



ROBUST ESTIMATION IN NEURAL NETWORKS

JAN KALINA

Motivation

The task is approximation of an unknown continuous real function f , which is modelled as a response of some given independent variables (regressors). This task of function approximation, together with a prediction for new observations, can be solved by neural networks. Particularly multilayer feedforward neural networks are suitable for function approximation in various applications. Most commonly, they use an identical activation function in this task, i. e. they can be described as

$$\hat{f}(x) = w^T x + b, \quad x \in \mathbb{R}^p,$$

where unknown function f is estimated by \hat{f} , $x \in \mathbb{R}^p$ are regressors and $w \in \mathbb{R}^p$ with $b \in \mathbb{R}$ are unknown constants (parameters).

Because the common approach to fitting neural networks is vulnerable to the presence of outlying values (outliers) in the data, the aim will be to propose and investigate a robust approach, particularly based on the least weighted squares (LWS) estimator. In both linear and nonlinear regression, the LWS estimator attains a high robustness (for data contaminated by outliers) as well as high efficiency (for non-contaminated data), if suitable weights are used.

A critical description of the back-propagation algorithm

The back-propagation algorithm is commonly used for function approximation networks. It minimizes the total error computed across all data values of the training data set. The algorithm is based on the least squares method, which is optimal for normally distributed random errors in the data. After an initiation of the values of the parameters, the forward propagation is a procedure for computing weights for the neurons sequentially in particular hidden layers. This leads to computing the value of the output and consequently the sum of squared residuals computed for the whole training data set. To reduce the sum of squared residuals, the network is sequentially analyzed from the output back to the input. Particular weights for individual neurons are transformed using the optimization method of the steepest gradient.

Available robust approaches for neural networks

A robust version of fitting multilayer feedforward networks for the task of function approximation for contaminated data was described for specific kinds of neural networks so far, mainly for multilayer feedforward networks. Neural networks based on robust multivariate estimation using the minimum covariance determinant (MCD) estimator were studied in [6]. Other approaches included the study of M-estimators within neural networks and their influence function [2,5], i.e. a local measure of their robustness. Neural networks based on nonlinear regression were studied in [7], where the nonlinear least trimmed squares (LTS) estimation was performed instead of estimating parameters of neural networks by means of the traditional nonlinear least squares.

Outline of the work

1. Basics of neural networks. Multilayer feedforward networks.
2. Back-propagation algorithm for estimating parameters of multilayer feedforward networks.
3. Numerical experiments on various data sets. A study of sensitivity of the networks to the presence of outliers.
4. Basics of robust regression estimation. The least weighted squares in the linear and nonlinear regression model, various weighting schemes. Implementation.
5. Proposal of a robust back-propagation algorithm based on the least trimmed squares.
6. Implementation of the robust approach to training multilayer feedforward networks.
7. Numerical experiments on various data sets, comparison of classical and robust approaches.
8. Conclusions. Hopes and challenges of the robust estimation within the task of training neural networks.

Possible applications

All possible tasks of function approximation with data contaminated by outlying values, e.g. in engineering, medicine, econometrics or management etc.

References

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