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ON THE ORIGIN AND EARLY STAGES OF THE METEOR STREAMS

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ABSTRACT

1. *Cometary outbursts and the origin of meteor streams*

Cometary outburts, pointed out by Richter [12], are studied on the basis of recent investigations by Whitney [13]. Unlike Whitney's assumption of a uniformity of dimensions, a wide range of radii is supposed for the ejected particles. Assuming the distribution law $N(s)\,\mathrm{d}s \sim s^{-4}\,\mathrm{d}s$ to be valid over the range from 10^{-5} cm to 1 cm, we obtain $7 \cdot 10^{11}$ g for the entire mass of particles ejected at an outburst. The dynamical effect of such an outburst upon the comet's motion is negligible. The ejected particles can produce a remarkable meteoric shower as long as they occupy a space of the same order of dimensions as do the Draconids. A permanent stream cannot be generated by a single outburst.

In order to explain the existence of the Draconids, it is probably necessary to postulate an ejection of 10^{11} g of meteors per revolution of the parental comet. This hypothesis seems to be plausible. Internal forces far fainter than those operating at the outburst would suffice to account for such a process. Slow ejections supposed here cannot manifest itselves in the motion of the comet, but they may be detected photometrically and spectroscopically.

2. *Ejection theory of the formation of the meteor streams*

An analysis of the Draconids and Leonids shows that the ejection velocities are probably very low. In this case, simple formulae derived in 2.2 can be applied to in computing the orbits of the ejected meteors. The newly formed swarm is very thin, but the meteors become rapidly dispersed along the orbit of the comet. Four simple models of meteor swarms after ejection are considered and the distribution of meteors along the orbit investigated.

3. *Local perturbations of meteor streams*

An approximate analytical method is derived to account for local perturbations of meteor streams due to a close approach of a major planet. The cases of the Lyrids and Draconids are investigated. The great importance of planetary

perturbations is shown numerically. It is concluded that the Draconids observed in 1933 and 1946 could hardly have originated before the close approach of the parental comet to Jupiter in 1898.

4. *Mass and density of meteor streams*

A method of calculating the total mass and density of meteor streams is developed and applied to the Draconids of 1933. From the visual and telescopic observations it is found that the probable mass of this swarm is of the order of 10^{12} g. Although the spatial density inside the concentrated cloud is considerable, the total mass is far lower than that of the Geminids or Perseids. This, again, may be due to the fact that the stream is still being formed.

INTRODUCTION

Meteor streams are very particular and interesting systems of particles. As was proved by the author in 1950, (1), mutual gravitational attraction between the particles is compensated by the radiation pressure, so that there probably exists no force holding the particles together. Thus the meteors form an organized system simply because the dispersive forces are rather weak to scatter them rapidly enough. Naturally, over periods of thousands of years, the evolution of the streams becomes perceptible so that it can be traced more easily than the evolution of other celestial bodies.

On the other hand, it is evident that the age of the observed swarms must be far lower than that of the planets. Although the processes of the formation of the meteor swarms may proceed before our eyes, they are still rather obscure to us. Most of the meteor research workers agree that the streams (or at least many of them) have their origin in the disintegration of the comets; yet there is much controversy as to the process of formation.

The views maintained by various authors at the present time may be perhaps summed up into the following four hypotheses:

(*1*) Collision hypothesis. — *It was concluded by* WHIPPLE *and* HAMID (*2*) *that the Taurids originated in collisions of the parental comet Encke with asteroidal bodies. They considered this process to be rather exceptional. A few years earlier* GUIGAY (*3*) *attempted to explain the features of the Perseid stream in terms of a collision, but his conclusions seem to have been disproved by* AHNERT-ROHLFS (*4*). *However,* ORLOV (*5*) *believes that, in general, the meteor streams and even the comets originate in collisions of large parental comets with minor interplanetary bodies — asteroids and meteorites. His hypothesis is strongly supported by a recent paper of* BABADJAN (*6*) *on the Perseids.*

(*2*) Disintegration of the comets by tidal actions by the Sun and planets. — *This is the idea expressed already by* SCHIAPARELLI (*7*) *and still* *considered as a possible explanation, although little has been done in this respect in recent years.*

(*3*) Continuous escape of the meteors from the nucleus. — *If the nucleus is considered as a huge cluster of solid bodies of various sizes, we must admit internal motions of the kind found in the globular clusters. The velocity of some particles may exceed the velocity of escape and they leave the nucleus. The dispersion of small particles may be supported by frequent collisions within the nucleus. This is the idea expressed recently by* DUBJAGO (*8*).

(*4*) Ejection hypothesis. — *The idea that the meteors are emitted from the nucleus by internal forces in the comet was formulated by* BREDICHIN (*9*) *who considered the anomalous tails of the comets as strong streams of heavy particles expelled by internal forces. A modern form of the ejection theory is due to* WHIPPLE (*10*), *who, on the basis of his icy-conglomerate comet model, assumes that the meteors are expelled together with gas clouds set free by a mighty evaporation of the nuclear ices. The ejection theory is supported by the investigations of* DUBJAGO (*11*) *and* WHIPPLE (*10*) *on the anomalous motions of periodic comets. Another support may be seen in the papers by* RICHTER (*12*) *on cometary outbursts if* WHITNEY'*s interpretation* (*13*) *is correct.*

As the comets are very unstable bodies and the meteor streams are simply products of disintegration, several processes of their formation are possible; yet one of them must be the prevailing one. Without attempting to go into these involved problems, the author would like to explain briefly why he is inclined to prefer the ejection hypothesis. The compact clouds of the Draconids and Leonids may be looked upon as typical meteor swarms at an early stage of evolution. The formation of the clouds can hardly be explained in terms of a collision because a collision involves a much more pronounced dispersion of the swarm. The large dispersion of the Perseids, Orionids etc. may well be explained in terms of planetary perturbations of an originally thin stream as shown by AHNERT (*4*) *and* HAMID (*14*). *Moreover, it is doubtful whether the spatial density of larger interplanetary bodies is sufficient to account for the great number of meteor streams.*

A continuous escape of meteors from the nucleus as described in (*8*) *is always to be expected but it cannot probably generate more than a thin* *ring of very scattered meteors along the orbit. Concentrated clouds of meteors cannot be explained in this way.*

It is difficult to decide between the ejections and the tidal disintegration. Both effects may be operating in many comets. The outbursts pointed out by RICHTER *(12) and the natural explanation of the anomalies of motion (10, 11) seem to favour the former hypothesis. No doubt it depends very much on the structure of the cometary nuclei of which we know very little. If the nucleus were a cluster of bodies, it would be particularly liable to tidal forces; if the nucleus were monolithic, it would be fairly stable.*

These problems, no doubt, require more observational facts and deeper theoretical discussions. The present paper is a contribution to the discussion.

Some considerations about the origin of the meteor streams are presented in the two first chapters. They were stimulated mainly by the recent work of RICHTER *(12) and* WHITNEY *(13) on cometary outbursts. The second chapter contains theoretical investigations on the form of the meteor swarms, based on the ejection hypothesis of their formation.*

The last two chapters constitute, as a matter of fact, two individual papers. They were included mainly because they are referred to in the discussion of the origin of the streams. But the main aim of the third chapter is to show that the structure of the swarms may be strongly affected by the perturbations due to large planets — it seems to be the first quantitative treatment of this problem. Finally, the method of computing the total mass and spatial density of the swarms and its application to the Draconids may be of some interest.

It should be remarked that the present considerations — just like other recent papers — are based on classical ÖPIK*'s and* WATSON*'s values for the masses of the meteors. If the dimensions were as large and the densities as low as indicated recently by* WHIPPLE *(15) and* JACCHIA *(16), some of the conclusions should be revised.*

COMETARY OUTBURSTS AND THE ORIGIN OF METEOR STREAMS

1.1 *Cometary outbursts*

Recently, attention has been called to cometary phenomena which may be considered as a direct evidence of an ejection of matter from cometary nuclei.

It was RICHTER [12] who showed that occasionally violent outbursts are observed in comets. The phenomenon appears to have always the same course of events. Inside a diffuse coma, a strong stellar nucleus is formed within a few hours. The brightness of the comet increases rapidly, sometimes by as much as 8^m. The nucleus expands into a planetary disc; the proceeding expansion makes it then hazy and more diffuse so that eventually the comet returns to its previous appearance and brightness. The spectrum of the expanding nucleus is entirely continuous so that the increase in brightness is evidently due to solid particles reflecting the sunlight. It can be hardly doubted that vast amounts of solid particles are ejected during such an outburst.

WHITNEY [13] concluded that the outbursts can be explained in terms of WHIPPLE's icy-conglomerate comet nucleus. He supposes that the expanding halo consists essentially of meteoritic dust particles, the total mass being of the order of 10^{12} g. The observed expansion velocities range, according to Richter, between 40 m/sec and 7 km/sec, the average velocity being of the order of 10^4 m/sec. Then the kinetic energy of the halo comes out to be of the order of 10^{21} ergs. Whitney shows that if the nuclear albedo of the comet Schwassmann-Wachmann dropped suddenly by a factor of 2, the nucleus would, within several hours, absorb energy enough to eject a halo. Thus a rapid evaporation of frozen gas which, mixed with meteoric material, forms the nucleus according to Whipple, could probably fully account for the outburst phenomena.

These considerations seem to favour the hypothesis that meteor streams are formed by cometary ejections. There arises a question whether the observed outbursts might be considered as a kind of a process of formation of the meteor swarms.

Whitney assumed all the ejected particles to be of the same radius and discussed three models with particles with radii of 10^{-3}, 10^{-4} and 10^{-5} cm respectively. These dust particles are much smaller than observable meteors. The assumption of the uniform dimensions can, however, hardly be accepted. More probably, larger particles, too, are ejected simultaneously with dust. Let us suppose that a similar distribution law is valid for the ejections at the observed outbursts as for meteor streams. The distribution of meteors in a stream like the Draconids (sect. 4·2) roughly satisfies the law

$$Z(m) = Z(0) \,.\, 10^{0 \cdot 4\, m} \,, \tag{1-1}$$

where all meteors in the interval of magnitude from $m - 0{\cdot}5$ to $m + 0{\cdot}5$ are counted as of the magnitude m. Using Watson's relation between mass and magnitude and supposing the density of a meteoric body to be 1, we have the following functional dependence of the number of meteors with radii s between s and $s + \mathrm{d}s$ on the radius:

$$N(s)\,\mathrm{d}s = N(1) s^{-4}\,\mathrm{d}s \;,$$

where $N(1)$ corresponds to the semidiameter of 1 cm. This is the distribution law we shall suppose to be valid also for the halo particles.

I shall suppose that the ejected cloud of particles contains particles of radii between $s = 10^{-5}$ cm and $s = 1$ cm. In agreement with Whitney, the lower limit is estimated from the fact that the observed halos were not definitely coloured, so that presence of a large amount of particles with dimensions of the same order as the wavelength of the incident light is improbable. The upper limit was put to 1 cm because larger meteors are very rare in meteor streams and because it cannot be expected that large particles could be shot out of the cometary nuclei with a sufficient velocity.

The total mass of the halo is given by the integral

$$M_H = \int_{10^{-5}}^{1} M(1)s^3 N(1)s^{-4}\,\mathrm{d}s \tag{1-2}$$
$$= M(1)N(1)\ln 10^5$$
$$= 48{\cdot}4N(1)\ .$$

The value of $N(1)$ follows from the considerations about the brightness of the halo. Denote by m_H the total stellar magnitude of the halo, by $m(1)$ the apparent magnitude of a particle of 1 cm radius at the same distance. Then, because the intensity of reflected light varies as s^2, we have

$$m_H = m(1) - 2{\cdot}5 \log \frac{J_H}{J(1)} \tag{1-3}$$
$$= m(1) - 2{\cdot}5 \log \int_{10^{-5}}^{1} N(1)\, s^{-2}\,\mathrm{d}s\ .$$

Integrating, we have

$$\log N(1) = 0{\cdot}4[m(1) - m_H] - 5\ . \tag{1-4}$$

Table I contains data of four well-observed outbursts, according to Richter. Here, r and Δ are the distances of the comet from the Sun and from the Earth respectively and α is the phase-angle of the cometary nucleus. Supposing, in accord with Wihtney, the albedo of the particles to be that of Ceres, and the phase-angle dependence of its

TABLE I

Data on cometary outbursts

comet	outburst	r	Δ	α
1899 I Swift	1899 VI. 4.	1·29	0·58	52°
1892 III Holmes	1893 I. 16.	2·67	2·40	22
1884 I Pons — Brooks............	1883 IX. 23.	2·20	2·16	30
1884 I Pons — Brooks............	1884 I. 1.	0·87	0·64	80
1925 II Schwassmann-Wachmann ..	1933 I. 20.	7·3	6·3	0

TABLE II

Photometric data on the outbursts

comet	outburst	$m(1)$	m_0	m_{max}	Δm	m_H	$m(1)$-m_H
Swift	1899	42·3	6·4	4·4	2	4·6	37·7
Holmes	1893	46·2	12	7·5	4·5	7·5	38·7
Pons-Brooks..........	1883	45·9	13	8	5	8	37·9
Pons-Brooks..........	1884	42·4	8·2	6·9	1·3	7·3	35·1
Schw.-Wach.	1933	49·5	17	12·5	4·5	12·5	37·0

brightness as derived for the Moon by RUSSELL [17], we can derive the apparent brightness of a particle of 1 cm semidiameter. The results are summarized in Table II in the column headed $m(1)$. The brightness of the halo m_H can be computed from the observed increase in brightness Δm and from the observed maximum brightness $m_{\max}$ or original brightness of the comet m_0, respectively.

It may be observed that, except for the last but one fainter outburst, the value of $m(1) - m_H$ is about the same in all cases. Let us accept its mean value to be 38. Then from (1-4) we obtain

$$N(1) = 1{\cdot}5 \,.\, 10^{10} \ .$$

Inserting this into (1-2), the total mass of the halo comes out to be

$$M_H = 7 \,.\, 10^{11}\, \mathrm{g} \ .$$

It is to be noted, however, that the law

$$N(s) \sim s^{-4}$$

has been found, in sect. 4·2, to be valid for $m \leqq 9$ or $s \geqq 2{\cdot}5 \,.\, 10^{-2}$ cm only. There are no proofs of its validity beyond this limit and, as a matter of fact, it appears more probable that the increase in the number of meteors with decreasing diameter becomes much slower for fainter meteors.

Let us therefore consider another distribution, defined by the law

$$N(s)\,\mathrm{d}s \sim s^{-4}\,\mathrm{d}s \text{ for } 1 > s > 10^{-2}\text{ cm},$$
$$N(s)\,\mathrm{d}s = \text{const. } \mathrm{d}s \text{ for } 10^{-2} > s > 10^{-5}\text{ cm}.$$

Then we have

$$M_H = \int_{10^{-5}}^{10^{-2}} N(10^{-2})M(1)s^3\,\mathrm{d}s + \int_{10^{-2}}^{1} N(1)M(1)s^{-1}\,\mathrm{d}s \qquad (1\text{-}5)$$
$$= 21N(1)\ ,$$

where now $N(1)$ is defined by the equation

$$\log\ [\int_{10^{-2}}^{1} N(1)s^{-2}\,\mathrm{d}s + \int_{10^{-5}}^{10^{-2}} N(10^{-2})s^2\,\mathrm{d}s] = 0{\cdot}4[m(1) - m_H] \qquad (1\text{-}6)$$

or

$$\log N(1) = 0{\cdot}4[m(1) - m_H] - 2{\cdot}12\ .$$

Thus we obtain $N(1) = 10^{13}$

and $M_H = 2\,.\,10^{14}$ g .

No doubt this value is too large, being within 10% of the total mass of a smaller comet. Thus the former assumption appears to be more probable. If our fundamental concept of the nature of the cometary outbursts is correct, it must be probably supposed that large amounts of very small particles are present in the ejected halo but are rapidly dispersed afterwards so that they are less abundant in the observed meteor streams. A force acting in this way is known — it is the dynamical effect of solar radiation (Poynting-Robertson effect). This effect, however, acts rather slowly and an additional force may be postulated.

Returning to the former hypothesis about the distribution law, we see that the total mass of an average halo comes out to be about 10^{12} g. Assuming an average ejection velocity of 0·3 km/sec, which appears to be quite reasonable, the total energy involved is found to be of the order of 10^{21} ergs. These values agree fully with Whitney's results, although the present assumptions are more general.

1.2 *Dynamical effect of the ejection on comet's motion*

The law of the conservation of momentum requires a drag acting upon the nucleus during the ejection, directed oppositely to the direction of ejection. The change in the semimajor axis a or in the period T is most easily detectable and will be investigated here.

The ordinary equations of celestial mechanics give for the variation of a:

$$da = 2 \frac{a^2}{k\sqrt{p}} [e \int R \sin w \, dt + \int S(1 + e \cos w) \, dt]$$

or, for T,

$$\frac{dT}{T} = \frac{3}{2\pi} \cdot \frac{T}{\sqrt{ap}} [e \int R \sin w \, dt + \int S(1 + e \cos w) \, dt] . \qquad (1\text{-}7)$$

The orbital elements are represented by the usual symbols; w is the true anomaly. The components of the disturbing function are: R, the component along the radius vector and S, the tangential component. The orthogonal component W does not enter the equation for a. The integration extends over the time interval during which the ejections take place.

In the case of the cometary outbursts considered here, the duration δt of the explosion was very short so that w may certainly be taken constant. Thus we have

$$\frac{dT}{T} = \frac{3T}{2\pi\sqrt{ap}} [e \sin w_0 \overline{R} \, \delta t + \frac{p}{r_0} \overline{S} \, \delta t] . \qquad (1\text{-}8)$$

It remains to determine $\overline{R} \, \delta t$ and $\overline{S} \, \delta t$. Denote by M_0 the total mass of the comet. Then is it clearly

$$M_0 \overline{R} \, \delta t = - Q_R$$
$$M_0 \overline{S} \, \delta t = - Q_S , \qquad (1\text{-}9)$$

where Q_R and Q_S are the components of the total momentum of the ejected particles relatively to the nucleus, or

$$Q_R^2 + Q_S^2 + Q_W^2 = M_H^2 \, c^2 ,$$

c being the ejection velocity.

In order to determine the values of the components of Q, denote by φ the angle of the ejection velocity vector $\vec{c}$ of a particle with the direction of $\vec{S}$ and by ψ the corresponding angle with $\vec{R}$. Then it is obviously

$$Q_R = \int_0^{\pi} M(\psi)\, c \cos \psi \, \mathrm{d}\psi$$

$$Q_S = \int_0^{\pi} M(\varphi)\, c \cos \varphi \, \mathrm{d}\varphi \; . \tag{1-10}$$

The distribution functions $M(\psi)$ and $M(\varphi)$ depend upon the assumed form of the ejection. The matter becomes particularly simple if all the particles are ejected into a small space angle. In the case of such an ejection towards the Sun, $Q_R = M_H\, c$, while $Q_S = 0$; the values are interchanged if all particles were ejected forwards along the tangent to the orbit.

But the observed halos appear to be ejections into a wide space angle. Suppose that the particles are ejected with equal density into a space angle Φ. Then the mass ejected into a unit space angle is $\frac{M_H}{\Phi}$. The particles having an ejection velocity component $c \cos \varphi$ are contained within a narrow strip corresponding to an element of the space angle $\mathrm{d}\Phi$ which is equal to

$$\mathrm{d}\Phi = K(\varphi) \sin \varphi \, \mathrm{d}\varphi \;\; ,$$

and similarly for the other component. Thus it is

$$Q_R = \frac{M_H}{2}\, c \int_0^{\pi} K(\psi) \sin 2\psi \, \mathrm{d}\psi$$

and (1-11)

$$Q_S = \frac{M_H}{2}\, c \int_0^{\pi} K(\varphi) \sin 2\varphi \, \mathrm{d}\varphi \; .$$

If the particles are emitted isotropically into all directions, then in both integrals $K = 2\,\pi$ and it follows immediately $Q_R = Q_S \doteq 0$. An isotropic ejection does not influence the motion of the comet, however great quantity of matter is ejected. The only influence is that, in the relation between a and T,

$$\frac{2\pi}{T} = \frac{k\sqrt{1 + \mu_0}}{a^{3/2}} ,$$

where μ_0 is the mass of the comet expressed in the units of the Sun's mass, μ_0 is diminished by the ejection. However, μ_0 itself is so small, that its change by the outburst cannot cause an observable effect in the period of the comet.

Now let us suppose that the particles are ejected into the hemisphere

$$0 < \varphi < \frac{\pi}{2}. \text{ Then } \Phi = 2\pi ; \quad K(\varphi) = 2\pi \text{ for } 0 < \varphi < \frac{\pi}{2}$$

and again $K(\varphi) = 0$ for $\frac{\pi}{2} < \varphi < \pi$. Then we have

$$Q_S = \frac{M_H c}{2} \int_0^{\frac{1}{2}\pi} \sin 2\varphi \, d\varphi = \frac{1}{2} M_H c , \tag{1-12}$$

while $K(\psi) = \pi$ for $0 < \psi < \pi$ so that evidently $Q_R = 0$.

Quite similarly, if the particles are ejected into the hemisphere facing the Sun, it is

$$Q_R = -\frac{1}{2} M_H c \quad , \quad Q_S = 0 .$$

Thus, in the former case, we have

$$\overline{R} \, \delta t = 0 \quad , \quad \overline{S} \, \delta t = -\frac{1}{2} \frac{M_H}{M_0} c$$

and in the latter case,

$$\overline{R} \, \delta t = +\frac{1}{2} \cdot \frac{M_H}{M_0} c \quad , \quad \overline{S} \, \delta t = 0 .$$

Because the total mass of a comet may be put to about $5 \cdot 10^{15}$ g, it may be roughly taken $M_H/M_0 = 10^{-4}$. Again, suppose $c = 0{\cdot}3$ km/sec $\doteq 2 \cdot 10^{-4}$ astro. units of velocity. Thus we obtain the following formulae:

Ejection into the hemisphere forwards or backwards:

$$\frac{dT}{T} = \frac{\mp 3}{2\pi} \cdot \frac{T\sqrt{p}}{r_0\sqrt{a}} \cdot 10^{-8} . \qquad (1\text{-}14)$$

ejection into the hemisphere towards the Sun:

$$\frac{dT}{T} = \frac{+3}{2\pi} \cdot \frac{Te}{\sqrt{ap}} \sin w_0 \cdot 10^{-8} \qquad (1\text{-}13)$$

The change in the periods of revolution (in days) for 3 observed outbursts is given in Table III.

TABLE III

Change in the period due to the outbursts

comet	outburst	ejection towards the Sun		ejection backwards	
		dT/T	dT_{days}	dT/T	dT_{days}
Holmes	1893	$+1{\cdot}4 \cdot 10^{-6}$	$+0{\cdot}004$	$4{\cdot}1 \cdot 10^{-6}$	0·01
Pons-Brooks	1883	$-2{\cdot}2 \cdot 10^{-5}$	$-0{\cdot}58$	$1{\cdot}7 \cdot 10^{-5}$	0·44
Schw.-Wach.	1933	$< 6{\cdot}0 \cdot 10^{-8}$	$< 4{\cdot}10^{-4}$	$3{\cdot}8 \cdot 10^{-6}$	0·02

1.3 *Comparison with meteor streams*

It has been found that, assuming the same distribution law as for the Draconids, an average outburst results into an ejection of about $7 \cdot 10^{11}$ g of meteor mass. Direct comparison of this value with the meteor streams is hardly possible, for it includes a large amount of meteor dust. Let us therefore first calculate the total mass of ejected particles of the size of observable meteors. Taking 9^m or 10^m as the limit for good observation of meteors, including telescopic observations, we can, using WATSON's values, express the total mass of observable meteors by the integral

$$M_{vis} = \int_{10^{-2}}^{1} M(1)N(1)s^{-1}\, ds \qquad (1\text{-}15)$$

$$= -\, 6{\cdot}3 \cdot 10^{10} \ln 10^{-2}$$

$$= 2{\cdot}9 \cdot 10^{11}\,\text{g} .$$

If such a mass were dispersed into the volume occupied by the Draconid swarm (sect. 4·2), i. e. $6{\cdot}6 \cdot 10^{33}$ cm^3, the density would be $4{\cdot}4 \cdot 10^{-23}$ g . cm^{-3}.

This is probably about ten times smaller density than that of the visible Draconid meteors. Remembering the great displays of that stream, we see that a single cometary outburst would secure a fine meteor shower so long as the ejected meteors are close together. The velocity of ejection, however, appears to be at least ten times the ejection velocity of the Draconids [18] so that the dispersion of such a swarm would proceed very rapidly.

If the swarm occupied a volume like the Perseids ($2{\cdot}4 \cdot 10^{38}$ cm^3), the density would become as low as 10^{-27} g . cm^{-3} which is about 10^{-3} the density of the Perseids. In such a case the swarm ejected at a single outburst would be unobservable.

Let us now take an actual swarm, e. g. the Draconids. The total mass of meteors brighter than 9^m can be estimated at 10^{12} g (sect. 4·2). It was found that the age of the swarm is very low and that the shower has formed probably during the 20th century [18]. Another support to this conclusion was brought about by JEVDOVKIMOV [27], who pointed out that the present swarm must have formed after 1899, because in 1897—1899 the parental comet passed close (0·2 a. u.) to Jupiter and the swarm would have been dispersed.

This conclusion appears to be confirmed by a rough quantitative discussion of the perturbations in sect. 3·4 and will probably be more safely proved by a more detailed treatment which is just being worked at.

Assuming that the Draconid stream has formed since 1900 we conclude that 10^{12} g of meteors had to be ejected during about 8 periods of the comet. Thus about 10^{11} g of meteors are required to be ejected per revolution. Again, the ejection velocity can be hardly expected to be greater than 30 m/sec (sect. 2·2). Thus the total kinetic energy of the meteors emitted during one period of revolution comes out to some $5{\cdot}10^{17}$ ergs, while the outbursts considered above require about 10^3 times more energy. Moreover, all the energy involved in the outburst is dissipated within a time of the order of a day, while in the case of the meteor swarms it may be supposed that the meteors are ejected continually during many weeks. Thus, for example, the comet Giacobini-Zinner is for more than 200 days nearer the Sun than 2 astronomical units, which distance may be considered as a limit within which the slow ejections may be considerable. Thus the ejection of the Draconids requires an amount of about $2 \cdot 10^{15}$ ergs per day, which is less than 10^{-5} the power required to explain the cometary outbursts. The persistent streams, e. g. the Perseids, contain about 10^2 to 10^3 times more meteoric material the Draconids. But here it may be assumed that the swarm was being formed in the course of a longer time than the Draconids. Yet even if we had to assume that the rate of dissipation of energy per revolution was greater than in the case of the Draconids, the forces involved would still be far smaller than those operating in the outbursts.

Thus it may be concluded: The formation of the meteor streams by cometary ejections involve forces much weaker than the observed cometary outbursts. Because the outbursts are a fairly frequent phenomenon, it may be supposed that less violent ejections leading to the formation of the meteor swarms are quite ordinary phenomena. This, in the author's opinion, may be considered as a strong support to the ejection theory of the formation of the meteor streams.

1.4 *Effect of slow ejections upon comet's motion and brightness*

In the end, let us inquire whether the assumed slow ejections can be detected by the observations of the comets.

Let us first consider the effect upon the period of revolution. The following formula was established above:

$$\frac{\mathrm{d}T}{T} = \frac{3}{2\pi} \cdot \frac{T}{\sqrt{ap}} \left[e \int_{t_1}^{t_2} R \sin w \, \mathrm{d}t + \int_{t_1}^{t_2} S(1 + e \cos w) \, \mathrm{d}t \right] \quad (1\text{-}16)$$

The interval of integration extends over the whole arc along which the ejections take place; the result is then the change of the period per one revolution. It will be convenient to assume that the ejections are perceptible, in the case of the comet Giacobini-Zinner, as long as

$r < p = 1{\cdot}71$. According to this asumption, the period of activity is as long as 210 days.

Assume first that the rate of emission of the meteor particles per unit of time is constant throughout the whole period of activity. Then the integrals in (1-16) reduce to

$$\int_{t_1}^{t_2} \sin w \,\mathrm{d}t = \frac{\sqrt{p}}{ke} \int_{p}^{q} \mathrm{d}r + \frac{\sqrt{p}}{ke} \int_{q}^{p} \mathrm{d}r = 0 \,,$$

and

$$p \int_{t_1}^{t_2} \frac{\mathrm{d}t}{r} = \frac{p}{na} \int_{E(w=-\frac{1}{2}\pi)}^{E(w=\frac{1}{2}\pi)} \mathrm{d}E = \frac{p\sqrt{a}}{k} \,(\pi - 2\varepsilon)\,. \quad ^{1)}$$

Now we have, analogically to (1-9),

$$\int_{t_1}^{t_2} S_0 \,\mathrm{d}t = S_0 (t_2 - t_1) = -\frac{Q_s}{M_0}$$

so that

$$S_0 = \frac{-Q_s}{210 M_0}$$

time being expressed in days throughout.

Thus the equation (1-16) becomes

$$\frac{\mathrm{d}T}{T} = -\frac{1}{140 M_0} \,\frac{T\sqrt{p}}{\pi k}\,(\pi - 2\varepsilon)\, Q_s \,. \tag{1-17}$$

It seems to be more probable, however, that the ejections are the more abundant the nearer the comet comes to the Sun. Let us therefore assume that R and S vary inversely with r^2. In this case we have again

$$\int_{t_1}^{t_2} R \sin w \,\mathrm{d}t = R_0 \int_{t_1}^{t_2} \frac{\sin w}{r^2} \,\mathrm{d}t = 0 \,,$$

---

1) Here $\varepsilon = \arcsin e$.

while

$$\int_{t_1}^{t_2} S_0 \frac{p}{r^3}\, dt = \frac{S_0\sqrt{p}}{k} \int_{-\frac{1}{2}\pi}^{+\frac{1}{2}\pi} \frac{1}{r}\, dw = \frac{S_0}{k\sqrt{p}} (\pi + 2e)\,.$$

Again, according to (1-9),

$$-\frac{Q_s}{M_0} = \int_{t_1}^{t_2} S\, dt = S_0 \int_{t_1}^{t_2} \frac{1}{r^2}\, dt = \frac{\pi S_0}{k\sqrt{p}}\,.$$

Inserting into (1-16), the final form of the formula will be:

$$\frac{dT}{T} = -\frac{3a}{\pi k\sqrt{p}} (\pi + 2e) \frac{Q_s}{M_0}\,. \tag{1-18}$$

As the two final formulae (1-17) and (1-18) contain but Q_s, it is evident that the emission towards the Sun considered in Section 1·2 does not imply a change in the period of revolution of the emitting comet. Thus we restrict ourselves to the case of ejections ahead into the hemisphere $0 < \varphi < \frac{1}{2}\pi$. In such a case,

$$Q_s = \tfrac{1}{2} M_H c\,,$$

where M_H is now the total mass of the meteoric material emitted per revolution. According to what has been said, this number is to be taken equal to 10^{11} g. Thus it may be assumed $M_H/M_0 = 2\,.\,10^{-5}$. Further, the ejection velocity will be taken to be 30 m/sec or $1{\cdot}7\,.\,10^{-5}$ astronomical units per day. It follows therefore

$$\frac{Q_S}{M_0} = 1{\cdot}7\,.\,10^{-10}\,.$$

Inserting this and the elements of the comet Giacobini-Zinner into the formulae (1—17) and (1-18) respectively, we obtain

$$\frac{dT}{T} = -\,1{\cdot}1\,.\,10^{-7} \quad \text{or} \quad dT = -\,0{\cdot}00026^{d}$$

and, according to the latter formula,

$$\frac{dT}{T} = -\,1{\cdot}2\,.\,10^{-7} \quad \text{or} \quad dT = -0{\cdot}00028^{d}\,.$$

We realize that the effect of a slow formation of the swarm is below the limits of the accuracy of the observation even if it accumulates in successive revolutions. The things are a little better for comets of longer periods, but the effect remains still unobservable. Supposing the same conditions as above, the change of period of comet Halley would still be only $-0^{d}_{.}024$.

Thus it is evident that slow ejections, strong enough to form meteor swarms, can hardly be detected by their influence on cometary motions.

1.5 *Effect of slow ejection upon comet's brightness*

The matter is different as to the influence of the assumed ejections upon comet's brightness and it will be seen that moderate ejections may be ocassionally detected photometrically.

From (1-2) and (1-4) we have approximately

$$m(1) - m_H = 8{\cdot}3 + 2{\cdot}5 \log M_H \ . \tag{1-19}$$

Suppose we observe the comet at a distance of $\Delta = 1$ and $r = 1$. In this case, $m(1) = 41{\cdot}6$. Putting $M_H = 10^{11}$ g, we obtain $m_H = 5{\cdot}8$. We see that a halo at this distance, containing a mass of 10^{11} g, would be considerably bright.

However, it is evident that such a halo could not have existed round the comet Giacobini-Zinner 1946c. 23 visual estimates of its perihelion brightness (corrected to $\Delta = 1$), collected by VANYSEK and ŠIROKY (19), give $m = 11{\cdot}5$ as the most probable total magnitude of the comet. If we assume that the overwhelming part of its light was due to the ejected particles (which, of course, cannot be true), the total mass of the ejected material would be $m_H = 5 \,.\, 10^8$ g. On the other hand, $M_H = 10^{11}$ g is required to explain the existence of the Draconids. If we accept Whipple's suggestion that the ejections are perceptible within a sphere of a radius of about $r = 2$ only, we must expect that about half the material ejected during one revolution will shine in the comet's coma at the perihelion. Thus we have a discrepancy of about 3 orders. This may be diminished by an assumption that the ejections take place along the whole orbit with about the same power. In this case, about 2 . 10^9 g of particles would be expected to shine at the perihelion. This would lead to a perihelion magnitude of 10^m or brighter, which is still untenable. Moreover, the hypothesis of a continuous ejection during the whole revolution is very doubtful even if we realize, from the observed outbursts of the comet Schwassmann-Wachmann, that ejections need not be limited to a close vicinity of the Sun.

We might also assume that the Draconids were ejected in several separate outbursts of the kind considered by Richter and Whitney. Yet there is no evidence of such outbursts in the case of the comet Giacobini-Zinner.

Another hypothesis is worth considering more thoroughly. Let us, on the one hand, admit that the present shower could not have been formed before the near approach to Jupiter in 1898. On the other hand, according to what was said above, the ejections seem to be unsufficient. Then we perhaps ought to conclude that the swarm was formed during the close approach of the comet to Jupiter by its tidal action upon the comet.

Although this suggestion is very serious, there seems to be an explanation in terms of the ejections, namely, if we suppose another distribution of radii of the particles. Let us suppose that the distribution law

$$N(s)\,\mathrm{d}s = s^{-4}N(1)\,\mathrm{d}s$$

is valid only in the interval $1 > s > 10^{-2}$. Suppose further that within the interval $10^{-2} > s > 10^{-5}$ the number of particles is constant. This assumption does not contradict the observations of the Draconids, nay, it seems to be more plausible than a general validity of the $s^{-4}\,\mathrm{d}s$ law.

On these assumptions, the formula (1-19) takes now the form

$$m(1) - m_H = 2 + 2{\cdot}5 \log M_H \ . \tag{1-20}$$

Taking again $M_H = 10^{11}$, we get $m_H = 12{\cdot}1$ so that in this case the halo would contribute to the total brightness by about 33 % more than the comet itself. This is probably still too much, but a moderate lowering of the mass of the halo would make the result more probable without violating our general idea. Thus, for example, a reduction of the assumed mass to a half — which may very well be in accord with the observations of the Draconids — reduces the contribution of the halo to 40 % of the intrinsic brightness of the comet.

Another point may be of interest. The ejection velocities in the observed great outbursts range, according to Richter's list, between 0·3 and about 7 km/sec. The ejection assumed here to explain the formation of the meteor swarms appear to be slower, the velocities being of the order of metres per second. Now let us admit, according to what was shown above, that both kinds of emissions differ in the distribution of the radii of the particles. No doubt the observed velocities refer to those particles the contribution of which to the reflected light of the halo is the largest. In the outbursts, these are the smallest particles, $\sim 10^{-5}$ cm in diameter. On the other hand, the estimated velocities of ejection for observed meteor streams refer to particles of about 10^{-1} or 10^{-2} cm in diameter.

Suppose now, according to Whipple's ideas, that the meteoric particles are expelled from the comet by the outward pressure of the evaporizing gas. If so, the force acting upon a particle is probably proportional to the area of its cross-section. Supposing that the same momentum has been transferred to all particles, we may easily see that the ejection velocity is inversely proportional to the diameter. Thus the average ejection velocity of the visible meteors should be about 10^{-3} that of the velocity obtained for the outbursts, i. e., of the order of metres per second, in accord with our concept about the formation of the meteor streams. Thus the different ejection velocity may be simply a consequence of the differing radii of the particles. One of the main differences between an outburst and a slow ejection may be in the different distribution of the ejected particles according to their diameters.

A further advance in solving these problems requires a deeper study of the nature of the comets as well as a thorough study of the telescopic meteors in the meteor streams. A detailed photometric and spectroscopic investigation of the comets seems to be of a particular value in the efforts to observe directly the suggested process of the formation of meteor streams. This may be shown more clearly by considering the following example.

Suppose the comet emits particles along an arc of its orbit in the vicinity of the perihelion. According to the formulae (1-5) and (1-6), the relation between the instantaneous mass of the halo $M_H(w)$ and its brightness $J_H(w)$ may be written thus:

$$J_H(w) = 6{\cdot}3\, J(1)\, M_H(w)\,, \tag{1-21}$$

where by the symbol w for the true anomaly it should be marked that both quantities depend upon the position of the comet in its orbit. Denoting now by $J_0(1)$ the brightness of a unit particle at a distance of 1 astronomical unit from the Sun, we may write

$$J_H(w) = 6{\cdot}3\, J_0(1)\, \frac{M_H(w)}{r^2}\,. \tag{1-22}$$

Let us now consider again the comet Giacobini-Zinner. In accord with the considerations of Section 1·4, suppose the comet is active in emitting particles along the arc $-\frac{1}{2}\pi < w < \frac{1}{2}\pi$. Suppose further that the ejection velocity remains constant, while the instantaneous amount of the emitted particles varies as r^{-2}.

Moreover, we must bear in mind that each particle remains in the visible halo disc for a limited time only; at a certain distance from the nucleus the cloud of the particles becomes so dispersed that they cease contributing to the total light of the halo. We may tentatively assume that the semidiameter of the visible halo is of the order of 10^4 km. Assuming an ejection velocity of a few m/sec, we realize that the life time of the particles is of the order of 100 days which is nearly the time necessary for the comet to pass from $w = -90°$ to the perihelion.

On the basis of these assumptions, the function $M_H(w)$ may now be constructed. Let us first take

$$M_H(w) = H_0 \frac{1}{r^2}\,, \tag{1-23}$$

where

$$H_0 \,.\, \int_{t_1}^{t_2} \frac{dt}{r^2} = M_H\,,$$

M_H being, as before, the total mass ejected per one revolution. The integral gives

$$H_0 = \frac{M_H k\sqrt{p}}{\pi} .$$

Thus we obtain:

$$\text{For} \quad w < -\tfrac{1}{2}\pi , \qquad M_H(w) = 0 . \tag{1-24}$$

$$\text{For} \quad \tfrac{1}{2}\pi < w < 0,$$

$$M_H(w) = \frac{M_H\, k\sqrt{p}}{\pi} \int_{t_1}^{t(w)} \frac{dt}{r^2} = M_H \cdot \left(\frac{1}{2} + \frac{w}{\pi}\right) . \tag{1-25}$$

After the perihelion passage, the function is slightly more involved owing to the escape of the meteors ejected at the beginning. At a time $t > t_0$ (t_0 being the time of the perihelion passage), the number of escaped meteors is

$$M_H \left[\frac{1}{2} + \frac{1}{\pi} w(t - t_0 + t_1)\right] . \tag{1-26}$$

Here the symbol $w(t - t_0 + t_1)$ refers to the true anomaly that the comet had at the time $t_1 + (t - t_0)$, i. e. before the perihelion passage. Thus for $0 < w < \frac{1}{2}\pi$ the function has the form

$$M_H(t) = M_H \left[w(t) - w(t - t_0 + t_1)\right] \frac{1}{\pi} . \tag{1-27}$$

It is evident that for $w = \frac{1}{2}\pi$, $M_H(w) = \frac{1}{2}M_H$.

Finally for $w > \frac{1}{2}\pi$, no more particles are ejected, and the function is of the form

$$M_H(w) = M_H \left[\frac{1}{2} - \frac{1}{\pi} w(t - t_0 + t_1)\right] , \tag{1-28}$$

where, of course, now $t - t_0 > t_0 - t_1$ so that $M_H(w)$ is diminishing rather rapidly; it becomes zero for $t = 2(t_2 - t_0) + t_0$.

The values of the function $\frac{M_H(w)}{r^2}$ are given in Table IV.

The intrinsic brightness of the comet can be expressed by means of the well-known formula

$$J_K = \frac{J_{K0}}{\Delta^2 r^n} ,$$

TABLE IV

Effect of the halo upon the comet's brightness

r	$F(w)$	$\frac{F(w)}{r^2}$	J_K	J_H	m_K	m_{KH}
3·00	0	0	0·001	0	19·1	19·1
2·50	0	0	0·002	0	18·4	18·4
2·00	0	0	0·009	0	16·7	16·7
1·75	0	0	0·026	0	15·6	15·6
1·50	0·062	0·028	0·053	0·022	14·8	14·4
1·30	0·145	0·086	0·132	0·069	13·8	13·4
1·10	0·281	0·232	0·339	0·186	12·8	12·3
1·00	0·500	0·500	0·600	0·400	12·2	11·6
1·10	0·619	0·511	0 339	0·409	12·8	11·9
1·30	0·645	0·382	0·132	0·306	13·8	12·5
1·50	0.614	0·273	0,053	0·218	14·8	13·0
1·75	0·470	0·154	0·026	0·123	15·6	13·7
2·00	0·334	0·084	0·009	0·067	16·7	14·4
2·50	0·092	0·015	0·002	0·012	18·4	16·2
3·00	0	0	0·001	0	19·1	19·1

so that the total brightness of the comet with the halo, reduced to $\Delta = 1$, may be expressed as follows:

$$J_{KH} = \frac{J_{K0}}{r^n} + 6{\cdot}3 J_0(1) \frac{M_H(w)}{r^2} \,. \tag{1-29}$$

The perihelion brightness of the comet Giacobini-Zinner, which corresponds to $r = 1$, is

$$J_{KH} = J_{K0} + 6{\cdot}3 J_0(1) \,.\, \tfrac{1}{2} M_H \,. \tag{1-30}$$

According to Vanýsek and Široký [19], the perihelion magnitude of the comet was $m_{KH} = 11{\cdot}6$. Assume further the exponent of r to be $n = 6$, which is roughly the proper value for a periodic comet and is indeed close to Vanýsek's value 6·6. Suppose now that the halo contributed by 40% to the total brightness at the perihelion. This assumption leads to $M_H = 10^{11}$g. At the perihelion, where $5 \,.\, 10^{10}$ g of particles are shining in the halo, the magnitude of the halo itself would be 12·6. Table IV contains, besides the values of the function

$F(w) = \frac{M_H(t)}{M_H}$, the brightness of the comet itself J_K as well as that of the halo J_H, both in the units of the total perihelion brightness. In the two last columns there are the magnitude of the comet without the halo m_K and the total magnitude of the comet with halo m_{KH}. Both quantities are plotted against the radius-vector in Fig. 1.

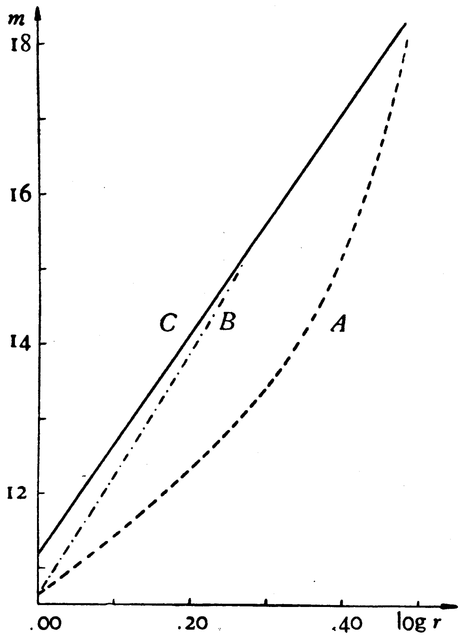

Fig. 1.

Brightness of a model comet with halo: C intrinsic brightness of the comet, B comet with halo before the perihelion passage, A comet with halo after the perihelion passage. (Jasnost modelu komety s halem: C vlastní jasnost komety, B kometa s halem před perihelem a A po perihelu.)

Fig. 1 shows a perceptible assymetry of the magnitude before and after the perihelion. If an observer tried to express the brightness in terms of the law $J = J_0 r^{-n}$, he would get $n = 6{\cdot}4$ before the perihelion, but only $n = 3{\cdot}5$ for the first weeks after the perihelion. At $r = 2$, the comet is by $2{\cdot}3^m$ brighter after the perihelion that before. By about the same amount the total brightness exceeds the intrinsic brightness of the comet at this place. At the perihelion, the halo makes the comet brighter by $0{\cdot}6^m$. Such a halo could be probably detected by a careful photometric and spectroscopic study. Naturally, our assumption that the halo shares by 40 % in the total brightness at the perihelion is rather bold. — If this contribution is decreased to 10 %, the halo will increase the perihelion brightness by $0{\cdot}1^m$ only and the maximum increase in brightness due to the halo would be about $0{\cdot}8^m$. These figures look rather hopeless. Nevertheless we must bear in mind that the brightness of the halo depends to a great extent upon the dimensions of particles in it; an ejection of dust would be more easily observable than an ejection of the same mass of larger meteors. Although the ejections by which meteor swarms are being formed may evidently proceed without being observed, it seems to be worth while to look for them: photometrically and spectroscopically, they may sometimes be detected.

2. EJECTION THEORY OF THE FORMATION OF THE METEOR STREAMS

2·1. *Orbit of a single meteor*

2·11 The general case

In general, the orbital elements of an ejected meteor can be readily calculated by means of the coordinates and velocity components. The former are given simply by the position of the comet at the instant of ejection, while the latter are obtained by adding the components of the ejection velocity to those of the original orbital velocity of the parental comet.

Let us denote the point where the ejection took place by E. The position of this point in the orbit of the comet is determined by its radius-vector r_0 and by the true anomaly w_{0E}. The orbital velocity of the comet at this point be v_0. Let us now introduce two rectangular coordinate systems with the common centre at the point E.

System (I): The $\xi\eta$-plane coinciding with the orbital plane of the comet; the ξ-axis in the direction of the instantaneous motion of the comet; the η-axis perpendicularly to it, positive towards the Sun; the ζ-axis perpendicular to the comet's orbital plane, positive to the left (i. e., northwards in direct orbits).

The direction and magnitude of the ejection velocity vector $\vec{c}$ can be determined by introducing the angle φ between $\vec{c}$ and the ξ-axis and the angle Θ of the η-axis with the projection of $\vec{c}$ into the $\eta\zeta$-plane. Then the components of the resulting velocity of the ejected meteor are

$$\begin{aligned} \dot{\xi} &= v_0 + c \cos \varphi \\ \dot{\eta} &= c \sin \varphi \cos \Theta \\ \dot{\zeta} &= c \sin \varphi \sin \Theta \end{aligned} \qquad (2\text{-}1)$$

System (II): The xy-plane coinciding with the orbital plane of the comet; the y-axis identical with the radius-vector, positive away from the Sun. The x-axis perpendicular to the radius-vector, positive in the direction of the comet's motion. The z-axis indentical with the ζ-axis of the system (I). Denote further by ψ the angle between $\vec{c}$ and the y-axis, by Φ the angle of the projection, into the xz-plane, of $\vec{c}$ with the x-axis and by ϑ the angle of $\vec{c}$ with the xy-plane.

Denoting the ejection velocity components by c_r, c_w and c_b (thus indicating their relation to the radius-vector r, true anomaly w and the binormal b), we have

$$\begin{aligned} v_r &= v_{0r} + c \cos \psi = \dot{r} \\ v_w &= v_{0w} + c \sin \psi \cos \Phi = r\dot{w} \\ v_b &= c \sin \psi \sin \Phi = r\dot{\vartheta} \end{aligned} \qquad (2-2)$$

The two systems are connected by the following transformation equations:

$$\begin{aligned} x &= \xi \sin \tau + \eta \cos \tau \\ y &= \xi \cos \tau - \eta \sin \tau \\ z &= \zeta \end{aligned}$$

vhere

$$\operatorname{tg}\tau = \frac{p}{re \sin w}, \quad 0 \leqq \tau < \pi .$$

At the perihelion, $\tau = 90°$ and we have

$v_w = v_0 + c \cos \varphi$, $\quad v_r = c \sin \varphi \cos \Theta$, $\quad v_b = c \sin \varphi \sin \Theta$.

In Table V, several orbits are computed for special directions of the ejection for the parental comets of the Draconids and Perseids. An ejection velocity of $c = 3$ km/sec is assumed and the ejection is supposed to take place at the perihelion. The cases investigated here are:

A_1 Meteor ejected directly forwards in the direction of the comet's motion: $\varphi = 0^0$

A_2 Meteor ejected directly backwards: $\varphi = 180°$

B_1 Meteor ejected along the radius-vector, away from the Sun: $\varphi = 90°$, $\psi = 0°$.

TABLE V

Dispersion of meteors due to quick ejections

Meteor	a	e	p	i	ω	Ω	T
Draconids:	comet 3·51	0·717	1·71	30·7	171·8	196·2	6·59
A_1	83	0·989	1·98	30·7	171·8	196·2	759
A_2	1·86	0·465	1·46	30·7	171·8	196·2	2·5
B_1	3·65	0·729	1·71	30·7	161·5	196·2	6·97
B_2	3·65	0·729	1·71	30·7	182·1	196·2	6·97
C_1	3·65	0·727	1·71	26·4	170·6	197·6	6·97
C_2	3·65	0·727	1·71	35·0	172·7	195·1	6·97
Perseids:	comet 24·3	0·960	1·89	113·6	152·8	137·5	119·6
A_1	hyperb.	1·25	2·16	113·6	152·8	137·5	hyperb.
A_2	3·16	0·695	1·63	113·6	152·8	137·5	5·6
B_1	32·3	0·970	1·89	113·6	144·6	137·5	182
B_2	32·3	0·970	1·89	113·6	160·9	137·5	182
C_1	32·3	0·970	1·90	110·0	153·5	139·5	182
C_2	32·3	0·970	1·90	117·2	151·9	135·5	182

B_2 Meteor ejected towards the Sun: $\varphi = 90°$, $\psi = 180°$.

C_1 Meteor ejected perpendicularly to the plane of the comet's motion: $\varphi = 90°$, $\vartheta = 90°$.

C_2 Meteor ejected perpendicularly to the orbital plane in the opposite direction: $\varphi = 90°$, $\vartheta = -90°$.

2·12 FORMULAE FOR SMALL EJECTION VELOCITIES

If the ejection velocity is small compared with the orbital velocity of the comet, the square of the ratio c/v can be neglected and differential formulae can be derived, giving directly the deviation of the elements of the meteors from those of the parental comet.

a) The semimajor axis a

Let us sconsider the coordinate system (I). Squaring and adding the equation (2-1), we obtain

$$v^2 = v_0^2 + c^2 + 2v_0 c \cos \varphi \ .$$

Again, expressing everywhere the velocities in the units of the circular velocity at $a = 1$ (i. e. 29·765 km/sec), we can write

$$v^2 = \frac{2}{r_0} - \frac{1}{a} \qquad v_0^2 = \frac{2}{r_0} - \frac{1}{a_0} \ .$$

Thus the final formula is

$$\frac{1}{a} = \frac{1}{a_0} - 2v_0 c \cos \varphi - c^2 \ . \tag{2-3}$$

Here it is advisable to retain the term c^2, for it causes no difficulties in numerical calculation and it can influence the result, at least in the case of comets of long periods.

The new semimajor axis will be the largest for $\varphi = 0°$ (ejection in the direction of the comet's motion) and the smallest for $\varphi = 180°$ (ejection velocity opposite to the comet's motion). The change of a for $\varphi = 90°$ is negligible for small ejection velocities.

b) The period of revolution T

From the well-known formula connecting a and T, we get readily

$$\delta T = \frac{3}{2} \cdot \frac{T}{a} \cdot \delta a \ . \tag{2-3a}$$

It may be noted here that for $c \cos \varphi > 0$, that is for $\varphi < 90°$, we obtain $a > a_0$ or $dT > 0$; that is, meteors ejected forwards obtain longer periods, and will come back to the perihelion later than the comet. On the other hand, meteors ejected backwards will be observed ahead of the comet at its next apparition.

c) The parameter p

We shall first choose the orbital plane of the comet as a fundamental plane for the angular elements of the orbits of the ejected meteors.

Thus we introduce relative elements i', Ω' etc. which are later to be transformed into the ecliptical elements i, Ω and ω.

The point, where the meteor was ejected, is the ascending (for $c_b > 0$) or descending (for $c_b < 0$) node of its orbit upon that of the comet. If we choose the ascending node to be the origin of the longitudes measured along the comet's orbit, it may be easily inferred that the integrals of areas can be written as follows:

$$r_0 v_w = p^{\frac{1}{2}} \cos i' \qquad (2\text{-}4)$$
$$r_0 |v_b| = p^{\frac{1}{2}} \sin i' \ .$$

In order to obtain the change of the parameter p, we square and add the equations and have

$$p = r_0^2 (v_w^2 + v_b^2) \ .$$

For small ejection velocities, the formula for the variation of p is obtained by differentiation, thus:

$$\delta p = 2 v_{0w} r_0^2 c_w \ . \qquad (2\text{-}5)$$

d) The relative inclination i'

Defining the inclination i', as usual, by the restriction $0^\circ < i' < 180^\circ$, we can determine its value by dividing the equations (2-4), taking here the absolute value of $|c_b|$. However, when passing to the ecliptical elements, it appears to be more convenient to define i' simply by the relation

$$\operatorname{tg} i' = \frac{c_b}{v_w} \, ,$$

or approximately,

$$i' = 206\ 265 \, \frac{c_b}{v_{0w}} \, , \qquad (2\text{-}6)$$

where i' is now expressed in seconds of arc. It is now $i' > 0$ for $c_b > 0$ and vice versa; this definition will be tacitly kept in section f). Note that for small ejection velocities, retrograde orbits relatively to the comet's orbit are impossible.

e) The elements e and ω

In order to get the variations of e and ω, it is convenient to introduce two new elements h and k by the following definition:

$$h = e \sin w_E$$
$$k = e \cos w_E \ ,$$

where w_E is the true anomaly (w_{0E} in the comet's orbit, w_E in that of the meteor) of the point of ejection E.

From the polar ellipse equation we get

$$k = \frac{p}{r} - 1 \ ,$$

or

$$k_0 = \frac{p}{r_0} - 1$$

and

$$\delta k = \frac{\delta p}{r_0} = 2 r_0 v_{0w} c_w \ . \tag{2-7}$$

Similarly, by differentiating the ellipse equation, we obtain

$$h = v_r \sqrt{p}$$
$$h_0 = v_{0r} \sqrt{p_0}$$
$$\delta h = \frac{r_0^2}{\sqrt{p_0}} v_{0w} v_{0r} c_w + \sqrt{p_0}\, c_r \ . \tag{2-8}$$

It is evident that

$$e_0 \delta e = h_0 \delta h + k_0 \delta k \ ,$$
$$e_0^2 \delta w_E = k_0 \delta h - h_0 \delta k \ . \tag{2-9}$$

f) Ecliptical elements

Now, we can pass to the usual ecliptical elements. From Fig. 2 it follows:

$$\cos i = \cos i_0 \cos i' - \sin i_0 \sin i' \cos (w_{0E} + \omega_0)$$

or approximately

$$\delta i = i' \cos (w_{0E} + \omega_0) \ . \tag{2-10}$$

Similarly,

$$\sin(\Omega - \Omega_0)\sin i = \sin(w_{0E} + \omega_0)\sin i'$$

or

$$\delta\Omega = i' \frac{\sin(w_{0E} + \omega_0)}{\sin i_0} . \qquad (2\text{-}11)$$

Finally,

$$\cos(w_E + \omega) = \cos(\Omega - \Omega_0)\cos(w_{0E} + \omega_0) + \sin(\Omega - \Omega_0)\sin(w_{0E} + \omega_0)\cos i_0 .$$

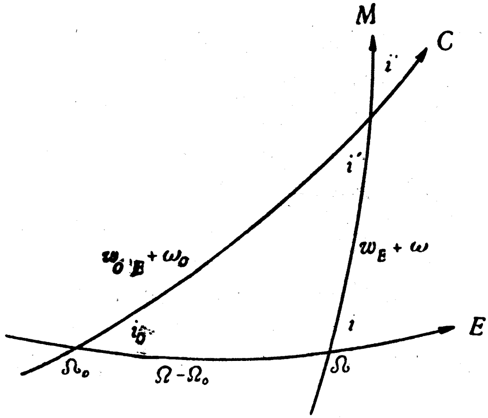

Fig. 2.

In the majority of the cases, the following approximation will hold good:

$$\delta\omega = -\delta w_E - \delta\Omega\cos i_0 . \qquad (2\text{-}12)$$

g) Simplification for the perihelion

It is probable that the ejections are most violent and frequent in the vicinity of the perihelion. For the case of a perihelion ejection, the equations will be considerably simplified, for

$$w_{0E} = 0, \quad v_{0w} = v_0, \quad r_0 = q_0, \quad \tau = 90° .$$

Thus we have

$$\frac{1}{a} = \frac{1}{a_0} - 2v_0 c\cos\varphi - c^2 , \qquad (2\text{-}13)$$

$$\delta p = 2q_0^2 v_0 c \cos \varphi \qquad (2\text{-}14)$$

$$\delta e = 2q_0 v_0 c \cos \varphi \qquad (2\text{-}15)$$

$$i' = \frac{c_b}{v_0} \qquad (2\text{-}16)$$

$$e_0^2 \, \delta w_E = c_r \sqrt{p_0} \qquad (2\text{-}17)$$

$$\delta i = i' \cos \omega_0 \qquad (2\text{-}18)$$

$$\delta \Omega = i' \frac{\sin \omega_0}{\sin i_0} \qquad (2\text{-}19)$$

$$\delta \omega = - \delta w_E - \delta \Omega \cos i_0 \, . \qquad (2\text{-}20)$$

It may be interesting to note that in this case the deviations δa, δe and δp depend on the same ejection velocity component $c \cos \varphi$ only.

TABLE VI

Deviations of elements due to slow ejections

Elements \ Shower	Geminids	Draconids	Perseids
a_0	1·396	3·514	24·27
e_0	0·900	0·717	0·960
p_0	0·265	1·709	1·887
T_0	$1^{\mathrm{a}}.65$	$6^{\mathrm{a}}.59$	$119^{\mathrm{a}}.6$
i_0	23·28	30·44	113·6
a_M	1·446	3·626	31·47
a_m	1·349	3·408	19·76
δp	±0·0005	±0·0087	±0·0087
δe	±0·0035	±0·0087	±0·0090
i'	$\pm 3'.1$	$\pm 8'.4$	$\pm 7'.9$
δi	$\pm 2'.5$	$\pm 9'.3$	$\pm 7'.0$
$\delta \Omega$	$\pm 4'.6$	$\pm 2'.5$	$\pm 3'.9$
$\delta \omega$	$\pm 10'.1$	$\pm 23'.1$	$\pm 17'$
T_M	$1^{\mathrm{a}}.74$	$6^{\mathrm{a}}.90$	$176^{\mathrm{a}}.6$
T_m	$1^{\mathrm{a}}.57$	$6^{\mathrm{a}}.29$	$87^{\mathrm{a}}.8$

h) Numerical application

Table VI gives the results of application of the above formulae to three typical showers: the Geminids, Draconids and Perseids. An ejection velocity of 100 m/sec and ejection at the perihelion were assumed. The table gives the maximum posible deviations in each element.

2·2 *Considerations on the value of the ejection velocity*

The value of the ejection velocity can be found by means of an analysis of the orbits of the ejected meteors. But Tables V and VI show that the departures of the elements of the meteors from those of the parental comet are rather minute. At the present time, it is photography only that can, in some cases, secure the accuracy required to detect the small differences in the elements. However, the position and shape of the meteor orbits can be very seriously affected by the perturbing action of the planets. An analysis of very dispersed meteor showers becomes extremely intricate and the results are not always quite reliable.

This is the case of the Perseids. Bredichin, attempting to account for their long duration, postulated ejection velocities as high as 3 or even 6 km/sec. Hamid [14] and Ahnert-Rohlfs [4] showed that the long duration may well be explained in terms of planetary perturbations. From a direct analysis of several individual photographic meteor orbits, Babadjan [6] found ejection velocities of about 1·5 km/sec, i. e. much nearer to Bredichin's values than to the value postulated by Hamid (metres per second only). The problem is whether the perturbations were properly allowed for.

The author believes that an investigation of meteor streams of a recent origin is more conclusive. Table VI shows that even a small ejection velocity causes a rapid dispersion of meteors along the orbit, while the cross-section of the stream near perihelion is small. Let us assume that the Draconids have been ejected with a velocity of 100 m/sec into all directions. According to Table VI, the dispersion in the node will be $2\delta\Omega = 5'$, which means that the Earth passes through the swarm within two hours, which is about the actual duration of the shower. But the meteors disperse rather rapidly along the orbit. On the same assumption, it may be shown that within 700 years, a closed ring of meteors is formed. But the actual Draconids appear to form rather an isolated cloud near the comet. It is evident that the age of such a cloud must be very low. Particularly the meteors observed in a close vicinity of the comet (e. g. those observed in 1946, following the comet by only 16 days) must have been ejected quite recently and it is probable that their orbits have suffered about the same perturbations as the comet itself. If so, the deduced ejection velocities will be more reliable.

TABLE VII

Distances of meteor clouds from comets

Shower		distance ΔM comet-meteors (years)	sT_0c_w
Leonids	1866.......	0·81	0·0188
Draconids	1933.......	0·219	0·0159
Draconids	1946.......	0·041	0·0030

The determination of the value of the ejection velocity would be particularly simple if we succeeded to obtain precisely the difference in the semimajor axes or periods between the meteors and the comet. If we observe a meteor cloud at a distance ΔM from the comet (ΔM is measured in days, for example by the difference of the perihelion or node passages), we can write, according to (2-3a):

$$\Delta M = s\,\mathrm{d}T = 3sT_0a_0v_0c_w\ , \tag{2-21}$$

where s is the number of periods elapsed since the ejection, or sT_0 is the age of the swarm in years. Table VII contains data on three great meteor clouds, from which the value of the ejection velocity may be derived.

In the case of the great Leonid cloud of 1866, there exists an estimate of dT. The parental comet 1866 I has a period of 33·18 [20], while the period of the cloud was found to be 33·25 [21]. If the difference is real, we get from (2-21) $s = 12$ or $sT_0 = 400^a$ and the ejection velocity comes out to be about 0·75 m/sec. The age will be probably considered as rather low, but in this case the ejection velocity would be even smaller.

There are, unfortunately, no precise determinations of the velocity of the Draconids, so that the equation (2-21) may furnish a rough estimate of the upper limit for the ejection velocity only. The reader will probably agree that an age of only 1 year is too low for the cloud observed in 1946; yet $sT_0 = 1^a$ leads to $c = 90$ m/sec. We see again that the ejection velocity of the Draconids was rather low. The author attempted to derive the ejection velocity from the difference of the position of the observed radiant from the theoretical one and found $c = 30$ m/sec to be the most probable value [18]. The corresponding very small age $sT_0 = 3^a$ indicates that even this value is rather overestimated.

Thus the author is inclined to believe that the ejection velocities are low, in general fairly below 1 km/sec, and that the high dispersion of some streams is due to external forces and is an evidence of that the dispersed streams are already at a late stage of evolution.

2·3 *Form of the stream after ejection*

It was shown in the previous section that the ejected meteors are dispersed along the comet's orbit, but do not deviate considerably from it. Thus, in investigating the dimensions and form of the shower immediately after the ejection, it is sufficient to study the distribution of the semimajor axes.

Let us first suppose that the whole swarm of particles was ejected within a short time, practically at the same moment. As mentioned in section 1·1, such cases were actually observed. Suppose further that all particles were ejected with the same velocity. It will be shown that in this case the form of the stream depends greatly upon the distribution of the directions of the ejection velocity vectors. Let us discuss several examples.

2·31 Isotropic ejection

Suppose that the meteors were emitted into all directions in the same amount. Denote the quantity of meteors ejected into an unit space angle by N_0; the total number of emitted particles is then $4\pi N_0$.

As evident from the equation (2-3), the value of the semimajor axis depends but on φ; in the interval between φ and $\varphi + \mathrm{d}\varphi$, the number of emitted particles is

$$N(\varphi)\,\mathrm{d}\varphi = 2\pi N_0 \sin\varphi\,\mathrm{d}\varphi.$$

Differentiating (2-3) we get

$$\mathrm{d}a = 2a^2 v_0 c \sin\varphi\,\mathrm{d}\varphi\ .$$

Thus the relative number of particles having semimajor axes between a and $a + \mathrm{d}a$ is

$$n(a)\,\mathrm{d}a = \frac{N(a)\,\mathrm{d}a}{4\pi N_0} = \frac{\mathrm{d}a}{4v_0 c a^2}\,. \tag{2-22}$$

This distribution law is valid within the limits a_M and a_m given by

$$\frac{1}{a_M} = \frac{1}{a_0} - c^2 - 2v_0 c$$

and (2-23)

$$\frac{1}{a_m} = \frac{1}{a_0} - c^2 + 2v_0 c\ .$$

As a numerical example, let us take the case of the comet Pons-Brooks, the observed outburst of which actually took place near the perihelion. According to Bobrovnikov, we shall take the ejection velocity to be $c = 0{\cdot}3$ km/sec $\sim 10^{-2}$ in our units of velocity. We have $a_0 = 17{\cdot}25$, $v_0 = 1{\cdot}50$, from which it follows $a_m = 11{\cdot}38$, $a_M = 35{\cdot}88$. Dividing the interval $< 11{\cdot}38,\ 35{\cdot}88 >$ into ten equal parts, we obtain the following approximate relative number of meteors in each of them:

TABLE VIII

Dispersion of semimajor axes — meteors ejected by an outburst of comet Pons-Brooks

a	$n(a)\,.\,\Delta a$	a	$n(a)\,.\,\Delta a$
11·38		23·63	
	0·258		0·066
13·83		26·08	
	0·181		0·055
16·28		28·53	
	0·134		0·046
18.73		31·98	
	0·103		0·039
21·18		33·43	
	0·082		0·034
23·63		35·88	

As evident, orbits of shorter periods are overwhelming. In connection with this, there arises the following question: Is the number of the meteors ahead of the comet equal to that of the meteors behind the comet?

In order to answer this question, let us consider the two integrals:

$$N_{\text{ahead}} = \int_{a_m}^{a_0} N(a)\,\mathrm{d}a = \frac{\pi N_0}{v_0 c}\left(\frac{1}{a_m} - \frac{1}{a_0}\right)$$

$$N_{\text{behind}} = \int_{a_0}^{a_M} N(a)\,\mathrm{d}a = \frac{\pi N_0}{v_0 c}\left(\frac{1}{a_0} - \frac{1}{a_M}\right)$$

Inserting from (2-23), we obtain the ratio

$$\frac{N_{\text{ahead}}}{N_{\text{behind}}} = \frac{2v_0 c - c^2}{2v_0 c + c^2} \doteq 1 - \frac{c}{v_0} \quad \text{(for small ejection velocities)} \qquad (2\text{-}24)$$

Thus the meteors are not exactly equally distributed on both sides of the parental comet; the meteors behind the comet are slightly more numerous. The difference is small provided $c \ll v_0$. In our case, when

$c : v_0 = 1 : 150$, the above ratio is 0·9933. Thus the meteors behind the comet are by some 0·3 % of the total number more numerous than those preceding the comet.

However, it is more advisable to know the distribution of the periods of revolution. Expressing T in years, we have simply

$$T^2 = a^3$$

and the distribution law becomes

$$n(T)\,\mathrm{d}T = \frac{N(T)\,\mathrm{d}T}{4\pi N_0} = \frac{\mathrm{d}T}{6v_0cT^{5/3}}\,. \tag{2-25}$$

The quantity directly observed is the distance, along the orbit, of the meteors from the comet. This may be measured as the time difference of the passages through the node of the comet and of the meteors in question respectively. It is obvious that the difference, $\Delta M \equiv L$, is given by the product

$$L = s(T - T_0)\,,$$

where s is the number of the periods T_0 elapsed since the ejection.

Using now the distribution function $N(L)$, the distribution of the meteors will be given by the law

$$\frac{N(L)\,\mathrm{d}L}{4\pi N_0} = \frac{\mathrm{d}L}{6v_0cT^{5/3}}\,, \tag{2-26}$$

where $\mathrm{d}L = s\,\mathrm{d}T$. This formula is valid between the limits

$$L_M = s(T_M - T_0) \quad \text{and} \quad L_m = s(T_m - T_0).$$

As numerical examples, let us put $s = 1$ and compute the distribution of meteors in the following cases:

(1) Ejections by observed outbursts of the comets Pons-Brooks and Holmes, supposing $c = 0{\cdot}3$ km/sec.

(2) Perihelion ejections from the parental comets of the Geminids, Draconids, Leonids, and Perseids, the ejection velocity being supposed to be 10 m/sec.

Evidently the swarm behind a comet stretches farther backwards from it, while the swarm ahead of a comet is more closely packed to the comet. Considering the same interval $\mathrm{d}T$ at the same absolute distance from the comet on either side, we realize that the density of the meteor material is far greater ahead of the comet than behind it. Considering meteors at a distance $\Delta T = T - T_0$ from the comet, the ratio of the densities is

$$\frac{N(T)\,\mathrm{d}T_{\text{ahead}}}{N(T)\,\mathrm{d}T_{\text{behind}}} = \left(\frac{T_0 + \Delta T}{T_0 - \Delta T}\right)^{5/3},$$

valid for $0 < \Delta T < T_0 - T_m$. For ΔT greater than this value but smaller than $T_M - T_0$, there are meteors only behind the comet. The total extent of the stream depends perceptibly on the original period

TABLE IX

Distribution of periods at isotropic ejection

Comet Pons-Brooks, $c =$ 0·3 km/sec		Comet Holmes, $c =$ 0·3 km/sec	
T	$n(T)\,.\,\Delta T$	T	$n(T)\,.\,\Delta T$
a		a	
213·89		7·41	
	0.029		0·089
196·34		7·31	
	0·033		0·092
178·79		7·20	
	0.039		0·094
161·25		7·10	
	0·046		0·096
143·70		7·00	
	0·055		0·098
126·15		6·90	
	0·071		0·101
108·60		6·79	
	0·092		0·103
91·05		6·69	
	0·127		0·106
73·51		6·59	
	0·188		0·109
55·96		6·49	
	0·318		0·112
38·41		6·38	

Table IX (continued)

$c = 10$ m/sec							
Geminids		Draconids		Leonids		Perseids	
T	$n(T).\Delta T$	T	$n(T).\Delta T$	T	$n(T).\Delta T$	T	$n(T).\Delta T$
a		a		a		a	
1·664		6·614		33·60		123·4	
	0·099		0·0996		0·0983		0·095
1·662		6·609		33·51		122·7	
	0·099		0·0997		0·0985		0·096
1·661		6·603		33·43		121·9	
	0·099		0·0998		0·0989		0·097
1·659		6·598		33·34		121·2	
	0·100		0·0999		0·0994		0·098
1·658		6·593		33.25		120·4	
	0·100		0·1001		0·0998		0·099
1·656		6·587		33·17		119·7	
	0·100		0·1002		0·1002		0·100
1·655		6·582		33·08		119·0	
	0·100		0·1004		0·1006		0·102
1·654		6·576		33·00		118·2	
	0·101		0·1005		0·1010		0·103
1·652		6.571		32·91		117·5	
	0·101		0·1006		0·1014		0·104
1·650		6·562		32·82		116·7	
	0·101		0·1008		0·1019		0·105
1·649		6·560		32·73		116·0	

of the comet, as it is clearly seen from the Tables VIII and IX. For example, the same ejection velocity causes the short-period Draconid meteors to be dispersed along an arc corresponding to 0·05 years, while the Perseids are scattered so considerably that the meteors will be coming next to the perihelion continually for a period of 7·5 years. It is to be realized that far away from the Sun, the orbits are very scattered, but they form a thin bundle in the vicinity of the perihelion, where they are actually observed.

2·32 Ejection towards the Sun

As a matter of fact, an isotropic ejection is not the most probable kind of ejection. It is more probable that the meteors are most intensely ejected on the sun-side of the cometary nucleus. Let us discuss a simple example of such an ejection.

Suppose the ejection takes place instantaneously at the perihelion. Suppose the meteors are ejected into a limited cone only, the axis of which is directed towards the Sun, i. e. it lies in the orbital plane and is perpendicular to the direction of the comet's motion. Let the vertex angle of the cone be 90°, so that the total number of ejected meteors is now $\pi(2 - \sqrt{2})\, N_0$ if N_0 is again the number ejected into an unit space angle.

It is evident that the angle of the ejection velocity vector with the direction of the comet's motion can lie between the limits $\left\langle \frac{\pi}{4}, \frac{3\pi}{4} \right\rangle$ only. Thus we have

$$\frac{1}{a_M} = \frac{1}{a_0} - c^2 - v_0 c\sqrt{2} \qquad \frac{1}{a_m} = \frac{1}{a_0} - c^2 + v_0 c\sqrt{2}\,.$$

The number of meteors with φ lying between φ and $\varphi + \mathrm{d}\varphi$ is

$$N(\varphi)\,\mathrm{d}\varphi = 2N_0 H \sin\varphi\, \mathrm{d}\varphi\ ,$$

where H depends upon φ and its value is given by the simple relation

$$\cos H = \frac{1}{\sqrt{2}\sin\varphi}\,.$$

Thus the relative number of meteors in the interval between T and $T + \mathrm{d}T$ is given by the formula

$$n(T)\,\mathrm{d}T = \frac{N(T)\,\mathrm{d}T}{\pi(2 - \sqrt{2})\, N_0} = \frac{2H(T)\,\mathrm{d}T}{3\pi(2 - \sqrt{2})\, v_0 c T^{5/3}} \qquad (2\text{-}27)$$

where, however, H depends upon T. Numerical example is given in Table X for two streams (comet Pons-Brooks, $c = 0{\cdot}3$ km/sec and comet Giacobini-Zinner, $c = 0{\cdot}01$ km/sec).

TABLE X

Ejections towards the Sun

Comet Pons-Brooks, $c = 0{\cdot}3$ km/sec		Draconids, $c = 10$ m/sec	
T	$n(T)\,.\,\Delta T$	T	$n(T)\,.\,\Delta T$
a 142·1		a 6·608	
	0·014		0·051
132·8		6·604	
	0·029		0·087
122·6		6·600	
	0·044		0·104
112·9		6·596	
	0·059		0·114
103·2		6·592	
	0·081		0·119
93·4		6·588	
	0·103		0·120
83·7		6·584	
	0·125		0·118
74·0		6·580	
	0·154		0·111
64·3		6·576	
	0·191		0·099
54·5		6·572	
	0·206		0·077
44·8		6·569	

In general, it may be said that the form of the shower in this case is similar to that of an isotropic ejection. The meteors are distributed nearly equally on both sides of the comet, but the shower is denser, it occupies a smaller arc along the orbit and the distribution of the meteors is somewhat different.

2.33 EJECTION IN CASE OF A NEGATIVE ROTATION

Let us now suppose that the meteors are ejected into the same space angle, but the axis of the cone is now turned by 45° towards the direction of the instantaneous motion of the comet. This may be approximately the case if the nucleus rotates in the opposite direction with respect to the comet's motion and the axis of rotation is perpendicular to the orbital plane. In this case, the limits for φ are 0 and $\frac{1}{2}\pi$; thus the extremes of the semimajor axis are given by the formulae

$$\frac{1}{a_M} = \frac{1}{a_0} - c^2 - 2v_0c, \quad \frac{1}{a_m} = \frac{1}{a_0} - c^2 .$$

The distribution law is the same as in the previous case, except that the limits of its validity are different and that H is now given by

TABLE XI

Ejections in case of negative rotation

Comet Pons-Brooks, c = 0·3 km/sec		Draconids, c = 10 m/sec	
T	$n(T) \cdot \Delta T$	T	$n(T) \cdot \Delta T$
a		a	
213·9		6·614	
	0·064		0·151
199·7		6·612	
	0·071		0·137
185·5		6·609	
	0·078		0·126
171·3		6·606	
	0·086		0·117
157·0		6·604	
	0·094		0·108
142·8		6·601	
	0·103		0·096
128·6		6·598	
	0·112		0·087
114·4		6·595	
	0·121		0·074
100·2		6·593	
	0·131		0·057
86·0		6·590	
	0·143		0·037
71·8		6·587	

the equation

$$\cos H = \mathrm{tg}\,\frac{\varphi}{2}\,.$$

Numerical application to the same cases as before is given in Table XI. We realize that the swarm concentrates only behind the comet. The density distribution is again different from the previous cases.

TABLE XII

Ejections in case of positive rotation

Comet Pons-Brooks, $c =$ 0·3 km/sec		Draconids, $c =$ 10 m/sec	
T	$n(T)\,.\,\Delta T$	T	$n(T)\,.\,\Delta T$
a		a	
71·8		6·587	
	0·016		0·030
68·4		6·584	
	0·031		0·059
65·1		6·582	
	0·045		0·071
61·8		6·579	
	0·059		0·087
58·4		6·576	
	0·075		0·099
55·1		6·573	
	0·094		0·109
51·8		6·571	
	0·116		0·120
48·4		6·568	
	0·143		0·130
45·1		6·565	
	0·180		0·141
41·7		6·562	
	0·234		0·153
38·4		6·560	

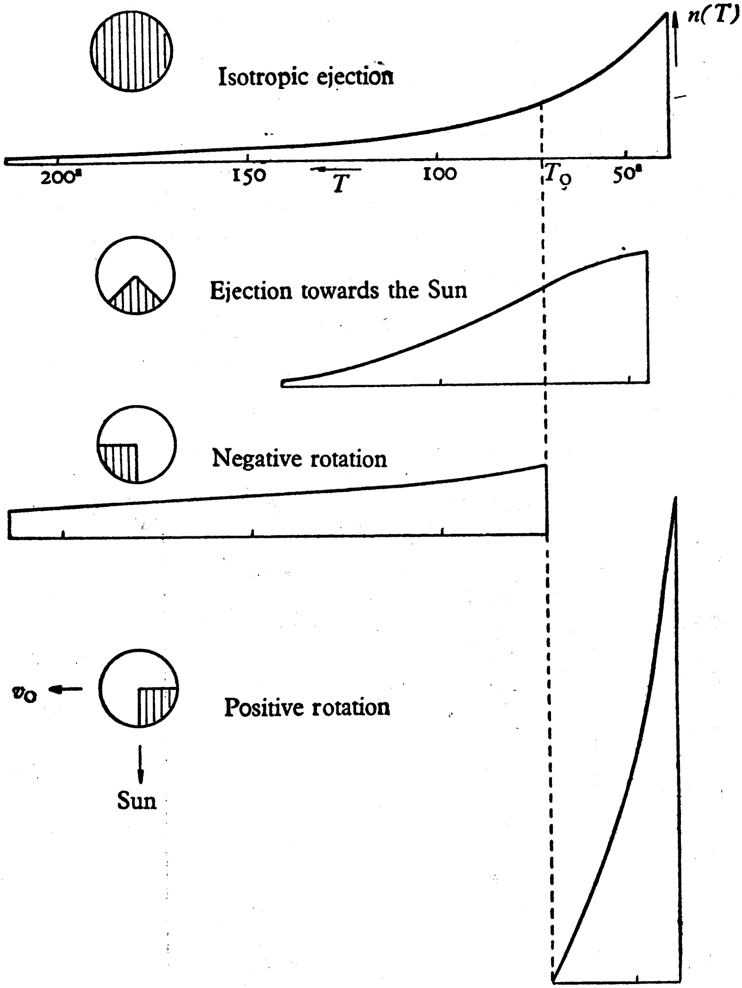

Fig. 3.

Models of hypothetical meteor swarms formed by a perihelion outburst of comet Pons-Brooks. Abscissae: periods of revolution in years (T_0 period of the comet), ordinates: spatial density of meteors $n(T)$ in arbitrary units. Circles in the left represent schematically the form of the ejected halo in its section by the orbital plane. The arrows indicate the direction of the Sun and of the comet's orbital motion (v_0).

Modely hypothetických meteorických rojů, vzniklých výbuchem komety Pons-Brooks v perihelu. Úsečky: oběžná doba v rocích (T_0 oběžná doba komety), pořadnice jsou úměrné hustotě meteorů v daném místě. Kruhy vlevo schematicky vyznačují druhy ejekce; představují průřez vzniklého hala dráhovou rovinou. Vyznačeny jsou směr ke Slunci a směr pohybu komety (v_0).

2.34 Ejection in the case of a positive rotation

Suppose finally that the circumstances of the ejection are the same as in the previous case, but the rotation of the cometary nucleus is positive (i. e. it rotates in the same direction as the comet moves).

The limits of φ are now $\frac{1}{2}\pi$ and π, and the extreme values of a are

$$\frac{1}{a_M} = \frac{1}{a_0} - c^2, \quad \frac{1}{a_m} = \frac{1}{a_0} - c^2 + 2v_0 c .$$

The distribution law is the same as (2-27), but

$$\cos H = \operatorname{cotg} \frac{\varphi}{2} ,$$

and naturally the limits of the validity of the law are other. See again numerical example in Table XII. Now, the meteors are strongly concentrated on the forward side of the comet, the swarm precedes the comet and practically no meteors are observed behind it.

We may easily imagine that if the meteors are ejected forwards or backwards into a limited space angle, the case may come that an isolated cloud of meteors is formed in front of or behind the comet, separated from it by a gap where there are practically no meteors.

2·35 Actual form of the streams

No doubt the types of ejections considered here are rough approximations only. Disregarding occasional great outbursts of some comets, the ordinary ejections are probably slow processes, repeated at each return of the comet to the Sun. The activity of the comet in emitting meteor particles may extend along a considerable arc of the orbit. Moreover, the particles are very probably not ejected with the same velocity. Rough consideration makes it probable that the ejection velocity is inversely proportional to the diameter of the particle (sect. 1·5). All this makes the swarm much more diffuse.

Nevertheless, it cannot be excluded that a similarity can be found between our models and the actual streams. As to the Draconids, dense clouds of meteors appear to be located on both sides of the parental comet. If so, the swarm may be thought as have been formed by an isotropic ejection or by an ejection of the form considered in 2·32.

On the other hand, according to ABELMANN [22], the great cloud of Leonids appears to be located behind the comet only. This form of the swarm can be explained in terms of an ejection into a limited space angle in the forward direction (sect. 2·33). It appears to be a gap between the comet and the main meteor cloud. If the gap is real, it may be explained by the assumption that the cone into which the meteors were ejected did not contain the direction $\varphi \doteq 0$ and that the meteors were ejected with about the same velocities and probably during a short period of time.

3. LOCAL PERTURBATIONS OF THE METEOR STREAMS

3·1 *Importance of local perturbations*

Major planets execute a perceptible perturbation effect on the orbits of many meteor streams. During long periods of ages, the shape and position of a meteoric ring change so perceptibly that the conditions of visibility may be quite altered [23]. This kind of perturbations is generally termed secular perturbations; in our case, it refers to a swarm as a whole. Moreover, the secular perturbations of the orbits of the individual meteors are always a little different for different particles; these differential perturbations probably play the most important role in changing an originally tenuous swarm into a wide stream.

A different effect arises if the orbit of the swarm crosses or comes near the orbit of a major planet. Then the meteors that happen to come near the planet are so strongly perturbed and dispersed that the continuous ring of meteoric matter is heavily destroyed. Naturally such an approach only influences a small part of the whole ring, nevertheless repeated approaches may disturb considerably the structure of the stream. The gaps or again accumulations in the ring can become very impressive for an observer on the Earth, because, owing to the small dimensions of the Earth, a moderate shift of a meteoric filament can make the Earth miss the meteors or again to meet suddenly a swarm not known before.

In order to investigate the importance of local perturbations, let us take two examples, the Lyrids and the Draconids. The former stream forms a more or less continuous ring. If the orbit of the parental comet, Thatcher-Baeker 1861 I, is correct and if the meteors follow it fairly exactly, they come very near the orbit of Saturn. As a matter of fact, every 30 years the planet comes so close to the comet's orbit, that the meteors pass through the sphere of activity of Saturn.

The Draconids seem to form a condensed cloud in the vicinity of the parental comet Giacobini-Zinner. The comet came close to Jupiter in 1898 and it may be expected that the swarm of meteors moving then in its vicinity was strongly perturbed.

In order to obtain a qualitative picture of the perturbative effect of Saturn and Jupiter respectively, let us apply LAPLACE's concept of the *sphere of activity* [24]. Let us suppose that the meteors moved unperturbed outside the sphere of activity, while inside it, the planet was the central body and the action of the Sun may be neglected.

No doubt, such a scheme of perturbations is too rough and cannot secure sufficient accuracy for a detailed investigation in either case. Surely the perturbations outside the sphere of activity cannot be neglected. The only way to attain a degree of accuracy needed in a quantitative discussion is mechanical quadrature. Yet the present procedure, being far shorter, can well provide us with a general picture of the influence of the local perturbations.

3·2 *Method of computation*

The method of computing the local perturbations within the sphere of activity may now be briefly described; a more extended account was given elsewhere [26].

Let ϱ be the radius of the sphere of activity of a planet whose mass is m (Sun's mass $=1$). The rectangular ecliptical coordinates and velocity components of the disturbed body at the moment T_1 when it enters the sphere of activity are x_1, y_1, z_1 and $\dot{x}_1, \dot{y}_1, \dot{z}_1$ respectively. The corresponding quantities referring to the planet at this moment will be denoted by capitals. We further write the relative coordinates and velocity components

$$\xi = x - X\,, \quad \eta = y - Y\,, \quad \zeta = z - Z$$
$$\dot{\xi} = \dot{x} - \dot{X}\,, \quad \dot{\eta} = \dot{y} - \dot{Y}\,, \quad \dot{\zeta} = \dot{z} - \dot{Z}\,.$$

Thus we have at the moment T_1

$$\varrho^2 = \xi_1^2 + \eta_1^2 + \zeta_1^2 \tag{3-1}$$

and the relative velocity at the same moment is

$$w^2 = \dot{\xi}_1^2 + \dot{\eta}_1^2 + \dot{\zeta}_1^2 \tag{3-2}$$

The relative orbit is a hyperbola. Denote its elements by a', e', i', Ω', and ω'. The known formulae of hyperbolic motion are in our case

$$\begin{aligned} \xi\dot{\eta} - \eta\dot{\xi} &= (mp')^{\frac{1}{2}} \cos i' \\ \eta\dot{\zeta} - \zeta\dot{\eta} &= (mp')^{\frac{1}{2}} \sin \Omega' \sin i' \\ \xi\dot{\zeta} - \zeta\dot{\xi} &= (mp')^{\frac{1}{2}} \cos \Omega' \sin i' \end{aligned} \tag{3-3}$$

$$w^2 = m\left(\frac{2}{\varrho} + \frac{1}{a'}\right) = \dot{\varrho}^2 + \frac{mp'}{\varrho^2}\,. \tag{3-4}$$

Here p' is the parameter of the planetocentric orbit. As before, the unit of velocity is so chosen that the gravitational constant $k = 1$.

Let $-\psi$ be the true anomaly, in the planetocentric orbit, of the point where the meteor enters the sphere of activity; owing to symmetry, the true anomaly of the point of emersion is $+\psi$. If we denote the values referring to the latter point by the indices *2*, we can evidently write

$$\varrho^2 \cos 2\psi = \xi_1\xi_2 + \eta_1\eta_2 + \zeta_1\zeta_2\,. \tag{3-5}$$

Again, differentiating the polar equation of the hyperbola and using Kepler's second law, we obtain

$$\dot{\varrho} = \left(\frac{m}{p'}\right)^{\frac{1}{2}} e' \sin \psi\,,$$

from which we conclude that

$$\varrho\dot{\varrho} \equiv \varrho\dot{\varrho}_1 = \xi_1\dot{\xi}_1 + \eta_1\dot{\eta}_1 + \zeta_1\dot{\zeta}_1 = -\,\varrho\dot{\varrho}_2 = -\,(\xi_2\dot{\xi}_2 + \eta_2\dot{\eta}_2 + \zeta_2\dot{\zeta}_2). \tag{3-6}$$

The well-known formulae connecting the position elements with the rectangular coordinates give for the point of immersion:

$$\begin{aligned} \xi_1 &= \varrho[\cos(\omega' - \psi)\cos\Omega' - \sin(\omega' - \psi)\sin\Omega'\cos i'], \\ \eta_1 &= \varrho[\cos(\omega' - \psi)\sin\Omega' + \sin(\omega' - \psi)\cos\Omega'\cos i'], \\ \zeta_1 &= \varrho\sin(\omega' - \psi)\sin i'. \end{aligned} \tag{3-7}$$

The coordinates of the point of emersion are given by the same equations except that $-\psi$ is to be replaced by $+\psi$. But they can be expressed in terms of ξ_1, η_1, ζ_1 if we write $\omega' + \psi = (\omega' - \psi) + 2\psi$ and use the equations (3-7):

$$\begin{aligned}
\xi_2 &= \xi_1 \cos 2\psi + \sin 2\psi\,(\eta_1 \cos i' + \zeta_1 \cos \Omega' \sin i')\,,\\
\eta_2 &= \eta_1 \cos 2\psi + \sin 2\psi\,(-\xi_1 \cos i' + \zeta_1 \sin \Omega' \sin i')\,,\\
\zeta_2 &= \zeta_1 \cos 2\psi + \sin 2\psi\,(-\xi_1 \cos \Omega' \sin i' - \eta_1 \sin \Omega' \sin i')\,.
\end{aligned} \tag{3-8}$$

It will be useful to have the reverse transformation:

$$\begin{aligned}
\xi_1 &= \xi_2 \cos 2\psi + \sin 2\psi\,(-\eta_2 \cos i' - \zeta_2 \cos \Omega' \sin i'),\\
\eta_1 &= \eta_2 \cos 2\psi + \sin 2\psi\,(\xi_2 \cos i' - \zeta_2 \sin \Omega' \sin i'),\\
\zeta_1 &= \zeta_2 \cos 2\psi + \sin 2\psi\,(\xi_2 \cos \Omega' \sin i' + \eta_2 \sin \Omega' \sin i').
\end{aligned} \tag{3-9}$$

The elements of the relative (planetocentric) orbit can now be eliminated by means of equations (3-3) and (3-4). After some improvements, we get, instead of (3-8), equations of the form

$$\xi_2 = \xi_1 \cos 2\psi + \frac{\sin 2\psi}{(mp')^{\frac{1}{2}}}\,(\xi_1 \varrho \dot{\varrho} - \dot{\xi}_1 \varrho^2) \tag{3-10}$$

and, instead of (3-9),

$$\xi_1 = \xi_2 \cos 2\psi + \frac{\sin 2\psi}{(mp')^{\frac{1}{2}}}\,(\xi_2 \varrho \dot{\varrho} + \dot{\xi}_2 \varrho^2). \tag{3-11}$$

Similar equations hold for the other two coordinates.

We shall form another three pairs of equations by adding and subtracting the corresponding equations in (3-10) and (3-11). After some improvements we obtain three pairs of the following form:

$$\begin{aligned}
(\xi_2 - \xi_1)\,[(mp')^{\frac{1}{2}} \cos \psi + \varrho \dot{\varrho} \sin \psi] + (\dot{\xi}_2 + \dot{\xi}_1)\,\varrho^2 \sin \psi = 0\,,\\
(\xi_2 + \xi_1)\,[(mp')^{\frac{1}{2}} \sin \psi - \varrho \dot{\varrho} \cos \psi] - (\dot{\xi}_2 - \dot{\xi}_1)\,\varrho^2 \cos \psi = 0\,.
\end{aligned} \tag{3-12}$$

The polar equation of hyperbola, its derivative and the formula for the relative velocity furnish the following relations:

$$\varrho \cos \psi = \frac{1}{e'}\,(p' - \varrho)\,,$$

$$\varrho \sin \psi = \frac{1}{e'} \left(\frac{p'}{m}\right)^{\frac{1}{2}} \varrho \dot{\varrho}\,.$$

$$\frac{mp'}{\varrho^2} = w^2 - \dot{\varrho}^2 .$$

Inserting into the previous equations, we get the following final formulae for the unknown quantities ξ^2, $\dot{\xi}^2$ etc.:

$$\xi_2\left(w^2 - \frac{m}{\varrho}\right) + \dot{\xi}_2\varrho\dot{\varrho} = \xi_1\left(w^2 - \frac{m}{\varrho}\right) - \dot{\xi}_1\varrho\dot{\varrho} , \tag{3-13}$$

$$\xi_2\varrho\dot{\varrho} - \dot{\xi}_2\frac{\varrho^3}{m}\left[(w^2 - \dot{\varrho}^2) - \frac{m}{\varrho}\right] = -\xi_1\varrho\dot{\varrho} - \dot{\xi}_1\frac{\varrho^3}{m}\left[(w^2 - \dot{\varrho}^2) - \frac{m}{\varrho}\right] .$$

It remains to calculate the time T_2 of the emersion out of the sphere of activity. Hyperbolic motion is described by the following laws:

$$\varrho = a'(e' \sec F - 1), \quad \varrho \cos \psi = a'(e' - \sec F) \tag{3-14}$$

where F is an auxiliary anomaly and can best be calculated from the relation

$$\cos F = \frac{e'}{1 + \frac{\varrho}{a'}} . \tag{3-15}$$

The time required for the body to pass from the pericentrum to the point with the true anomaly ψ is given by

$$\frac{km^{\frac{1}{2}}}{a'^{\frac{3}{2}}} t = e' \tan F - \frac{1}{\mathrm{mod}} \log \tan\left(\frac{\pi}{4} + \frac{F}{2}\right) .$$

Here mod is the modulus of logarithms to base 10.

Thus the time interval during which the body moves inside the sphere of activity is

$$T_2 - T_1 = \frac{2a'^{\frac{3}{2}}}{km^{\frac{1}{2}}}\left[e' \tan F - \frac{1}{\mathrm{mod}} \log \tan\left(\frac{\pi}{4} + \frac{F}{2}\right)\right] , \tag{3-16}$$

where F is given by (3-15); it is to be taken $0 < F < \frac{1}{2}\pi$. Applying the formulae (3-4) and (3-14), the planetocentric elements a' and e' can be eliminated and the formula (3-15) becomes

$$\cos F = \frac{\sqrt{\frac{\varrho}{m}\left(w^2\frac{\varrho}{m} - 2\right)\left((w^2 - \dot{\varrho}^2\right) + 1}}{w^2\frac{\varrho}{m} - 1} . \tag{3-17}$$

Thus, eventually,

$$T_2 - T_1 = \tag{3-18}$$

$$= \frac{2}{km^{\frac{1}{2}}\left(\frac{w^2}{m} - \frac{2}{\varrho}\right)^{\frac{3}{2}}} \left[\left(w^2 \frac{\varrho}{m} - 1\right) \sin F - \frac{1}{\text{mod}} \log \tan \left(\frac{\pi}{4} + \frac{F}{2}\right)\right].$$

3·51 Synopsis of the Formulae

The subsequent numerical application will proceed along the following lines:

Having calculated several sets of simultaneous positions of the planet and the perturbed body, we determine, by interpolation, the moment T_1 when the rectangular heliocentric ecliptical coordinates satisfy the relation

$$(X_1 - x_1)^2 + (Y_1 - y_1)^2 + (Z_1 - z_1)^2 = \varrho^2 .$$

For this moment, we compute the velocity components of both bodies and then pass to relative coordinates by putting

$$\xi = x_1 - X_1, \ldots, \dot{\xi} = \dot{x}_1 - \dot{X}_1, \ldots$$

We now compute the following quantities:

$$\varrho\dot{\varrho} = \xi_1\dot{\xi}_1 + \eta_1\dot{\eta}_1 + \zeta_1\dot{\zeta}_1 ,$$

$$w^2 = \dot{\xi}_1^2 + \dot{\eta}_1^2 + \dot{\zeta}_1^2 ,$$

$$J = gw^2 - 1 ,$$

$$\cos F = \frac{\sqrt{g(J-1)(w^2 - \dot{\varrho}^2) + 1}}{J} \qquad 0 < F < \frac{\pi}{2} .$$

Then the inquired quantities follow from the formulae

$$\xi_2(w^2 - f) + \dot{\xi}_2\varrho\dot{\varrho} = \xi_1(w^2 - f) - \dot{\xi}_1\varrho\dot{\varrho} ,$$

$$\xi_2\varrho\dot{\varrho} - \dot{\xi}_2 h[(w^2 - \dot{\varrho}^2) - f] = - \xi_1\varrho\dot{\varrho} - \dot{\xi}_2 h[(w^2 - \varrho^2) - f]$$

and from the two similar systems obtained by interchanging everywhere ξ and $\dot{\xi}$ by η and $\dot{\eta}$ or by ζ and $\dot{\zeta}$ respectively.

The corresponding time T_2 is obtained from the formula

$$T_2 = T_1 + \frac{d}{\left(\frac{w^2}{m} - \frac{2}{\varrho}\right)^{\frac{3}{2}}} \cdot \left[J \sin F - \frac{1}{\text{mod}} \log \tan\left(\frac{\pi}{4} + \frac{F}{2}\right)\right].$$

The constants d, f, g and h used above are defined thus:

$$d = \frac{2}{km^{\frac{1}{2}}} \text{ with } \quad k = 0{\cdot}017202$$

if time is expressed in mean solar days,

$$f = \frac{m}{\varrho}, \qquad g = \frac{\varrho}{m}, \qquad h = \frac{\varrho^3}{m}.$$

The following obvious relations were used as useful checks on numerical results:

$$\Sigma\xi_2^2 = \varrho^2, \qquad \Sigma\xi_1\dot{\xi}_1 + \Sigma\xi_2\dot{\xi}_2 = 0,$$
$$\Sigma\dot{\xi}_2^2 = w^2, \qquad \Sigma\xi_1\dot{\xi}_2 + \Sigma\xi_2\dot{\xi}_1 = 0.$$

Finally, we find out the position of the disturbing planet at the moment T_2 in the ephemeris and return to the heliocentric coordinates:

$$x_2 = X_2 + \xi_2, \quad x_2 = \dot{X}_2 + \dot{\xi}_2,$$

from which we can derive the new orbital elements, which are considered as unperturbed in the course of the further motion of the considered body.

3·3 *The Lyrids*

As mentioned above, the Lyrids are supposed here to follow exactly the orbit of the comet Thatcher-Baeker derived by Oppolzer [20]. Saturn passed close to this orbit last time in 1940. For eight positions of the planet at that epoch (Table XIII), points in the orbit of the stream were found that satisfy the condition that their distance from the planet is equal to the radius of the sphere of activity of Saturn (Table XIV; v is the true anomaly in the orbit of the stream). According to Laplace

TABLE XIII

Coordinates and velocity components of Saturn

J. D. 2429000 +	X_1	Y_1	Z_1	X_1	Y_1	Z_1
630·5	+8·11647	+4·51195	—0·40178	—0·17555	+0·28282	+0·00217
635·5	8·10147	4·53561	0·40157	0·17640	0·28234	0·00221
640·5	8·08627	4·55994	0·40140	0·17733	0·28185	0·00225
650·5	8·05578	4·60848	0·40102	0·17901	0·28086	0·00234
660·5	8·02490	4·65675	0·40060	0·18074	0·27987	0·00242
680·5	7·96236	4·75285	0·39922	0·18424	0·27784	0·00260
700·5	7·89861	4·84325	0·39884	0·18760	0·27577	0·00277
720·5	7·83371	4·94294	0·39734	0·19107	0·27365	0·00294

[24], $\varrho = rM^{\frac{2}{5}}$, where r is the radius-vector of the planet and M its mass. In our case, $r = 9{\cdot}3$ so that $\varrho = 0{\cdot}3555$.

Tables XV and XVI contain the positions and velocities of the Lyrids and Saturn respectively at the moment the meteors leave the sphere of activity. From them, the new orbits of the eight various groups of the Lyrids can be computed (Table XVII).

The results of the computations of the perturbations can be summarized as follows:

(1) The longitude of the ascending node of the Lyrids is only slightly affected so that the date of activity of the shower should be almost invariable. This seems to be in accord with observation.

(2) The periods of revolutions are very strongly altered and lie within the range from 350 to 530 years, while the unperturbed period is about 415 years. In consequence of this, the perturbed meteors are swept out of the orbit of the stream and the corresponding portion of the orbit is devoid of meteors. This gap in the meteoric ring of the Lyrids should cause deep minima of activity, when crossing the Earth's orbit.

(3) The perturbed particles pass through the node far from the Earth, as is evident from the value of the radius-vector, R, of their orbits in the node (Table XVII). The comet's orbit has $R = 1{\cdot}003$ and the Earth comes as near to it as 300,000 km, while the heavily disturbed particles pass by as much as 0·12 astro. units farther.

TABLE XIV

Planetocentric coordinates and velocity components of the Lyrids

J. D. 2429000+	ξ_1	η_1	ζ_1	w	$\dot{\xi}_1$	$\dot{\eta}_1$	$\dot{\zeta}_1$	v
630·5	—0·32914	+0·12452	+0·05074	0·30047	+0·53151	—0·04007	+0·12790	144°18′00″
635·5	—0·34393	+0·08052	+0·03962	0·30220	+0·53323	—0·03907	+0·12782	144 13 15
640·5	—0·35196	+0·04036	+0·03045	0·30383	+0·53484	—0·03817	+0·12775	144 09 33
650·5	—0·35376	—0·03018	+0·01890	0·30652	+0·53749	—0·03661	+0·12761	144 04 22
660·5	—0·34314	—0·09223	+0·01116	0·30891	+0·53982	—0·03526	+0·12750	144 01 06
680·5	—0·29517	—0·19828	+0·00450	0·31296	+0·54375	—0·03297	+0·12730	143 58 44
700·5	—0·21537	—0·28276	+0·00994	0·31596	+0·54664	—0·03119	+0·12716	144 01 20
720·5	—0·09683	—0·34069	+0·02790	0·31791	+0·54851	—0·03001	+0·12706	144 09 58

TABLE XV

Coordinates of the Lyrids at the moment of emersion

T_1 2429000 +	$T_2 - T_1$	ξ_2	η_2	ζ_2	$\dot{\xi}_2$	$\dot{\eta}_2$	$\dot{\zeta}_2$
630·5	65·23	+0·28564	+0·07604	+0·19624	+0·53227	—0·04366	+0·12349
635·5	69·88	+0·29898	+0·03156	+0·18964	+0·53450	—0·04224	+0·12133
640·5	71·15	+0·30537	—0·00729	+0·18202	+0·53945	—0·03948	+0·11930
650·5	70·81	+0·30367	—0·07242	+0·17018	+0·54103	—0·03257	+0·12018
660·5	68·02	+0·29028	—0·13059	+0·15832	+0·54418	—0·03016	+0·12363
680·5	57·08	+0·23944	—0·22905	+0·12910	+0·54682	—0·02966	+0·12627
700·5	39·70	+0·15808	—0·30345	+0·09669	+0·54855	—0·02940	+0·12684
720·5	14·43	+0·03926	—0·34827	+0·05943	+0·54855	—0·02994	+0·12699

Table XVI

Coordinates of Saturn at the moment of emersion

T_1 2429000 +	T_2	X_2	Y_2	Z_2	$\dot{X}_2$	$\dot{Y}_2$	$\dot{Z}_2$
630·5	655·73	+7·91385	+4·82559	—0·39903	—0·18679	+0·27627	+0·00273
635·5	665·38	+7·88290	+4·87143	—0·39856	—0·18844	+0·27526	+0·00281
640·5	671·65	+7·86261	+4·90108	—0·39830	—0·18950	+0·27459	+0·00287
650·5	680·81	+7·83111	+4·94660	—0·39782	—0·19144	+0·27357	+0·00295
660·5	688·52	+7·80731	+4·98056	—0·39747	—0·19236	+0·27279	+0·00301
680·5	69' ·58	+7·77736	+5·02317	—0·39700	—0·19389	+0·27181	+0·00309
700·5	699·70	+7·76867	+5·03530	—0·39688	—0·19433	+0·27153	+0.00311
720.5	694·93	+7·78619	+5·01072	—0·39713	—0·19344	+0·27210	+0·00307

Table XVII

New (perturbed) heliocentric orbits of the Lyrids

T_1 2429000 +	ω	Ω	i	a	T	e	R
630·5	211°54′10″	31°04′10″	80°34′45″	50·25	356[a]	0·98280	0·9325
635·5	211 34 00	31 08 20	80 37 40	49·60	349	0·98307	0·9033
640·5	211 12 55	31 09 25	79 59 30	51·98	375	0·98427	0·8802
650·5	211 54 10	31 13 00	78 00 00	61·43	482	0·98640	0·9070
660·5	212 54 10	31 13 30	78 04 50	65·19	526	0·98642	0·9606
680·5	213 27 00	31 12 20	78 47 40	62·70	487	0·98500	0·9972
700·5	213 26 40	31 11 15	79 16 50	59·67	461	0·98368	1·0030
720·5	213 26 40	31 10 30	79 38 00	57·11	432	0·98355	1·0030
comet	213 26 45	31 10 10	79 46 05	55·68	416	0·98346	1·0030

The observations of the Lyrids, collected by GUTH [25], show occasional high maxima and again deep minima of activity. The minima can be explained by the present theory of Saturn's perturbing effect, but not the maxima. It must be remembered, however, that the orbit of the Lyrids is rather uncertain, that the picture may be altered by considering a fairly thick ring instead of a linear distribution of meteors as was done above, and that the perturbations by Saturn were accounted for only approximately and those by Jupiter have been neglected. Thus the author believes that the most important result is that a major planet, coming near a stream, can very heavily disturb the structure of the shower.

3·4 *The Draconids*

The orbit of the parental comet of the Draconids, Giacobini-Zinner, is known much more accurately than it was in the previous case. The same applies to the orbit of the swarm. Thus the study of the close approach of the comet to Jupiter towards the end of the past century bears much more reliable results. The perturbing action of Jupiter on the comet itself was thoroughly studied by JEVDOKIMOV [27], who derived the osculating elements valid for August 23, 1897 (before the approach) and for March 12, 1899 (after approach) (Table XXII). On October 30, 1898, the comet came as near to Jupiter as 0·2 astro. units.

As evident from the table, the perturbations were considerable. It can be expected that meteors moving then in the vicinity of the comet were heavily perturbed too and dispersed. I have computed, by the method described in section 3·2, the perturbations of meteors moving in the orbit of the comet, but preceding it (case *A*) or following it (case *B*) by 40 days; thus the original orbits of the meteors were supposed identical with that of the comet before its approach to Jupiter. The latter was obtained by backward computation (*C* in Table XXII).[1])

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[1]) The swarm ahead of the comet (position *A*) did not enter the sphere of activity of Jupiter. Thus, in our rough scheme of the perturbations, it must be considered as unperturbed and its elements in 1899 equal to those in 1897.

TABLE XVIII

Coordinates and velocity components of Jupiter at immersion

	T_1	X_1	Y_1	Z_1	$\dot{X}_1$	$\dot{Y}_1$	$\dot{Z}_1$
C	2414547·890	—5·15013	—1·69362	—0·60058	+0·13947	—0·36073	—0·15819
B	534·543	—5·18131	—1·61052	—0·56417	+0·13216	—0·36307	—0·15901

TABLE XIX

Position and velocity of the Draconids at immersion

	x_1	y_1	z_1	$\dot{x}_1$	$\dot{y}_1$	$\dot{z}_1$	w
C	—5·190776	—2·02588	—0·64413	+0·20899	—0·18289	+0·06566	0·29422
B	—5.37028	—1·85344	—0·70283	+0·18237	—0·19268	+0·06226	0·28376

TABLE XX

Position and velocity of the Draconids at emersion

	T_2	x_2	y_2	z_2	$\dot{x}_2$	$\dot{y}_2$	$\dot{z}_2$
C	2414646·480	—4·79171	—2·30282	—0·53437	+0·25992	—0·14606	+0·06024
B	654·390	—4·89871	—2·20720	—0·57582	+0·27207	—0·15540	+0·05365

Table XXI

Position and velocity of Jupiter at emersion

	T_2	X_2	Y_2	Z_2	$\dot{X}_2$	$\dot{Y}_2$	$\dot{Z}_2$
C	2414646·480	—4·86864	—2·28858	—0·86275	+0·19203	—0·33985	—0·15052
B	654·390	—4·84220	—2·33477	—0·88321	+0·19612	—0·33790	—0·14994

Table XXII

New (perturbed) elements of the orbits of Draconids

Object	ω	Ω	i	a	T	e	q	R
Comet, osc. 1897	167·563	198·834	33·422	3·6348	6·9327	0·6727	1·1896	1·201
C, osc. 1898·7	168·20	198·20	32·84	3·599	6·828	0·690	1·116	1·125
B, osc. 1899·0	172·45	198·03	29·15	3·717	7·166	0·697	1·125	1·130
Comet, osc. 1899	171·100	197·407	29·843	3·4915	6·5240	0·7331	0·9319	0·932

It is evident that both the comet and the meteors are strongly perturbed, and, what is the most important thing, the perturbations are different so that the system of bodies becomes strongly dispersed. The meteors moving by only 40 days before or behind the comet are shifted so that they pass the node much farther from the Earth than the comet. It may be expected, moreover, that a cluster of meteors at the places A or B becomes strongly dispersed so that the density of these meteor swarms is strongly reduced.

However, the quantitative discussion is an approximative one for two reasons: First, the theory of perturbations is an approximation only. The perturbations outside the sphere of activity cannot be negligible. This is clearly seen from the discrepancy between the elements of the comet deduced by our method (C in Table XXII) and the elements obtained by Jevdovkimov by means of numerical integration of the perturbations over the period from August 1897 to March 1899. It is seen that the results differ but that the present theory accounts correctly for the order of the perturbations.

The second fact is that the meteors ejected from the comet cannot have exactly the same orbit as the comet and that the minute differences become quite perceptible near aphelion, i. e. just near Jupiter's orbit. Then the perturbation can be fairly different.

But it is evident that both these circumstances cannot alter the main conclusion: The meteors, even close at the comet, became very dispersed by Jupiter's action in 1898. Now, if we observe concentrated swarms of meteors close to the comet (80 days behind it in 1933, 16 days behind in 1946)—and these swarms follow almost exactly the comet's orbit, we must conclude that they must have been generated after the great approach to Jupiter or during it.

Thus we see that the Draconid stream is of quite a recent origin or is still being formed — and this is a very important fact which must be borne in mind when we discuss the origin of meteor showers. Here we are dealing with a swarm the origin of which can be much easier investigated than in other cases.

4. MASS AND DENSITY OF THE METEOR STREAMS

The density of the particles in a meteor stream and the total mass of the stream can be determined from the observed frequencies and from the space occupied by the stream. Such calculations were recently performed by LEVIN [28] and LOVELL [29]. Here a new value for the Draconids will be derived, based on visual and telescopic observations. The method of deriving these results will be different from those used by the mentioned authors and should be described in the first place.

4·1 *Method of computation*

The method used here is, in principle, due to KRESÁK [30] and some modifications only were performed by the writer. Nevertheless, it is considered necessary to describe it briefly, because the original treatment has not yet appeared in a form accessible to foreign readers.

The computation consists of several successive steps:

(1) An area of the atmosphere is to be outlined in which the hourly rates of meteors are to be determined by observation.

(2) Owing to decreasing sensitivity of the eye for fainter meteors, the observed numbers must be multiplied by factors, depending upon the magnitude of the meteors, in order to get actual hourly rates in the observed region.

(3) The estimated magnitudes of the meteors must be corrected for the influence of different zenith distances.

(4) Having now the hourly rate as a function of the zenith magnitude, we can pass from the zenith magnitudes to the masses of the meteors.

(5) By adding the respective masses of meteors of individual magnitudes, we obtain the total mass of meteors that appeared in the observed region during an hour. Knowing the area of the observed region, the position of the radiant and the velocity of the meteors, we can calculate the volume of the space occupied by the meteors and calculate the spatial density.

(6) By multiplying this value by the total volume occupied by the swarm, we obtain the total mass.

Let us now discuss the successive stages of computation more thoroughly.

4·11 The delimitation of the region to be watched

As the observation near the horizon is practically impossible and would make the subsequent reduction too complicated, it is best to restrict the area under observation by a small circle parallel to the horizon. From the reasons to be explained in sect. 4·13, the limiting zenith distance of 56°32′ was chosen. Restricting ourselves to the area of the sky above this circle, we can compute easily that 0·449 of the entire hemisphere is watched.

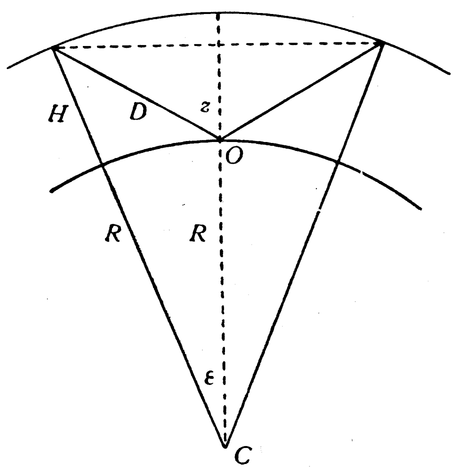

Fig. 4.

In order to know the area of the basis of this atmospheric cape, S, we find, by means of Fig. 4, the following formula:

$$S = (R + H)^2 \sin^2 \varepsilon \,, \quad (4\text{-}11)$$

where R is the radius of the Earth, H is the average height of the observed meteors above the earth and the angle at the Earth's centre, ε, is given by the formula

$$\sin(z - \varepsilon) = \frac{R}{R + H} \sin z \,. \tag{4-2}$$

Assuming $H = 90$ km, we obtain for $z = 56°32'$

$$S = 57\,700 \text{ km}^2 \,.$$

4·12 Correction for the sensitivity of the eye

The relation between the observed and true number of meteors as a function of their apparent magnitude was studied by many authors. The coefficients derived by Kresák apply immediately to the standard group of experienced observers at the Skalnaté Pleso Observatory, but will be used here, too. The coefficients, reducing the rate observed by a single observer to the entire hemisphere of the sky, are given in Table XXIII. Denote the observed hourly rate by $f(m)$. Then the true hourly rate in our cape is

$$F(m) = 0{\cdot}449\, c(m)\, f(m) \,. \tag{4-3}$$

Table XXIII

Kresák's coefficients c(m)

m	$c(m)$
0	2·14
1	2·62
2	3·74
3	6·02
4	9·89
5	18·3

4·13 Correction of the magnitudes to the zenith

Two factors make the meteors fainter than they would be if observed in the zenith: the distance and the atmospheric extinction. In order to allow for the latter, a numerical table of the normal extinction was used, giving for $z = 50°$ a correction of the observed magnitude m to the zenith magnitude m_z

$$m - m_z = 0{\cdot}14^m \,.$$

The influence of a different distance of the meteor from the observer is more serious, but may be readily allowed for. By Fig. 5, the distance of a meteor at a zenith distance z from the observer O is

$$D = (R + H)\frac{\sin \varepsilon}{\sin z}. \tag{4-4}$$

Denote the ratio of the distance D to the distance of a meteor in the zenith, H, by x. Then we have the obvious relation

$$m_z = m - 5 \log x . \tag{4-5}$$

According to this relation, each observed meteor can be corrected to the zenith magnitude. There is, however, a difficulty with the meteors that escaped our observation but must be counted with according to 4·12. Thus it is more advisable to apply the correction statistically. Let us divide the observed cape into a number of belts, bordered by the circles at which the correction is just $m_z - m = -0{\cdot}5^{\rm m}$, $m_z - m = -1{\cdot}5^{\rm m}$ etc. For all meteors within a given belt, an average correction will be applied. Denote by z_i the zenith distance corresponding the correction $-\frac{1}{2}i$ in magnitudes. Then in the cape $0 < z < z_1$ the correction will be 0, in the cape $z_1 < z < z_3$ the correction is $-1^{\rm m}$, for $z_3 < z < z_5$ the correction is $-2^{\rm m}$ etc.

By the equation (4-5), the correction of $\frac{1}{2}i$ corresponds to an x_i given by

$$\log x_i = \frac{i}{10}, \tag{4-6}$$

and the corresponding zenith distance z_i follows from the equation

$$\cos z_i = \frac{2h + 1 - x_i^2}{2hx_i}, \tag{4-7}$$

where it was put $h = R : H$. Taking again $H = 90$ km, we can form the following small table:

i	$m_z - m$	x_i	z_i
	m		
1	$-0{\cdot}5$	1·259	37°26′
3	$-1{\cdot}5$	1·995	59°59′
5	$-2{\cdot}5$	3·162	71°41′

Allowing for the extinction, we get the following final values:

i	$m_z - m$	z_i	correction	area
	^{m}			
0	0·0	0°		
			0^m	0·188
1	−0·5	35°42′		
			−1	0.261
3	−1·5	56°32′		
			−2	0·185
5	−2·5	68°30′		

The restriction of the cape under observation to $z \leqq 56°32'$ can now be explained. For larger zenith distances, the correction would be -2^m or larger. Thus meteors of an apparent magnitude 5^m would have a zenith magnitude of 3^m. But the number of the 5^m meteors is determined very poorly by visual observations and this uncertainty would influence even the 3^m zenith magnitude meteors. Thus it is better to restrict the observations to the cape where the corrections do not exceed -1^m.

Assume now that the meteors are distributed at random all over the hemisphere. Then the number of meteors in a given belt will be proportional to the area of the belt. Taking the area of the hemisphere as unit, we get the areas of the belts given in the last column of the above table.

Denoting the true hourly rate of meteors of a zenith magnitude m_z by $Z(m_z)$, we have

$$Z(m_z) = 0{\cdot}188\, c(m)\, f(m) + 0{\cdot}261\, c(m+1)\, f(m+1)\,. \qquad (4\text{-}8)$$

4·14 Relation between mass and magnitude

The mass of a meteor depends on its zenith magnitude as well as on its geocentric velocity. A functional dependence of the form

$$\log M = -0{\cdot}4 m_z - \gamma \log \frac{w}{w_0} + \log M(0, w_0) \qquad (4\text{-}9)$$

seems to be generally accepted, but there are still considerable doubts as to the value of the exponent of the velocities, γ, and as to the mass of a zero magnitude meteor at a given velocity. Öpik [31] takes $w_0 =$

$= 58$ km/sec and assumes $M(0,58) = 76$ mg, while WATSON [32] assumes $M(0,58) = 250$ mg. Quite recently WHIPPLE put $M(0,28) = 20$ g, a value which could considerably alter many of the results of meteor astronomy. Yet this hypothesis has still to be tested. It will be perhaps best to maintain Watson's value in order to compare the present calculations with other authors.

Another difficulty is met when dealing with the influence of the velocity upon the brightness of the meteor. The exponent figuring in the dependence law, γ, is given values between 0 and 3. There is, in my opinion, more reason to accept a value nearer to 3, but the question is far from being settled. In the present calculations, two alternatives were computed: $\gamma = 1$ and $\gamma = 3$ respectively.

As a slight modification, we must bear in mind that in the definition of the zenith magnitude, Öpik assumes the distance of a meteor in the zenith to be 100 km, while I have reduced the magnitudes to a distance of 90 km. Taking this into account and accepting Watson's value of 250 mg for a zero magnitude meteor moving with a velocity of 58 km/sec, we can write the final formula as follows:

$$\log M(m_z, w) = -0{\cdot}4m_z - \gamma \log \frac{w}{58} - 0{\cdot}5\,. \tag{4-10}$$

4·15 SPATIAL DENSITY

The total mass of meteors entering within an hour the observed cape of the atmosphere is

$$= \sum_{m_1}^{m_2} Z(m_z)\, M(m_z)\,. \tag{4-11}$$

This estimate is necessarily limited to the magnitudes visually observed, as indicated by the limits. Experience shows that within this range, the functional dependence $Z(m_z)$ can be well expressed by the formula

$$\log Z(m_z) = m_z \log \varkappa + \log Z(0)\,, \tag{4-12}$$

where $\varkappa$ is a positive number, characterizing the shower in question.

Using this and (4-10), formula (4-11) can be rewritten thus:

$$\mu = \sum_{m_1}^{m_2} Z(0)\, M(0)\, 10^{(\log\varkappa - 0\cdot4)m_z}\,,$$

or

$$\mu = \frac{1 - 10^{(\log\varkappa - 0\cdot4)(m_2 - m_1)}}{1 - 10^{\log\varkappa - 0\cdot4}} \,.\, Z(0)\, M(0)\,. \qquad (4\text{-}13)$$

It is, however, obvious that this value is lower than the actual mass of the shower entering the observed region of the atmosphere, because a very limited range of magnitudes can be observed. For visual observations, the lower limit of magnitudes is $m_z = 4^{\mathrm{m}}$. Telescopic observations shift this limit to some 9^{m}, but they are, unfortunately, very rare. No doubt, the mass of the meteors fainter than these limits cannot be neglected. On the other hand, we can safely put $m_1 = -2^{\mathrm{m}}$ as the upper limit in formula (4-11); brighter meteors are too rare so that the general law (4-12) cannot be applied to them and they must be included individually, if necessary.

In the case of the Draconids (4·2), telescopic observations suggest that the value of $\varkappa$ is sensibly the same over the range from 0^{m} to 7^{m}. But in many other cases, visual data for meteors of 4^{m} and 5^{m} seem to indicate that $\varkappa = 1$ for fainter meteors [35]. There are theoretical reasons in favour of the reality of these "kinks" of the number-magnitude dependence curve [33, 34]. But visual estimates at 4^{m}—5^{m} are too uncertain and only radar or telescopic observations can settle the question of reality of the kinks. Unless this is done, we must always bear in mind that the results of the visual statistics can furnish only the lower limit of mass and density of the meteor showers.

In order to estimate the uncertainty, let us suppose that $\varkappa = 2\cdot5$ in the visual range, which corresponds nearly to the actual value for various streams. Then the total mass of meteors of a given magnitude between -2^{m} and 4^{m} is the same and formula (4-11) gives

$$\mu_v = 7M(4)\, Z(4)\,.$$

Suppose now that the same $\varkappa$ holds even for fainter meteors down a magnitude m_0. Then the neglected mass of telescopic meteors, μ_t, would be

$$\mu_t = \frac{m_0 - 4}{7}\, \mu_v\,.$$

Theoretically, m_0 might be as low as 30^m; fainter meteors are repelled by light pressure. If so, visual meteors would represent only about 20% of the total mass of the swarm. It may be, however, expected that other forces, e.g. the Poynting-Robertson effect, will sweep even much brighter meteors out of the shower. If $m_0 = 10^m$, the visual meteors would contribute by about 50% to the total mass.

However, if the kinks at about 4^m are real, we can put $\varkappa = 1$ for fainter meteors and have

$$\mu_t = M(4)\,Z(4) \sum_1^{\infty} 10^{-0.4\,m}$$
$$= \frac{2}{3}\,M(4)\,Z(4)\,,$$

or telescopic meteors would contribute no more than some 10% to the entire mass.

In general, we can say that the value of the mass of a swarm depends sensibly upon the unknown amount of telescopic meteors and dust but the order of this value is probably well determined by visual observations.

In order to obtain spatial density, it is now necessary to determine the space occupied by the meteors that entered the observed area within an hour. This volume is 3600 $Sw \cos z$, where w is the geocentric velocity of the shower and z is the zenith distance of the radiant. Thus the density is, taking $S = 57\,700$ km²,

$$\delta = 4{\cdot}815\,.\,10^{-24}\,\frac{\mu}{w \cos z} \quad [\mathrm{g}\,.\,\mathrm{cm}^{-3}] \qquad (4\text{-}14)$$

4·16 Total mass of the swarm

The total mass can be estimated provided we know the volume of the space occupied by the swarm. The simplest assumption is that the cross-section of the stream is circular and the Earth passes through its centre[2]). Let it take the Earth n days to pass through the stream,

[2]) These assumptions are probably roughly fulfilled in the case of the Draconids, investigated here. In general, the Earth passes rather far from the central filament of the stream and the formulae would be more involved.

the radiant of which is at an elongation η from the apex. Thus the area of the cross-section is, in km²,

$$C = \frac{1}{4} \pi \,.\, n^2 \,.\, 30^2 \,.\, 86\,400^2 \,.\, \sin^2 \eta$$
$$= 5{\cdot}28 \,.\, 10^{12} n^2 \sin^2 \eta \,. \tag{4-15}$$

Supposing the period of revolution of the swarm (or of its parental comet) is T years, we may put the volume V of the swarm to

$$V = 365{\cdot}24\, Tv\, 86400\, C$$
$$= 1{\cdot}665 \,.\, 10^{35} Tvn^2 \sin^2 \eta \quad [\mathrm{cm}^3] \,, \tag{4-16}$$

where v is heliocentric velocity of the shower at the node where it meets the Eearth. The total mass can now be simply expressed by the formula

$$\mu_0 = V\bar{\delta} = 8{\cdot}0 \,.\, 10^{11} \frac{Tvn^2 \sin^2 \eta}{w \cos z} \bar{\mu} \,, \tag{4-17}$$

provided we insert here an average value of μ. This value depends on the distribution of meteors in the cross-section as well as along the orbit and may be obtained from the observed frequency curves. Let us measure the distances along the orbit by the difference of the perihelion passages of the parental comet (or a fixed point in the orbit) and of the meteors in question, L, measured in years. The instantaneous distance of the Earth from the central orbit of the stream can be expressed in terms of the time, t, needed to travel this distance. Then the mean value of μ can be formally expressed in terms of the following double integral:

$$\bar{\mu} = \frac{1}{nT} \int_0^T \int_0^n \mu(t, L)\, \mathrm{d}t\, \mathrm{d}L \,. \tag{4-18}$$

4·2 *The Draconids*

The observations of the magnificent display in 1933 enable us to calculate directly the total mass of the visual as well as the telescopic meteors. The function $Z(m_z)$ was investigated by FL. WATSON Jr. [35].

His results, based on visual observations of DE ROY [36] and telescopic observations of SANDING and RICHTER [37] will be revised, but essentially confirmed here.

Reduction of the visual observations by the single observer proceeds easily along the lines explained in the previous sections. Table XXIV gives in successive columns the following data:

$N(m)$ actual numbers of meteors observed by de Roy during the period from 19^h15^m to 19^h45^m, when there was no moonlight and no clouds.

$f(m)$ the average hourly rate of apparent magnitudes. From the frequency curve given by Watson in Fig. 1 of his paper, it follows that the duration of the shower was about 2 hours and that de Roy would have seen roughly 5·6 times more meteors if he had watched the whole display. Thus the relation is

$$f(m) = 2{\cdot}8N(m) .$$

$F(m)$ the average hourly rate of meteors that entered the cape defined in 4·11, computed according to the section 4·12.

$Z(m_z)$ the average true hourly rate of meteors of zenith magnitude m_z calculated according to the section 4·13. The last column contains the decadic logarithm of this quantity.

The telescopic observations present a greater difficulty. They were made by means of a 70 mm telescope and the field was 6°, centered

TABLE XXIV

Visual observations of Draconids, 1933

m	$N(m)$	$f(m)$	$F(m)$	$Z(m_z)$	$\log Z(m_z)$
0	1	2·8	2·7	14·5	1·16
1	7	19·6	23·1	142	2·15
2	30	84	141	454	2·66
3	90	252	680	975	2·99
4	95	266	1180	1390	3·14
5	68	190	1560	—	—
6	7	19·6	—	—	—

at the north pole. The reduction to the zenith magnitude is easy, the correction being -1^m all over the observed field. In order to allow for faint meteors that escaped observation, the same principles were maintained as in section 4·12. In accord with Watson, an allowance of $3{\cdot}5^m$ was made for the use of the telescope; thus it was assumed that the same coefficients apply to the telescopic meteors of a magnitude of $m + 3{\cdot}5$ as to visual meteors of magnitude m. As only the ratios of the coefficients are needed here, the meteor rates have been related to that of the meteors of 4^m.

In order to reduce the rates of the telescopic meteors to the cape of the atmosphere defined in section 4·11, it seems to be preferable to determine the proper coefficient empirically, by considering the visual and telescopic rates of meteors of $m_z = 4^m$. In table XXIV, we get $Z(4) = 1390$, while the telescopic observations give $Z'(4) = 6{\cdot}3$. Thus the ratio would be 220. But it seems to be probable that the visually estimated rate is underestimated. Therefore we should better extrapolate $Z(4)$ from the visually estimated rates $Z(m)$ for $m = 1$ to $m = 3$. In this case, we obtain $Z(4) = 2250$ and the coefficient turns out to be 350. This value has been used.[3])

The results of the described procedure are summarized in Table XXV, the columns being as follows:

$N(m)$ the number of telescopic meteors actually observed between 19^h25^m and 20^h25^m,

$f(m)$ the average hourly rate of apparent magnitudes. As the telescopic observations lasted longer, the factor reducing the rates to the whole duration of the shower is about 1·5, so that the hourly rate is $f(m) = 0{\cdot}75N(m)$.

$F(m)$ the true hourly rate of apparent magnitudes, the escaped meteors having been allowed for. The rates have been related to those of $m = 4^m$ by multiplying them by a coefficient $t(m)$ the definition

[3]) Dr. Kresák and M. Kresáková recently called attention to the fact that the observed frequencies of telescopic meteors are rather considerably influenced by the relation of the angular length to the magnitude [38]. However, their investigations probably do not immediately apply to such slow meteors as the Draconids are. As the observations of Sanding and Richter do not provide the necessary data, this effect has been neglected. It would probably make μ a little greater.

of which is

$$t(m) = \frac{c(m - 3{\cdot}5)}{c(4 - 3{\cdot}5)} \,.$$

$Z(m_z)$ the true hourly rates of zenith magnitudes in the same region as the visual rates $Z(m_z)$ refer to. Thus

$$Z(m_z) = 350F(m + 1) \,.$$

The last column contains $\log Z(m_z)$.

By means of the method of least squares, it was found (after approximate weights have been applied to) that, within the range from 0^m to 8^m, the function $Z(m)$ can best be represented by the equation

$$\log Z(m) = 1{\cdot}72 + 0{\cdot}41m \,. \tag{4-19}$$

The total mass of these meteors is now given by the formula

$$\mu = Z(0)\, M(0) \left(\frac{w_0}{w}\right)^\gamma \sum_{m=0}^{m=8} 10^{0{\cdot}01m}$$
$$= 164 \left(\frac{w_0}{w}\right)^\gamma .$$

Taking successively $\gamma = 1$ and $\gamma = 3$, we have, because of $w =$ $= 23$ km/sec,

$$\mu = 414\,\text{g} \quad \text{or} \quad \mu = 2630\ \text{g respectively.}$$

Table XXV

Telescopic observations of Draconids, 1933

m	$N(m)$	$f(m)$	$F(m)$	$Z(m_z)$	$\log Z(m_z)$
3	1	0·8	—	—	—
4	1	0·8	0·8	2200	3·34
5	6	4·5	6·3	11000	4·04
6	20	15	31·5	27000	4·43
7	29	22	77	23100	4·36
8	15	11	66	32200	4·51
9	11	8·2	(98)	—	—

As the duration of the shower was only 2^h, the average value of $\cos z$ brings no difficulty with it and was calculated to 0·81.

By formula (4-14), the average density comes out to be

$$\bar{\delta} = 1{\cdot}1 \,.\, 10^{-22}\ \mathrm{g\,.\,cm^{-3}} \quad \text{for} \quad \gamma = 1$$

or

$$\bar{\delta} = 6{\cdot}8 \,.\, 10^{-22}\ \mathrm{g\,.\,cm^{-3}} \quad \text{for} \quad \gamma = 3\,.$$

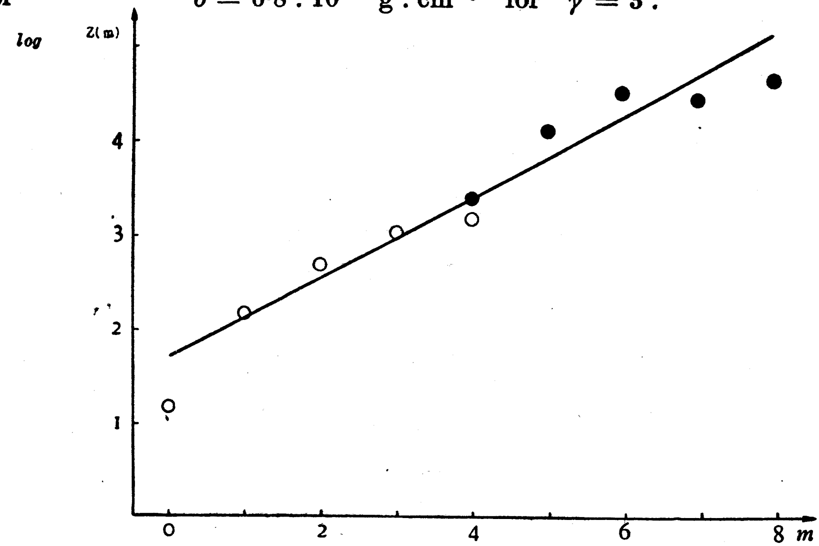

Fig. 5.
Number-luminosity curve of the Draconids in 1933. Open circles: visual observations, full circles: telescopic observations.
(Závislost počtu meteorů na jasnosti u Draconid z r. 1933. Prázdná kolečka: visuální pozorování, plná: teleskopická pozorování.

The swarm of the above density does not appear to be spread over an arc of the orbital ellipse larger than some 200 days. Taking now $n = 0{\cdot}08$ days and $\sin \eta = 0{\cdot}52$, the volume turns out to be

$$V = 6{\cdot}6 \,.\, 10^{33}\ \mathrm{cm^3}\,.$$

Thus the total mass of the Draconid stream is

$$\mu_0 = 7{\cdot}1 \,.\, 10^{11}\ \text{g provided } \gamma = 1$$

or

$$\mu_0 = 4{\cdot}5 \,.\, 10^{12}\ \text{g provided } \gamma = 3\,.$$

The latter value seems to be nearer reality. Nevertheless it is believed that even this figure is rather low. The meteors fainter than 8^m have not been taken into account, but their mass may be considerable. Also the contribution of meteors brighter than 0^m is not negligible, however rare they were in 1933. Moreover, the volume occupied by the swarm may be larger. Thus it can be concluded that a value within ten times the above value is the most probable.

This above value agrees exactly with the estimate made by LOVELL [29].

It is important to note that the total mass of permanent swarms like the Perseids or Geminids comes out to be by about two or three orders higher than that of the Draconids. If we assumed that the swarm of the Draconids is distributed with equal density along the whole orbital ellipse (which, of course, is improbable), the total mass would increase about eight times. Even in this case, it would be fairly lower than the total mass of the permanent showers. We must conclude that either such a faint comet as that of Giacobini-Zinner cannot generate a great stream — or that the Draconids are just being formed. The latter idea appears to be more probable in regard of what was said in the previous chapters.

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VZNIK A RANÁ VÝVOJOVÁ STADIA METEORICKÝCH ROJŮ

1. *Výbuchy v kometách a vznik meteorických rojů*

Richter [12] upozornil na občasné prudké výbuchy v jádrech komet. Whitney [13] dospěl k závěru, že výbuchy lze vysvětlit silným vyvrhováním meteorického prachu. Předpokládal, že všechny vyvržené částice mají stejný rozměr a odvodil, že typický výbuch znamená vyvržení asi 10^{12} g meteorického prachu.

Je však správnější předpokládat, že vyvržené částice mají různé rozměry. Předpokládal jsem rozdělovací funkci ve tvaru $N(s)\,ds \sim \sim s^{-4}\,ds$. Tento zákon platí pro Draconidy v oboru asi mezi $s = 1$ cm a $s = 10^{-2}$ cm. Předpokládáme-li jeho platnost až po $s = 10^{-5}$ cm, vyjde celková hmota vyvrženého hala $M_H = 7.10^{11}$ g. Kdybychom předpokládali, že tento zákon platí pouze v témž oboru jako byl nalezen u Draconid a počet meteorů slabších než odpovídá $s = 10^{-2}$ cm předpokládali konstantní, byla by celková hmota $M_H = 2\,.\,10^{14}$ g. To je číslo příliš vysoké, protože se blíží hmotě celé malé komety. Musíme tedy předpokládat, že při výbuchu je vyvrhováno mnoho meteorického prachu.

V odst. 1.2 jsou uvažovány dynamické účinky výbuchu. Tabulka III ukazuje, že zpětným impulsem se oběžná doba změní jen o zlomek dne, takže účinek je patrně nepozorovatelný.

Kdyby se částice, vyvržené při výbuchu, rozptýlily do prostoru téhož objemu jako zaujímají Draconidy, daly by větší částice při setkání se Zemí vznik meteorickému dešti o hustotě asi 1/10 Draconid, tedy velmi silnému. Rozptyl částic však velmi rychle pokračuje a jakmile zaujmou takový prostor jako na př. Perseidy, klesne hustota asi na tisícinu hustoty Perseid. Jediným výbuchem by tedy snad mohl vzniknout malý hustý meteorický oblak, nikoliv však permanentní roj.

Úvahy z kapitol 2, 3, a 4 vedou k závěru, že Draconidy patrně vznikly teprve v tomto století. To by vyžadovalo, aby v každém oběhu kometa průměrně vyvrhla asi 10^{11} g meteorů, máme-li vznik roje vysvětlit ejekcemi. Protože ejekční rychlosti jsou u Draconid patrně nejméně desetkrát menší než u výbuchů, vyžadoval by tento proces síly jen asi milionkrát menší než výbuchy. Takové děje jsou jistě velmi dobře možné.

V odst. 1.4 ukazuji, že předpokládané ejekce malou rychlostí se nemohou projevit v pohybu komety. Není však vyloučeno, že by se mohly projevit fotometricky v nesymetrickém průběhu jasnosti komety před a po perihelu.

2. *Ejekční theorie tvoření meteorických rojů*

V odst. 2.1 jsou odvozeny vzorce pro výpočet dráhy meteoru, vyvrženého z komety. Je-li ejekční rychlost malá v porovnání s dráhovou rychlostí komety v místě ejekce, jsou vzorce poměrně jednoduché. Numerická aplikace (tab. VI) ukazuje, že dráhy meteorů po ejekci malou rychlostí se jen nepatrně liší od dráhy mateřské komety. Rozdíl oběžných dob má však za následek, že meteor se od komety při každém oběhu vždy více vzdaluje. Tvar vzniklého roje je tedy určen především rozdělením oběžných dob meteorů.

Při úvahách o ejekční rychlosti u pozorovatelných rojů můžeme nejlépe určit její hodnotu u mladých rojů, jež ještě nejsou rozptýleny planetárními poruchami. Takovými roji jsou velké oblaky Draconid a Leonid. Autor odvodil u Draconid rychlosti nejvýše 30 m/sec a u Leonid spíše pod 1 m/sec. (Odst. 2.2).

Tvar roje po ejekci závisí hlavně na velikosti a směru ejekční rychlosti pro jednotlivé meteory. V odst. 2.3 jsou propočítány 4 modely za předpokladu, že ejekční rychlost je pro všechny meteory stálá. Uvažována je: (2.31) isotropní ejekce (t. j. ejekce do všech směrů), (2.32) ejekce směrem ke Slunci, (2.33) ejekce při záporné rotaci jádra, takže většina meteorů je vyvržena pod úhlem 45° ke směru pohybu komety, (2.34) ejekce při kladné rotaci, kdy většina meteorů je vyvržena pod úhlem 135° ke směru pohybu. Rozložení meteorů vzhledem ke kometě je také znázorněno na obr. 3. Odvozeny jsou také vzorce pro porovnání rozložení meteorů před a za kometou.

3. *Lokální poruchy meteorických rojů*

Po vzniku roje mají meteory tendenci se rychle rozložit rovnoměrně podél dráhy komety. Lokální poruchy při průchodu planety v těsné blízkosti dráhy roje však mohou místně velmi rozrušit strukturu meteorického prstenu.

Pro výpočet poruch jsem odvodil přibližnou metodu, založenou na Laplaceově principu sféry aktivity. Touto methodou byly vypočteny poruchy Lyrid při průchodu sférou aktivity Saturna. Každých třicet let projde Saturn v okolí předpokládané dráhy roje a změní oběžné doby meteorů ze 415 let až na 350 nebo 530 let. Silně rušené meteory procházejí při tom uzlem blíže ke Slunci než nerušené a míjejí Zemi. Tím by bylo možno vysvětlit občasná nápadná minima činnosti Lyrid. Potíž je však v tom, že dráha Lyrid je ve skutečnosti velmi špatně známa.

Druhou aplikací je výpočet poruch komety Giacobini-Zinnerovy a Draconid při těsném přiblížení k Jupiteru r. 1897—1899. Jupiter působí značně rozdílné poruchy na různých místech kolem komety, takže meteorické oblaky kolem ní se musily rozptýlit. Pozorované husté roje z r. 1933 a 1946 buď tedy vznikly při setkání slapovým rozpadem komety nebo vznikly až v tomto století ejekcemi. Roj Draconid se patrně stále ještě tvoří.

4. *Hmota a hustota meteorických rojů*

Je vyložena methoda výpočtu hmoty a hustoty rojů z visuálních a teleskopických pozorování. Uvažuji vliv meteorů, slabších než je mezná hvězdná velikost daná citlivostí oka. Metoda je aplikována na určení hmoty a hustoty Draconid z r. 1933. Provedl jsem revisi závislosti počtu meteorů na jasnosti, kterou odvodil Watson. Nejpravděpodobnější hmota roje je řádu 10^{12} g. To je pouze řádově setina až tisícina hmoty Geminid nebo Perseid. Tuto okolnost možno vysvětlit buď tím, že tak slabá kometa jako kometa Giacobini-Zinnerova nemůže dát vznik velikému roji, nebo tím, že roj Draconid se teprve tvoří. Této druhé domněnce nasvědčují určení stáří v kap. 2.

ВОЗНИКНОВЕНИЕ И РЫННИЕ СТАДИИ РАЗВИТИЯ МЕТЕОРНЫХ ПОТОКОВ

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1. *Взрывы в кометах и возникновение метеорных потоков*

Рихтер [12] обратил внимание на повременные сильные взрывы, происходящие в ядрах комет. Уитней [13] пришел к выводу, что эти взрывы можно объяснить сильным извержением метеорной пыли. Он предполагал, что все выброшенные частицы по своим размерам одинаковы и из этого вывел заключение, что при типичном взрыве извергается около 10^{12} г метеорной пыли.

Однако, было бы более правильным предполагать, что выброшенные частицы по своим размерам не одинаковы. По предположению автора функция распределения была представлена в виде $N(s)\,\mathrm{d}s \sim s^{-4}\,\mathrm{d}s$. Этот закон остается в силе и для Драконид в пределах приблизительно между $s = 1$ см и $s — 10^{-2}$ см. Если предположить его действительность до предела $s — 10^{-5}$ см, то количетсво массы изверженного гала будет равно $M_H = 7\,.\,10^{11}$ г. Если предположить, что этот закон действителен лишь для того же предела, как это было установлено у Драконид и что количество метеоров более слабых, чем это соответствует значению $s — 10^{-2}$ см — постоянно, то количество всей массы M_H равнялось бы $2\,.\,10^{14}$ г. Это число слишком велико ввиду того, что его порядок и порядок массы кометы почти одинаковы. Итак, мы допжны предполагать, что при взрыве обильно извергается метеорная пыль.

В параграфе 1.2 были приведены соображения, касающиеся динамических влияний взрыва. Табл. III показывает, что в результате обратного импульса время обращения изменяется лишь на некоторую часть дня, так что эти влияния очевидно незаметны.

Если бы частицы, выброшенные при взрыве, рассеялись в пространстве, размеры которого совпадали бы с размерами объема пространства, занятого Драконидами, то более крупные частицы при встрече с Землей являлись бы причиной метеорного дождя с плотностью около $^1/_{10}$ Драконид. Это значит, что дождь был бы очень обилен. Однако, частицы рассеиваются очень быстро и как только они займут пространство, равное пространству занятому, например, Персеидами, то плотность уменьшится и будет приблизительно равна одной тысячной доли плотности Персеид. В результате взрыва могло бы в таком случае образоваться небольшое густое метеорное облако, но ни коим образом не перманентный поток.

На основании соображений, приведенных в пар. 2, 3 и 4, можно сделать вывод, что Дракониды очевидно возникли не раннее нынешнего века. Если объяснить образование потока, как результат процесса извержений, то необходимо, чтобы было удовлетворено условие, в силу которого комета при каждом своем обращении выбрасывала бы в среднем приблизительно 10^{11} г метеоров. Ввиду того, что скорости извержений у Драконид, очевидно, минимально в десять раз меньше скоростей взрывов, то сила, требйемая для этого процесса, равнялась бы приблизительно лишь одной миллионной доли силы, необходимой для взрывов. Такие явления, наверное, вполне возможны.

В пар. 1·4 было показано, что предполагемое извержение, происшедшее с небольшой скоростью, не могут проявиться в движениях кометы. Однако, не исключено, что они могли бы проявиться фотометрическим образом в несимметричном изменении блеска кометы перед прохождением через перигелий и после его прохождения.

2. *Теория образования метеорных потоков в результате извержений*

В параграфе 2·1 были приведены выведенные формулы для вычисления орбиты метеора, выброшенного кометой. Если скорость извержений по сравнению со скоростью движения кометы по орбите в месте извержения не велика, то формулы сравнительно просты. Числовые данные (табл. VI) показывают, что орбиты метеоров после извержения, происходящего с небольшой скоростью, лишь незначительно отличаются от орбиты кометы-родоначальницы. Результатом различий во времени обращения является, однако, то обстоятельство, что метеор при каждом своем обращении неизменно более и более от кометы удаляется. Итак, вид образовавшегося потока определяется прежде всего распределением времени обращения метеоров.

Что касается соображений относительно скорости извержений у наблюдаемых потоков, то ее значение лучше всего определить для молодых потоков, которые еще не были рассеяны в результате возмущений под действием притяжения планет. Такими потоками являются крупные облака Драконид и Леонид. Автор вывел скорости у Драконид, равняющиеся максимально 30 м/сек, а у Леонид — скорее меньшие, чем 1 м/сек. (Пар. 2·2).

Вид потока после извержения зависит главным образом от величины и направления скорости извержения отдельных метеоров. В пар. 2.3 были расчитаны 4 модели, предполагая, что скорость извержения для всех метеоров неизменна.

Были приведены рассуждения, касаюащиеся вопросов: (2-31) изотропных извержений (т. е. извержений во всевозможных направлениях), (2-32) извержений в направлении к Солнцу, (2—33) извержений при отрицательном вращении ядра, т. е. таких извержений, при которых большинство метеоров выбрасывается под углом 45⁰ в направлении движения кометы, (2-34) извержений при положительном вращении кометы, т. е. таких извержений, при которых большинство метеоров выбрасывается под углом 135⁰ в направлении движения кометы. Расположение метеоров относительно кометы было также приведено на рис. 3. Равным образом были выведены формулы, применяемые при сравнении расположения метеоров перед кометой и за ней.

3. *Местные возмущения метеорных потоков*

После образования метеорного потока у метеоров проявляется тенденция к быстрому и равномерному расположению вдоль орбиты кометы. Местные возмущения при прохождении планеты в непосредственной близости к орбите потока, однако, могут вызвать сильные местные возмущения в структуре метеорного кольца.

Автором был разработан приближенный метод для вычисления возмущений, основанный на принципе Лапласа относительно сферы активности. По этому методу были вычислены возмущения Лирид при их прохождении через сферу активности Сатурна. Через каждых 30 лет Сатурн пройдет в окрестностях предполагаемой орбыты потока и изменит время обращения метеоров с 415 лет на 350—530 лет. При этом сильно возмущенные метеоры проходят узлом на расстоянии более близком Солнцу, чем невозмущенные метеоры, в силу чего первые минуют Землю. Таким образом можно было бы объяснить повременные, заметные минимумы деятельности Лирид. Трудности, однако, заключаются в том, что орбита Лирид в действительности очень мало известна. Дальнейшим применением метода, приведенного в пар. 2, является вычисление возмущений кометы Джиакобини-Циннера и Драконид при их приближении к Юпитеру, имевшим место в 1897—1899 годах. Юпитер вызывает значительно друг от друга отличающиеся возмущения на различных местах вокруг кометы, так что метеорные облака вокруг нее должны рассеиваться. Итак, плотные потоки, наблюдавшиеся в 1933 и 1946 годах, образовались или в результате механического распада при встрече с Юпитером, или в результате извержений, имевших место уже в настоящем столетии. Поток Драконид, очевидно, находится все еще в стадии образования.

4. *Масса и плотность метеорных потоков.*

Здесь излагается метод вычисления массы и плотности потоков, разработанный на основании визуальных и телескопических наблюдений. Речь идет о влиянии метеоров, более слабых, чем предельная магнитуда, данная чувствительностью глаза. Этот метод был использован для определения массы и плотности Драконид в 1933 году. Автор произвел снова вычисления в зависимости количества метеоров от блеска, выведенной Уотсоном. Наиболее вероятной массой потока была бы масса 10^{12} г порядка. Это представляет с точки зрения порядка массы лишь величину от одной сотой до одной тысячной доли массы Геминид или Персеид. Это обстоятельство можно объяснить или тем, что такая слабая комета, как комета Джиакобини-Циннера, не может вызвать образования большого потока, или тем, что поток Драконид находится в стадии образования. Это последнее предположение опирается на определение возраста, приведенное в пар. 2.

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