Institute of Mathematics CAS, v. v. i.

Identification number: 67985840
Address: 115 67 Praha 1, Žitná 609/25

Annual report on activities and economic management in 2021

English summary

The Annual report was discussed by the Supervisory Board of the Institute on May 20, 2022 and approved by the Board of the Institute on May 30, 2022.

Contents

1 The Institute

The Institute of Mathematics of the Czech Academy of Sciences, v. v. i. (“the Institute” or “IM”), is a public research institution according to the Act No. 341/2005 Coll.

The founder of the Institute is the Czech Academy of Sciences seated at Praha 1, Národní street 1009/3, ZIP code 117 20.

The Institute was founded in order to carry out scientific research in the field of mathematics, to contribute to the utilisation of its research results, and to provide the research infrastructure.

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IN: 67985840
TIN: CZ67985840

1.1 Foundation deed (unofficial translation)

Based upon Act No. 283/1992 Coll., on the Czech Academy of Sciences, as subsequently amended, and upon Act No. 341/2005 Coll., on public research institutions, as subsequently amended, and further, in accordance with the Statutes of the Czech Academy of Sciences issued on 24 May 2006, the Czech Academy of Sciences (hereinafter CAS) hereby issues the Full Text of the Foundation Deed of the Institute of Mathematics of the CAS (in Czech "Matematický ústav AV ČR, v. v. i.") dated 28 June 2006 (English version dated 20 December 2006), as subsequently amended by the resolution of the twentieth session of the Academy Council of the CAS held on 15 July 2014 which changed English translation of the name of the CAS from the Academy of Sciences of the Czech Republic to the Czech Academy of Sciences as of 1 January 2015:

I.

(1) The Institute was incorporated into the Czechoslovak Academy of Sciences (hereinafter CSAS) under the name the Mathematical Institute of the CSAS by a resolution of the third plenary meeting of the Government Commission for the Establishment of the CSAS held on 30 March 1952, which took effect on 1 January 1953. Under section 18 (2) of Act No. 283/1992 Coll., the Institute became an entity of the CAS as of 31 December 1992.

(2) Under Act No. 341/2005 Coll., the legal status of the Mathematical Institute of the CAS has been transformed from a state contributory organisation into a public research institution (abbreviated in Czech as v. v. i.) from 1 January 2007.

II.

(1) The Institute of Mathematics of the CAS (hereinafter IM) is established for an indefinite period as a legal entity with identification number 67985840, and is located in Prague 1, Žitná 609/25, Postal Code 115 67.

(2) The founder of the IM is the CAS, an organisational body of the state, identification number 60165171, headquartered in Prague 1, Národní 1009/3, Postal Code 117 20.

III.

(1) The purpose for which the IM has been established is to carry out scientific research in the field of mathematics, to contribute to the utilisation of its research results, and to provide the research infrastructure.

(2) The principal activity of the IM is scientific research in the fields of mathematics and its applications. The IM contributes to raising the level of knowledge and education and to utilising the results of scientific research in practice. It acquires, processes and disseminates scientific information, issues scientific and professional publications (monographs, journals, proceedings, etc.). It provides scientific assessments, professional opinions and recommendations, consulting and advisory services. In cooperation with universities, the IM carries out doctoral study programmes and provides training for young scientists. Within the scope of its activity, the IM promotes international cooperation, including the organisation of joint research projects with foreign partners, participation in exchange programmes for scientists and the exchange of scientific information, as well as the preparation of joint publications. The IM organises scientific meetings, conferences and seminars on the national and international levels and provides the infrastructure for research, including the provision of accommodation for its employees and guests. It pursues its aims both independently and in cooperation with universities and other research and professional institutions.

IV.

(1) The director, the Board and the Supervisory Board are the bodies of the IM. The director is the statutory body of the IM and is entitled to act on behalf of the IM.

(2) Basic organisational units of the IM are scientific departments responsible for research and development, and service departments responsible for provision of the infrastructure.

(3) The detailed organisational structure of the IM is regulated by rules of organisation issued by the director after being approved by the Board.

V.

The foundation deed in its present form took effect on 1 January 2015.

Prof. Jiří Drahoš
President of the CAS

1.2 Governing bodies (as of December 31, 2021)

Director: Doc. RNDr. Tomáš Vejchodský, Ph.D.

Deputy Director: Doc. Dr. Ing. Miroslav Rozložník, DSc.

Board of the Institute:

Chair:	RNDr. Martin Markl, DrSc.
Vice-chair:	Vojtěch Pravda, Ph.D., DSc.
Members at large:	Prof. RNDr. Zuzana Došlá, CSc., DSc. (Masaryk University)
	Prof. RNDr. Pavel Drábek, DrSc. (University of West Bohemia in Pilsen)
	Prof. RNDr. Eduard Feireisl, DrSc.
	Prof. RNDr. Stanislav Hencl, Ph.D., DSc. (Charles University)
	Prof. RNDr. Michal Křížek, DrSc.
	Prof. Wieslaw Kubiś, Ph.D.
	RNDr. Šárka Nečasová, CSc., DSc.
	Doc. Mgr. Milan Pokorný, Ph.D., DSc. (Charles University)
	Ing. Jakub Šístek, Ph.D.

Supervisory Board:

Chair:	Prof. Ing. Michal Haindl, DrSc. (Academy Council of the CAS)
Vice-chair:	Mgr. Alena Pravdová, Ph.D.
Members at large:	Prof. RNDr. Jan Hamhalter, CSc. (Czech Technical University in Prague)
	Prof. RNDr. Luboš Pick, CSc., DSc. (Charles University)
	Ing. Július Štuller, CSc. (Institute of Computer Science of the CAS)

The director of the Institute cooperated with the Board of the Institute and relied on an informal advisory board formed by the chair of the Board Martin Markl, deputy director Miroslav Rozložník, the scientific secretary and project manager Beata Kubiś, head of the Administration Department Jan Bíža, head of the IT Department Martin Jarník.

The **Board of the Institute** held six meetings, four of them remotely. The topics the Board discussed and approved included among others:

- budget of the Institute for 2021 and the medium-term financial outlook for 2022–2023,
- annual report on activities and economic management in 2020 and auditor's report on financial statements,
- proposal of T. Bice for the Lumina quaeruntur Premium,
- selection of I. Gudoshnikov for the postdoctoral positions in the Academy's programme supporting prospective human resources,
- recommendation to invite Prof. Ramamohan Paturi (University of California, San Diego La Jolla, USA) for the Eduard Čech Distinguished Visitor position,
- proposal of T. Kania for the Otto Wichterle Award to promising young scientists.

The **Supervisory Board** held four meetings, two of them remotely. The topics they discussed and approved included among others:

- discussion about the proposal of the budget of the Institute for 2021 and the medium-term financial outlook for 2022–2023, about the annual report on activities and economic management in 2020 and about auditor's report on financial statements,
- approval of l amendments extending lease agreement with the Institute of Material Physics, Czech Academy of Sciences in Brno,
- approval of lease agreements and amendments extending lease agreements for flats in the Institute's building,
- approval of the updates of the contract for the lease of space for the SPIN flower shop.

1.3 Evaluation

The process of evaluation of research and professional activity of research-oriented institutes of the Czech Academy of Sciences for the period 2015–2019 culminated by the online presentation of the Institute of Mathematics to the evaluation committee on March 8, 2021. Preparations for the evaluation began in 2019, and the documents for Phase I were submitted in March 2020. However, a covid-19 pandemic started at that time, and the Academic Council decided to postpone all remaining evaluation deadlines. The full text of the application was submitted at the end of November 2020. The presentation of the Institute was originally planned as face-to-face, but due to the unfavourable epidemic situation, the Academic Council decided that it would be replaced by a videoconference meeting.

During the entire evaluation process, the management of the Institute, in close cooperation with the heads of departments, prepared all documents and presentations. In accordance with the organizational structure, the Institute of Mathematics submitted for evaluation six teams. The following teams were registered in the field of mathematics: Abstract Analysis, Algebra, Geometry and Mathematical Physics, Differential Equations and Theory of Integral, Evolutionary Differential Equations, and Numerical Analysis. The team of Mathematical Logic and Theoretical Computer Science was registered in the field of computer science.

In Phase I, anonymous foreign evaluators assessed a prescribed number of selected outputs of individual teams and classified them into five categories: 1. world-leading, 2. internationally excellent, 3. recognized internationally, 4. recognized acceptable, 5. below the standards. Out of the total number of 115 evaluated outputs of the Institute of Mathematics, 28.7% were evaluated in the first category and 53.9% in the second category. In comparison with the results of the whole Czech Academy of Sciences, where 15.7% were evaluated in the first category and 43.4% in the second category, the Institute of Mathematics achieved results high above the average.

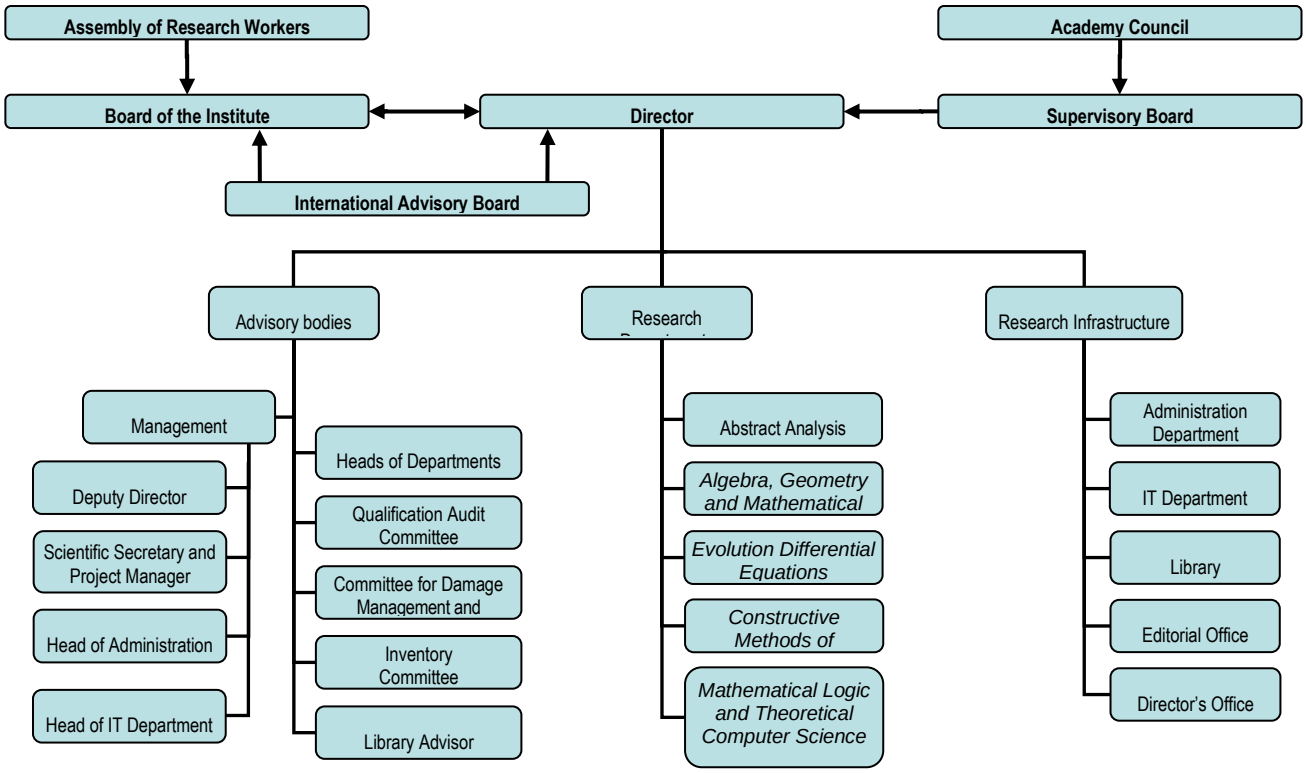
In Phase II, the committee for mathematics and computer science led by Professor Edwin Hancock evaluated the Institute and all its teams. The committee prepared the final report based on the results of Phase I, all submitted materials, and presentations of the director and team leaders. In the report, the committee recommended increasing the support of the team of Mathematical Logic and Theoretical Computer Science and keeping the support for other evaluated teams except for the Differential Equations and Theory of Integral team. The committee did not provide any recommendation for this team because it was already restructured during the evaluation process. Overall, the Institute of Mathematics was evaluated very well. The management of the Institute will in the near future use inspiring comments and recommendations to eliminate shortcomings and to further increase the quality and performance.

1.4 Structure

On December 2, 2020, the Board of the Institute discussed and approved the proposal of the Director to change the organizational structure by removing the Branch in Brno with effect on April 1, 2021. After several researchers working in Brno left this department, its existence as an independent research department became unsustainable. Its remaining employees were formally moved to Prague departments according to their specialization, however their workplace remained in Brno. The updated organizational structure is depicted in the scheme below.

Institute of Mathematics, Czech Academy of Sciences

Scheme of the organizational structure



The Institute publishes three internationally recognized mathematical journals:

- Czechoslovak Mathematical Journal
- Mathematica Bohemica
- Applications of Mathematics

The director nominates the Editorial Boards and the Editors-in-Chief.

The Institute is maintaining and developing the Czech Digital Mathematics Library DML-CZ accessed at https://dml.cz and participates in the development of the European Digital Mathematics Library EuDML accessed at https://eudml.org. The Institute operates the Prague Zentralblatt Editorial Group contributing to the production of the database zbMATH.

2 Research activities

2.1 Characteristics of the principal activity

The principal activity of the IM is to support fundamental research in the fields of mathematics and its applications, and to provide necessary infrastructure for research. The IM contributes to raising the level of knowledge and education and to utilising the results of scientific research in practice. It acquires, processes and disseminates scientific information including scientific publications (journals, proceedings, monographs etc.). In cooperation with universities, the IM carries out doctoral study programmes and provides training for young scientists. The IM promotes international cooperation, including the organisation of joint research projects with foreign partners and participation in exchange programmes. The IM organises scientific meetings, conferences and seminars on the national and international levels.

Research in the Institute focuses on mathematical analysis (differential equations, numerical analysis, functional analysis, theory of function spaces), algebraic and differential geometry, mathematical physics, mathematical logic, complexity theory, combinatorics, set theory, numerical linear algebra, general and algebraic topology, optimization and control.

2.2 Departments

Abstract Analysis

Main research themes of the department members can be described as the study and classification of mathematical structures, using advanced methods of logic, set theory, and category theory, as well as modern tools of mathematical analysis and algebra. Abstract analysis refers to these areas of science where mathematical logic plays a significant role, even though it is not the main object of study. These areas include descriptive set theory, topology, Banach space theory, and the theory of C* algebras.

Algebra, Geometry and Mathematical Physics

The department consists of researchers interested in algebraic and differential geometry and in closely related areas of mathematical physics. The research is focused on mathematical aspects of modern theoretical models of physics of microcosmos and cosmology related to logical correctness of physical hypotheses and mathematical models aiming at understanding the nature of matter and space. Research topics include representation theory and its applications to algebraic geometry, homological algebra, algebraic topology, applied theory of categories, tensors classification, generalized theory of gravitation, and study of Einstein equations.

Constructive Methods of Mathematical Analysis

The department focuses on mathematical modelling of complex physical processes that involve an immense amount of data and require advanced implementations on parallel computer architectures. The main topics include theory and applications of numerical methods for partial differential equations, a posteriori error analysis, computational methods of numerical linear algebra, matrix theory, domain decomposition and multilevel methods. Another topic is presented by methods of flow-filed analysis, mostly for vortex identification. Members of the department are involved in the Jindřich Nečas Centre for Mathematical Modeling (http://ncmm.karlin.mff.cuni.cz/) and in the network for industrial mathematics EU-MATHS-IN.CZ (http://www.eu-maths-in.cz/), part of the European network EU-MATHS-IN (http://eu-maths-in.eu/).

Evolution Differential Equations

The department focuses on qualitative theory of partial differential equations in mechanics and thermodynamics of continuum, in biology, chemistry and other natural and technical sciences. The research aims at verification of correctness and other fundamental properties of mathematical models and at the possibility of providing theoretical predictions of future development of systems without the full knowledge of the initial state. The core are work equations describing motion of various kinds of

fluids, including exchange heat and interaction with solid bodies. The attention is also paid to processes in solid materials, focusing on mathematical modelling of memory in multifunctional substances, on dynamical behaviour of bodies in contact with neighbourhood, and on phase transitions. Several members investigate modern theory of integration in connection with ordinary differential equations.

Mathematical Logic and Theoretical Computer Science

The research conducted in the department concerns several loosely connected areas. The main ones are theoretical computer science and mathematical logic; other important areas are combinatorics, control theory, automata theory and differential geometry. The main topic in theoretical computer science is computational complexity, which is connected with another topic, proof complexity, an area of research on the border of theoretical computer science and mathematical logic. Other main topics of mathematical logic studied in the department are set theory and formal arithmetic.

2.3 Research centres

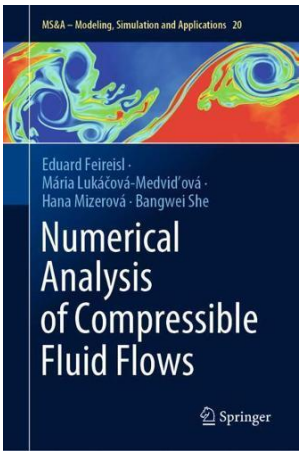
Jindřich Nečas Centre for Mathematical Modeling (http://ncmm.karlin.mff.cuni.cz/) is a consortium of the Institute of Mathematics, the Faculty of Mathematics and Physics of the Charles University, and the Institute of Computer Science CAS. It was established in 2013 to continue the efforts of a joint project funded by the Ministry of Education, Youth and Sports in 2005–2011. Its general goal is to establish a strong research team in the field of mathematical properties of models in continuum mechanics and thermodynamics, developed by an intensive collaboration of important research groups at participating institutions and their goal-directed collaboration with top experts from abroad. Organization of lecture courses and the everyday interaction with PhD and undergraduate students aims at upbringing new generation of competent scientists and forming a basis for a strong and stable research team.

DIMATIA – Centre for Discrete Mathematics, Theoretical Computer Science and Applications (http://dimatia.mff.cuni.cz/) is a consortium of the Faculty of Mathematics and Physics of the Charles University, the Institute of Mathematics and the Institute of Chemical Technology in Prague. It was established in 1996 with the aim to foster research in all fields of discrete mathematics and its modern applications and relationship to computer science, operations research and fields as diverse as biology, chemistry and social sciences. The centre organizes a continuing programme of workshops, conferences and research visits, postdoctoral positions announced and jointly supported by the partners and short-term visits of senior researches. DIMATIA created an extensive international network with 13 further research institutions.

2.4 Research output

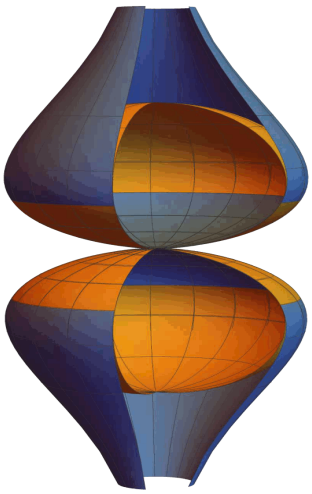
In 2021, members of the Institute published the total of 179 journal and proceedings papers, including three monographs. The following 13 results were selected to illustrate the output. The detailed information about all publications is available at Institute's web site http://www.math.cas.cz/ in section Research / Publications.

[1] **Feireisl, E**. – Lukáčová-Medviďová, M. – **Mizerová, H.** – **She, B.** *Numerical Analysis of Compressible Fluid Flows*. Springer, Cham, 2021.



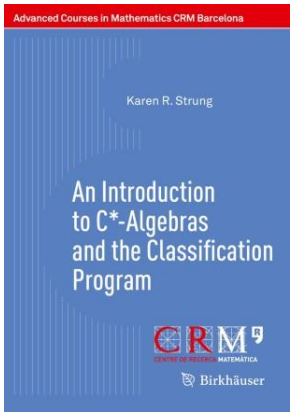
This book is devoted to the numerical analysis of compressible fluids in the spirit of the celebrated Lax equivalence theorem.The book contains original theoretical material based on a new approach to generalized solutions (dissipative or measure-valued solutions). The concept of a weak-strong uniqueness principle in the class of generalized solutions is used to prove the convergence of various numerical methods. The problem of oscillatory solutions is solved by an original adaptation of the method of K-convergence. An effective method of computing the Young measures is presented. Theoretical results are illustrated by a series of numerical experiments..

[2] Kolář, I. – **Málek, T.** – Mazumdar, A. *Exact solutions of nonlocal gravity in a class of almost universal spacetimes*, Phys. Rev. D 103 (2021), Article ID 124067.



We studied exact solutions of the infinite derivative gravity (IDG). This theory improves Einstein's general relativity by introducing additional corrections involving an infinite number of covariant derivatives. Although, IDG is a classical field theory, it shares some interesting properties of quantum theories due to its nonlocality arising from the infinite number of derivatives. For example, smoothing of spacetime singularities. The nonlocality of IDG however makes it significantly difficult to find exact solutions of the IDG field equations. So far, all known solutions of IDG were either approximative nonlocal solutions of linearized theory or exact but local solutions lacking the nonlocal behavior. We managed to successfully employ the results of our previous works dealing with the so-called almost universal spacetimes. These spacetimes are characterized by the fact that they linearize the field equations of any geometric theory of gravity, but at the same time, they do not reduce the number of covariant derivatives. This approach thus allowed us to find exact nonlocal IDG solutions describing gravitational waves generated by a massless particle propagating on a maximally symmetric background spacetime with an arbitrary cosmological constant..

[3] **Strung K. R.** *Introduction to C*-Algebras and the Classification Program*. Birkhäuser, Cham, 2021.



This book is directed towards graduate students that wish to start from the basic theory of C*-algebras and advance to an overview of some of the most spectacular results concerning the structure of nuclear C*-algebras. The text is divided into three parts. First, elementary notions, classical theorems and constructions are developed. Then, essential examples in the theory, such as crossed products and the class of quasidiagonal C*-algebras, are examined, and finally, the Elliott invariant, the Cuntz semigroup, and the Jiang-Su algebra are defined. It is shown how these objects have played a fundamental role in understanding the fine structure of nuclear C*-algebras. To help understanding the theory, plenty of examples, treated in detail, are included.

[4] Aksteiner, S. – Andersson, L. – Bäckdahl, T. – **Khavkine, I.** – Whiting, B. *Compatibility complex for black hole spacetimes*, Commun. Math. Phys. 384 (2021) 1585–1614.

The dynamical variables in linearized General Relativity come in equivalence classes generated by infinitesimal coordinate transformations (through the Killing operator). Any physically observable quantity that locally depends on the dynamical variables must depend only on the equivalence class, and hence must annihilate the Killing operator. All such physically observable quantities can be

generated from the components of a "compatibility operator" for the Killing operator, which strongly depends on the background spacetime geometry. We have explicitly constructed such a compatibility operator on the background of a Kerr black hole, which is of importance in the astrophysics of gravitational waves. The construction employed algebraic tools from homological algebra and geometric tools from spinor calculus.

[5] Carson, E. – Lund, K. – **Rozložník, M.** *The stability of block variants of classical Gram-Schmidt*, SIAM J. Matrix Anal. Appl. 42 (2021), 1365–1380.

The block version of the classical Gram--Schmidt (BCGS) method is often employed to efficiently compute orthogonal bases for Krylov subspace methods and eigenvalue solvers, but a rigorous proof of its stability behavior has not yet been established. It is shown that the usual implementation of BCGS can lose orthogonality at a rate worse than $O(\varepsilon)\kappa^2(X)$, where X is the input matrix and úroveň (ε) is the unit roundoff. A useful intermediate quantity denoted as the Cholesky residual is given special attention and, along with a block generalization of the Pythagorean theorem, this quantity is used to develop more stable variants of BCGS. These variants are proven to have $O(\varepsilon)\kappa^2(X)$ loss of orthogonality with relatively relaxed conditions on the intrablock orthogonalization routine satisfied by the most commonly used algorithms. A variety of numerical examples illustrate the theoretical bounds.

[6] Chiodaroli, E. – **Kreml, O.** – **Mácha, V.** – **Schwarzacher, S.** *Non-uniqueness of admissible weak solutions to the compressible Euler equations with smooth initial data*, Trans. Am. Math. Soc. 374 (2021), 2269–2295.

We consider the isentropic Euler equations of gas dynamics in the whole two-dimensional space and we prove the existence of a initial datum which admits infinitely many bounded admissible weak solutions. Taking advantage of the relation between smooth solutions to the Euler system and to the Burgers equation we construct a smooth compression wave which collapses into a perturbed Riemann state at some time instant . In order to continue the solution after the formation of the discontinuity, we adjust and apply the theory developed by De Lellis and Székelyhidi [Ann. of Math. (2) 170 (2009), no. 3, pp. 1417–1436; Arch. Ration. Mech. Anal. 195 (2010), no. 1, pp. 225–260] and we construct infinitely many solutions. We introduce the notion of an admissible generalized fan subsolution to be able to handle data which are not piecewise constant and we reduce the argument to finding a single generalized subsolution.

[7] Dolgov, S. – **Vejchodský, T.** *Guaranteed a posteriori error bounds for low rank tensor approximate solutions*, IMA J. Numer. Anal. 41 (2021), 1240–1266.

The paper derives guaranteed a posteriori upper bound on the error of approximate solutions of elliptic partial differential equations computed by low-rank tensors. These methods solve efficiently high-dimensional problems, but the solution process cumulates a sequence of errors. The proposed bound determines a posteriori the total error of the computed solution and finds whether it is sufficiently accurate.

[8] **Gavinsky, D.** *Bare quantum simultaneity versus classical interactivity in communication complexity*, IEEE Trans. Inform. Theory 67 (2021), 6583–6605.

Článek uvádí jeden z nejsilnějších známých příkladů kvalitativní převahy kvantové komunikace: bipartitní komunikační problém, který má účinný kvantový souběžný (totiž SMP) protokol, ale žádný účinný klasický obousměrný protokol.

[9] Krawczyk, A. – **Kubiś, W.** *Games with finitely generated structures*, Ann. Pure Appl. Logic 172 (2021), Article ID 103016.

We study the abstract Banach-Mazur game played with finitely generated structures instead of open sets. We characterize the existence of winning strategies aiming at a single countably generated structure. We also introduce the concept of weak Fraïssé classes, extending the classical Fraïssé theory, revealing its relations to our Banach-Mazur game. Finally, we exhibit connections between the universality number and the weak amalgamation property.

[10] Krejčí, P. – **Monteiro, G. A.** – Recupero, V. *Explicit and implicit non-convex sweeping processes in the space of absolutely continuous functions*, Appl. Math. Optim. 84 (2021), 1477–1504..

We show that sweeping processes with prox-regular constraints generate a strongly continuous input-output mapping in the space of absolutely continuous functions. This happens to be an interesting tool to address implicit problems in the non-convex case without using other kinds of regularization approaches.

[11] **Müller, V.** – Tomilov, Y. *On the interplay between operators, bases, and matrices*. J. Funct. Anal. 281 (2021), Article ID 109158.

Given a bounded linear operator T on a separable Hilbert space, we develop an approach allowing one to construct a matrix representation for T having certain specified algebraic or asymptotic structure. We obtain matrix representations for T with preassigned bands of the main diagonals, with an upper bound for all of the matrix elements, and with entrywise rational-like lower and upper bounds for these elements. In particular, we substantially generalize and complement our results on diagonals of operators and other related results. Moreover, we obtain a vast generalization of a theorem by Stout (1981), and (partially) answer his open question. Several of our results have no analogues in the literature.

[12] **Pravda, V.** – **Pravdová, A.** – Podolský, J. – Švarc, R. *Black holes and other spherical solutions in quadratic gravity with a cosmological constant*, Phys. Rev. D 103 (2021), Article ID 064049.

Conformal-to-Kundt metric ansatz is employed to study static spherically symmetric solutions to the vacuum field equations of quadratic gravity in the presence of a cosmological constant Λ. We arrive to several new classes of solutions to the field equations of quadratic gravity. In particular, we obtain various non-Einstein generalizations of the Schwarzschild, (anti-)de Sitter, Nariai, and Plebański-Hacyan spacetimes. Interestingly, some classes of solutions allow for an arbitrary value of Λ, while other classes admit only discrete values of Λ. We determine which classes contain the Schwarzschild-(A)dS black hole as a special case and briefly discuss the physical interpretation of the spacetimes. In the discussion of physical properties, we focus on the generalization of the Schwarzschild-(A)dS black hole, namely the Schwarzschild-Bach-(A)dS black hole, which possesses one additional Bach parameter. We also study its basic thermodynamical properties and observable effects on test particles caused by the presence of the Bach tensor..

[13] Reveliotis, S. – **Masopust, T.** – Ibrahim, M. *Polynomial-Time Optimal Liveness Enforcement for Guidepath-based Transport Systems*, Nonlinear Anal., Hybrid Syst. 41 (2021), Article ID 101058.

The addressed problem is a basic reachability problem regarding the traffic management of multi-agent systems that had stood open in the literature for a long time. The polynomial-time complexity of the derived algorithm is a surprising result when assessed within the scope of similar reachability problems. The methodological framework and tools developed to deliver the result are pretty complex and adapt and integrate concepts and results drawn from formal languages and automata theory, graph theory, and some classical perspectives coming from the area of deadlock avoidance and liveness-enforcing supervision of sequential resource allocation systems. From the standpoint of practical relevance, the results find immediate applicability not only in unit-load material handling systems, such as automated guided vehicles or overhead monorail systems used in many production and distribution facilities, but also in robotics, in the representation of some board games, in the programming of animations developed by the current video game industry, or in the modeling and analysis of the elementary physical operations taking place in quantum computing.

2.5 Projects

1 project Praemium Academiae funded by the Czech Academy of Sciences

- Operadic categories and their applications (2019–2024, M. Markl)

2 grant projects for the support of excellence in basic research EXPRO funded by the Czech Science Foundation:

- 20-31529X Abstract convergence schemes and their complexities (2020–2024, W. Kubiś)

- 19-27871X Efficient approximation algorithms and circuit complexity (2019–2023, P. Hrubeš)

10 standard grant projects funded by the Czech Science Foundation:

- 21-02411S Solving ill posed problems in the dynamics of compressible fluids (2021–2023, E. Feireisl)
- 20-14736S Hysteresis modeling in mathematical engineering (2020–2022, G. Monteiro)
- 20-13778S Symmetries, dualities and approximations in derived algebraic geometry and representation theory (2020–2022, L. Positselski)
- 20-01074S Adaptive methods for the numerical solution of partial differential equations: analysis, error estimates and iterative solvers (2020–2022, T. Vejchodský)
- 19-09659S Exact solutions of gravity theories: black holes, radiative spacetimes and electromagnetic fields (2019–2021, V. Pravda)
- 19-04243S Partial differential equations in mechanics and thermodynamics of fluids (2019–2021, Š. Nečasová)
- 19-05497S Complexity of mathematical proofs and structures (2019–2021, E. Jeřábek)
- 18-00580S Function spaces and approximations (2018–2020, extended to 2021, A. Gogatishvili)
- 18-07776S Higher structures in algebra, geometry and mathematical physics (2018–2020, extended to 2021, M. Markl)
- 18-05974S Oscillations and concentrations versus stability in the equations of mathematical fluid dynamics (2018–2020, extended to 2021, E. Feireisl)

4 junior grant projects funded by the Czech Science Foundation:

- 20-17488Y Applications of C*-algebra classification: dynamics, geometry, and their quantum analogues (2020–2022, K. Strung)
- 19-05271Y Groups and their actions, operator algebras, and descriptive set theory (2019–2021, M. Doucha)
- 19-07129Y Linear-analysis techniques in operator algebras and vice versa (2019–2021, T. Kania)
- 18-01472Y Graph limits and inhomogeneous random graphs (2018–2020, extended to 2021, J. Hladký)

2 international grant projects funded by the Czech Science Foundation

- 19-06175J Compositional Methods for the Control of Concurrent Timed Discrete-Event Systems (2019–2021, J. Komenda)
- 18-01953J Geometric methods in statistical learning theory and applications (2018–2020, extended to 2021, H. V. Le)

1 international grant project evaluated on the basis of the LEAD Agency principle funded by the Czech Science Foundation

- 20-22230L Banach spaces of continuous and Lipschitz functions (2020–2022, W. Kubiś)

2 projects in the Structural Funds Operational Programme Research, Development and Education, funded by the European Commission, operated by the Ministry of Education, Youth and Sports

- CZ.02.2.69/0.0/0.0/18_054/0014664 Institute of Mathematics CAS goes for HR Award
- implementation of the professional HR management (2020–2022, team: L. Bauerová, B. Kubiś, M. Rozložník, K. Strung, T. Vejchodský)
- CZ.02.2.69/0.0/0.0/16_018/0002713 Doctoral School for Education in Mathematical Methods and Tools in HPC (2017–2022, T. Vejchodský)

2 MOBILITY projects funded by the Ministry of Education, Youth and Sports:

- 8J20FR007 Mathematics of diffuse interface models (2020–2021, E. Feireisl)
- 8J20AT022 Hysteresis in hypo-plastic models (2020–2021, G. Monteiro)

1 INTER-EXCELLENCE project funded by the Ministry of Education, Youth and Sports:

- LTAUSA19098 Verification and Control of Networked Discrete-Event-Systems (2020–2022, J. Komenda)

A detailed information on the projects is available at the Institute's web site http://www.math.cas.cz/ in the section Research / Grants.

2.6 International conferences and workshops organized by the Institute

Fluids under Control, Prague, 23.–27. 8. 2021
https://prague-sum.com/

Workshop on Generic Structures, Praha, 24.–30. 10. 2021
https://gens.math.cas.cz/

Complexity Theory with a Human Face, 2nd Edition, Špindlerův Mlýn*,* 17.–20. 10. 2021
https://users.math.cas.cz/~talebanfard/workshop21/

Mathematics in Industry 2021, Praha, 6. 12. 2021,
http://workshop.math.cas.cz/MathInIndustry2021/

Mathematics for Industry 2021, Praha, 7. 12. 2021,
https://calendar.math.cas.cz/sites/default/files/Bobcat_Program_07_12_2021_0.pdf

3rd Chinese-Czech Conference on Mathematical Fluid Mechanics, (virtually on Zoom), Praha, 15.–16. 12. 2021,
https://www2.karlin.mff.cuni.cz/~tumak3am/download/C-C-workshop-program.pdf

2.7 International collaboration

An extensive international collaboration in 2018 is documented by the following facts:

- 33 visitors to the Institute
- 62 research visits abroad
- 6 international conferences and meetings organized or co-organized by the Institute
- 56 memberships in editorial boards of international scholarly journals

The Institute is a corporate member of the following organizations:

- The Union of Czech Mathematicians and Physicists
- The European Mathematical Society
- ERCOM (European Research Centres on Mathematics)
- European Digital Mathematics Library Initiative
- EU-MATHS-IN (European Service Network of Mathematics For Industry and Innovation) – through the national network EU-MATHS-IN.CZ

2.8 Cooperation with universities in education

Members of the Institute held a number of courses for students at Czech and foreign universities, supervised 30 PhD students. The Institute is accredited for 18 PhD programmes jointly with the Charles University and the University of West Bohemia.

PhD students trained in the Institute in cooperation with the universities:

Azhar Abekova, L. U. Gumilov Eurasian National University, consultant A. Gogatishvili
David Adamadze, I. Javakhishvili Tbilisi State University, supervisor A. Gogatishvili
Jiří Balun, Palacký University Olomouc, supervisor T. Masopust
Danica Basarić, Technische Universität Berlin, supervisor E. Feireisl
José Andrés Oyarce Boggioni, Universidad del Bío-Bío, Chile, cosupervisor R. Hakl
Jaroslav Bradík, Silesian university in Opava, supervisor M. Engliš
Matěj Dolník, Technical University Brno, supervisor A. Lomtatidze
Zadira Ermiashova, L. U. Gumilov Eurasian National University, consultant A. Gogatishvili
Martin Fencl, University of West Bohemia Plzeň, supervisor M. Kučera
Lukáš Folwarczný, Charles University, supervisor P. Pudlák

Sviatoslav Gryaznov, St. Petersburg Department of Steklov Mathematical Institute of Russian Academy of Sciences, cosupervisor E. Jeřábek
Martin Hanek, Czech Technical University Prague, supervisor specialist J. Šístek
Nilasis Chaudhuri, Technische Universität Berlin, supervisor E. Feireisl
Erfan Khaniki, Charles University, supervisor P. Pudlák
David Kokoška, Charles University, supervisor M. Ortaggio
Ziemowit Kostana, Uniwersytet Warszawski, supervisor W. Kubiś
Natalia Maślany, Uniwersytet Jagielloński, Krakow, supervisor T. Kania
David Matejov, Charles University, supervisor I. Khavkine
Ruben Medina, Universidad de Granada, supervisor P. Hájek
Josef Navrátil, FJFI ČVUT v Praze, supervisor M. Kučera
Andres Quiles, Universidad Politècnica de València, supervisor P. Hájek
Paulina Radecka, Uniwersytet S. Wyszynskiego, Warszawa, supervisor W. Kubiś
Ana Radošević, University of Zagreb, supervisor Š. Nečasová
Jan Scherz, Charles University and Universität Würzburg, consultant Š. Nečasová
Aravindhan Srinivasan, Charles University, supervisor M. Ortaggio
Tomáš Tintěra, Charles University, supervisor V. Pravda
Dominik Trnka, Masaryk University in Brno, Přírodovědecká fakulta, supervisor M. Markl
George Turner, Charles University, supervisor V. Pravda
Dávid Uhrik, Charles University, supervisor D. Chodounský
Xingchen Yu, Nanjing University of Information Science and Technology, cosupervisor R. Hakl

2.9 Awards

Šárka Nečasová, Praemium Academiae awarded by the Czech Academy of Sciences.

Tomasz Kania, Otto Wichterle Award, Czech Academy of Sciences. Award given to young scientists under 35 years for achievement of excellent results in their disciplines.

Martin Doubek, Branislav Jurčo, Martin Markl a **Ivo Sachs,** Prize of the Dean of the Faculty of Mathematics and Physics, Charles University for the best book publication in 2020 in the category Monographs for the book *Algebraic Structure of String Field Theory.* Lecture Notes in Physics, 973. Cham: Springer, 2020.

Jan Papež, Prize of the Dean of the Faculty of Mathematics and Physics, Charles University for the best evaluated teachers in the academic year 2020/21, for teaching the courses Analysis of matrix computations 1 a Linear algebra 1.

2.10 Further activities

The prestigious annual **Eduard Čech Lecture** devoted to the memory of the eminent Czech mathematician and founder of the Institute was created to attract excellent foreign mathematicians and to further stimulate creative environment at the Institute. The seventeenth Eduard Čech Distinguished Visitor of the Institute of Mathematics was A. Novotný (Institut de Mathématiques de Toulon, France), who has agreed to come despite the problematic situation.The stay of previous distinguished visitor S. Todorčević (University of Toronto, Centre National de la Recherche Scientifique, Paris, and Matematički Institut SANU, Belgrade) did not happen due epidemilogic situation. The next distinguished visitor will be R. Paturi (University of California, San Diego La Jolla, USA).

The Institute organized traditional Open Houses as a part of the scientific festival Week of Czech Academy of Sciences. According to our statistics during first five days in November 905 high-school students and other visitors watched 34 lectures and attended 8 excursions of the Institute.

The Institute continued in providing professional and financial support to the Mathematical Olympiad, particularly in preparation of the national representatives to the International Mathematical Olympiad.

3 Economic management

3.1 Assets

The Institute owns the estate, parcel no. 2120, and the building, house no. 609/25, on that land. Total area of residential and non-residential premises is 3,341 square metres. Part of the ground floor in the front building of 64 square metres is leased for commercial purpose; further two rooms and one storeroom are leased for non-commercial purpose to the Union of Czech Mathematicians and Physicists. In the rear building there are five flats leased mostly to employees of the Institute. All other spaces in both buildings (2,836 square metres in total) are used for the purpose of the Institute.

The book value of the compound to the day of 31 December 2021 was 43 673 thousand CZK, its remaining book value was 19 835 thousand CZK.

Further tangible fixed assets is formed mostly by devices and IT equipment with the book value 8 868 thousand CZK to the date 31 December 2021, remaining book value was 702 thousand CZK.

3.2 Expenses and revenues

Principal entries (in thousands of CZK)

Total expenses	**110,155**
Purchases of materials, electricity, gas	3,102
Maintenance and reconstructions	1,232
Travel expenses	1,974
Other services	5,219
Personal expenses	96,921
Other expenses	593
Depreciation	977
Total revenues	**110,155**
Sales of periodicals	1,611
Other revenues	3,254
Institutional subsidies (from the budget of the Czech Academy of Sciences)	65,498
Grants	39,709
Earnings before taxes	**0**

The total revenues compared to the year 2020, increased by 2.1%. This was mainly due to the increase of the institutional subsidies provided by the Czech Academy of Sciences and the increase of grant sources provided mainly by the Czech Science Foundation and Ministry of Education, Youth, and Sports.

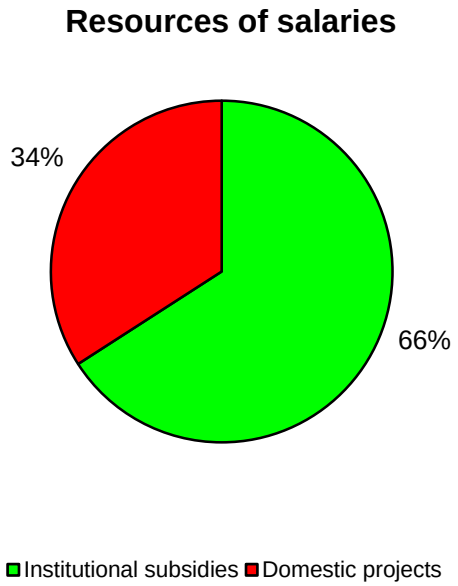
3.3 Personnel and salaries

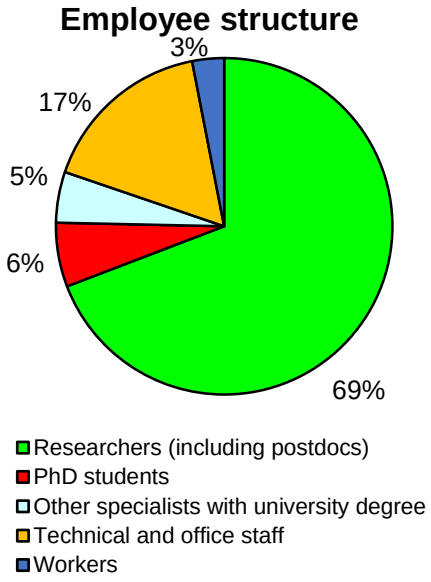
The average number of employees amounted to 96.20 FTE (almost the same as in previous year).

The personnel expenses of 96,921 thousand CZK represented 88,0% of total operating expenses.

The average monthly salary from all resources – institutional, project and commercial – was 60,108 CZK. This represents a negligible increase compared to the previous year, however, the average salary considerably increased already in 2020.

During 2021, 16 vacancies (4 research fellows, 6 postdocs, 6 Ph.D. students) were filled, with exception of research fellow positions mostly for two-year contracts. Eight researchers, 6 postdocs and 3 Ph.D. students terminated employment in 2021.





In line with the general approach of the Czech Academy of Sciences, research staff in the Institute is employed on fixed-term contracts and recruited in open competitions advertised at the Institute's web site and at the job server of the European Mathematical Society. Applicants are directed to the web site with detailed information and to the specialised web system for submitting applications and reference letters (https://application.math.cas.cz/Positions.html). The system enables a preliminary remote discussion of the heads of departments and of the selection committee members and facilitates the subsequent assessment of applications.

Tomáš Vejchodský

Doc. RNDr. Tomáš Vejchodský, Ph.D.
Director