar{y} + (q_{1r} + q_{2r})\sigma_\varepsilon \Phi^{-1}(\epsilon), & \\
& b_{rj} - {}_r b\lambda_j = q_{1r} - q_{2r}, & r = 1, \ldots, s, \\
& \varphi(\mathbf{1}^T\lambda_j) = \varphi, & \\
& \lambda_j \geq 0, q_{kr} \geq 0, h_{ki} \geq 0, & k = 1, 2.
\end{aligned}
$$

Furthermore, the optimal solution $(\lambda_j^*, \mathbf{q}_{1j}^*, \mathbf{q}_{2j}^*, \mathbf{h}_{1j}^*, \mathbf{h}_{2j}^*, \phi_j^*)$ of output oriented problem (1.12) (alternatively the optimal solution $(\lambda_j^*, \mathbf{q}_{1j}^*, \mathbf{q}_{2j}^*, \mathbf{h}_{1j}^*, \mathbf{h}_{2j}^*, \theta_j^*)$ of input oriented problem (1.14)) is used to evaluate the technical efficiency of the DMU$_j$. The DMU$_j$ is α–stochastic efficient, when the following two conditions are satisfied:

1. $\phi_j^* = 1$ $(\theta_j^* = 1)$;
2. $\mathbf{1}^T(\bar{X}\lambda_j^* - \bar{x}_j) + \mathbf{1}^T(\phi_j^*\bar{y}_j - \bar{Y}\lambda_j^*) + |\mathbf{1}^T(A\lambda_j^* - a_j) + \mathbf{1}^T(\phi_j^* b_j - B\lambda_j^*)|\sigma_\varepsilon\Phi^{-1}(\alpha) \geq 0$ $(\mathbf{1}^T(\bar{X}\lambda_j^* - \theta_j^*\bar{x}_j) + \mathbf{1}^T(\bar{y}_j - \bar{Y}\lambda_j^*) + |\mathbf{1}^T(A\lambda_j^* - \theta_j^* a_j) + \mathbf{1}^T(b_j - B\lambda_j^*)|\sigma_\varepsilon\Phi^{-1}(\alpha) \geq 0)$.

As mentioned in the section on efficiency measure introduction, a class of weakly efficient DMUs can be defined. The analyzed DMU$_j$ is identified as weakly efficient when the optimal solution of the associated problem satisfies $\phi_j^* = 1$ or $\theta_j^* = 1$.

1.6 Chance constrained DEA model

As in the section on almost 100% chance constrained models, I also assume the same disturbance structure for chance constrained efficiency models and the following chance constrained version of the DEA model can be derived:

$$
\begin{aligned}
\min_{\lambda_j}\quad & \mathbf{1}^T(\bar{X}\lambda_j - \bar{x}_j) + \mathbf{1}^T(\bar{y}_j - \bar{Y}\lambda_j) & \\
s.t.\quad & Prob({}_i\tilde{x}\lambda_j < \tilde{x}_{ij}) \geq 1 - \alpha, & i = 1, \ldots, m; \\
& Prob({}_r\tilde{y}\lambda_j > \tilde{y}_{rj}) \geq 1 - \alpha, & r = 1, \ldots, s; \\
& \varphi(\mathbf{1}^T\lambda_j) = \varphi & \\
& \lambda_j \geq 0; &
\end{aligned}
\tag{1.15}
$$

To relate Problem (1.15) to the definition of chance constrained efficiency domination introduced in definition (4), I state the following theorem:

Theorem 3. *Let DMU$_j$ be an α-stochastically constrained efficient. Then for all λ_j such* *that*

$$\begin{aligned}
&Prob({}_i\tilde{x}\lambda_j \leq \tilde{x}_i^*) \geq 1-\alpha, \quad i=1,\ldots,m;\\
&Prob({}_r\tilde{y}\lambda_j \geq \tilde{y}_j^*) \geq 1-\alpha, \quad r=1,\ldots,s;\\
&\varphi(\mathbf{1}^T\lambda_j)=\varphi,\ \lambda_j \in \mathbb{R}^n_+, \qquad \lambda_j \geq 0,
\end{aligned} \tag{1.16}$$

we have $\mathbf{1}^T(\bar{X}\lambda_j - \bar{x}_j) + \mathbf{1}^T(\bar{y}_j - \bar{Y}\lambda_j) = 0.$

Proof: Suppose there exists λ_j^* such that it fulfills constraints (1.16) and $\mathbf{1}^T(\bar{X}\lambda_j^* - \bar{x}_j) + \mathbf{1}^T(\bar{y}_j - \bar{Y}\lambda_j^*) > 0$. Then there exists s_r^+ or $s_i^- \in \mathbb{R}_+$, $s_r^+,\ s_i^- > 0$ such that $Prob({}_r\tilde{y}\lambda_j^* - \tilde{y}_{rj} \geq s_r^+) \geq 1-\alpha$ or $Prob(\tilde{x}_{ij} - {}_i\tilde{x}\lambda_j^* \geq s_i^-) \geq 1-\alpha$. According to definition (4) the DMU$_j$ is dominated by the point $(\tilde{X}\lambda_j^*, \tilde{Y}\lambda_j^*)$ and this contradicts the assumption in the theorem that DMU$_j$ is α–chance constrained efficient. □

Applying the same orientation procedure as for the almost 100% chance constrained problems the two stage problems are derived. As for problem (1.2) the dual problem to problem (1.15) can be derived and the optimal solutions are used to identify the supporting hyperplanes to analyzed DMUs and to set up the production possibility frontier estimate.

The same linearization procedure as was used to linearize problem (1.2) and described in the previous section is applied after the two stage problem is merged in one one–stage optimization problem. The following oriented and linearized chance constrained models are derived:

Output oriented model

$$\begin{aligned}
\max_{\lambda_j, q_{kr}, h_{ki}, \phi_j}\ & \phi_j - \epsilon(\mathbf{1}^T(\bar{X}\lambda_j - \bar{x}_j) + \mathbf{1}^T(\phi_j\bar{y}_j - \bar{Y}\lambda_j) + \\
& \qquad -\epsilon(\sum_{r=1}^{s}(q_{1r}+q_{2r}) + \sum_{i=1}^{m}(h_{1i}+h_{2i}))
\end{aligned} \tag{1.17}$$

$$
\begin{aligned}
s.t.\quad & {}_i\bar{x}\lambda_j \le \bar{x}_{ij} + (h_{1i}+h_{2i})\sigma_\varepsilon\Phi^{-1}(\alpha), && i=1,\dots,m,\\
& {}_ia\lambda_j - a_{ij} = h_{1i} - h_{2i}, && i=1,\dots,m,\\
& \bar{y}_j\lambda_j \le \phi_{j\,r}\bar{y} + (q_{1r}+q_{2r})\sigma_\varepsilon\Phi^{-1}(\alpha), && r=1,\dots,s,\\
& \phi_j b_{rj} - {}_rb\lambda_j = q_{1r} - q_{2r}, && r=1,\dots,s,\\
& \varphi(\mathbf{1}^T\lambda_j) = \varphi,\\
& \lambda_j \ge 0, q_{kr} \ge 0, h_{ki} \ge 0, && k=1,2,\\
& && i=1,\dots,m,\\
& && r=1,\dots,s.
\end{aligned}
$$

Input oriented model

$$
\begin{aligned}
\min_{\lambda_j,q_{kr},h_{ki},\theta_j}\quad & \theta_j + \epsilon(\mathbf{1}^T(\bar{X}\lambda_j - \theta_j\bar{x}_j) + \mathbf{1}^T(\bar{y}_j - \bar{Y}\lambda_j)) + \\
& \qquad\qquad + \epsilon(\sum_{r=1}^{s}(q_{1r}+q_{2r}) + \sum_{i=1}^{m}(h_{1i}+h_{2i}))
\end{aligned}
\tag{1.18}
$$

$$
\begin{aligned}
s.t.\quad & {}_i\bar{x}\lambda_j \le \theta_j\bar{x}_{ij} + (h_{1i}+h_{2i})\sigma_\varepsilon\Phi^{-1}(\alpha), && i=1,\dots,m,\\
& {}_ia\lambda_j - \theta_j a_{ij} = h_{1i} - h_{2i}, && i=1,\dots,m,\\
& \bar{y}_j\lambda_j \le {}_r\bar{y} + (q_{1r}+q_{2r})\sigma_\varepsilon\Phi^{-1}(\alpha), && r=1,\dots,s,\\
& b_{rj} - {}_rb\lambda_j = q_{1r} - q_{2r}, && r=1,\dots,s,\\
& \varphi(\mathbf{1}^T\lambda_j) = \varphi,\\
& \lambda_j \ge 0, q_{kr} \ge 0, h_{ki} \ge 0, && k=1,2,\\
& && i=1,\dots,m,\\
& && r=1,\dots,s.
\end{aligned}
\tag{1.19}
$$

Similarly, as in the previous section these models can be compared to DEA models summarized in Table 1.1 and as for Problems (1.14) and (1.12), the optimal solution $(\lambda_j^*, q_{11}^*, \mathbf{q}_{1j}^*, \mathbf{q}_{2j}^*, \mathbf{h}_{1j}^*, \mathbf{h}_{2j}^*, \phi_j^*)$ of problem (1.17), $((\lambda_j^*, \mathbf{q}_{1j}^*, \mathbf{q}_{2j}^*, \mathbf{h}_{1j}^*, \mathbf{h}_{2j}^*, \theta_j^*)$ for problem (1.18)) can be used to evaluate the efficiency of DMU_j as in the previous section.

The DMU_j is chance constrained efficient if the following two conditions are satisfied:

1. $\phi_j^* = 1$ $(\theta_j^* = 1)$;

2. All expected values of slacks and excess are zero: $\mathbf{1}^T(\bar{X}\lambda_j^* - \bar{x}_j) = 0$ and $\mathbf{1}^T(\phi_j^*\bar{y}_j - \bar{Y}\lambda_j^*) = 0$ $(\mathbf{1}^T(\bar{X}\lambda_j^* - \theta_j^*\bar{x}_j) = 0$ and $\mathbf{1}^T(\bar{y}_j - \bar{Y}\lambda_j^*) = 0)$.

To simplify the evaluation of efficiency score the following two efficiency measures for stochastic models which are stochastic equivalents for measures introduced by Tone

(1993), are proposed:

$$\text{Input oriented:} \quad \chi_j = \left(\theta_j^* + \frac{\mathbf{1}^T(\bar{X}\lambda_j^* - \theta_j^*\bar{x}_j)}{\mathbf{1}^T\bar{x}_j}\right)\frac{\mathbf{1}^T\bar{y}_j}{\mathbf{1}^T\bar{Y}\lambda_j^*},$$

$$\text{Output oriented:} \quad \tau_j^{-1} = \left(\phi_j^* - \frac{\mathbf{1}^T(\phi_j^*\bar{y}_j - \bar{Y}\lambda_j^*)}{\mathbf{1}^T\bar{y}_j}\right)\frac{\mathbf{1}^T\bar{x}_j}{\mathbf{1}^T\bar{X}\lambda_j^*}.$$

The proposed efficiency measures τ and χ have the following properties:

1. $0 \leq \tau_j, \chi_j \leq 1$
2. $\chi_j = 1$, $\tau_j = 1 \Leftrightarrow$ DMU$_j$ is chance constrained efficient
3. τ_j and χ_j are units invariant measures
4. τ_j and χ_j are monotonic increasing in inputs and outputs
5. τ_j and χ_j are decreasing in the relative values of the slacks
6. $\tau_j = \phi_j^*$, $\chi_j = \theta_j^* \Leftrightarrow$ the expected values of all slacks are zero.

These measures make it easier to evaluate the efficiency score of DMU$_j$ because they take into account the values of maximal proportional increase and the slacks (residuals) values.

1.7 Introducing returns to scale

As mentioned in the second section, the CCR model was designed to analyze the technology with property of constant returns to scale. Later, the BCC model and its variations were developed by Banker, Charnes, and Cooper (1984) to analyze the production function with variable returns to scale. Here, I follow this concept to introduce the variable returns to scale into the stochastic framework. The following definition uses the expected values to define types of returns to scale:

Definition 5. Returns to scale. Let the DMU$_j$ be stochastically efficient and the point $Z_\delta = ((1+\delta)\bar{x}_j, (1+\delta)\bar{y}_j)$ is a point in δ–neighborhood of $(\bar{x}_j, \bar{y}_j)$:

- The Non–Decreasing returns to scale are present $\Leftrightarrow \exists\, \delta^* > 0$ such that $Z_\delta \in T_\varphi$ for $\delta^* > \delta \geq 0$ and $Z_\delta \notin T_\varphi$ for $-\delta^* < \delta < 0$

- The Constant returns to scale are present $\Leftrightarrow \exists\, \delta^* > 0$ such that $Z_\delta \in T_\varphi$ for $\mid \delta \mid < \delta^*$

- The Non–Increasing returns to scale are present $\Leftrightarrow \exists\, \delta^* > 0$ such that $Z_\delta \notin T_\varphi$ for $\delta^* > \delta \geq 0$ and $Z_\delta \in T_\varphi$ for $-\delta^* < \delta < 0$.

The differences in types of returns to scale are reflected by different shapes of the production possibility set frontier that is set up by the intersection of supporting hyperplanes identified by optimal solutions of multiplier formulation of the DEA models. In the case of constant returns to scale (the CCR model by Charnes, Cooper, and Rhodes (1978)) the envelopment surface consists of a single half line that passes through the origin as shown in Figure 1.4. In the case of variable returns to scale, the production frontier is a piecewise linear set. Therefore, Figure 1.4 also shows the production possibility frontier of the model with the variable returns to scale that is referred to as the BCC model (Banker, Charnes, and Cooper (1984)) and in Figure 1.5 the BCC frontier is related to the frontier under the assumption of increasing returns to scale. These frontiers of production possibility set under various types of returns to scale are parameterized via the selection of φ and constraint type associated with the φ as follows:

$$\varphi = \begin{cases} 0 & \text{Constant returns to scale (CCR model)} \\ 1 & \text{Variable returns to scale (BCC model).} \end{cases}$$

Since the α–stochastically efficient point $(\tilde{x}_j, \tilde{y}_j)$ satisfies condition 1.9, for the point $Z_\delta = ((1+\delta)\bar{x}_j, (1+\delta)\bar{y}_j)$ can be derived

$$\begin{aligned} c^T(1+\delta)\tilde{x}_j + d^T(1+\delta)\tilde{y}_j + f_j + (1+\delta)\Phi^{-1}(\epsilon)\sigma_\varepsilon \mid c^T a_j + d^T b_j \mid \;\; &= \\ = (1+\delta)(c^T\tilde{x}_j + d^T\tilde{y}_j + f_j + \Phi^{-1}(\epsilon)\sigma_\varepsilon \mid c^T a_j + d^T b_j \mid) - \delta f_j = -\delta f_j & \end{aligned} \tag{1.20}$$

and the point $Z_\delta \in T_\varphi$ if and only if $-\delta f_j \geq 0$. Using definition 5, the relations between the type of the returns to scale and the sign of f_j is revealed and these relations are summarized in Table 1.2 together with choice of constrain on intensity variable vector λ_j.

1.8 Summary of SDEA models

In the previous sections, the oriented SDEA models were derived and these models are summarize in Table 1.4. It should be stressed that even the models using the same efficiency dominance definition but with different orientation choice result in different efficiency scores. Therefore, the choice of the efficiency dominance type, returns to scale and projection path to the envelopment surface (the set of dominating points in the production possibility set) are crucial for the efficiency analysis and the choice should reflect the aims of analyzis.

The returns to scale choice affects the shape of the production possibility set envelopment. The restrictions on returns to scale are related to four types of the envelopment surface shape through the geometry of the production possibility set and these restrictions are interpreted as the restriction on intensity variable λ in the envelopment problem or a restriction on supporting hyperplanes in the multiplier problem.

The evaluation of the efficiency score is based on distance measurement between the point that represents DMU and the associated point on the envelopment surface. This distance measure used in additive models is the most simple efficiency measure. A more sophisticated efficiency measure is created using the measure of maximal proportional inputs reduction (output augmentation) while keeping the levels of outputs (inputs) fixed. This proportional input (output) scaling approach is interpreted as the selection of a projection path towards the envelopment surface and results in the creation of oriented SDEA models.

The use of Non–Archimedean infinitesimal ϵ is closely related to the unit invariance property of the objective function values of the derived models because the result of multiplication by ϵ is not unit dependent. The use of unit invariant models also delivers the possibility of units of measurement change to avoid numerical problems [e.g., tiny diagonal matrices] when the SDEA models are solved.

Table 1.4 compares the derived SDEA with the most popular DEA models that appear in the present studies on efficiency evaluation. The additional SDEA models can be derived as extensions of models covered in this chapter using the extensions procedures for the DEA models.

1.9 Method for SDEA model solving

To solve the linear optimization problems associated with the derived SDEA models the variant of the interior point method (IPM) is used because it is less computationally costly than the simplex methods when large sized problems are solved. For the purpose of the IPM employment the linearized problems 1.12 and 1.14 can be easily transformed to the standard linear programming form:[9]

$$\begin{array}{llcll} \text{Primal:} & \min_{\mathbf{x}} \mathbf{c}^T \mathbf{x} & & \text{Dual:} & \max_{\mathbf{y},\mathbf{z}} \mathbf{b}^T\mathbf{y} \\ & \text{s.t. } \mathbf{A}\mathbf{x} = \mathbf{b}, \mathbf{x} \geq 0 & & & \text{s.t. } \mathbf{A}^T\mathbf{y} + \mathbf{z} = \mathbf{c}, \mathbf{z} \geq 0. \end{array} \tag{1.21}$$

Using the complementarity constraint $\mathbf{z}^T\mathbf{x} = 0$ (equivalent to duality gap condition $\mathbf{c}^T\mathbf{x} - \mathbf{b}^T\mathbf{y} = 0$) together with the feasibility constraints the following optimality condition for problem 1.21 is stated as

$$\begin{pmatrix} \mathbf{A}\mathbf{x} - \mathbf{b} \\ \mathbf{A}^T\mathbf{y} + \mathbf{z} - \mathbf{c} \\ \mathbf{z}^T\mathbf{x} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ 0 \end{pmatrix}, \tag{1.22}$$

where $\mathbf{z}, \mathbf{x} \geq 0$. To solve problem 1.22, I use Mehrotra's predictor–corrector algorithm that belongs to the class of the central path following IPM algorithms.[10] This primal–dual algorithm uses the combination of Newton's direction (duality gap reduction direction) and centering direction to solve the sequence of problems that comes from problem 1.22, where the complementarity constraint is modified to $\mathbf{x}_k^T\mathbf{z}_k = \mu_k$ and sequence $\{\mu_k\}$ converges to 0 for $k \to \infty$. So, the IPM algorithm generates an infinite sequence of points that converges to an optimal solution and the iteration process stops when the iterations are sufficiently close to the optimal solution or the limit for the number of iterations is reached. The advantage of the primal–dual version of the interior point method is that the primal and dual problem 1.21 are solved simultaneously.

Further, the IPM solutions satisfy the strong complementarity slackness condition (SCSC). The SCSC solution is the solution with the maximal product of the positive components of the optimal solution and therefore it is the optimal solutions with a minimal number of zero components. The SCSC property of optimal solutions helps to eliminate interpretation problems when the optimal solution to the DEA model are rendered as the shadow prices of inputs and outputs.[11]

[9] In the case of linearized stochastic problems, vectors $\mathbf{x}$, $\mathbf{c}$, $\mathbf{z} \in \mathbb{R}^{n+3(m+s)+1}$; vectors $\mathbf{y}$, $\mathbf{b} \in \mathbb{R}^{2(m+s)+1}$ and matrix $\mathbf{A} \in \mathbb{R}^{(2(m+s)+1)\times(n+3(m+s)+1)}$.

[10] The solver for the stated oriented SDEA models is constructed using the procedures package known as PCx linear solver obtained from Optimization Technology Center at Argonne National Laboratory and Northwestern University.

[11] For more details on the use of interior point methods solutions of the DEA related problems see Brázdik (2001).

1.10 Indonesian rice farms efficiency

To demonstrate the use of the oriented SDEA models, the results from the proposed SDEA models are compared to the DEA and SFA results. This comparison is motivated by Horrace and Schmidt's (1996) work, where parametric methods for efficiency estimation are compared using data on Indonesian rice farms. To compare with results presented in Druska and Horrace's (2004) methodological work on spatial effects in the SFA framework, I use the same data set to compute the SDEA and DEA scores.

Indonesia is the biggest rice importer in Asia at the same time almost 70% of the country's 213 million people are farmers, hence the identification of the linkages between different factors and rice yield in the West Java area is the subject of many studies on farming efficiency [e.g. Wadud (2002) and Daryanto, Battese, and Fleming (2002a)]. For research purposes, the Indonesian Ministry of Agriculture surveyed rice farms over six growing periods (3 wet and 3 dry periods) in six villages in the area of the Cimanuk River basin in West Java. The data set from this survey is filtered for outliers that reported yields over the maximum hectare yields reached in laboratory conditions. After this correction, the panel used for analysis is balanced and describes the production mixes of 160 rice farms with average yield of 3265.20 kg/ha that resemble the observed average yields in this area.

For the purpose of comparison with the SFA results, I use the same inputs and outputs to specify the inputs–output production mixes of the surveyed rice farms as were used in the SFA study by Druska and Horrace (2004). The considered inputs include total area of rice cultivation in hectares (Size), seed in kilograms (Seed), urea in kilograms (Urea), phosphate in kilograms (Phosphate) and total labor (Labor). As the measure of output the total output of rough rice in kilograms (Gross yield) is used and the summary statistics for the used inputs and output are presented in Table 1.5. All of the production factors exhibit very high variation and presence of noise that influence efficiency evaluation is expected. The presence of noise provides rationale for use of the SDEA approach.

To calculate the DEA efficiency scores, the output oriented DEA model presented in Table 1.1 is used. The α–stochastic efficiency of farms is evaluated by use of the linearized output oriented SDEA model described by problem 1.12. Moreover, I also compute the time average DEA efficiency scores and the DEA scores calculated using the mean values of farms' production mixes. The average DEA score for a rice farm is calculated by averaging the farm's efficiency scores when the data set is considered as a sample of 960 individual observations. The DEA–mean score is calculated using a sample with 160 observations, where each farm is characterized by mean values of its production mix characteristics.

For all data envelopment models, I consider the cases of normal (denoted by subscript N or $Norm$) and log–normal (denoted by subscript LN or $LogN$) distribution of the farms' inputs and outputs. Under the assumption of log–normal distribution, inputs and output are transformed by taking logs, therefore the efficiency scores are no more scale of operations invariant. The DEA and SDEA efficiency scores are calculated under assumption of constant returns to scale (choice $\varphi = 0$ and denoted by CCR) and variable returns to scale ($\varphi = 1$, BCC). The efficiency scores estimated by almost 100% chance constrained SDEA models are reported for $\alpha = 0.05$ as a level of modeler's risk because calculations shows that for higher levels the SDEA method suffers from a loss of discriminatory power and too many DMUs are evaluated as efficient.

The descriptive statistics of the computed DEA, SDEA and SFA efficiency scores are summarized in Table 1.6 and compared to Druska and Horrace's (2004) SFA scores FE and $FEsp$ that are estimated by the fixed effect method and fixed effect method with correction for spatially corrected errors, respectively. Table 1.6 reports higher mean values of efficiency scores for data envelopment approaches than for SFA scores. These SDEA and DEA results suggest that Indonesian rice farms are operating closer to the production frontier than in the SFA studies. Wadud (2002) observes a similar pattern for Bangladesh rice farms efficiency scores and he reports 0.80 as the mean score for the SFA and 0.86 and 0.91 for the CCR and BCC data envelopment models, respectively. From this comparison, I deduce that on average the considered Indonesian rice farms were operating at lower efficiency levels than rice farms in Bangladesh. As Table 1.6 reports, scores calculated by data envelopment approaches show a variance twice as high as scores calculated by the SFA. This is contrary to results by Wadud (2002), Ferro–Luzzi et al. (2003) and Jaforullah and Premachandra (2003) that report comparable variance for SFA and DEA efficiency scores.

Further, to highlight differences in efficiency scores among the used approaches, Table 1.7 compares efficiency scores for group of chosen DMUs. These DMUs were chosen according to the SFA efficiency scores estimates by Druska and Horrace (2004) to represent farms with the highest, median and the lowest technical efficiency scores. Due to the differences in nature of the compared methods differences in efficiency scores estimates are expected. However, the differences in efficiency rankings presented in Table 1.8 indicate inconsistency of efficiency evaluation across the assessed methods.

The nature of the SFA approach allows only one DMU to achieve a score of 1 while the data envelopment approaches assign efficiency score 1 to all DMUs on the production possibility frontier. Therefore, the peak at 1 with height proportional to the numbers of DMUs identified as efficient occurs in distribution of efficiency scores calculated by use of the data envelopment approaches. Keeping this fact in mind, the shapes of efficiency score distributions displayed in Figure 1.6, Figure 1.7 and Figure 1.8 can be compared. Examination of these figures reveals that the shape of the SFA efficiency score distribution function is matched at best by the distribution function estimate for the DEA average efficiency score under assumption of linearly distributed production characteristics for constant (CCRnorm) and variable (BCCnorm) returns to scale specification.

Due to the aforementioned differences in nature of efficiency scores, the results' consistency among the used approaches should be assessed through correlation of efficiency rankings rather than an efficiency scores. For ranking correlation evaluation, Spearman's (1904) correlation coefficient is used because its important feature is lower sensitivity to extreme values when compared to the standard correlation coefficient. Further, by evaluating the significance of calculated rankings correlations the hypothesis that considered rankings are not correlated is tested. Table 1.9 presents correlation coefficients for rankings generated using DEA on mean values, oriented SDEA and SFA efficiency scores. In Table 1.10, correlation coefficients for DEA on mean values, the oriented SDEA, and SFA efficiency rankings are summarized.

When the rankings correlation coefficients presented in Table 1.9 and Table 1.10 are assessed, I conclude that higher level of rankings consistency is observed between SFA efficiency rankings and data envelope analysis rankings than between SFA and SDEA rankings. The highest DEA–mean ranking correlation coefficients values are 0.72 and 0.55 and the values $0.85, 0.82$ for average DEA scores are substantially higher than the highest values $0.25, 0.24$ of the SFA–SDEA correlation coefficients. The presented SFA and DEA rankings correlation results correspond to findings in recent studies on the SFA and DEA ranking consistency. Wadud (2002) reports the highest correlation coefficients values ranging from 0.61 to 0.83, Jaforullah and Premachandra (2003) report 0.74 and Ferro–Luzzi et al. (2003) report significant correlation coefficients between SFA and DEA ranking in range from 0.594 to 0.677.

The purpose of this section was to improve the stochastic non–parametric approach for efficiency evaluation by introducing frontier projection direction. Therefore, the improvement in consistency of the SFA and SDEA results is expected. Contrary to this expectation, more consistency (in terms of significance of correlation coefficients and their absolute values) is found between the SFA and DEA (SFA–average DEA in range 0.11, 0.85, SFA–DEA mean in -0.22, 0.72) rankings than between the SFA–SDEA rankings (from -0.08 to 0.25). The observed low consistency of SFA–SDEA rankings may be a consequence of the high variance of the rice production characteristics that affects the accuracy of efficiency dominating set approximation. This conclusion originates from comparison of the DEA on mean values and SDEA efficiency rankings, where rankings correlations are insignificant or low and simultaneously the SDEA approach is derived from DEA on mean values approach by including correction for variance in data. Therefore, high values of the ranking correlation between SDEA and DEA–mean rankings are expected to be achieved when considered DMUs are characterized by random variables with low variances.

1.11 Conclusion

In the theoretical part of this chapter, I reviewed the technique used to derive linear deterministic equivalents to Huang and Li's (2001) SDEA models and this technique was used to develop the oriented stochastic DEA models and to describe their properties. Using the techniques of stochastic problems linearization the proposed oriented SDEA models were linearized, so the solver based on the interior point method for linear problems can be used to solve linear programming problems associated with the models. The created solver for problems associated with the SDEA and DEA models implements the primal–dual interior point method algorithm.

The empirical part of this chapter was motivated by Horrace and Schmidt's (1996) comparison of SFA methods. This part presents results of the technical efficiency evaluation of Indonesian rice farms by SDEA and DEA models. Further, efficiency rankings were constructed and compared with the SFA rankings constructed by Druska and Horrace (2004). While I was able to reject the hypothesis that the DEA, SDEA and SFA rankings are independent in the majority of the considered cases the consistency of results from the SFA and oriented SDEA models is questionable due to the low values of ranking correlation coefficients. Assessing the results of the DEA on the mean values approach, I conclude that in this data set the low rankings consistency originate from high variance present in the data. In spite of the low consistency of the SFA–SDEA approach the findings on the SFA–DEA rankings correlation are consistent with the recent studies on the SFA and DEA comparisons, e.g. Wadud and White (2000a) and Jaforullah and Premachandra (2003) that report considerable consistency of efficiency rankings.

1.A Figures and Tables

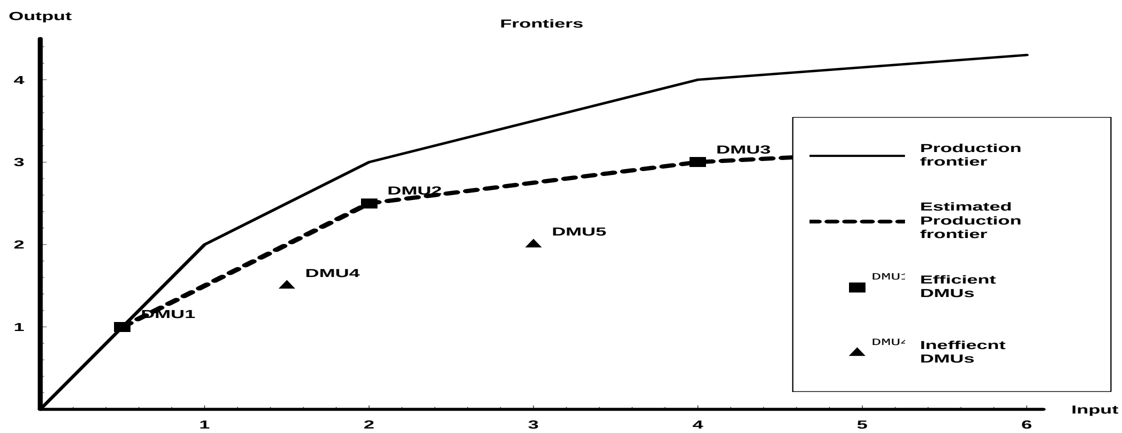

Figure 1.1: DEA estimate of production possibility frontier

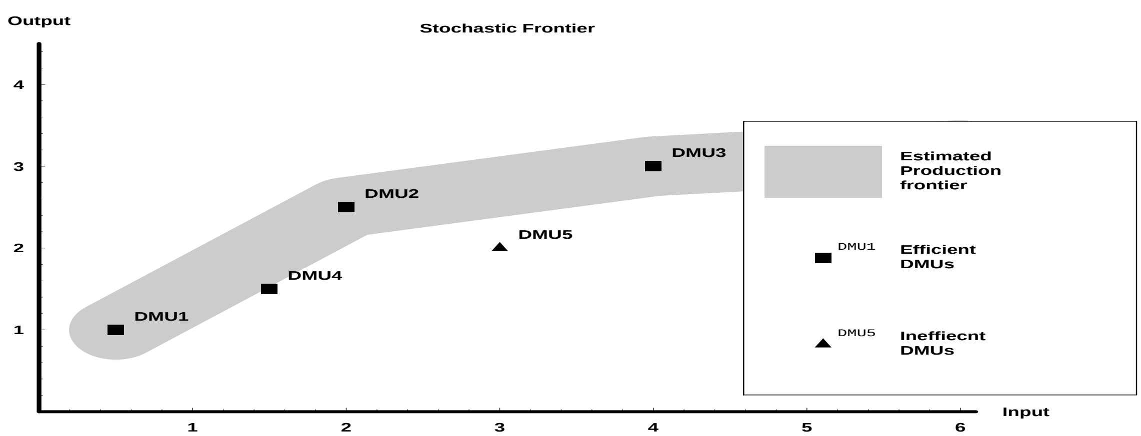

Figure 1.2: Set of α–stochastic dominant points

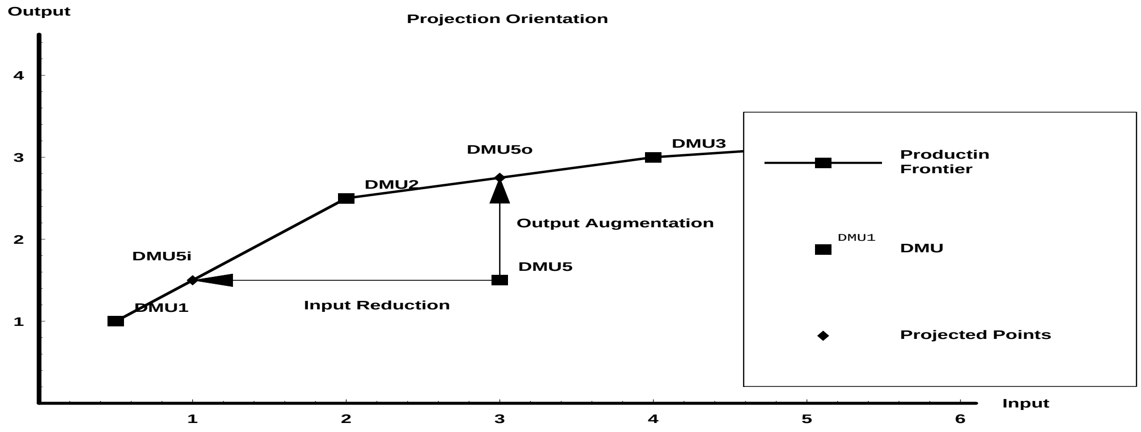

Figure 1.3: Projection on the production possibility frontier

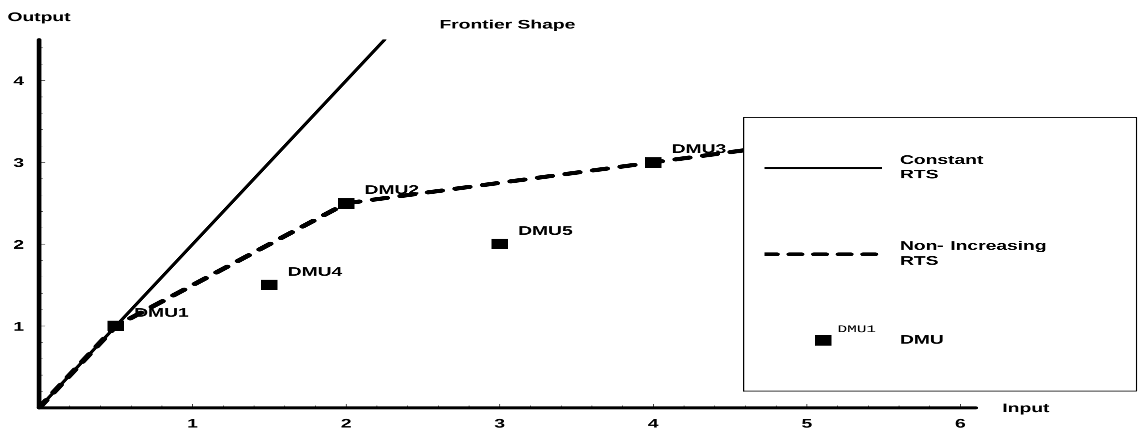

Figure 1.4: Returns to scale – Constant, Non–Increasing

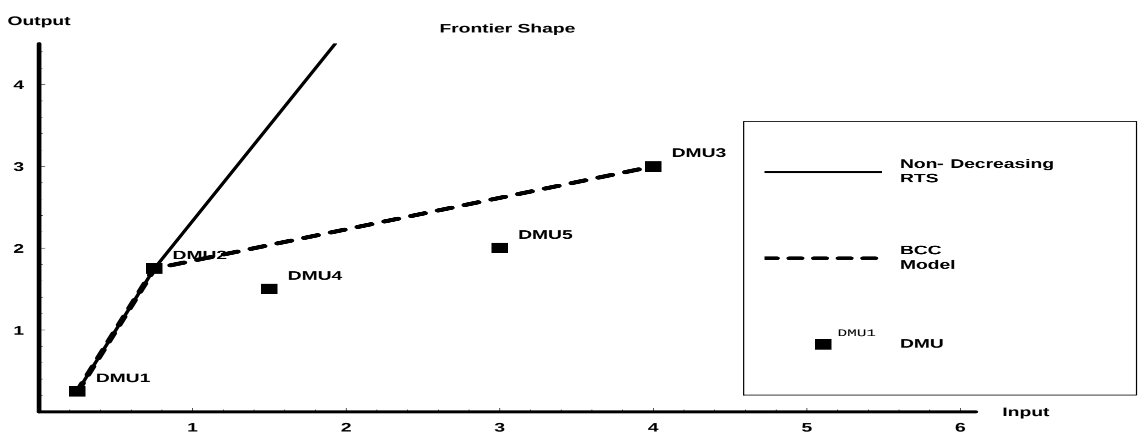

Figure 1.5: Returns to scale – Non–Decreasing, BCC model

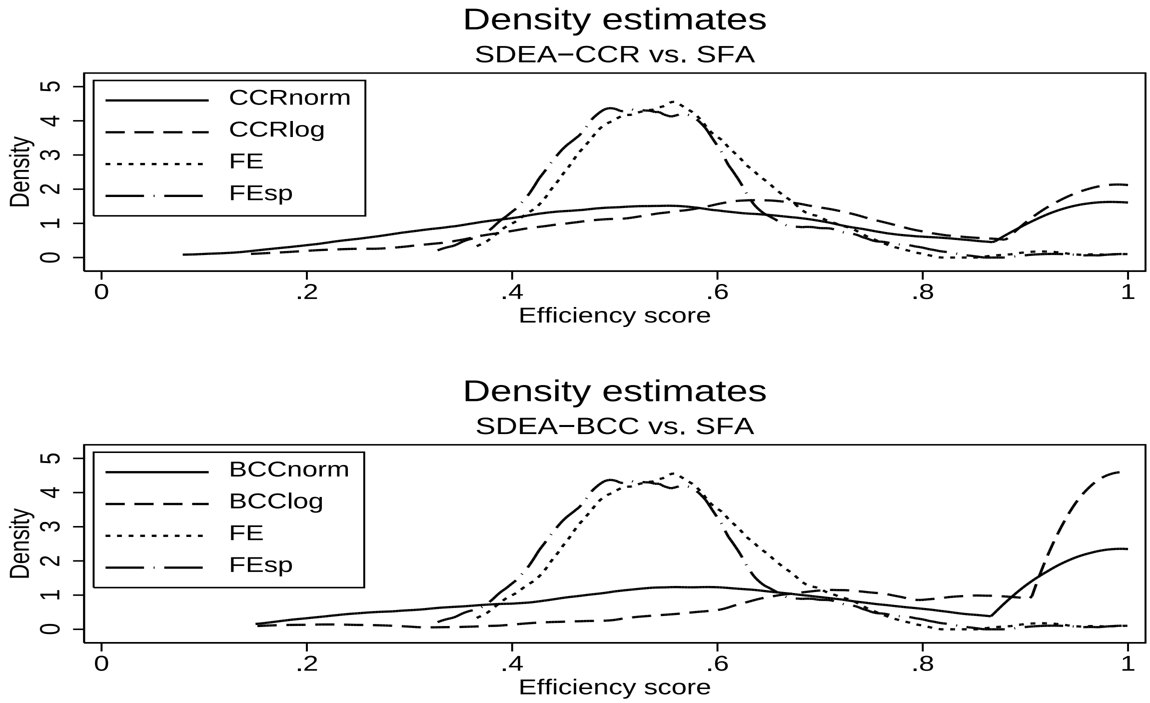

Figure 1.6: Kernel density estimates SDEA vs. SFA

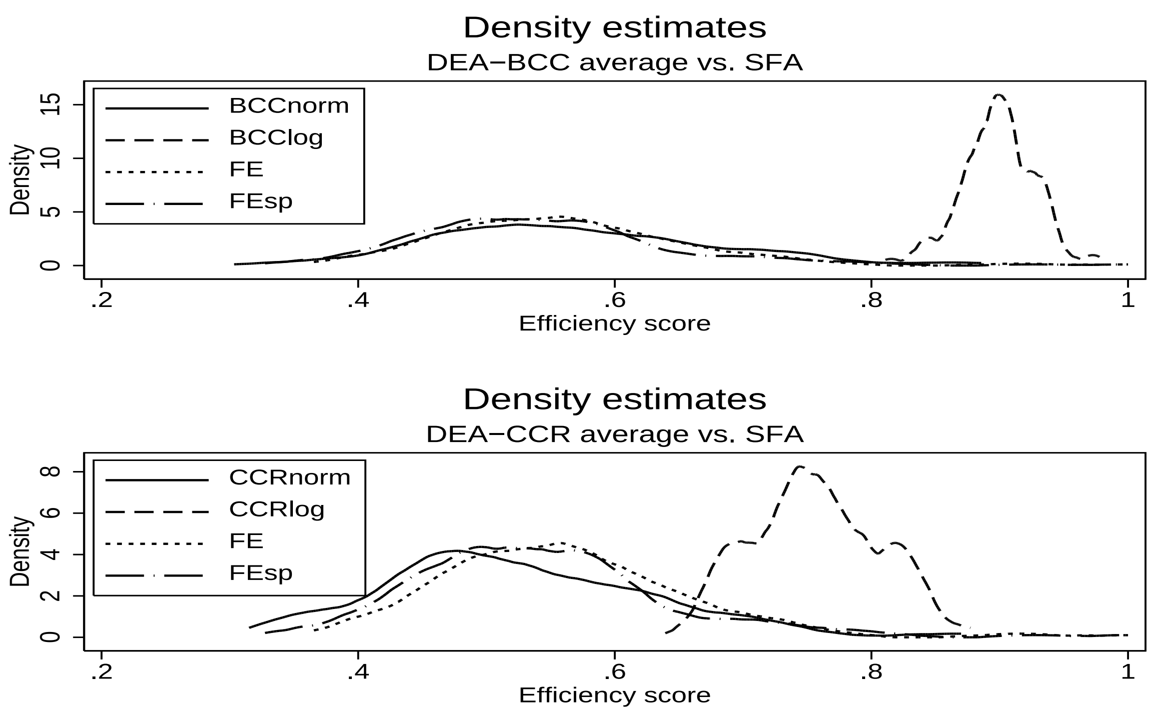

Figure 1.7: Kernel density estimates DEA average vs. SFA

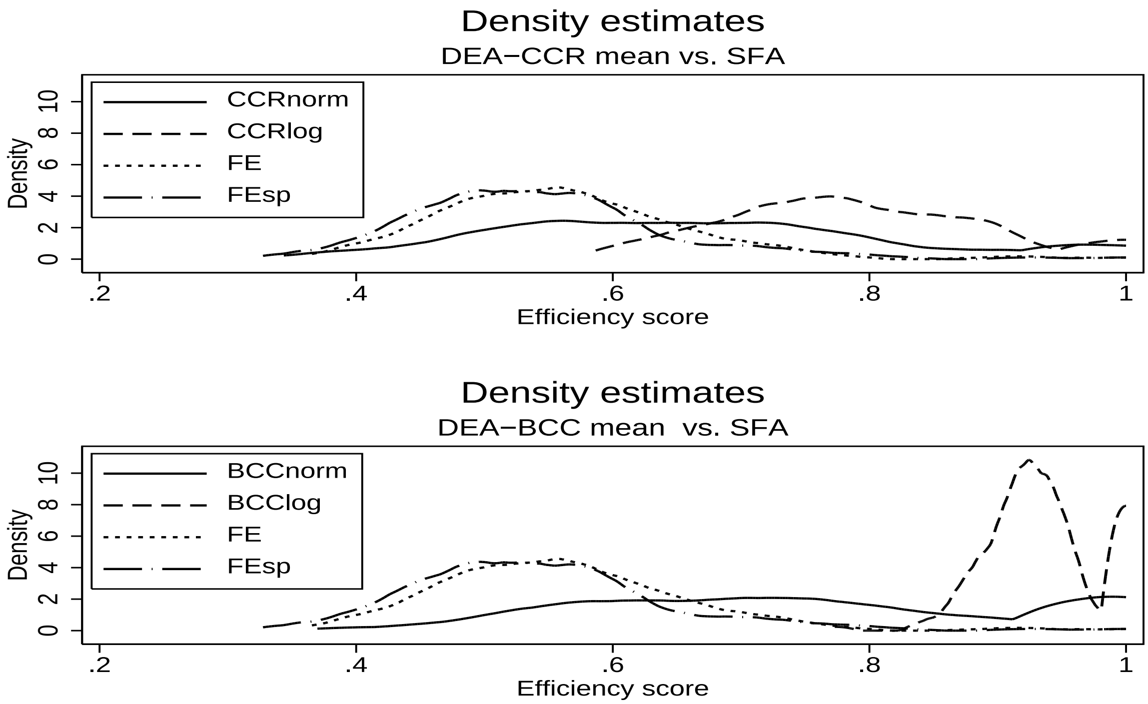

Figure 1.8: Kernel density estimates DEA–mean vs. SFA

Output oriented model		
$\max_{\lambda_j,\phi_j}$	$\phi_j + \epsilon(\mathbf{1}^T(X\lambda_j - x_j) + \mathbf{1}^T(\phi_j y_j - Y\lambda_j))$	
$s.t.$	${}_i x\lambda_j < x_{ij},$	$i = 1, \ldots, m;$
	${}_r y\lambda_j > \phi_j y_{rj},$	$r = 1, \ldots, s;$
	$\varphi(\mathbf{1}^T\lambda_j) = \varphi;$	
	$\lambda_j \geq 0$	
Input oriented model		
$\min_{\lambda_j,\theta_j}$	$\theta_j - \epsilon(\mathbf{1}^T(X\lambda_j - \theta_j x_j) + \mathbf{1}^T(y_j - Y\lambda_j))$	
$s.t.$	${}_i x\lambda_j < \theta_j x_{ij}$	$i = 1, \ldots, m;$
	${}_r y\lambda_j > y_{rj}$	$r = 1, \ldots, s;$
	$\varphi(\mathbf{1}^T\lambda_j) = \varphi;$	
	$\lambda_j \geq 0$	

Table 1.1: Generalized versions of input and output oriented DEA models

Model (Orientation)	Returns to scale	Constraint	Hyperplane(s)
CCR model			
(Input, Output)	Constant	None, $\varphi = 0$	Passes trough origin
BCC model			
(Input, Output)	Variable	$\mathbf{1}^T\lambda_j = 1$	Not constrained
SDEA models			
(Input)	Non–Decreasing	$\mathbf{1}^T\lambda_j \geq 1$	$f_j^* \geq 0$
(Input)	Non–Increasing	$\mathbf{1}^T\lambda_j \leq 1$	$f_j^* \leq 0$
(Input)	Constant	None	$f_j^* = 0$
(Output)	Non–Decreasing	$\mathbf{1}^T\lambda_j \geq 1$	$f_j^* \leq 0$
(Output)	Non–Increasing	$\mathbf{1}^T\lambda_j \leq 1$	$f_j^* \geq 0$
(Output)	Constant	None	$f_j^* = 0$

Table 1.2: Returns to scale

Output oriented model

First stage	Second stage
$\max_{\lambda_j,\phi_j} \phi_j$	$\max_{\lambda_j} Prob(\mathbf{1}^T(\tilde{X}\lambda_j - \tilde{x}_j) + \mathbf{1}^T(\hat{\phi}_j\tilde{y}_j - \tilde{Y}\lambda_j)) - \alpha$
s.t. $Prob({}_i\tilde{x}\lambda_j < \tilde{x}_{ij}) \geq 1-\epsilon$	s.t. $Prob({}_i\tilde{x}\lambda_j < \tilde{x}_{ij}) \geq 1-\epsilon$
$Prob({}_r\tilde{y}\lambda_j > \phi\tilde{y}_{rj}) \geq 1-\epsilon$	$Prob({}_r\tilde{y}\lambda_j > \hat{\phi}_j\tilde{y}_{rj}) \geq 1-\epsilon$
$\varphi(\mathbf{1}^T\lambda_j) = \varphi$	$\varphi(\mathbf{1}^T\lambda_j) = \varphi$
$\lambda_j \geq 0$	$\lambda_j \geq 0$
	$i = 1,\ldots,m;\ r = 1,\ldots,s.$

Input oriented model

First stage	Second stage
$\min_{\lambda_j,\theta_j} \theta_j$	$\max_{\lambda_j} Prob(\mathbf{1}^T(\tilde{X}\lambda_j - \hat{\theta}_j\tilde{x}_j) + \mathbf{1}^T(\tilde{y}_j - \tilde{Y}\lambda_j)) - \alpha$
s.t. $Prob({}_i\tilde{x}\lambda_j < \theta_j\tilde{x}_{ij}) \geq 1-\epsilon$	s.t. $Prob({}_i\tilde{x}\lambda_j < \hat{\theta}_j\tilde{x}_{ij}) \geq 1-\epsilon$
$Prob({}_r\tilde{y}\lambda_j > \tilde{y}_{rj}) \geq 1-\epsilon$	$Prob({}_r\tilde{y}\lambda_j > \tilde{y}_{rj}) \geq 1-\epsilon$
$\varphi(\mathbf{1}^T\lambda_j) = \varphi$	$\varphi(\mathbf{1}^T\lambda_j) = \varphi$
$\lambda_j \geq 0$	$\lambda_j \geq 0$
	$i = 1,\ldots,m;\ r = 1,\ldots,s.$

Table 1.3: Two stages of oriented almost 100% confidence chance constrained models

Model (Orientation)	Returns to Scale	Envelopment Type	Range	Units Invariant	Involves Non–Archimedean
Additive	Variable	Piecewise linear	objective value≤ 0	No	No
	Constant	Piecewise linear		No	No
Almost 100% confidence	Constant	St. Hyperplane	objective value$\leq \sigma_{\varepsilon}\Phi^{-1}(\epsilon)$	No	Yes
additive model; Problem (1.7)	Variable	St. Hyperplanes	$\mid \mathbf{1}^T(A\lambda_j - a_j) + \mathbf{1}^T(b_j - B\lambda_j) \mid$	No	Yes
BCC model (input)	Variable	Piecewise linear	$0 < \theta \leq 1$	Yes	Yes
BCC model (output)	Variable	Piecewise linear	$1 \leq \phi$	Yes	Yes
CCR model (input)	Constant	Piecewise linear	$0 < \theta \leq 1$	Yes	Yes
CCR model (output)	Constant	Piecewise linear	$1 \leq \phi$	Yes	Yes
Almost 100% confidence					
oriented models,	Variable	St. Hyperplanes	$0 < \theta \leq 1,\ 1 \leq \phi$	Yes	Yes
Problems (1.14),(1.12)	Constant	St. Hyperplane	$0 < \theta \leq 1,\ 1 \leq \phi$	Yes	Yes
(input, output)					

Table 1.4: Comparison of models

Data summary statistics

Variable	Obs.	Mean	Std. Dev.	Minimum	Maximum
Size	960	0.4398	0.5607	0.0140	5.3220
Seed	960	18.4708	46.6819	1.0000	1250.0000
Urea	960	96.5250	130.3932	1.0000	1250.0000
Phosphate	960	33.8072	48.3489	0.0000	700.0000
Labor	960	394.2240	496.0169	17.0000	4774.0000
Gross yield	960	1413.9340	1966.0950	42.0000	20960.0000

Table 1.5: Indonesian rice farm summary statistics

Efficiency scores summary statistics

Model	Obs	Mean	Std. Dev.	Minimum	Maximum
DEA					
BCC_{Norm}	960	0.5672	0.2044	0.1912	1
CCR_{Norm}	960	0.5256	0.1943	0.1775	1
BCC_{LogN}	960	0.8987	0.0565	0.6484	1
CCR_{LogN}	960	0.7561	0.0817	0.5143	1
DEA–mean					
BCC_{Norm}	160	0.7641	0.1723	0.3698	1
CCR_{Norm}	160	0.6721	0.1616	0.3436	1
BCC_{LogN}	160	0.9360	0.0427	0.7730	1
CCR_{LogN}	160	0.7918	0.1026	0.5867	1
SDEA					
BCC_{Norm}	160	0.7343	0.2614	0.1500	1
CCR_{Norm}	160	0.6594	0.2569	0.0791	1
BCC_{LogN}	160	0.8714	0.1867	0.1519	1
CCR_{LogN}	160	0.7260	0.2331	0.1456	1
SFA					
FE	160	0.5613	0.0992	0.3655	1
$FEspatial$	160	0.5435	0.1023	0.3274	1

Table 1.6: Efficiency scores summary statistics

Efficiency scores

Score	Farm	SFA		SDEA				DEA average efficiency score				DEA–mean			
		FE	FE_{sp}	CCR_N	BCC_N	CCR_{LN}	BCC_{LN}	CCR_N	BCC_N	CCR_{LN}	BCC_{LN}	CCR_N	BCC_N	CCR_{LN}	BCC_{LN}
High	164	1.0000	1.0000	0.6660	0.7109	0.6442	0.6808	0.8635	0.8613	0.7782	0.9690	1.0000	1.0000	0.7362	1.0000
	118	0.9323	0.9269	0.6875	1.0000	1.0000	1.0000	0.8699	0.8754	0.7926	0.9778	1.0000	1.0000	0.7853	1.0000
	152	0.8993	0.8152	0.4109	0.6398	0.2872	0.2940	0.7922	0.8269	0.8595	0.9707	1.0000	1.0000	1.0000	1.0000
	153	0.7717	0.7487	0.7604	0.7899	0.9128	1.0000	0.6589	0.6710	0.7734	0.9347	0.8717	0.8768	0.7528	0.9459
Medium	40	0.5535	0.5824	0.9622	1.0000	1.0000	1.0000	0.5969	0.6298	0.7348	0.9118	0.8476	0.8590	0.6776	0.9787
	101	0.5518	0.5282	0.5967	0.6117	0.8212	1.0000	0.5117	0.5252	0.6864	0.9028	0.6680	0.7005	0.6893	0.9311
	80	0.5518	0.5166	0.2974	0.3012	0.5673	0.7255	0.5528	0.6064	0.7741	0.8842	0.5723	0.6305	0.8240	0.9205
	149	0.5495	0.5173	1.0000	1.0000	1.0000	1.0000	0.4588	0.5494	0.8046	0.8789	0.5981	1.0000	0.8589	0.8544
Low	86	0.3980	0.3907	1.0000	1.0000	0.5822	1.0000	0.3351	0.3527	0.7280	0.8381	0.3859	0.4478	0.7608	0.8452
	143	0.3837	0.3596	0.4127	0.4960	1.0000	1.0000	0.3150	0.3539	0.7438	0.8202	0.4933	0.5247	0.7591	0.8722
	117	0.3790	0.3713	1.0000	1.0000	1.0000	1.0000	0.3944	0.4998	0.6907	0.8109	0.5387	0.8970	0.8572	0.8800
	45	0.3655	0.3274	0.4770	0.6235	0.5744	0.7485	0.3814	0.5945	0.8252	0.8474	0.4896	1.0000	0.8862	1.0000

Note: Farm identification number is from original sample.

Table 1.7: Comparison of technical efficiency scores

Efficiency rankings

Score	Farm	SFA		SDEA				DEA average efficiency score				DEA–mean			
		FE	FE_{sp}	CCR_N	BCC_N	CCR_{LN}	BCC_{LN}	CCR_N	BCC_N	CCR_{LN}	BCC_{LN}	CCR_N	BCC_N	CCR_{LN}	BCC_{LN}
High	164	1	1	71	84	96	138	2	3	54	3	1	1	111	1
	118	2	2	67	1	1	1	1	2	39	1	1	1	81	1
	152	3	3	131	96	155	157	3	4	3	2	1	1	1	1
	153	4	7	56	74	54	1	19	27	60	17	23	48	97	61
Medium	40	79	48	42	1	1	1	41	44	109	51	25	51	140	34
	101	80	82	88	103	61	1	82	100	144	70	76	96	134	81
	80	81	91	148	148	120	123	56	54	59	115	111	116	56	102
	149	82	89	1	1	1	1	117	82	33	125	103	1	44	157
Low	86	157	154	1	1	114	1	158	159	114	156	157	157	91	158
	143	158	158	130	126	1	1	160	158	96	159	142	149	93	152
	117	159	157	1	1	1	1	145	115	142	160	125	45	46	147
	45	160	160	115	99	116	121	148	60	18	153	144	1	28	1

Note: Farm identification number is from original sample.

Table 1.8: Comparison of technical efficiency rankings

Efficiency rankings correlations

	SDEA CCR_N	BCC_N	CCR_{LN}	BCC_{LN}	DEA average efficiency score CCR_N	BCC_N	CCR_{LN}	BCC_{LN}	SFA FE	FE_{sp}
SDEA										
CCR_N	1.00									
BCC_N	0.85***	1.00								
CCR_{LN}	0.49***	0.43***	1.00							
BCC_{LN}	0.39	0.46	0.62***	1.00						
DEA av.										
CCR_N	0.28***	0.28***	-0.04	0.00	1.00					
BCC_N	0.28***	0.32***	-0.03	0.02	0.85***	1.00				
CCR_{LN}	0.08*	0.05*	0.08*	-0.04	0.30***	0.51***	1.00			
BCC_{LN}	0.23***	0.27***	-0.08**	-0.01	0.84***	0.80***	0.23***	1.00		
SFA										
FE	0.25***	0.24***	-0.02	-0.02	0.82***	0.71***	0.29***	0.85***	1.00	
FE_{sp}	0.21***	0.23***	-0.08***	-0.07**	0.79***	0.62***	0.11***	0.82***	0.89***	1.00

Note: ***,** and * coefficient significance at 1%,5% and 10% level.

Table 1.9: Spearman ranking correlation coefficients and significance levels

Efficiency rankings correlations

	SDEA CCR_N	BCC_N	CCR_{LN}	BCC_{LN}	DEA–mean CCR_N	BCC_N	CCR_{LN}	BCC_{LN}	SFA FE	FE_{sp}
SDEA										
CCR_N	1.00									
BCC_N	0.85***	1.00								
CCR_{LN}	0.49***	0.43***	1.00							
BCC_{LN}	0.39	0.46	0.62***	1.00						
DEA mean										
CCR_N	0.44***	0.41***	0.11***	0.05*	1.00					
BCC_N	0.46***	0.50***	0.15***	0.11***	0.64***	1.00				
CCR_{LN}	0.03	-0.02	0.14***	-0.02	-0.02	0.30***	1.00			
BCC_{LN}	0.29***	0.31***	0.01	0.06*	0.56***	0.76***	0.24***	1.00		
SFA										
FE	0.25***	0.24***	-0.02	-0.02	0.72***	0.55***	0.04	0.54***	1.00	
FE_{sp}	0.21***	0.23	-0.08***	-0.07**	0.71***	0.44***	-0.22***	0.41***	0.89***	1.00

Note: ***,** and * coefficient significance at 1%,5% and 10% level.

Table 1.10: Spearman ranking correlation coefficients and significance levels

Chapter 2

Factors affecting efficiency of West Java rice farms

The main objective of this chapter is to investigate the inverse relationship between farm size and efficiency that has became almost a "stylized fact" in the literature on agricultural development. The recent literature focused on agricultural economics in developing countries [e.g., Binswanger, Deininger, and Feder (1995, Barrett (1996, Towsend, Kirsten, and Vink (1998, Helfand and Levine (2004)] indicate that the size–productivity relation is more complex and caution must be used when advocating policies for agricultural development. This analysis supports the hypothesis that the size–productivity relation is not straightforward negative and for small farms (less than 5 hectares) there exists a threshold size over which efficiency growth is observed with increasing farm size.

Recently, the Data Envelopment Analysis (DEA) studies [Dhungana, Nuthall, and Nartea (2004, Sang and Hyunok (2004, Krasachat (2004, Umetsu, Lekprichkui, and Chakravorty (2003); and Wadud and White (2000b)], with focus on the evaluation of rice farms' efficiency, are motivated by the importance of rice production in the economies of Asian countries. I focus on Indonesian rice production in the West Java area. West Java province is the home of intensification programs and agricultural development institutions in Indonesia and the interest in this area is stressed by the fact that farmers from Java island produced over 60% of Indonesia's total rice output at the time of the survey. Therefore, the aim of this chapter is to evaluate the technical efficiency of rice farms. To do this, the DEA approach is employed for evaluation of technical and scale efficiency of farms.

The analysis of technical and scale efficiency is followed by the analysis of farm characteristics and efficiency score relations. To evaluate these relations, a panel data version of the Tobit model is used. The evaluation of the effect of the farm specific factors on the efficiency scores is focused on the farm size–productivity relation. Also, the effect of the later stage of the Indonesian government intensification program, known as BIMAS, on technical efficiency impact is investigated.

Further, analysis presented in this chapter illustrates how to test hypotheses related to the DEA performance measures using the data set used in the previous chapter and that was the focus of recent studies [Horrace and Schmidt (1996, Druska and Horrace (2004)] on methodological issues related to production frontier estimation. Horrace and Schmidt (1996) compare various stochastic frontier methods (SF) with regard to constructed confidence intervals for performance score estimates and they prefer to use the SF methods for testing hypotheses related to performance scores because the DEA does not provide confidence intervals for performance measures. However, Simar and Wilson (2000) show how a simple underlying model of data generating process defines a statistical model, allowing determination of the statistical properties of the nonparametric estimators in the multi–output and multi–input case.

This chapter is organized as follows. The next section reviews the history of intensification program aims and rice production technology during the "Green Revolution" period. The third section gives a review of DEA methodology used to evaluate farm's efficiency scores and Tobit estimation technique used to estimate the effects of characteristics on the efficiency score. The fourth section presents results from calculation of technical efficiency measures and estimation of its determinants. The last section summarizes the results and their relations to intensification policies.

2.1 Rice farming in Indonesia

The following review is focused on the main objectives of the BIMAS intensification program. Also, in this section factors related to technical inefficiency of rice farming are discussed. In the data subsection, a description of analyzed data is given.

While in the 1960's agriculture contributed 51% to Indonesian GDP and according to Pearson et al. (1991), despite output growth of agricultural productivity the contribution to GDP decreased to 31% by the end of the 1970's and further to 25% by the end the 1980's. Even though this decline of contribution to GDP, the importance of rice for the economy is stressed by the fact that it contributes 50% of Indonesian agriculture production because rice is a staple food. Also, in rice growing areas it is a major source of income for the farmers. Therefore, a critical part of the economy stabilization process are stable and low rice prices that became goals of agriculture intensification programs.

To stabilize rice prices and increase output of domestic rice producers, the Indonesian government heavily supported the rice farming sector by subsidizing inputs for agricultural production and consumer prices of rice were held below world market prices [Erwidodo, Sudaryanto, and Bahri 1999]. Pearson et al. (1991) illustrate this situation by the fact that in the 1970s, the Indonesian rice price averaged 30 % below the world market price. Due to the costs of subsidization and the importance of rice for food supply as well as threat of famine, the Indonesian government claimed self–sufficiency as a national objective.

To meet this long term objective, the Indonesian government has been allocating a sizable amount of its budget to the agricultural sector since the beginning of the 1970s. These funds were used to introduce various intensification programs (e.g., BIMAS, INMAS and IPM) within the last thirty years. The effects of these programs were following typical patterns for the introduction of new technology. The early and late stages showed small productivity growth while the most rapid growth is observed in the middle period. This is due to low implementation of new methods in the early stages and then due to the fact that the productivity limits of the new technology were reached in the later period (e.g., Umetsu, Lekprichkui, and Chakravorty 2003).

Indonesia used to import 25% of all rice traded in the world market in the 1960s and early 1970s, but exported small amounts in the late 1980s. This change, known as the "Green Revolution" is a result of adopting new rice production techniques, modern rice varieties and organizational changes that were introduced as a result of intensification programs. According to Lokollo's (2002) report, in the mid 1980s Indonesia changed its position from a net rice importer to being self–sufficient. Despite this production growth and increase of rice production, the population growth pressure reverted the self–sufficiency trend and in the late 1980s Indonesian production was again not sufficient to meet domestic demand for rice and Indonesia returned to a net importer position.

The first efforts of the Indonesian government to improve rice production technology are dated to the 1950s. These efforts included development of irrigation systems, establishment of "paddy centers" and soil conservation. The growth of rice production until the late 1960s was driven through enlargement of rice production areas by conversion from sugar–growing land while the rice yield stagnated at 2 tons per hectare.

Often by use of force, the new high–yielding rice varieties (HYV), fertilizers and pesticides were introduced into the production process in the beginning of intensification programs. Also, credit programs for farmers forced them to purchase input packages, and they had to take the prescribed package of seeds, fertilizers and pesticides. Inputs for rice production were distributed through the village administration. The village administration forced (by cutting down crop of those who were not growing rice with the assistance of the army) farmers to plant rice instead of growing more profitable crops. Moreover, this administration often decided to spray large areas with pesticides by use of planes.

As Lokollo (2002) or Daryanto, Battese, and Fleming (2002b) review, more farmer friendly intensification programs were introduced later, e.g., BIMAS (seeds and fertilizer, technical know–how, credit and guaranteed markets) and INMAS (extension of BIMAS, subsidized fertilizes and pesticides). In the late 1970s, extensions of the BIMAS program in form of the INSUS [in irrigated areas], and OPSUS [inputs for farms for free according local resource endowment] programs for groups of farmers were introduced. These programs focused on the management of farms and planning. To promote coordination of farmers and to capture economies of scale, another extension of the BIMAS program was introduced in the form of the SUPRA INSUS program in the late 1980s.

In the 1990s Indonesia suffered from a deep political, economic and financial crisis. As Erwidodo, Sudaryanto, and Bahri (1999) review, the Indonesian government was also forced to reform its agricultural policies. This led to agricultural liberalization because the regulatory body (National Logistic Agency, BULOG) was seen as the main source of agricultural distortions. Liberalization included elimination of the state monopoly on agricultural imports, introduction of international and provincial tariffs and the reduction of trade restrictions on a number of agricultural products. Since 1998, the fertilizer distribution monopoly was eliminated and fertilizers are traded at market prices. Further reforms include promotion of adequate incentives to rice farmers, changes in the role of government in marketing and food distribution and further reduction of non–tariff barriers for agricultural markets.

Recently, the main objective has not been to attain zero a import position of rice but to adequately feed the population and reduce poverty. This goal should be achieved by reducing distortions to the farming inputs market that result from heavy subsidization of fertilizer and pesticide. These reforms should be followed by an increase in competition in the agricultural sector, which should promote more efficient use of production factors. Erwidodo, Sudaryanto, and Bahri (1999) conclude that despite the unclear results of the introduced agricultural reforms in the near–term, there remains a potential source of future economic growth.

As it follows from the above intensification program review, the BIMAS program [Bimbingan Masai or "mass guidance" intensification program] was the most important ingredient of the rice development policy in the 1970s and its influence on productivity increase declined in the 1980s after most farmers adopted HYVs and were capable of funding the production inputs from rice farming profits. According to Pearson et al. (1991), in 1969 yield on *sawah* in Java was on average 2.6 tons of rice per hectare, and until 1987 these yields had increased to about 5 tons per hectare.

The most significant factor of this increase in rice productivity in the period in 1970s and 1980s was the spread of high–yield rice varieties. By the mid–1980s, 85% of rice farmers used high yield variety seeds, compared with 50% in 1975. This was a result of the promotion of HYVs together with subsidized fertilizers, pesticides, and credit through the "mass guidance" intensification program. During 1970s, Indonesian farmers increased their consumption of pesticides sevenfold and their consumption of fertilizers fourfold, even though Indonesian farmers used only 20–25% of the amounts used by farmers in Japan, Taiwan or South Korea; see Table 6.6 in Barker, Herdt, and Rose (1985). The later introduced extensions of the BIMAS program continued to offer technical assistance to farmers unfamiliar with the new cultivation techniques.

The general belief of farmers involved in the BIMAS program was that more agrochemical inputs (fertilizers and pesticides) will lead to even higher yields. (Gallagher) explains that the massive use of subsidized pesticides (farmers paid only 10 to 20 % of the world price of pesticides) led to outbreaks in rice production when more than one million of hectares were infested by pests, e.g., insects like brown planthopper. The applied pesticides damaged the rice ecosystems so much that beneficial predators and parasites were destroyed; therefore, migrating pests survived without any mortality and destroyed crops. To help reduce pesticide use, in 1989 the subsidy on pesticides was eliminated. (Gallagher) concludes that since 1989 no outbreaks have occurred and farmers were able to increase yields without increased pesticide use.

The aforementioned problem of heavy pesticide use is only one from a range of socio–economic and demographic factors that determine efficiency of rice farms. Literature on technical efficiency of rice farms [Wadud and White 2000b; Daryanto, Battese, and

Fleming 2002b] lists factors like credit availability, farm size, weather, topography and poor soils as the principal production constraints. Technical factors include irrigation (often not functional in the dry season when the irrigation system is in short supply of water), plot size and land degradation. Especially during the wet season, the quality of roads and communication facilities are constraining the movement of inputs to the paddies that results in crop losses. Also non–physical factors like experience, age, years of schooling, ownership structure and information availability are considered as relevant, e.g., Parikh, Ali, and Shah (1995); Dhungana, Nuthall, and Nartea (2004); Timmer (1971); and Dhungana, Nuthall, and Nartea (2004).

2.1.1 Data description

The data used in this chapter were previously used by Druska and Horrace (2004) and Horrace and Schmidt (1996) in their studies on theoretical developments of methods for stochastic frontier analysis (SFA) and in the previous chapter.

The used panel data come from an individual rice farm survey by the Indonesian Ministry of Agriculture that begun in 1977. These farms were selected from six villages [Wargabinangun, Lanjan, Gunungwangi, Malausma, Sukaambit, Ciwangi] in Cinamuk River Basin area in West Java, and farms were surveyed over six growing periods (three wet and three dry periods). These villages are a sample of heterogenous environment with various altitudes (sea level, central area of West Java and highland) and the villages infrastructure (both in low and highlands, where not all villages are accessible by all–weather local roads).

The sample used for analysis covers 160 farms after I removed outliers (performance outliers and errors in data) according to yield per hectare criterion and comparison of net and gross yield of farms. After this correction, the used data still contains farms with a wide range of characteristics.

Table 2.1 summarizes of descriptive statistics of used inputs and outputs. Land is considered as the most important input, and it is represented as the size of rice farms in hectares. Approximately 90% of farms in the sample are smaller than 2 hectares. As reported by Fredierick and Worden (1992) and Pakpahan (1992), the 1973 and 1983 agricultural census showed that about 44% percent of all farm households were either landless or operated holdings too small (0.5 hectare) to meet more than subsistence requirements. The census shows that average farm size in Java was 0.66 hectare, while in other parts of the archipelago and outer islands the farms were larger and the average size ranged from about 1.33 to 2.71 hectares. At the same time, the average size of rice farms in Thailand was 2.9 hectares and 8.7 hectares in the USA. Ray (1998) summarizes that the low value of per capita land holdings is transformed into the fact that a significant fraction of farms are owner–operated. The other contractual arrangement of land renting in Asia that occurs frequently is sharecropping under which tenants cede to the landlord a prescribed fraction of his crop. Ray (1998) reports that 60 % of tenanted land in Indonesia is tenanted under the sharecropping arrangement. In the analyzed sample, one third of farmers operate at least a part of their land under share tenancy.

Based on previous research on rice farms in Asia [e.g., Erwidodo 1990, Umetsu, Lekprichkui, and Chakravorty 2003 and Krasachat 2004], I use quantity of seeds, urea, triple superphosphate (TSP) and labor to quantify the rest of the inputs that characterize production technology. I abstract from the role of mechanization or use of animals as production inputs because from Barker, Herdt, and Rose's (1985) review of mechanization studies follows that almost no change occurred in cropping intensity after the introduction of tractors for land preparation. Moreover, they report a field experiment which compared alternative land preparation techniques and failed to show any difference in wetland rice yields.

In the sample, the employment of HYVs is still very low but tends to increase over the observed periods. Close to one third of farmers used HYVs in the first observed season, and the use of HYVs is increased to 50% in the last period. According to statistics presented by Lokollo (2002) this reflects the overall process of HYV employment, when in 1974 33% of farmers employed modern rice varieties and employment was increased to 77% of farmers by 1989. The use of the HYVs is one of the rice production growth drivers, when HYVs yielded on average approximately 1.4 times more rice than traditional varieties in the 1970s in Asia.

Total quantity of urea and phosphate are used to measure the amount of fertilizers applied by farmer because the use of fertilizer make a substantial contribution to the rice yield increase. But as Barker, Herdt, and Rose's (1985) estimations of yield response to amount of fertilizer show, this contribution decreases with an increase in the level of applied fertilizer.

Labor includes both family and hired labor in rice production and is measured by man–hours. Labor is used to repair dikes; raise, pull and transplant seedlings; harvest and thresh. The rice production in Indonesia is characteristic by its very high labor intensity and very low level of mechanization; when in this area there was only 1 tractor available per 200 hectares. Therefore, land preparation in wetland cultivation area on Java remains largely unmechanized during the considered period and Pearson et al.'s (1991) estimate based on calculations from survey data place tractor use on about 7% of total cultivated area in 1987. Barker, Herdt, and Rose (1985) reports that in the 1970s innovative farmers on Java used 200–250 days of labor to cultivate 1 hectare of rice. On average, Indonesian farmers in the analyzed sample used 173 man–days per hectare, but this is still three times more than reported for Thailand and Burma (Table 3.5 in Barker, Herdt, and Rose 1985) and approximately two times more than Umetsu, Lekprichkui, and Chakravorty (2003) report for the Philippines. Due to the low employment of mechanization, the considered production mix does not include tractor or animal work.

In this chapter, two definitions of a farm's outputs are used to assess the robustness of the results with respect to production mix specification. In the model, referred to as one–output, a farm's output is described only by the gross observed rice production in kilograms. Due to high labor intensity of rice harvesting farmers, usually hire share-croppers to harvest rice. The harvesting cost is paid in terms of rough rice harvested. Therefore, the gross rice production can be decomposed into net yield and rice used to cover the harvest costs measured in kilograms of rice and the this model is referred to as a two–output model.

In the second stage of analysis, the effect of the type of rice variety together with land status (owner, sharecropper) and type of the BIMAS program participation [non-BIMAS farmer, mixed, BIMAS farmer] is examined. In the analyzed sample, farmers tend to drop out from the program. In the first period 66% of farmers are not taking part in the program while in the last period 87% are not. Further, I also investigate the influence of the price (in Rupiah per kilogram) of seeds, urea and phosphate on the technical efficiency scores because due to low prices farmers tend to overuse cheap inputs. Overuse of inputs may lead to a decrease in productivity rather than to an increase as in the case of pesticide use. In this analysis, the use of chemical protection of plants is measured by pesticide costs (in thousands of Rupiah).[1]

[1]In the late 1970s, 1000 Indonesian Rupiah had a value of approximately 2 USD.

2.2 Methodology

In this chapter, a two–stage procedure is employed to evaluate the effects of rice farm characteristics on the efficiency of production mixes used by farms. In the first stage, the performance of the decision making unit (DMU, farm) is calculated by the non–parametric approach based on Farrell's (1957) measures of efficiency by Farrell (1957) and Farrell and Fieldhouse (1962). This approach to measurement of technical efficiency is one of the most popular approaches in recent performance analysis studies.

In Farrell's (1957) concept, the overall efficiency (OE) is a multiplicative combination of technical (TE) and allocative efficiency (AE), so that OE=TE*AE. Allocative efficiency measures the extent to which an analyzed DMU produces its outputs in a proportion that minimizes costs of production, assuming that the unit is already fully technically efficient. Technical efficiency measures the extent to which inputs are converted to outputs relative to the best practice and does not depend on prices of inputs and outputs as does Hanoch and Rothschild's (1972) non–parametric concept for testing hypotheses about production relations.

In Farrell's (1957) concept, the farmer's decision process may fail in two different ways. Economic theories usually consider the case when the marginal product of some or all factors are not equal to their marginal costs, then the allocative decision is inefficient. The second case considers the failure to produce the maximum possible output from a given mix of inputs and this means that the technical decision is inefficient. In this work, technical efficiency serves as a proxy for overall efficiency because in environment where input and output prices are heavily distorted by various subsidization, schemes allocative efficiency does not work as a good measure of efficiency.

In the first stage of the analysis, the technical efficiency of individual farms is evaluated by the data envelopment approach (DEA). Since the production frontier in the DEA approach is deterministic, the resulting efficiencies contain noise from data. Therefore, in the second stage of this analysis, the features of the operating environment (farm characteristics) are used to explain the computed technical efficiency scores by estimating an efficiency model. As it follows from the DEA efficiency score definition, the DEA score falls between the 0 and 1, making the dependent variable (efficiency score from the first stage of analysis) a limited dependent variable. Therefore, the Tobit model is suggested [e.g., Cooper 1999; Grigorian and Manole 2002] as an appropriate model in the second stage of analysis when considering the effects of farm's characteristics on the a farm's efficiency score.

2.3 Efficiency measurement

The DEA approach introduced in a seminal paper by Charnes, Cooper, and Rhodes (1978) uses linear programming to pursue Farrell's (1957) concept of technical efficiency to evaluate performance. Charnes, Cooper, and Rhodes's (1978) approach deals with multiple inputs and multiple output technology by computing the maximal performance score for each decision making unit relative to all other units in the sample. For each unit, the unit's performance score is calculated by comparing its production mix with an efficient unit (located on the technology frontier) or with convex combination of different efficient units (weighted mix of other decision making units).

The common feature of estimation techniques based on Farrell's (1957) efficiency definition is that the information is extracted from extreme observations in the sense of technical efficiency, to form the best practice production frontier. This makes DEA scores sensitive to errors in data. However, the main advantage of the DEA approach is that it does not require the assumption of a functional form for the specification of the input–output relation.

Technical efficiency is considered in terms of the optimal combination of inputs to achieve a given level of output (an input–orientation) or the optimal output that can be produced given a set of inputs (an output–orientation). This analysis is focused on input–oriented models, where DMU's ability to consume the minimum input given the level of outputs that should be attained is considered. The input orientation is more appropriate in this case because the output level is given by the target of rice production that should attain the self–sufficient level (zero imports). The decision on the orientation of DEA models is also supported by considering the degree of farmer's control over variables in DMU's production mix (rice farm). Rice farmers have more control over their inputs than their outputs. Therefore, as in other agricultural productivity studies [e.g., Wadud and White (2000b, Davidova and Latruffe (2003); and Krasachat (2004)], the input–oriented DEA model is used.

When using the DEA approach, the set of n homogenous farms described by an input vector $x_j = (x_{1j}, \ldots, x_{mj})^T \in \mathbb{R}^m_+$ of m inputs are employed to produce s outputs in amounts described by vector $y_j = (y_{1j}, \ldots, y_{sj})^T \in \mathbb{R}^s_+$.[2] Therefore, data on production

[2] Here, $\mathbb{R}_+$ means the set of positive real numbers and $\mathbf{1}$ is a column vector of ones.

process observations consist of n pairs of input–output vectors $(x_j, y_j) \in \mathbb{R}_+^{m+s}$ and by aggregating these vectors, the following matrix notation is used to describe inputs $X_{m\times n} = (x_1, \ldots, x_n)$ and outputs by matrix $Y_{s\times n} = (y_1, \ldots, y_n)$.

The DEA methodology approach developed by Charnes, Cooper, and Rhodes (1978) and reviewed by Seiford and Thrall (1990) and by Charnes et al. (1994) show that Farrell's (1957) input–oriented efficiency measure for the DMU_j is found as an optimal solution to the following linear programming problem (model):

$$\begin{aligned} \min_{\lambda_j,\theta_j,e_j,s_j} \quad & \theta_j \\ s.t. \quad & X\lambda_j + e_j = \theta_j x_j, \\ & y_j - Y\lambda_j + s_j = 0, \\ & \varphi(\mathbf{1}^T\lambda_j) = \varphi, \\ & \lambda_j, e_j, s_j \geq 0, \end{aligned} \tag{2.1}$$

where $\lambda_j \in \mathbb{R}_+^n$; $\theta_j \in \mathbb{R}_+$; $e_j \in \mathbb{R}_+^m$; $s_j \in \mathbb{R}_+^s$ and φ is 0 for the model (CCR model) with constant returns to scale introduced by Charnes, Cooper, and Rhodes (1978) and 1 for the model (BCC model) with variable returns to scale by Banker, Charnes, and Cooper (1984). For the DMU_j the optimal value θ_j^* measures the maximal equi–proportional input reduction without altering the level of outputs. The vector λ_j^* of intensity variables indicates participation of each considered farm in the construction of the virtual reference farm that the DMU_j is compared with.

Problem 2.1 is solved n times to generate the optimal values of the objective function and the elements of intensity variables vector λ for each farm.[3] In the DEA literature [e.g., Charnes et al. 1994; Banker, Charnes, and Cooper 1984], the efficiency of the DMU_j is evaluated using the optimal solution $(\lambda_j^*, \theta_j^*, e_j^*, s_j^*)$ of Problem 2.1 under the assumption of the selected returns to scale (RTS) type according to the following theorem:

Theorem 4. *Efficient DMU_j : The DMU_j is DEA efficient if both of the following conditions are satisfied: 1) $\theta_j^* = 1$; and 2) all values of slacks are zero: $\mathbf{1}^T e_j^* = 0$ and $\mathbf{1}^T s_j^* = 0$. Otherwise the DMU_j is inefficient.*

If the DMU_j is identified as inefficient according to Theorem 4, optimal values of non–proportional slacks e_j^*, s_j^* and the optimal value θ_j^* identify the sources and levels of

[3] For more information on solving DEA models, see chapter "Computational aspects of DEA approach" in Charnes et al. (1994).

present inefficiency and the following input–oriented efficiency measure by Tone (1993) that accounts for the presence of proportional and non–proportional slacks:

$$\chi_j \;=\; \left(\theta_j^* - \frac{\mathbf{1}^T e_j^*}{\mathbf{1}^T x_j}\right) \frac{\mathbf{1}^T y_j}{\mathbf{1}^T Y \lambda_j^*.} \tag{2.2}$$

Properties of Tone's (1993) efficiency measure guarantee that this efficiency measure uniquely identifies the efficient DMU$_j$ when $\chi_j = 1$. Further, the properties of χ_j (monotonically increasing in values of inputs and outputs; decreasing in the relative values of the slacks; and units' invariancy) provide rationale for the use of this efficiency measure to create efficiency ranking for the analyzed DMUs.

Solving the CCR version of the problem 2.1 ($\varphi = 0$), the total technical efficiency measure $\phi_j^*(CCR)$ is obtained by comparing of small scale units with large scale units and vice versa without considering the economies of scale. This may be inappropriate for all of the farms in the sample; therefore, the BCC model ($\varphi = 1$ in problem 2.1) that allows for variations in the RTS is considered. The BCC model formulation allows one to calculate the pure technical efficiency $\phi_j^*(BCC)$ and decompose the technical efficiency score into pure technical efficiency and scale efficiency (SE). Evaluation of the scale efficiency measure of the DMU$_j$ assumes calculation of $\phi_j^*(BCC)$ and $\phi_j^*(CCR)$ and the scale efficiency measure is calculated as in the summary of SE calculation methods by Löthgren and Tambour (1996):

$$SE_j \;=\; \frac{\phi_j^*(CCR)}{\phi_j^*(BCC)}. \tag{2.3}$$

The value of the SE measure is interpreted in the following way: if $SE_j = 1$ then the DMU$_j$ is considered as a scale efficient unit and this unit shows constant returns to scale property (CRS); if $SE_j < 1$ then the production mix of the DMU$_j$ is not scale efficient.

Scale inefficiencies arise because of the presence of either decreasing (DRS) or increasing (IRS) returns to scale. As largely outlined in the DEA literature [e.g. Färe and Grosskopf 1994; Zhu and Shen 1995; and Löthgren and Tambour 1996], returns to scale characterize locally the production frontier so that they can be solely computed with respect to originally efficient DMUs or projections (equi–proportional inputs reduction) of inefficient DMUs belonging to the production possibility set.

Following the Löthgren and Tambour's (1996) review of identification of the RTS type procedures, the method of the sum of the intensity variables is employed. This method originates from Banker, Charnes, and Cooper's (1984) analysis of the CCR model by Charnes, Cooper, and Rhodes (1978). The ability to determine the RTS type of the DMU by Banker, Charnes, and Cooper's (1984) method was later questioned by Färe and Grosskopf (1994) and an improved method of sum of the intensity variables is given, as in Zhu and Shen (1995), by the following theorem:

Theorem 5. *Sum of intensity variables method: For the specific* DMU_j, *let us define* $SE_j = \frac{\theta_j^*(CRS)}{\theta_j^*(VRS)}$. *We have* $SE_j = 1$ *iff the* DMU_j *exhibits CRS; otherwise if* $SE_j < 1$, *then* $\sum \lambda_j^* < 1$ *iff the* DMU_j *exhibits IRS;* $\sum \lambda_j^* > 1$ *iff the* DMU_j *exhibits DRS.*

An important part of the DEA is the analysis of efficiency score sensitivity with respect to model specifications. In this chapter, the comparison of the stochastic frontier method with the DEA and the stochastic DEA approach presented in the previous chapter is utilized. For analysis of efficiency determinants, the additive formulation of production function is used because this formulation (piecewise linear envelopment surface) is more consistent (in terms of rank correlation) with stochastic frontier analysis than the model with multiplicative formulation (piecewise Cobb–Douglas envelopment surface) as shown in the previous chapter. Further, the robustness of calculated efficiency rankings is analyzed with respect to model specification by use of two different output specifications. The consistency of efficiency ranking is evaluated by using a rank correlation coefficient by Spearman (1904) and the hypothesis of rank independence is tested. Spearman's (1904) rank correlation coefficient is used because its important feature is lower sensitivity to extreme values when compared with the standard correlation coefficient.[4]

2.4 Tobit model

The goal of the second stage is to explore relationships between the technical efficiency measure and other relevant variables such as size, rice variety used, BIMAS participation or intensity of factor employment. Some of the considered factors are neither inputs or outputs of the production process, but rather circumstances faced by decision makers, e.g., wet growing period, prices of inputs or location of paddy.

The used two stage procedure originates from Timmer's (1971) idea for the explanation of aggregated (at state level) technical efficiency of individual farmers. Kumar

[4] For implementation details of Spearman's (1904) rank correlation coefficient, see Stata Corporation (2003).

and Russell (2002) used this procedure to regress the change in efficiency against the output per worker to show that output per worker is positively related with the change in the technology index constructed by using the DEA. Further, Cooper (1999) argues that the second stage regression is useful for checking the consistency of the DEA results and identification of explanatory variables. Moreover, as Fried, Schmidt, and Yaisawarng (1999) summarize, an advantage of the two-stage approach is that the influence of the external variables on the production process can be tested in terms of both sign and significance. However, they point out that the disadvantage is that the second stage regression ignores the information contained in the slacks and surpluses and this may bias the parameter estimates and give misleading conclusions regarding the impact of each external variable on efficiency. Therefore, they proposed a four stage process to correct the measure of technical efficiency for the presence of slacks. Fried et al. (2002) present an improved version of Fried, Schmidt, and Yaisawarng's (1999) technique for incorporating environmental effects and statistical noise into a producer performance evaluation based on data envelopment analysis (DEA) where the slacks are decomposed to a part attributable to environmental effects, a part attributable to managerial inefficiency and to a part attributable to statistical noise.

Let us assume that the efficiency of farms could be presented, in a simplified setting suggested by many studies [e.g., Parikh, Ali, and Shah 1995; Hallam and Machado 1996; Llewelyn and Williams 1996; Shafiq and Rehman 2000; and Grigorian and Manole 2002] by the following function:

$$\chi_{jt} = E(F_{jt}, P_{jt}, X_t, \epsilon_{jt}),$$

where χ_{jt} is the measure of farm j efficiency in period t, F_{jt} is a vector of farm j specific variables, P_{jt} is a vector of economic factors, X_t is a vector of period t external factors that are likely to affect the efficiency of farm j; β_j is a vector of parameters to be estimated and ϵ_j is the part attributable to statistical noise.

The DEA approach provides efficiency measure χ_{jt} with distribution bounded between 1 and 0. Alternatively, the efficiency scores are censored at 0.9 when assuming that there is not too much difference between fully efficient farms and over 90% efficient farms. In this case the ordinary least squares method can not be applied because the expected errors will not equal zero, and so standard regression will provide a biased estimate. Therefore, the limited dependent variable approach is preferred and the Tobit model is applied.

Following Kmenta (1990) and Wooldridge (2002), the model can be written in following way:

$$\chi^*_{jt} \quad = \quad \alpha^T F + \beta^T P + \gamma^T X + \varepsilon_{jt}, \tag{2.4}$$

where χ^*_{jt} is a latent variable that refers to the technical efficiency of rice farms and x are explanatory variables. However, due to nature of the efficiency measure, the following is observed:

$$\begin{aligned} \chi_{jt} &= 0 && \text{if} \quad \chi_{jt} \leq 0 \\ \chi_{jt} &= \chi^*_{jt} && \text{if} \quad 0 < \chi_{jt} < 1 \\ \chi_{jt} &= 1 && \text{if} \quad 1 \leq \chi_{jt}. \end{aligned} \tag{2.5}$$

To estimate the effects of farm characteristics on the technical efficiency score, the Tobit and random–effect Tobit models are used. The random–effect Tobit model captures individual–specific effects, assuming no correlation between the individual–specific effects and explanatory variables. The random–effect Tobit model for efficiency scores is considered in the following form:

$$\chi^*_{jt} = \alpha^T F + \beta^T P + \gamma^T X + \nu_j + \epsilon_{jt}$$

assuming that χ_{jt} is censored at 0 and 1 (0.9 respectively). In here random–effects, ν_j, are iid $N(0, \sigma^2_\nu)$ and ϵ_{jt} are iid $N(0, \sigma^2_\epsilon)$ independently of ν_j. Assessed models are estimated using the maximum likelihood estimation procedures implemented in STATA.

Here, the fixed–effect Tobit model is not used to model the efficiency score, as there does not exist a sufficient statistic that allows the fixed–effect to be conditioned out of the likelihood. Unconditional fixed–effect Tobit models may be fitted by using the Tobit model with an individual indicator. However, these estimates are biased. According to Greene (2004), the variance estimator (crucial parameter for inference and analysis purposes) in the Tobit model is affected specially in samples with a small number of time periods observed, as in the case of this analysis.

However, it is possible to control for correlation with unobserved heterogeneity because Wooldridge (2002) suggests that in this case one should utilize an assumption presented by Mundlak (1978). Mundlak (1978) assumed that unobserved heterogeneity can be modelled as a function of the means of included regressors. So, the following relation is assumed: $\nu_j = \bar{\alpha}^T \bar{F}_j + \bar{\beta}^T \bar{P}_j + \bar{\gamma}^T \bar{X}_j + \delta_j$. Here, δ_j is assumed to be a part of a farm's unobserved heterogeneity such that it is uncorrelated with regressors F, P, X and $\bar{F}_j, \bar{P}_j, \bar{X}_j$, where $\bar{F}_j, \bar{P}_j, \bar{X}_j$, are vectors of farm j means for individual regressors over the observed growing periods. After, the additional set of mean regressors is included, the efficiency equation can be estimated by the random–effect Tobit approach.

2.5 Technical efficiency

As mentioned in previous sections, the technical efficiency and pure technical efficiency scores are evaluated by use of the input–oriented DEA models via solving Problem 2.1 for two different output specifications under the assumption of a period specific production frontier. The model with the output specified by gross rice production is referred to as the one–output model and the model with harvest cost and net rice used to specify production output is referred to as the two–outputs model. Further, for the two–outputs specification, efficiency scores were calculated under the assumption of the time invariant production frontier (pooled sample, referred to as the pooled DEA).

The DEA estimates of technical efficiency are summarized in Table 2.2. The differences in efficiency score (χ) and technical efficiency score (θ) result from the presence of positive non–proportional slacks (e, s). From comparison of χ and θ values, it can be observed that these non–proportional slacks are less important than equi–proportional reduction of inputs (θ).

From comparison of the reported technical efficiency scores with Krasachat's (2004) results for Thai rice farms, it can be concluded that West Javan and Thai rice farms are operating approximately at the same level of relative efficiency. Krasachat (2004) reports an average technical efficiency score of 0.74 for Thai farms while in the analyzed sample of West Javan, farms the technical efficiency ranges from 0.60 to 0.77 (under the assumption of the time varying production possibility frontier). Also, the technical efficiency scores of West Javan rice farms are lower than technical efficiency scores of rice farm in Bangladesh reported by Wadud and White (2000b), where the average technical efficiency ranges from 0.86 to 0.91 and standard deviation ranges from 0.10 to 0.12.

With awareness of the fact that Llewelyn and Williams (1996) used an output–oriented measure, these results can be liken to results presented in Llewelyn and Williams's (1996) study on multi–product food–crop producing farms (58.1% of their production can be attributed to rice) in East Java during the 1994 growing season. Llewelyn and Williams

(1996) reports farms' technical efficiency in the range from 0.95 to 0.98 with standard deviation ranging from 0.019 to 0.043. Also, the histograms of computed technical efficiency scores plotted in Figure 2.1 and 2.2 illustrate the observed high degree of diversity in farms' performance. In both figures, the typical pattern of the DEA efficiency measures characterized by a peak at one is observed. From a comparison of standard deviation values, it follows that productivity performance of West Java rice farms was much more heterogenous than in other countries at that time and in East Java in early 1990s. Therefore, it is appropriate to conjecture that the low average technical efficiency performance of West Java farms is caused by high heterogeneity of rice farming practices in Indonesia in the late 1970s.

Assessing the scale efficiency results reported in Table 2.2, one can conclude that scale inefficiency is not the major source of Indonesian rice farm inefficiency. The average scale efficiency value of 0.90 is comparable to scale efficiency scores of farms in Thailand [0.96 reported by Krasachat (2004)] and Bangladesh [0.91 reported by Wadud and White (2000b)]. The international comparison of the RTS identification is presented Table 2.3. These results shows that most of the farms in West Java and Bangladesh operate in the production possibility region with decreasing returns to scale property. While in the case of Thailand and East Java, most of the farms are operating in either the constant or increasing returns to scale region of their production possibility set.

From these results it follows that increases in inputs intensity leads to less than a proportional increases in the outputs because farmers were not using the proper mix of inputs that could generate constant or increasing returns to scale of operations. Technical efficiency results suggest that at the time of the survey, it was more beneficial to drive the efficiency improvements through the employment of "best practise" technology than trying to exploit the scale of operations. Because the size of operations considered by government programs, further analysis examines the size of the operations–productivity relation in detail in the following section.

The consistency of DEA results with respect to specification of the input–output relation is evaluated by comparing efficiency rankings. To compare SFA and DEA results, the DEA rank is constructed using the average efficiency score computed over the considered growing periods. Table 2.4 reports rank correlation coefficients for models with a time varying production frontier that ranges from 0.73 to 0.97. Also, high values of ranking correlation coefficients (0.65–0.93) under the assumption of a common frontier for all periods reported in Table 2.5 support the hypothesis of robust input–output specifications. The box plots in Figure 2.3 show development of technical and pure technical efficiency over the observed growing periods. These box plots show that there no significant technological change over the observed periods. This result is also supported by an analysis of the Malmquist productivity index of technological change, where the index of geometric average technology change is 0.978 and the average index of efficiency change is 1.007 (the unity value of index means no change). Further, the DEA rankings are compared with the SFA rankings estimated by Druska and Horrace (2004). According to the literature on parametric and non–parametric methods comparison, e.g., Wadud and White (2000b), a high level of DEA–SFA ranking consistency is observed. Because in each case the majority of the farms are scale inefficient and operating in the decreasing returns to scale region, the following analysis is focused on the efficiency scores obtained from two–output models under variable returns to scale.

2.6 Factors associated with efficiency

Using the efficiency scores from the model with a time varying production frontier and assessing characteristics of inefficient and efficient farms summarized in Table 2.6, it seems that larger farm size, lower usage of fertilizers and higher pesticides costs tend to be associated with the technical efficiency of farms. To provide a closer look on shifts in distribution of efficiency, box–plots in Figure 2.4 illustrate the relation of mean values of efficiency score (under CRS and VRS assumption) according to categories of ownership, variety type and BIMAS participation. Even partial application of high yielding varieties shifts farms towards higher efficiency. Mixing types of land status is reflected in a shift towards less efficiency. This may reflect frictions originating from heterogenous ownership structures of the land. An striking distributional shift occurs when participation in an intensification program with efficiency is considered. The downward shift may be attributed to the fact that farmers were receiving the same package of inputs that were not efficient production mixes for all of them due to the heterogeneity of conditions. Also, participating farmers due to easy availability of inputs [e.g., pesticides] may tend to overuse these inputs.

For a more detailed analysis of factors related to technical efficiency, a Tobit model is used. To do this the efficiency is tracked over time under a time variant and invariant production possibility frontier. In the case of the time varying frontier, the efficiency of farm may not be directly compared with the efficiency of another farm in different time (including itself) because the farm is in each period compared to different "best practice" farms. However, this analysis is beneficial for assessing the relative performance improvements. When a pooled production frontier is used, the efficiency of a farm may be directly compared and tracked over time because the production possibility frontier is constructed by use of the same best performers in all periods. Using this approach, the downward efficiency shift is observed in the case when all DMUs in some period faced an unfavorable production condition, e.g., the third and fourth period in Figure 2.3. To control for these unfavorable conditions, time dummies (t3, t4) are introduced.

In the recent literature on agricultural development [Pearson et al. (1991, Towsend, Kirsten, and Vink (1998, Llewelyn and Williams (1996, Davidova and Latruffe (2003); and Helfand and Levine (2004)], the most common variables used to asses the factors associated with farms' efficiency cover characteristics like farm size, age of farmers, schooling of the farmers and employment level of machinery. The Tobit regression defined by equation 2.4 is estimated for all combinations of frontier types and corrections of efficiency scores (censoring bound).

The factors analyzed can be divided into three groups: farm specific variables (intensity of inputs – labor, fertilizers, seeds and farm size; organizational structure – land status, BIMAS participation, rice variety used), economic factors (prices of some inputs) and environmental factors (wet–dry period, village). Due to the assumption of homogeneity of inputs in all six villages (particulary land quality, sea level), village dummies are include into the models to control for differences across villages.

Table 2.8 reports the results of the Tobit and random–effect Tobit estimations and Table 2.9 reports the results of the random–effect estimation when Mundlak's (1978) correction is applied. In all estimated models, only significant the effect of geographical location is found for Ciwangi village. This reflects the fact that Ciwangi village is located in the center part of West Java island with an average altitude of 375 meters, while the rest of the villages are located along the oast (10–15 meters above sea level) or in the central area of island (600–1000 meters above sea level). The difference between the DEA approach and the stochastic frontier analysis is illustrated by low significance of location effect when DEA is used, while Druska and Horrace (2004) report that SFA scores show significant spatial effect.

All the coefficients related to the intensity of input use per hectare have the expected sign, and high consumption of input per unit of size may indicate wastage of the considered input. Sizes of the effects indicate possible substitutability between labor and biochemical inputs (fertilizers and seeds) when searching for efficiency improvements as mentioned by Barker, Herdt, and Rose (1985) in the chapter on trends in labor use. They also mention that experiments on proper timing and placement of fertilizer suggest that fertilizer inputs can be reduced as much as one third without lowering yields.

As it follows from the estimation results, the effect of the wet season is not clear because several opposing effects occur. It would be natural to expect that a significant positive effect of the wet season is due to water demanding nature of rice. The conjecture is that the positive effect of wet weather is ruled out by the facts that most of the areas lack a reliable transportation system (paved roads) during the wet season and farmers are not capable of delivering proper care to paddies. Also, flooding and lodging can affect yields when severe weather occurs, as mentioned by Pearson et al. (1991).

The prevailing positive but not significant effect of a shift towards land tenancy can be explained by Timmer's (1971) reasoning that ownership status might be associated with the extra effort and motivation of tenant farmers who are attempting to save enough capital to buy their own land. However, Pearson et al. (1991) mention that sharecropping contracts were often arranged so that the benefits of higher returns to land go to owners rather than tenants and this discouraged tenants from increasing their productivity. Also, Umetsu, Lekprichkui, and Chakravorty (2003) and Helfand and Levine (2004) identify a similar negative relationship between landlord share and efficiency; therefore, to assess the effect of land ownership in West Java rice farming, more details on contract arrangement are needed. From the view of principal–agent theories, the trade–off between the insurance and incentive aspects in contracts is the most crucial information. And the simple principal–agent models illustrate how sharecropping arises when landlords are unsure about the true ability and can not observe the productivity of their tenants, as in Ray (1998).

Further, the estimation result suggest that a significant positive performance gain comes from employing modern high–yielding varieties. This result is also supported by the observed rapid and widespread replacement of traditional seed varieties with short-duration HYVs during the period 1969–1980. The use of HYVs has transformed the nature of wetland rice agriculture in Indonesia from one of low yields, nonuse of purchased inputs, and single annual rice crops to one of high yields, high levels of purchased inputs, and multiple rice crops. So, self–sufficiency was attained in the beginning of the 1980s.

As mentioned in the review, the BIMAS program was an important ingredient of rice development policy in the beginning of the 1970s, while its importance declined by the 1980s after most farmers adopted HYVs and were capable of funding inputs from rice profits. The negative effect of BIMAS participation it not so surprising because the intensification programs provided farmers with a technology package that included input recommendations; subsidized credit, fertilizer and pesticides in prescribed composition.[5] Also, this result supports the hypothesis that in the later period of the intensification program the positive effects from introducing HYVs reached their limits. Further, because choice of ownership type, HYV employment and program participation is suspected for possible endogeneity, Table 2.7 reports the results of exogeneity test statistics by Smith and Blundell (1986). In all cases, we accepted exogeneity of explanatory variables.

Assessing the positive coefficients of seed and urea price, it can be concluded that an increase in these factor prices has a significant impact on increasing efficiency, which can support the thesis that the goal of technology improvement is to reduce costly inputs. The negative effect of fertilizer price on farm efficiency (attaining the given yield level) is the result of low fertilizer use. Barker, Herdt, and Rose (1985) document decreasing returns to scale in yield with respect to fertilizer use. Together with the fact that farmers in Indonesia were applying very low levels of fertilizers compared to industrialized countries' farmers [Japan, South Korea], this indicates that the negative effect of reduced fertilizer use prevails over any positive effect originating from more efficient use of fertilizers.

The opposite effect is observed in the case of pesticides costs (thousands of rupiah per hectare) because pesticides are used to prevent losses while the initial application of fertilizers always increases crop yield. Also as mentioned in the section on rice farming, low prices of pesticides lead to overuse, which has negative effects on the yield due to environment degradation. Generalizations about the technical efficiency response to the use of pesticide treatment are difficult to make because of the high number of interacting factors [weather, type of pests, variety resistance].

Farm size in Indonesia has been assessed since the 1960s (Basic Agrarian Law), since this law was imposed, the average farm size has tended to increase. Farm size is an important production factor because it affects the way of farming. Farm size in Java was much smaller (on average 0.439 hectare in the analyzed sample) than on the outer islands. Pakpahan (1992) reports, using the Agricultural census that the average size of land holding was 1.77 ha in 1973 and 1.78 ha in 1983. This difference provides rationale for the limits imposed by Basic Agrarian Law, which sets the minimum and maximum

[5] For more details on this intensification package contents, see e.g., Pearson et al. (1991, Barker, Herdt, and Rose (1985); and Lokollo (2002).

size of 2 and 20 ha, respectively.

Because of the focus on the relation of farm size to efficiency, the quadratic term was added, as in Wadud and White (2000b), to capture non-linearities that were usually not explored in works that identified a negative relationship between farm size and productivity. The negative effect of size on productivity is consistent with the fact that land is considered as an input, and with empirical findings for Asian countries summarized by Ray (1998). Assessing the positive sign for the quadratic term (Size^2), it can be concluded that there exists a threshold size and farms larger than this threshold show a positive relationship between farm size and productivity. These thresholds are calculated using calculus and for a time varying frontier range 1.26–1.44 ha, 1.71–1.88 ha when Mundlak's correction is used, and the average threshold size is 1.60 ha. For the time invariant frontier, the average threshold size is 1.67 ha, while thresholds range from 1.45 to 1.62 ha and 1.68–1.94 ha for estimations with Mundlak's correction. The computed threshold sizes are very similar to the size of rice farms in other parts of Indonesia (outer islands) or East Asia and this result can be used to advocate the intensification programs and legal restrictions with aims to increase the size of rice farms.

Further, these results coincide with Wadud and White's (2000b) findings that, on average, farmers with lower land fragmentation (greater plot size) more likely have the opportunity to apply new technologies such as tractors or irrigation, resulting in the higher efficiency of their farms. Also, Pearson et al. (1991) and Ray (1998) note that especially the small size of plots and the impracticality of using tractors in hilly areas, are the main constraints on mechanization of land preparation. Under the objective of increasing farm size even pooling of smaller farms may be beneficial because with an increase in farm size, employment of mechanization will allow an increased production of rice and small landowners would lend their plots to larger landowners because the returns from land renting will increase. However, constraints on greater tractor use (especially, on the outer islands) are probably more varied due to topographic limitations and greater difficulty in obtaining and servicing tractors.

Analyzing the time evolution of efficiency scores summarized in Table 2.8, the sign of the estimated coefficient indicates that the relative technical efficiency was only slightly increasing during the end of the 1970s–beginning of the 1980s. When the time evolution of efficiency scores under time varying frontier is considered this observation indicates that adoption of efficient techniques is not the major factor for increase in farms's efficiency and it supports the view that the increase in rice production was driven by expansion of the cultivated area. Assessing these results, it is observed that there exist periods where the significant decrease in efficiency is observed which suggests that positive productivity effects of the green revolution were not fully realized for some years after initial increase in productivity. These results are consistent with other studies of technological change in less developed countries that indicated declining agricultural productivity. For example, Fulginiti and Perrin (1997) confirmed findings that on average, agricultural productivity have declined in these countries, especially during 1961–1973, but also during 1974–1985. His findings reveal that the declining productivity during 1974–1985 period characterized even those countries such as Pakistan and the Philippines, where green–revolution varieties of wheat and rice became widely adopted since the 1960s.

Finally, the estimations results reveal consistently significant positive relationship between the share of family labor and efficiency measure in all estimated models. As found by Dhungana, Nuthall, and Nartea (2004) this tend to negate the belief that farmers in developing countries are operating inefficiently due to excessive use of family labor. As it was mentioned in the data description section, the timing for delivering the proper care to rice plants matters. Therefore, the positive relation between share of family labor and efficiency may be explained as the result of seasonal labor scarcity when the farmers with larger families are able to deliver their family labor at the time when the demand for labor culminates.

Ray (1998) argues that in the world with unemployment that for somebody who hires labor the opportunity costs of additional unit of labor are still at market wage rate, while for family labor the opportunity costs are lower because of possibility of unemployment. He argues that this lead to higher employment of family labor by farmers with small size plot. Therefore, the observed positive relation of share of family labor to efficiency is not surprising and due to the substitutability of inputs the small size farmers deliver more care to the plants are able to increase the efficiency of other production factors without increasing the intensity of use of these factors.

2.7 Conclusion

In this chapter, I analyze performance of West Java rice farms during the late periods [end of 1970's – beginning of 1980's] of intensification program known as BIMAS. The applied non–parametric approach is more suitable to analyze production processes in developing countries where the availability of data is limited and production technologies are less understood. The analysis of technical efficiency scores reveals that farmers could benefit from adoption of the best practice methods of production because the results indicate a wide differences in efficiency across farms. On average, the analyzed farms were relatively inefficient with potential for reducing their inputs from 23 to 42 % to grow the same amount of rice. Decomposing the technical efficiency into pure technical efficiency and scale efficiency it can be concluded that the majority of farms operate at or close to full scale efficiency. So, farmers that are operating technically inefficiently are doing so because of employment of technically inefficient production mixes rather than the size of their operations. Further, up to 77% of scale inefficient farms shows decreasing returns to scale.

The second stage analysis of the factors associated with observed technical efficiency score indicates what aspects of the considered rice farms could be targeted in order to improve farm efficiency. The employment of modern varieties had a positive and significant effect on the rice farms performance but the time pattern of productivity suggest that during the considered period the yield potential of introduced modern varieties was exhausted.

The surprising result is that the participation in intensification program did not provided significantly positive effects on employment of the best practice farming technologies. Similarly as in Daryanto, Battese, and Fleming (2002b), the predominance of negative relationships between technical efficiency and participation in intensification program suggest that the program has often failed to increase the technical efficiency of rice farms in West Java. The main assumption of the intensification program (BIMAS) approach was that small scale farmer productivity could be raised if they had better access to certain inputs and used them according to a set of prescribed instructions but the factors which affects the decision on factors intensities differs significantly among farmers. To be successful, future intensification programs should recognize these differences and be personalized to accommodate them. For personalization the detailed data on farmer characteristics (education, age and family size of farmers); infrastructure of villages (irrigation, types of roads); and mechanization used (water pumps, tractors or buffalos) should be analyzed for effects on technical efficiency.

The main result of the size–efficiency relation analysis suggests that it is misleading to generalize the inverse relationship between farm size and productivity as it is noted in recent agricultural studies, e.g. Towsend, Kirsten, and Vink (1998) and Helfand and Levine (2004). The non–linearity in this relation is identified and it allows for calculation of threshold size over which the size–efficiency relation turns to be positive. The calculated threshold size coincides with average sizes of rice farms on the other Indonesian islands and in other Asian countries. Assessing this fact, the increase in farms size (pooling plots) looks beneficial for further increase in production of rice. Also, when the plot sizes will be increased the production of rice can be mechanized and this can induce further growth of rice production. When farm size increase is considered, policy makers should be aware of decreasing returns to scale because for the majority of the West Java farms the increase in farms size without change in the relative input levels will lead to the decrease in the technical efficiency. Therefore, the assessment of yields increase to attain self–sufficiency in rice production should distinguish between enlarging farm size, and the efforts to increase technical efficiency of the small size farms.

A suggestion that can be drawn from the presented analysis is that the future intensification programs have to take into account the capacity of farmers for applying the available technology more efficiently. Therefore, the policies aimed to spread the efficient technology should improve the access to personalized intensification programs, or by increasing the educational levels of farmers, as many studies on farming performance suggest, e.g. Llewelyn and Williams (1996), Dawson and Lingard (1991) and Dhungana, Nuthall, and Nartea (2004).

2.A Figures and Tables

Variables	Farms	Periods	Mean	Std. Dev.	Min	Max
Inputs						
Land (hectares)	160	6	0.439	0.560	0.014	5.322
Seed (kg)	160	6	18.470	46.681	1.000	1250.000
Urea (kg)	160	6	96.525	130.393	1.000	1250.000
Phosphate (kg)	160	6	33.807	48.348	0.000	700.000
Labor (hours)	160	6	394.224	496.016	17.000	4774.000
Outputs						
Gross yield (kg)	160	6	1414.205	1966.252	42.000	20960.000
Net Yield (kg)	160	6	1248.825	1675.924	42.000	17610.000
Harvest costs (kg)	160	6	165.380	302.433	0.000	3350.000

Table 2.1: Input–Output summary

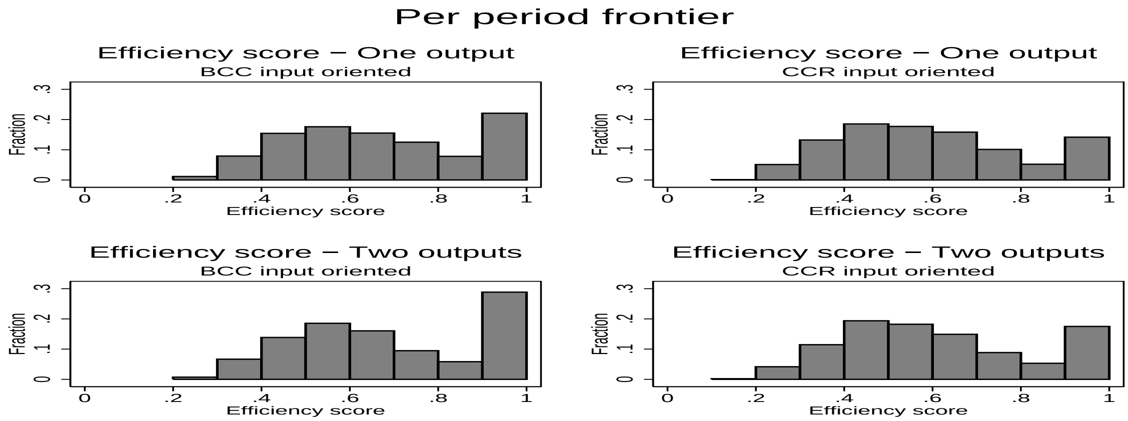

Figure 2.1: Histograms of efficiency scores (χ_j)

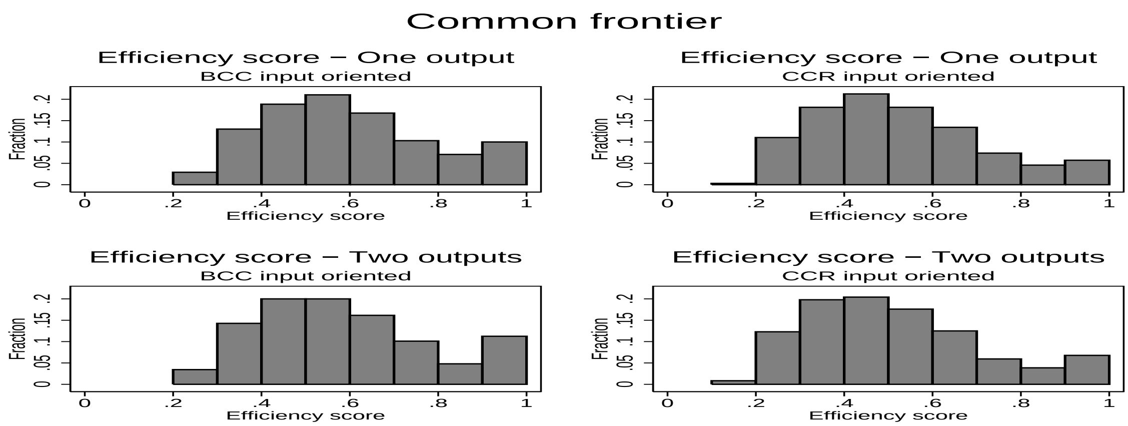

Figure 2.2: Histograms of efficiency scores (χ_j) for pooled sample

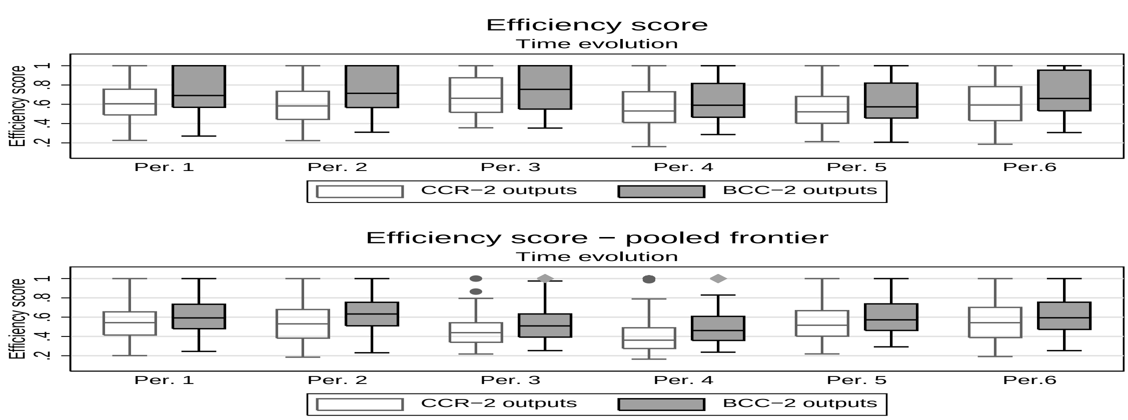

Figure 2.3: Mean efficiency score over time

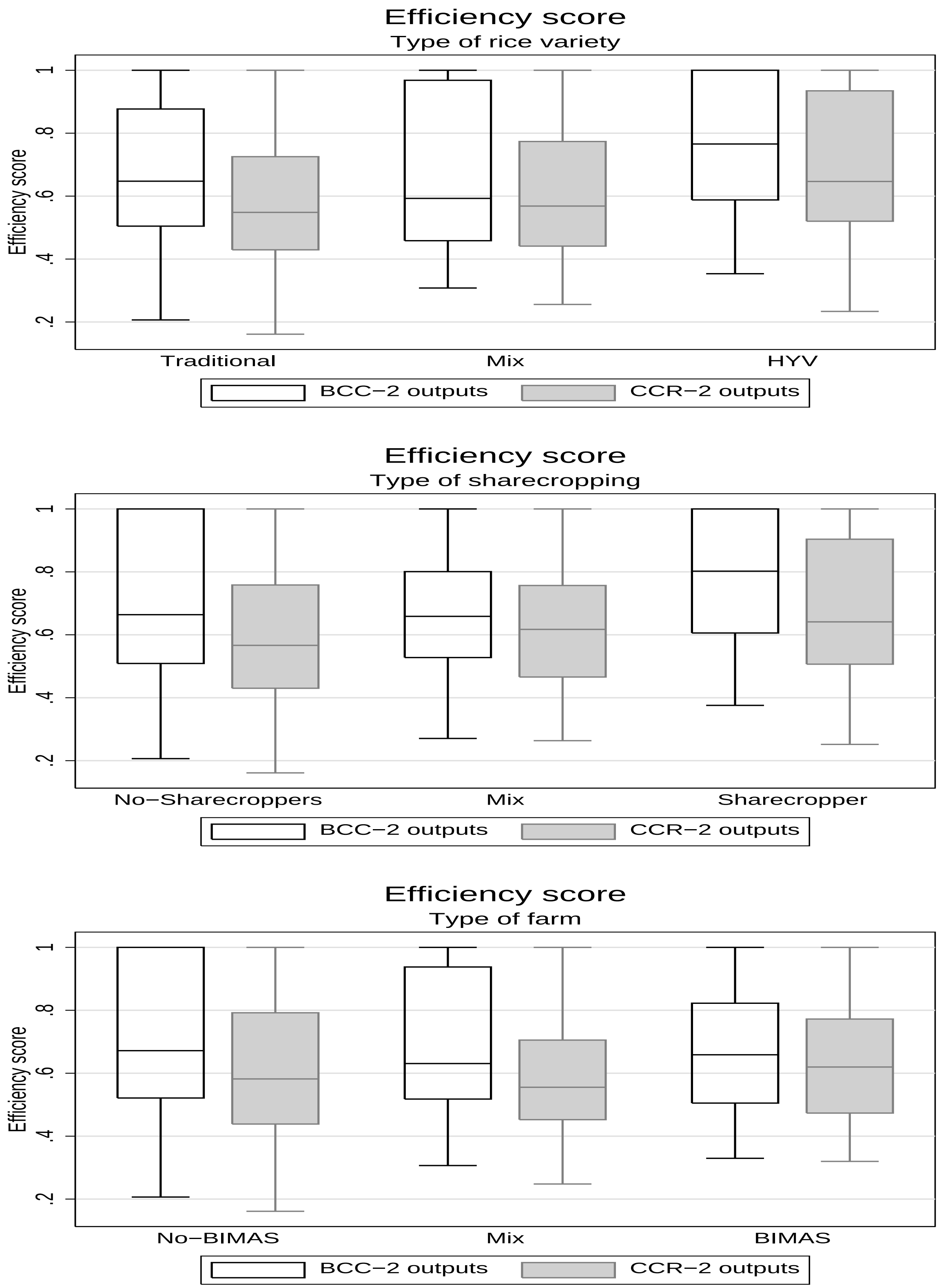

Figure 2.4: Efficiency scores by farm characteristics

Model		Obs.	Mean	Std.Dev.	Min	Max
One–output	χ–CCR	960	0.6016	0.2158	0.1869	1
	θ–CCR	960	0.6750	0.1956	0.2553	1
	χ–BCC	960	0.6777	0.2149	0.2056	1
	θ–BCC	960	0.7457	0.1922	0.3227	1
	Scale efficiency	960	0.9074	0.1190	0.4029	1
Two–outputs						
	χ–CCR	960	0.6199	0.2221	0.1612	1
	θ–CCR	960	0.7069	0.1942	0.2795	1
	χ–BCC	960	0.7016	0.2216	0.2065	1
	θ–BCC	960	0.7757	0.1884	0.3294	1
	Scale efficiency	960	0.9126	0.1123	0.4493	1
Two–outputs – pooled frontier						
	χ–CCR	960	0.5155	0.2024	0.1647	1
	θ–CCR	960	0.5866	0.1948	0.2116	1
	χ–BCC	960	0.5913	0.2012	0.2309	1
	θ–BCC	960	0.6533	0.1988	0.2591	1
	Scale efficiency	960	0.9003	0.1183	0.3618	1

Table 2.2: Efficiency scores (χ) and technical efficiency (θ) summary statistics

Model	DRS	CRS	IRS
One–output	66%	12%	22%
Two–outputs	62%	16%	22%
Two–outputs – pooled frontier	77%	5%	18%
Thailand*	19%	32%	49%
Bangladesh**	63%	16%	21%

* From Krasachat (2004), ** From Wadud and White (2000b)

Table 2.3: Returns to scale summary

Rankings	One–output		Two–outputs		SFA
	CCR	BCC	CCR	BCC	
One–output					
CCR	1.0000				
BCC	0.7377	1.0000			
Two–outputs					
CCR	0.9714	0.7318	1.0000		
BCC	0.7520	0.9726	0.7632	1.0000	
SFA	0.8521	0.6080	0.8248	0.6114	1.0000

Note: In all cases the hypothesis of rank independence was rejected at the 1% significance level.

Table 2.4: Spearman rank correlation coefficients

Rankings	Two–outputs CCR	Two–outputs BCC	Two–outputs – pooled CCR	Two–outputs – pooled BCC	SFA
Two–outputs					
CCR	1.0000				
BCC	0.7377	1.0000			
Two–outputs – pooled frontier					
CCR	0.9342	0.6195	1.0000		
BCC	0.7736	0.9235	0.7300	1.0000	
SFA	0.8521	0.6080	0.8248	0.6114	1.0000

Note: In all cases the hypothesis of rank independence was rejected at the 1% significance level.

Table 2.5: Spearman rank correlation coefficients

Inefficient production mixes					
Variable	Obs	Mean	Std. Dev.	Min	Max
Size	711	0.3977	0.4029	0.0360	3.6430
Land status	711	1.3713	0.6097	1	3
Variety	711	1.5218	0.8503	1	3
BIMAS	711	1.3417	0.6301	1	3
Seed per ha	711	43.5229	38.9072	13.0841	857.1429
Urea per ha	711	237.8890	107.3938	6.9930	712.2507
Phosphate per ha	711	98.1660	70.1368	0.0000	418.9944
Labor per ha	711	1060.4180	463.1572	314.0625	3414.6340
Family labor ratio	711	0.5122	0.2701	0.0006	1.0000
Yield per ha	711	3048.3050	1064.2220	630.6667	6305.7320
Pesticides costs	711	459.2194	1755.3570	0.0000	24000
Efficient production mixes					
Variable	Obs	Mean	Std. Dev.	Min	Max
Size	249	0.5599	0.8551	0.0140	5.3220
Land status	249	1.3574	0.6874	1	3
Variety	249	1.8313	0.9649	1	3
BIMAS	249	1.2610	0.5536	1	3
Seed per ha	249	43.6059	33.9238	4	350.1401
Urea per ha	249	206.9264	131.4522	0.8748	682.7586
Phosphate per ha	249	70.0780	76.5883	0.0000	375.9398
Labor per ha	249	990.7551	516.3687	108.0000	2966.6670
Family labor ratio	249	0.5854	0.3193	0.0002	1.0000
Yield per ha	249	3884.5560	1467.2710	400.0000	7910.3450
Pesticides costs	249	1017.4500	5113.0330	0.0000	62600

Table 2.6: Efficient vs. inefficient production mixes

Model	variable	Test stat.	P-value	exogeneity
Probit	variety	0.1765	0.6744	accepted
	land status	1.0751	0.2998	accepted
	BIMAS	1.0573	0.3038	accepted
Tobit	variety	1.4556	0.2279	accepted
	land status	0.8322	0.3619	accepted
	BIMAS	2.4549	0.1175	accepted

Table 2.7: Smith-Blundell test of exogeneity for time invariant frontier

Variable	Tobit corrected	Tobit original	Panel data Tobit corrected	Panel data Tobit original	Tobit – pooled corrected	Tobit – pooled original	Panel data Tobit –pooled corrected	Panel data Tobit –pooled original
Land status	0.01485	0.00921	0.0206	0.01608	0.01244	0.01196	0.01412	0.01365
	[0.01241]	[0.01422]	[0.01339]	[0.01534]	[0.00928]	[0.01015]	[0.00997]	[0.01088]
Variety type	0.04907***	0.05383***	0.04961***	0.05385***	0.04119***	0.04396***	0.04133***	0.04390***
	[0.01376]	[0.01586]	[0.01357]	[0.01563]	[0.01044]	[0.01141]	[0.01030]	[0.01128]
BIMAS	-0.03128**	-0.03658**	-0.02545*	-0.03085*	-0.02738***	-0.03247***	-0.02453**	-0.02984**
	[0.01353]	[0.01558]	[0.01432]	[0.01647]	[0.01032]	[0.01130]	[0.01088]	[0.01190]
Wet period	-0.0214	-0.01345	-0.02154	-0.01315	0.00692	0.00714	0.00619	0.00627
	[0.02026]	[0.02334]	[0.01907]	[0.02201]	[0.01541]	[0.01685]	[0.01458]	[0.01600]
Size	-0.19627***	-0.20257***	-0.18978***	-0.19922***	-0.14774***	-0.14682***	-0.14945***	-0.15032***
	[0.04573]	[0.05246]	[0.04893]	[0.05600]	[0.03248]	[0.03497]	[0.03421]	[0.03682]
$Size^2$	0.07438***	0.08065***	0.06603***	0.07244***	0.04858***	0.05063***	0.04611***	0.04854***
	[0.01449]	[0.01650]	[0.01506]	[0.01710]	[0.00931]	[0.00986]	[0.00947]	[0.01005]
Fam. lab/Tot. lab.	0.14400***	0.17678***	0.14518***	0.18010***	0.08898***	0.09789***	0.08333***	0.09287***
	[0.03278]	[0.03769]	[0.03505]	[0.04028]	[0.02466]	[0.02694]	[0.02630]	[0.02868]
Seed per ha.	-0.0003	-0.0003	-0.00037*	-0.00038*	-0.00033**	-0.00035**	-0.00036**	-0.00038**
	[0.00020]	[0.00023]	[0.00019]	[0.00022]	[0.00015]	[0.00017]	[0.00015]	[0.00016]
Urea per ha.	-0.00024***	-0.00027***	-0.00031***	-0.00034***	-0.00027***	-0.00029***	-0.00031***	-0.00033***
	[0.00008]	[0.00009]	[0.00008]	[0.00009]	[0.00006]	[0.00006]	[0.00006]	[0.00007]
Phosphate per ha.	-0.00037***	-0.00045***	-0.00027**	-0.00034**	-0.00023**	-0.00025**	-0.00019*	-0.00021*
	[0.00013]	[0.00015]	[0.00013]	[0.00015]	[0.00010]	[0.00011]	[0.00010]	[0.00011]
Labor per ha.	-0.00009***	-0.00009***	-0.00009***	-0.00009***	-0.00009***	-0.00009***	-0.00009***	-0.00009***
	[0.00002]	[0.00002]	[0.00002]	[0.00002]	[0.00001]	[0.00001]	[0.00001]	[0.00001]
Phosphate price	-0.01215***	-0.01411***	-0.01216***	-0.01429***	-0.01151***	-0.01251***	-0.01151***	-0.01262***
	[0.00316]	[0.00366]	[0.00312]	[0.00361]	[0.00244]	[0.00268]	[0.00241]	[0.00264]
Seed price	-0.00004	-0.00009	0.00005	0.00001	-0.00009	-0.00015	-0.00005	-0.00011
	[0.00020]	[0.00023]	[0.00020]	[0.00023]	[0.00015]	[0.00017]	[0.00015]	[0.00017]
Urea price	0.00740**	0.00844**	0.00800**	0.00933**	0.00565**	0.00607**	0.00616**	0.00672**
	[0.00329]	[0.00380]	[0.00324]	[0.00375]	[0.00254]	[0.00278]	[0.00250]	[0.00275]
Pesticide cost	0.00520***	0.00588***	0.00462**	0.00524**	0.00510***	0.00588***	0.00511***	0.00595***
	[0.00195]	[0.00225]	[0.00189]	[0.00218]	[0.00149]	[0.00163]	[0.00144]	[0.00158]
v2dum	0.00671	-0.00911	0.00724	-0.00703	0.01767	0.02275	0.01808	0.02348
	[0.03278]	[0.03765]	[0.04147]	[0.04743]	[0.02450]	[0.02671]	[0.03059]	[0.03299]
v3dum	-0.01483	-0.03266	-0.02021	-0.03891	-0.02377	-0.03058	-0.02591	-0.03337
	[0.03751]	[0.04322]	[0.04399]	[0.05047]	[0.02835]	[0.03093]	[0.03284]	[0.03555]
v4dum	-0.0203	-0.04288	-0.03115	-0.05552	-0.00408	-0.00677	-0.00915	-0.01296
	[0.04141]	[0.04773]	[0.04786]	[0.05496]	[0.03134]	[0.03426]	[0.03580]	[0.03885]
v5dum	0.03985	0.02376	0.02921	0.01116	0.02182	0.01874	0.01631	0.01209
	[0.03825]	[0.04397]	[0.04621]	[0.05290]	[0.02873]	[0.03140]	[0.03425]	[0.03709]
v6dum	0.09297**	0.08592*	0.08536*	0.07728	0.08166***	0.08298**	0.07666**	0.07729**
	[0.04097]	[0.04713]	[0.04709]	[0.05398]	[0.03088]	[0.03373]	[0.03512]	[0.03809]
t	0.00114	0.00411	-0.00216	0.00027	0.02031***	0.02349***	0.01823**	0.02128***
	[0.01001]	[0.01152]	[0.00982]	[0.01131]	[0.00758]	[0.00828]	[0.00745]	[0.00817]
t3	-0.01239	-0.01968	-0.00383	-0.00962	-0.18757***	-0.20436***	-0.17964***	-0.19600***
	[0.03612]	[0.04160]	[0.03493]	[0.04029]	[0.02722]	[0.02977]	[0.02643]	[0.02900]
t4	-0.14961***	-0.16709***	-0.13720***	-0.15251***	-0.22122***	-0.23597***	-0.21271***	-0.22723***
	[0.03514]	[0.04045]	[0.03399]	[0.03916]	[0.02660]	[0.02911]	[0.02582]	[0.02834]
Constant	1.22415***	1.33253***	1.17699***	1.27925***	1.17662***	1.23888***	1.14502***	1.20686***
	[0.14783]	[0.17047]	[0.14764]	[0.17023]	[0.11183]	[0.12237]	[0.11178]	[0.12248]
se	0.21706***	0.25190***			0.16970***	0.18640***		
	[0.00632]	[0.00717]			[0.00424]	[0.00461]		
σ_u			0.08417***	0.09557***			0.06078***	0.06418***
			[0.01039]	[0.01200]			[0.00770]	[0.00853]
σ_e			0.20036***	0.23325***			0.15829***	0.17480***
			[0.00627]	[0.00716]			[0.00431]	[0.00472]
Observations	960	960	960	960	960	960	960	960
Llikelihood	-175.25	-268.49	-159.26	-253.49	179.73	110.7	193.92	123.01
Censored	277	249	277	249	108	93	108	93

Standard errors in brackets, significant at 10%; ** significant at 5%; *** significant at 1%

Table 2.8: Tobit regression results

	Time varying frontier		Time varying frontier		Pooled frontier		Pooled frontier	
Variable	corrected	original	corrected	original	corrected	original	corrected	original
Land status	0.04597***	0.04689**	0.03881**	0.03915**	0.02254*	0.02289	0.02066	0.02115
	[0.01702]	[0.01953]	[0.01683]	[0.01936]	[0.01315]	[0.01446]	[0.01276]	[0.01404]
Variety type	0.04703***	0.04946***	0.04806***	0.05100***	0.03323***	0.03482***	0.04052***	0.04246***
	[0.01470]	[0.01689]	[0.01453]	[0.01673]	[0.01142]	[0.01253]	[0.01108]	[0.01217]
BIMAS	-0.00766	-0.01349	-0.0145	-0.02067	-0.01526	-0.0207	-0.01895	-0.02427*
	[0.01736]	[0.01997]	[0.01713]	[0.01976]	[0.01359]	[0.01492]	[0.01317]	[0.01448]
Wet period	0.0144	0.0255	-0.01837	-0.00848	-0.01803	-0.02103	0.00713	0.00757
	[0.01559]	[0.01793]	[0.01946]	[0.02246]	[0.01209]	[0.01329]	[0.01492]	[0.01640]
Size	-0.20886***	-0.23580***	-0.22449***	-0.25330***	-0.11080**	-0.11658**	-0.17450***	-0.18404***
	[0.06116]	[0.07004]	[0.06143]	[0.07049]	[0.04362]	[0.04729]	[0.04341]	[0.04704]
Size^2	0.05663***	0.06495***	0.05921***	0.06792***	0.03213***	0.03477***	0.04506***	0.04845***
	[0.01755]	[0.01996]	[0.01748]	[0.01992]	[0.01104]	[0.01183]	[0.01103]	[0.01179]
Fam. lab/Tot. lab.	0.17658***	0.22079***	0.15409***	0.19654***	0.08730**	0.09937***	0.07259**	0.08430**
	[0.04452]	[0.05119]	[0.04408]	[0.05078]	[0.03442]	[0.03779]	[0.03345]	[0.03676]
Seed per ha.	-0.00048**	-0.00050**	-0.00049**	-0.00052**	-0.00043***	-0.00047***	-0.00040***	-0.00044***
	[0.00021]	[0.00024]	[0.00020]	[0.00023]	[0.00016]	[0.00017]	[0.00015]	[0.00017]
Urea per ha.	-0.00044***	-0.00050***	-0.00043***	-0.00049***	-0.00040***	-0.00043***	-0.00039***	-0.00042***
	[0.00009]	[0.00010]	[0.00009]	[0.00010]	[0.00007]	[0.00008]	[0.00007]	[0.00007]
Phosphate per ha.	-0.00005	-0.00009	-0.00013	-0.00018	0.00002	0.00002	-0.00012	-0.00014
	[0.00014]	[0.00017]	[0.00014]	[0.00017]	[0.00011]	[0.00012]	[0.00011]	[0.00012]
Labor per ha.	-0.00010***	-0.00011***	-0.00010***	-0.00010***	-0.00009***	-0.00009***	-0.00009***	-0.00009***
	[0.00002]	[0.00002]	[0.00002]	[0.00002]	[0.00002]	[0.00002]	[0.00001]	[0.00002]
Phosphate price	-0.01074***	-0.01282***	-0.01156***	-0.01380***	-0.00651**	-0.00740***	-0.01137***	-0.01264***
	[0.00331]	[0.00382]	[0.00338]	[0.00391]	[0.00259]	[0.00285]	[0.00260]	[0.00286]
Seed price	0.00027	0.00028	0.00019	0.00018	0.00055***	0.00054***	0.00002	-0.00002
	[0.00019]	[0.00022]	[0.00021]	[0.00024]	[0.00015]	[0.00016]	[0.00016]	[0.00018]
Urea price	0.01076***	0.01271***	0.00848**	0.01016**	0.01165***	0.01282***	0.00686**	0.00768***
	[0.00345]	[0.00398]	[0.00349]	[0.00404]	[0.00271]	[0.00298]	[0.00269]	[0.00296]
Pesticide cost	0.00330*	0.00367	0.00382*	0.00422*	0.00499***	0.00589***	0.00500***	0.00590***
	[0.00199]	[0.00229]	[0.00196]	[0.00226]	[0.00156]	[0.00171]	[0.00151]	[0.00166]
v2dum	0.00136	-0.01532	0.00093	-0.01589	0.01519	0.01919	0.01511	0.01914
	[0.04203]	[0.04791]	[0.04183]	[0.04772]	[0.03169]	[0.03414]	[0.03165]	[0.03409]
v3dum	0.0047	-0.01164	0.00358	-0.01306	-0.01603	-0.01968	-0.01737	-0.021
	[0.07319]	[0.08362]	[0.07285]	[0.08327]	[0.05558]	[0.05996]	[0.05553]	[0.05989]
v4dum	0.03286	0.01413	0.03156	0.0127	0.03132	0.0355	0.02932	0.03374
	[0.08311]	[0.09499]	[0.08271]	[0.09458]	[0.06313]	[0.06814]	[0.06307]	[0.06807]
v5dum	0.09582	0.08559	0.09468	0.08431	0.06364	0.06822	0.06183	0.0667
	[0.06754]	[0.07710]	[0.06721]	[0.07676]	[0.05113]	[0.05518]	[0.05107]	[0.05511]
v6dum	0.12386	0.11088	0.12236	0.10908	0.11422*	0.11969*	0.11206*	0.11781*
	[0.08574]	[0.09794]	[0.08534]	[0.09753]	[0.06509]	[0.07026]	[0.06503]	[0.07018]
t	-0.01798***	-0.01811**	-0.00653	-0.00487	-0.02852***	-0.02932***	0.01545*	0.01803**
	[0.00687]	[0.00790]	[0.01045]	[0.01205]	[0.00534]	[0.00587]	[0.00797]	[0.00876]
t3			0.0108	0.00686			-0.16787***	-0.18292***
			[0.03661]	[0.04224]			[0.02783]	[0.03060]
t4			-0.11784***	-0.12990***			-0.19980***	-0.21278***
			[0.03558]	[0.04098]			[0.02713]	[0.02983]
Constant	1.48242***	1.62337***	1.47043***	1.61236***	1.57939***	1.67962***	1.47045***	1.56163***
	[0.44797]	[0.51156]	[0.44623]	[0.50980]	[0.33885]	[0.36559]	[0.33872]	[0.36542]
σ_u	0.07355***	0.08316***	0.07436***	0.08397***	0.05421***	0.05679***	0.05682***	0.05969***
	[0.01029]	[0.01186]	[0.01007]	[0.01163]	[0.00802]	[0.00894]	[0.00761]	[0.00845]
σ_e	0.20252***	0.23515***	0.19922***	0.23182***	0.16358***	0.18039***	0.15787***	0.17433***
	[0.00630]	[0.00717]	[0.00620]	[0.00707]	[0.00444]	[0.00486]	[0.00429]	[0.00470]
Observations	960	960	960	960	960	960	960	960
Number of farms	160	160	160	160	160	160	160	160
Llikelihood	-155.8	-247.8	-144.87	-238.27	173.55	104.43	201.01	130.46
Censored	277	249	108	93	277	249	108	93

Standard errors in brackets, significant at 10%; ** significant at 5%; *** significant at 1%

Table 2.9: Tobit regression results: Mundlak's correction

Chapter 3

Announced regime switch: Optimal policy for transition period

It is not rare for monetary authority to consider a switch in the focus of their monetary policy. One of the most interesting cases is a switch to a regime of managed, pegged exchange rate or even fixed exchange rate. The motivation for switch may stem from international treaties or beliefs of central bankers about the benefits of a new monetary policy regime. New members of the European Union have agreed on joining the European monetary union (EMU) in the accession treaty. The ERM II accession process asks them to maintain stability of the exchange rate over the evaluation period. This periods usually ends with the adoption of the common currency, e.g. Malta, Slovenia and Slovakia as the most recent cases.

Countries like Bulgaria and Estonia voluntarily decided to set-up a currency board even before entering the evaluation period. The decision to manage or to peg the exchange rate is based on their belief that a currency board is advantageous for small open economies. Also, there exist countries that find their own monetary policy difficult to sustain, e.g., Sweden and Finland in the early 1990's. Countries like these opt for managing their exchange rate in order to achieve macroeconomic stability during currency distress. Regardless, the motivation for the policy switch, the newly adopted policy rule in the aforementioned cases, is usually a sort of nominal exchange rate peg.

Many recent works in monetary economics that focus on the choice of monetary policy study the properties of alternative monetary policy rules by analyzing macroeconomic stability [Collard and Dellas (2002)]; using the loss function of the monetary authority

[Santacreu (2005)]; or the welfare function of households [Gali and Monacelli (2005)] to identify the optimal policy. These studies consider models with a given monetary policy rule and there is no change of rule possible. Therefore, these analyses can be considered as static in form of rule. The static comparison does not determine if it is worth to switch to another policy rule, while it omits the loss occurring over the transition.

The aforementioned points motivate me to focus on the analysis of small open economy behavior over the transition period towards the exchange rate peg. An important issue is how announcing the adoption of the exchange rate peg affects the properties of the business cycles of the small open economy.

I address these issues using the standard stochastic general equilibrium model of the small open economy, e.g., Justiniano and Preston (2004), Gali and Monacelli (2005) and Cuche-Curti, Dellas, and Natal (2008). To simplify my analysis, I decided to use the model by Justiniano and Preston (2004), where all goods are tradable. However, this model uses a Calvo type rigidities as the more complex models do. To provide a specific example, I identify the large economy as the Euro area and the small open economy as the Czech Republic. While the Czech Republic is a representative country that aims to adopt the common currency, it also copes with the limitations of its own independent monetary policy.

For a better description of the Czech Republic monetary policy, I close the model by monetary policy of forecasted inflation targeting. Also, structural parameters of the model are estimated for the Czech Republic.

The novelty presented in this chapter is the approach to modeling the transition period when the change in the monetary regime type is announced. As Farmer, Waggoner, and Zha (2007) summarize, recent works rely on Markov switching processes to account for changes of policy rule. Generally, the solution is computed by as a average of separate models weighted by the probability matrix of the process. Instead of the Markov switching process, I extend the standard model with a binary indicator of the regime that identifies the operative monetary policy. Moreover, in my simulations the change in the regime indicator is credibly announced in advance. Therefore, a model with this indicator offers an alternative approach that more closely models the commitment to the regime change than models based on Markov process.

For my analysis of the macroeconomic stability over the transition, I assume that the monetary authority follows an optimal policy with respect to the loss function for the monetary authority as in Laxton and Pesenti (2003) and Santacreu (2005). As Cuche-

Curti, Dellas, and Natal (2008) and Dellas and Tavlas (2003) summarize, there is no straightforward recommendation for the type of the optimal policy. The optimal policy choice depends on many factors like the presence and origin of rigidities and structural shocks. Therefore, I solve for the optimal policy that takes a simple form where monetary authority reacts to deviations output gap, inflation and change in nominal exchange rate.

Moreover, as Cuche-Curti, Dellas, and Natal (2008) point out, the simple form of the optimal policy avoids questioning information capabilities of the monetary authority. To identify the simple optimal monetary policy for the transition period for various preferences on inflation, output and policy stability, the utility has one degree of freedom as in Santacreu (2005).

The goal of monetary policy for the transition is still to support macroeconomic stability. However, it is also important to know how these policies change the characteristics of the business cycles. To analyze these changes, I compute and analyze the correlations of business cycles as described by inflation, output and interest rate.

The rest of the chapter is organized as follows. Section 3.1 presents the model of rule switch. In section 3.2, the parameters estimation is presented. Basic characteristics and properties of the model are presented in section 3.3. Section 3.4 presents the macroeconomic stability results obtained and section 3.6 concludes. All figures can be found in the appendix sections.

3.1 Model

The basics of the model are taken from Justiniano and Preston (2004). The used model consists of a small open economy (domestic) and the rest of the world (foreign). The domestic economy is characterized by the existence of habit formation and indexation of prices to inflation. The fundamental model is based on the work of Gali and Monacelli (2002) and Monacelli (2005), where micro-foundations for the small open economy model are summarized and incomplete pass-through is discussed. The following sections provide commented derivations of the structural equations of Justiniano and Preston's (2004) model. Further, the modification of monetary policy and approach to modeling the transition period is described in the separate subsection.

3.1.1 Households

The considered small open economy is populated by a representative household that maximizes its lifetime utility function

$$E_t \sum_{t=0}^{\infty} \beta^t e^{g_t} \left[\frac{(C_t - H_t)^{1-\sigma}}{1-\sigma} - \frac{{N_t}^{1+\varphi}}{1+\varphi} \right], \tag{3.1}$$

where β, $0 < \beta < 1$, is the utility discount factor; σ and φ are the inverse of elasticities of the inter-temporal substitution and labor supply, respectively; N_t is total labor effort; $g_t = \rho_g g_{t-1} + \varepsilon_t^g$ is a preference shock, and $\varepsilon_t^g \sim N(0, \sigma_g^2)$; C_t is the consumption of a composite good; $H_t = hC_{t-1}$ is the external habit taken as exogenous by household as presented by Fuhrer (2000). The parameter h indexes the importance of habit formation. The household consumes a Dixit-Stiglitz composite of the home and foreign good:

$$C_t \;=\; [(1-\alpha)^{\frac{1}{\eta}} (C_t^H)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (C_t^F)^{\frac{\eta-1}{\eta}}]^{\frac{\eta}{\eta-1}}, \tag{3.2}$$

where α is the share of the imported good in domestic consumption and $\eta > 0$ is the intra-temporal elasticity of substitution between the domestic and foreign good.

Given the specification of the household's preferences, the minimization of expenditures for the given level of consumption C_t implies, as in Walsh (2003), the following aggregate domestic consumer price index (CPI):

$$P_t \;=\; [(1-\alpha)(P_t^H)^{1-\eta} + \alpha (P_t^F)^{1-\eta}]^{\frac{1}{1-\eta}}, \tag{3.3}$$

where P_t^H and P_t^F are prices of the domestic and foreign Dixit-Stiglitz composite good used to produce the final composite good C_t.

In aggregate, the household maximizes lifetime utility according to the following budget constraint:

$$P_t C_t + Q_{t,t+1} D_{t+1} \;\leq\; D_t + W_t N_t + T_t, \tag{3.4}$$

where W_t is the nominal wage; D_{t+1} is the nominal pay-off received in the period $t+1$ acquired from the portfolio held at the end of the period t, and $Q_{t,t+1}$ is the value of the discount factor of this portfolio; T_t are transfers that include taxes/subsidies and profits collected from domestic firms and importers.

Given the Dixit-Stiglitz aggregation, households optimally (cost minimization) allocate their aggregate expenditures for the foreign and domestic good according to the following demand functions:

$$
\begin{aligned}
C_t^H &= (1-\alpha)\left(\frac{P_t^H}{P_t}\right)^{-\eta} C_t \\
C_t^F &= \alpha\left(\frac{P_t^F}{P_t}\right)^{-\eta} C_t. \qquad (3.5)
\end{aligned}
$$

The first order necessary conditions imply the domestic Euler equation in the following form:

$$
\lambda_t E_t[Q_{t,t+1}] = \beta E_t[\lambda_{t+1}\frac{P_t}{P_{t+1}}], \qquad (3.6)
$$

where λ_t is the Lagrange multiplier associated with a budget constraint. This equation is used in the following section to link the domestic and foreign economy.

3.1.2 International arrangements

The real exchange rate is defined as the ratio of foreign prices in domestic currency to the domestic prices $\hat{q}_t \equiv \hat{e}_t \frac{P_t^*}{P_t}$, where $\hat{e}_t$ is the nominal exchange rate (in terms of the domestic currency per unit of foreign currency); P_t^* is the foreign consumer price index and P_t is the domestic consumer price index given by equation (3.3). An increase in $\hat{e}_t$ coincides with an depreciation of the domestic currency.[1] Further, I assume that $P_t^* = P_t^{F*}$ (P_t^{F*} is the price of the foreign good in a foreign currency), the law of one price gap is given by $\Psi_t^F = \hat{e}_t \frac{P_t^*}{P_t^F}$, as in Monacelli (2005). The law of one price gap represents a wedge between the foreign price of a foreign good P_t^{F*} and price of the foreign good when sold on the domestic market P_t^F by importers [see Lubik (2005) for details]. The law of one price (LOP) holds when $\Psi_t^F = 1$; for $\Psi_t^F > 1$, importers realize losses due to increasing costs of imported goods; when $\Psi_t^F < 1$, importers enjoy profits.

The foreign economy is identical in preferences, therefore optimality conditions are similar to the domestic optimality conditions. The foreign economy is considered to be large and the domestic good takes only a negligible fraction of its consumption. Therefore, the foreign composite consumption bundle can be simplified and only foreign produced good are considered in the overall foreign consumption. Further, under the assumption

[1] The superscript * denotes "foreign" equivalents of domestic variables throughout this chapter.

of complete international financial markets, arbitrage implies that the marginal utility of consumption in a foreign economy is proportional to that in a domestic economy. Using the domestic Euler equation (3.6), the following condition is derived:

$$\beta E_t[\frac{\lambda_{t+1}}{\lambda_t}\frac{P_t}{P_{t+1}}] = \quad E_t[Q_{t,t+1}] \quad = \beta E_t[\frac{\lambda^*_{t+1}}{\lambda^*_t}\frac{P^*_t}{P^*_{t+1}}\frac{\hat{e}_{t+1}}{\hat{e}_t}]. \tag{3.7}$$

Defining the gross nominal return on the portfolio as $R_t^{-1} = E_t[Q_{t,t+1}]$, the risk sharing condition (3.7) equation implies the following uncovered interest rate parity (UIP) condition:

$$E_t[Q_{t,t+1}(R_t - R^*_t(\frac{\hat{e}_t}{\hat{e}_{t+1}}))] \quad = \quad 0. \tag{3.8}$$

The uncovered interest rate parity places a restriction on the relative movement of the domestic and foreign interest rate and on the nominal exchange rate. However, the interest rate parity can be distorted by a risk premium shock. Therefore, as in Kollmann (2002), a shock that captures deviations from purchasing power parity and not already explained endogenously through imperfect pass-through, such as a time varying risk premium, is added into the log-linearized form of the model. Moreover, the risk premium is constant in the steady state and equation (3.8) collapses to the standard uncovered interest rate parity equation for the nominal exchange rate in the steady state.

Finally, the terms of trade are defined as the relative price of imports in terms of exports:

$$S_t \quad = \quad \frac{P^F_t}{P^H_t}. \tag{3.9}$$

Note that changes in the terms of trade may reflect future changes in the competitiveness of an economy. The depreciation of the exchange rate induces an increase in import prices and deterioration of terms of trade. However, the depreciated exchange rate restores competitiveness of the economy since demand for cheaper exports grows and import demand from domestic consumers decreases.

3.1.3 Firms

In this economy, the nominal rigidities driving the price adjustment occurs due to monopolistic competition in the good market. Suppose there is a continuum of domestic firms indexed by i, $0 \leq i \leq 1$. A typical firm i in the home country produces a differentiated good with constant returns to scale according to the following production function:

$$Y_t(i) = A_t N_t(i),$$

where $N_t(i)$ is labor supplied by a household to firm i; A_t is a common stationary productivity process that follows $log(A_t) = a_t = \rho_a a_t + \varepsilon_t^a$, where $\varepsilon_t^a \sim N(0, \sigma_a^2)$ is an exogenous productivity shock common to all firms. The firm's index can be dropped, while in the symmetric equilibrium all choices of the firms are identical. According to the production function, the representative firm faces real marginal costs $MC_t = \frac{W_t}{P_t A_t}$, where W_t is the nominal wage.

Here, the domestic inflation rate is defined as $\pi_t^H = log(P_t^H / P_{t-1}^H)$. Firms producing a domestic good are monopolistically competitive with a Calvo-style price setting using the inflation indexation. Further, only a fraction $(1-\theta^H)$ of firms are allowed to set their price $P_t^{H,new}$ optimally in the considered period. The remaining fraction θ^H, $0 \leq \theta^H < 1$ sets its price according to the following indexation rule:

$$log(P_t^H(i)) = log(P_{t-1}^H(i)) + \delta \pi_{t-1}^H,$$

where $0 \leq \delta < 1$ is the degree of indexation. Therefore, the aggregate price index is evolving according to the following relation:

$$P_t^H = \left[(1-\theta^H)(P_t^{H,new})^{(1-\varepsilon)} + \theta^H \left(P_{t-1}^H \left(\frac{P_{t-1}^H}{P_{t-2}^H} \right)^{\delta} \right)^{(1-\varepsilon)} \right]^{1/(1-\varepsilon)}, \qquad (3.10)$$

where $\varepsilon > 1$ is the elasticity of substitution between the varieties of goods produced by domestic firms. Firm i, setting its price in period t and following the indexation rule in all subsequent periods T, $T \geq t$, faces the following demand curve in period T:

$$y_T^H(i) = \left(\frac{P_t^{H,new}(i)}{P_T^H} \left(\frac{P_{T-1}^H}{P_{t-1}^H} \right)^{\delta} \right)^{-\varepsilon} (C_T^H + C_T^{H*}),$$

where C_t^H is domestic demand and C_t^{H*} is foreign demand for the composite domestic good. While firm i is maximizing its present value by maximizing the value of the real profits stream, the firm's price-setting problem in period t is to solve:

$$\max_{P_t^H(i)} \quad E_t \quad \sum_{T=t}^{\infty}(\theta^H)^{T-t}Q_{t,T}y_t^H(i)\left[P_t^{H,new}(i)\left(\frac{P_{T-1}^H}{P_{t-1}^H}\right)^{\delta} - P_t^H MC_T\right]$$

subject to the aforementioned demand curve. This implies the following first-order condition:

$$E_t\sum_{T=t}^{\infty}(\theta^H)^{T-t}Q_{t,T}y_t^H(i)\left[P_t^{H,new}(i)\left(\frac{P_{T-1}^H}{P_{t-1}^H}\right)^{\delta} - \frac{\varepsilon}{1-\varepsilon}P_t^H MC_T\right] = 0,$$

where MC_T are real marginal costs in the period of price decision.

Similarly, as in the domestic good production, the nominal rigidities in the foreign good sector are resulting from staggered price setting and monopolistic competition. Foreign good retailers import foreign goods so that the law of one price holds "at the docks" and resell them in a monopolistically competitive market. To set their prices, importers also use Calvo pricing with indexation to past inflation of imported good prices, which is defined as $\pi_t^F = log(P_t^F/P_{t-1}^F)$.

Again, only a fraction $(1-\theta^F)$ of importers are allowed to set their new price $P_t^{F,new}$ optimally in each period. The fraction θ^F, $0 \leq \theta^F < 1$ of importers just updates its price according to the following indexation rule:

$$log(P_t^F(i)) = log(P_{t-1}^F(i)) + \delta\pi_{t-1}^F,$$

where the same degree of indexation δ as for domestic producers is assumed. The foreign good price index is evolving according the following relation:

$$P_t^F = \left[(1-\theta^F)(P_t^{F,new})^{(1-\varepsilon)} + \theta^F\left(P_{t-1}^F\left(\frac{P_{t-1}^F}{P_{t-2}^F}\right)^{\delta}\right)^{(1-\varepsilon)}\right]^{1/(1-\varepsilon)}.$$

Similarly, importer i, who is setting its price in period t, faces the following demand curve in period T, $T \geq t$:

$$y_T^F(i) \quad = \quad \left(\frac{P_t^{F,new}(i)}{P_T^F}\left(\frac{P_{T-1}^F}{P_{t-1}^F}\right)^{\delta}\right)^{-\varepsilon} C_T^F, \tag{3.11}$$

as for the domestic good, in here $\varepsilon > 1$ is a parameter describing the substitution between the varieties of foreign goods. Therefore, the importer's price-setting problem in period t is to maximize

$$E_t \sum_{T=t}^{\infty} (\theta^F)^{T-t} Q_{t,T} y_t^F(i) \left[P_t^{F,new}(i) \left(\frac{P_{T-1}^F}{P_{t-1}^F} \right)^{\delta} - \hat{e}_T P_t^F MC_T \right]$$

subject to the aforementioned demand equation (3.11). This implies the following first-order condition:

$$E_t \sum_{T=t}^{\infty} (\theta^F)^{T-t} Q_{t,T} y_t^F(i) \left[P_t^{F,new}(i) \left(\frac{P_{T-1}^F}{P_{t-1}^F} \right)^{\delta} - \frac{\varepsilon}{1-\varepsilon} \hat{e}_T P_t^F MC_T \right] = 0,$$

and the new optimal price $P_t^{F,new}(i)$ is the solution to this equation. The presence of monopolistic competition results in deviations from the law of one price in the short run, while a complete pass-through is reached in the long-run as presented in Monacelli (2005).

3.1.4 Equilibrium

Equilibrium requires that all markets clear. The good market clearing condition in the domestic economy is given by the following equation:

$$Y_t^H = C_t^H + C_t^{H*}. \tag{3.12}$$

Under the assumption of a large foreign economy, market clearing in the foreign economy gives $Y_t^* = C_t^*$. Households, which are assumed to have identical initial wealth, make identical consumption and portfolio decisions. So, the following analysis considers a symmetric equilibrium, domestic producers, importers, and foreign firms also behave identically. Therefore, the individual index can be dropped and the representative household, representative firm, and the single good in each sector can be used for the model solution. In period t the representative domestic producers set common prices P_T^H. Importers also set a common price P_t^F, so do the foreign producers when setting P_t^*. Finally, as in Gali and Monacelli (2002) and Justiniano and Preston (2004), I assume that the government off-sets distortions originating from monopolistic competition in the goods markets by a subsidy/transfer that is financed through a lump-sum tax T_t on representative household.

3.1.5 A log-linearized model

To analyze the behavior of the underlying model, an approximation around the non-stochastic steady state of the presented model is obtained as in Justiniano and Preston (2004). For any variable, the lowercase letters denote the log-deviation from the steady state of their uppercase counterparts in the frictionless equilibrium. The non-stochastic steady state is characterized by setting all shocks to zero for all periods.

As in Justiniano and Preston (2004), I assume a zero inflation steady state, so that $\pi_t = \frac{P_t}{P_{t-1}} = \frac{P_t^H}{P_{t-1}^H} = \frac{P_t^F}{P_{t-1}^F} = 1$, and for the steady state of the nominal interest rate $1 + i_t = \frac{1}{\beta}$.

Linearizing the domestic good market clearing condition (3.12) together with a linearized version of the demand functions (3.5) implies

$$(1-\alpha)c_t = y_t - \alpha\eta(2-\alpha)s_t - \alpha\eta\psi_t^F - \alpha y_t^*, \tag{3.13}$$

where $\psi_t^F = (e_t + p_t^*) - p_t^F$ is a log-linear approximation of the law of one price, and $s_t = p_t^F - p_t^H$ is a log-linear approximation of the terms of trade given by equation (3.9). Time differentiating of the terms of trade definition implies

$$\Delta s_t = \pi_t^F - \pi_t^H. \tag{3.14}$$

Using the log-linearized equations of the law of one price gap and terms of the trade, the following link between the terms of trade and the real exchange rate can be derived:

$$q_t = \psi_t^F + (1-\alpha)s_t. \tag{3.15}$$

The log-linear approximation to the optimality conditions of domestic firms for price setting, the law of motion for the domestic producers price, and the domestic price index given by equation (3.10) imply the following hybrid Philips curve:

$$\pi_t^H - \delta\pi_{t-1}^H = \frac{1-\theta^H}{\theta^H}(1-\theta^H\beta)mc_t + \beta E_t[(\pi_{t+1}^H - \delta\pi_t^H)], \tag{3.16}$$

where the marginal costs is

$$mc_t = \varphi y_t - (1+\varphi)a_t + \alpha s_t + \sigma(1-h)^{-1}(c_t - hc_{t-1}). \tag{3.17}$$

The log-linear form of the real marginal costs mc_t of the representative firm originates from the log-linearization of the aggregate production function and the household's optimality condition for labor choice.

Similarly, the optimality condition for the pricing problem of retailers results in the following Philips curve:

$$\pi_t^F - \delta\pi_{t-1}^F = \frac{1-\theta^F}{\theta^F}(1-\theta^F\beta)\psi_t^F + \beta E_t[(\pi_{t+1}^F - \delta\pi_t^F)]. \tag{3.18}$$

Following the arguments of Justiniano and Preston (2004) and the derivation by Gali and Monacelli (2002), the complete markets assumption together with condition (3.7) imply the following relation for the log-linear approximation of the Euler equation (3.6):

$$c_t - hc_{t-1} = y_t^* - hy_{t-1}^* + \sigma^{-1}(1-h)[\psi_t^F + (1-\alpha)s_t] + \sigma^{-1}(1-h)g_t. \tag{3.19}$$

The log-linear approximation of the uncovered interest rate parity equation (3.8) gives $i_t - i_t^* = E_t\Delta e_{t+1}$. As mentioned in the previous section, to capture the deviations from UIP, a risk premium shock ϵ_t is added into equation (3.8); $\epsilon_t = \rho_s\epsilon_{t-1} + \varepsilon_t^s$, here $\varepsilon_t^s \sim N(0,\sigma_s^2)$. Using the definition of the real exchange rate,

$$\Delta e_t = \Delta q_t + \pi_t - \pi_t^*, \tag{3.20}$$

the following equation is derived:

$$(i_t - E_t\pi_{t+1}) - (i_t^* - E_t\pi_{t+1}^*) = E_t\Delta q_{t+1} + \epsilon_t. \tag{3.21}$$

The risk premium shock ϵ_t is zero in the steady state, so the steady state equation (3.21) collapses to a standard uncovered interest rate parity equation. Also, note that the positive (negative) values of Δe_t reflect domestic currency depreciation (appreciation).

Finally, the approximations of the CPI equation (3.3) and the change in terms of trade (3.14) give the following relation:

$$\pi_t = \pi_t^H + \alpha\Delta s_t. \tag{3.22}$$

Since the goods produced in the home economy represent only a small fraction of the foreign economy consumption, I consider the large foreign economy as exogenous to the domestic economy. Therefore, I assume that the paths of foreign variables π_t^*, y_t^*, and i_t^* are determined by the following VAR process:

$$\pi_t^* = \omega_\pi^\pi \pi_{t-1}^* + \omega_y^\pi y_{t-1}^* + \omega_i^\pi i_{t-1}^* + \varepsilon_t^\pi, \tag{3.23}$$

$$y_t^* = \omega_\pi^y \pi_{t-1}^* + \omega_y^y y_{t-1}^* + \omega_i^y i_{t-1}^* + \varepsilon_t^y, \tag{3.24}$$

$$i_t^* = \omega_\pi^i \pi_{t-1}^* + \omega_y^i y_{t-1}^* + \omega_i^i i_{t-1}^* + \varepsilon_t^i, \tag{3.25}$$

where ε_t^π, ε_t^y, and ε_t^i; $\varepsilon_t^y \sim N(0,\sigma_y^2)$, $\varepsilon_t^\pi \sim N(0,\sigma_\pi^2)$, and $\varepsilon_t^i \sim N(0,\sigma_i^2)$, represent the independent structural shocks that drive the foreign economy.

3.1.6 Model of the transition period

The description of the model is closed by describing the behavior of the domestic monetary authority. While the Czech central bank reacts to the forecasted inflation, I deviate from Justiniano and Preston (2004) in my analysis. As discussed by Carlstrom and Fuerst (2000), I assume that the monetary authority acts according to expected inflation rather than using the actual level of inflation. To keep my analysis simple, I assume that the monetary authority is forward looking only for one period ahead.

The focus of this chapter is to analyze macroeconomic stability during the transition. The economy begins in time $t = 1$, when it is announced that the regime will change in period $T, T > 1$. To simplify the analysis, I also assume that the monetary authority follows the same policy rule over all periods of the transition, $t \leq T$.

So, the monetary policy rule for the model of the transition period takes the following form:

$$\begin{aligned} i_t &= regime_t(\rho_i i_{t-1} + \rho_\pi E_t[\pi_{t+1}] + \rho_y y_t + \rho_e \Delta e_t + \varepsilon_t^m) + \\ &+ (1 - regime_t)\widehat{\rho}_e \sum_{j=t}^{\infty} \left(\frac{1}{2}\right)^{t-j} \Delta E_t[e_j], \end{aligned} \tag{3.26}$$

where $0 \leq \rho_i < 1$, $\rho_\pi > 1$, $\rho_y > 0$ and $\rho_e \geq 0$ are weights describing the responses of the domestic monetary authority; and ε_t^m, $\varepsilon_t^m \sim N(0,\sigma_m^2)$ is the shock capturing errors arising from the description of the monetary policy. In here, the effective monetary regime is selected via the regime indicator. In my experiment when the change is announced in the first period, the indicator is defined as follows:

$$regime_t = \begin{cases} 1, & \text{if } t < T; \\ 0, & \text{if } t \geq T, \end{cases}$$

where T is the announced time of regime change.

By varying values of the rule parameters ρ_π, ρ_y and ρ_e in rule (3.26), I am able to model a wide range of monetary policies for the transition ($t < T$), e.g. inflation targeting or exchange rate targeting. Further, the only objective of the post-transition monetary regime $t \geq T$, is to off-set all the foreseen changes in the nominal exchange rate. This regime is characterized by $\widehat{\rho_e}$, which measures the off-setting of the change in the nominal exchange rate. To keep the level of exchange rate volatility reasonably low, I set $\widehat{\rho_e} = 2.0$.

The introduction of the regime indicator transforms the problem of modeling an announced change to a problem of foreseen changes in the indicator. To model the announced changes in the indicator, I extend the state space of the model by an information buffer of length N, where $N > T$. This information buffer is capable of storing information for N periods ahead and takes the following form:

$$\begin{aligned} regime_t &= inf_{t,1} \\ inf_{t,1} &= inf_{t-1,2} + \nu_{t,1} \\ inf_{t,2} &= inf_{t-1,3} + \nu_{t,2} \\ &\vdots \\ inf_{t,N-1} &= inf_{t-1,N} + \nu_{t,N-1} \\ inf_{t,N} &= \nu_{t,N}, \end{aligned} \tag{3.27}$$

where $inf_{t,i}$, $i \in 1, \ldots, N$ are the new endogenous variables, and $\nu_{t,i}$, $i \in 1, \ldots, N$ are the announcement shocks, such that $\nu_{t,i}$ takes values 0 and 1 for all $i = 1, \ldots, N$ and $t > 0$. The initial condition for the buffer is $inf_{0,i} = 0$ and $\nu_{0,i} = 0$, $\forall i \in 1, \ldots, N$.

In the experiment, I focus on the perfectly credible announcements. Therefore, I can think about $\nu_{t,i}$s as random variables with zero mean and zero variance. However, by varying the assumption about information shocks, it is possible to model the uncertainty about keeping the commitment of the policy rule switch announced by the monetary authority. The higher the uncertainty about keeping commitments, the higher value of information shock variance should be used.

The announcement of the regime change in $t = 1$ is modeled by the realization of the information shocks $\nu_{t,i}$ $i \in 1, \ldots, N$ according to the following scheme:

$$\nu_{1,i} = \begin{cases} 1, & i \leq T; \\ 0, & i > T, \end{cases} \tag{3.28}$$

and $\nu_{t,i} = 0$, $\forall i$ and in the all subsequent periods t, $1 < t \leq T$. This realization of information shocks describes a one-time announcement of a policy rule switch in period T without any further changes of transition length.

The model of the transition period consists of equations (3.13)–(3.25), the monetary policy rule (3.26), the information buffer given by equations (3.27), and definitions of the AR(1) processes for technology and preference shocks.

Further, I assume that there are no shocks (for $t \geq T$) to risk premium when the regime of off-setting of the exchange rate changes is adopted. So, the risk premium shock ϵ_t described by equation (3.21) will become $\epsilon_t = \rho_s \epsilon_{t-1}$. To make this change foreseen in the model of transition, the AR(1) process for risk premium shock ϵ_t in equation (3.21) will become $\varepsilon_t = \rho_s \varepsilon_{t-1} + regime_t \varepsilon_t^s, \varepsilon_t^s \sim N(0, \sigma_s^2)$ since $t > T$.

The construction of the policy indicator $regime_t$ creates non-linearities in the monetary policy rule and risk premium process. Therefore, to solve and simulate the transition period model, the second order approximation is used. The model is solved by Dynare++.[2] A brief description of the computation of the transition period model is presented in Appendix (3.A).

3.2 Estimation

To provide a specific example, in my analysis I estimate the parameters of the model using data on the Czech Republic. In recent literature, Bayesian methods are considered an attractive tool for estimating a model's parameters, especially in open economy modeling. The most recent examples include Smets and Wouters (2003), who estimate the Eurozone model; Lubik and Schorfheide (2003) and Lubik and Schorfheide (2005), who analyze the behavior of the monetary authority; and Ireland (2004).

Due to the short span of the Czech data sample, I prefer Bayesian methods because

[2] Dynare++, developed by Kameník (2007), is a standalone C++ version of Dynare. Dynare is the pre-processor and collection of Matlab routines introduced by Juillard (1996), Collard and Juillard (2001b) and Collard and Juillard (2001a).

it allows me to incorporate information from previous studies in the form of informative priors on parameter values. This approach is preferred because the use of priors makes the estimation results more stable.

Model M and its associated parameters Θ can be estimated using the method outlined by An and Schorfheide (2007). In the Bayesian context, given a prior $p(\Theta)$ and a sample of data Y, the posterior density of the model parameters Θ is evaluated, and it is proportional to the likelihood of the data multiplied by the prior $p(\Theta)$:

$$p(\Theta|Y,M) \quad \propto \quad L(\Theta|Y,M)p(\Theta),. \tag{3.29}$$

The goal of the Bayesian estimation is to estimate the posterior distribution and to find such parameter estimates that given the model, the likelihood value $L(\Theta|Y,M)$ is maximized.

The Bayesian estimation procedure consists of the following three steps. In the first step, the model is extended for a measurement block that links model variables to data. The extended model is solved. In the second step, the fact that the solution of the model is in the form of a state space model is exploited. This allows me to compute the likelihood function of the underlying model by use of the Kalman filter, the observed data, and priors. The objective is to maximize the value of likelihood as the function of the model parameters. The second step results in the maximum-likelihood estimates of the model parameters. The objective of these estimation steps is to get parameter values for this model.

In the third step, the likelihood function conditional on a parameters estimate is combined with the prior distribution of parameters to obtain the posterior density function. The Metropolis-Hastings (MH) algorithm, which is an implementation of the Monte Carlo Markov chain (MCMC) method, is used to estimate the posterior distributions. The objective of the posterior distributions computation is to evaluate the sensitivity of the results to my choice of priors and optimization algorithm settings.

3.2.1 Data and priors

The used data sample covers a period of an CPI inflation targeting regime from its introduction in 1998 until the third quarter of 2007. Over this period changes in the inflation target occurred. However, the nature of the regime was not changed thus this does not lead to structural changes. Therefore, I can abstract from the effects of a decreasing inflation target. The detailed description of data and transformations used are summarized in Appendix 3.B.1.

The domestic block of the underlying model is estimated using the de-trended data on output growth, inflation, the nominal interest rate, terms of trade, and the real exchange rate. The foreign block is described by the de-trended series of effective output, inflation, and the nominal interest rate. The effective series are constructed as a sum of the trade partners series weighted by the export shares.

Model variables are expressed in percentage deviations from a steady state. The data series are related to model variables via a block of measurement equations. The measurement block connects the model variables with the observed data using the measurement error. The block of measurement equations and measurement errors characteristics are summarized in Appendix 3.B.2.

The choice of parameter priors is derived from previous studies [Lubik and Schorfheide (2003); Natalucci and Ravenna (2003); Justiniano and Preston (2004); and Musil and Vašíček (2006)] and is guided by the following considerations. The choice of prior distributions reflects the restrictions on the parameters such as non-negativity deviations or interval constraints. Therefore, for parameters constrained to the $\langle 0, 1 \rangle$ interval, the beta distribution is used. Prior distributions for standard deviations of shocks have been set to inverse gamma. Similarly, for parameters taking positive values, the gamma distribution is used. The standard deviation of priors also reflects my beliefs about confidence in the priors, and I decided to use loose priors rather than tighter ones. Tables 3.3 and 3.4 provide an overview of my choice of priors. Further, I assume $\beta = 0.99$ (strict prior), which implies an annual interest rate of about 4% in a steady state.

The model for estimation is closed by the simple monetary policy rule given as follows:

$$i_t = \rho_i i_{t-1} + \rho_\pi E_t[\pi_{t+1}] + \rho_y y_t + \rho_e \Delta e_t + \varepsilon_t^m, \tag{3.30}$$

and the risk premium process is given by equation (3.8) is used. The estimated model also does not include the information buffer.

For construction of the joint probabilistic distribution, I assume that the priors are independent of each other to simplify the use of the MCMC algorithm. The Dynare toolbox to estimate the presented model. Given the data and priors, I generated 300,000 draws for each of the 7 Markov chains using the MH algorithm. While acceptance rates between 20% and 40% are considered as reasonable for distribution sampling, I set the scaling parameter for jumping distribution in MH so that the average acceptance rate is 0.35.

3.2.2 Estimation results

The estimation results are summarized in Tables 3.3 and 3.4 in appendix 3.B.3. The analysis of the posterior distributions for each estimated parameter does not indicate the presence of computational problems.

The openness parameter α is estimated to be 0.35, implying 0.54 for a steady state ratio of domestic to foreign goods in the domestic consumption basket. The estimated value is very close to openness estimates by Natalucci and Ravenna (2003) and Musil and Vašíček (2006). These works base their estimates on imports share in consumption rather than on imports share in gross domestic product. The openness parameter is also in accordance with the value 0.27 of foreign-domestic good substitution η because it indicates low willingness of households to substitute domestic for foreign goods.

The value 0.92 of inverse elasticity of inter-temporal substitution σ implies inter-temporal elasticity of 1.08. This value of elasticity indicates that households are concerned about their consumption path and they are willing to substitute today's consumption for the future one. The acceptance of consumption changes is consistent with a low value of habit persistence. Also, the value of inverse elasticity of labor substitution, $\sigma = 1.08$, implies non-elasticity of the labor supply. The increase in real wage by 1% implies just 0.92% increase in the labor supply. I believe that this value is consistent with the low labor mobility that characterizes Czech labor market, especially at the beginning of the considered period.

According to the estimation results, interest rate smoothing ρ_i takes just a slightly higher value (0.58) than my prior (0.50). The reaction to inflation and the output gap deviation are taking values 1.38 and 0.47, respectively. These values of ρ_π and ρ_y reveal that the monetary authority places 2.9 more weight on keeping future inflation stable than closing the output gap. Moreover, the low value of reaction to the deviation of the nominal exchange rate ρ_e reflects the inflation targeting focus declared by the Czech National Bank.

My priors for the price stickiness parameters θ's are chosen based on Lubik and Schorfheide (2005), and they reflect the evidence on US prices. The prior value of price indexation to inflation is set to 0.70, while studies exists where the value of indexation is set to unity. My estimation results show that there is a low fraction of domestic firms (estimate of θ_H takes value 0.26) that optimize their prices every quarter. This is consistent with estimates using the European data presented by Smets and Wouters (2003). Approximately the same fraction of importers optimize their prices every period so the average contract length is approximately 4 quarters. The value of inflation indexation δ means that the price of the good is updated by half of price level change. I find it consistent with my estimates of the low frequency of price optimization. The estimated value of 0.56 for inflation indexation δ is almost three times as high as the estimates reported by Justiniano and Preston (2004).

I assume a high persistency of technological, risk premium and taste shocks, so the priors are set to 0.85. However, estimates show that the most persistent shock is the preference shock with a value of 0.95 for ρ_g. This indicates that impacts of the preference shocks are not temporary but near permanent. I believe that the low persistency of technological shock, taking value 0.83, with a large standard deviation of technological shock, reflects the structural changes of Czech industry over the considered period.

For the foreign block, I assume the autocorrelation of foreign shocks to be 0.7 [used by Natalucci and Ravenna (2002)], while I find the values of Justiniano and Preston (2004) quite low. However, my estimation results show little persistency in the foreign inflation series. The foreign monetary policy described by equation (3.25) reveals persistency close to the prior value, thus indicating significant interest rate smoothing in the Eurozone. Only, the foreign output series reveal persistency higher then a prior values, and the value of 0.93 is in accordance with estimates for developed economies, like the USA.

Priors and estimates of the standard deviation of structural shocks are summarized in Table 3.4. These results show that the preference shock ε_t^g is most volatile. However, this does not mean that the preference shock is the main driving force of the variables of my interest. Using variance decomposition, I found that the preference shock generates only 7.5% of inflation volatility, 4.5% of output growth, and 7.3% of nominal interest rate variance. Due to the high value of openness, I determined that the risk premium shock generates 26% of domestic CPI inflation variance. However, for the estimated coefficients, variance decomposition shows that the foreign shocks are not the main drivers of domestic variables volatility. The shocks to foreign inflation and interest rates are responsible for approximately 11.3%, respectively 2.8% of domestic inflation variance.

To evaluate empirical properties of the generic model, Table 3.1 compares moments of the time series used for estimation with moments of the variables of the estimated model.

	Data		Model	
Variable	Std. dev.	Corr.	Std. dev.	Corr.
Output growth	1.05	1.00	2.28	1.00
Nominal interest rate	1.38	-0.53	0.53	-0.35
CPI inflation	3.14	-0.12	3.34	-0.06
Change in nominal ex. rate	8.37	0.17	8.12	0.11
Real ex. rate	3.48	0.17	6.87	0.01
Foreign output gap	0.81	0.02	0.74	0.03
Foreign inflation	0.66	0.21	0.81	-0.02
Foreign nom. int. rate	0.65	-0.03	0.73	-0.02

Table 3.1: Moments summary

This comparison shows that the model exhibits more volatile output and real exchange rate series and excess interest rate smoothing. However, the estimated model matches the properties of the foreign series.

Finally, to evaluate the amount of information included in the observed series, I use a comparison of priors and posteriors distributions. This comparison helps to gain insight about the extent to which the data provide information about the estimated parameters. According to plots presented in Figure 3.1, I conclude that some of the priors are significantly updated by information included in the data.

3.3 Impulse response analysis

The goal of the following comparison is to point to differences induced by adding the possibility of a policy rule switch in the estimated model [model with the monetary policy rule (3.26)]. Therefore, the models of the announced change of monetary policy are calibrated with the same parameters values as the benchmark model. Figures (3.2)–(3.8) present impulse response functions of the following four models: estimated model (dash-doted red line); model of switch in 4 (solid magenta line); 8 (dashed blue line); and 40 (dotted black line) periods. The results are presented as quarterly percentage deviations from the steady state.

Figure 3.2 depicts responses to the 1% domestic technology shock to ε_t^a. As it is expected for the case of a supply shock, output increases and inflation decreases. Via the uncovered interest rate parity relation, the decrease in the domestic inflation is accompanied with a currency appreciation (since the inflation and interest rate of a foreign economy does not react to domestic shocks). The monetary authority decreases interest rates. Due to the currency appreciation and the fact that importers do not update their prices immediately for lower input cost, the law-of-one-price (LOP) gap closes, eliminating importer profits. The presence of habit formation supports hump-shaped consumption profile because households gradually adjust their consumption profile. However, an update of imported good prices, with slowing currency appreciation and real depreciation, restrain the rise in demand for the foreign good. As inflation in the imported good sector rises, the steady state is established.

In the case of the estimated model (dash-dotted red line), due to the absence of regime change, much stronger appreciation is observed. The price rigidity in imported goods sector and appreciation leads to a long period deflation of imported goods prices. Due to low inflation, authority responds with expansive monetary policy. The main difference in responses between the model of announced rule switch and the model of independent monetary policy is in the extent of response to technology shocks.

Figure 3.3 presents responses to the domestic taste shock ε_t^g. This shock initiates an increase in domestic inflation and output as expected in the case of demand shock. Because of the initial currency appreciation, which results from an expected hike in interest rates, importers decrease the prices of their goods. The foreign goods become cheaper and this supports increase in demand for foreign good. Due to output rigidities, the increase in output follows with lag. In response to inflation and output increases, the domestic monetary authority increases the interest rate. Due to the price indexation of import prices to CPI inflation, the initial response of the LOP gap is negative and importers enjoy profits.

For the benchmark model, the import price decrease has a larger extent than in the case of a rule switch and this makes households increase their demand for a foreign good. This results from the reaction of the monetary authority, which can not rely on the expectations formed according to exchange rate stabilizing policy. Moreover, the extent of these deviations is very small.

Figure 3.4 presents responses to the risk premium shock ε_t^s. In the case of an announced change in monetary regime, this leads to initial depreciation and an immediate increase in the interest rate to prevent further depreciation and a rise in inflation. For the models of the policy switch, the monetary authority strongly increases the interest rate in order to offset the change in the nominal exchange rate immediately. However, due to the extent of the depreciation and the inflation indexation of import prices, a significant increase in the price of imported goods is observed. In here, the main difference between the models is the extent of the initial depreciation.

In the case of a monetary policy shock ε_t^m, as shown in Figure 3.5, the shape of the responses does not differ much between models of transition because of the low persistency of the shock, and the steady state is quickly established. A positive monetary policy shock is equivalent to a contractionary policy. Therefore, output decreases in line with consumption as inter-temporal substitution motivates households to postpone consumption. The induced appreciation results in a drop of price of imports. The estimated change model initially reacts with much stronger appreciation, leading to a significant drop in inflation and output, therefore expansionary policy is conducted in the following periods. However, the steady state is established within periods.

Responses to a foreign inflation shock ε_t^π are presented in Figure 3.6. In models of transition, an increase in the foreign inflation rate leads to an immediate appreciation of the domestic currency (implied by UIP). An increase in price of imports supports domestic inflation rise. The monetary authority has to react with contractionary policy, which suppresses output. But this deviation is very small. In the estimated model initial appreciation is very strong, so the real exchange rate together with contractionary policy does not allow for the initial increase in output fueled by increased foreign demand.

Figure 3.7 depicts responses to the foreign positive output shock ε_t^y. An increase in foreign economic activity leads to an increase in demand for the domestic goods and domestic inflation, so domestic output rises in response to this shock. High foreign demand leads to increase of foreign good prices, leading to imported goods price increase which together with domestic inflation delivers domestic currency depreciation via UIP. Depreciation eliminates importer profits and is followed by a large increase in domestic interest rates.

For the foreign output shock, the main differences in responses occur in the initial period, where more extensive depreciation is observed for the estimated model in the period following the shock realization. Therefore, the monetary authority responds with contractionary policy.

Finally, Figure 3.8 depicts responses to the positive shock to foreign interest rate ε_t^i. The UIP implies an initial depreciation of domestic currency because of the negative interest rate differential. Domestic currency depreciation is able to support an initial increase in foreign demand that fuels domestic output and inflation increase. The domestic monetary authority reacts with contractionary monetary policy in the following periods. However, even through interest rate increases, the analysis of the LOP gap shows that importers are facings losses. This means that importers are bearing the costs of depreciation due to the high rigidity of import prices.

3.4 Macroeconomic stability

As discussed in the previous section, impulse response functions mostly differ in the extent of the deviations in reactions to shocks. Therefore, I focus on volatility of inflation, output gap, and the exchange rate change.

Focus on macroeconomic stability was used as the standard approach in the early literature on monetary policy evaluations. It simplifies the analysis because of the independence from the welfare function specification. I believe it can still offer interesting comparisons, as recently presented by Cuche-Curti, Dellas, and Natal (2008) and Collard and Dellas (2002).

However, due to the volatility trade-offs between variables, a simple comparison of volatilities does not straightforwardly identify the regime that delivers the highest level of macroeconomic stability. As Cuche-Curti, Dellas, and Natal (2008) summarize, an exchange rate peg can outperform a flexible exchange rate regime under assumptions of a stable external environment and that the main source of nominal rigidity is in the goods market. They also find that policies ignoring movements in the exchange rate can be dominated by a simple exchange rate targeting policy. Also, Dellas and Tavlas (2003) show that pegging of the exchange rate may be beneficial in the presence of nominal rigidities.

Therefore, for the purpose of monetary regime comparison, I use the traditional form of the per-period loss function [e.g., as in Laxton and Pesenti (2003) and Santacreu (2005)]:

$$L_t = \tau Var(\pi_t) + (1-\tau)Var(y_t) + \frac{\tau}{4}Var(\Delta i_t), \tag{3.31}$$

where $\tau \in <0,1>$ is used to describe the preferences of monetary authority about inflation output and monetary policy stability. To compute the loss over the transition, β is used as the discount factor and the overall loss is computed as a discounted sum of per period losses.

Using the loss function, I compute optimal policies that minimize the value of the loss by choice of the weights ρ_i, ρ_π, ρ_y and ρ_e for the monetary policy rule given by equation

(3.26).

In this experiment, the variances from the estimated model are used as the initial conditions for recursive computation, as described in Appendix 3.A. Further, I compute the optimal policy for various lengths of transition. I also repeat the minimization problem for the various specifications of preferences of the monetary authority by varying τ. The resulting loss is shown in Figure 3.9.

It can be observed that a longer transition period leads to lower values of loss. Also, as the monetary authority becomes more concerned about the output volatility (low values of τ), the authority is generally achieving lower loss.

Figure 3.10 shows the parameters of the optimal policy rule for the transition period as the function of transition length and preferences specification. The plot for the interest rate smoothing parameter ρ_i shows that for all transition periods, the policy rigidity is steeply increasing as the inflation stability is gaining higher weight. The plot for the choice of the inflation targeting parameter ρ_π does not show much variance over the considered transition lengths. Intuitively, as the weight on inflation in loss function specification is getting higher (τ increases), ρ_π is also increasing.

Further, for ρ_y the value of output gap targeting is varying among transition lengths and preferences specifications. Also, intuitively when output stability is extremely preferred the ρ_y reaches the upper constraint. It seems that there is a trade-off between the output gap and a change in nominal exchange rate targeting while as preferences are shifted towards inflation, stability ρ_e decreases. This can be explained by the foreign shock absorbing nature of the exchange rate. Lower values of exchange rate targeting provide a more flexible exchange rate, which is able to absorb the foreign inflation movements. At the same time, the changes in exchange rate can affect domestic output via the foreign demand. Therefore to avoid increase in the domestic output volatility, ρ_y is increasing.

3.4.1 Variance decomposition

As in Collard and Dellas (2002) and in order to better understand the forces that drive change in the business cycle behavior, change in the origins of the variance is analyzed. I analyze the changes in variance decomposition between the estimated model and the model of post-transition ($t \geq T$). I report the changes in variance contribution shock to the volatility of variables in Table 3.2. These changes are computed as a difference of shock contribution to the total variance of the considered variable (in percents) in the estimated model and in the model of post-transition regime. In here, a positive value signals an increase in the contribution to volatility in the model of the post-transition regime.

Variable	Shocks						
	ε^a	ε^m	ε^g	ε^s	ε^π	ε^y	ε^i
Δe_t	-1.4	-16.4	-64.3	-9.8	16.4	41.1	40.4
i_t	-19.5	-1.5	-7.3	-59.5	11.9	52.4	23.5
mc_t	-1.2	-18.0	45.6	-10.7	0.2	-14.7	-1.3
π_t	-6.0	-43.9	84.1	-26.4	1.0	-6.1	-2.7
pi_t^F	-2.3	-16.9	-69.1	-10.2	51.0	39.8	7.7
pi_t^H	-3.4	-18.6	41.2	-11.2	0.4	-7.3	-1.1
ψ_t^F	-0.2	-18.2	-69.2	-10.8	80.7	4.7	12.9
y_t	-0.1	-1.7	2.7	-1.0	0.1	0.2	-0.1

Table 3.2: Variance decomposition: Changes

The negative change in the contribution of the monetary policy shock and risk premium originates from the design of my experiment when these shocks are eliminated in the post-transition model. The 64.3% decrease in the contribution of the taste shock ε^a to the volatility of change in the exchange rate shows that the exchange rate operates as a shock absorber in the estimated model. The taste shock ε^g become the dominant source of the domestic and CPI inflation volatility in the model of the post-transition regime, as the increases by 41.2% and 84.1% show. So offsetting of the nominal exchange rate changes makes the stability of inflation significantly more vulnerable to the domestic preference shock that acts as a demand shock in the estimated model.

As the exchange rate become less volatile in the model of the post-transition regime, foreign shocks become the major sources of macroeconomic volatility. The source of volatility in LOP gap (ψ_t^F) shifts from domestic preference and monetary shock towards foreign inflation shock (80.7%) and foreign interest rate (12.9%). This indicates that profits of importers become very sensible to shocks originating in the foreign economy in the post-transition period. This also applies for imported inflation because importers' profits are closely connected with changes in foreign price level. The reason for this change is that the stable exchange rate is not able to work as a shock absorber for foreign shocks. Therefore, all foreign shocks are directly transferred to the domestic economy.

A significant shift in sources of volatility occurs for domestic interest rates as the monetary policy focuses on the exchange rate. For the interest rate, all domestic sources of volatility are eliminated and volatility is almost fully driven by foreign shocks; 87.8% shift toward foreign shocks. This originates from the increase in exchange rate stability while the domestic economy becomes more vulnerable to foreign demand shocks. Also, the quite high persistency of foreign output and interest rate shocks is the reason that these shocks generate a large fraction (75%) of the domestic interest rate volatility.

There are no important shifts in sources of output gap volatility over the regimes. Output volatility remains mainly driven by preference, technology and foreign output shocks that act as the demand shock. As the contribution of the supply shock ε^a to interest rate is decreased, I can conclude that the demand shocks will be the dominant source of volatility.

3.4.2 Business cycles correlations

In the previous sections, my examples show how macroeconomic volatility is changing over the transition period. Also, the comparison of an estimated and a post-transition regime provides a closer look at the changes in the sources of inflation. As the adoption of a pegged or fixed exchange rate strengthens the links between economies, the transmission of disturbances is also increased. According to theories of currency areas, business cycle synchronization is a necessary condition for successful implementation and sustainability of pegged or fixed exchange rate regimes.

This section is devoted to the analysis of changes in the synchronization of business cycles between a small and large economy. Therefore, Figures 3.11–3.13 show the evolution of the correlations with foreign variables over the various transition period lengths; 2, 4, 8 and 12 quarters. To compute the correlations, the optimal policies for these lengths are used. For these computations $\tau = 0.75$ is chosen to reflect the preference for inflation stability as observed in the estimated rule, where the inflation targeting weight ρ_π is 2.9 times higher than output gap weight ρ_y.

As shown in Figure 3.11, the correlation of foreign inflation and exchange rate movements is suddenly changed to a value close to zero after the regime switch because under the post-transition rule changes in the exchange rate are significantly eliminated. This indicates that the exchange rate loses its shock-absorbing nature. As expected, domestic inflation is becoming more correlated with foreign inflation over the transition period via the imported goods channel. Interestingly, at the end of the transition period this correlation drops temporarily. A similar pattern is observed for the correlation of foreign inflation and domestic nominal interest rate. This indicates that the monetary authority trades-offs exchange rate inflation targeting for exchange rate stability at the end of transition. After transition is over, the increase of this correlation continues as domestic monetary authority has to follow changes in imported goods prices while these are not absorbed by the exchange rate movements.

As shown in Figure 3.12, a steep increase in the correlation of foreign and domestic interest rate is observed. As the focus of a post-transition regime is a stable exchange rate, domestic monetary policy has to eliminate the pressures for exchange rate change originating from change in foreign interest rate that is transferred via UIP. The steep increase in the foreign interest rate and changes in nominal exchange rate is also observed. Over the transition the domestic monetary authority does allow for changes in the exchange rate that helps as a shock absorber for foreign shocks. Therefore, the correlation of foreign interest rate and domestic CPI inflation is close to zero or negative. However, the focus on stability of the exchange rate eliminates this shock absorbing feature so the steep increase in this correlation is achieved after the regime change. Figure 3.12 shows that the domestic monetary authority strongly reacts to changes in foreign interest rate. Also, domestic output is getting more positively correlated with foreign interest rate, while the UIP implies more depreciation pressures as a reaction to the foreign interest rate increase. However, these changes in correlation are relatively small.

Further, Figure 3.13 shows a correlation with foreign output. Also, in here an increase in domestic-foreign output synchronization is observed. These correlation changes are small while the increase in CPI inflation-foreign output correlation signals that the price is increased in response to higher foreign demand for domestic goods. Therefore, the positive value of foreign output-domestic interest rate correlation over the transition is a result of inflationary pressures that originate from changes in foreign demand. These pressures require a response by the domestic monetary authority to suppress inflation. Also the negative value of the exchange rate-foreign output correlation shows that the exchange rate is helping to absorb the output shock. Figure 3.13 also shows a drop in correlation of domestic nominal interest and exchange rates with foreign output at the end of transition. This shows that in the last periods of transition, the domestic monetary policy is less contractive while the changes in foreign demand are absorbed by the exchange rate.

3.5 Policy implications

A very important concern of the monetary authority of a small open economy is its influence on inflation and output. Figure 3.14 shows the evolution of the correlation of inflation, output and exchange rate changes with domestic nominal interest rates over the transition. In these plots, the optimal policies for various lengths of the transition are considered as in the previous section.

The inflation-interest rate correlation drops mainly in the initial and late phase of the transition. The initial drop is originating from the announcement of the policy rule change. At this point, households realize that in future the inflation stability will be not the main concern of the monetary authority. The plot for inflation-interest rate correlation shows that the monetary authority loses its control over domestic CPI inflation rapidly in the transition. The second drop in its influence over inflation occurs in the last periods of the transition when monetary policy is at the most contractive level for output.

Consistently with the experiment design, interest rate gets more correlated with the changes in the exchange rate over the transition. This correlation reaches almost unity in the post-transition regime, as the increase in the domestic interest rate is used to eliminate the depreciation of the exchange rate.

Interestingly, the correlation of output and interest rate is initially negative, as the increase in interest rate leads to a contraction of output. As the output-interest rate plot in Figure 3.14 shows, monetary policy is gaining more contractionary power towards the end of the transition. However, after the regime is changed, the increasing interest rate losses its contractionary nature. This loss originates from the nature of the new regime, under which the increase in interest rate is closely related to depreciation under the post-transition regime, as the interest-exchange rate plot shows.

3.6 Conclusions

In this chapter, I analyze the effects of an announced transition towards the regime of pegged exchange rate for the small open economy. Therefore, the model of the credible and foreseen regime switch is needed to create. I do this by extending the standard model of the small open economy with the binary regime indicator and information buffer that makes the changes of indicator foreseen.

In the presented model of transition towards the pegged exchange rate, the announcement of the change is modeled as the realization of information shocks that are entering the information buffer.

To parameterize the model, its parameters are estimated via Bayesian method using data on the Czech Republic. The properties of the estimated model are examined via the impulse response functions. The impulse responses are computed for the estimated model with respect to the various lengths of the transition toward the pegged exchange rate regime.

Further, setting up the ad-hoc loss function allows me to compute simple optimal policies for the transition period with the respect to preferences for inflation-output stabilization and length of transition. Generally, the optimal policies are able to deliver a lower loss for long transition periods and under the strong focus on output stability. The monetary policies delivering the lowest loss are characterized by very low interest rate smoothing and low weight on inflation targeting.

The business cycle synchronization analysis shows that there are significant changes in the correlations of inflation, interest rate and exchange rate changes. The correlation of domestic variables and the interest rate shows that in the last period of transition, the contractionary effect of the interest rate is reaching its maximum. While after the adoption of the rule of the pegged exchange rate, increases in the interest rate becomes a sign of expansion as the result of reaction to expected depreciation.

3.A Transition period model

The solution of the transition period model given by equations (3.13)–(3.25), and equations (3.27) takes the following general form:

$$x_t = F(x_{t-1}, \varepsilon_t, \nu_t),\ 0 < t \leq T$$

where x_t is the vector of the model variables, $\varepsilon_t = \{\varepsilon_t^\pi, \varepsilon_t^y, \varepsilon_t^i, \varepsilon_t^a, \varepsilon_t^m, \varepsilon_t^g, \varepsilon_t^s\}$ is the vector of foreign and domestic structural shocks, $\nu_t = \{\nu_{t,1}, \ldots, \nu_{t,N}\}$ is the vector of information shocks, and $F(.)$ is the second-order polynomial. However, due to the independence of information and structural shocks after the evaluation of information shocks (an announcement of the transition), the system will be become linear. The evaluation takes the form given by scheme (3.28) and $\nu_{t,i} = 0$, $\forall i$ and for all subsequent periods t, $1 < t \leq T$. Therefore, the transition period model with a given length of the transition period takes the following form:

$$x_t = A_t x_{t-1} + B\varepsilon_t,\ 0 \leq t \leq T \tag{3.32}$$

where matrices A_t, $t = 0, \ldots, N$ and matrix B depend on the structural parameters of the model and the transition period length. Matrix B is time invariant while the structural shocks are independent. However for $t_1, t_2 > T$, I have $A_{t_1} = A_{t_2}$ because ν_t for $t > 1$ is a vector of zeros and after period T the information buffer is filled only with zeros.

The state-space solution conditional on evaluation of the information shocks is used to simulate the model and compute the covariance matrices Σ_t. To compute the covariance matrix Σ_t recursively the following formula is used:

$$\Sigma_t = A_t \Sigma_{t-1} A_t^T + BVar(\varepsilon_t)B^T, \; 0 < t \leq T \tag{3.33}$$

where Σ_0 is the covariance matrix from the model estimated on data, $Var(\varepsilon_t)$ is time invariant covariance matrix of structural shocks. Further, to compute the evolution of variance after the change of regime, the following recursive formula for $t > T$ is used:

$$\Sigma_{t+1} = A^f \Sigma_t A^{f^T} + B^f Var(\varepsilon_t) B^{f^T}, \; t > T \tag{3.34}$$

where matrices A^f and B^f are taken from the solution of the model with the monetary policy rule given by equation (3.26) for $regime_t = 0$.

3.B Estimation

3.B.1 Data description

All data in the estimation are from the Czech National Bank database. Series are seasonally adjusted with TRAMO/Seats and X12. All observed series are measured at quarterly frequency and filtered. Series are in logs; therefore they can be interpreted as the percentage deviations from steady state levels.

- Domestic output growth (ΔGDP_t) is the HP de-trended annualized logarithm of real GDP growth.
- Domestic CPI inflation deviation (PI_t) is the HP de-trended annualized quarterly growth rate of the logarithm of the consumer price index (CPI).
- Foreign good inflation (PIF_t) is the HP de-trended annualized quarterly logarithm of the growth rate of imported good price (in domestic currency) index.
- Nominal interest rate (RS_t) is the HP de-trended annualized quarterly value of the 3-month PRIBOR.
- Real exchange rate (Q_t) is the HP de-trended quarterly value of the real exchange rate.
- Foreign output gap (GDP_t^*) is the real GDI gap for an effective Eurozone created by the use of the export values weights and de-trended by the Kalman filter.
- Foreign real interest rate (RS_t^*) is the HP de-trended annualized quarterly value of the 3-month EURIBOR.

- Foreign inflation (PI_t^*) is the HP de-trended annualized quarterly growth rate in the log of consumer price index for the effective Eurozone (export weights).

All series used for estimation cover the period from the first quarter of 1998 to the second quarter of 2007.

3.B.2 Measurement block

For my estimation the following measurement block is used to relate model variables to observed time series data:

$$\begin{aligned}
\Delta GDP_t &= 4*(y_t - y_{t-1} + \varepsilon_t^a) + \varepsilon_t^{GDP} \\
PI_t &= 4*\pi_t + \varepsilon_t^{PI} \\
PIF_t &= 4*\pi_t^F + \varepsilon_t^{PIF} \\
RS_t &= 4*i_t + \varepsilon_t^{RS} \\
Q_t &= q_t + \varepsilon_t^{Q} \\
PI_t^* &= 4*pi_t^* + \varepsilon_t^{PI^*} \\
RS_t^* &= 4*i_t^* + \varepsilon_t^{RS^*} \\
GDP_t^* &= y_t^* + \varepsilon_t^{GDP^*},
\end{aligned}$$

where I assume that $\varepsilon_t^{GDP}, \varepsilon_t^{PI}, \varepsilon_t^{PIF}, \varepsilon_t^{RS}, \varepsilon_t^{Q}, \varepsilon_t^{PI^*}, \varepsilon_t^{RS^*}, \varepsilon_t^{GDP^*}$ are independent normally distributed with zero mean. For estimation I assume that the standard deviations of the measurement errors take following values 0.25, 0.5, 0.3, 2.0, 1.0, 0.1, 0.1, 0.1 (in the given order).

3.B.3 Priors and posteriors

The following tables summarize the distribution type and parameters choice (mean, and standard deviation) of prior distributions used to estimate the parameters of posterior distributions (mode and standard deviation).

Variable	Description	Prior			Posterior	
		Distr.	Mean	s.d.	Mode	s.d.
β	Discount factor		0.99			
α	Degree of openness	Beta	0.40	0.05	0.35	0.04
η	Elasticity of F-H substitution	Gamma	1.50	0.50	0.27	0.07
δ	Degree of inflation indexation	Beta	0.70	0.10	0.56	0.13
σ	Inverse elasticity of substitution	Gamma	0.90	0.50	0.92	0.29
φ	Inverse elasticity of labor supply	Gamma	1.50	0.50	1.08	0.48
θ_F	Calvo pricing - foreign	Beta	0.50	0.10	0.22	0.04
θ_H	Calvo pricing - domestic	Beta	0.50	0.10	0.26	0.04
h	Degree of habit formation	Beta	0.80	0.10	0.65	0.11
ρ_i	Interest rate smoothing	Beta	0.50	0.05	0.58	0.04
ρ_π	Response to inflation	Gamma	1.50	0.20	1.38	0.23
ρ_y	Response to output gap	Gamma	0.50	0.10	0.47	0.09
ρ_e	Response to ex. rate change	Gamma	0.10	0.05	0.04	0.02
ω_{11}	Foreign VAR	Normal	0.70	0.30	0.18	0.18
ω_{12}	Foreign VAR	Normal	0.00	0.20	0.10	0.04
ω_{13}	Foreign VAR	Normal	0.00	0.20	-0.14	0.16
ω_{21}	Foreign VAR	Normal	0.50	0.30	-0.07	0.22
ω_{22}	Foreign VAR	Normal	0.70	0.20	0.93	0.06
ω_{23}	Foreign VAR	Normal	-0.10	0.20	-0.09	0.18
ω_{31}	Foreign VAR	Normal	1.50	0.20	0.27	0.09
ω_{32}	Foreign VAR	Normal	0.50	0.20	0.05	0.02
ω_{33}	Foreign VAR	Normal	0.70	0.30	0.58	0.13
ρ_a	Technology - VAR(1)	Beta	0.85	0.10	0.83	0.11
ρ_s	Ex. rate risk - VAR(1)	Beta	0.85	0.10	0.59	0.20
ρ_g	Taste shock - VAR(1)	Beta	0.85	0.10	0.95	0.02

Table 3.3: Results from posterior parameters (parameters)

Variable	Description	Prior			Posterior	
		Distribution	Mean	s.d.	Mode	s.d.
ε^π	Foreign inflation	$Gamma^{-1}$	0.60	0.50	0.18	0.02
ε^y	Foreign demand shock	$Gamma^{-1}$	0.30	0.50	0.30	0.03
ε^i	Foreign monetary shock	$Gamma^{-1}$	0.30	0.50	0.08	0.01
ε^a	Domestic technology shock	$Gamma^{-1}$	0.80	0.50	0.25	0.03
ε^m	Domestic monetary shock	$Gamma^{-1}$	0.30	0.10	0.44	0.07
ε^g	Domestic preference shock	$Gamma^{-1}$	1.50	0.50	3.07	0.43
ε^s	Risk premium shock	$Gamma^{-1}$	1.00	0.50	0.34	0.05

Table 3.4: Estimation summary: Standard deviation of structural shocks

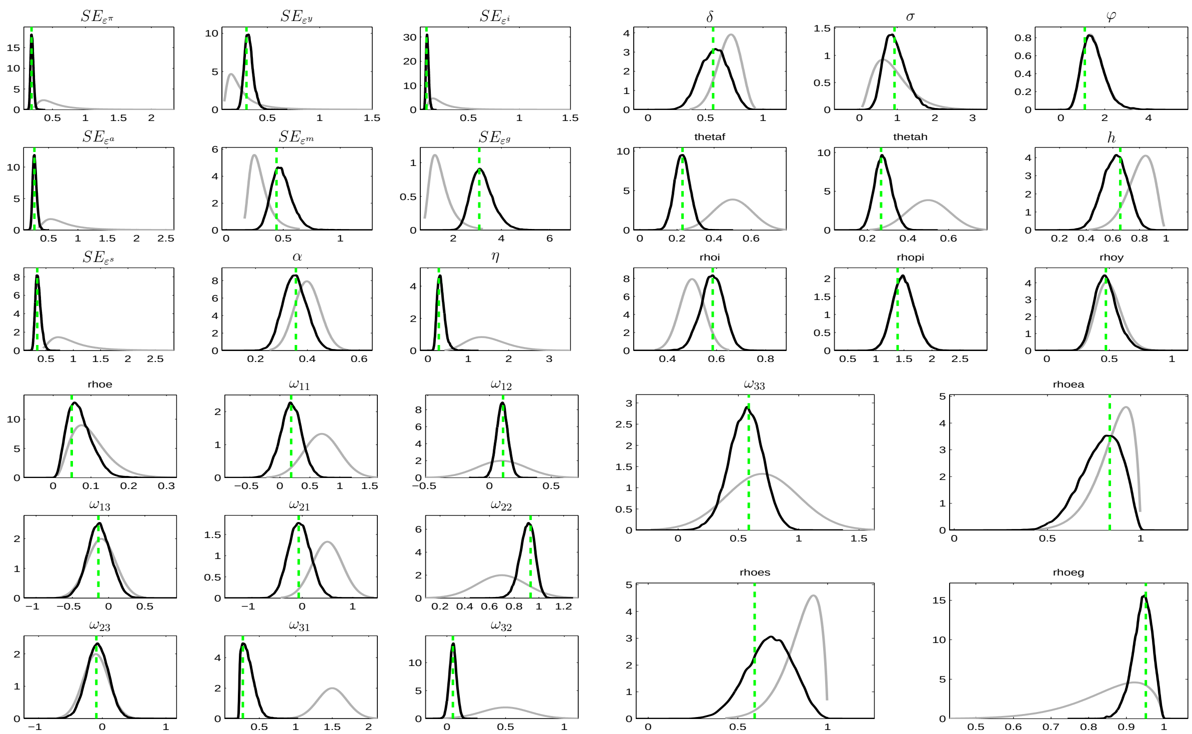

Figure 3.1: Priors and posterior distributions

3.C Impulse response functions

Here, the dash-dotted red line represents an estimated model; the magenta solid line is for regime switch in 4; the dashed blue line in 8; and the dotted black line in 40 periods. The results are presented as quarterly percentage deviations from the steady state.

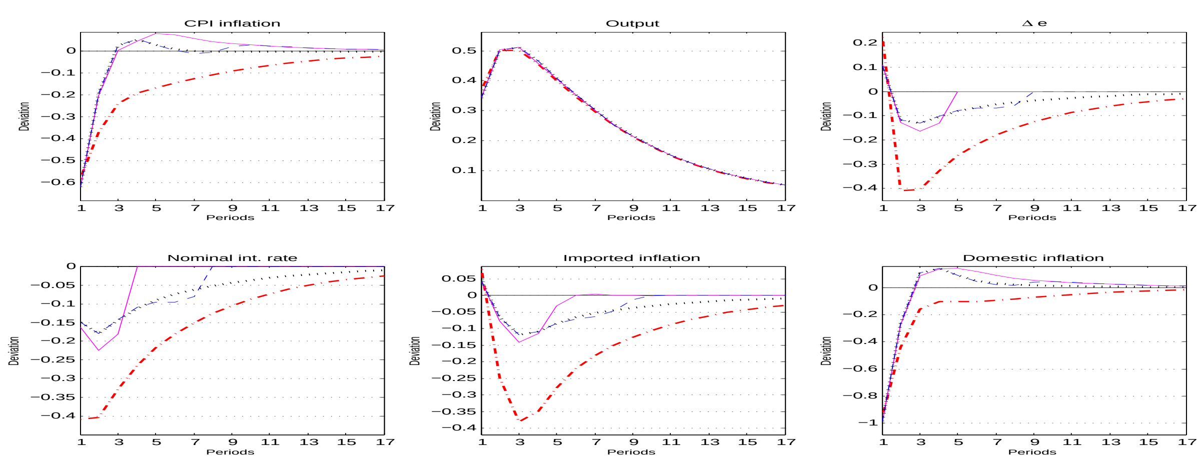

Figure 3.2: IRF comparison - Response to technology shock ε^a

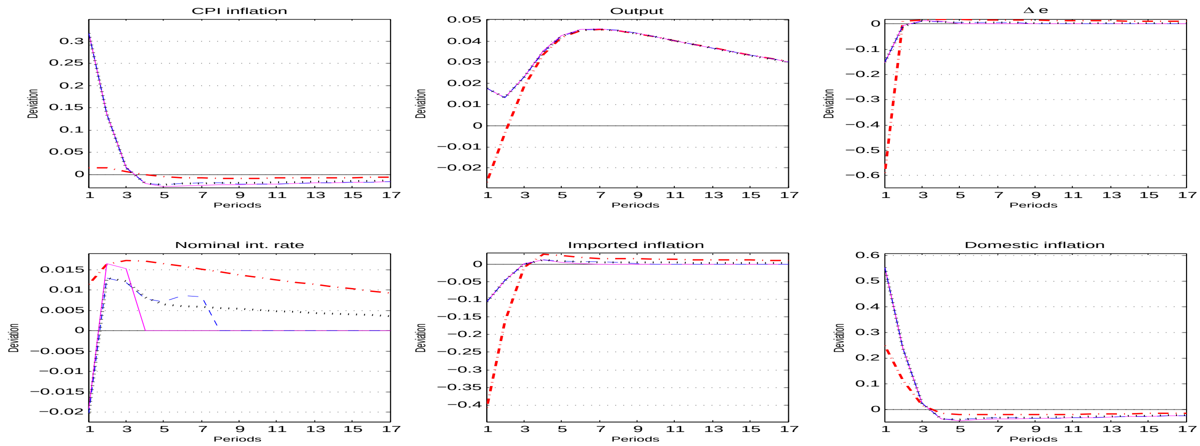

Figure 3.3: IRF comparison - Response to preference shock ε^g

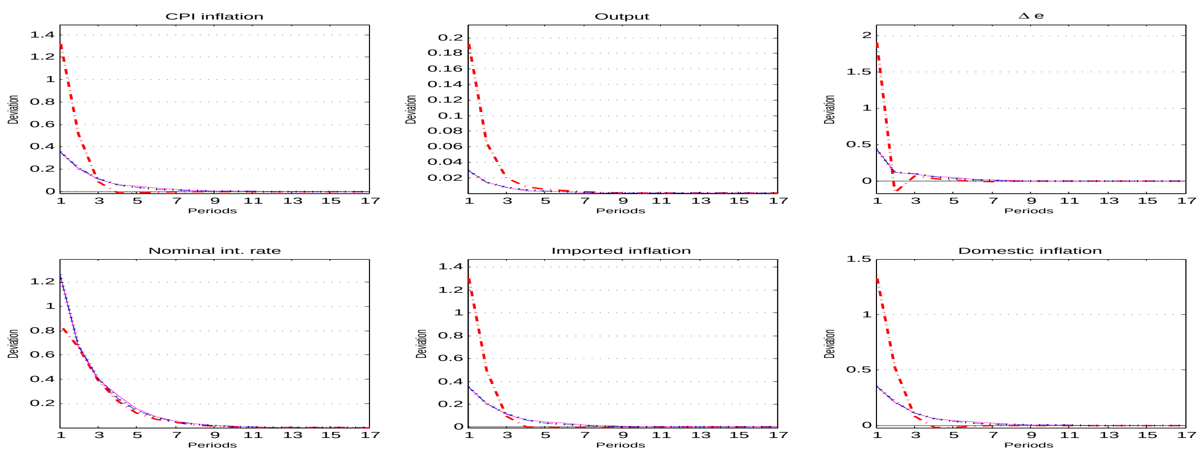

Figure 3.4: IRF comparison - Response to risk premium shock ε^s

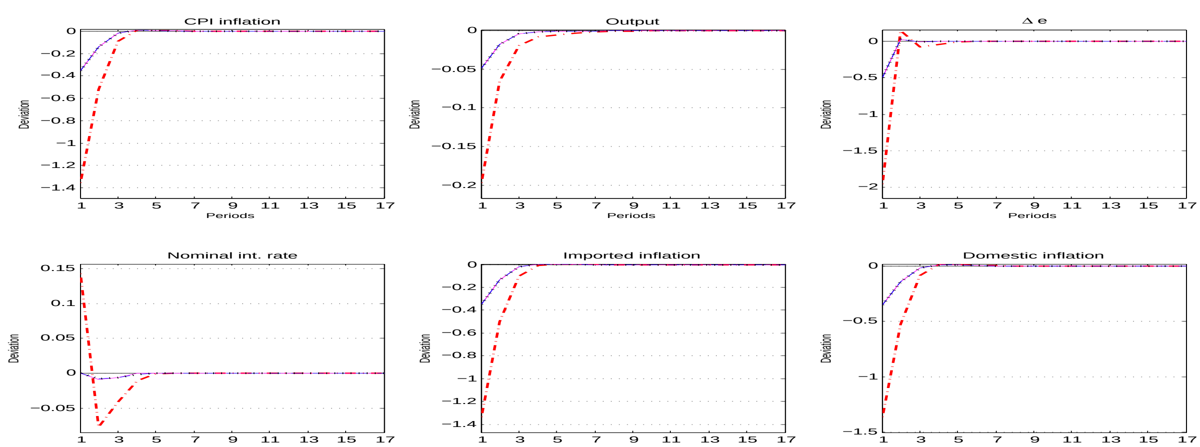

Figure 3.5: IRF comparison - Response to policy shock ε^m

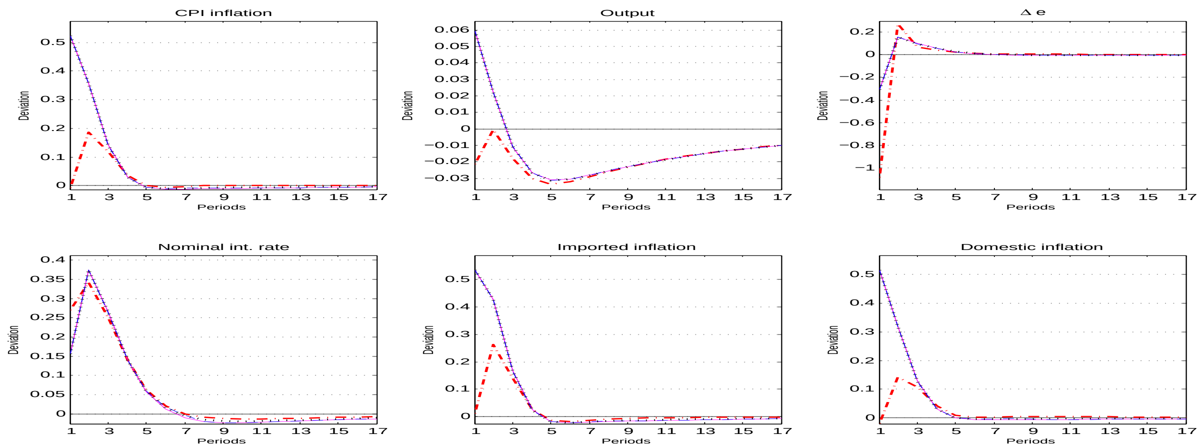

Figure 3.6: IRF comparison - Response to foreign inflation ε^{π}

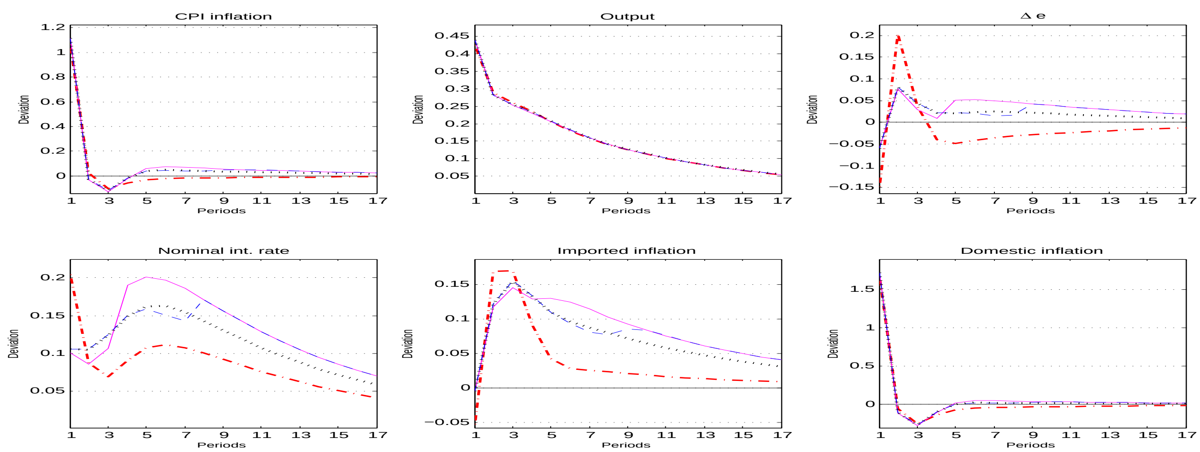

Figure 3.7: IRF comparison - Response to foreign output ε^{y}

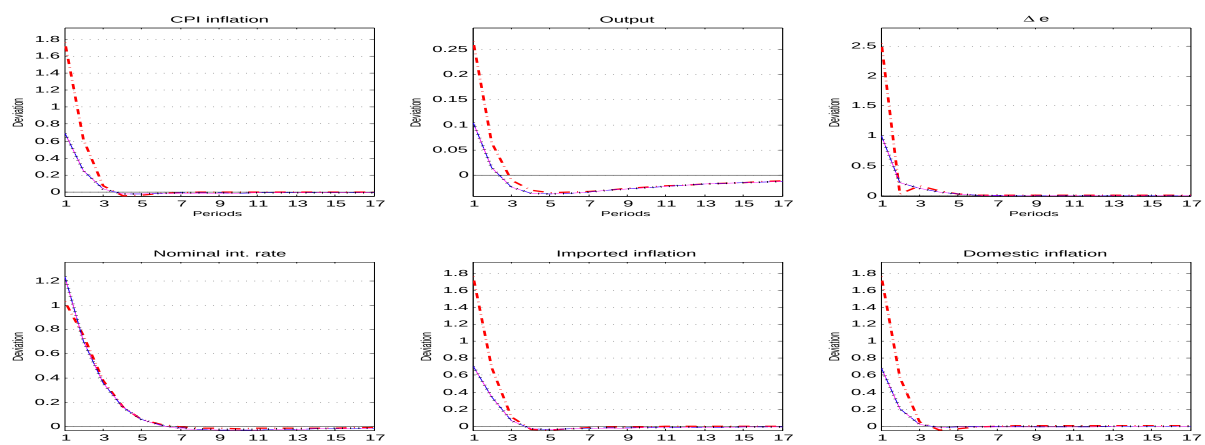

Figure 3.8: IRF comparison - Response to foreign interest rate ε^{i}

3.D Volatility and loss evaluation

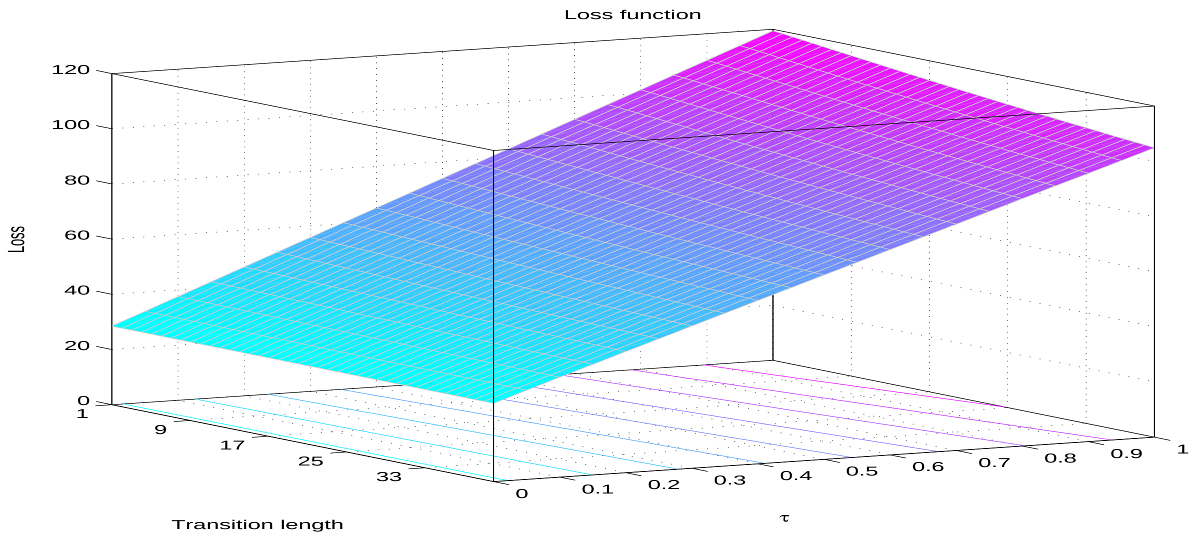

Figure 3.9: Loss function: Different specifications and transition lengths

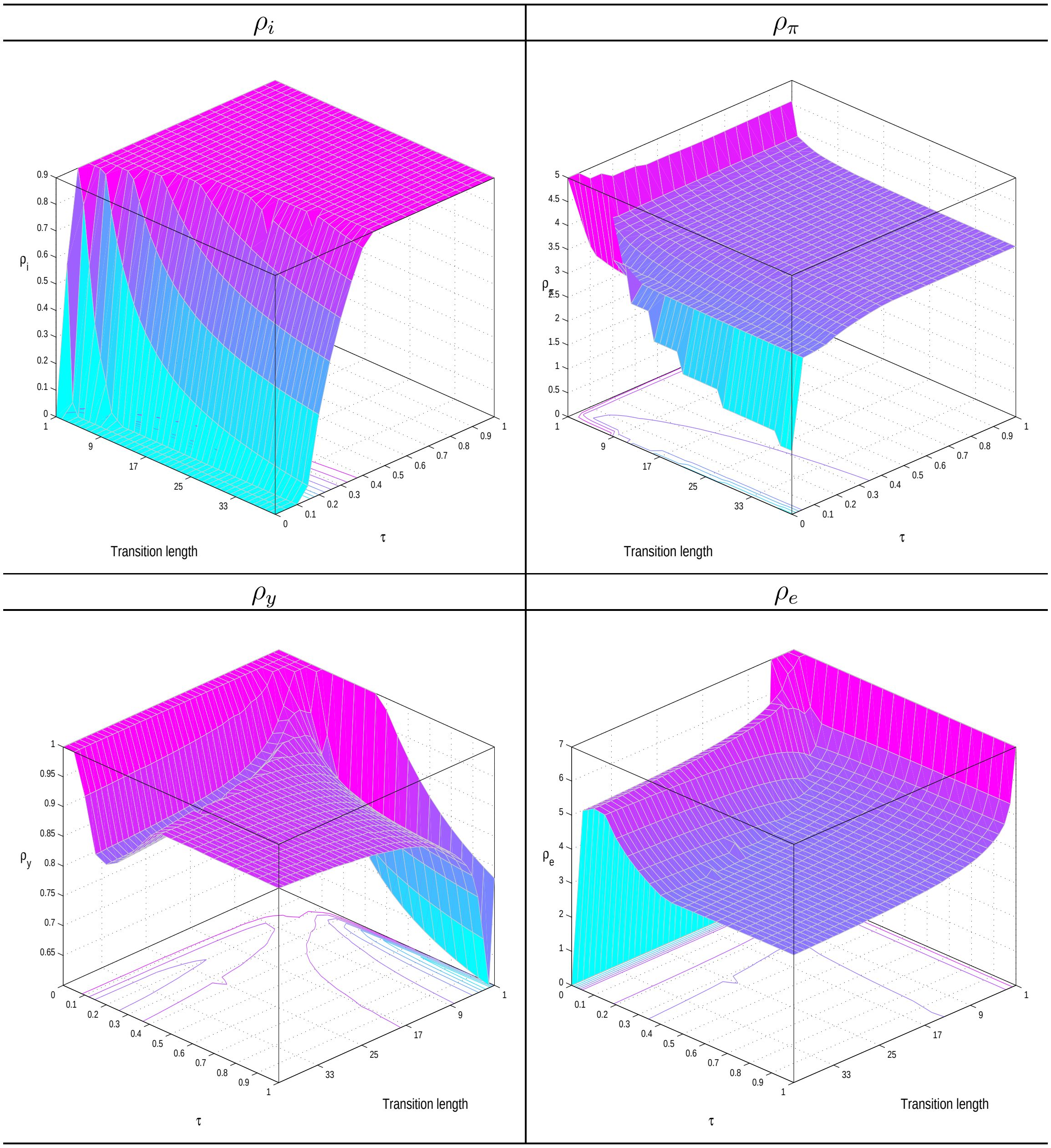

Figure 3.10: Optimal policies: Different specifications and transition lengths

3.E Cycles synchronization

Here, the dash-dotted red line is for a policy switch in 2 periods; the magenta solid line is for regime switch in 4; dashed blue line in 8; the dotted black line in 12 periods. The results are presented as quarterly percentage deviations from the steady state.

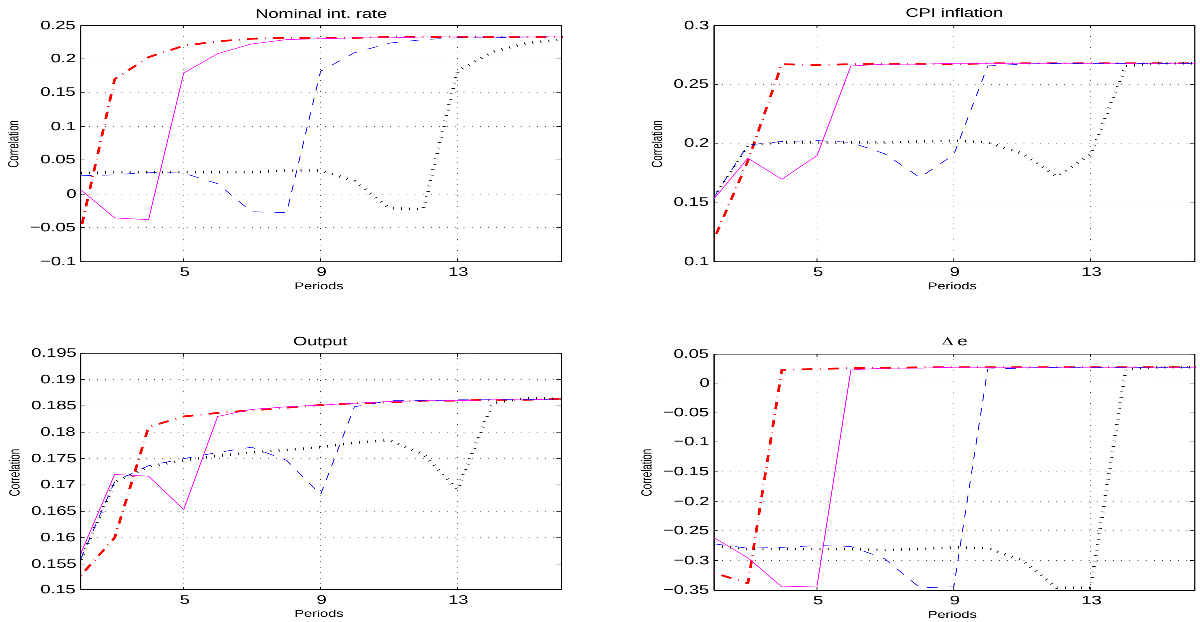

Figure 3.11: Correlation: π_t^*

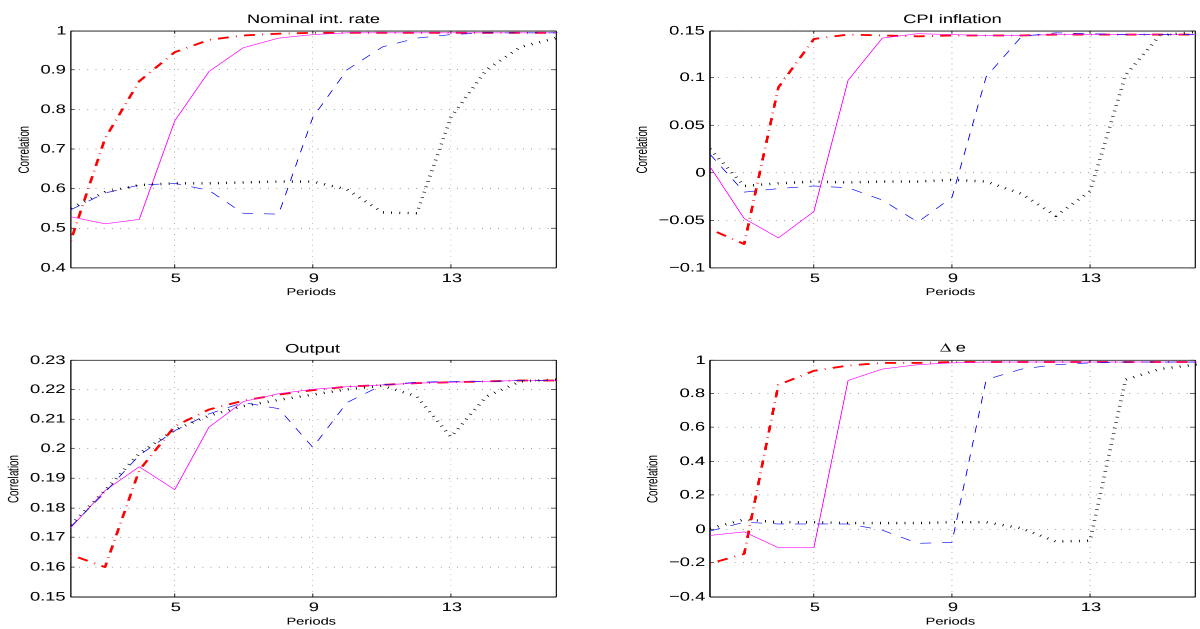

Figure 3.12: Correlation: i_t^*

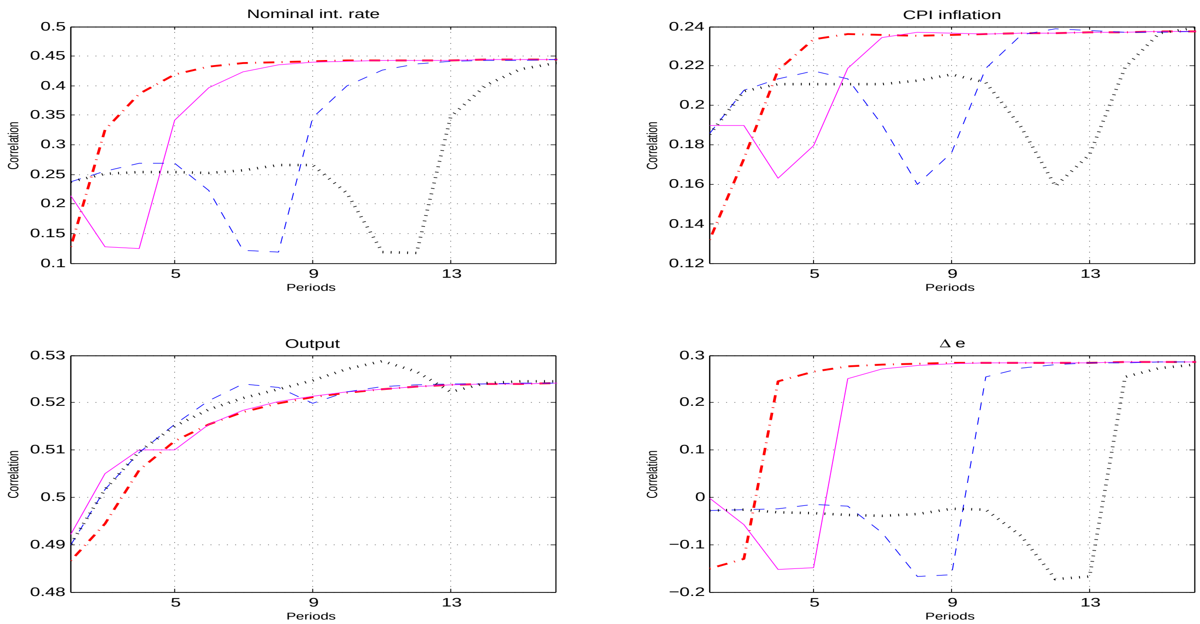

Figure 3.13: Correlation: y_t^*

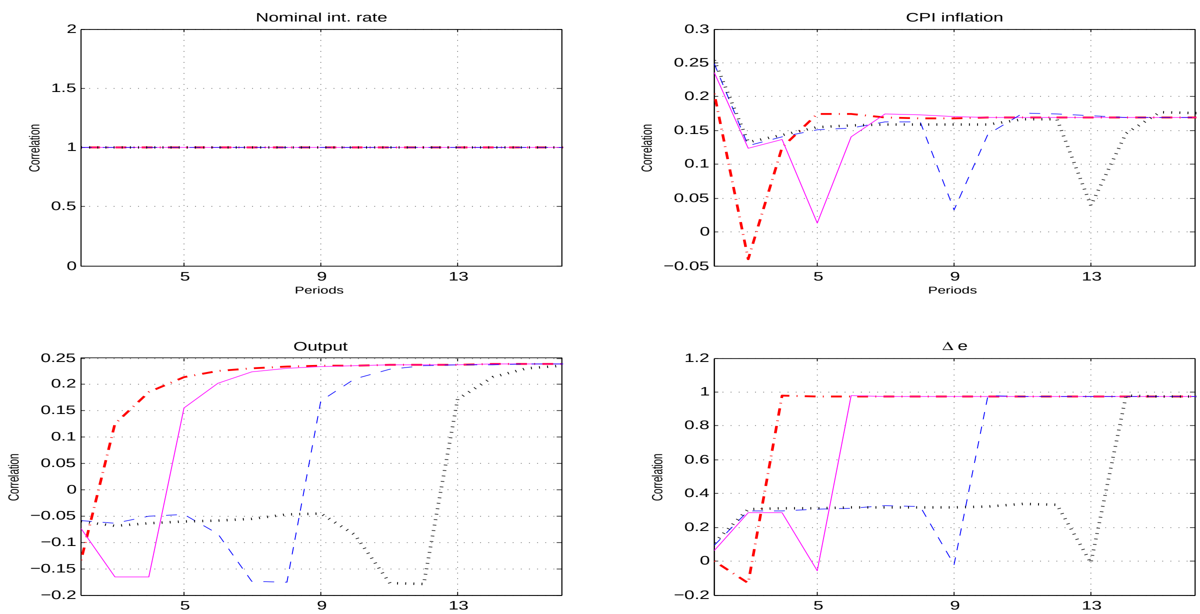

Figure 3.14: Correlation: i_t

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