Czech Science Foundation - Part C

Description and Substantiation of the Project

Applicant: RNDr. Pavel Krejčí, CSc.

Name of the Project: Mathematical Modeling of Processes in Hysteretic Materials far from Equilibrium

1 Summary of present knowledge.

The occurrence of hysteresis is common in many fields of science, e.g., in ferromagnetism, ferroelectricity, micromagnetics, superconductivity, solid-solid phase transitions, and elastoplasticity, to name only a few. Dynamical systems exhibiting hysteresis are characterized by an input-output behavior that can no longer be represented by a relation in form of a simple function or of a multivalued graph. Instead, such systems carry a *memory* of earlier states that is reflected by complicated *nested loops* in the input-output behavior. Figure 1 shows a typical diagram of the dependence between the magnetic field h and magnetization m in a ferromagnetic material.

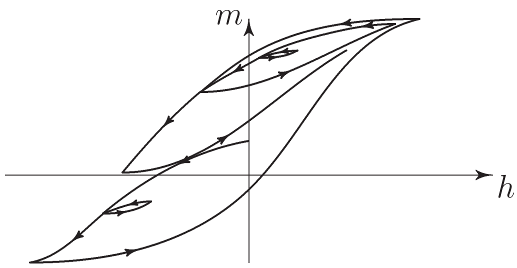

Figure 1: Hysteresis in ferromagnetism.

Despite its steadily increasing importance for industrial applications, the analysis of nonequilibrium systems with hysteresis is an area in which few researchers work systematically. The thermodynamics of hysteresis requires a very large (usually infinite-dimensional) state space of memory configurations having only limited regularity. In particular, the presence of memory has the consequence that the input-output behavior of hysteretic nonlinearities is of an intrinsically *nondifferentiable* nature and *nonlocal in time.* These facts render the mathematical treatment of partial differential equations containing hysteretic nonlinearities extremely difficult: nondifferentiability and nonlocality in time entail a *loss of compactness*, so that

– standard techniques for the derivation of a priori estimates do not apply,

– for limit processes with hysteresis nonlinearities the usual approach using weak convergence in L^p - spaces does not work; instead, *uniform* convergence with respect to the time variable is mandatory.

As a consequence, new techniques have to be designed to recover the compactness necessary for existence proofs, which is a challenging mathematical task. Following the pioneering works by L. Prandtl [26], F. Preisach [27], and A. Yu. Ishlinskii [9], in the first half of the last century, the mathematical modeling and treatment of hysteresis phenomena was initiated by the group around M. A. Krasnosel'skii in the early sixties, which culminated in the monograph [10]. Krasnosel'skii and his co-workers introduced the notion of *hysteresis operators* that turned out to be a very fruitful new mathematical concept. Later, these studies were continued and extended by other mathematicians, see the monographs [2, 13, 30]. We also mention the contributions [4, 22, 23, 28] by physicists and engineers.

We consider a special class of problems exhibiting hysteresis, namely *oscillatory elastoplastic processes*. It is well known that plastic deformations lead to *energy dissipation* and *material fatigue*. The effects of energy exchange between heat and mechanical energy, thermal stresses, and material fatigue therefore have to be taken into account.

The existing mathematical literature on the dynamics of elastoplastic processes with hysteresis appears to be very scarce, while there are many papers on quasistatic approaches; hence, any new mathematical results on the dynamics will be *innovative*. In particular, methods for estimating the lifetime of oscillating thermoelastoplastic structures under material fatigue are of great importance in applications.

A classical hysteresis-type model for one-dimensional elastoplasticity was introduced by L. Prandtl and A. Yu. Ishlinskii. In their model, the relation between (one-dimensional) strain ε and stress σ is given in the form of the so-called *Prandtl-Ishlinskii operator*

$$\sigma = \mathcal{P}[\varepsilon] = \int_0^\infty \varphi(r)\mathfrak{s}_r[\varepsilon]\,\mathrm{d}r\,. \tag{1.1}$$ 1.2

Here, φ is a nonnegative weight function, and $\mathfrak{s}_r$ represents the one-dimensional *elastic-ideally plastic element* or *stop operator* with the threshold $r > 0$ (see Figure 2 below). This model can easily be generalized to higher dimensions. as the solution operator of the variational inequality

$$\begin{cases} \frac{1}{r}\boldsymbol{\sigma}_r(t) \in Z & \forall t \in [0,T]\,, \\ \left\langle \partial_t(\boldsymbol{\varepsilon} - \boldsymbol{\sigma}_r), \frac{1}{r}\boldsymbol{\sigma}_r - \tilde{\boldsymbol{\sigma}} \right\rangle \geq 0 \quad \text{a.e.} & \forall \tilde{\boldsymbol{\sigma}} \in Z\,, \end{cases} \tag{1.2}$$ 1.3

where Z is a convex closed set.

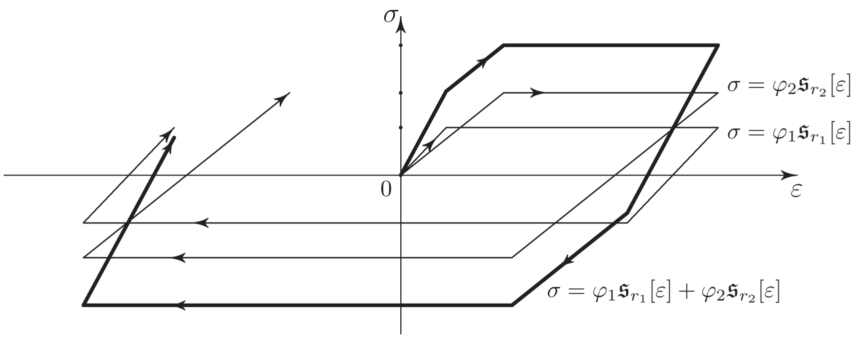

Figure 2: The stop operators and their combinations.

Although the Prandtl-Ishlinskii description of elastoplasticity as a superposition of infinitely many stop operators having different thresholds is very imaginative and easily understood, engineers very often prefer classical engineering approaches like the three-dimensional *von Mises* or *Tresca* models. One reason for their reluctance to use the Prandtl-Ishlinskii approach is the disadvantage that the weight function φ is not known a priori and must be identified.

Recently, in a series of groundbreaking papers, a new theory of oscillating elastoplastic beams and plates has been developed by the P. Krejčí and J. Sprekels. They demonstrated that the three-dimensional single-yield von Mises criterion leads after a dimensional reduction to a multi-yield Prandtl-Ishlinskii operator

- *for which the weight function could be determined explicitly.*

The new theory is based on the idea that a lower-dimensional observer does not see anymore the sharp transition from the purely elastic to the purely plastic regime as in the von Mises model.

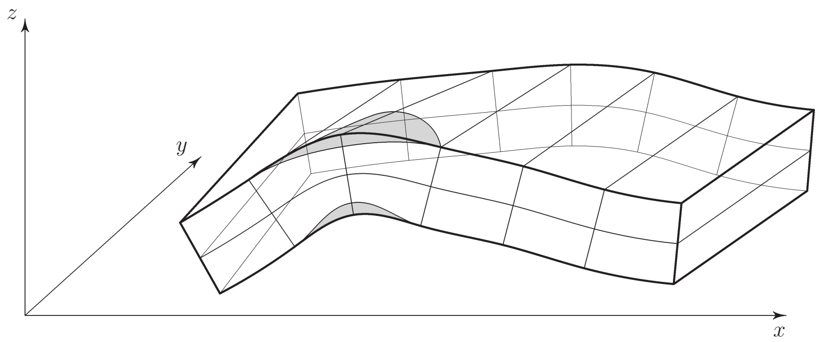

Figure 3. A plate section with grey plasticized zone.

Indeed, if a plate is subject to bending, small plasticized zones start forming at first near the boundary, and then propagate to the interior, which still preserves a partial elasticity (see Figure 3). Hence, when the transversal variable is eliminated, this gradual plasticizing is expressed in terms of the Prandtl-Ishlinskii superposition of single-yield stop operators that are successively activated (see Figure 2).

2 The aim of the project and schedule, expected outputs

The *main aim* of this proposal is to develop in a concentrated effort a consistent thermodynamic theory of oscillating thermoelastoplastic plates under material fatigue. The *basic modeling assumption* consists in replacing the elastoplastic constitutive law (1.1) by

$$\boldsymbol{\sigma} \;=\; \mathbf{B}(m)\,\boldsymbol{\varepsilon} + \int_0^\infty \varphi(\theta, r)\,\mathfrak{s}_{rZ}[\boldsymbol{\varepsilon}]\;\mathrm{d}r\,, \tag{2.1}$$

de6

where $\theta(x,t) > 0$ is the absolute temperature, and where $m(x,t) \geq 0$ represents the accumulated fatigue at the space point x during the time interval $[0,t]$. If the same scaling hypothesis as in [3] for elastic plates is applied to the elastoplastic case, then, upon normalizing physical constants to unity (which has no bearing on the mathematical analysis), the resulting *prototypical system* of partial differential equations is of the form

$$\partial_{tt} w \; - \; \partial_{tt} \Delta w + \mathbf{D}_2^* \boldsymbol{\sigma} \; = \; g \, , \tag{2.2}$$

$$\partial_t \mathcal{U}[\theta, \boldsymbol{\varepsilon}] + \operatorname{div} \mathbf{q} = \langle \boldsymbol{\sigma}, \partial_t \boldsymbol{\varepsilon} \rangle \; , \tag{2.3}$$

$$\boldsymbol{\sigma} \; = \; \mathbf{B}(m) \, \boldsymbol{\varepsilon} + \int_0^\infty \varphi(\theta, r) \, \mathfrak{s}_{rZ}[\boldsymbol{\varepsilon}] \; \mathrm{d}r \, , \tag{2.4}$$

$$\boldsymbol{\varepsilon} \; = \; \mathbf{D}_2 w \, , \tag{2.5}$$

$$\mathbf{q} = -\kappa \nabla \theta \, . \tag{2.6}$$

Here, (2.2)–(2.3) represent the balance laws of linear momentum and internal energy, (2.4)–(2.6) are constitutive equations, and appropriate initial and boundary conditions have to be prescribed. The main goal is to establish existence, uniqueness, and stability results for solutions to the above system of PDEs with hysteresis operators, and to design new robust numerical schemes for a reliable simulation. Extensions of the model to the Mindlin-Reissner plate theory and to curved elastoplastic structures are to be carried out by analogy to the elastic cases considered in [24, 25].

Notice that (2.5)–(2.9) is a highly nonlinearly coupled system of partial differential equations of hyperbolic-parabolic type in which hysteretic nonlinearities occur at different places. The treatment of this system constitutes an extremely difficult mathematical task.

The *main aim* is to expand the theory developed in [8] along the following lines:

1. Extension of the modeling process and of the existence theory to the oscillations of more general mechanical structures (Mindlin-Reissner plates, plane arches, curved rods and shells).

2. Inclusion of energy dissipation and material fatigue.

3. Development of a thermodynamically consistent vector Prandtl-Ishlinskii-type model for thermoelastoplastic materials to account for the energy exchange between heat and mechanical energy in oscillating systems.

4. Study of the asymptotic behavior of elastoplastic structures with hysteresis, development of efficient numerical methods for the solution of the resulting systems of partial differential equations with hysteresis.

In the following, we give a rough outlook on the milestones of the proposal:

1st year:

Mathematical models for longitudinal or transversal oscillations will be derived and studied for elastoplastic Mindlin-Reissner plates in the isothermal situation.

2nd-3rd year:

Inclusion of temperature dependence and fatigue into the models. Applications to plates and planar arches. Development of numerical methods and first simulations.

4th-5th year:

Extension of the models to thermoelastoplastic rods and shells. Inclusion of further physical phenomena, e.g., the so-called *shape memory effect*. Further development of numerical methods and simulations.

3 Relevance of the topic

Any modeling and mathematical progress along these lines is of immediate *interdisciplinary* use, since the techniques to be developed cannot only be employed by engineers working in elastoplasticity but also by scientists in other fields in which hysteresis occurs. In particular, in the engineering community working in micromagnetics there is a long tradition to use hysteresis operators of Prandtl-Ishlinskii or Preisach type, and since the applicant is in close contact with this community since many years, it will directly profit from the proposed research.

4 Conceptual and methodical approaches

4.0.1 Modeling

Jointly with R. B. Guenther and J. Sprekels, P. Krejčí recently derived a new equation for isothermal elastoplastic Kirchhoff plate oscillations in the form (see [8])

$$\partial_{tt} w \;-\; \partial_{tt}\Delta w + \mathbf{D}_2^* \boldsymbol{\sigma} \;=\; g\,, \tag{4.1}$$

$$\boldsymbol{\sigma} \;=\; \mathbf{B}\boldsymbol{\varepsilon} + \mathcal{P}[\boldsymbol{\varepsilon}]\,, \tag{4.2}$$

$$\boldsymbol{\varepsilon} \;=\; \mathbf{D}_2 w\,. \tag{4.3}$$

Here, $\mathbf{D}_2$ is the differential operator $(\partial_{xx}, \partial_{yy}, \partial_{xy})$, and $\mathbf{D}_2^*$ its formal adjoint. Moreover, Δ is the Laplace operator, $\mathcal{P}$ represents a vectorial Prandtl-Ishlinskii operator of the form (1.1), $\mathbf{B}$ is a matrix representing *kinematic hardening*, and g is a given distributed load. The term $-\partial_{tt}\Delta w$ represents the contribution to the momentum balance that is due to the rotational inertia of the transversal fibers. The mass density is assumed constant and normalized to unity.

The evolution takes place in a two-dimensional domain $(x, y) \in \Omega$ representing the midsurface of the plate in reference configuration, and in a time interval $t \in [0, T]$. The system is complemented with suitable initial and boundary conditions.

It is well known that plastic deformations lead to *energy dissipation* and *material fatigue*. It is therefore very important to look for models that account for the energy exchange between heat and mechanical energy in oscillating mechanical systems, as well as for a possibly finite lifetime due to material fatigue.

An extension to constitutive laws like (2.1) cannot be done in an arbitrary way, since general thermodynamic principles have to be respected. Following the idea of [17] in the case without fatigue, we define the *specific free energy* $\mathcal{F}$ associated with the constitutive law (2.1) by

$$\mathcal{F}[\theta,\boldsymbol{\varepsilon}] \;=\; c_V\theta(1-\log(\theta/\theta_c)) + \frac{1}{2}\,\langle \mathbf{B}(m)\,\boldsymbol{\varepsilon},\boldsymbol{\varepsilon}\rangle + \frac{1}{2}\int_0^\infty \varphi(\theta,r)\;\langle \mathfrak{s}_{rZ}[\boldsymbol{\varepsilon}],\mathfrak{s}_{rZ}[\boldsymbol{\varepsilon}]\rangle\;\mathrm{d}r\,, \qquad (4.4) \qquad \boxed{\texttt{de7}}$$

with some reference temperature $\theta_c > 0$ and the specific heat $c_V > 0$. The *specific entropy* $\mathcal{S}$ then has the form

$$\mathcal{S}[\theta,\boldsymbol{\varepsilon}] \;=\; c_V\log(\theta/\theta_c) - \frac{1}{2}\int_0^\infty \partial_\theta\varphi(\theta,r)\;\langle \mathfrak{s}_{rZ}[\boldsymbol{\varepsilon}],\mathfrak{s}_{rZ}[\boldsymbol{\varepsilon}]\rangle\;\mathrm{d}r\,, \qquad (4.5) \qquad \boxed{\texttt{de8}}$$

and the *specific internal energy* $\mathcal{U}$ becomes

$$\mathcal{U}[\theta,\boldsymbol{\varepsilon}] \;=\; c_V\theta + \frac{1}{2}\,\langle \mathbf{B}(m)\,\boldsymbol{\varepsilon},\boldsymbol{\varepsilon}\rangle + \frac{1}{2}\int_0^\infty (\varphi(\theta,r)-\theta\partial_\theta\varphi(\theta,r))\;\langle \mathfrak{s}_{rZ}[\boldsymbol{\varepsilon}],\mathfrak{s}_{rZ}[\boldsymbol{\varepsilon}]\rangle\;\mathrm{d}r\;. \qquad (4.6) \qquad \boxed{\texttt{de9}}$$

The problem consists in solving the system (2.2)–(2.6) under appropriate initial and boundary conditions. Here, $\mathbf{q}$ is the heat flux, and $\kappa > 0$ is the heat conductivity (assumed constant).

In order that (2.2)–(2.6) be a complete system, we need a constitutive law for the fatigue parameter $m(x,t)$. It can be derived from the *Second Principle of Thermodynamics*, expressed by the *Claudius-Duhem inequality*

$$\psi(x,t) := \partial_t\mathcal{S}[\theta,\boldsymbol{\varepsilon}] + \mathrm{div}\left(\frac{\mathbf{q}}{\theta}\right) \geq 0\,, \qquad (4.7) \qquad \boxed{\texttt{de10}}$$

where $\psi(x,t)$ is the local (specific) entropy production rate. Formal computations yield

$$\begin{aligned} \theta\,\psi \;&=\; \theta\partial_t\mathcal{S}[\theta,\boldsymbol{\varepsilon}] + \langle\boldsymbol{\sigma},\partial_t\boldsymbol{\varepsilon}\rangle - \partial_t\mathcal{U}[\theta,\boldsymbol{\varepsilon}] - \frac{\mathbf{q}\cdot\nabla\theta}{\theta} \\ &=\; -\frac{1}{2}\,\langle\mathbf{B}'(m)\boldsymbol{\varepsilon},\boldsymbol{\varepsilon}\rangle\;\partial_t m + \int_0^\infty \varphi(\theta,r)\;\langle\partial_t(\boldsymbol{\varepsilon}-\mathfrak{s}_{rZ}[\boldsymbol{\varepsilon}]),\mathfrak{s}_{rZ}[\boldsymbol{\varepsilon}]\rangle\;\mathrm{d}r - \frac{\mathbf{q}\cdot\nabla\theta}{\theta}\,. \qquad (4.8) \end{aligned}$$

It remains to prescribe the evolution of the parameter m representing fatigue. To this end, we recall that in the paper [1], which deals with the so-called *rainflow method of cyclic fatigue accumulation*, it was shown that the accumulated fatigue in the isothermal case can (up to a suitable scaling) be identified with the dissipated energy. Mathematical consequences of this hypothesis have been investigated in the case of the wave equation in [12]. We pursue this idea also here in the nonisothermal case, and we assume a relation between m and ψ in the form

$$\partial_t m = C(\theta)\,\psi \qquad (4.9) \qquad \boxed{\texttt{de17}}$$

with a proportionality factor $C(\theta) > 0$. The evolution equation for m then becomes

$$\left(\frac{\theta}{C(\theta)} + \langle\mathbf{B}'(m)\boldsymbol{\varepsilon},\boldsymbol{\varepsilon}\rangle\right)\,\partial_t m = -\frac{\mathbf{q}\cdot\nabla\theta}{\theta} + \int_0^\infty \varphi(\theta,r)\;\langle\partial_t(\boldsymbol{\varepsilon}-\mathfrak{s}_{rZ}[\boldsymbol{\varepsilon}]),\mathfrak{s}_{rZ}[\boldsymbol{\varepsilon}]\rangle\;\mathrm{d}r\;. \qquad (4.10) \qquad \boxed{\texttt{de18}}$$

The right hand side of (4.10) is nonnegative by virtue of (1.2) and (2.6): indeed, since $\mathbf{0}\in Z$ for all relevant cases under investigation, we can conclude from (1.2) the energy dissipation rule for the stop operator in form of the *chain rule inequality*

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}\langle\mathfrak{s}_{rZ}[\boldsymbol{\varepsilon}]),\mathfrak{s}_{rZ}[\boldsymbol{\varepsilon}]\rangle\right) \leq \langle\mathfrak{s}_{rZ}[\boldsymbol{\varepsilon}]),\boldsymbol{\varepsilon}_t\rangle\,, \quad \text{a.e. in } (0,T)\,. \qquad (4.11) \qquad \boxed{\texttt{crstop}}$$

Assuming that $m(x,0) = 0$ and $\mathbf{B}'(0) = 0$, we see that for small times we have $\psi(x,t) \geq 0$ in agreement with the Clausius-Duhem inequality. Experiments show that under the influence of fatigue, the elastic response becomes weaker. This observation suggests to assume that the matrix $\mathbf{B}'(m)$ is negative semidefinite. Then, naturally, the coefficient in front of $\partial_t m$ in (4.10) may vanish in finite time and the solution has to be expected to blow up as a result of material failure.

4.0.2 Analytical Considerations

In this proposal, we plan to investigate the complex PDE systems that govern the oscillations of thermoelastoplastic mechanical structures like plates, arches, rods, and shells. A prototypical example is the system (2.2)–(2.6), which has to be coupled with (4.10) and suitable initial and boundary conditions. The main directions will be

- existence, regularity, uniqueness, and continuous data dependence of solutions,
- development of numerical methods and numerical simulation.

The analysis of the problem will be based on specific properties of hysteresis operators, in particular on the *second-order energy inequality* established by P. Krejčí in [11] and further developed in the monograph [13]. To explain the connection between the "usual" energy inequality and the second-order energy inequality, we consider in the isothermal case the scalar temperature-independent operator (1.1) and the associated internal energy operator

$$\mathcal{U}[\varepsilon] \; = \; \frac{1}{2}\int_0^\infty \varphi(r)\,|\mathfrak{s}_r[\boldsymbol{\varepsilon}]|^2 \,\mathrm{d}r \tag{4.12}$$

ene1

analogous to (4.6). Since hysteretic processes are irreversible, the *orientation of the hysteresis loops* is substantial. We see in Figures 1, 2 that both the clockwise and the counterclockwise orientations may occur. In agreement with the terminology introduced in [2], the operator $\mathcal{U}$ is a *clockwise potential* in the sense that

$$\mathcal{P}[\varepsilon](t)\cdot\frac{\mathrm{d}}{\mathrm{d}t}\varepsilon(t) - \frac{\mathrm{d}}{\mathrm{d}t}\mathcal{U}[\varepsilon](t) \;\geq\; 0 \tag{4.13}$$

e8

for every absolutely continuous input ε and for a. e. t in a given time interval. This is a consequence of the chain rule inequality (4.11) for the stop operator. The counterclockwise counterpart corresponding to Figure 1 with a constitutive mapping $m = \tilde{\mathcal{P}}[h]$ and a potential $\tilde{\mathcal{U}}$ would read

$$h(t)\cdot\frac{\mathrm{d}}{\mathrm{d}t}\tilde{\mathcal{P}}[h](t) - \frac{\mathrm{d}}{\mathrm{d}t}\tilde{\mathcal{U}}[h](t) \;\geq\; 0\,. \tag{4.14}$$

e8c

The left-hand side of (4.13) or (4.14) is the dissipation rate, defined as the difference between the work rate and the potential increment. It is indeed nonnegative in agreement with the Second Principle of Thermodynamics, similarly as the right-hand side of (4.10).

While the dissipation in (4.13) can be visualized by the *area* of hysteresis loops, the *second-order dissipation* is related to their *curvature*, and is characterized by the inequality

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{P}[\varepsilon](t)\,\frac{\mathrm{d}^2}{\mathrm{d}t^2}\varepsilon(t) - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\varepsilon(t)\,\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{P}[\varepsilon](t)\right) \;\geq\; 0 \tag{4.15}$$

e11

in the sense of distributions, as long as the loops are *clockwise convex*. The magnetic hysteresis on Figure 1 exhibits *counterclockwise convex* loops of small amplitude. The Prandtl-Ishlinskii operator $\mathcal{P}$ has the clockwise convexity property globally for all sizes of loops as in Figure 4.

There is an obvious formal similarity between (4.13) and (4.15) with one additional time derivative in each term, if we interpret the term

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\varepsilon(t)\,\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{P}[\varepsilon](t)$$

as a "second-order potential". For strictly convex operators, a lower bound of the cubic order is available, which makes it possible to derive strong a priori estimates for solutions to evolution equations with hysteresis and to prove the stability of the systems.

4.0.3 Numerical Methods

Numerical simulations will have to accompany the theoretical and modeling studies in parallel, in order to verify the models and to indicate where are necessary for a better agreement with physical reality. However, "ready-made" PDE solvers can only be used to a limited extent. The main reason for this is the complex memory storage and deletion exhibited by hysteretic systems. Similarly as in [29], where the spatially one-dimensional problem of isothermal elasto-plastic wave propagation was considered, not only *time* and *space*, but also the *memory* has to be discretized.

In [29], optimal error estimates were derived for a combined semi-implicit discrete scheme on a non-adaptive grid. This approach will be extended to the present situation. For adaptive grids, it will be necessary to use interpolation to create a memory for newly added grid points. It is an aim of the proposal to derive also for this situation local a posteriori error estimators, which may be used to monitor the local grid refinement.

5 Expected results and their use

Robust mathematical models are necessary for a reliable prediction of the long-time behavior of complex systems. As of today, a mathematically rigorous thermodynamic theory of continua with temperature-dependent mechanical memory, taking fatigue under cyclic loading into account, is to a large extent still missing. However, such a theory would be an important starting point for efficient simulations. Also, theoretical stability results for thermodynamically consistent systems of equations governing the physical processes in solids will also provide important information for the construction of numerical algorithms and for the derivation of error estimates.

6 International cooperation

Active cooperation in modeling of hysteretic material behavior exists with M. Brokate (TU Munich), J. Sprekels (WIAS Berlin), P. Colli, U. Stefanelli (Uni Pavia), K. Kuhnen (Uni Saarbruecken), S. Zheng (Fudan Uni, Shanghai), E. Rocca (Uni Milan).

7 Preparedness of the applicants and the hosting institution

P. Krejčí has been working systematically since 1986 in the modeling and analysis of nonequilibrium hysteretic processes in (thermo-)elastoplastic, ferromagnetic, piezoelectric, and other materials with rate-independent memory, as well as in phase transitions. His main contributions are related to quantitative energy dissipation estimates in oscillating systems with rate-independent memory.

Jana Kopfová and Michela Eleuteri have been working in the qualitative theory of partial differential equations with hysteresis.

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Czech Science Foundation - Part D
The Applicant and Joint Applicants
Applicant: RNDr. Pavel Krejčí, CSc.

Curriculum vitae

-1978 Graduated at the Faculty of Mathematics and Physics, Charles University, Prague

-1984 CSc. (= PhD), Czechoslovak Academy of Sciences, Prague

-1979-81 Research fellow, Institute for Fluid Dynamics, Czechoslovak Academy of Sciences, Prague

-1981-84 Research fellow, Mathematical Institute, Czechoslovak Academy of Sciences, Prague

-1984-96 Researcher, Mathematical Institute, Czechoslovak Academy of Sciences, Prague

-1990 Visiting associated professor, University of Wisconsin-Milwaukee, USA

-1991-93 A.v.Humboldt-Fellowship, Uni. Kaiserslautern and TU München

-1995 Visiting associated professor, Université de Technologie de Compiègne, France

-1997-2000 Researcher, WIAS Berlin, Germany

-2001 Research Prize of the Czech Minister of Education

-2001-03 Head of the research group "Evolution Equations", Mathematical Institute, Academy of Sciences of the Czech Republic, Prague

-from 2002 Editor-in-Chief of the journal "Applications of Mathematics", Prague

-2004-05 Researcher, WIAS Berlin, Germany and Mathematical Institute, Czechoslovak Academy of Sciences, Prague

-2005-09 Deputy head of the research group "Thermodynamic Modeling and Analysis of Phase Transitions", WIAS Berlin

-from 2009 Director of the Mathematical Institute, Academy of Sciences of the Czech Republic, Prague

Most cited publications

[1] P. Krejčí: Hysteresis, Convexity and Dissipation in Hyperbolic Equations. Gakuto Int. Series Math. Sci. & Appl., Vol. 8, Gakkōtosho, Tokyo 1996. 121 citations.

[2] P. Krejčí, K. Kuhnen: Inverse control of systems with hysteresis and creep. IEE Proc.– Control Theory Appl. **148** (2001), 185–192. 52 citations.

[3] P. Krejčí: Hysteresis and periodic solutions of semilinear and quasilinear wave equations. Math. Z. **193** (1986), 247–264. 26 citations.

[4] P. Krejčí, J. Sprekels: A hysteresis approach to phase-field models. Nonlin. Anal. **39** (2000), 569–586. 16 citations.

[5] J. Franců, P. Krejčí: Homogenization of scalar wave equations with hysteresis. Cont. Mech. & Ther. **11** (1999), 371–391. 13 citations.

[6] P. Krejčí: On solvability of equations of the 4^{th} order with jumping nonlinearities. Čas.

Pěst. Mat. **108** (1983), 29–39. 11 citations.

[7] M. Brokate, P. Krejčí, H. Schnabel: On uniqueness in evolution quasivariational inequalities. J. Convex Anal. **11** (2004), 111–130. 8 citations.

[8] P. Krejčí: Hysteresis memory preserving operators. Appl. Math. **36** (1991), 305–326. 7 citations.

18 refereed publications since 2004, 67 since 1983. I completely ignore what a "journal with IF" or an "acknowledged journal" is. The Web of Science registers a couple of hundred citations of my publications.

1-st joint applicant: Jana Kopfová

Curriculum vitae

- Undergraduate study at the Comenius University, Bratislava, Department of Mathematical Analysis (1985-1990). - Diploma Thesis: "Solution of non-linear diffusion problems using linear approximations" (in Slovak, Bratislava, 1990).

- Graduate study at the University of Alberta, Edmonton, Department of Mathematical Sciences (1993-1998). - Ph.D. Thesis: "Differential Equations involving Hysteresis" (Edmonton, 1998).

- Sessional instructor at the University of Alberta, Edmonton, Department of Mathematical Sciences (1998-1999).

- Mathematical Institute of the Silesian University in Opava (since 1999).

List of publications

(1) J. Kopfová, Semigroup Approach to the Question of Stability for a Partial Differential Equation with Hysteresis, J. of Math. Anal. and Appl.223, 272–287, (1998), 1 citation, IF: 0.872

(2) J. Kopfová, Uniqueness Theorem for a Cauchy problem with Hysteresis, Proc. AMS, 127 (12), 3527–3532, (1999), IF:0.520

(3) J. Kopfová and T. Kopf, Differential equations, hysteresis, and time delay. Z. Angew. Math. Phys. 53, no. 4, 676–691, (2002)., IF:0.872

(4) J. Kopfová, Periodic solutions and asymptotic behaviour of a PDE with hysteresis in the source term, Rocky Mountain J.Math. 36 (2006), no.2, 539–554, IF:0.267

(5) J. Kopfová, A convergence result for a spatially inhomogeneous Preisach operator, ZAMP, Vol. 58, Number 2 (2007), 350–356, 2 citations, IF: 0.872

(6) J. Kopfová, A homogenization result for a parabolic equation with Preisach hysteresis, ZAMM 87, Issue 5, (2007), 352–359, 2 citations, IF:0.550

(7) M. Eleuteri, J. Kopfová, P. Krejčí, On a model with hysteresis arising in magnetohydrodynamics, Physica B 403, pp 448–450, (2008), 1 citation, IF: 0.751

(8) M. Eleuteri, J. Kopfová, P. Krejčí: Magnetohydrodynamic flow with hysteresis, SIAM J. Math. Anal. (In print)

10 papers since 2004, 6 papers in journals with IP, 4 in referred journals and procedeings, 8

citations by WEB of science

2-st joint applicant: Michela Eleuteri

Curriculum vitae

- 2001 Graduated in Mathematics at the University of Parma (Italy) - 2006 Ph.D. in Mathematics, University of Trento (Italy) - 2006-2007 post-doc fellow (University of Trento and WIAS — Weierstrass Institute for Applied Analysis and Stochastics, Berlin) - 2008-2010 post-doc fellow (University of Trento - Italy)

List of most important publications

(1) M. Eleuteri: Hölder continuity results for a class of functionals with non standard growth, Boll. Unione Mat. Ital., Sez. B, Artic. Ric. Mat. (8), 7, (2004), no. 1, 129-157, 1citation

(2) M. Eleuteri, P. Krejčí: Asymptotic behaviour of a Neumann parabolic problem with hysteresis, ZAMM - Z. Angew. Math. Mech., 87, (2007), No. 4, 261-277, 1 citation, IF: 0.550

(3) M. Eleuteri: Well posedness results for a class of partial differential equations with hysteresis arising in electromagnetism, Nonlinear Anal., Real World Appl., 8, (2007), No. 5, 1494-1511, IF: 1.232

(4) M. Eleuteri, P. Krejčí: An asymptotic convergence result for a system of partial differential equations with hysteresis, Commun. Pure Appl. Anal., 6, (2007), No. 4, 1131-1143, IF: 0.609

(5) M. Eleuteri, O. Klein, P. Krejčí: Outward pointing inverse Preisach operators Physica B: Condensed Matter, Proceedings of the Sixth International Symposium on Hysteresis ad Micromagnetic Modeling, 403, no. 2, (2008), 254-256, IF: 0.751

(6) M. Eleuteri, J. Kopfová, P. Krejčí: On a model with hysteresis arising in magnetohydrodynamics Physica B: Condensed Matter, Proceedings of the Sixth International Symposium on Hysteresis ad Micromagnetic Modeling, 403, no. 3, (2008), 448-450, 1 citation, IF: 0.751

(7) M. Brokate, M. Eleuteri, P. Krejčí: On a model for electromagnetic processes inside and outside a ferromagnetic body, Math. Methods Appl. Sci., 31, (13), (2008), 1545-1567, IF: 0.594

(8) M. Eleuteri, J. Kopfová, P. Krejčí: Magnetohydrodynamic flow with hysteresis, SIAM J. Math. Anal. (In print)

Since 2004: 10 papers, 3 proceedings, 8 citations