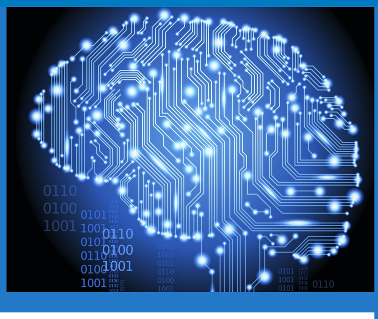
Department of Machine Learning

Institute of Computer Science, CAS Prague



Analysis of data-mining methods based on one-to-one neural networks.

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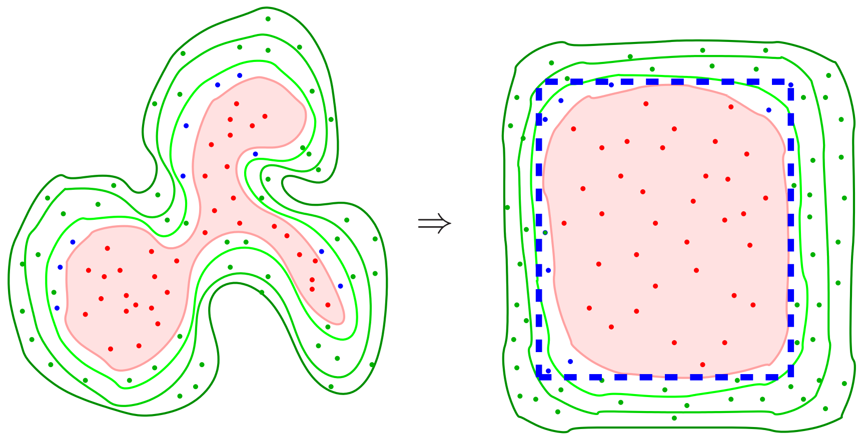
Topic Description

One-to-one property of mapping between two Euclidian spaces, used for the solution of the tasks of separation of sets, allows to analyse separated (or rather approximated) sets from their geometric point of view and define the “inside of the set“ in the ordinary sense of the word, and to select from the separated set patterns that are situated “inside“. In case of one-to-one mapping, this is made possible by the fact that every two disjunctive sets must be mapped again onto disjunctive sets.

Basic Principle of Utilizing One-to-one Mapping

The points in the border regions of the studied set (marked in green) are mapped into the “border” region of the one-to-one image of this set. The points we will regard as ”inner” – marked in red – are mapped into the ”inner” area of the set’s image. Thanks to the target shape of the mapped set, defined beforehand, it is possible to define the inner area of the mapped set in the image space (marked in blue), and the points mapped into this inner region may then be regarded as the inner points of the original set. The points marked in blue are, thanks to assumed continuity of the applied one-to-one mapping, still close to the inside of the set.



It is evident that by taking away points of the original set that will be mapped into the inner (blue) area, we will reduce the size of the original set, while retaining the information concerning its geometric shape. This procedure may be used for reducing the size of data sets. At the same time, we may define more “inner” sets, ordered by inclusion, and evaluate points of the original data set according to whether they belong to those “inner“ sets we have created. The actual values of this evaluation can then be perceived as a expectation measure that a given point constitutes an inner one. This may have a practical impact in such applications where the state of the system is continuously dependent on the parameters, and where it is, in specific areas, dependent on those parameters proportionately – e.g. the higher the value of the parameter the greater approaching the system to its undesirable state. If, furthermore, the system can be externally managed, one-to-one property may be utilized in the opposite direction, whereby we can keep the system’s parameters outside the blue area, which, in turn, guarantees existence of the system in an feasible set of states.

One-to-one Neural Network of the MPL Type

Artificial neural networks, which may be adjusted to perform one-to-one mapping, constitute one type of methods used for the separation of sets. One-to-one neural networks are derived from the standard models of multi-layered neural networks, while their topology and parametric space are modified to guarantee one-to-one mapping between the input and output space of a network. A major task in synthesizing these networks is to propose a suitable restriction of architecture and parameter in a way to achieve one-to-one property, with the broadest possible degree of freedom of the parameters used. Moreover, it is necessary to design a suitable algorithm for the network’s learning, when it is impossible to employ the standard gradient iteration methods, which do not reflect the specified restrictions of the parametric space.

Theorem of the One-to-one of Layered Neural Networks

Let us assume that $\mathbf{A}_i, \mathbf{H}_i, \mathbf{B} \in R^{n\times n}$, $\mathbf{f}_i \in R^n$, $F : R^n \Rightarrow R^n$, $\frac{\partial F_i}{\mathbf{x}_i} > 0$, $\frac{\partial F_i}{\mathbf{x}_j} = 0$, $i, j \in \{1, \dots, L\}$. Then the matrix $\sum_{i=1}^{L} \mathbf{A}_i\mathbf{D}_i\mathbf{H}_i + \mathbf{B}$ belongs to the class P for any positive diagonal matrices $\mathbf{D}_i$, $i \in \{1, \dots, L\}$ if and only if the equation

$$\sum_{i=1}^{L} \mathbf{A}_i F\left(\mathbf{H}_i\mathbf{x} - \mathbf{f}_i\right) + \mathbf{B}\mathbf{x} = \mathbf{c} \qquad (1)$$

has exactly one solution for any vector $\mathbf{c}$. Morever, this solution is continuously dependent on the vector $\mathbf{c}$.

Practical Applicability of One-to-one property

Practical applicability of one-to-one networks may be envisaged in areas where the need to respond to data in separated sets has a continuous nature and the data, which are inside the sets, are more critical than the data on the surface (e.g. in medical applications, tasks involving evaluation of data without categorial variables, systems monitoring technological processes continuously dependent on external parameters etc.).

Topics You Could Address

- General study of necesary and sufficient conditions for the one-to-one property of mapping. Stipulation of conditions for parametric space of mapping maintaining one-to-one property.
- Derivation of learning algorithms for one-to-one separation machines, for instance gradient methods with restricting conditions or other methods of global optimization facilitating capture of restricting conditions in a definition set.
- Application of one-to-one separators to selected real–life data.

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